Bluetooth MAC Spoofing System

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Abstract

1. Introduction

2. How to measure Speed using two BMS

We know that speed is

$$speed = \frac{\text{distance covered}}{\text{time taken}} \tag{1}$$

From Figure [1] it can be seen that distance covered $=$ distance between point A & point B whereas time taken $=$ time taken to travel from point A to point B. By putting the values of **distance covered** and **time taken** from Figure[1] in equation[1] we get

$$speed = \frac{\hat{d} - R_2 + R_1}{(\hat{t_2} - T_2) - (\hat{t_1} - T_1)} \tag{2}$$

whereas, $\hat{d}$ is distance between two sites. R_1 & R_2 is range of site 1 and site 2, respectively. $\hat{t_1}$ & $\hat{t_2}$ is Timestamp of detection of first packet at site 1 and site 2, respectively. T_1 & T_2 is time till first detection after entering into the range of site 1 and site 2, respectively.

For a single pass of a vehicle from site 1 to site 2, $\hat{d}$, $\hat{t_1}$ and $\hat{t_2}$ are constants. Whereas, R_1, R_2, and T_1, T_2 are random variables. Hence we need to compute the Expected value of speed.

$$E[speed] = E\left[\frac{\hat{d} - R_2 + R_1}{(\hat{t_2} - T_2) - (\hat{t_1} - T_1)}\right] \tag{3}$$

$$var[speed] = var\left[\frac{\hat{d} - R_2 + R_1}{(\hat{t_2} - T_2) - (\hat{t_1} - T_1)}\right] \tag{4}$$

Equation [5] and [6] can be written as

$$E[Z] = E\left[\frac{X}{Y}\right] \tag{5}$$

and

$$var[Z] = var\left[\frac{X}{Y}\right] \tag{6}$$

whereas $X = \hat{d} - R_2 + R_1$, $Y = (\hat{t_2} - T_2) - (\hat{t_1} - T_1)$ and $Z = speed$.

2.1. Computing E[Z] and VAR[Z]

If random variable Y is an arbitrary non-linear function of two random variables X_1, X_2

$$Y = g(X_1, X_2) \tag{7}$$

Then Taylor Series Expansion of Y around mean, μ_1, μ_2, will be

$$\begin{aligned} Y &= g(\mu_1, \mu_2) \\ &+ (X_1 - \mu_1)\frac{\mathrm{d}g}{\mathrm{d}X_1}\bigg|_{\mu_1,\mu_2} + (X_2 - \mu_2)\frac{\mathrm{d}g}{\mathrm{d}X_2}\bigg|_{\mu_1,\mu_2} + \\ &+ \frac{(X_1 - \mu_1)^2}{2!}\frac{\mathrm{d}^2 g}{\mathrm{d}X_1^2}\bigg|_{\mu_1,\mu_2} + \frac{(X_2 - \mu_2)^2}{2!}\frac{\mathrm{d}^2 g}{\mathrm{d}X_2^2}\bigg|_{\mu_1,\mu_2} \\ &+ (X_1 - \mu_1)(X_2 - \mu_2)\frac{\partial^2 g}{\partial X_1 \partial X_2}\bigg|_{\mu_1,\mu_2} + ... \end{aligned} \tag{8}$$

If, $g(X, Y) = \frac{X}{Y}$ then,

$$g(\mu_X, \mu_Y) = \frac{\mu_X}{\mu_Y} \tag{9a}$$

$$\frac{\partial^1 g}{\partial X^1}\bigg|_{\mu_X,\mu_Y} = \frac{1}{\mu_Y} \tag{9b}$$

$$\frac{\partial^1 g}{\partial Y^1}\bigg|_{\mu_X,\mu_Y} = \frac{-\mu_X}{\mu_Y^2} \tag{9c}$$

$$\frac{\partial^2 g}{\partial X^2}\bigg|_{\mu_X,\mu_Y} = 0 \tag{9d}$$

$$\frac{\partial^2 g}{\partial Y^2}\bigg|_{\mu_X,\mu_Y} = \frac{2\mu_X}{\mu_Y^3} \tag{9e}$$

$$\frac{\partial^2 g}{\partial X \partial Y}\bigg|_{\mu_X,\mu_Y} = \frac{-1}{\mu_Y^2} \tag{9f}$$

By putting the values from equation 9 into 8

$$g(X,Y) = \frac{\mu_X}{\mu_Y} + \frac{1}{\mu_Y}(X-\mu_X) - \frac{\mu_X}{\mu_Y^2}(Y-\mu_Y) + \frac{\mu_X}{\mu_Y^3}(Y-\mu_Y)^2 - \frac{1}{\mu_Y^2}(X-\mu_X)(Y-\mu_Y) + ... \tag{10}$$

Second order approximation of $E[g(X,Y)]$ is,

$$E\left[\frac{X}{Y}\right] \approx \frac{\mu_X}{\mu_Y} + \frac{\mu_X}{\mu_Y^3} var[Y] - \frac{1}{\mu_Y^2} cov[X,Y] \tag{11}$$

Similarly, first-order approximation of $var[g(X,Y)]$ is,

$$var\left[\frac{X}{Y}\right] \approx \frac{var[X]}{\mu_Y^2} - \frac{2\mu_X}{\mu_Y^3} cov[X,Y] + \frac{\mu_X^2}{\mu_Y^4} var[Y] \tag{12}$$

$$E[X] = \hat{d} - E[R_2] + E[R_1] \tag{13}$$

$$E[Y] = (\hat{t_2} - \hat{t_1}) - (E[T_2] - E[T_1]) \tag{14}$$

$$var[X] = var[R_1] + var[R_2] \tag{15}$$

$$var[Y] = var[T_1] + var[T_2] \tag{16}$$

$$cov[X,Y] = cov[R_1,T_1] + cov[R_2,T_2] \tag{17}$$

2.2. Computing *COV[RANGE, TTDD]*

This is a citation [1] and it is showing up. But I cannot add page numbers. one wonders why?

References

[1] H. Benaroya and S. M. Han. *Probability Models in Engineering and Science*. CRC Press, Taylor and Francis Group, 2005. Refer to page 168 for derivation of mean and variance of a general function of N RVs. 2