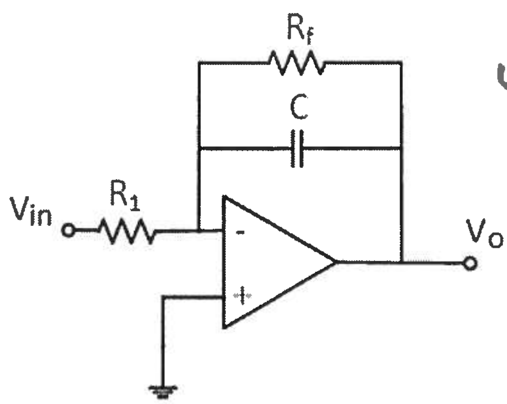
Week 3 Quiz Solutions

1. Considering the following circuit where C=200nF.



Lowpass Filter

$$\omega_b = \frac{1}{R_F C} \qquad H(\omega) = \frac{-R_F}{R_1} \frac{1}{R_F C j\omega + 1}$$

$$\text{gain} = \frac{-R_F}{R_1}$$

To get a bandwidth of ω_b=500rad/s and a passband gain of -400, Determine the appropriate value of R_f and R_1. Enter the value of R_1 in ohms

$$500 = \frac{1}{R_F(200 \times 10^{-9})} \Rightarrow R_F = \frac{1}{500 \cdot 200 \cdot 10^{-9}}$$

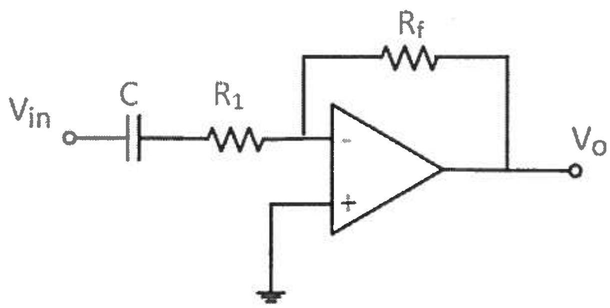
$$= 10000\,\Omega$$

$$-400 = \frac{-10000}{R_1} \Rightarrow R_1 = 25\,\Omega$$

2. For the above circuit in question 1, enter the value of R_f in <u>KILO</u> ohms.

$10\,k\Omega$

3. For the following circuit, R_1 =100Ω. Select the values of R_f and C to have a passband gain of -100 and a corner frequency of 1,000 rad/s. Enter the value of C in uF.



High Pass Filter

$$H(\omega) = \frac{-R_F C j\omega}{R_1 C j\omega + 1}$$

$$\text{gain} = \frac{-R_F}{R_1}$$

$$\omega_c = \frac{1}{R_1 C}$$

$$1000 = \frac{1}{100\,C}$$

$$\Rightarrow C = 1 \times 10^{-5}\,F$$

$$= 10\,\mu F$$

4. From the above circuit in question 3, enter the value of the value of R_f in kΩ.

$$-100 = \frac{-R_F}{100} \Rightarrow R_F = 10\,k\Omega$$

5. What is the DC gain of a filter circuit that has the given transfer function?

$$H(\omega) = \frac{10}{j\omega + 5}$$

DC gain = $|H(\omega)|$ for $\omega = 0$

$$\frac{10}{j0+5} = \boxed{2}$$

6. What is the bandwidth in radians-per-second of a filter circuit that has the given transfer function?

$$H(\omega) = \frac{10}{j\omega + 5}$$

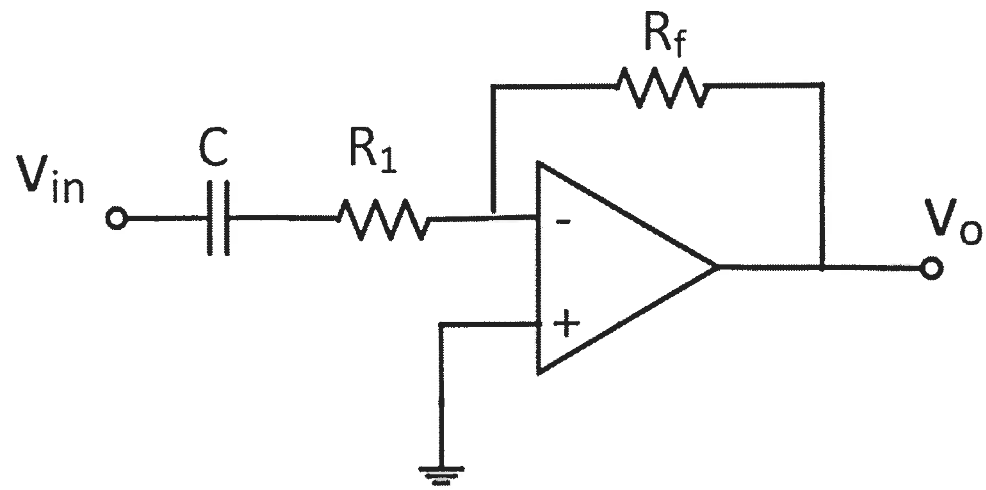
rewrite as

$$\frac{10}{5} \frac{1}{j\omega\frac{1}{5}+1} \Rightarrow$$

Low pass filter with gain of 2 and bandwidth of $\boxed{5 \frac{rad}{s}}$

$$2 \frac{1}{\frac{j\omega}{5}+1}$$

7. A sine wave with amplitude 2 V and frequency 1358 Hz is applied to the circuit shown. If R_f = 10 kΩ , R_1 = 1.5 kΩ , and C = 0.1 μF , what is the amplitude of the output sine wave?



High pass filter

$$H(f) = -\frac{R_F}{R_1} \frac{jf/f_o}{jf/f_o + 1} \qquad f_o = \frac{1}{2\pi R_1 C}$$

$$= \frac{1}{2\pi \, 1500 \, 0.1\times10^{-6}}$$

$$= 1061.03 \text{ Hz}$$

$$|H(1358)| = \left| -\frac{10}{1.5} \frac{j1358/1061.03}{j\frac{1358}{1061.03} + 1} \right|$$

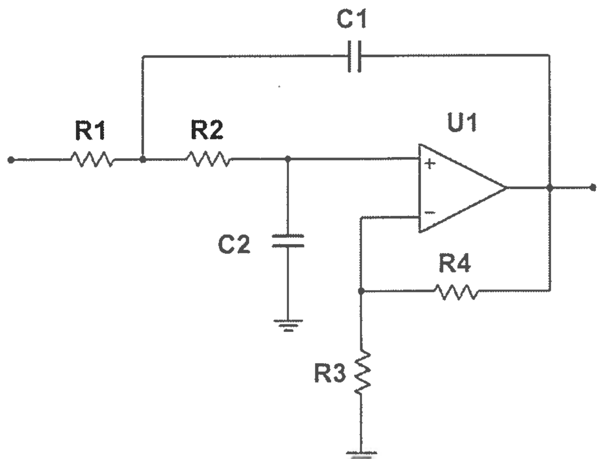
$$= 6.6667 \frac{1358/1061.03}{\sqrt{1^2 + \left(\frac{1358}{1061.03}\right)^2}} = 6.6667 \frac{1.2799}{1.6242}$$

$$= 5.25$$

$V_o = V_{IN} \times \text{Gain} = 2\times5.25 = \boxed{10.5V}$

8. In the filter circuit shown, R1 = 10kΩ, R2 = 10kΩ, R3 = 4kΩ, R4 = 6.8kΩ, C1 = 5.3nF, and C2 = 5.3nF. 1 nF = 1 nanofarad = 10^{-9} farads. What is the midband gain of this filter? Calculate your answer to at least the nearest tenth.

Sallen – Key Low Pass



$$K = 1 + \frac{R_4}{R_3} = 1 + \frac{6.8}{4} = \boxed{2.7}$$

9. For the above circuit in question 8, what is the resonance frequency in **kilohertz**? Calculate your answer to at least the nearest tenth.

$$f_0 = \frac{1}{2\pi\sqrt{R_1 R_2 C_1 C_2}} = \boxed{3.0\,kHz}$$

10. For the above circuit in question 8, what is the quality factor? Calculate your answer to at least the nearest tenth.

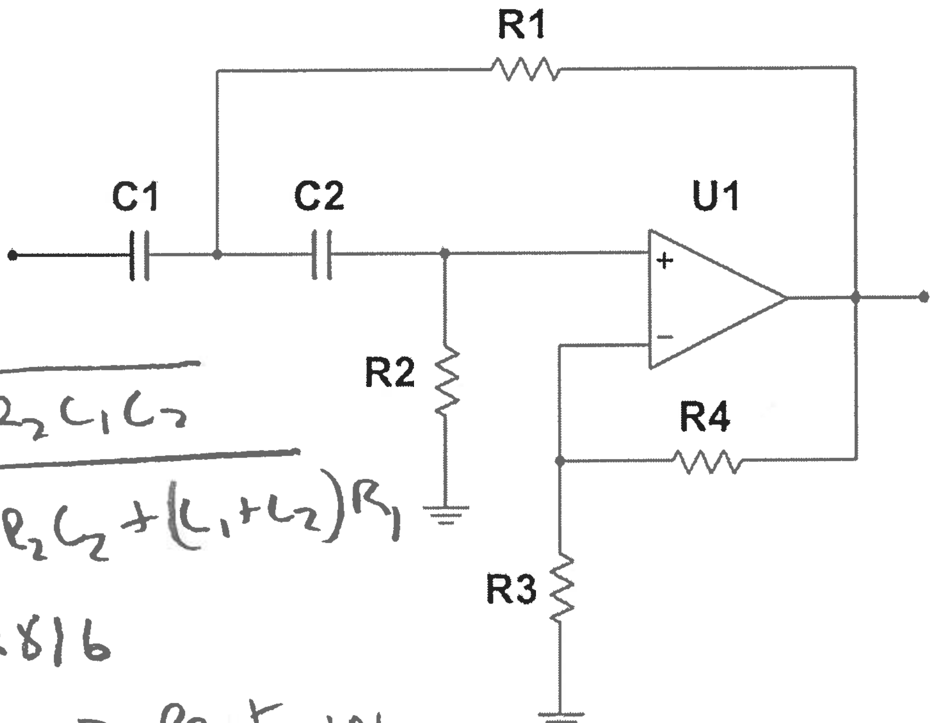
Since $R_1 = R_2 = R$ & $C_1 = C_2 = C$

$$K = 3 - \frac{1}{Q} \Rightarrow \frac{1}{Q} = 3 - 2.7 = 0.3$$

$$\boxed{Q = 3.3}$$

or can use $Q = \dfrac{\sqrt{R_1 R_2 C_1 C_2}}{(1-K)R_1 C_1 + (R_1+R_2)C_2}$

11. In the filter circuit shown, R1 = 10kΩ, R2 = 10kΩ, R3 = 2kΩ, R4 = 2kΩ, C1 = 0.15 μF, and C2 = 0.1 μF. 1 μF = 1 microfarad = 10^{-6} farads. Which of the following are **not true** for the circuit shown.



$$Q = \frac{\sqrt{R_1 R_2 C_1 C_2}}{(1-K)R_2 C_2 + (C_1 + C_2) R_1} = 0.816 > \frac{1}{\sqrt{2}} \Rightarrow$$ Peak in magnitude of transfer function

T ☑ There is a peak in the transfer function of this filter.

F ☒ The DC gain is 2 $\quad 1 + \frac{R_4}{R_3} = 1 + \frac{2}{2} = 2$ = Passband gain, NOT DC gain.

T → ☑ The resonance frequency increases as the value of C2 is decreased.

F ☒ The resonance frequency is approximately 130 radians per second.

$$\rightarrow \frac{1}{2\pi\sqrt{R_1 R_2 C_1 C_2}} = 129.9\ Hz$$ (Not rad/s)

$$f_o = \frac{1}{2\pi\sqrt{R_1 R_2 C_1 C_2}} \qquad C_2 \downarrow \Rightarrow f_o \uparrow$$