Jeide, Matthew Date 11/14/2024 Period 2

W14 DE 2.1.4 Circuit Simplification: Boolean Algebra

Read the introduction in each section carefully.

INTRODUCTION

In the 1800s, Mathematician George Boole developed a unique mathematical system, named Boolean algebra, and published his work as *An Investigation of the Laws of Thought, on Which Are Founded the Mathematical Theories of Logic and Probabilities.* At the time, it had no practical application, but many years later his system became a crucial mathematical tool to design digital logic circuits.

Similar to how *traditional* algebra has rules to simplify algebraic expressions, Boolean algebra has theorems and laws to simplify expressions to design logic circuits. Through logic expression simplification, a logic circuit can be reduced to a simpler version that performs the same function. The advantage of a simpler circuit is that it contains fewer gates, is easier to build, and costs less to manufacture.

In this activity you learn how to apply the theorems and laws of Boolean algebra to simplify logic expressions and digital logic circuits.

Procedure

Justify the answer in pink. Make sure to show all your work and explain which Boolean Theorems were used. Make sure to show how it was simplified. This work must be neat.

1. $$F_1 = A\left(\overline{A} + A\,B\right)$$

$F_1 = AB$

$F_1 = A(\overline{A} + AB) = A(\overline{A} + B)) = A\overline{A} + AB = 0 + AB = AB$

2. $$F_2 = X\,\overline{Y}\,Z + \overline{X}\,\overline{Y}\,Z + \overline{X}\,Y\,Z$$

$F_2 = \overline{Y}Z + \overline{X}Z$

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$F_2 = XY\overline{Z} + \overline{X} * \overline{Y}Z + \overline{X}YZ = X\overline{Y}Z + \overline{X}(\overline{Y}Z + YZ) = X\overline{Y}Z + \overline{X}(Z(\overline{Y} + Y)) = X\overline{Y}Z + \overline{X}Z$

$F_2 = X\overline{Y}Z + \overline{X}Z = Z(\overline{X} + X\overline{Y}) = Z(\overline{X} + \overline{Y}) = Z\overline{X} + Z\overline{Y}$

3 $\mathbf{F_3 = J\,K + J\,\overline{K}}$

$\mathbf{F_3 = J}$

$F_3 = JK + J\overline{K} = J(K + \overline{K}) = J * 1 = J$

4 $\mathbf{F_4 = (B + \overline{B})\,(A\,\overline{B} + A\,\overline{B}\,\overline{C})}$

$\mathbf{F_4 = A\overline{B}}$

$F_4 = (1)(\overline{B}(A + A\overline{C})) = \overline{B}(A(1 + \overline{C})) = \overline{B}(A(1)) = \overline{B}A$

5 $\mathbf{F_5 = (X + \overline{Y})\,(X + Y)}$

$\mathbf{F_5 = X}$

$F_5 = XX + XY + X\overline{Y} + Y\overline{Y} = X + XY + X\overline{Y} + 0 = X(1 + Y + \overline{Y}) = X$

6 $\mathbf{F_6 = J\,K + (\overline{J} + \overline{K})\,L + J\,K}$

$\mathbf{F_6 = JK + L}$

$F_6 = JK + L(\overline{J} + \overline{K}) = JK + L$

If (L’ + K’) is not true, JK is true, it’s redundant.

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7 $\mathrm{F}_7 = \mathrm{R\,S} + \mathrm{R}\,(\mathrm{S} + \mathrm{T}) + \overline{\mathrm{S}}\,(\mathrm{S} + \mathrm{U})$

$\mathbf{F_6 = JK + L}$

I DON'T THINK THAT'S TRUE FOR F7 WOOOOOOOOOOOOOOOO

$F_7 = RS + RS + RT + 0 + \overline{S}U = RS + RT + \overline{S}U = R(S + T) + \overline{S}U$

8 $\mathrm{F}_8 = (\mathrm{N} + \overline{\mathrm{N}}\,\mathrm{M})\;(\overline{\mathrm{N}} + \mathrm{N}\,\mathrm{M})\;(\mathrm{N} + \mathrm{M})$

$\mathbf{F_8 = M}$

$F_8 = (N + M)(\overline{N} + M)(N + M) = (N + M)(\overline{N} + M) = N\overline{N} + NM + \overline{N}M + MM = 0 + NM + \overline{N}M + M$

$F_8 = M(N + \overline{N} + 1) = M(1) = M$

Almost as important as being able to use the laws of Boolean algebra (associative, commutative, or distributive) to simplify logic expressions, it is critical that you can identify them. **Identify the law of Boolean algebra that each of the following equalities is based on.**

9 $\overline{\mathrm{A}}\,\mathrm{B} + \mathrm{A}\,\overline{\mathrm{C}} + \overline{\mathrm{B}}\,\mathrm{C} = \mathrm{A}\,\overline{\mathrm{C}} + \overline{\mathrm{B}}\,\mathrm{C} + \overline{\mathrm{A}}\,\mathrm{B}$

Commutative Law

10 $(\mathrm{D}\,\overline{\mathrm{E}})\;(\mathrm{F}\,\overline{\mathrm{G}}) = \mathrm{D}\;(\overline{\mathrm{E}}\,\mathrm{F})\;\overline{\mathrm{G}}$

Associative Law

11 $((\mathrm{R} + \overline{\mathrm{S}}) + \mathrm{T}) + \overline{\mathrm{U}} = (\mathrm{R} + \overline{\mathrm{S}}) + (\mathrm{T} + \overline{\mathrm{U}})$

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Associative Law

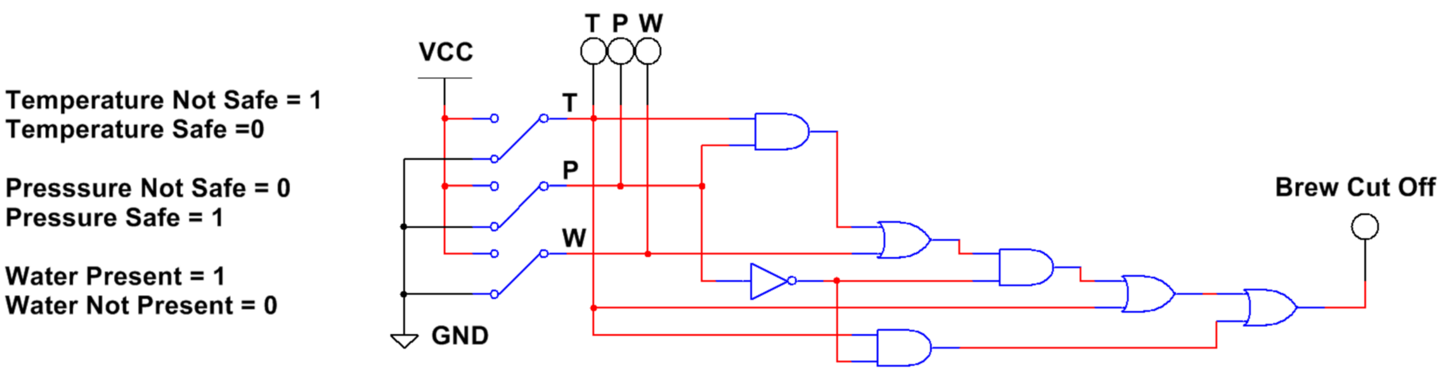
12 $(J + \overline{K})\ (\overline{L} + M) = J\,\overline{L} + J\,M + \overline{K}\,\overline{L} + \overline{K}\,M$

Distributive Law

13 $R\ (\overline{S}\,T\ +\ S\,\overline{V})\ \ =\ R\,\overline{S}\,T\ +\ R\,S\,\overline{V}$

Distributive Law

Now that you've practiced simplifying logic expressions, apply your knowledge to simplifying a circuit. Refer to figure 1 that shows a poorly designed AOI circuit that is part of a coffee vending machine based on the design statement "If the temperature is too high or the pressure is not below the safe value with water present, the brew sensor cuts off the brew process."



14. Write the *un-simplified* logic expression for the output ***Brew Cut Off***.

$$F_{14} = (\overline{P}(TP + W) + T) + \overline{P}T$$

15. Using the theorems and laws of Boolean algebra, simplify the logic expression ***Brew Cut Off***. Write your logic expression in Sum-of-Products (SOP) form.

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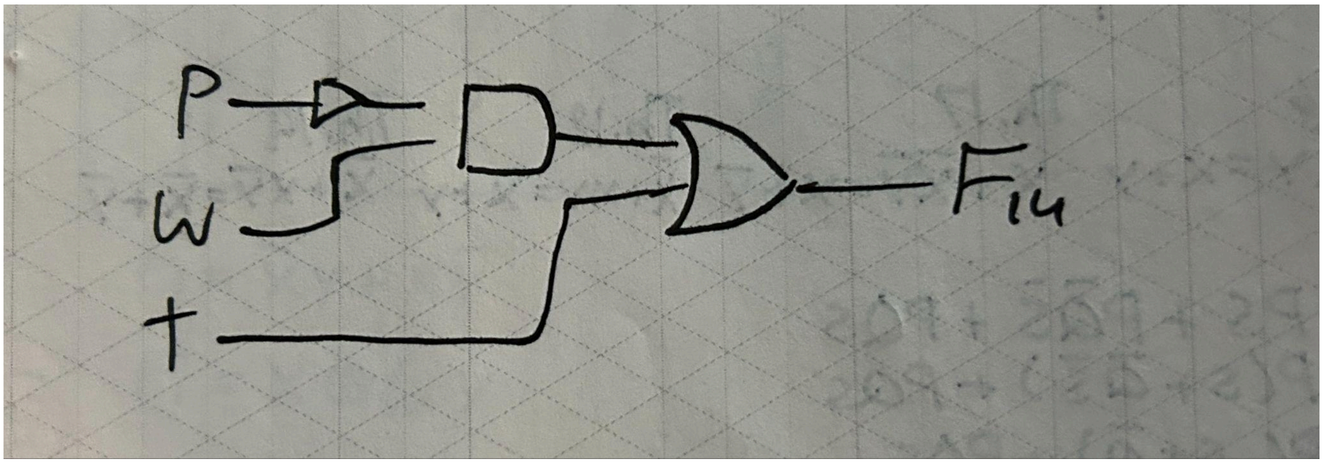
$$F_{14} = (\overline{P}(TP + W) + T) + \overline{P}T = (\overline{P}TP + \overline{P}W) + T) + \overline{P}T = ((0 + \overline{P}W) + T) + \overline{P}T$$

$$F_{14} = (\overline{P}W + T) + \overline{P}T = \overline{P}W + T + \overline{P}T = \overline{P}W + T(1 + \overline{P}) = \overline{P}W + T$$

16. In your notebook, draw an AOI circuit that implements the simplified logic expression ***Brew Cut Off***. Assume that only 2-input AND gates (74LS08), 2-input OR gates (74LS32), and inverters (74LS04) are available.



CONCLUSION

A. Describe the process you would use to simplify a logic expression using Boolean algebra.

Simplify a logic expression by applying Boolean laws step-by-step until no further reductions are possible.

B. How do you know when you are finished simplifying and have arrived at the simplest equation?

You're finished when I say you're finished.

You're finished when the expression has the fewest possible terms and can't be simplified further without changing its function.

C. Other than using Boolean algebra, how could you prove that two circuits are equivalent?

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Use a truth table to compare the two functions.

D. If you worked for a company that manufactured the coffee vending machine that used the poorly designed circuit, how much money would your new design save the company annually if each GATE cost 15¢ and the company made 500,000 vending machines per year?

7 * 0.15 = 1.05 USD
1.05 USD * 500000 = 525000 USD

3 * 0.15 = 0.45 USD
0.45 * 500000 = 225000 USD

525000 - 225000 = 300000 USD

USING MY BRILLIANT DESIGN THE COMPANY WOULD SAVE 300,000 USD BECAUSE MY GATE USES HALF THE GATES THE ORIGINAL POOR DESIGN HAD. WOOOOOOOOO