# Value-Based Model Free RL

# Last Time

- Policy Optimization
- Policy Gradient ← 1st order
- Tricks for Policy Gradient

- log derivative
- causality
- baseline subtraction

# Map

# Map

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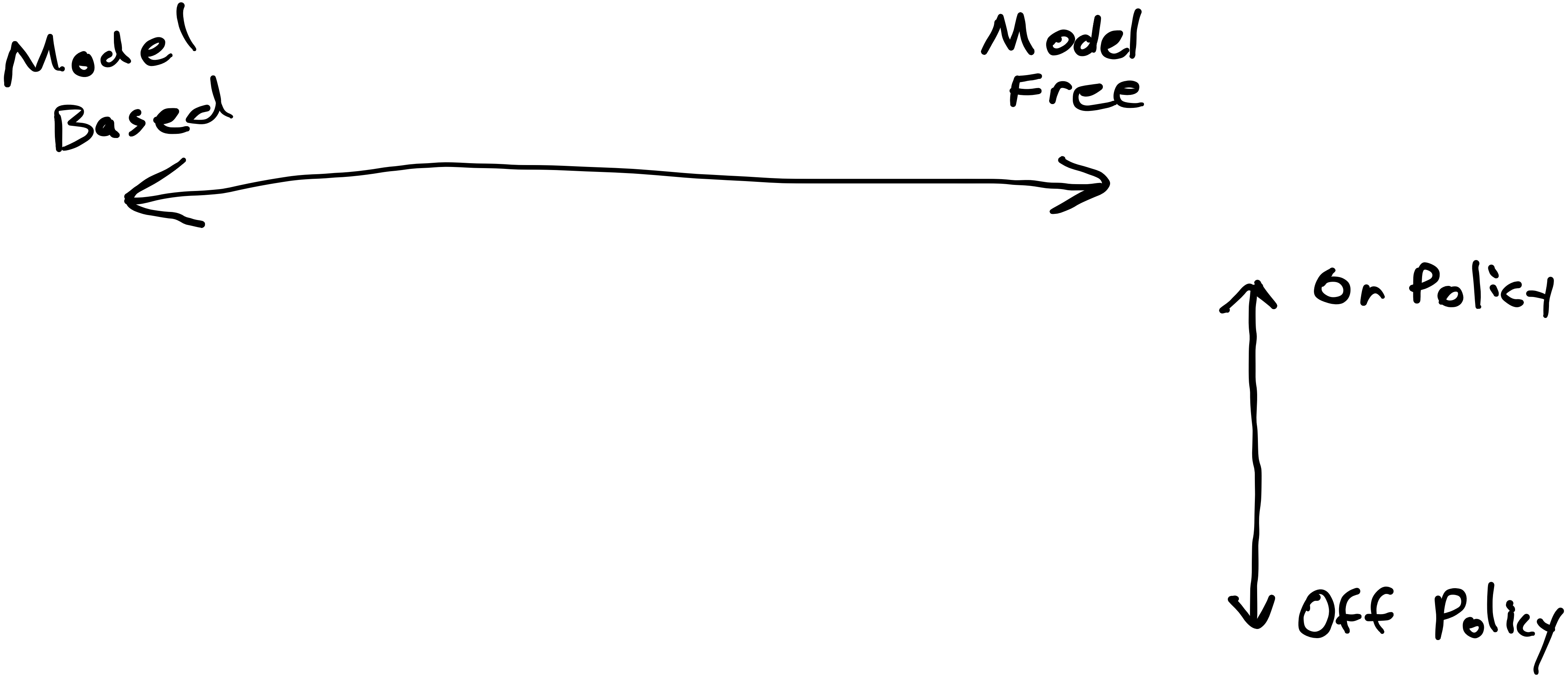

# Map

Model Based

Model Free

On Policy

Off Policy

MLMBTRL
(learn T,R)

# Map

Model Based

Model Free

learn Q

learn π

On Policy

Off Policy

MLMBTRL
(learn T,R)

# Map

Model Based

Model Free

learn Q

learn $\pi$

On Policy

Policy Gradient

Off Policy

MLMBTRL (learn T,R)

# Map

Model Based $\longleftrightarrow$ Model Free

learn Q

SARSA

learn $\pi$

Policy Gradient

On Policy

Off Policy

MLMBTRL

(learn T, R)

# Map

Model Based

Model Free

learn Q

learn π

On Policy

SARSA

Policy Gradient

MLMBTRL (learn T,R)

Q-learning

Off Policy

# Today

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- Basic On- and Off-Policy **value based** model free RL algorithms

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- Basic On- and Off-Policy **value based** model free RL algorithms
- Tricks for tabular value based RL algorithms

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- Basic On- and Off-Policy **value based** model free RL algorithms
- Tricks for tabular value based RL algorithms
- Understanding of On- vs Off-Policy

# Why learn Q?

$\hat{T}, \hat{R}$ $\hat{U}$ $\hat{\pi}$ $\hat{Q}$

$$\pi(s) = \operatorname{argmax}\left(\hat{R}(s,a) + \gamma\, E\left[\hat{U}(s')\right]\right)$$

$\hat{T}$

$$\pi(s) = \operatorname*{argmax}_a \hat{Q}(s,a)$$

# Incremental Mean Estimation

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$$\hat{x}_m = \frac{1}{m} \sum_{i=1}^{m} x^{(i)}$$

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$$
\begin{aligned}
\hat{x}_m &= \frac{1}{m} \sum_{i=1}^{m} x^{(i)} \\
&= \frac{1}{m} \left( x^{(m)} + \sum_{i=1}^{m-1} x^{(i)} \right)
\end{aligned}
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&= \hat{x}_{m-1} + \frac{1}{m} \left( \underbrace{x^{(m)} - \hat{x}_{m-1}}_{\text{TD}} \right)
\end{aligned}
$$

step size

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\end{aligned}
$$

```julia
function simulate!(π::MonteCarloTreeSearch, s, d=π.d)
    if d ≤ 0
        return π.U(s)
    end
    𝒫, N, Q, c = π.𝒫, π.N, π.Q, π.c
    𝒜, TR, γ = 𝒫.𝒜, 𝒫.TR, 𝒫.γ
    if !haskey(N, (s, first(𝒜)))
        for a in 𝒜
            N[(s,a)] = 0
            Q[(s,a)] = 0.0
        end
        return π.U(s)
    end
    a = explore(π, s)
    s′, r = TR(s,a)
    q = r + γ*simulate!(π, s′, d-1)
    N[(s,a)] += 1
    Q[(s,a)] += (q-Q[(s,a)])/N[(s,a)]
    return q
end
```

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```

loop

$$\hat{x} \leftarrow \hat{x} + \alpha\left(x - \hat{x}\right)$$

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end
```

$$\boxed{\begin{array}{l}\text{loop} \\ \quad \hat{x} \leftarrow \hat{x} + \alpha\left(x - \hat{x}\right)\end{array}}$$

"Temporal Difference (TD) Error"

# Q Learning

Want $\qquad Q(s,a) \leftarrow Q(s,a) + \alpha\left(\hat{q}((s,a,r,s')) - Q(s,a)\right)$

$$
\begin{aligned}
Q(s,a) &= R(s,a) + \gamma E\left[V(s')\right] \\
&= R(s,a) + \gamma E\left[\max_{a'} Q(s',a')\right] \\
&= E\left[r + \gamma \max_{a'} Q(s',a')\right]
\end{aligned}
$$

# Q learning and SARSA

$(s, a, r, s')$ $(s, a, r, s', a')$

# Q learning and SARSA

## Q-Learning

$$Q(s,a) \leftarrow 0$$

$$s \leftarrow s_0$$ $\leftarrow a \leftarrow \varepsilon\text{-greedy}$

loop

$$a \leftarrow \operatorname{argmax} Q(s,a) \text{ w.p. } 1-\epsilon, \quad \text{rand}(A) \text{ o.w.}$$

$$r \leftarrow \text{act!}(\text{env}, a)$$

$$s' \leftarrow \text{observe}(\text{env})$$ $\leftarrow a' \leftarrow \varepsilon\text{-greedy}$

$$Q(s,a) \leftarrow Q(s,a) + \alpha\,(r + \gamma \max_{a'} Q(s',a') - Q(s,a))$$ $\leftarrow Q(s,a) \leftarrow Q(s,a) + \alpha(r + \gamma\, Q(s',a') - Q(s,a))$

$$s \leftarrow s'$$ $\leftarrow a \leftarrow a'$

# Q learning and SARSA

Q-Learning

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loop

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$r \leftarrow \text{act!}(\text{env}, a)$

$s' \leftarrow \text{observe}(\text{env})$

$Q(s,a) \leftarrow Q(s,a) + \alpha\ (\underbrace{r + \gamma \max_{a'} Q(s',a') - Q(s,a)}_{TD})$

$s \leftarrow s'$

# Illustrative Problem

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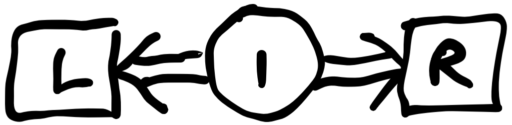

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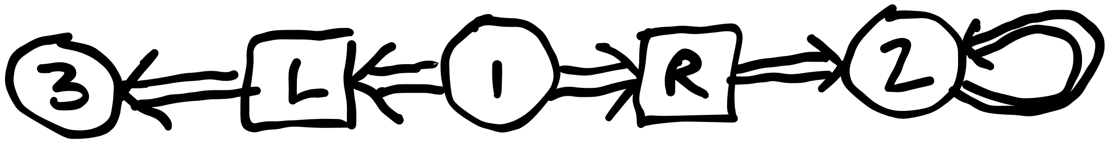

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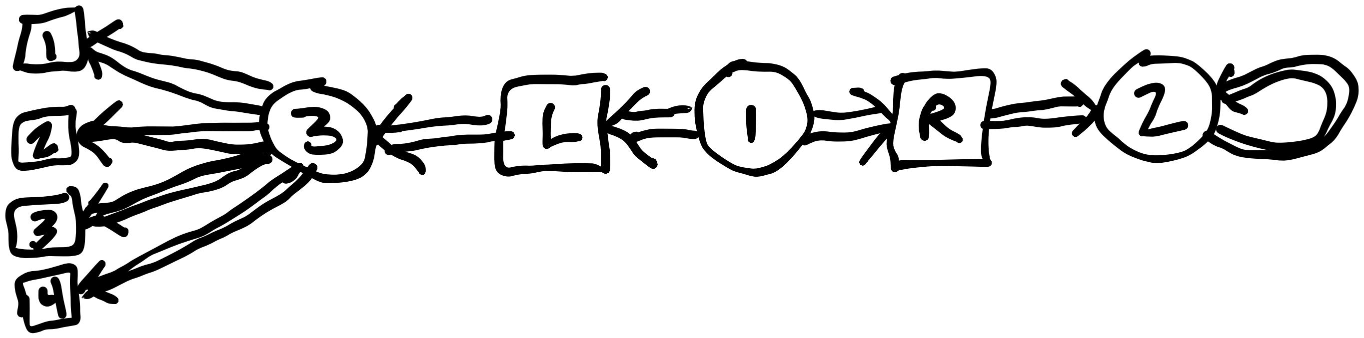

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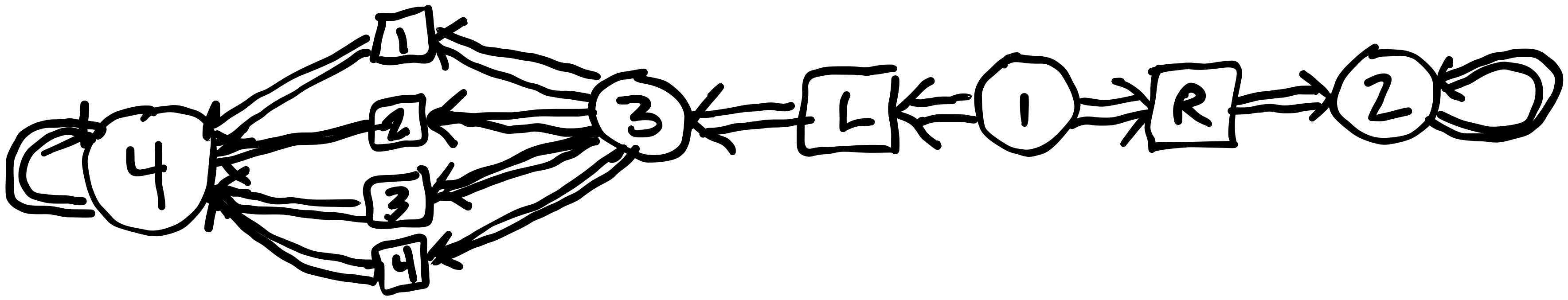

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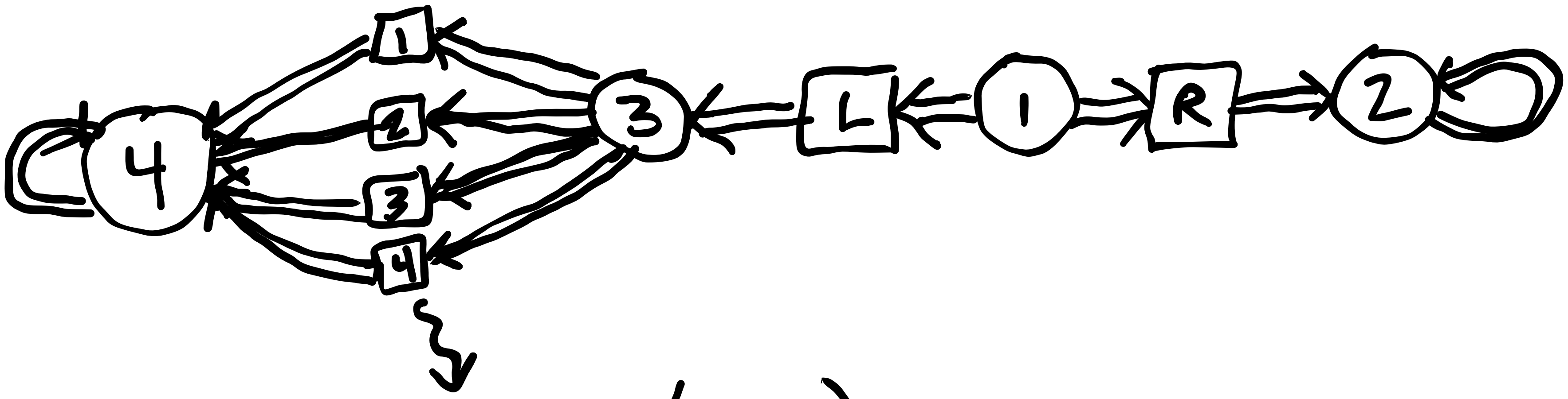


Actions 1-4: $r \sim \mathcal{N}(-0.1, 1)$

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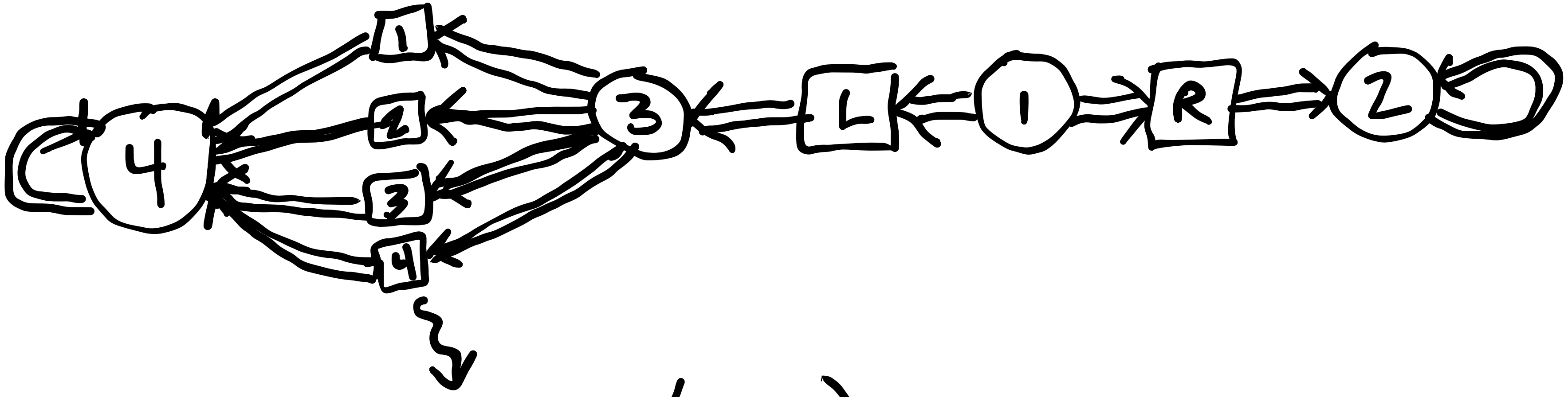


Actions 1-4: $r \sim \mathcal{N}(-0.1, 1)$

1. After a few episodes, what is Q(3, a) for a in 1-4?

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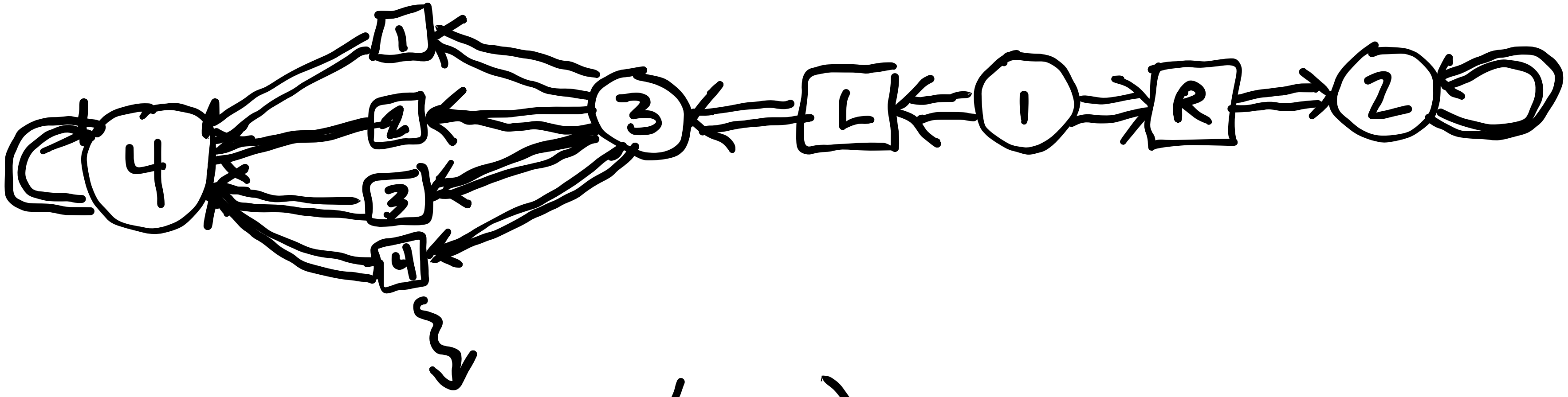
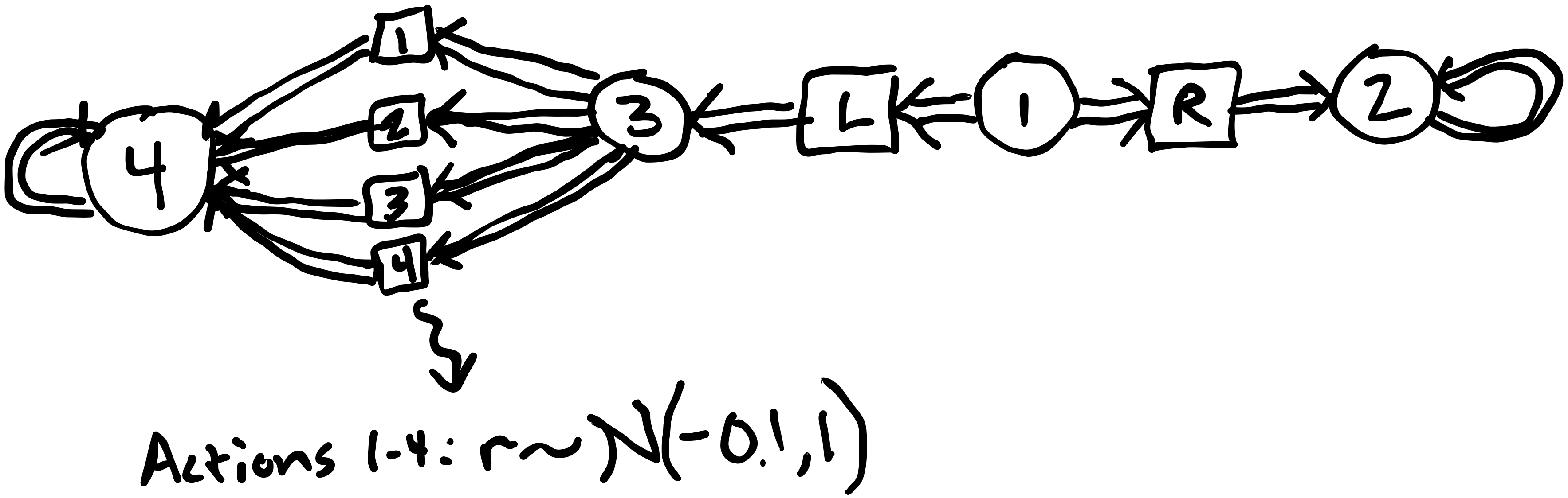



1. After a few episodes, what is Q(3, a) for a in 1-4?
2. After a few episodes, what is Q(1, L)?

$$Q(1, L) \leftarrow \cdots \max_{a'} Q(3, a')$$

# Illustrative Problem




1. After a few episodes, what is Q(3, a) for a in 1-4?
2. After a few episodes, what is Q(1, L)?
3. Why is this a problem and what are some possible solutions?

# Big Problem: Maximization Bias

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Even if all $Q(s', a')$ unbiased, $\max_{a'} Q(s', a')$ is biased!

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Solution: Double Q Learning $Q_1$, $Q_2$

$$Q_1(s,a) \leftarrow Q_1(s,a) + \alpha \left( r + \gamma\, Q_2 \left( s', \underset{a'}{\operatorname{argmax}} Q_1(s', a') \right) - Q_1(s,a) \right)$$

# Big Problem: Maximization Bias

Even if all $Q(s', a')$ unbiased, $\max_{a'} Q(s', a')$ is biased!

Solution: Double Q Learning $\quad Q_1, Q_2$

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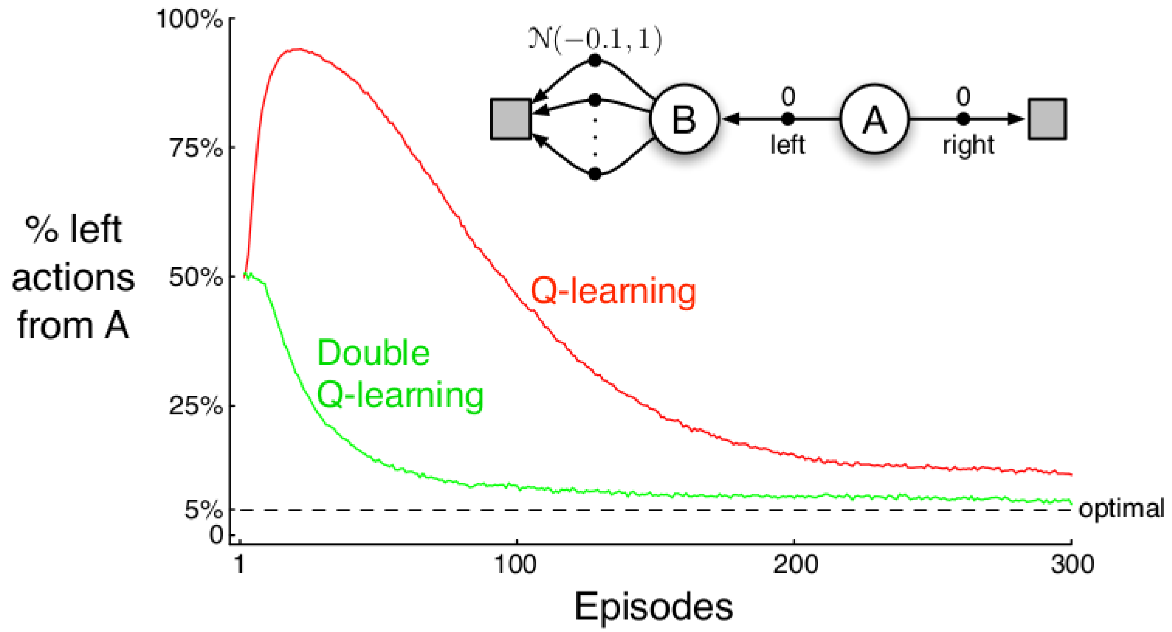
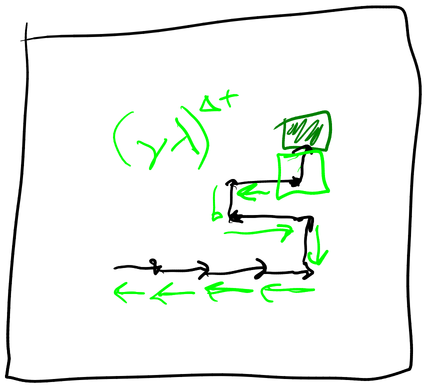


# Eligibility Traces



# SARSA-λ

$Q(s,a), N(s,a) \leftarrow 0$

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$Q(s, a), N(s, a) \leftarrow 0$

initialize $s, a, r, s'$

loop

$a' \leftarrow \text{argmax}\, Q(s', a)$ w.p. $1 - \epsilon, \quad \text{rand}(A)$ o.w.

$N(s, a) \leftarrow N(s, a) + 1$

TD

$\delta \leftarrow r + \gamma Q(s', a') - Q(s, a)$

$Q(s, a) \leftarrow Q(s, a) + \alpha\delta\, N(s, a) \quad \forall s, a$

$N(s, a) \leftarrow \gamma\lambda N(s, a)$

$s \leftarrow s', \quad a \leftarrow a'$

$r \leftarrow \text{act!}(\text{env}, a)$

$s' \leftarrow \text{observe}(\text{env})$

# Convergence

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- Q learning converges to optimal Q-values w.p. 1
  (Sutton and Barto, p. 131)

# Convergence

- Q learning converges to optimal Q-values w.p. 1 (Sutton and Barto, p. 131)
- SARSA converges to optimal Q-values w.p. 1 ***provided that***

  $\pi \rightarrow \text{greedy}$

  (Sutton and Barto, p. 129)

# On vs Off-Policy

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## On Policy

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On Policy

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# On vs Off-Policy

## On Policy

SARSA:

$$Q(s,a) \leftarrow Q(s,a) + \alpha\ (r + \gamma Q(s',a') - Q(s,a))$$

## Off Policy

# On vs Off-Policy

## On Policy

SARSA:

policy is good

$$Q(s,a) \leftarrow Q(s,a) + \alpha\ (r + \gamma Q(s',a') - Q(s,a))$$

## Off Policy

Q-learning:

$$Q(s,a) \leftarrow Q(s,a) + \alpha\ (r + \gamma \max_{a'} Q(s',a') - Q(s,a))$$

q values

# On vs Off-Policy

## On Policy

SARSA:

$$Q(s,a) \leftarrow Q(s,a) + \alpha\ (r + \gamma Q(s',a') - Q(s,a))$$

## Off Policy

Q-learning:

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Will eligibility traces work with Q-learning?

# On vs Off-Policy

## On Policy

SARSA:

$$Q(s,a) \leftarrow Q(s,a) + \alpha \ (r + \gamma Q(s',a') - Q(s,a))$$

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Q-learning:

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Will eligibility traces work with Q-learning?

Not easily

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## On Policy

SARSA:

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Q-learning:

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Will eligibility traces work with Q-learning?

Not easily

Policy Gradient:

$$\theta \leftarrow \theta + \alpha \sum_{k=0}^{d} \nabla_\theta \log \pi_\theta(a_k \mid s_k) R(\tau)$$

# Today

SARSA Q-learning

- Basic On- and Off-Policy **value based** model free RL algorithms
- Tricks for tabular value based RL algorithms
  SARSA – eligibility traces
  Q-learning – double Q-learning
- Understanding of On- vs Off-Policy