Exact POMDP Solutions: α-vectors

Recap

Recap

- POMDP

Recap

- POMDP $(S, A, O, R, T, Z, \gamma)$

Recap

- POMDP
- Belief Updates

$$(S, A, O, R, T, Z, \gamma)$$

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- POMDP
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$$(S, A, O, R, T, Z, \gamma)$$

$$b_t(s) = P(s_t = s \mid h_t)$$

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$$b_t(s) = P(s_t = s \mid h_t)$$

$$b' = \tau(b, a, o)$$

Recap

- POMDP
- Belief Updates

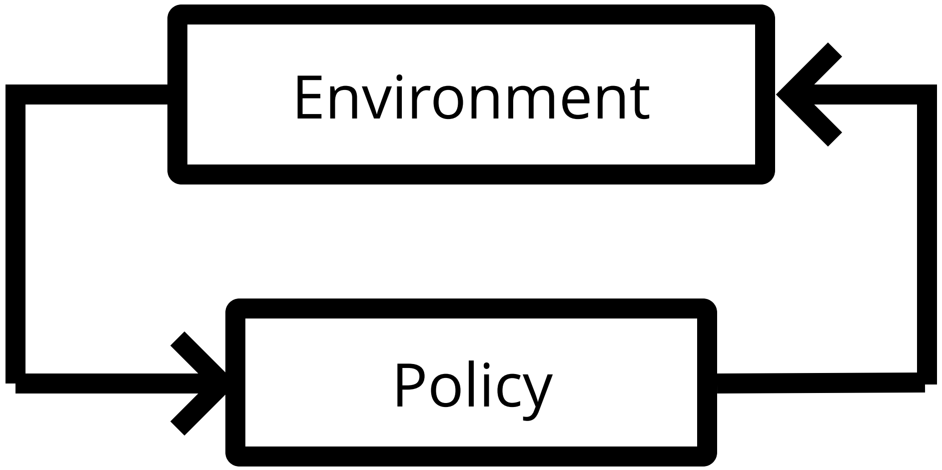
$$(S, A, O, R, T, Z, \gamma)$$

$$b_t(s) = P(s_t = s \mid h_t)$$

$$b' = \tau(b, a, o)$$

$$b'(s') \propto Z(o \mid a, s') \sum_s T(s' \mid s, a)\, b(s)$$

MDP Sense-Plan-Act Loop



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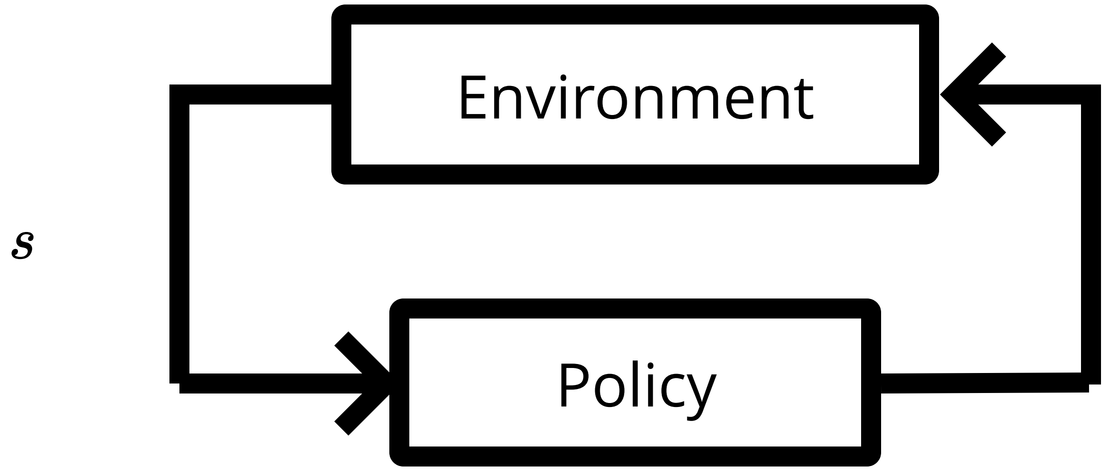

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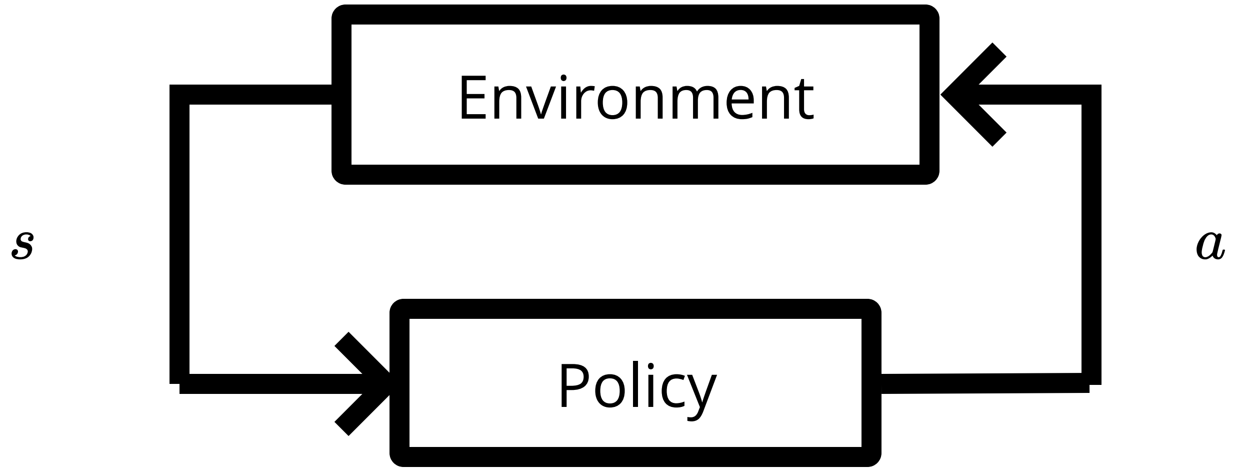
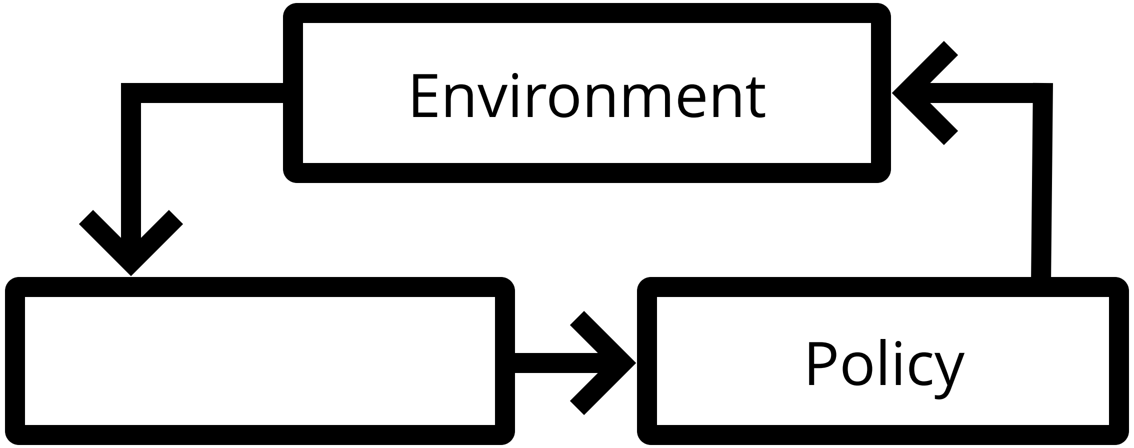


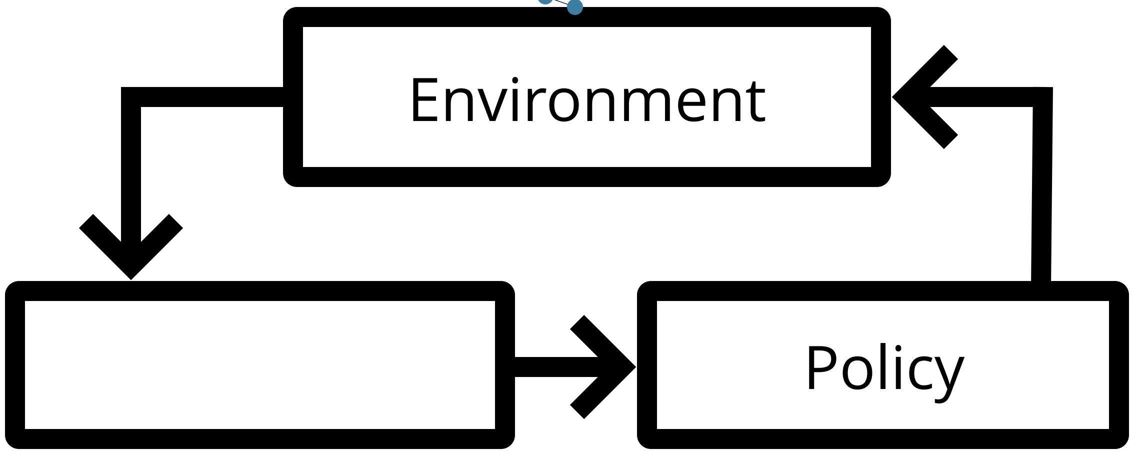
POMDP Sense-Plan-Act Loop



POMDP Sense-Plan-Act Loop

True State

$s = TL$



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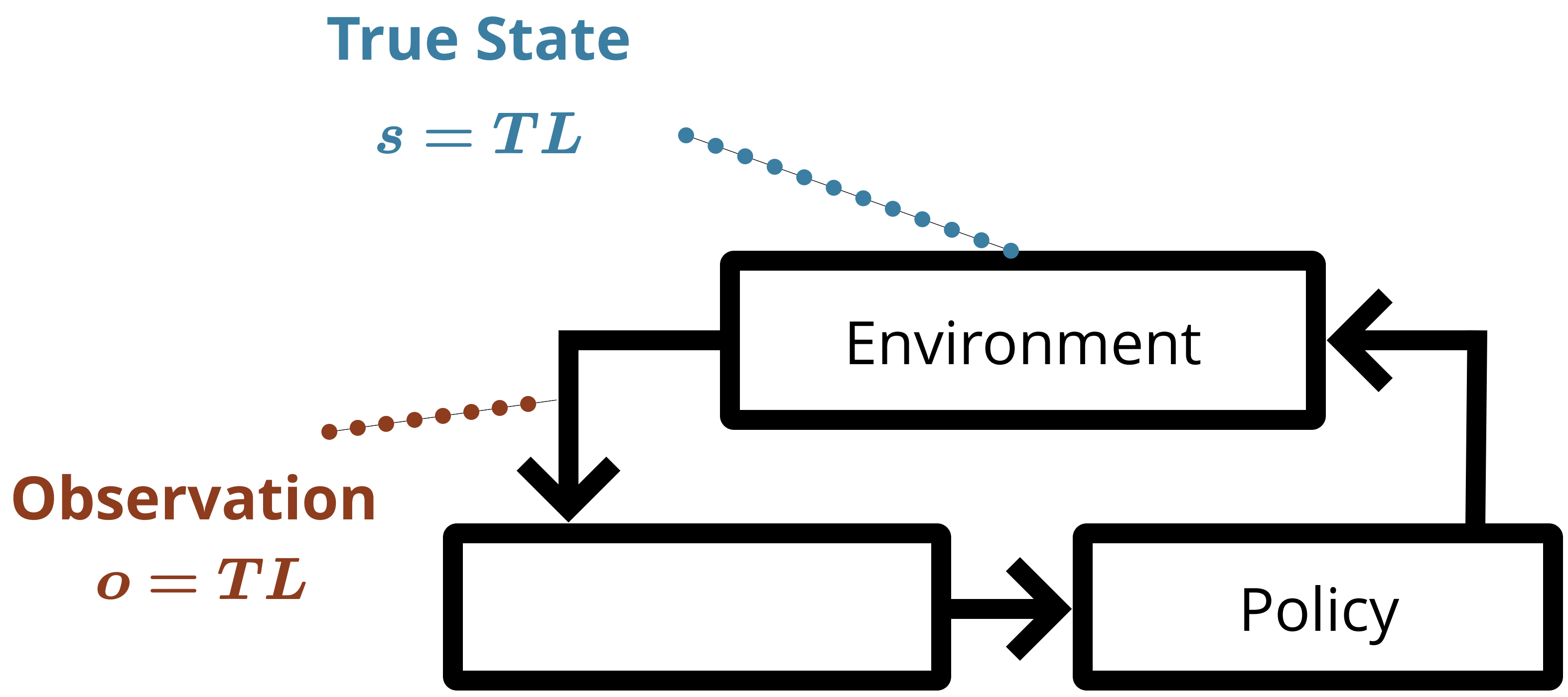
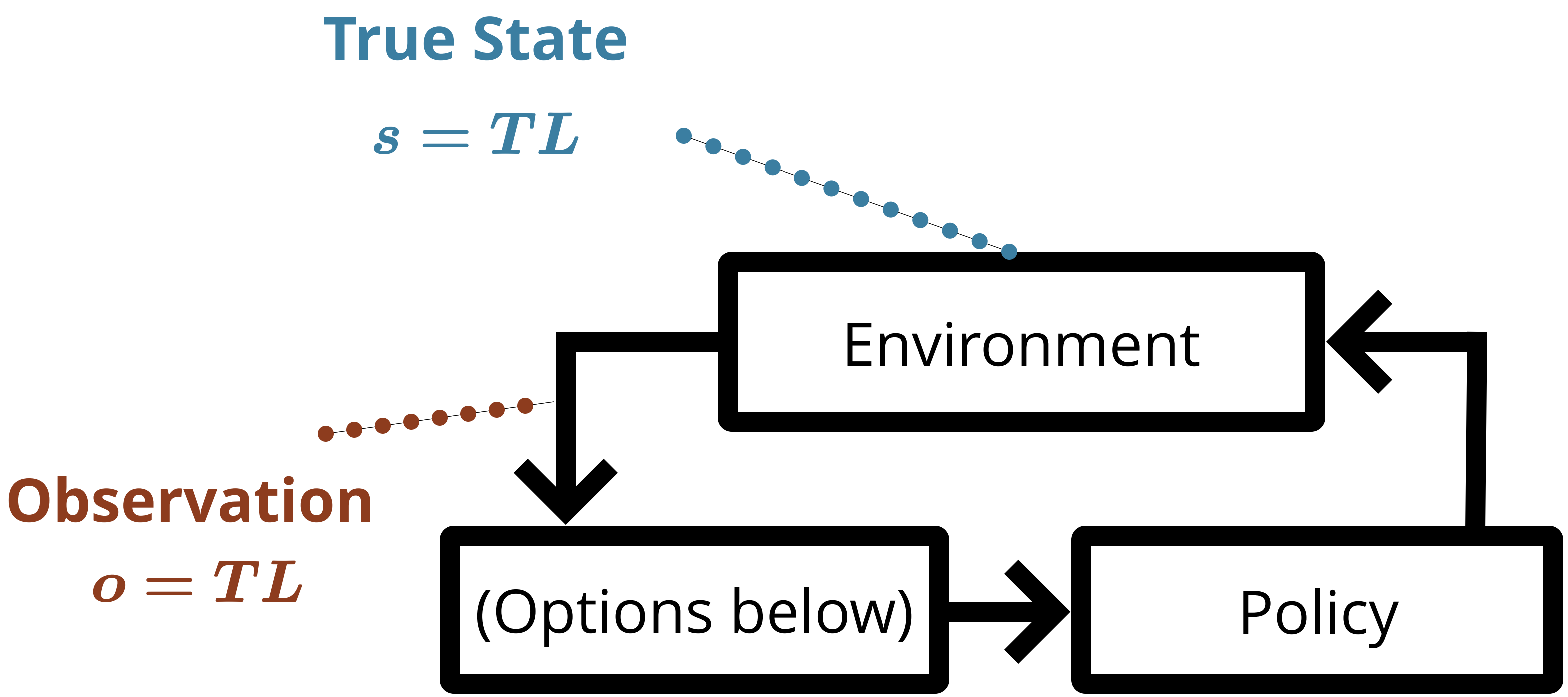
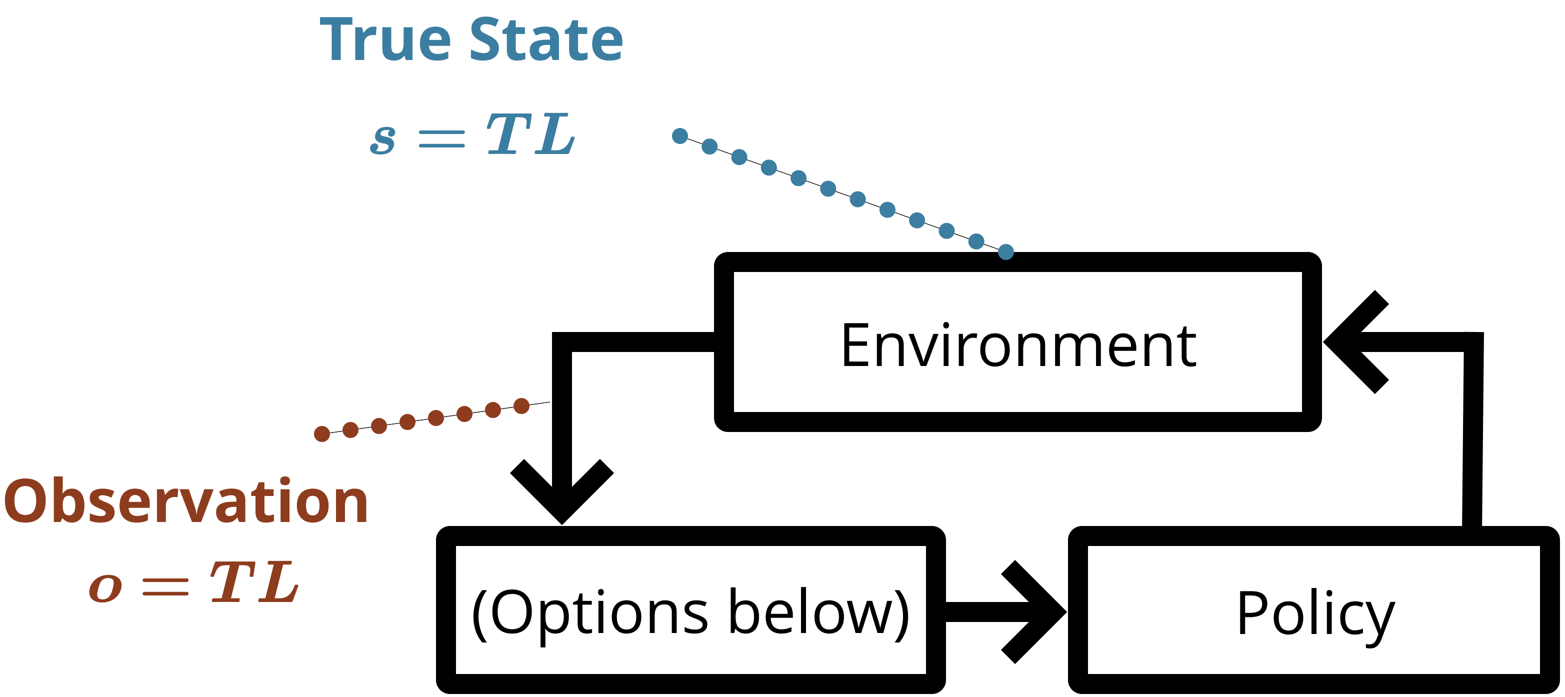


POMDP Sense-Plan-Act Loop



POMDP Sense-Plan-Act Loop

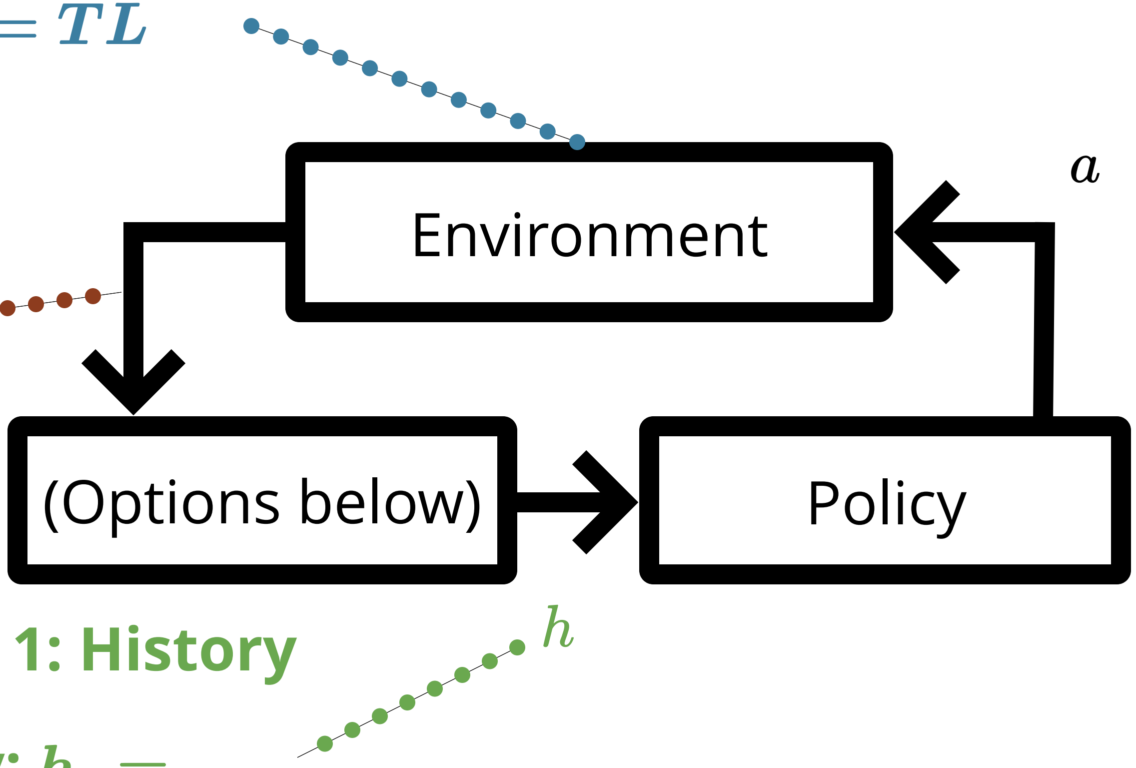


Option 1: History

POMDP Sense-Plan-Act Loop

True State

$s = TL$



Observation

$o = TL$

Option 1: History

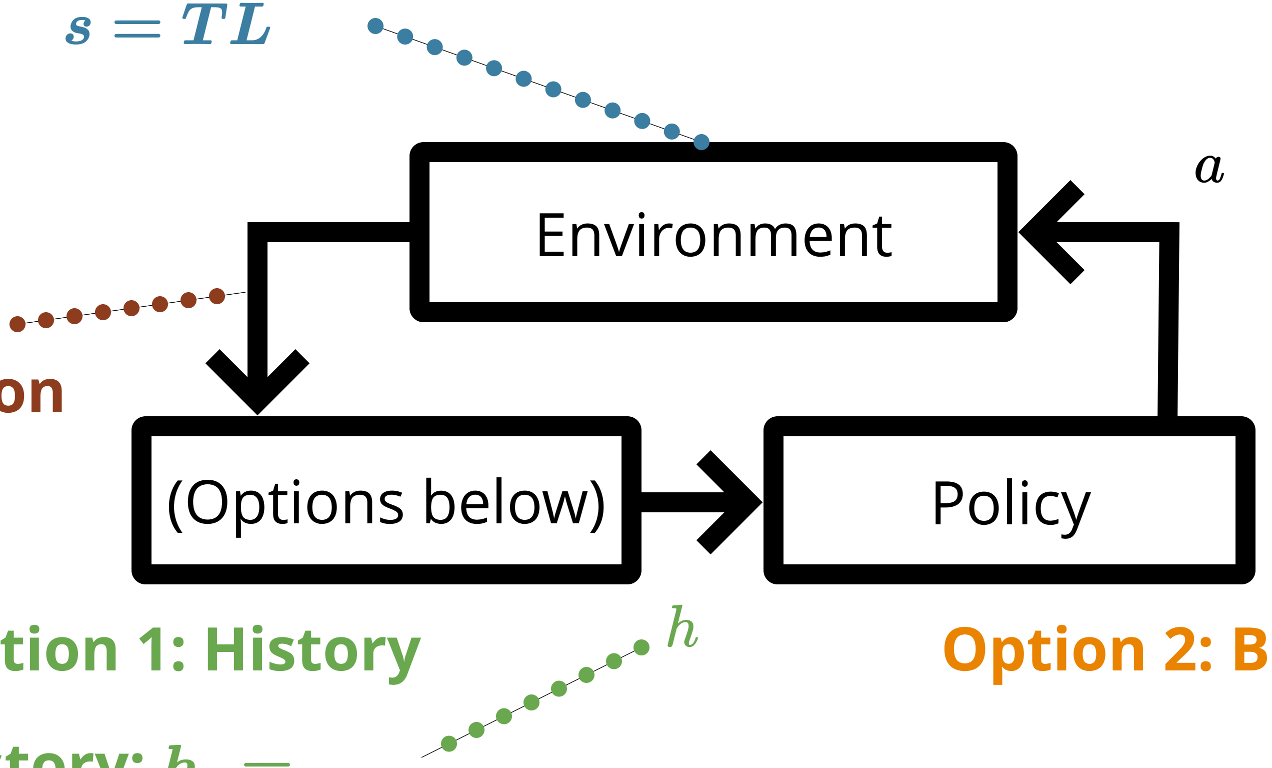
History: $h_t =$

$(b_0, a_0, o_1, a_1, \ldots a_{t-1}, o_t)$

POMDP Sense-Plan-Act Loop

True State

$s = TL$



Observation

$o = TL$

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History: $h_t =$

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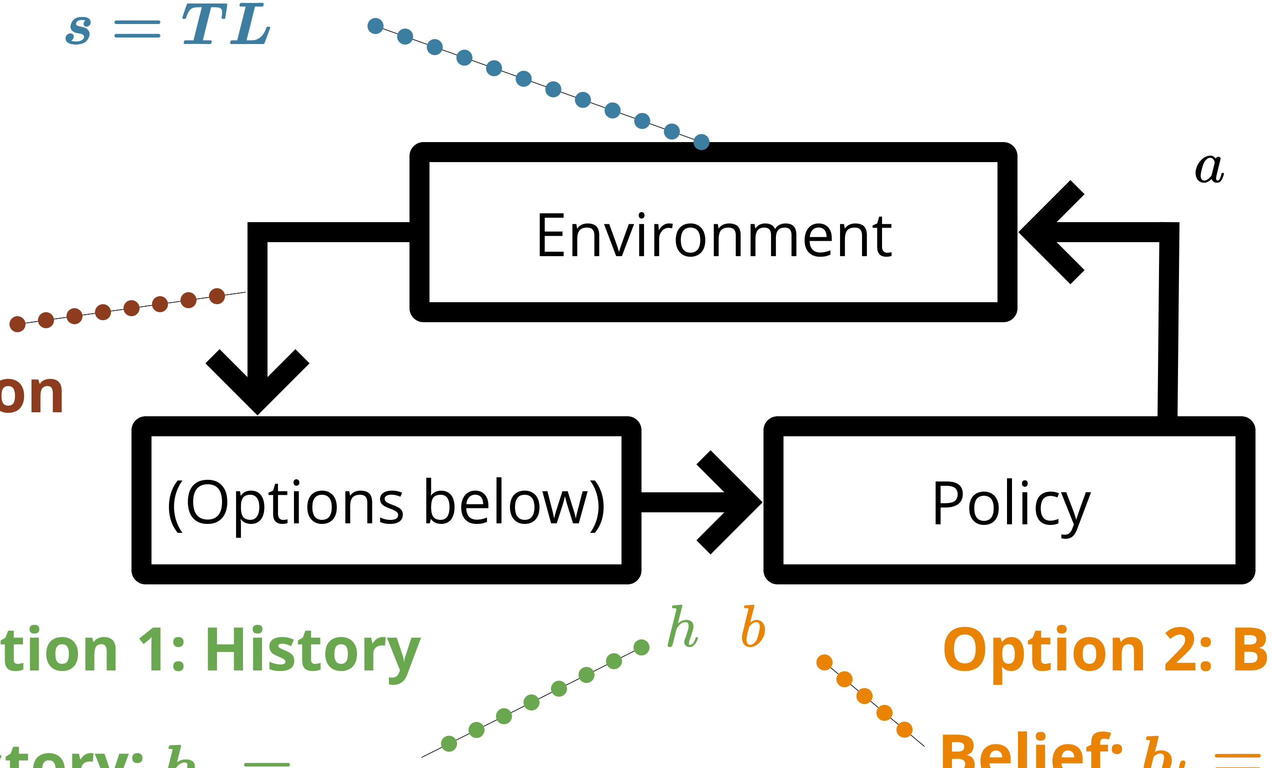
Option 2: Belief Updater

POMDP Sense-Plan-Act Loop

True State

$$s = TL$$



Observation

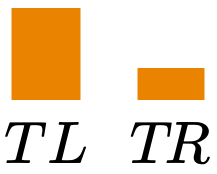
$$o = TL$$

Option 1: History

History: $h_t = (b_0, a_0, o_1, a_1, \ldots a_{t-1}, o_t)$

Option 2: Belief Updater

Belief: $b_t = P(s_t \mid h_t)$



Exercise 1: Crying Baby Belief Update

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$S = \{h, \neg h\}$

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$R(s, a) = R(s) + R(a)$

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$$R(s) = \begin{cases} -10 \text{ if } s = h \\ 0 \text{ otherwise} \end{cases}$$

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Exercise 1: Crying Baby Belief Update

$S = \{h, \neg h\}$ $\quad$ $T(h \mid h, \neg f) = 1.0$

$A = \{f, \neg f\}$ $\quad$ $T(h \mid \neg h, \neg f) = 0.1$

$O = \{c, \neg c\}$ $\quad$ $T(\neg h \mid \cdot, f) = 1.0$

$$R(s, a) = R(s) + R(a)$$

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$Z(c \mid \cdot, h) = 0.8)$

$Z(c \mid \cdot, \neg h) = 0.1$

$$b'(s') \propto Z(o \mid a, s') \sum_s T(s' \mid s, a)\, b(s)$$

$b'(h) \propto Z($

Exercise 1: Crying Baby Belief Update

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$$Z(c \mid \cdot, h) = 0.8)$$
$$Z(c \mid \cdot, \neg h) = 0.1$$
$$\gamma = 0.9$$

$$b'(s') \propto Z(o \mid a, s') \sum_s T(s' \mid s, a)\, b(s)$$

Starting at a $b(h) = 0$, calculate b' with $a = \neg f$ and $o = c$.

$$b'(h) \propto \underset{Z(c \mid \neg f,\, h)}{0.8} + \Big(\underset{T(h \mid \neg h, \neg f)}{0.1} \underset{b(\neg h)}{(1)} + \underset{T(h \mid h, \neg f)}{1.0}\, (0) \Big)$$

$$b'(h) \propto 0.08$$

$$b'(\neg h) \propto 0.09$$

$$b'(h) = 0.08 / (0.08 + 0.09) = 47\%$$

$$b'(\neg h) = 53\%$$

Belief Dynamics

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$$b'(s') \propto Z(o \mid a, s') \sum_s T(s' \mid s, a)\, b(s)$$

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$b(TL)$

t

Belief Dynamics

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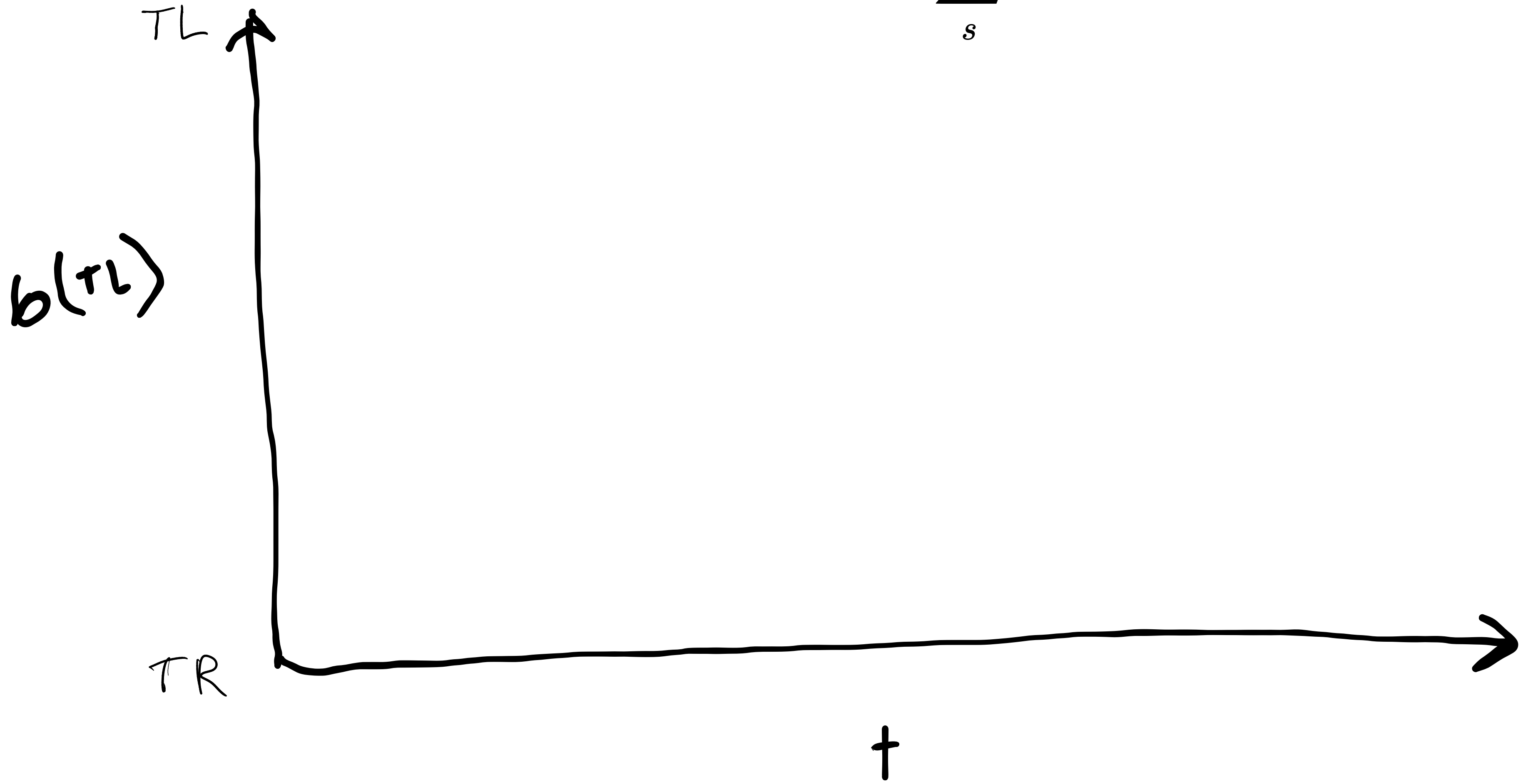

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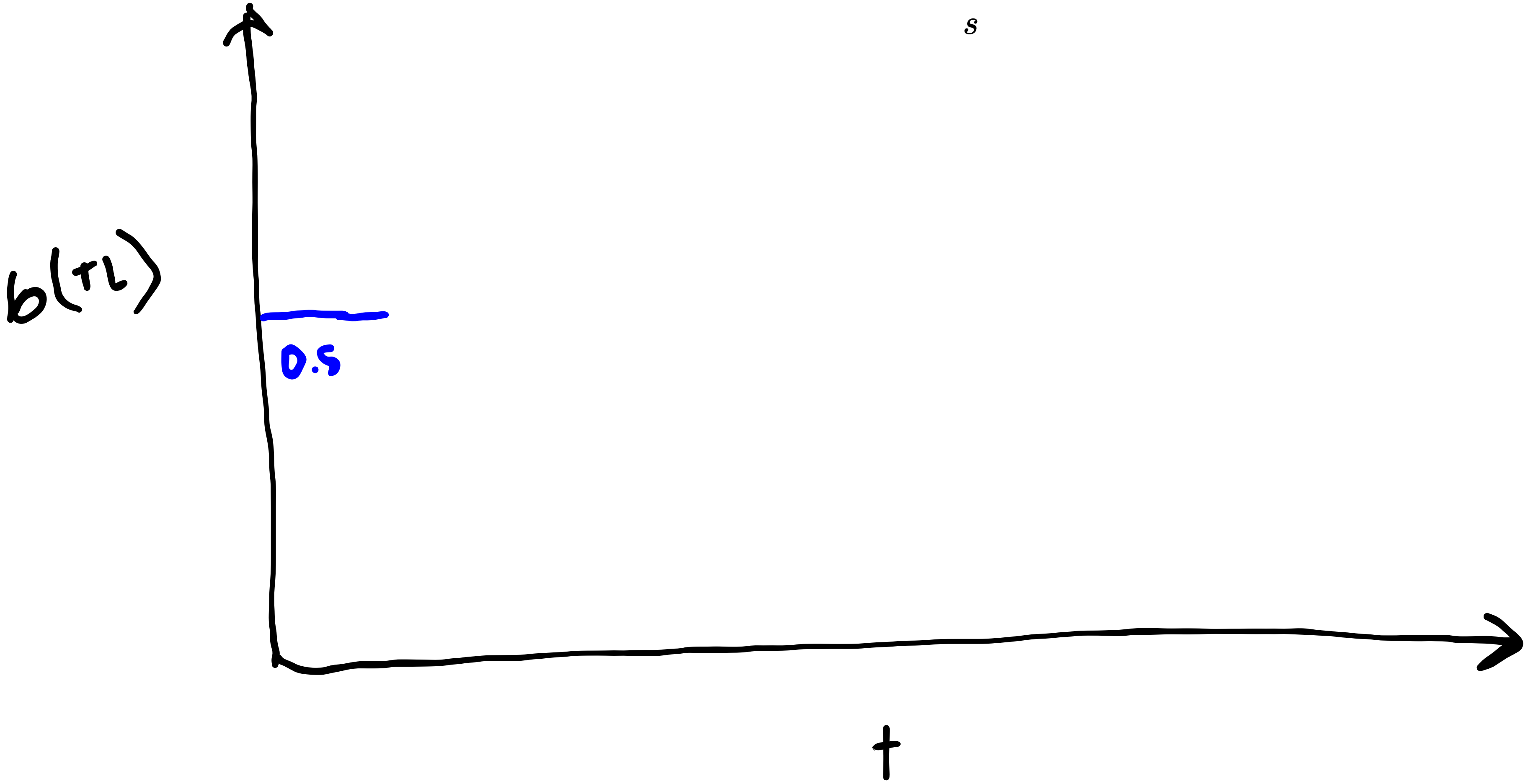

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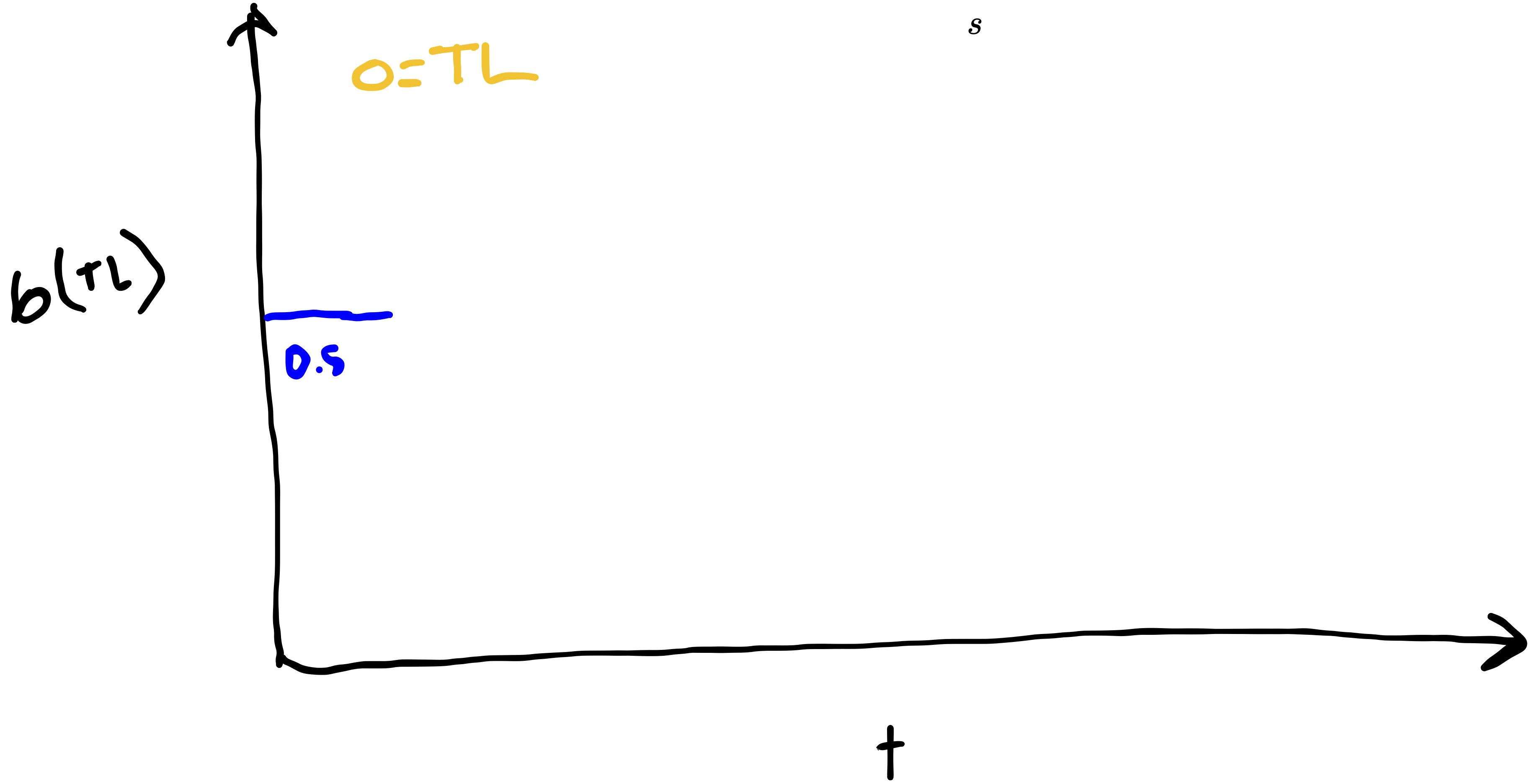

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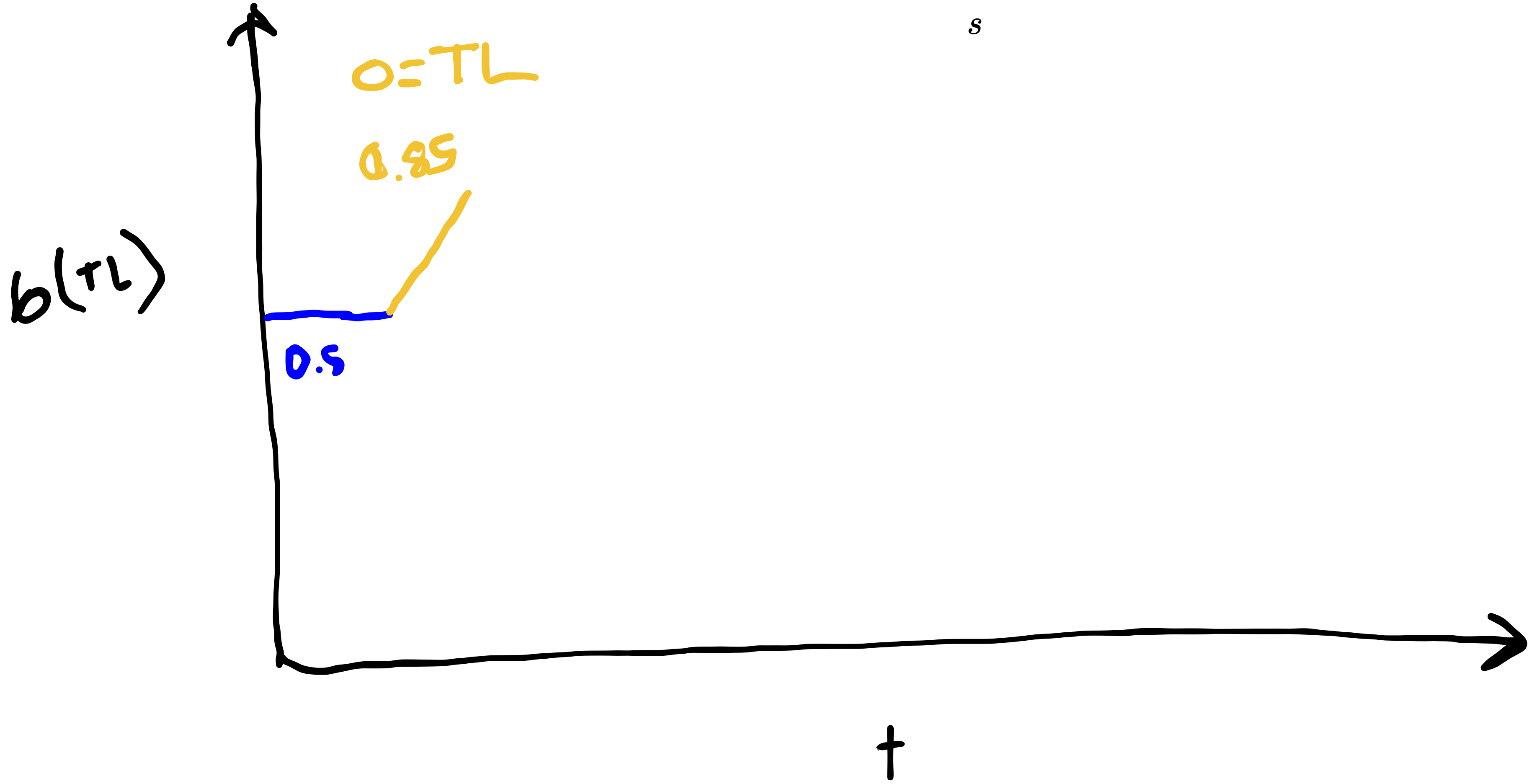

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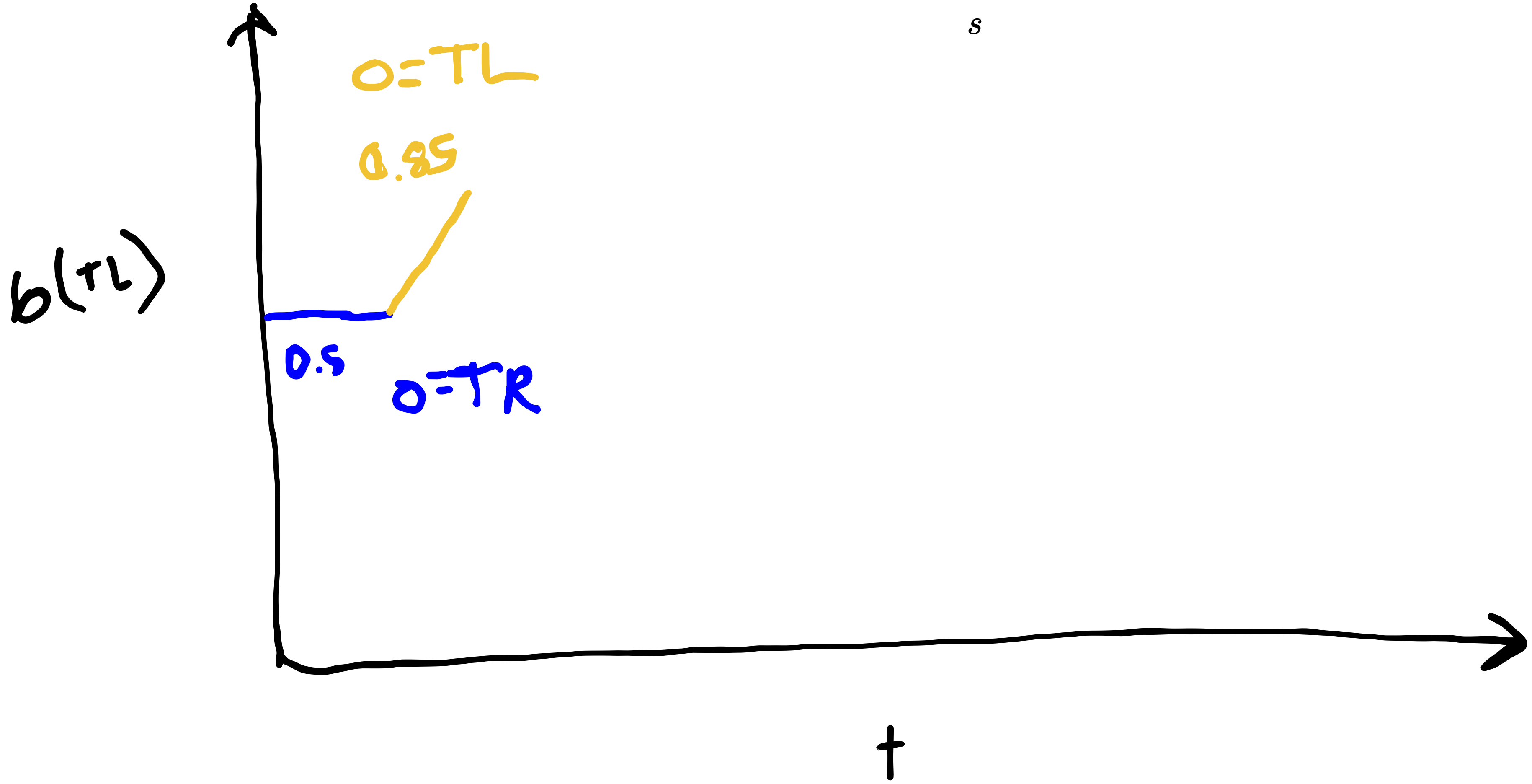

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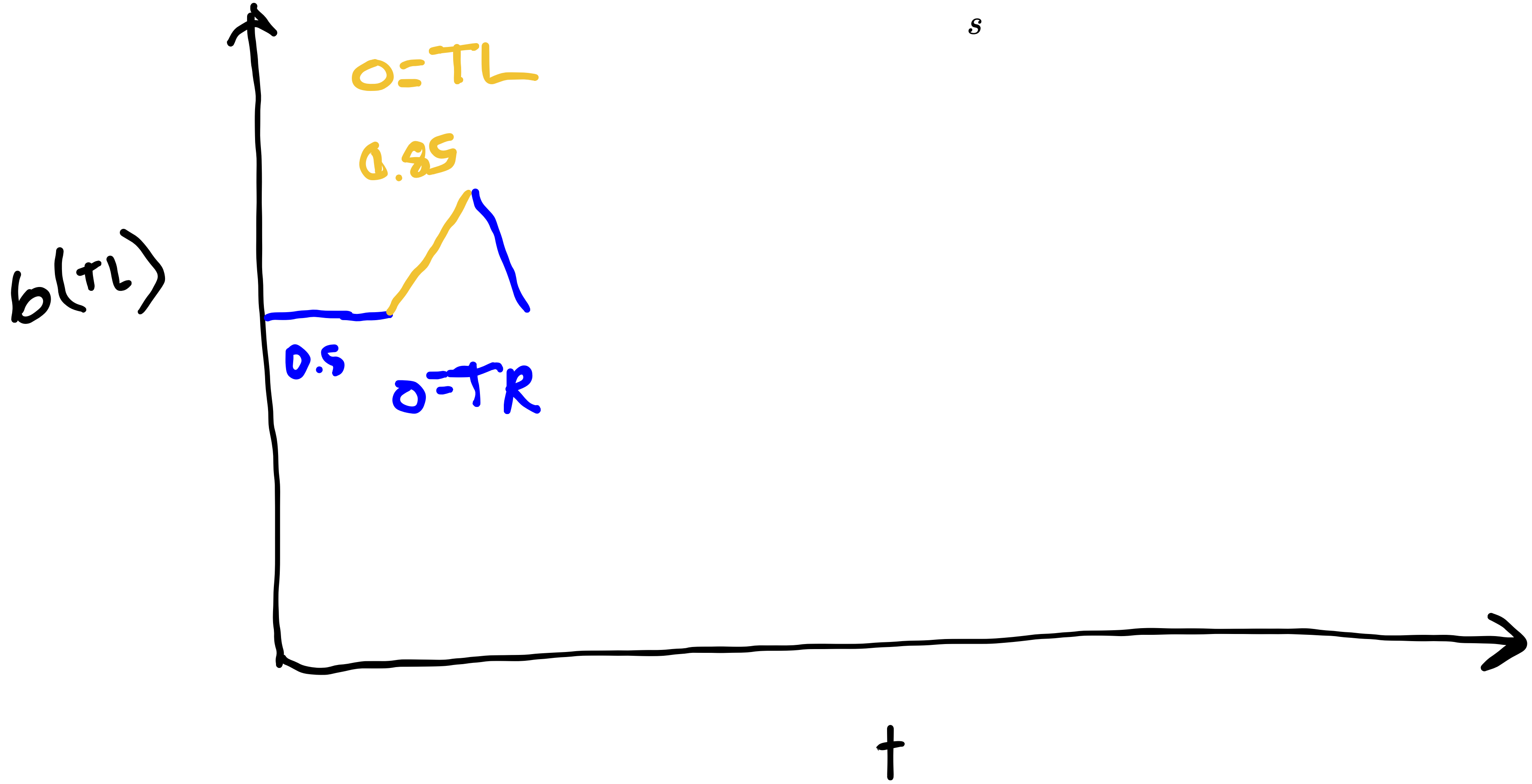

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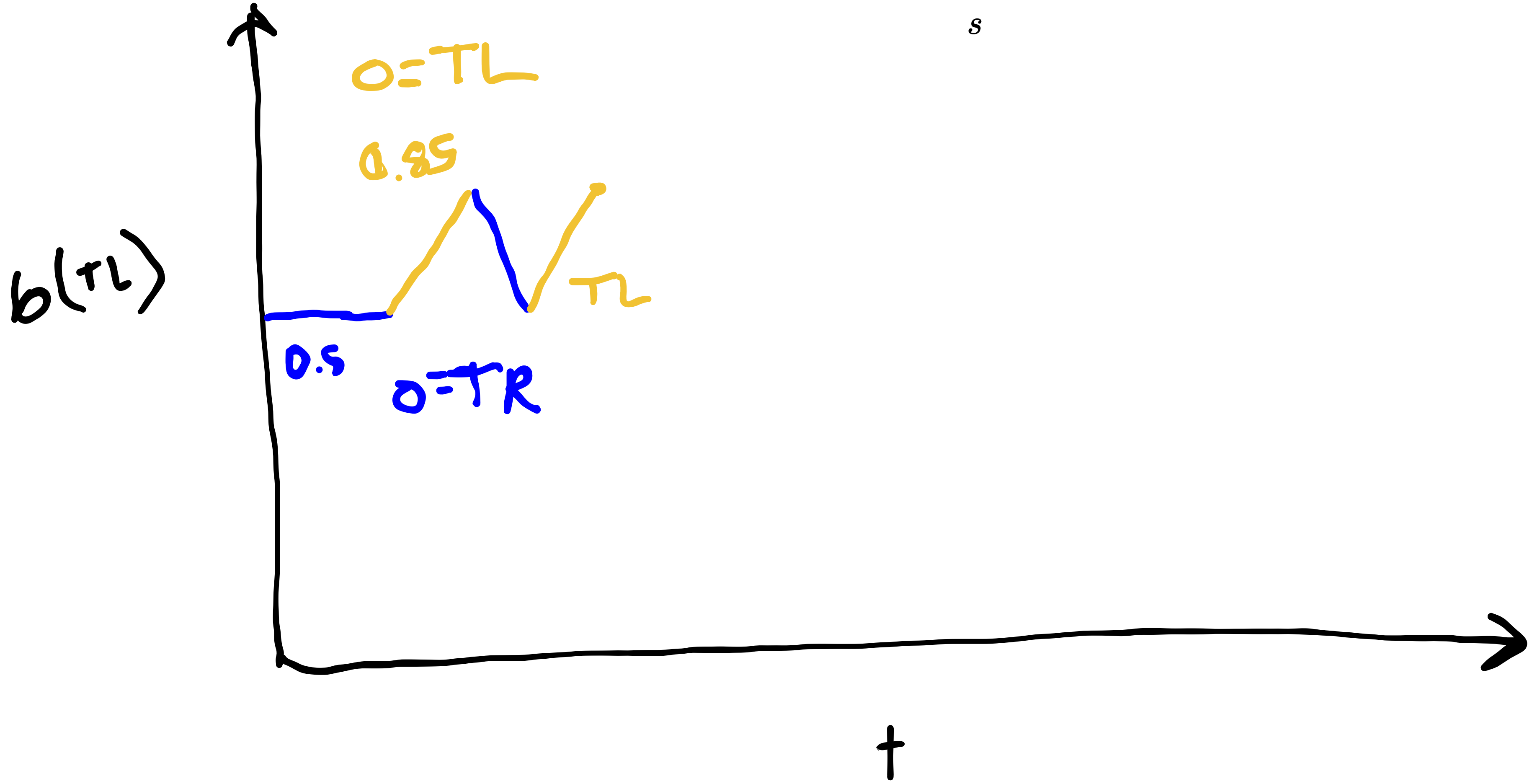

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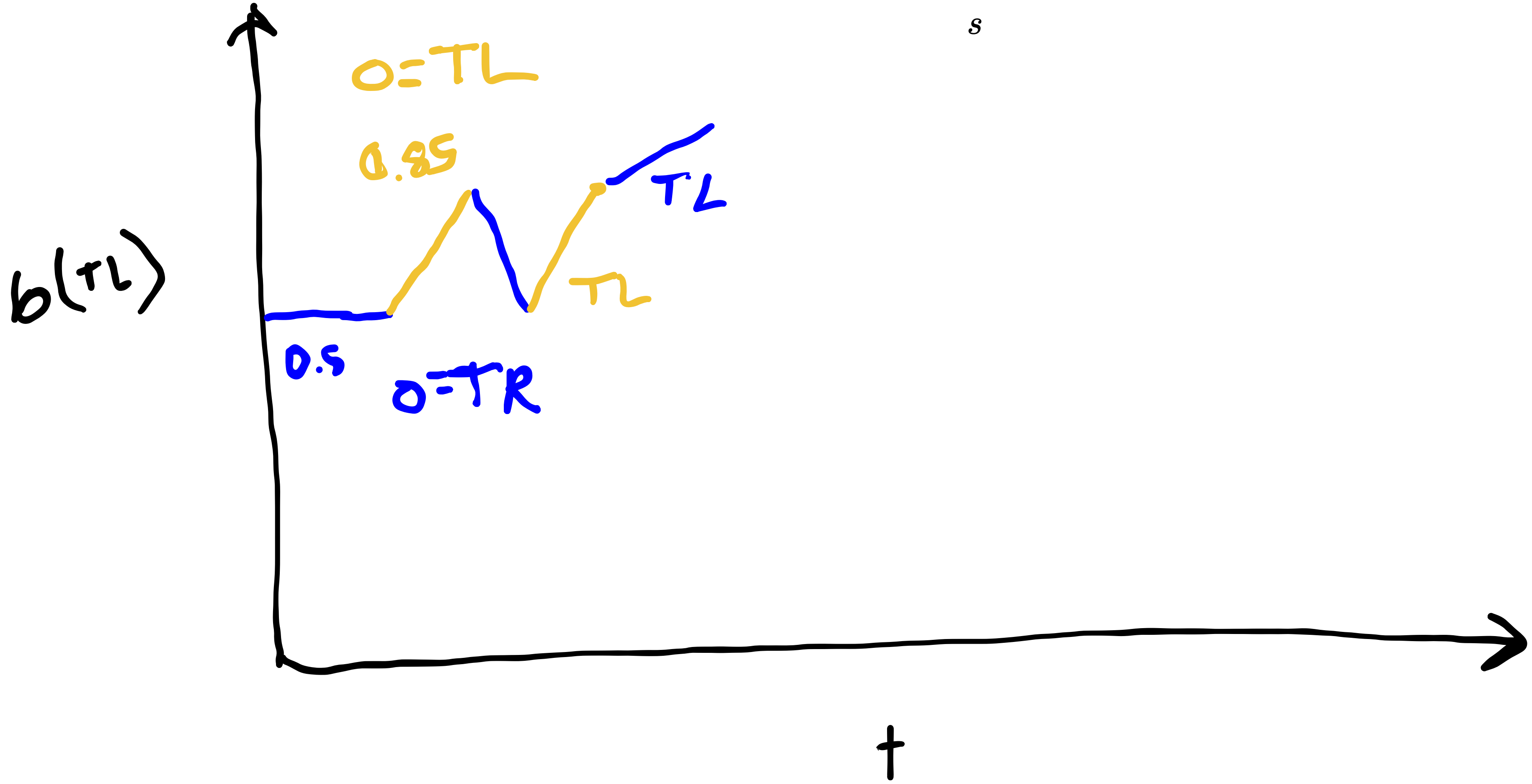

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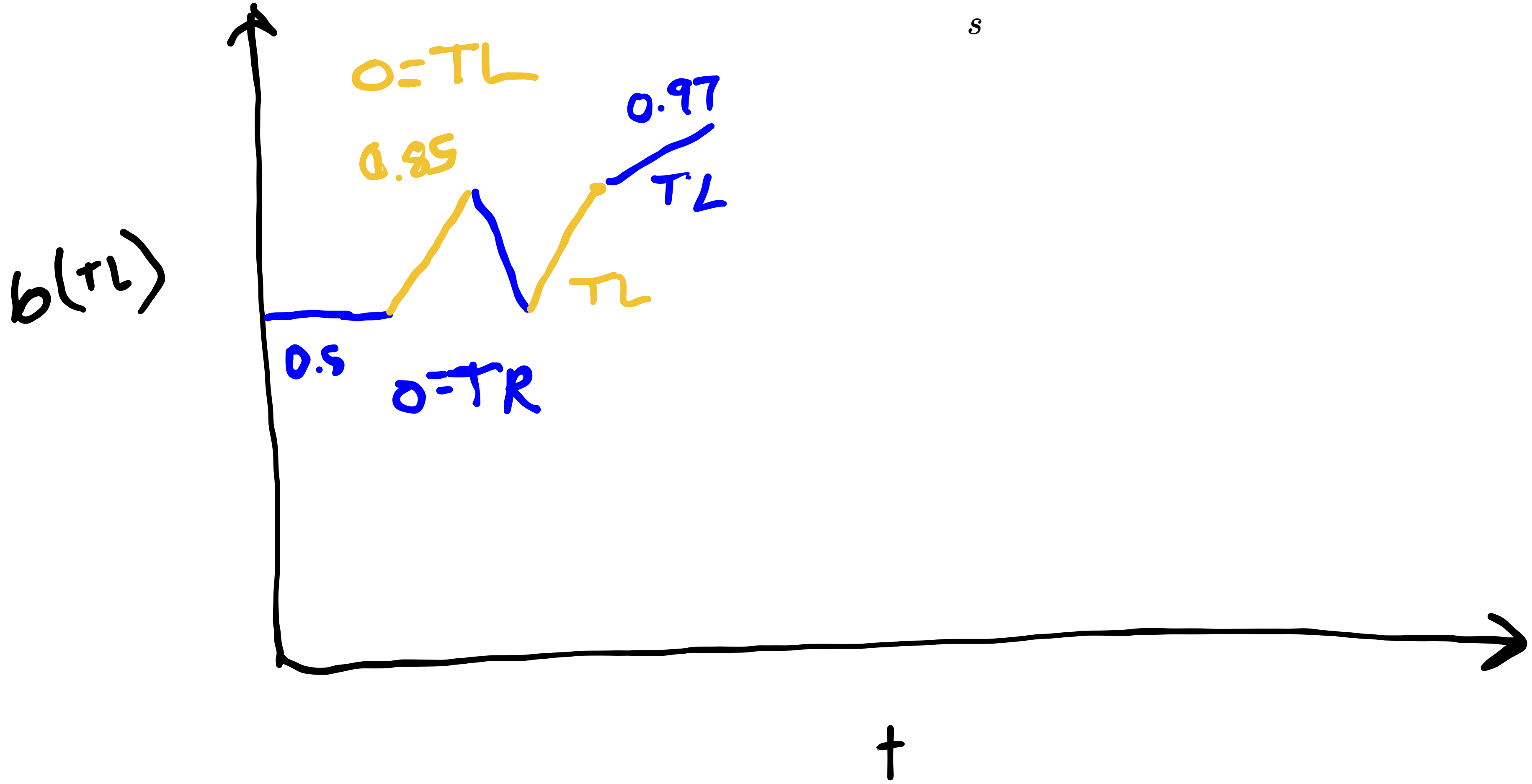

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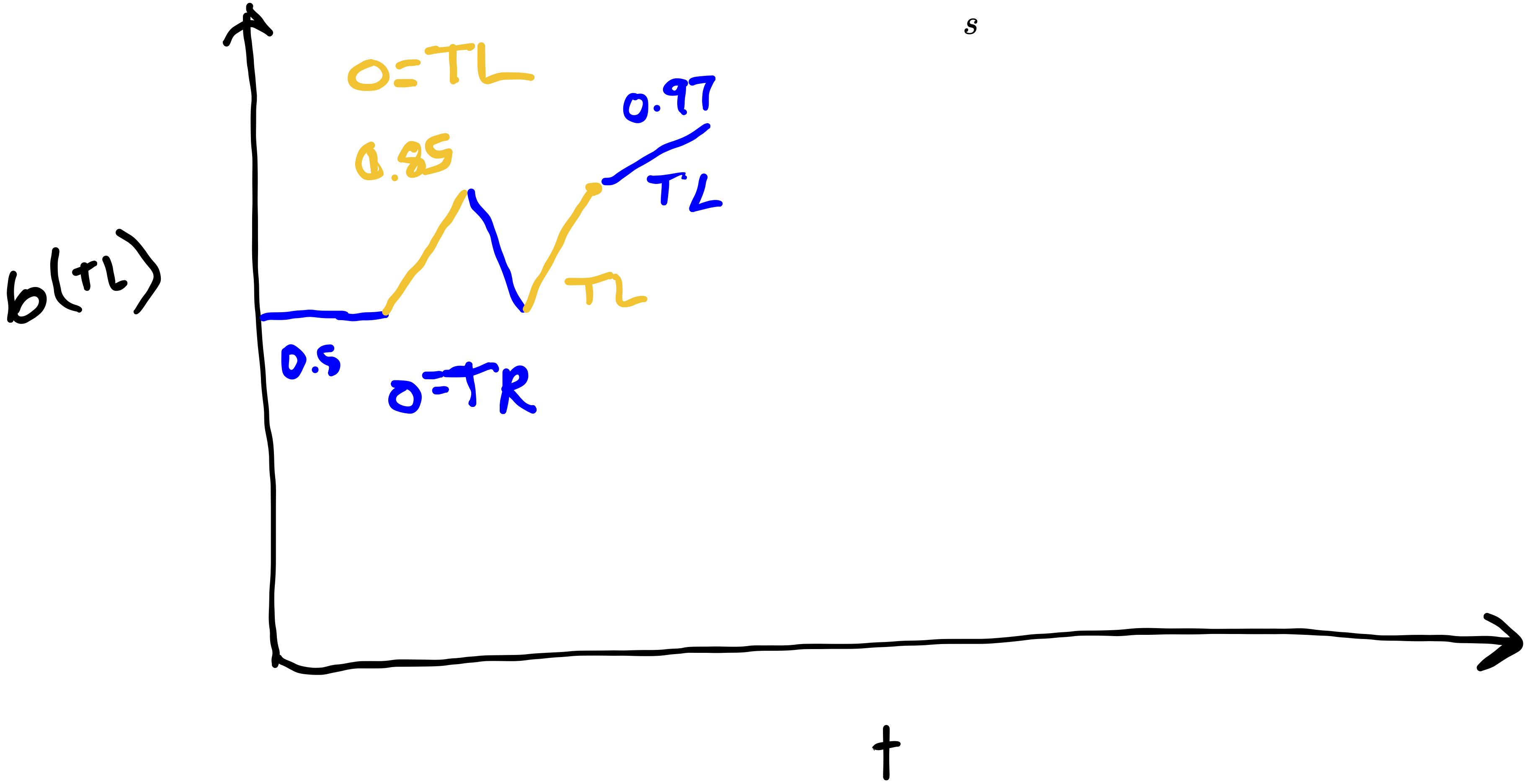

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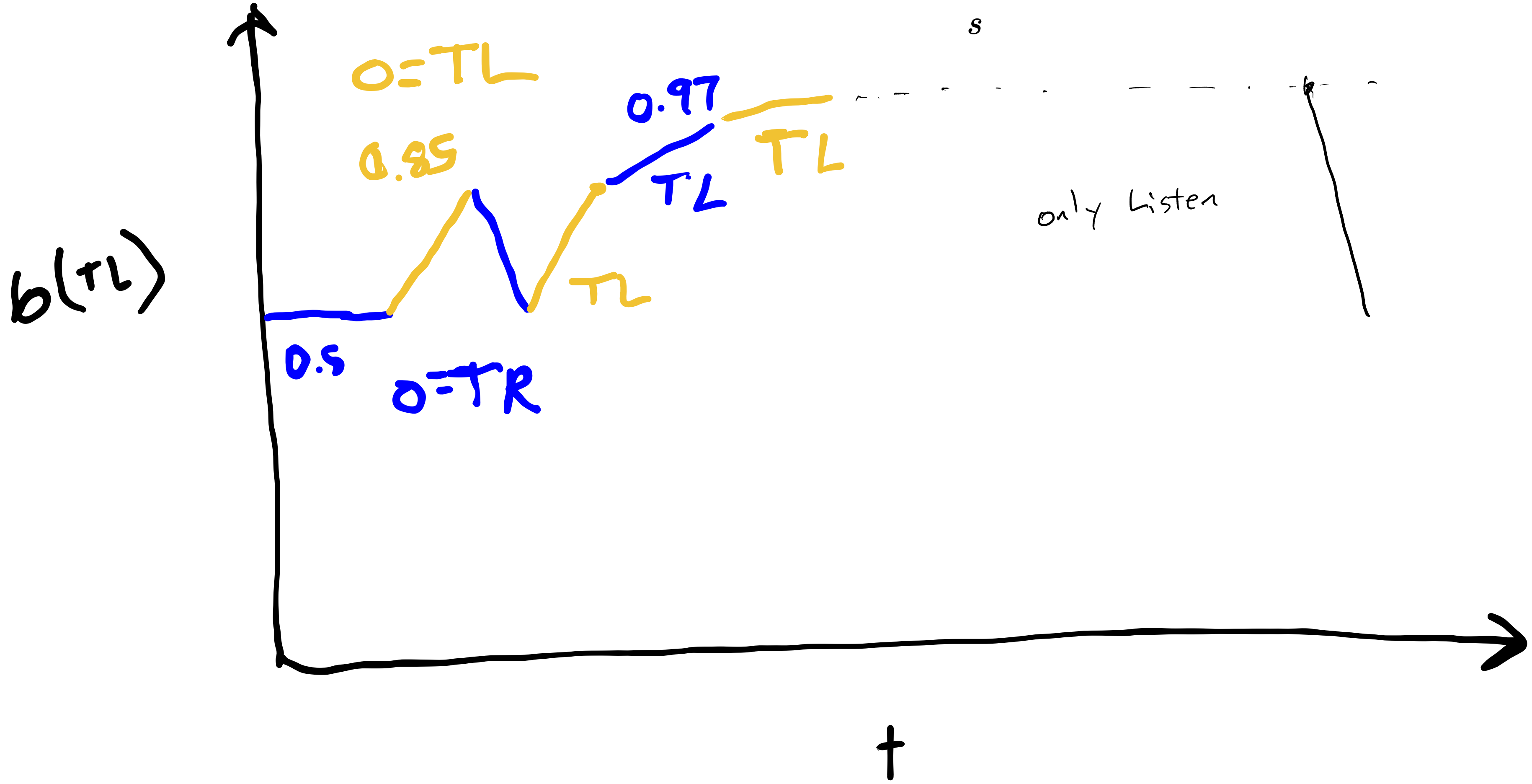
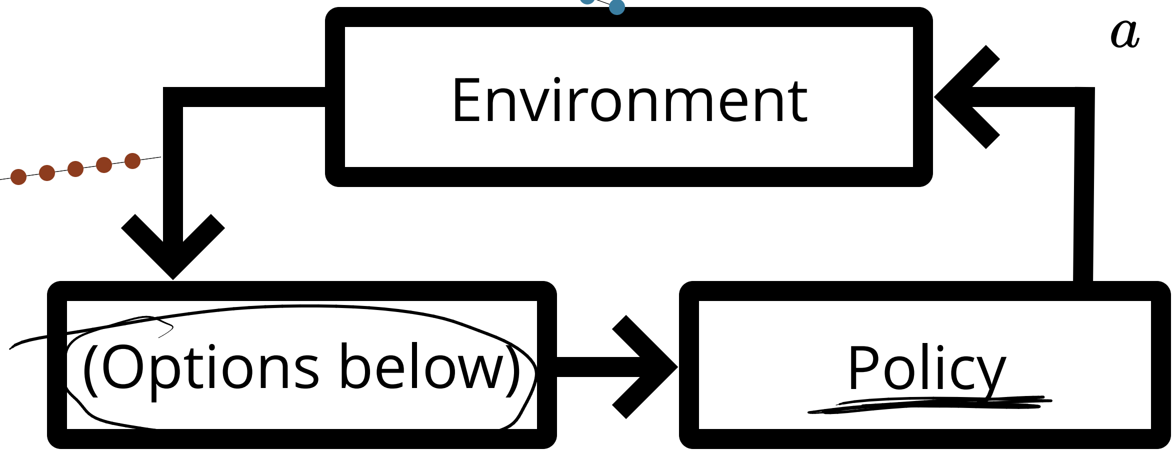


POMDP Sense-Plan-Act Loop

True State

$s = TL$



Observation

$o = TL$

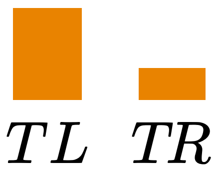
h b

Option 1: History

History: $h_t = (b_0, a_0, o_1, a_1, \dots a_{t-1}, o_t)$

Option 2: Belief Updater

Belief: $b_t = P(s_t \mid h_t)$



Guiding Quesiton

How do we calculate the optimal action in a POMDP?

Reward

+10	empty
-1	Listen
-100	tiger

One-step utility

$a = L$

$R(TR, L) = -1$

$R(TL, L) = -1$

$a = OL$

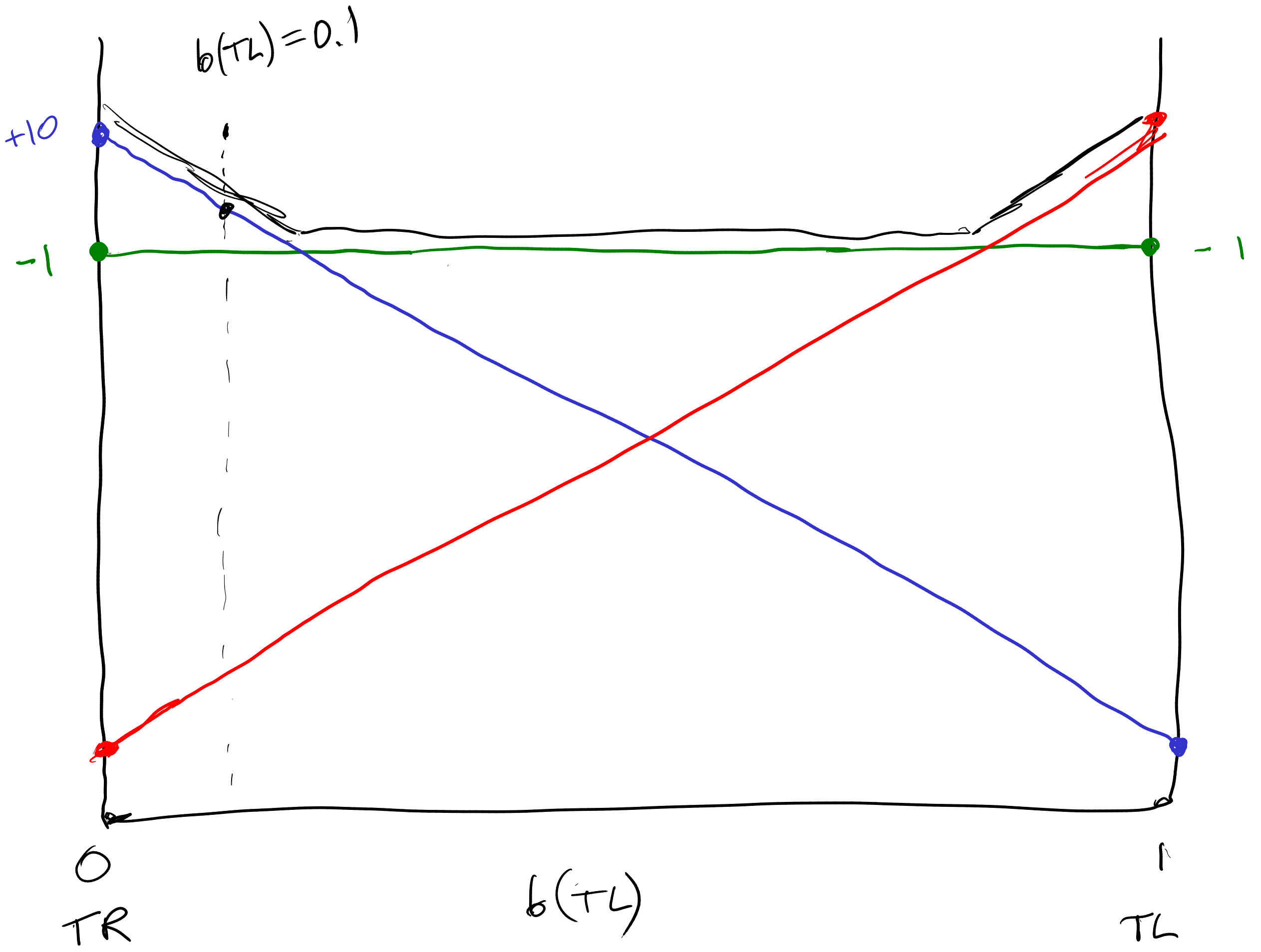
$R(TR, OL) = +10$

$R(TL, OL) = -100$

$a = OR$

$R(TR, OR) = -100$

$R(TL, OR) = 10$



$[R(s^1,a), R(s^2,a) \ldots]$

$[b(s^1), b(s^2) \ldots b(s^n)]$

$$R(b,a) = \bar{r}^a \cdot b$$

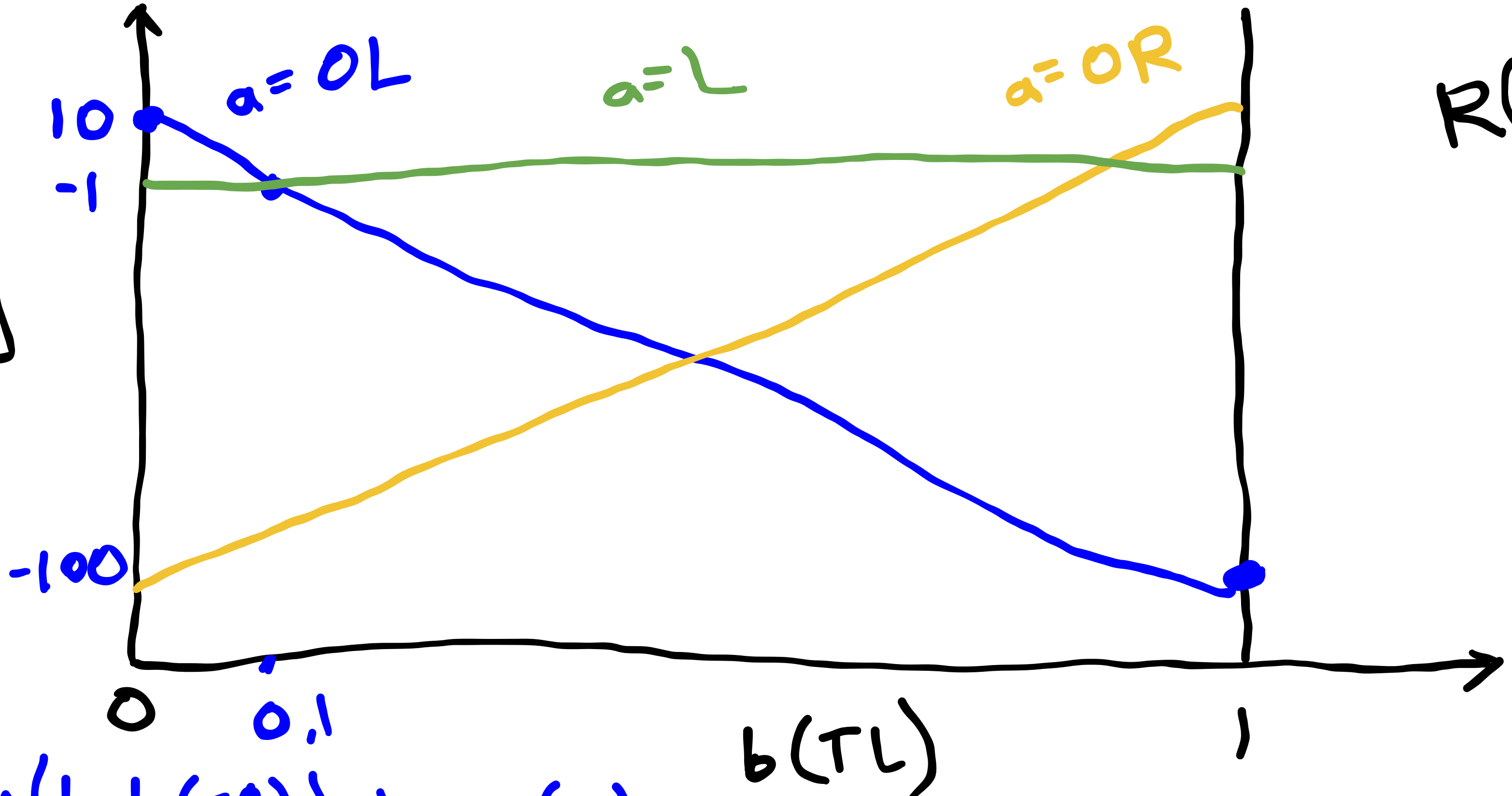
one-step α-vector

$$R(b,a) = \sum b(s) R(s,a) = b(TL) R(TL,a) + (1 - b(TL)) R(TR,a)$$

One-step utility

Reward: +10 empty door
-1 Listen
-100 Tiger

$R(b,a) = \underset{s\sim b}{E}[R(s,a)]$



$R(b,a) = r_a \cdot b$

α-vector

$R(b,a) = 10\cdot(1-b(TL)) - 100\, b(TL) = -110\, b + 10$

Exercise 2: Crying Baby 1-Step Utility

$S = \{h, \neg h\}$ $\quad T(h \mid h, \neg f) = 1.0$

$A = \{f, \neg f\}$ $\quad T(h \mid \neg h, \neg f) = 0.1$

$O = \{c, \neg c\}$ $\quad T(\neg h \mid \cdot, f) = 1.0$

$$R(s,a) = R(s) + R(a)$$

$$R(s) = \begin{cases} -10 \text{ if } s = h \\ 0 \text{ otherwise} \end{cases}$$

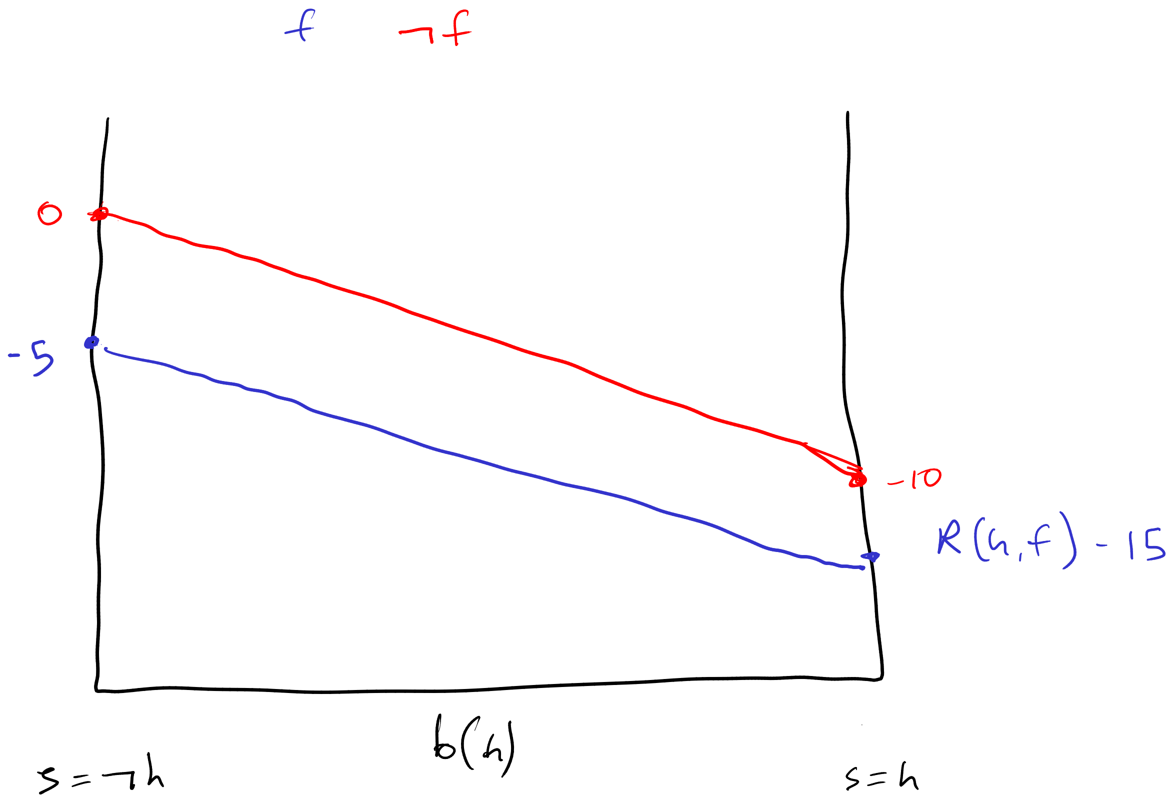
$$R(a) = \begin{cases} -5 \text{ if } a = f \\ 0 \text{ otherwise} \end{cases}$$

$$Z(c \mid \cdot, h) = 0.8)$$

$$Z(c \mid \cdot, \neg h) = 0.1$$

$$\gamma = 0.9$$

Draw the 1-step utility α-vectors for the Crying Baby problem.



Alpha Vectors for Conditional Plans

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Conditional Plans: fixed-depth history-based policies

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1 Step:

Alpha Vectors for Conditional Plans

Conditional Plans: fixed-depth history-based policies

1 Step: L OL OR

Alpha Vectors for Conditional Plans

Conditional Plans: fixed-depth history-based policies

1 Step: L OL OR

2 Step:

Alpha Vectors for Conditional Plans

Conditional Plans: fixed-depth history-based policies

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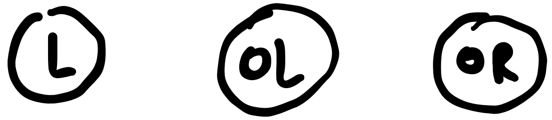
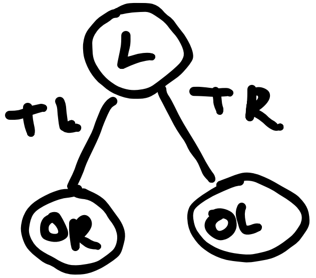


2 Step:



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Conditional Plans: fixed-depth history-based policies

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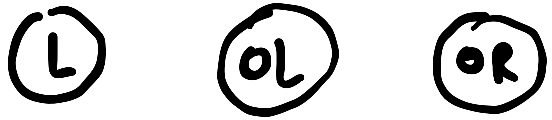

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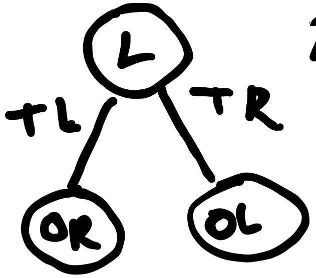

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Alpha Vectors for Conditional Plans

Conditional Plans: fixed-depth history-based policies

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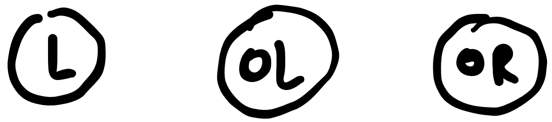

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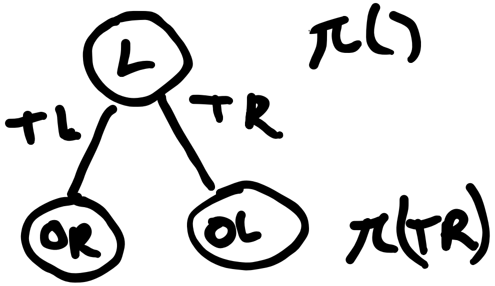

Alpha Vectors for Conditional Plans

Conditional Plans: fixed-depth history-based policies

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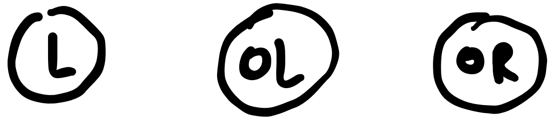

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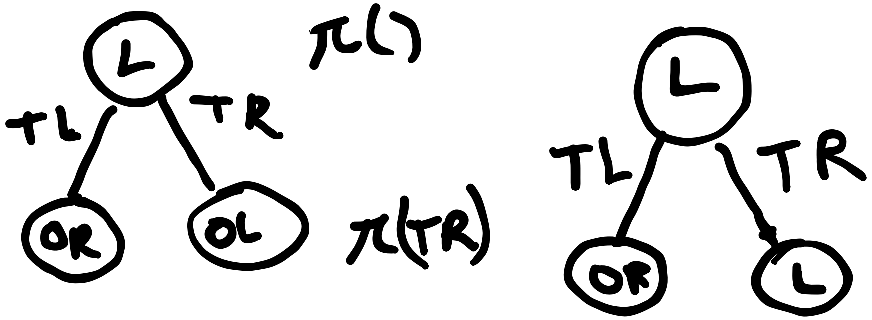

Alpha Vectors for Conditional Plans

Conditional Plans: fixed-depth history-based policies

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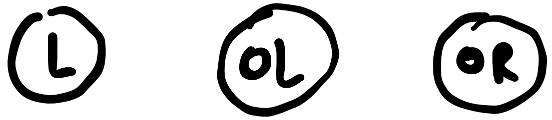

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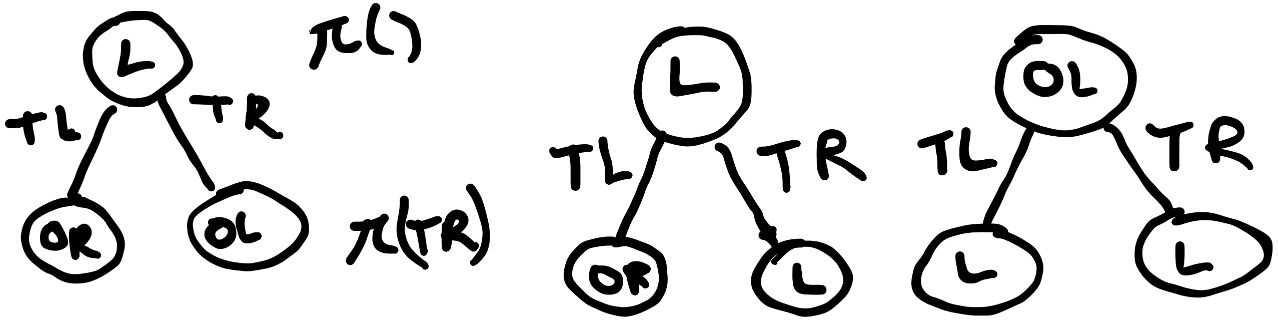
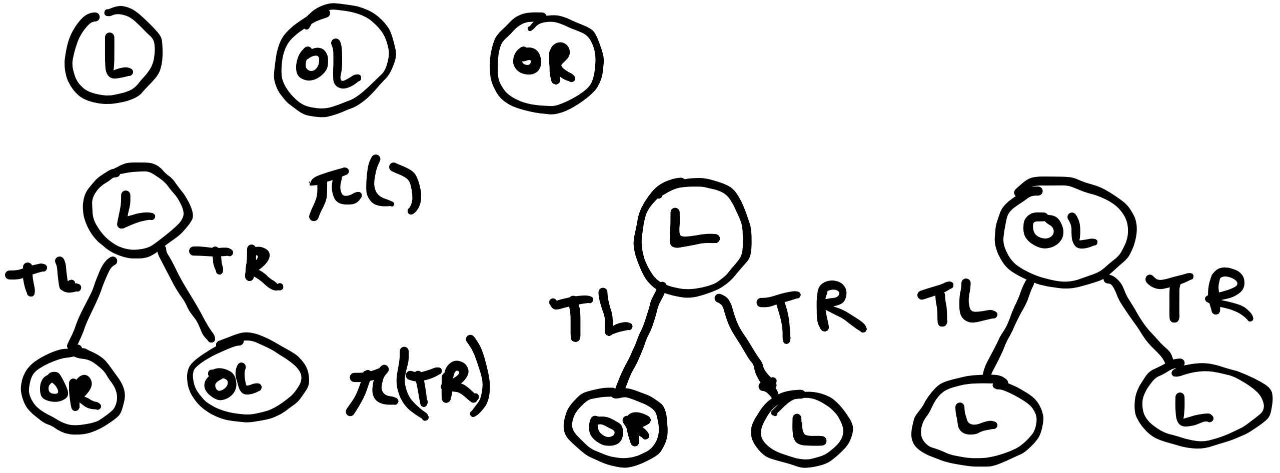


Alpha Vectors for Conditional Plans

Conditional Plans: fixed-depth history-based policies

1 Step:

2 Step:



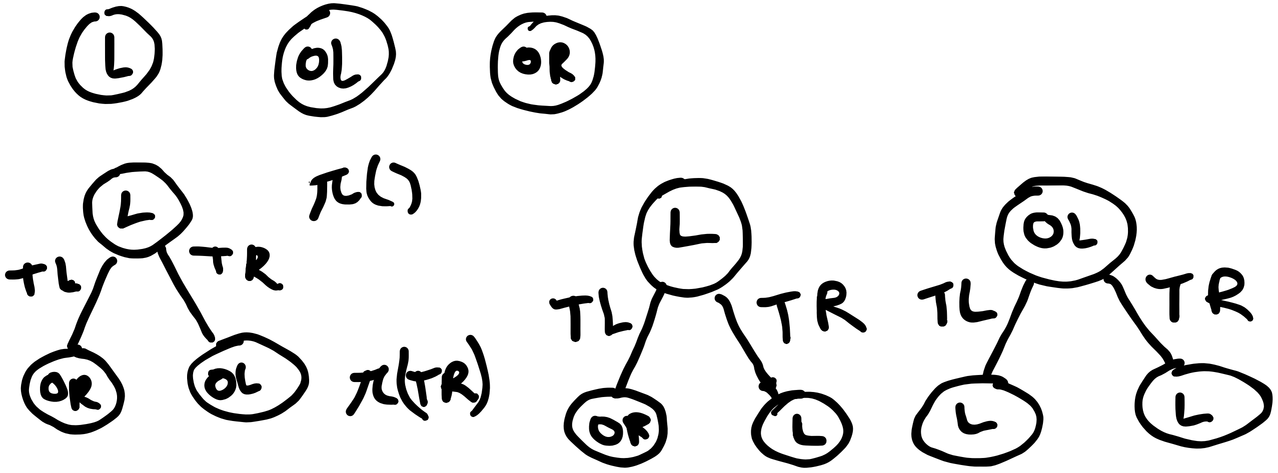
$$|A|^{\frac{(|O|^h-1)}{(|O|-1)}}$$

Alpha Vectors for Conditional Plans

Conditional Plans: fixed-depth history-based policies

1 Step:

2 Step:



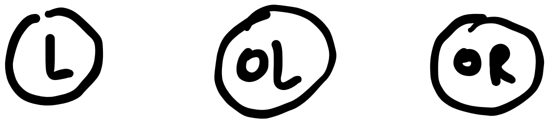
$|A|^{\frac{(|O|^h-1)}{(|O|-1)}}$

27 two step plans!

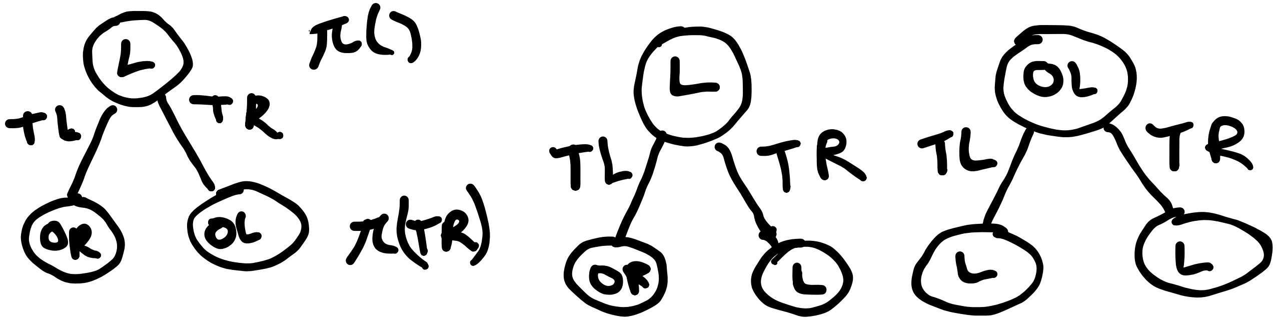
Alpha Vectors for Conditional Plans

Conditional Plans: fixed-depth history-based policies

1 Step:

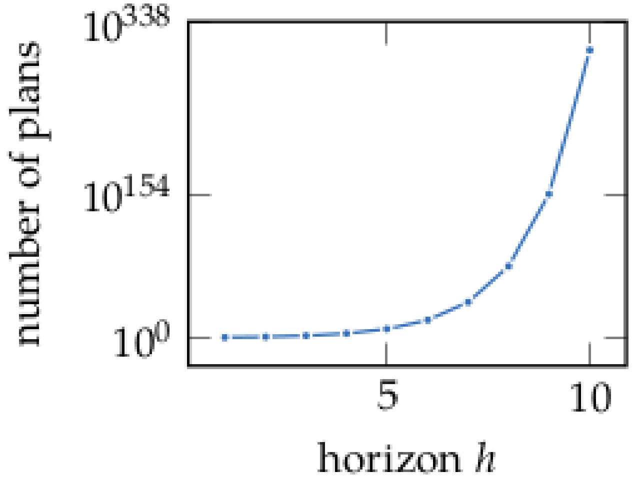


2 Step:



$$|A|^{\frac{(|O|^{h}-1)}{(|O|-1)}}$$

27 two step plans!



Alpha Vectors for Conditional Plans

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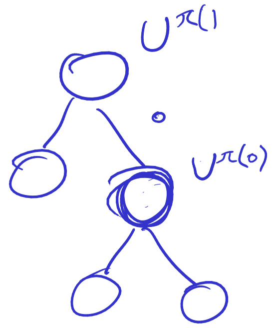
For 1-step: $U^{\pi}(s) = R(s, \pi())$

Alpha Vectors for Conditional Plans

For 1-step: $U^\pi(s) = R(s, \pi())$

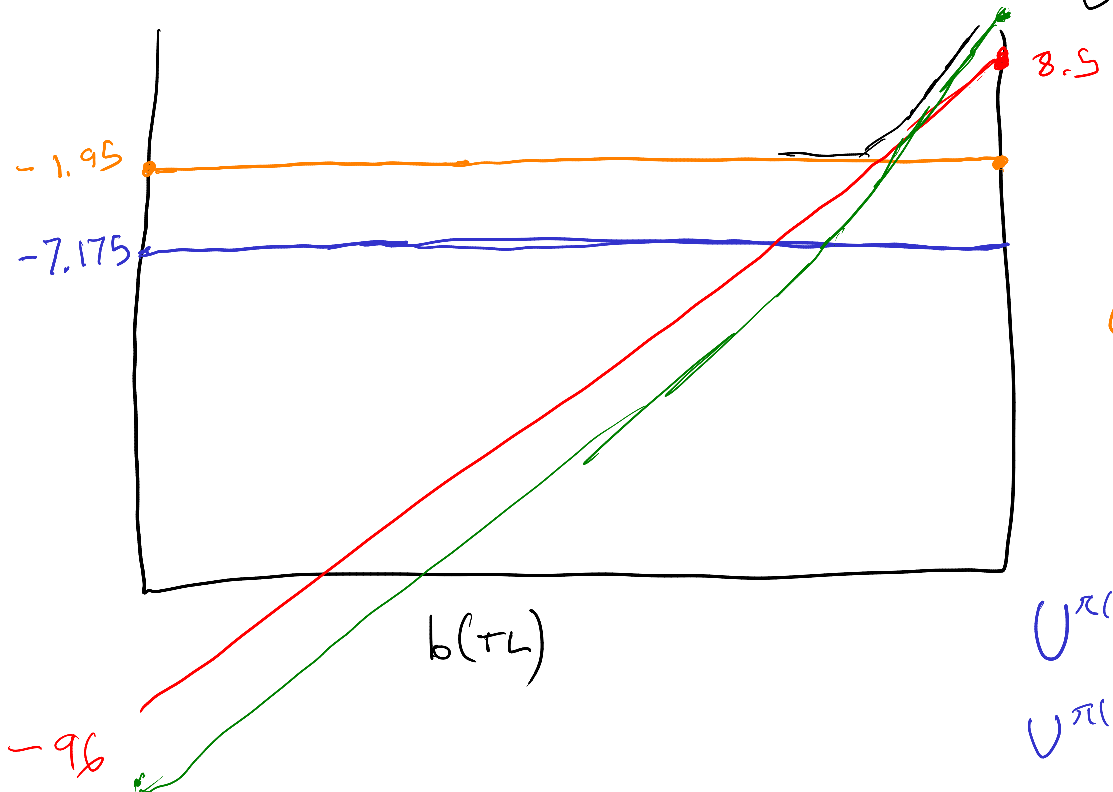
$$U^\pi(s) = R(s,\pi()) + \gamma \left[\sum_{s'} T(s' \mid s, \pi()) \sum_o \cancel{O}(o \mid \pi(), s') U^{\pi(o)}(s') \right]$$

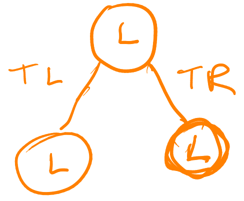


L π_L $U^{\pi_L()}(\cdot) = -1$

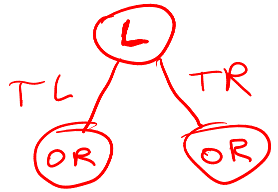
OL π_{OL} $U^{\pi_{OL}()}(TR) = 10$ $U^{\pi_{OL}()}(TL) = -100$

OR π_{OR} $U^{\pi_{OR}()}(TR) = -100$ $U^{\pi_{OR}()}(TL) = 10$

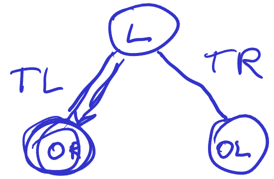


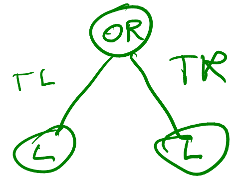


$U^{\pi()}(\cdot) = -1 + \gamma(-1)$



$U^\pi(TL) = -1 + \gamma\, 10 = 8.5$ $U^\pi(TR) = -1 + \gamma(-100) = -96$





$U^\pi(TL) = 9.5$

$U^\pi(TR) = -100.95$

$(T(TL \mid TL, L) = 1)$
$T(TR \mid TL, L) = 0)$

$U^{\pi()}(TL) = -1 + \gamma(0.85 \cdot 10 + 0.15 \cdot (-100))$

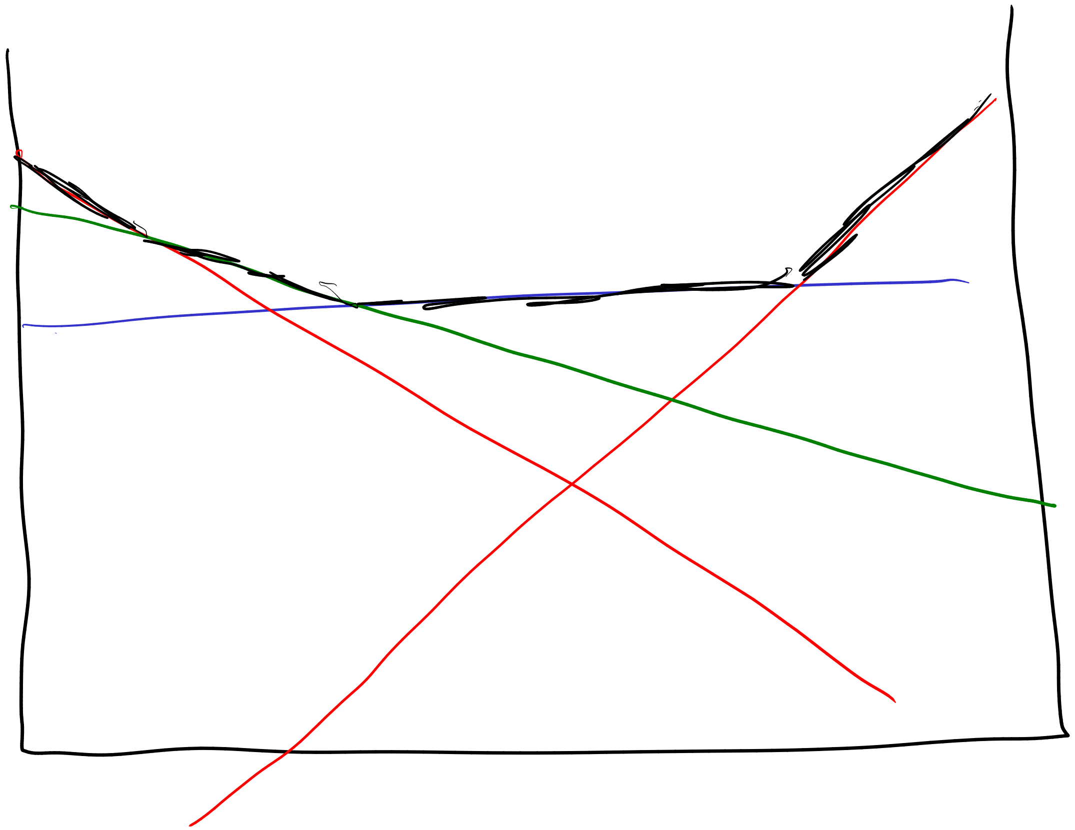
$\uparrow Z(TL \mid L, TL)$ $\uparrow U^{\pi(TL)}(TL)$

$= -7.175$

$U^{\pi()}(TR) = -7.175$

POMDP Value Functions

POMDP Value Functions



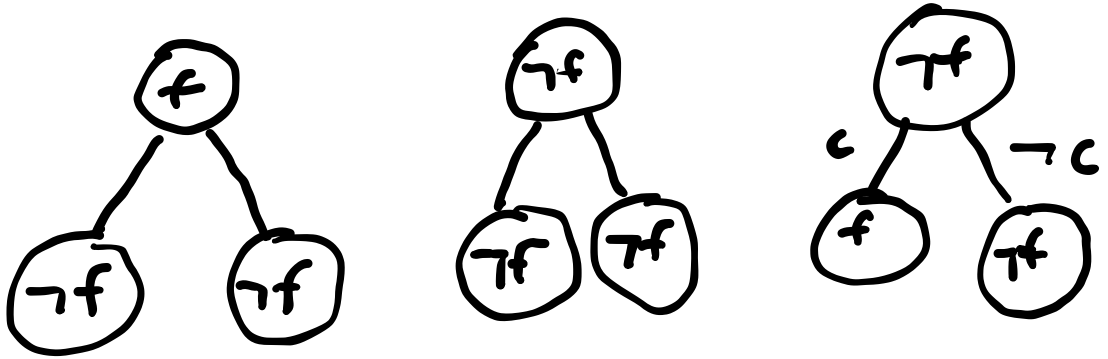
$$V^*(b) = \max_{\alpha \in \Gamma} \alpha^\top b$$

Exercise: 2-Step Crying Baby α Vectors

$$S = \{h, \neg h\} \qquad T(h \mid h, \neg f) = 1.0$$

$$A = \{f, \neg f\} \qquad T(h \mid \neg h, \neg f) = 0.1$$

$$O = \{c, \neg c\} \qquad T(\neg h \mid \cdot, f) = 1.0$$



$$R(s,a) = R(s) + R(a)$$

$$U^{\pi}(s) = R(s, \pi()) + \gamma \left[\sum_{s'} T(s' \mid s, \pi()) \sum_{o} O(o \mid \pi(), s') U^{\pi(o)}(s') \right]$$

$$R(s) = \begin{cases} -10 \text{ if } s = h \\ 0 \text{ otherwise} \end{cases}$$

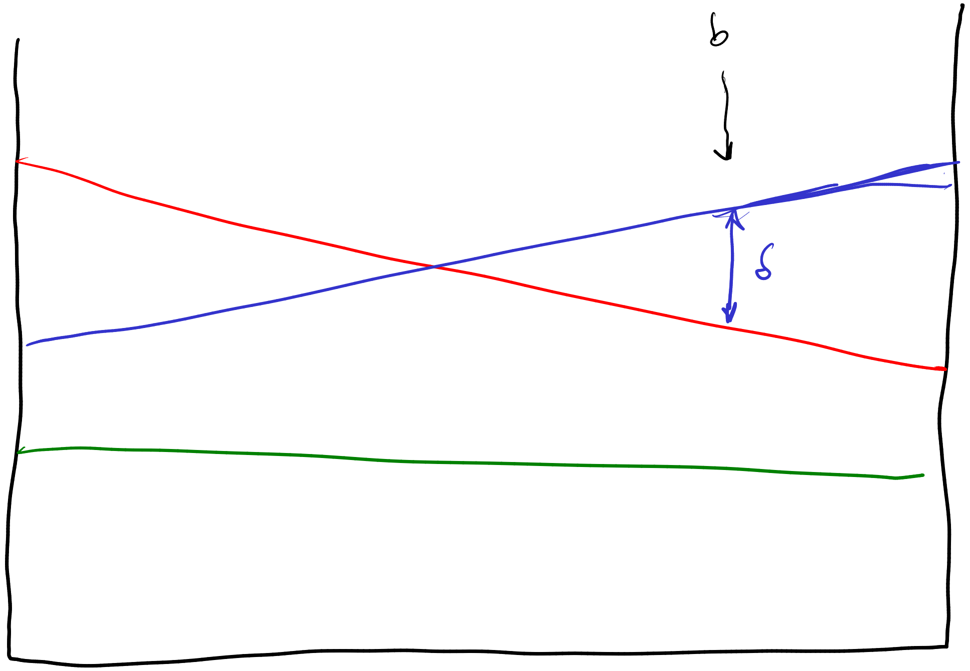
$$R(a) = \begin{cases} -5 \text{ if } a = f \\ 0 \text{ otherwise} \end{cases}$$

$$Z(c \mid \cdot, h) = 0.8)$$

$$Z(c \mid \cdot, \neg h) = 0.1$$

$$\gamma = 0.9$$

α-Vector Pruning



$$\underset{\delta,\, b}{\text{maximize}}\ \delta$$

subject to $b \geq 0$, $\mathbf{1}^T b = 1$ } enforce b is probability

$\alpha^T b > \alpha'^T b + \delta \quad \forall \alpha' \in \Gamma$

- If there is a positive δ solution then α is not dominated
- b is sometimes called the "witness"

Alpha Vector Expansion

POMDP Value Iteration (horizon d)

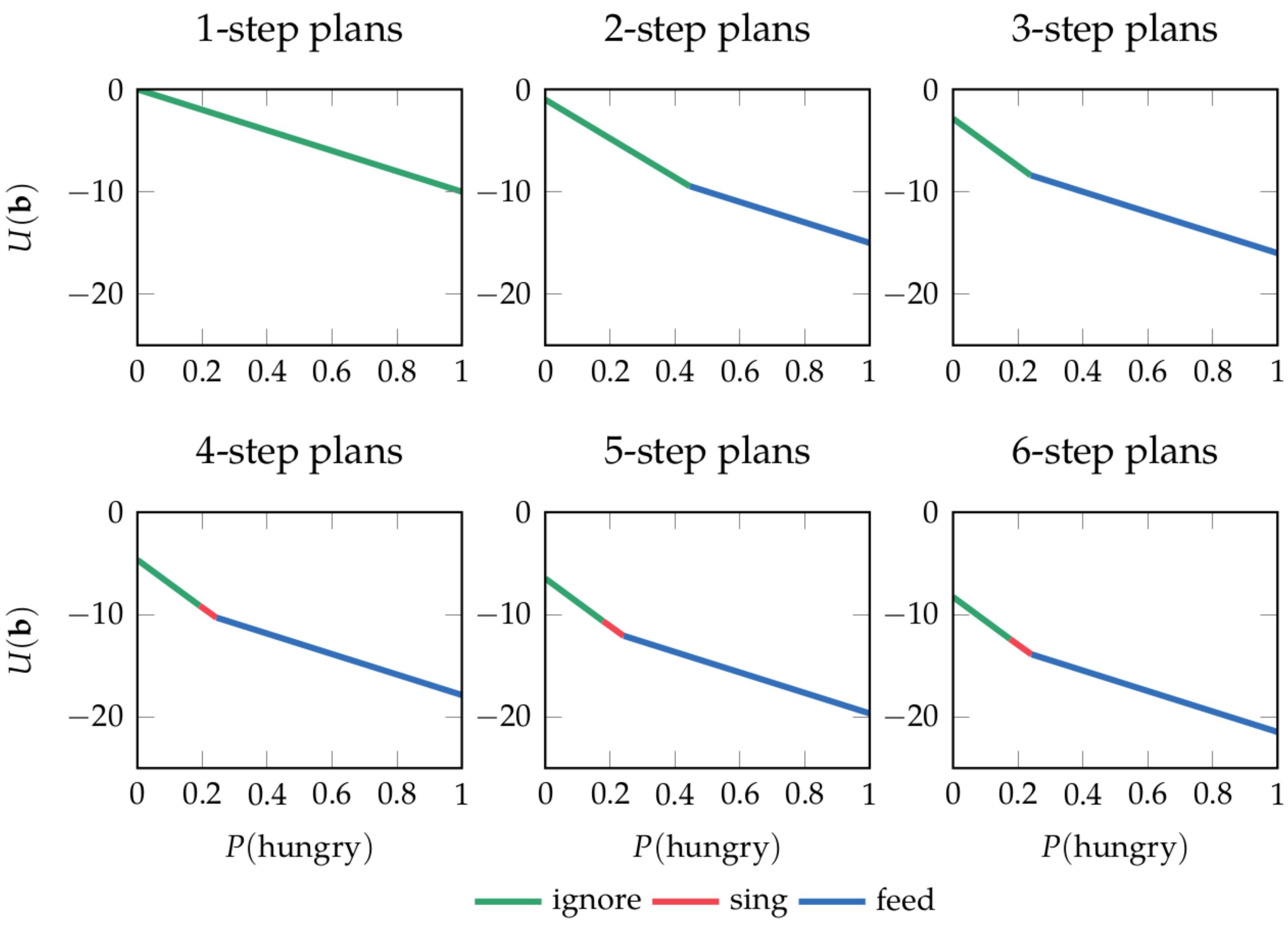
$\Gamma^0 \leftarrow \emptyset$

for $n \in 1 \ldots d$

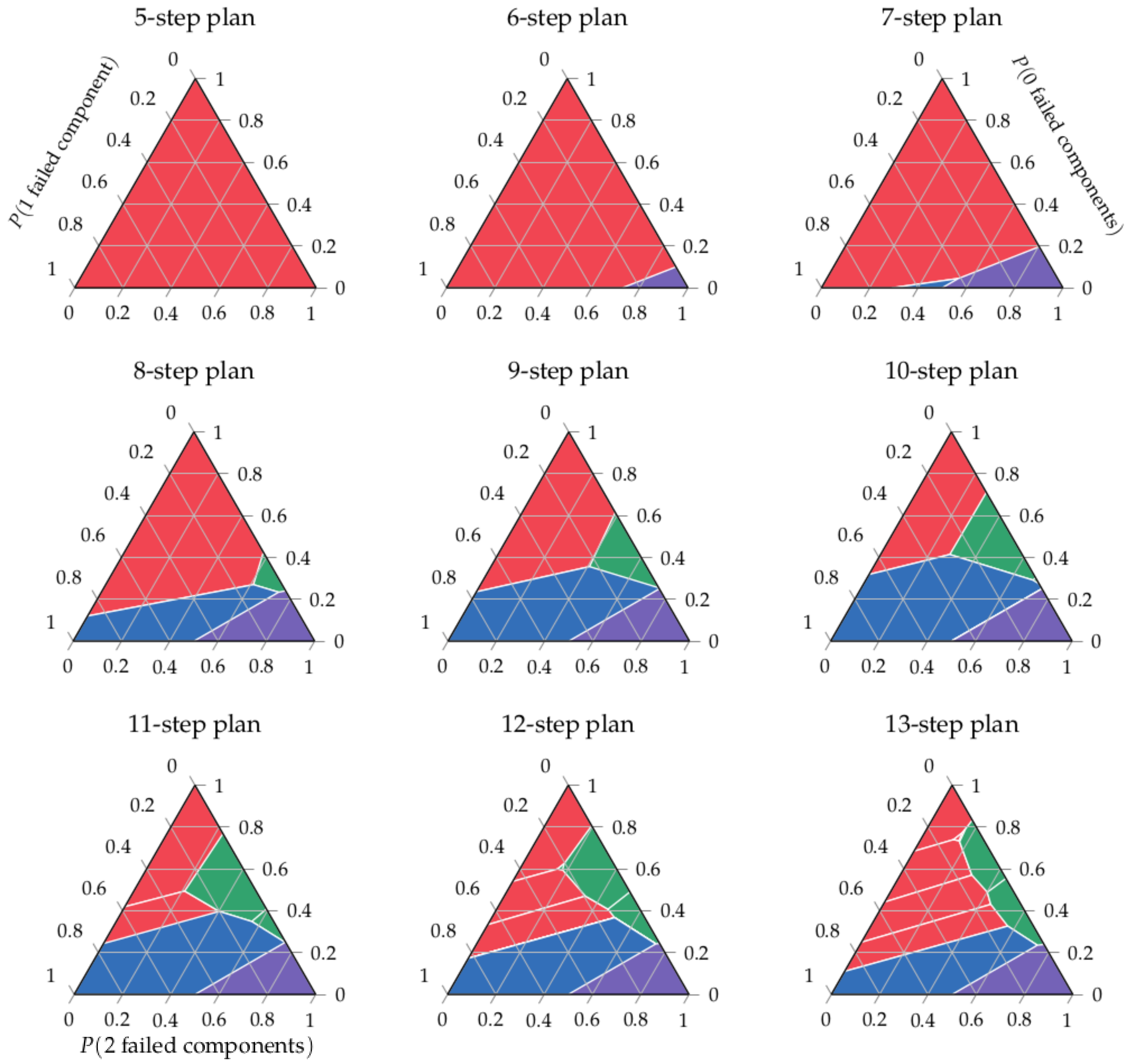
  Construct Γ^n by expanding with Γ^{n-1}

  Prune Γ^n

Finite Horizon POMDP Value Iteration



Finite Horizon POMDP Value Iteration



Recap

Recap

- A POMDP is an MDP on the ____________

Recap

- A POMDP is an MDP on the belief space

Recap

- A POMDP is an MDP on the belief space
- The value function of a discrete POMDP can be represented by a set of ___________________

Recap

- A POMDP is an MDP on the belief space
- The value function of a discrete POMDP can be represented by a set of α-vectors

Recap

- A POMDP is an MDP on the belief space
- The value function of a discrete POMDP can be represented by a set of α-vectors
- Each α vector corresponds to a ___________________

Recap

- A POMDP is an MDP on the belief space
- The value function of a discrete POMDP can be represented by a set of α-vectors
- Each α vector corresponds to a conditional plan