

6/26/14 20:36

# Epistemic Game Theory

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## Game Theory at Princeton

[http://upload.wikimedia.org/wikipedia/commons/0/04/Nassau\\_Hall\\_Princeton.JPG](http://upload.wikimedia.org/wikipedia/commons/0/04/Nassau_Hall_Princeton.JPG)  
<http://www.awesomesories.com/asset/view/John-Nash-Photo-as-a-Young-Man>  
<http://www.vmes.org/sites/classics/classics5.xhtml>



**John von Neumann:** “Each player must choose his strategy in ‘complete ignorance’”

**John Nash:** “[A] rational prediction should be unique”

## Von Neumann's General-Sum Theory

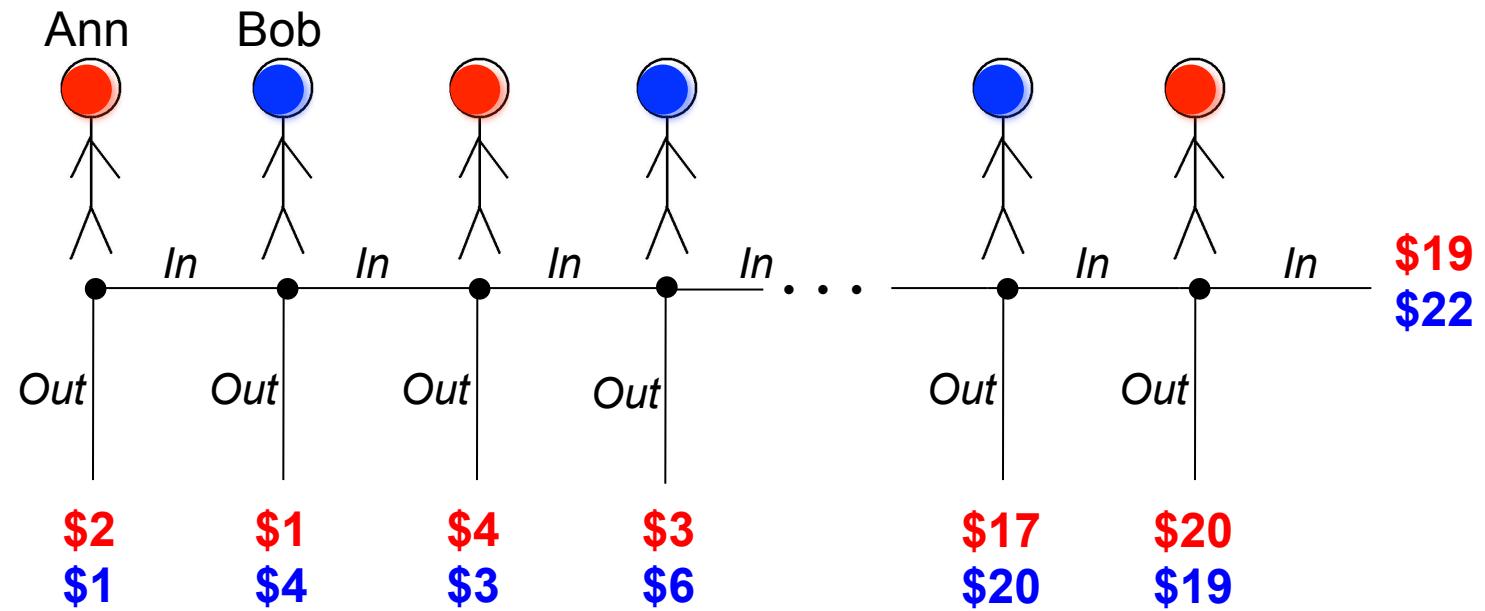
**57.5.2.** The domain of all characteristic functions (of all general  $n$ -person games) was characterized by the conditions (57:2:a), (57:2:c), which we restate:

$$(57:2:a) \quad v(\emptyset) = 0,$$

$$(57:2:c) \quad v(S \cup T) \geq v(S) + v(T) \quad \text{for} \quad S \cap T = \emptyset.$$

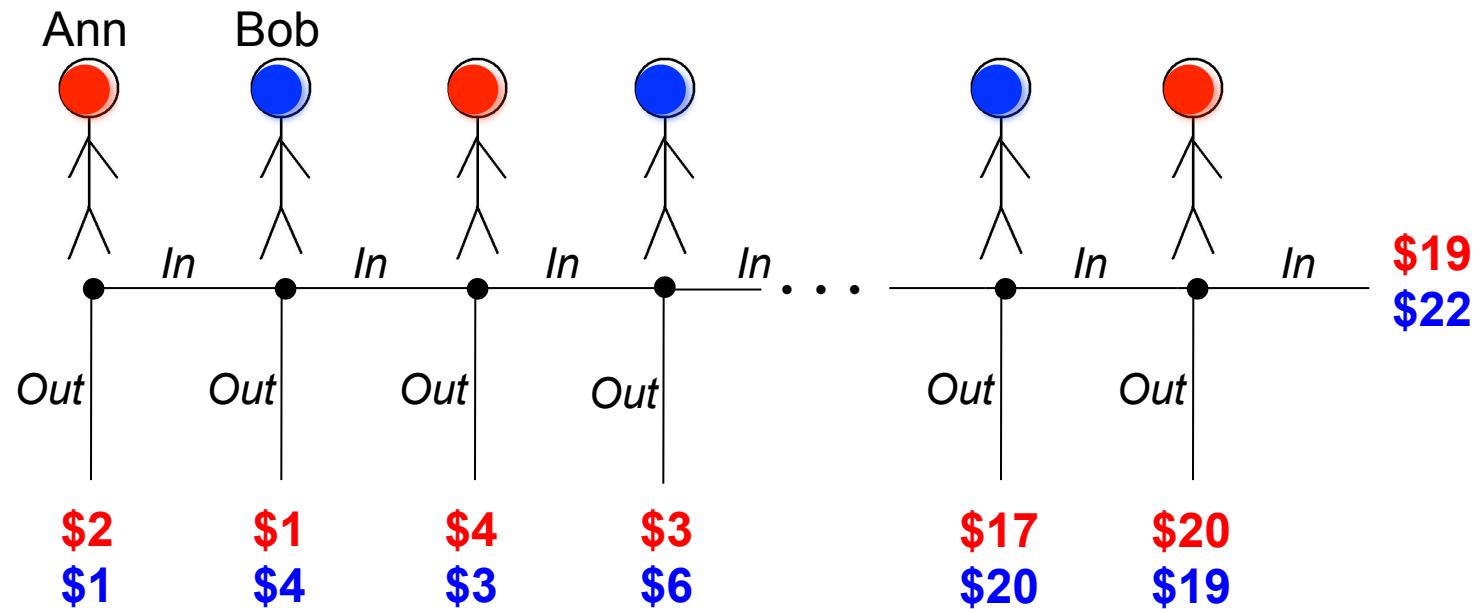
Among these the characteristic functions of zero-sum games and of constant-sum games form two special classes.

## Centipede



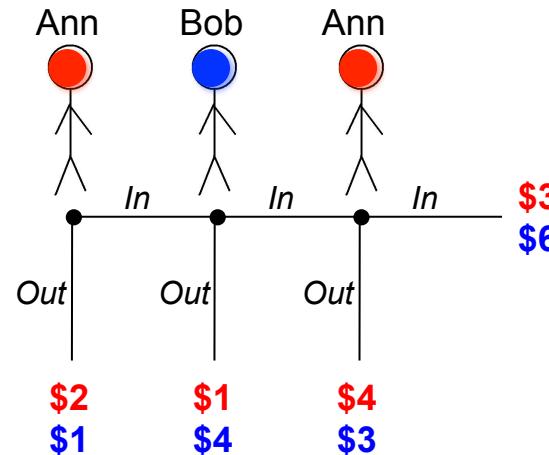
Rosenthal, R., "Games of Perfect Information, Predatory Pricing, and the Chain Store," *Journal of Economic Theory*,  
1980, 25, 92-100  
[http://commons.wikimedia.org/wiki/File:Stick\\_Figure.jpg](http://commons.wikimedia.org/wiki/File:Stick_Figure.jpg)

## Centipede



In any (pure or mixed) Nash equilibrium of Centipede, Ann chooses *Out* immediately

## Centipede: Epistemic Possibilities



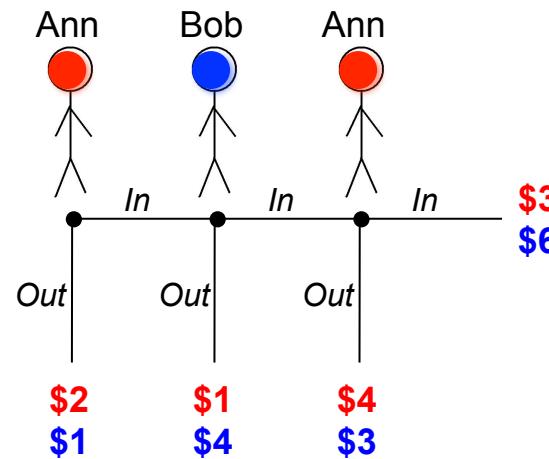
Ann plays *In*

She plays *In* because she thinks Bob thinks she will play *Out*, and that he will be surprised when she plays *In* and, perhaps, will think she will play *In* again

So, Ann plays *In* and plans then to play *Out*

Of course, Bob might play *Out*, in which case Ann will be surprised!

## Centipede: Formal Epistemics



Type  $t^a$

$T^b$	$u^b$	0	0
$t^b$		0	1
		<i>Out</i>	<i>In</i>
		$S^b$	

Type  $u^a$

$T^b$	$u^b$	1 [0]	0 [0]
$t^b$		0 [0]	0 [1]
		<i>Out</i>	<i>In</i>
		$S^b$	

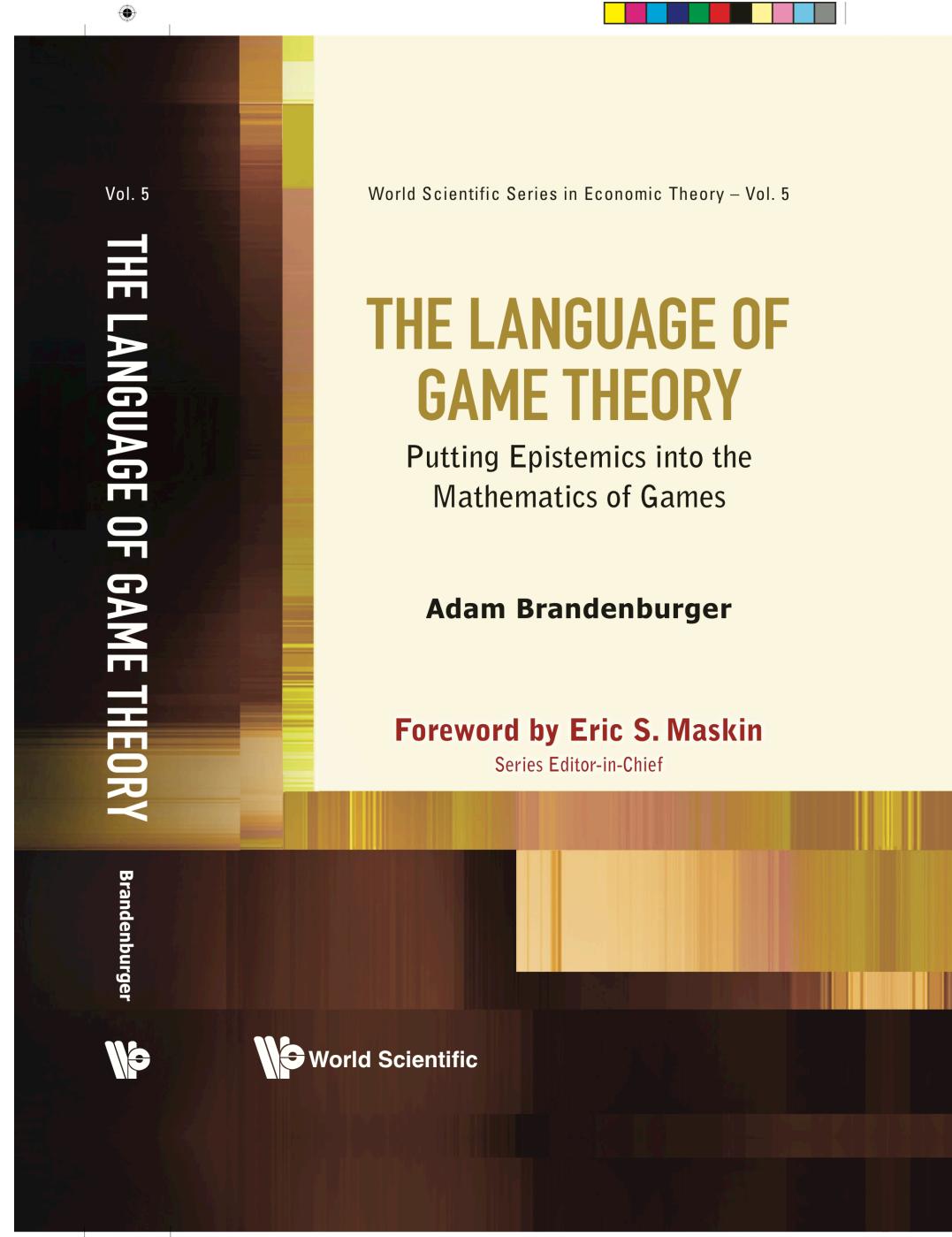
Type  $t^b$

$T^a$	$u^a$	1 [0]	0 [0]	0 [0]
$t^a$		0 [0]	0 [0]	0 [1]
		<i>Out</i>	<i>In-Out</i>	<i>In-In</i>
		$S^a$		

Type  $u^b$

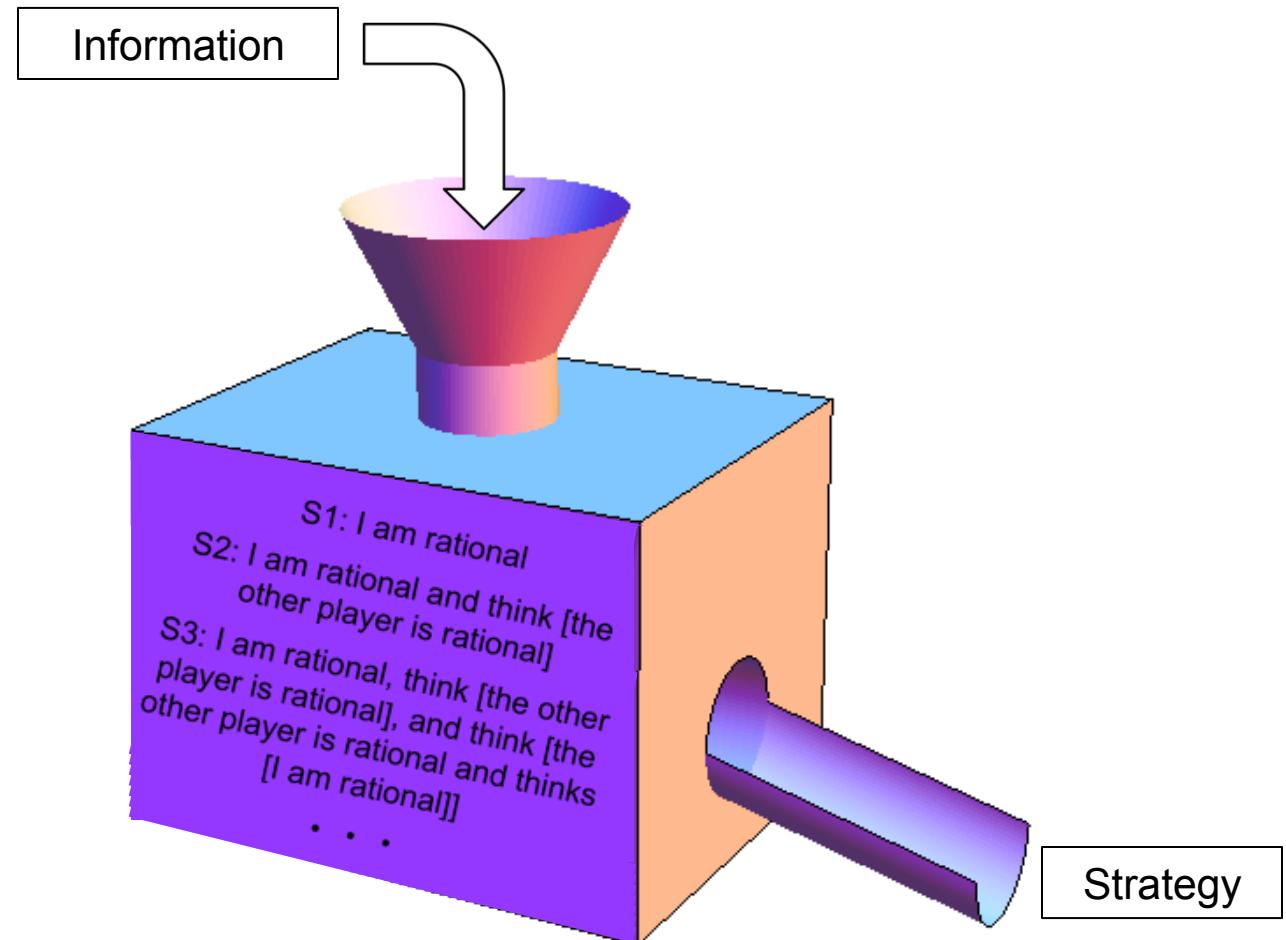
$T^a$	$u^a$	1 [0]	0 [0]	0 [0]
$t^a$		0 [0]	0 [1]	0 [0]
		<i>Out</i>	<i>In-Out</i>	<i>In-In</i>
		$S^a$		

Description Not  
Prediction



# An Ultimate Game Player?

Note: “Think” is short for “strongly believes”



# An Ultimate Game Player?

It is not true that

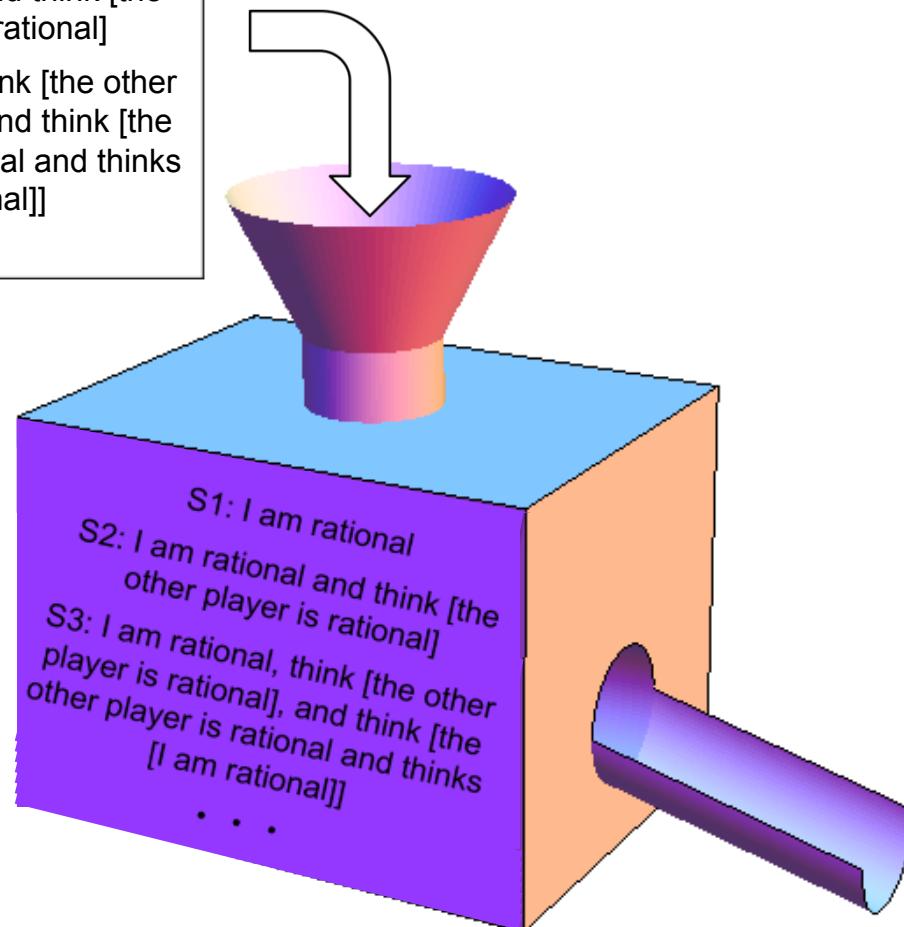
T1: I am rational

T2: I am rational and think [the other player is rational]

T3: I am rational, think [the other player is rational], and think [the other player is rational and thinks [I am rational]]

• • •

Note: “Think” is short for “strongly believes”



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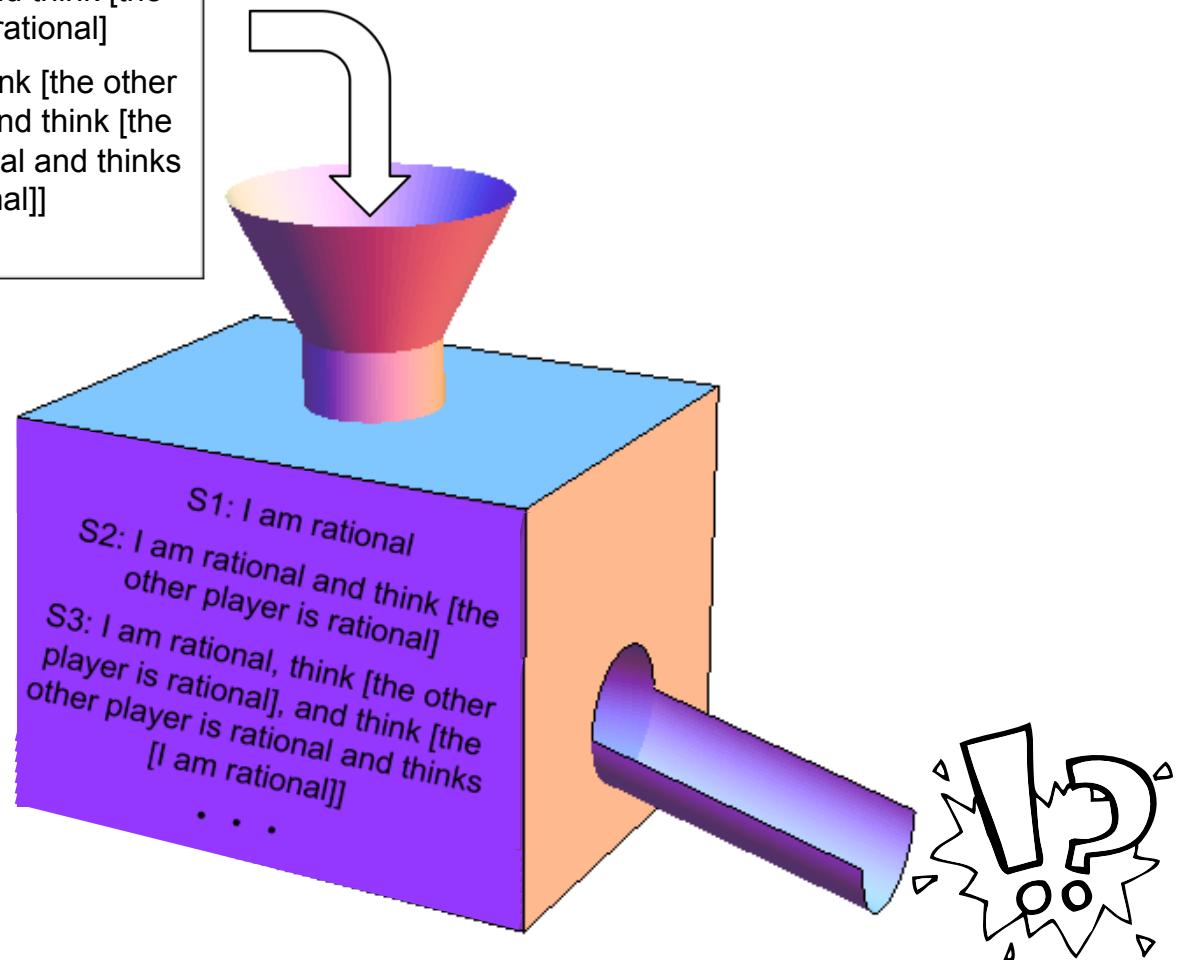
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Brandenburger, A., A. Friedenberg, And H.J. Keisler, "Admissibility in Games," *Econometrica*, 76, 2008, 307-352  
 Battigalli, P., and M. Siniscalchi, "Strong Belief and Forward-Induction Reasoning," *Journal of Economic Theory*, 106, 356-391

[http://mathinsight.org/exponential\\_function](http://mathinsight.org/exponential_function)

<http://www.shutterstock.com/pic-103764212/stock-vector-cartoon-question-and-exclamation-mark.html>

**From Sherlock  
to Mycroft**



## Experimental Findings in Kneeland (2013)

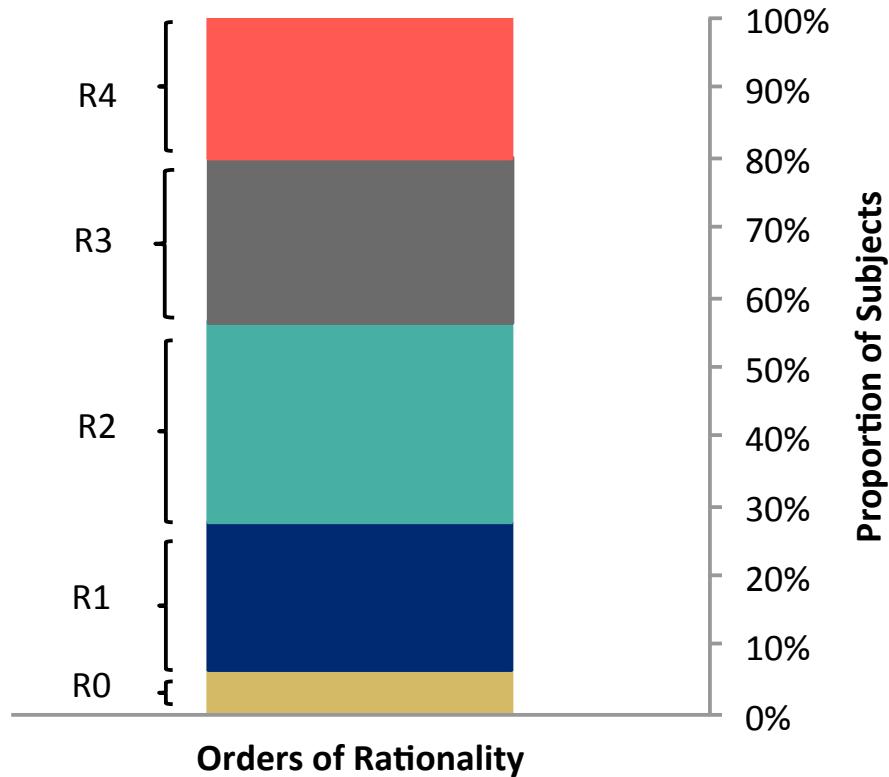


Diagram from Kneeland, T., "Testing Behavioral Game Theory: Higher-Order Rationality and Consistent Beliefs," 2013, terri.microeconomics.ca; used with permission

94 percent of the subjects engaged in first-order reasoning, 72 percent in second-order reasoning, 45 percent in third-order reasoning, and 20 percent in fourth-order reasoning.

## Some Questions

14

Empirical:

What happens if we put data such as these into epistemic game theory and use the resulting models in economics?

Theoretical:

What happens if we allow “intrinsic” correlation in games via correlation across players’ beliefs?

... and many more!