

Merge Sort & Heap Sort

2021년 1월 17일 일요일 오전 12:44

1. Merge Sort

: merge n sorted elements and n sorted elements to make the $2n$ elements sorted

ex)

1	3	4	9
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2	5	7	8
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Put the cursor at first place of each set.

⇒ 1 2 3 4 5 7 8 9

Write the smaller one,

and push the cursor pointing to it to right one at a time. Repeat.

ex)

3	7	4	5
---	---	---	---

1	8	9	2
---	---	---	---

 $N=8$

3	7	4	5
---	---	---	---

1	8	2	9
---	---	---	---

3	4	5	7
---	---	---	---

1	2	8	9
---	---	---	---

1 2 3 4 5 7 8 9

* $\log N$ steps

* N comparisons needed per step

⇒ $O(N \log N)$

→ Disadvantage: need $\times 2$ of memory space to write the array data in each step.

2. Heap Sort

* Min Heap (\Leftrightarrow Max Heap) (a.k.a.) Heap Tree

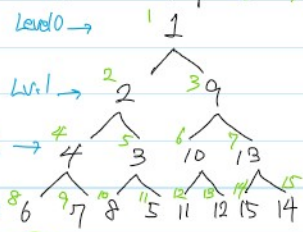
: Binary Tree. Root is minimum (\Leftrightarrow maximum)

: Every case \rightarrow Parent Node \leq Child Node (\geq)

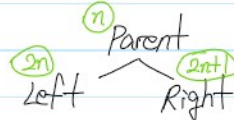
* Min-Heap Array example:

number that stores idx of end of min-heap $\{ \boxed{1}, 2, 9, 4, 3, 10, 13, 6, 7, 8, 5, 11, 12, 15, 14 \}$

ex) Min Heap (idx)



* Relationship between Parent & Child Nodes:
idx of parent = idx of child / 2



* Heap Sort: with an array given, $\left\{ \begin{array}{l} \textcircled{1} \text{ make it a min (or max) heap array} \\ \textcircled{2} \text{ sort the heap array made in } \textcircled{1} \end{array} \right.$

①: insertion algorithm

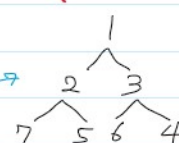
ex) initial array: $\{ 1, 7, 5, 6, 2, 1, 4, 3 \}$

min-heap completed until here.

- Compare the newly included number and its parent ($= \text{self}/2$)
- If the number is smaller, swap them and repeat until it goes to root.
- Increment $\text{arr}[0]$.

arr = $\{ 7, 1, 2, 3, 7, 5, 6, 4 \}$

min-heap completed!



* time complexity: $\log 1 + \log 2 + \log 3 + \log 4 + \dots + \log N$
 $= \log(N!) \leq \log N^N = N \log N$

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②: deletion algorithm

arr = {7, 1, 2, 3, 7, 5, 6, 4} → min-heap array
↑

- First element (root) is minimum. Swap it with last element of heap tree. Decrement arr[0] → last place is now not part of the heap tree.
 - When last element became the root, check if it's smaller than child nodes.
 - should be smaller or equal to min(left child, right child)
 - if not, swap it with min(left, right). And repeat this.
 - The back side of array, which is not part of the heap, is the sorted results. (in descending order)
- * time complexity: $\log(N!) \leq \log N^N = \underline{N \log N}$

∴ Time Complexity of Heap Sort: $N \log N + N \log N = \underline{2N \log N} \Rightarrow O(N \log N)$
 * But doesn't require additional memory space! → x2 longer than merge sort