Epistemic Game Theory

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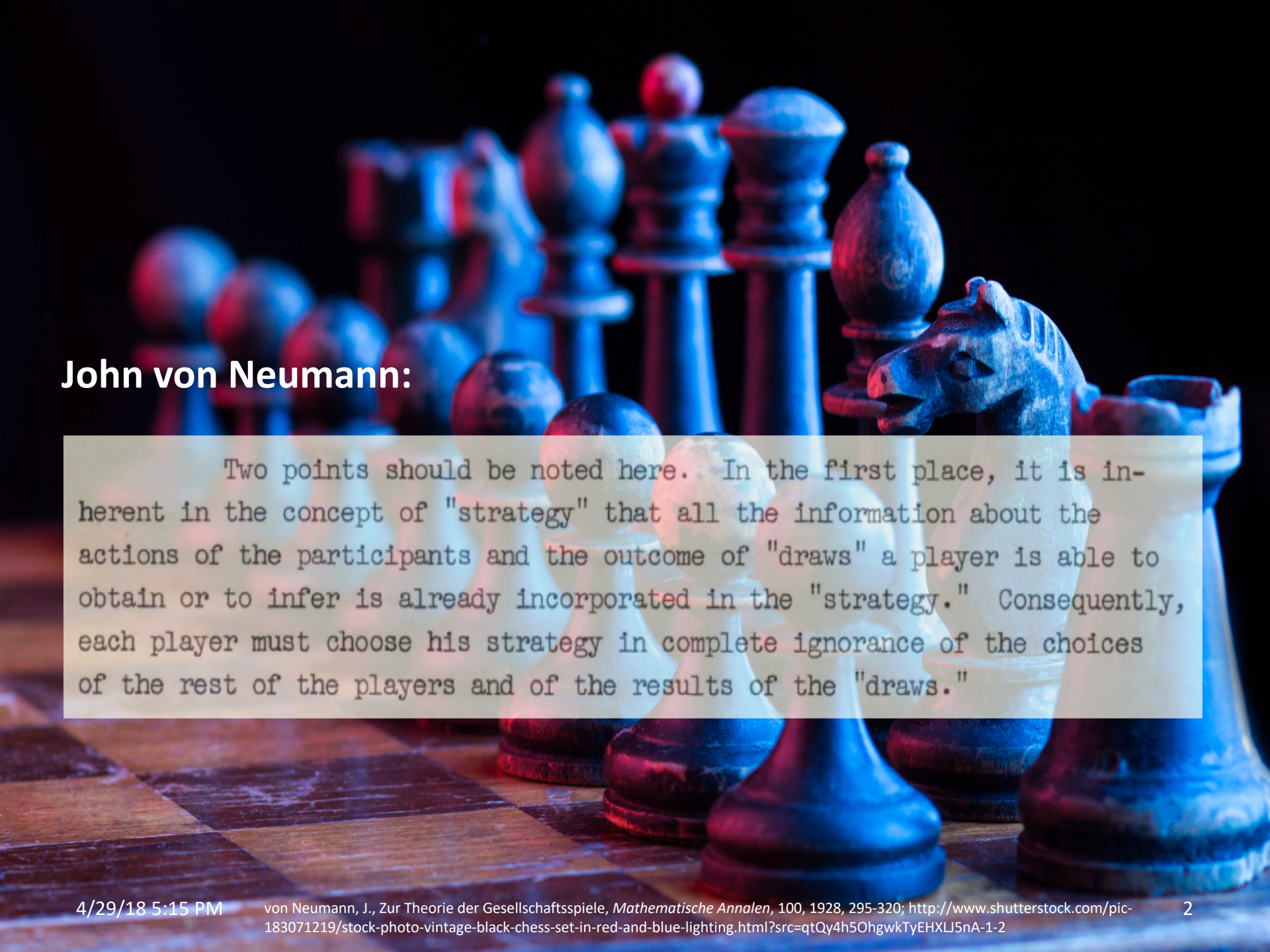


John von Neumann:

Two points should be noted here. In the first place, it is inherent in the concept of "strategy" that all the information about the actions of the participants and the outcome of "draws" a player is able to obtain or to infer is already incorporated in the "strategy." Consequently, each player must choose his strategy in complete ignorance of the choices of the rest of the players and of the results of the "draws."

von Neumann, J., Zur Theorie der Gesellschaftsspiele, *Mathematische Annalen*, 100, 1928, 295-320; http://www.shutterstock.com/pic-183071219/stock-photo-vintage-black-chess-set-in-red-and-blue-lighting.html?src=qtQy4h5OhgwkTyEHXLJ5nA-1-2

John Nash:

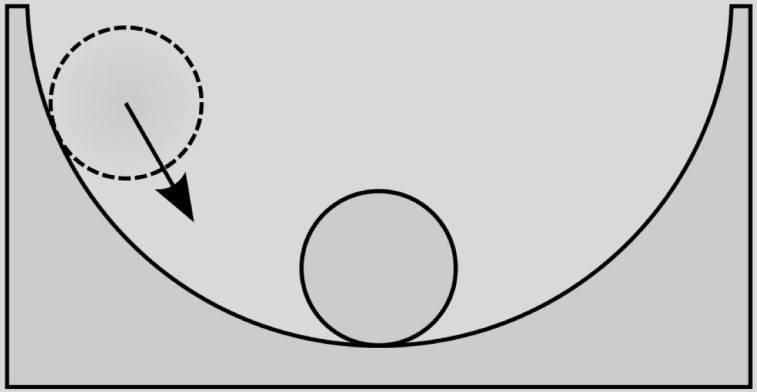
We proceed by investigating the questions what would be a "rational" prediction of the behavior to be expected of rational playing the game in question? By using the principles that a rational prediction should be unique, that the players should be able to deduce and make use of it, and that such knowledge on the part of each player of what to expect the others to do should not lead him to act out of conformity with the prediction, one is led to the concept of a solution defined before.

Nash, J., "Non-Cooperative Games," doctoral dissertation, Princeton University, 1950; http://www.shutterstock.com/pic-183071219/stock-photo-vintage-black-chess-set-in-red-and-blue-lighting.html?src=qtQy4h5OhgwkTyEHXLJ5nA-1-2

Nash assumed that each player has access to (knows) the actual strategy choices of the other players

Or: Each player knows the conjectures of the other players, knows that the other players know, and so on (Aumann and Brandenburger, 1995)

Perhaps, equilibrium can arise as an outcome of learning



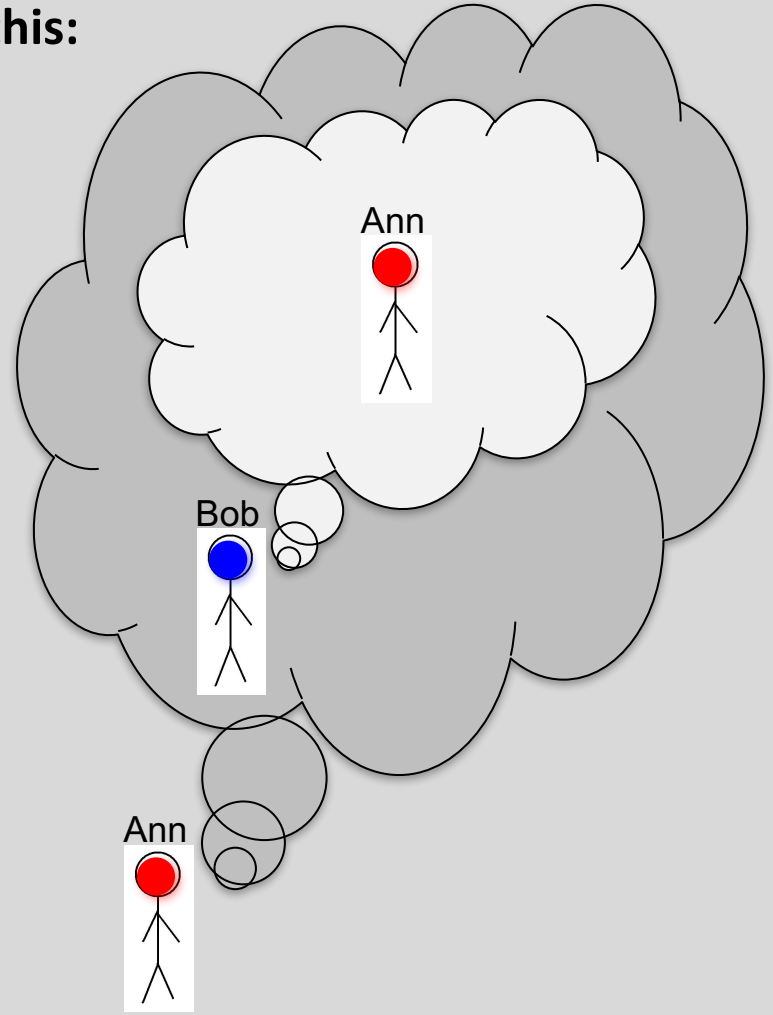
But, it seems unrealistic as an outcome of thinking

Counter-evidence to equilibrium came quickly --- see Flood (1958)

Aumann, R., and A. Brandenburger, "Epistemic Conditions for Nash Equilibrium," *Econometrica*, 63, 1995, 1161-1180; Flood, M., "Some Experimental Games," *Management Science*, 5, 1958, 5-26; https://commons.wikimedia.org/wiki/File:Stable_equilibrium.svg

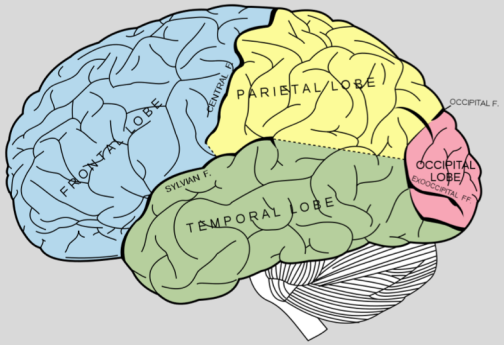
Let's build game theory making realistic information assumptions

We get a picture like this:

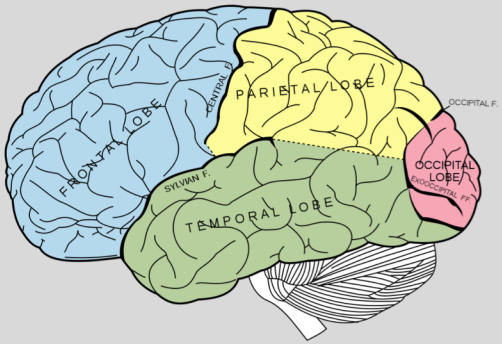


Do people actually think this way (or only in puzzle books)?

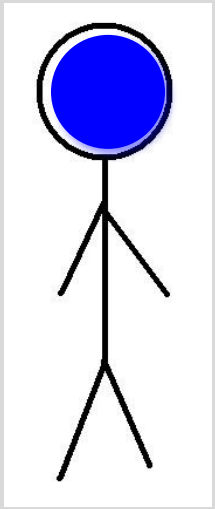
Yes, this is the cognitive process called Theory of Mind



Gallagher, H. et al., "Reading the Mind in Cartoons and Stories: An fMRI Study of 'Theory of Mind' in Verbal and Nonverbal Tasks," *Neuropsychologia*, 38, 2000, 11-21; http://upload.wikimedia.org/wikipedia/commons/thumb/1/1a/Gray728.svg/1024px-Gray728.svg.png



or



McCabe, K. et al., "A Functional Imaging Study of Cooperation in Two-Person Reciprocal Exchange," *PNAS*, 98, 2001, 11832-11835; Gallagher, H., et al., "Imaging the Intentional Stance in a Competitive Game," *NeuroImage*, 16, 2002, 814-821; Rilling, J., et al., "The Neural Correlates of Theory of Mind Within Interpersonal Interactions," *NeuroImage*, 22, 2004, 1694-1703;
http://upload.wikimedia.org/wikipedia/commons/thumb/1/1a/Gray728.svg/1024px-Gray728.svg.png

So, let's build **a model of beliefs, beliefs about beliefs, and so on** (Mertens and Zamir, 1985; Brandenburger and Dekel, 1993; and others)

Then do game theory using this model (call it **"epistemic game theory"**)

What do we get?

Maybe we get **iterated-dominance procedures** of various kinds (which go back to Gale, 1953) ...

Mertens, J-F., and S. Zamir, "Formulation of Bayesian Analysis for Games with Incomplete Information," *International Journal of Game Theory*, 14, 1985, 1-29; Brandenburger, A., and E. Dekel, "Hierarchies of Beliefs and Common Knowledge," *Journal of Economic Theory*, 59, 1993, 189-198; Gale, D., "A Theory of N-Person Games with Perfect Information," *Proceedings of the National Academy of Sciences*, 39, 1953, 496-501

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Iterated dominance may involve **correlation** in a player's beliefs about other players' strategies

	L	*R*
U	1, 1, 3	1, 0, 3
D	0, 1, 0	0, 0, 0

X

	L	*R*
U	1, 1, 2	0, 0, 0
D	0, 0, 0	1, 1, 2

Y

	L	*R*
U	1, 1, 0	1, 0, 0
D	0, 1, 3	0, 0, 3

Z

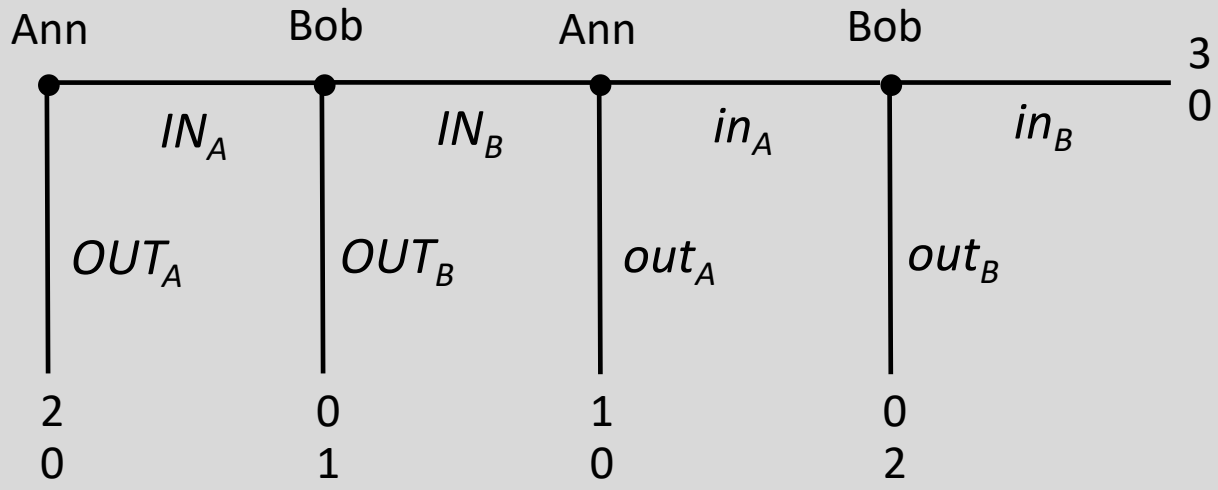
The strategy *Y* is optimal for Charlie if she puts probability ½ : ½ on (*U*, *L*) : (*D*, *R*)

There are no independent probabilities under which *Y* is optimal

But, surprise!

Iterated dominance may involve too much correlation --- i.e., correlation which cannot be explained via epistemic variables (Brandenburger and Friedenberg, 2008)

In the tree, the first guess is that levels of reasoning, formalized via epistemic game theory, yield **backward induction**



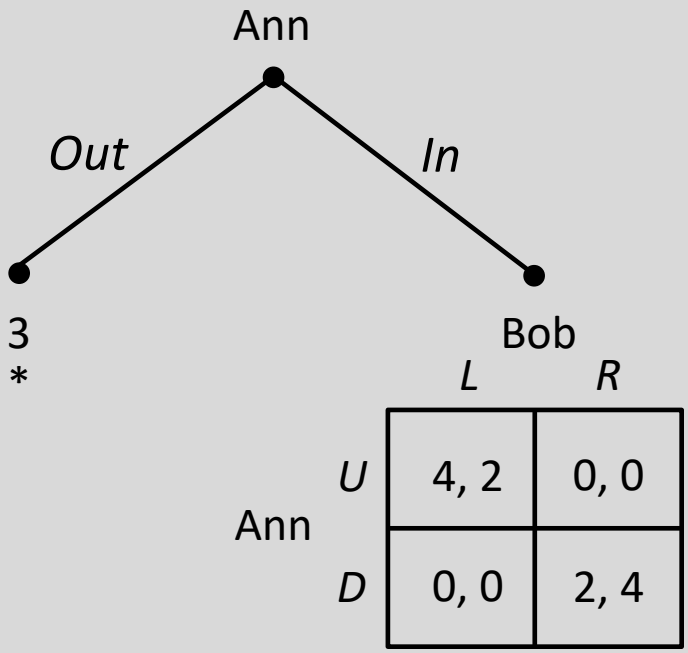
This is false, as this tree (Reny, 1992) indicates!

Backward induction preserves IN_A - out_A until the fourth round

But IN_A - out_A is dominated by OUT_A on the first round

Reny, P., "Backward Induction, Normal Form Perfection, and Explicable Equilibria," *Econometrica*, 60, 1992, 627-649

The second guess is an iterated-dominance procedure tailored to the tree, called **extensive-form rationalizability** (Pearce, 1984; Battigalli and Sinischalchi, 2002)

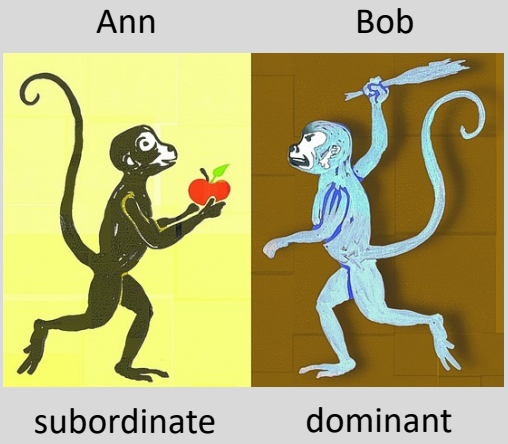


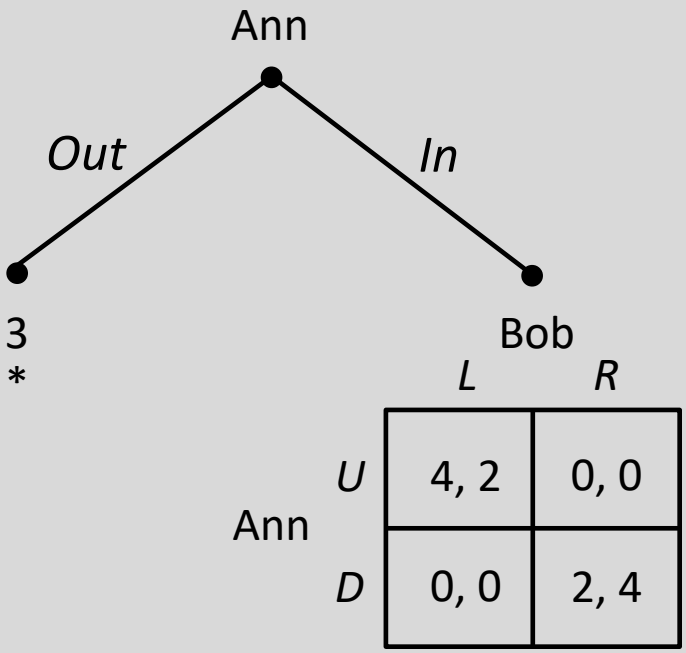
Round 1: Delete *In - D*
Round 2: Delete *R*
Round 3: Delete *Out*

Pearce, D., "Rational Strategic Behavior and the Problem of Perfection," *Econometrica*, 52, 1984, 1029-1050;
Battigalli, P., and M. Siniscalchi, "Strong Belief and Forward-Induction Reasoning," *Journal of Economic Theory*, 106, 2002, 356-391

But suppose it is believed (technically, common full belief) that Bob is a 'bully' --- i.e., that he will play *R*

Then, it is consistent with iterated-rationality assumptions for Ann to play *Out*

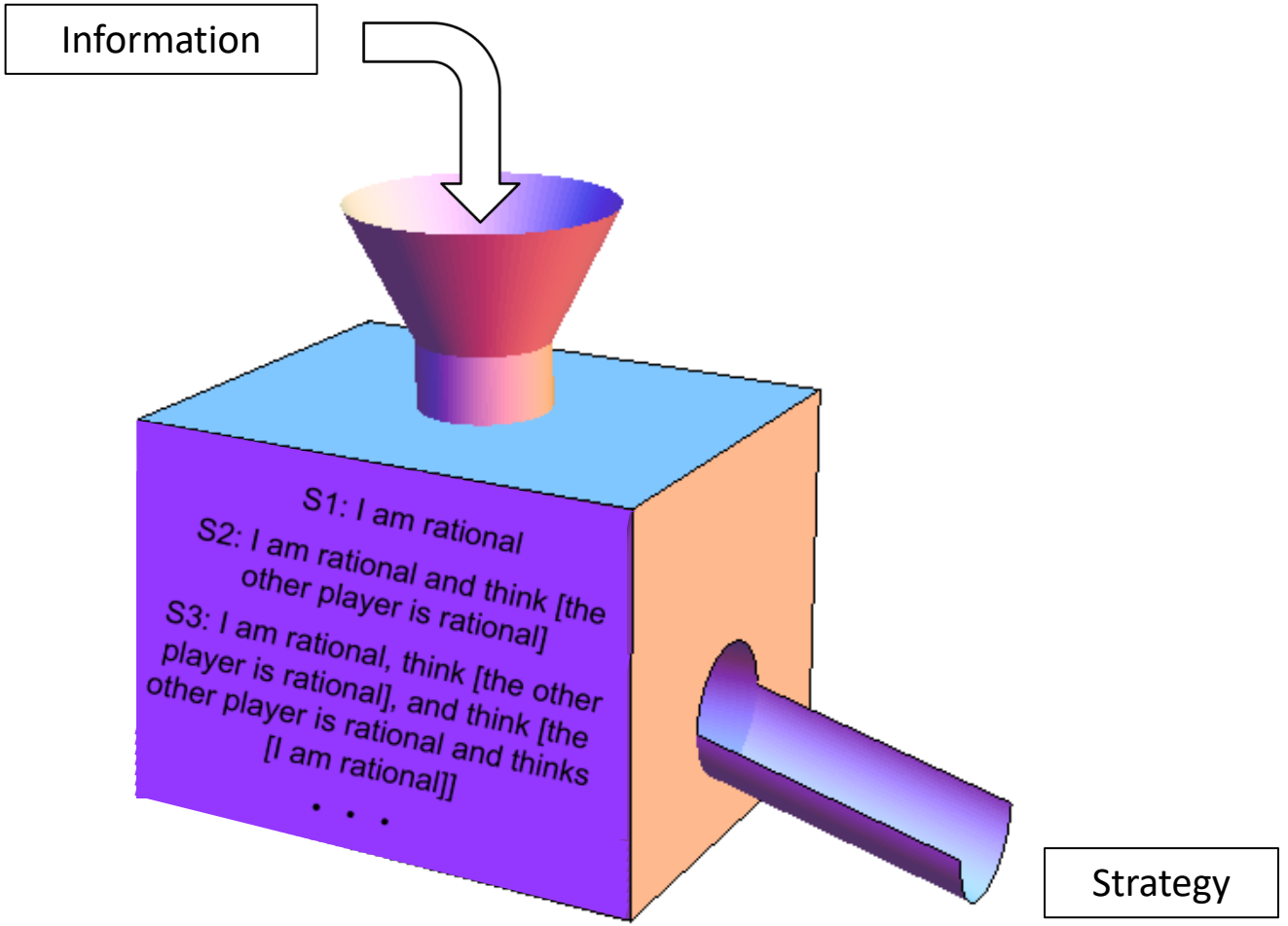




For a characterization of the strategies that can be played under ***m*-iterated rationality assumptions** in the tree (for different *m*), see Brandenburger, Danieli, and Friedenberg (2017)

Brandenburger, A., A. Danieli, and A. Friedenberg, "How Many Levels Do Players Reason? An Observational Challenge and Solution," 2017; https://si.wsj.net/public/resources/images/OB-SR153_ridley_G_20120420181525.jpg

What is the prediction of unlimited levels of reasoning?



Battigalli and Siniscalchi op. cit.; Brandenburger, A.,A. Friedenberg, and H.J. Keisler, "Admissibility in Games," *Econometrica*, 76, 2008, 307-352; http://mathinsight.org/exponential_function; http://www.shutterstock.com/pic-103764212/stock-vector-cartoon-question-and-exclamation-mark.html

It is not true that

T1: I am rational

T2: I am rational and think [the other player is rational]

T3: I am rational, think [the other player is rational], and think [the other player is rational and thinks [I am rational]]

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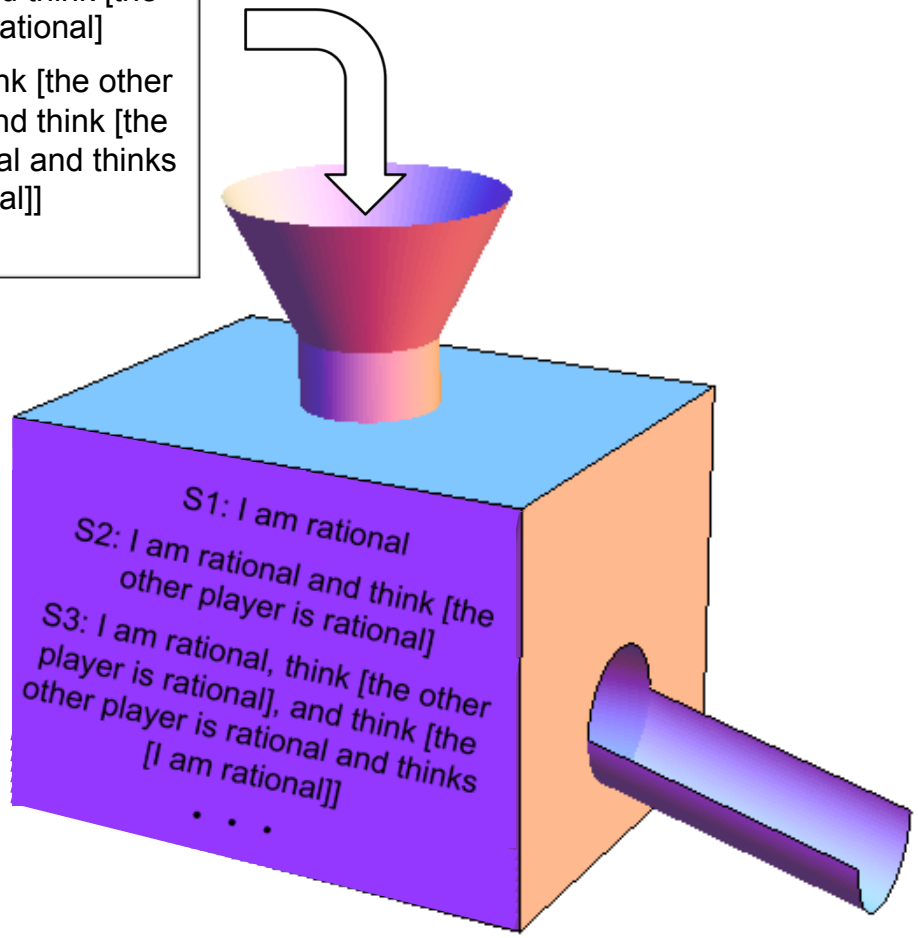

Strategy

Battigalli and Siniscalchi op. cit.; Brandenburger, A.,A. Friedenberg, and H.J. Keisler, "Admissibility in Games," *Econometrica*, 76, 2008, 307-352; http://mathinsight.org/exponential_function; http://www.shutterstock.com/pic-103764212/stock-vector-cartoon-question-and-exclamation-mark.html

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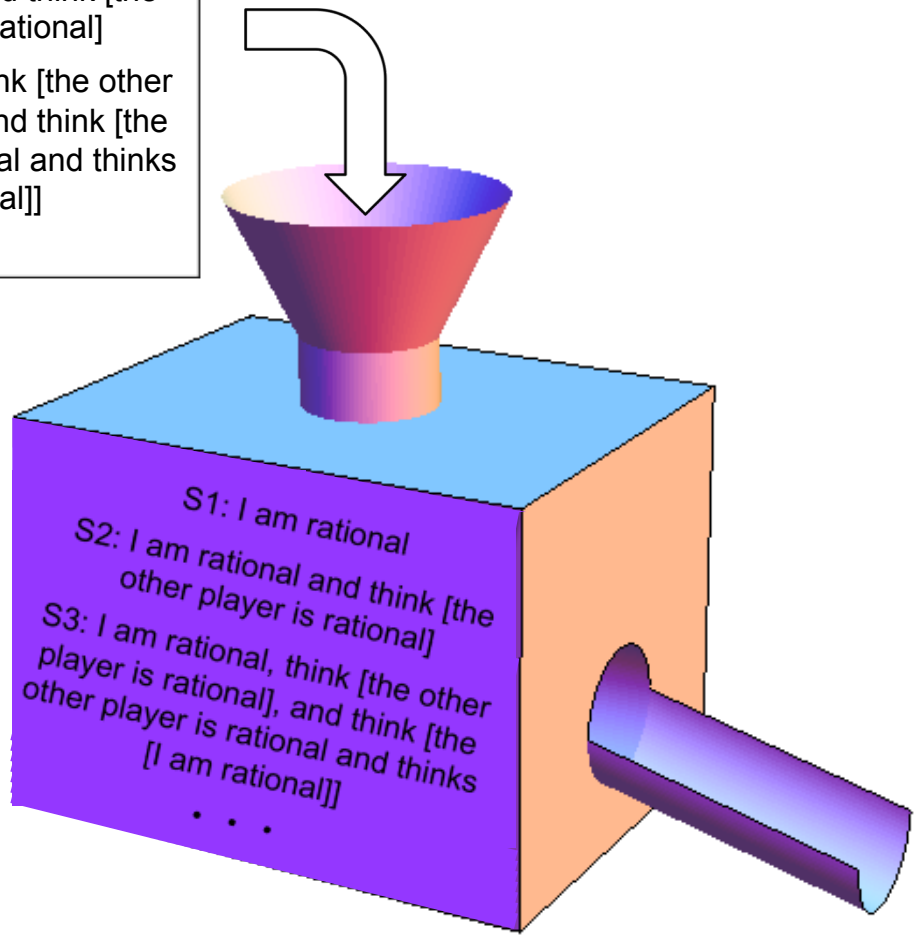

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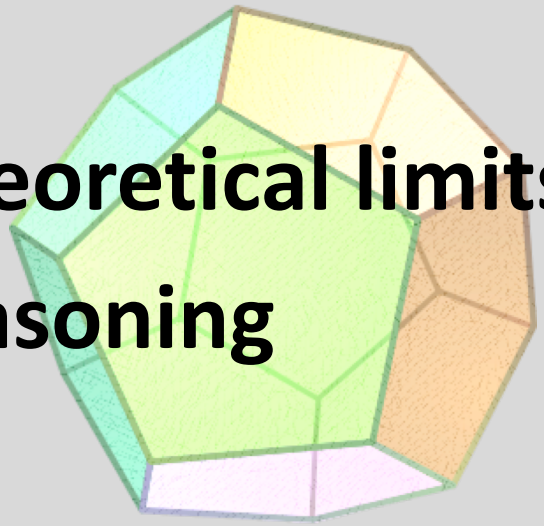
Theoretical limits to reasoning



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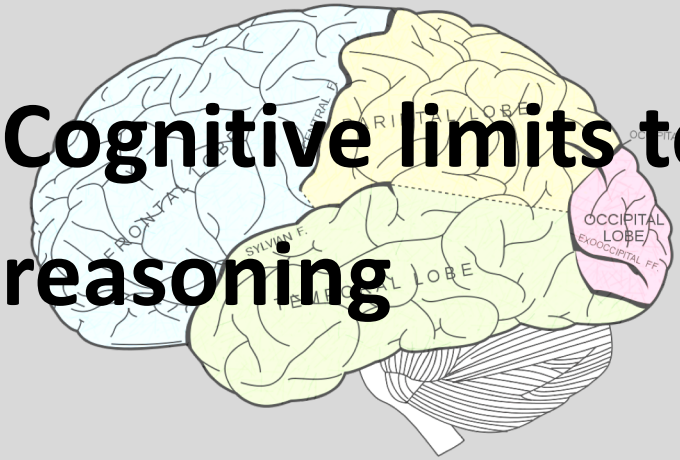
Cognitive limits to reasoning



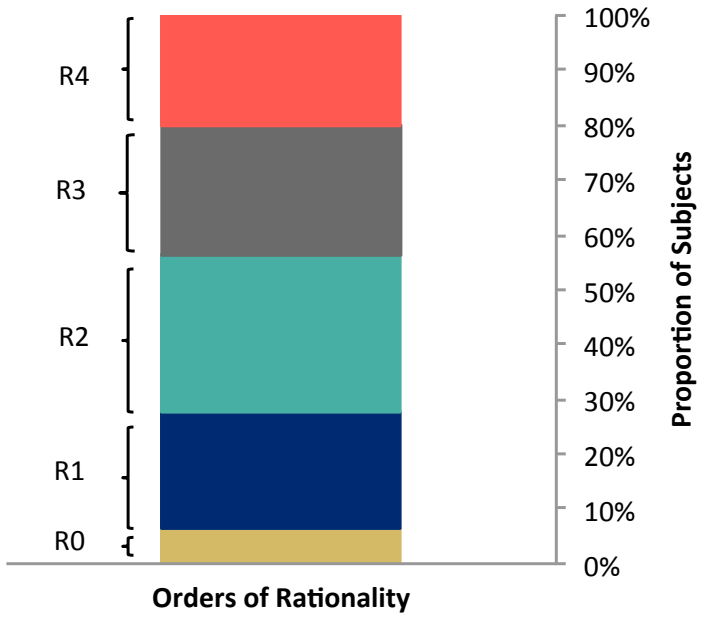
Shh! ... Platonic rationality in games does not even exist!

http://en.wikipedia.org/wiki/Platonic_solid#mediaviewer/File:POV-Ray-Dodecahedron.svg
http://upload.wikimedia.org/wikipedia/commons/thumb/1/1a/Gray728.svg/1024px-Gray728.svg.png

Where does this leave game theory?

We need more data on cognition in games

Kneeland (2015) is a breakthrough experimental design for estimating levels of reasoning in games



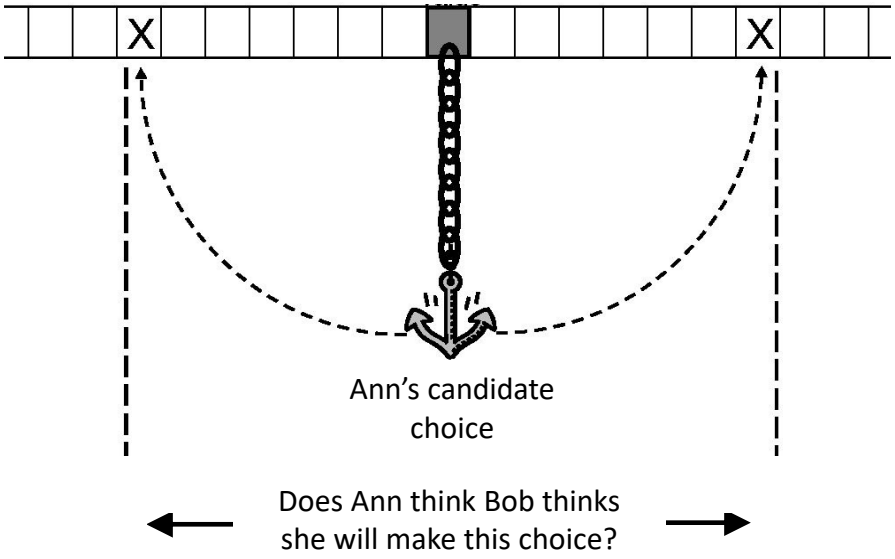
6 percent of subjects engaged in 0th-order reasoning

23 percent ...	1st-order ...
27 percent ...	2nd-order ...
22 percent ...	3rd-order ...
22 percent ...	4th-order ...

 Kneeland, T., "Identifying Higher-Order Rationality," *Econometrica*, 83, 2015, 2065-2079; diagram used with permission from Terri Kneeland

We need more theory on cognition in games

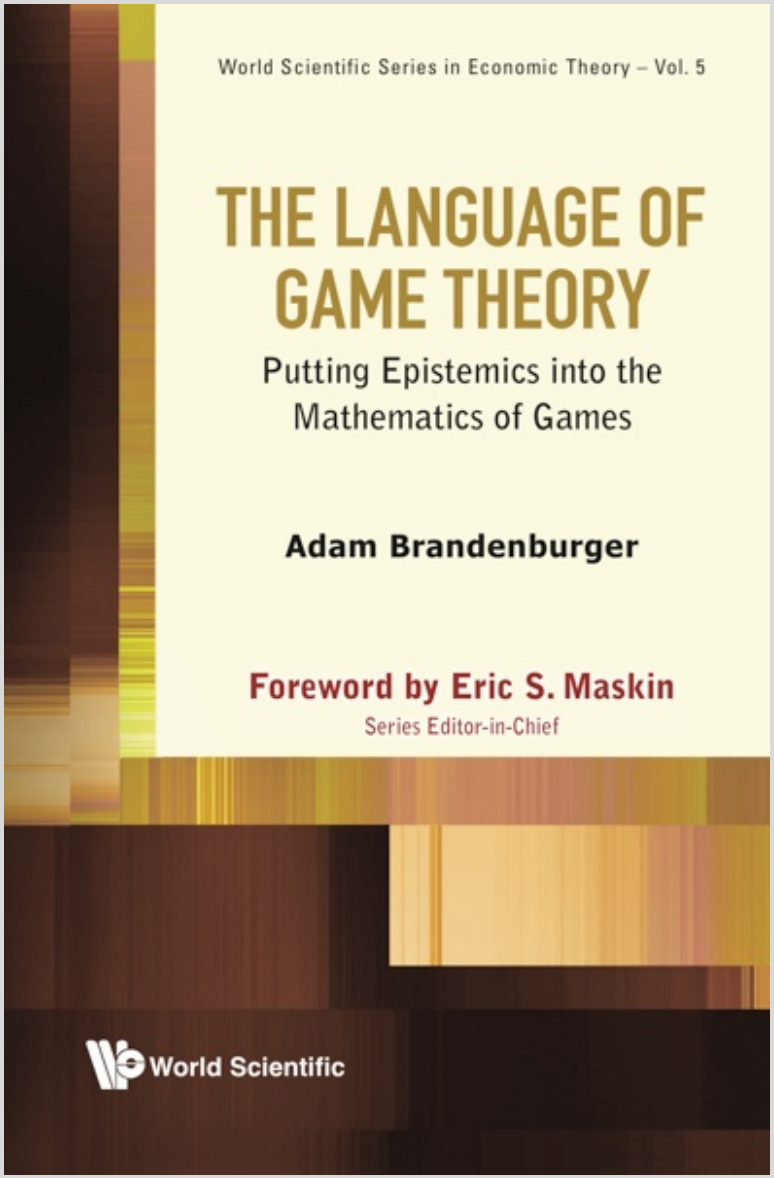
Neural data* suggest that forming hierarchies of beliefs may be an anchoring-and-adjustment process



This can be thought of as an internal equilibrium vs. disequilibrium process

It allows a simple case-counting argument for the complexity of higher-order beliefs (Brandenburger and Li, 2015)

* Bhatt, M., and C. Camerer, "Self-Referential Thinking and Equilibrium as States of Mind in Games: fMRI Evidence," *Games and Economic Behavior*, 52, 2005, 424-459; Brandenburger, A., and X. Li, "Thinking About Thinking and Its Cognitive Limits," 2015; diagram from http://www.gpmfirst.com/books/understanding-and-managing-risk-attitude/individual-risk-attitudes-and-heuristics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THE LANGUAGE OF GAME THEORY
Putting Epistemics into the Mathematics of Games
Adam Brandenburger
Foreword by Eric S. Maskin
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