UNIT 2: SETS, SET THEORY, TRUTH TABLES AND LOGIC

Exercises

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Unit 2: Sets, Set Theory, Truth Tables and Logic

Exercise 1

1. Given the following sets:

$$A = \{a, b, c, 2, 3, 4\} \qquad E = \{a, b, \{c\}\}$$
$$B = \{a, b\} \qquad F = \emptyset$$
$$C = \{c, 2\} \qquad G = \{\{a, b\}, \{c, 2\}\}$$
$$D = \{b, c\}$$

classify each of the following statements as true or false

(a)	$c \in A$	(g)	$D \subset A$	(m)	$B \subseteq G$
(b)	$c \in F$	(h)	$A \subseteq C$	(n)	$\{B\} \subseteq G$
(c)	$c \in E$	(i)	$D \subseteq E$	(o)	$D \subseteq G$
(d)	$\{c\} \in E$	(j)	$F \subseteq A$	(p)	$\{D\} \subseteq G$
(e)	$\{c\} \in C$	(k)	$E \subseteq F$	(q)	$G \subseteq A$
(f)	$B \subseteq A$	(l)	$B \in G$	(r)	$\{\{c\}\} \subseteq E$

Answers to Exercise 1

(a)	c ∈ A → TRUE	A = {a, b, c,2,3,4} c is in A
(b)	c ∈ F → FALSE	F = Ø (Empty set)
(c)	c ∈ E → FALSE	E = {a,b,{c}} We have set {c} not c directly
(d)	{c} ∈ E → TRUE	E = {a,b,{c}} We have set {c}
(e)	{c} ∈ C → FALSE	C = {c, 2} We have set {c} not c directly
(f)	B ⊆ A → TRUE	B = {a,b} A ={a,b,c, 2,3,4} All the elements of B are in A
(g)	D ⊂ A → TRUE	D = {b,c} A ={a,b,c, 2,3,4} All the elements of D are in A
(h)	A ⊆ C→ FALSE	A = {a,b,c,2,3,4} and C = {c, 2} A is not subset of C. All elements of A are not in C

(i)	D ⊆ E → FALSE	D = {b,c} and E = {a,b,{c}} D is not subset of E. All elements of D are in E.
(j)	F ⊆ A → TRUE	F = ∅ and A = {a,b,c,2,3,4} Empty set is a subset of every set.
(k)	E ⊆ F → FALSE	E = {a,b,{c}} and F = ∅ E is not an empty set and F is empty set
(l)	B ∈ G → TRUE	B = {a, b} and G={{a, b},{c,2}} {a,b} is there in G
(m)	B ⊆ G → FALSE	B = {a,b} and G={{a,b},{c,2}} Since G has sets and not individual elements
(n)	{B} ⊆ G → TRUE	B = {a,b} and G={{a,b},{c,2}} Since {B} is there in G
(o)	D ⊆ G → FALSE	D = {b,c}and G ={{a,b},{c,2}} The elements of D are not directly in G
(p)	{D} ⊆ G →FALSE	D = {b,c}and G ={{a,b},{c,2}} The elements of D are not directly in G
(q)	G ⊆ A → FALSE	G ={{a,b},{c,2}} and A ={a,b,c,2,3,4} G has sets and A has individual elements
(r)	{{c}} ⊆ E→ TRUE	E = {a,b,{c}} Since {c} is there in E

Exercise 4

Consider the following sets:

$$\begin{aligned} S1 &= \{\{\emptyset\},\{A\},A\} & S6 &= \emptyset \\ S2 &= A & S7 &= \{\emptyset\} \\ S3 &= \{A\} & S8 &= \{\{\emptyset\}\} \\ S4 &= \{\{A\}\} & S9 &= \{\emptyset,\{\emptyset\}\} \\ S5 &= \{\{A\},A\} \end{aligned}$$

Answer the following questions Remember that the members of a set are the items separated by commas, if there is more than one, between the outermost braces only; a subset is formed by enclosing within braces zero or more of the members of a given set, separated by commas.

(a) Of the sets $S1$ - $S9$ which are members of $S1$?

(b) which are subsets of $S1$?

(c) which are members of $S9$?

(d) which are subsets of $S9$?

(e) which are members of $S4$?

(f) which are subsets of $S4$?

Answers to Exercise 4

(a)	Of the sets S1-S9 which are members of S1	S3, S4, S8, S7
(b)	Which are subsets of S1	S1, S3, S4, S5, S6, S8
(c)	Which are members of S9?	S6, S7, S8
(d)	Which are subsets of S9 ?	S6, S7, S8
(e)	Which are members of S4 ?	S6
(f)	Which are subsets of S4 ?	S6

Exercise 2

i. For each clause (a) - (f) below, create truth tables for each to answer the question of when each statement is false.
   a. ~ P
   b. P ∧ Q
   c. P ∨ Q
   d. P → Q
   e. P ←→ Q
   f. P → (~ Q)

P	Q	~ P	P ∧ Q	P ∨ Q	P → Q	P ←→ Q	P → (~ Q)
T	T	F	T	T	T	T	F
T	F	F	F	T	F	F	T
F	T	T	F	T	T	F	F
F	F	T	F	F	T	T	T

ii. Consider the statement (~ Q) -> (~ P).
   a. When is it false?

The implication **(~Q) → (~P)** is **false only** when:

1. **~Q is True** (meaning Q is False)
2. **~P is False** (meaning P is True)

Thus, **(~Q) → (~P) is false when P is True and Q is False.**

   b. Now consider P → Q. When is it false?

The implication **P → Q** is **false only** when:

1. **P is True**
2. **Q is False**

   c. Do you believe these two compound statements mean the same thing?

(~Q) → (~P) is logically equivalent to P → Q because they yield the same truth table and can be transformed into one another using logical rules.

   d. Construct the truth table for the statement (~ Q) −> (~ P). Then revisit your answer to (c).

Truth table for (~Q) → (~P)

P	Q	~ Q	~ P	(~ Q) -> (~ P)
T	T	F	F	T
T	F	T	F	F
F	T	F	T	T
F	F	T	T	T

Truth table for P →Q

P	Q	P → Q
T	T	T
T	F	F
F	T	T
F	F	T

Since both truth tables are **identical**, this confirms that the two statements **(~Q) → (~P) and P → Q are logically equivalent**.

iii. Construct the truth table for P XOR Q.

P	Q	P XOR Q
T	T	F
T	F	T
F	T	T
F	F	F

iv. Construct the truth table for the below

a. ~ (P ∧ Q)

P	Q	P ∧ Q	~(P ∧ Q)
T	T	T	F
T	F	F	T
F	T	F	T
F	F	F	T

b. P ∨ (Q ∧ R)

P	Q	R	Q ∧ R	P ∨ (Q ∧ R)
T	T	T	T	T

T	T	F	F	T
T	F	T	F	T
T	F	F	F	T
F	T	T	T	T
F	T	F	F	F
F	F	T	F	F
F	F	F	F	F

c. P ∨ (Q ∨ R)

P	Q	R	Q ∨ R	P ∨ (Q ∨ R)
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	F	T
F	T	T	T	T
F	T	F	T	T
F	F	T	T	T
F	F	F	F	F

d. (P ∨Q) ∨ R

P	Q	R	P ∨ Q	(P ∨ Q) ∨ R
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	T	T
F	T	T	T	T
F	T	F	T	T
F	F	T	F	T
F	F	F	F	F

e. (P → Q) ∧ (Q → P)

P	Q	P →Q	Q→P	P → Q) ∧ (Q → P)
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T