CSP

June 21, 2021

[14]:
```
%matplotlib inline
from pathlib import Path
import numpy as np
import matplotlib.pyplot as plt
import subprocess
import pprint
import operator
import math
import copy
import json
import time
```

1 Utilities

[32]:
```
maximum_history_recording = 10000
class NumpyEncoder(json.JSONEncoder):
    """ Special json encoder for numpy types """
    def default(self, obj):
        if isinstance(obj, np.integer):
            return int(obj)
        elif isinstance(obj, np.floating):
            return float(obj)
        elif isinstance(obj, np.ndarray):
            return obj.tolist()
        return json.JSONEncoder.default(self, obj)


# printing result more prettier that 'pprint'!
def print_results(results):
    dumped = json.dumps(results, cls=NumpyEncoder)
    pprint.pprint(json.loads(dumped), width=50)


# visualiser
def draw_config(config, save=False, path=None ,file_name=None, show=True,␣
↪dpi=100):
    matrix    = config['map']
```

```python
lists     = matrix.tolist()
domains   = config['domains']
size = len(matrix)

fig, ax = plt.subplots(figsize=(size, size))
kwargs = {
    's'          : 1000,
    'c'          : '#bdc3c7',
    'marker'     : "s",
    'alpha'      : 0.5,
}

zeros = []
ones  = []
nans  = []
total = []

ax.set_xlim(-1, +1 * size)
ax.set_ylim(-1, +1 * size)


for row_idx, row in enumerate(lists):
    for col_idx, col in enumerate(row):
        cords = (row_idx, col_idx)
        if np.isnan(matrix[cords]):
            nans.append(cords)
        elif matrix[cords] == 0:
            zeros.append(cords)
        elif matrix[cords] == 1:
            ones.append(cords)
        total.append(cords)

zero_only_domain = []
one_only_domain = []
full_domain = []

for cell in nans:
    if len(domains[cell]) == 1 and 1 in domains[cell]:
        one_only_domain.append(cell)
    elif len(domains[cell]) == 1 and 0 in domains[cell]:
        zero_only_domain.append(cell)
    else:
        full_domain.append(cell)


kwargs['c'] = '#e74c3c'
```

```
    ax.scatter([cord[1] for cord in zero_only_domain], [cord[0] for cord in␣
↪zero_only_domain], **kwargs)

    kwargs['c'] = '#27ae60'
    ax.scatter([cord[1] for cord in one_only_domain], [cord[0] for cord in␣
↪one_only_domain], **kwargs)

    kwargs['c'] = '#b1a467'
    ax.scatter([cord[1] for cord in full_domain], [cord[0] for cord in␣
↪full_domain], **kwargs)


    kwargs['c'] = '#bdc3c7'
    kwargs['marker'] = 's'
    ax.scatter([cord[1] for cord in ones], [cord[0] for cord in ones], **kwargs)
    kwargs['c'] = '#27ae60'
    kwargs['marker'] = '$1$'
    ax.scatter([cord[1] for cord in ones], [cord[0] for cord in ones], **kwargs)


    kwargs['c'] = '#bdc3c7'
    kwargs['marker'] = 's'
    ax.scatter([cord[1] for cord in zeros], [cord[0] for cord in zeros],␣
↪**kwargs)
    kwargs['c'] = '#e74c3c'
    kwargs['marker'] = '$0$'
    ax.scatter([cord[1] for cord in zeros], [cord[0] for cord in zeros],␣
↪**kwargs)


    ax.invert_yaxis()
    if show:
        plt.show()
    if save:
        if not path:
            path = "../default_export_path/"
        Path(path).mkdir(parents=True, exist_ok=True)
        fig.savefig(f'{path}{file_name}', dpi=dpi)
        plt.close(fig)


# reports results and draw start and final states
def report(configurations, results, initial):
    assert len(results) == len(initial) , 'please provide valied inputs'
    for i in range(len(results)):
        print(f'#problem number {i} ------------------------')
        print(f'@successful: {results[i]["result"]["successful"]}')
```

```python
        print(f'@elapsed time: {round(results[i]["result"]["time"], 3)}␣
↪seconds')
        print(f'@start state:')
        s = len(configurations[i]['map'])
        mat = Matrix(s)
        mat.clear_populate(initial[i]['assignment'])
        print('--graphical')
        draw_config({'map': mat.matrix, 'domains': initial[i]['domains']})
        print('--textual')
        pprint.pprint(mat.matrix.tolist())
        print('\n')

        if results[i]["result"]["successful"]:
            print(f'@result:')
            print('--graphical')
            draw_config({'map': results[i]['result']['final'], 'domains':␣
↪results[i]['domains']})
            print('--textual')
            print_results(results[i]["result"]["final"])
            print('\n')

        else:
            print(f'@result:')
            print('NO solution found...')


# export each step image to make video from
def export_images(history, output_dir):
    print(f'exporting images in {output_dir}')
    for problem_index, problem_records in enumerate(history):
        print(f'exporting images for problem {problem_index}', end='')
        start = time.time()

        # too many images!
        if isinstance(problem_records, list):
            if len(problem_records) > maximum_history_recording:
                print(f', too many images [{len(problem_records)}] => aborted')
                continue
        else:
            print(f', too many images [{(problem_records)}] => aborted')
            continue

        counter = 0
        for index, record in enumerate(problem_records):

            path = f'{output_dir}/{problem_index}/'
            Path(path).mkdir(parents=True, exist_ok=True)
```

```
            try:

                size = len(configurations[problem_index]['map'])
                mat = Matrix(size)
                mat.clear_populate(record['assignment'])

                draw_config({'map': mat.matrix, 'domains': record['domains']},␣
 ↪save=True, path=path,
                            file_name=f'{index + 1:04d}.png', show=False,␣
 ↪dpi=70)
                counter += 1
            except Exception as e:
                print(e)

        end = time.time()
        print(f' ,elapsed time: {end - start}, images: {counter}')


def make_animation(frame_rates, base_dir):
    for problem_index, rate in enumerate(frame_rates):
        if rate:
            print(f'making animation for problem {problem_index}')

            input_path = f'../images/{base_dir}/{problem_index}'
            output_path = f'../animations/{base_dir}'
            file_name = f'animation-{problem_index}'

            Path(output_path).mkdir(parents=True, exist_ok=True)
            cmd = ['ffmpeg', '-y', '-framerate', str(rate) , '-i',␣
 ↪f'{input_path}/%04d.png', f'{output_path}/{file_name}.mp4']
            process = subprocess.run(cmd, capture_output=True, text=True)

print("done")
```

```
done
```

1.1 Constraints

[16]:
```
from typing import Generic, TypeVar, Dict, List, Optional, Tuple
from abc import ABC, abstractmethod

V = TypeVar('V') # variable type
D = TypeVar('D') # domain type

# Base class for all constraints
class Constraint(Generic[V, D], ABC):
    # The variables that the constraint is between
```

```python
    def __init__(self, variables: List[V]) -> None:
        self.variables = variables

    # Must be overridden by subclasses
    @abstractmethod
    def satisfied(self, assignment: Dict[V, D]) -> bool:
        ...

# In each column or row, the numbers of zeros and ones must be equal
class FirstConstraint(Constraint[Tuple, int]):
    def __init__(self, variables: List[Tuple], problem_size: int) -> None:
        super().__init__(variables)
        self.ps         = problem_size
        self.matrix     = Matrix(problem_size)

    # checks that the assignment satisfies the constraint or not
    # checks for equality of 'sum of each row and column' to 'numbers of cells␣
 ↪divided by two'
    def satisfied(self, assignment: Dict[Tuple, int]) -> bool:
        self.matrix.clear_populate(assignment)
        matrix = self.matrix.matrix

        for i in range(0, self.ps):
            if not np.isnan(matrix[i,:]).any():
                if np.sum(matrix[i,:]) != self.ps / 2.0:
                    return False

        for i in range(0, self.ps):
            if not np.isnan(matrix[:,i]).any():
                if np.sum(matrix[:,i]) != self.ps / 2.0:
                    return False
        return True


# each column an row must be unique
class SecondConstraint(Constraint[Tuple, int]):
    def __init__(self, variables: List[Tuple], problem_size: int) -> None:
        super().__init__(variables)
        self.ps         = problem_size
        self.matrix     = Matrix(problem_size)

    # checks that the assignment satisfies the constraint or not
    # checks for the uniqueness of each row and column in a pair
    def satisfied(self, assignment: Dict[Tuple, int]) -> bool:
        self.matrix.clear_populate(assignment)
        matrix = self.matrix.matrix
```

```
        for i in range(self.ps):  # generate pairs
            for j in range(i + 1, self.ps):
                if not np.isnan(matrix[i]).any() and np.isnan(matrix[j]).any():
                    if np.array_equal(matrix[i], matrix[j]):  # compare rows
                        return False


        for i in range(self.ps):  # generate pairs
            for j in range(i + 1, self.ps):
                if not np.isnan(matrix[:, i]).any() and np.isnan(matrix[:, j]).
↪any():
                    if np.array_equal(matrix[:,i], matrix[:, j]):  # compare␣
↪columns
                        return False
        return True


# In each column or row, there must not be more than two equal numbers(zero or␣
↪one)
class ThirdConstraint(Constraint[Tuple, int]):
    def __init__(self, variables: List[Tuple], problem_size: int) -> None:
        super().__init__(variables)
        self.ps         = problem_size
        self.matrix     = Matrix(problem_size)

    # checks that the assignment satisfies the constraint or not
    # checks for the triple similarity for each variable in the assignment
    def satisfied(self, assignment: Dict[Tuple, int]) -> bool:
        self.matrix.clear_populate(assignment)
        matrix = self.matrix

        for var in assignment:
            if matrix.left(var) == matrix.matrix[var] == matrix.right(var):
                return False

            if matrix.up(var) == matrix.matrix[var] == matrix.down(var):
                return False
        return True
```

1.2 CSP and solvers implementation

[17]:
```
# A helper class for this specific problem to make calculations easier
# used for representing of each state (via a matrix)
class Matrix():
    def __init__(self, size:int) -> None:
        self.matrix = np.empty((size, size))
```

```python
        self.matrix[:] = np.nan
        self.size = size

    def clear_populate(self, assignment: Dict[V, D]) -> None:
        self.matrix[:] = np.nan
        for variable in assignment:
            self.matrix[variable] = assignment[variable]

    def up(self, variable: V) -> V:
        if variable[0] == 0:
            return None
        else:
            new_variable = tuple(map(operator.add, variable, (-1,  0)))
            return self.matrix[new_variable]

    def down(self, variable: V) -> V:
        if variable[0] == self.size - 1:
            return None
        else:
            new_variable = tuple(map(operator.add, variable, (+1,  0)))
            return self.matrix[new_variable]

    def left(self, variable: V) -> V:
        if variable[1] == 0:
            return None
        else:
            new_variable = tuple(map(operator.add, variable, ( 0, -1)))
            return self.matrix[new_variable]

    def right(self, variable: V) -> V:
        if variable[1] == self.size - 1:
            return None
        else:
            new_variable = tuple(map(operator.add, variable, ( 0, +1)))
            return self.matrix[new_variable]

    def not_populated_adjacents(self, variable: V) -> List[V]:

        s = self.size - 1
        left_var  = tuple(map(operator.add, variable, ( 0, -1))) if variable[1] > 0 else None
        right_var = tuple(map(operator.add, variable, ( 0, +1))) if variable[1] < s else None
        down_var  = tuple(map(operator.add, variable, (+1,  0))) if variable[0] < s else None
        up_var    = tuple(map(operator.add, variable, (-1,  0))) if variable[0] > 0 else None
```

```python
        result = []
        if left_var  is not None and np.isnan(self.matrix[left_var ]): result.append(left_var)
        if right_var is not None and np.isnan(self.matrix[right_var]): result.append(right_var)
        if down_var  is not None and np.isnan(self.matrix[down_var ]): result.append(down_var)
        if up_var    is not None and np.isnan(self.matrix[up_var   ]): result.append(up_var)

        return result

    def adjacents(self, variable: V) -> List[V]:

        s = self.size - 1
        left_var  = tuple(map(operator.add, variable, ( 0, -1))) if variable[1] > 0 else None
        right_var = tuple(map(operator.add, variable, ( 0, +1))) if variable[1] < s else None
        down_var  = tuple(map(operator.add, variable, (+1,  0))) if variable[0] < s else None
        up_var    = tuple(map(operator.add, variable, (-1,  0))) if variable[0] > 0 else None

        result = []
        if left_var  is not None: result.append(left_var)
        if right_var is not None: result.append(right_var)
        if down_var  is not None: result.append(down_var)
        if up_var    is not None: result.append(up_var)

        return result


# A constraint satisfaction problem consists of variables of type V
# that have ranges of values known as domains of type D and constraints
# that determine whether a particular variable's domain selection is valid
class CSP(Generic[V, D]):
    def __init__(self, variables: List[V], domains: Dict[V, List[D]]) -> None:
        self.variables:   List[V]                          = variables # variables to be constrained
        self.domains:     Dict[V, List[D]]                 = domains   # domain of each variable
        self.constraints: Dict[V, List[Constraint[V, D]]] = {}        # constraints of each variable
```

```python
        self.problem_size: int                                  = int(math.
↪sqrt(len(variables)))
        self.matrix:       Matrix                               = Matrix(self.
↪problem_size)
        assert math.isqrt(len(variables)) , "please provide valid size of␣
↪variables"

        # these lists are for visualizing and do not participate in solving␣
↪procedure
        self.backtracking_history     = []
        self.backtracking_FC_history  = []
        self.backtracking_MAC_history = []
        self.history_gathering_time = 0

        # in start each variable has an empty list of constraints
        for variable in self.variables:
            self.constraints[variable] = []
            if variable not in self.domains:
                raise LookupError("Every variable should have a domain assigned␣
↪to it.")


    # Adding constraint to CSP problem
    # each constraint has a list target varibles,
    # we using that for assign 'the constraint' to each variable in that list
    def add_constraint(self, constraint: Constraint[V, D]) -> None:
        for variable in constraint.variables:
            if variable not in self.variables:
                raise LookupError("Variable in constraint not in CSP")
            else:
                self.constraints[variable].append(constraint)


    # Check if the value assignment is consistent by checking all constraints
    # for the given variable against it
    def consistent(self, variable: V, assignment: Dict[V, D]) -> bool:
        for constraint in self.constraints[variable]:
            if not constraint.satisfied(assignment):
                return False
        return True


    # Does 'Forward Checking' for values in 'domains' for adjacents of␣
↪'variable' in 'assignment'
    def forward_checking(self, domains: Dict[V, List[D]], assignment: Dict[V,␣
↪D], variable: Tuple) -> Dict[V, List[D]]:
```

```
        # the representaion of assignmetns in form of a matirx
        # clearing the previous matrix and fill it with new assignments
        self.matrix.clear_populate(assignment)
        matrix = self.matrix # get rid of 'self'

        new_domains = copy.deepcopy(domains)
        queue = matrix.not_populated_adjacents(variable)

        for adj in queue:
            bad_values_in_domain = []
            for value in new_domains[adj]:
                assignment[adj] = value
                if not self.consistent(adj, assignment):
                    bad_values_in_domain.append(value)

                del assignment[adj]

            new_values_in_domain = [d for d in new_domains[adj] if d not in␣
↪bad_values_in_domain]
            if not new_values_in_domain:
                return None

            new_domains[adj] = new_values_in_domain
        return new_domains


    # make variable1 arc consistent with respect to variable2
    # this method changes domains
    def arc_consistent(self, variable1: V, variable2: V, domains: Dict[V,␣
↪List[D]], assignment: Dict[V, D]) -> bool:
        removed = False
        for var1_value in domains[variable1]:
            fake_assignment = copy.deepcopy(assignment)
            fake_assignment[variable1] = var1_value

            found_indefectible_value = False
            for var2_value in domains[variable2]:
                fake_assignment[variable2] = var2_value
                if self.consistent(variable2, fake_assignment):
                    found_indefectible_value = True
                    break

            del fake_assignment[variable2]
            if not found_indefectible_value:
                domains[variable1].remove(var1_value)
                removed = True
```

```
        return removed


    # Gives a queue of paired arcs and makes the first arc arc_consistent to␣
↪the second arc
    # and considers cascading changes to domains
    def AC_3(self, queue: List[Tuple], domains: Dict[V, List[D]], assignment:␣
↪Dict[V, D]) -> bool:

        # the representaion of assignmetns in form of a matirx
        # clearing the previous matrix and fill it with new assignments
        self.matrix.clear_populate(assignment)
        matrix = self.matrix # get rid of 'self'

        while len(queue) > 0:
            pair = queue.pop(0)
            var1 = pair[0] # adj
            var2 = pair[1] # var
            domain_changed = self.arc_consistent(var1, var2, domains,␣
↪assignment)
            if not domain_changed:
                continue
            if len(domains[var1]) == 0:
                return False

            adjacents = matrix.adjacents(var1)
            adjacents.remove(var2)

            for adj in adjacents:
                queue.append((adj, var1))

        return True


    # MAC algorithm
    def MAC(self, domains: Dict[V, List[D]], assignment: Dict[V, D], variable:␣
↪Tuple) -> bool:

        # the representaion of assignmetns in form of a matirx
        # clearing the previous matrix and fill it with new assignments
        self.matrix.clear_populate(assignment)
        matrix = self.matrix # get rid of 'self'

        new_domains = copy.deepcopy(domains)
        adjacents = matrix.not_populated_adjacents(variable)
        queue = [(adj, variable) for adj in adjacents]
```

```
        if self.AC_3(queue, new_domains, assignment):
            return new_domains
        else:
            return None


    # Minimum Remaining Values
    def MRV(self, domain: List[D]) -> int:
        return len(domain)


    # Solver
    # using backtrack algorithm with MRV
    def backtracking_search(self, assignment: Dict[V, D] = {}) -> 
↪Optional[Dict[V, D]]:

        # return condition
        # assignment is complete if every variable is assigned (our base case)
        if len(assignment) == len(self.variables):
            return assignment

        # select a variable for assigning a value to it
        unassigned: List[V] = [v for v in self.variables if v not in assignment]
        unassigned.sort(key=lambda v: (self.MRV(self.domains[v]))) # MRV
        first: V = unassigned[0]

        # select a value from domain of variable
        for value in self.domains[first]:
            local_assignment = assignment.copy()
            local_assignment[first] = value

            copy_start = time.time()
            # recording this assignment and domains for visualization
            if isinstance(self.backtracking_history, list):
                if len(self.backtracking_history) < maximum_history_recording:
                    self.backtracking_history.append({
                        'assignment': copy.deepcopy(local_assignment),
                        'domains': copy.deepcopy(domains),
                    })
                elif len(self.backtracking_history) == 
↪maximum_history_recording:
                    del self.backtracking_history
                    self.backtracking_history = maximum_history_recording
            else:
                self.backtracking_history += 1
            copy_end = time.time()
            self.history_gathering_time += (copy_end - copy_start)
```

```
            if self.consistent(first, local_assignment): # there is no problem␣
↪with this assignment
                result: Optional[Dict[V, D]] = self.
↪backtracking_search(local_assignment)
                # if we're still consistent, we continue
                if result is not None:
                    return result

        return None


    # Solver
    # using backtrack algorithm with FC and MRV
    def backtracking_search_with_FC(self, assignment: Dict[V, D], domains:␣
↪Dict[V, List[D]]) -> Optional[Dict[V, D]]:

        # return condition
        # assignment is complete if every variable is assigned (our base case)
        if len(assignment) == len(self.variables):
            return assignment

        # select a variable for assigning a value to it
        unassigned: List[V] = [v for v in self.variables if v not in assignment]
        unassigned.sort(key=lambda v: (self.MRV(domains[v])))
        first: V = unassigned[0]

        # select a value from domain of variable
        for value in domains[first]:
            local_assignment = assignment.copy()
            local_assignment[first] = value

            # forward checking
            new_domains = self.forward_checking(domains, local_assignment,␣
↪first)
            if not new_domains:
                continue

            # recording this assignment and domains for visualization
            copy_start = time.time()
            self.backtracking_FC_history.append({
                'assignment': copy.deepcopy(local_assignment),
                'domains': copy.deepcopy(new_domains),
            })
            copy_end = time.time()
            self.history_gathering_time += (copy_end - copy_start)
```

```python
            result: Optional[Dict[V, D]] = self.
↪backtracking_search_with_FC(local_assignment, new_domains)
            if result is not None:
                return result

        return None


    # Solver
    # using backtrack algorithm with MAC and MRV
    def backtracking_search_with_MAC(self, assignment: Dict[V, D], domains:␣
↪Dict[V, List[D]]) -> Optional[Dict[V, D]]:

        # return condition
        # assignment is complete if every variable is assigned (our base case)
        if len(assignment) == len(self.variables):
            return assignment

        # select a variable for assigning a value to it
        unassigned: List[V] = [v for v in self.variables if v not in assignment]
        unassigned.sort(key=lambda v: (self.MRV(domains[v])))
        first: V = unassigned[0]

        # select a value from domain of variable
        for value in domains[first]:
            local_assignment = assignment.copy()
            local_assignment[first] = value
            local_domains = copy.deepcopy(domains)
            local_domains[first] = [value]

            # MAC
            new_domains = self.MAC(local_domains, local_assignment, first)
            if not new_domains:
                continue

            # recording this assignment and domains for visualization
            copy_start = time.time()
            self.backtracking_MAC_history.append({
                'assignment': copy.deepcopy(local_assignment),
                'domains': copy.deepcopy(new_domains),
            })
            copy_end = time.time()
            self.history_gathering_time += (copy_end - copy_start)

            result: Optional[Dict[V, D]] = self.
↪backtracking_search_with_MAC(local_assignment, new_domains)
            if result is not None:
```

```
                return result

        return None
```

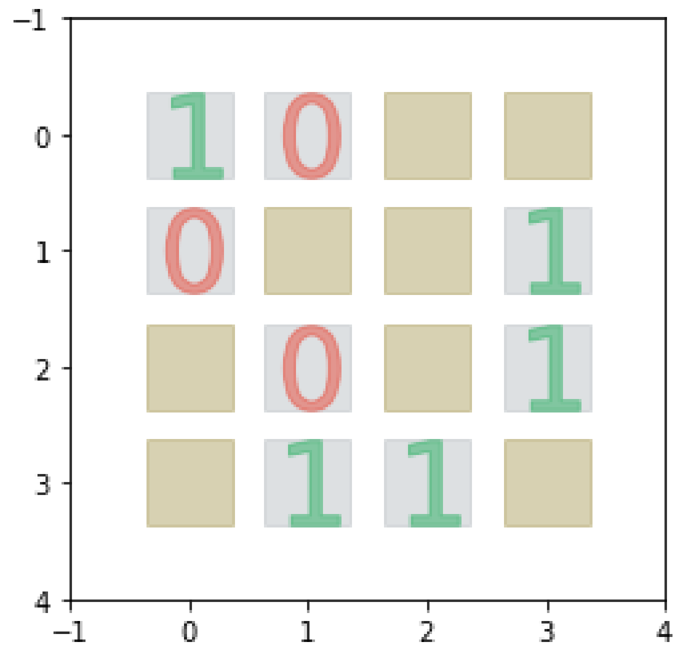
1.3 importing problems and initializing configurations

[18]:
```
input_files = [
    '../puzzles/puzzle0.txt',
    '../puzzles/puzzle1.txt',
    '../puzzles/puzzle2.txt',
    '../puzzles/puzzle3.txt',
    '../puzzles/puzzle4.txt',
    '../puzzles/puzzle5.txt',
]

configurations = []

# initializing 'map' for each config
for index, f in enumerate(input_files):
    configuration = {}
    with open(f, 'r') as file:
        lines = file.readlines()
        lines = [line.strip() for line in lines]

        first_line = lines.pop(0)
        row, col = first_line.split(' ')

        row, col = int(row), int(col)
        initial_map = np.empty((row, col))
        initial_map[:] = np.nan
        for i, line in enumerate(lines):
            temp = line.split(' ')
            for j, element in enumerate(temp):
                if element =='1':
                    initial_map[(i, j)] = 1
                elif element == '0':
                    initial_map[(i, j)] = 0
                else:
                    pass
        configuration['map'] = initial_map
    configurations.append(configuration)

# initializing 'domains', 'variables', 'asssignment' for each config
for config in configurations:
    initial_map = config['map']
    size = len(initial_map)
    r = range(0, size)
```

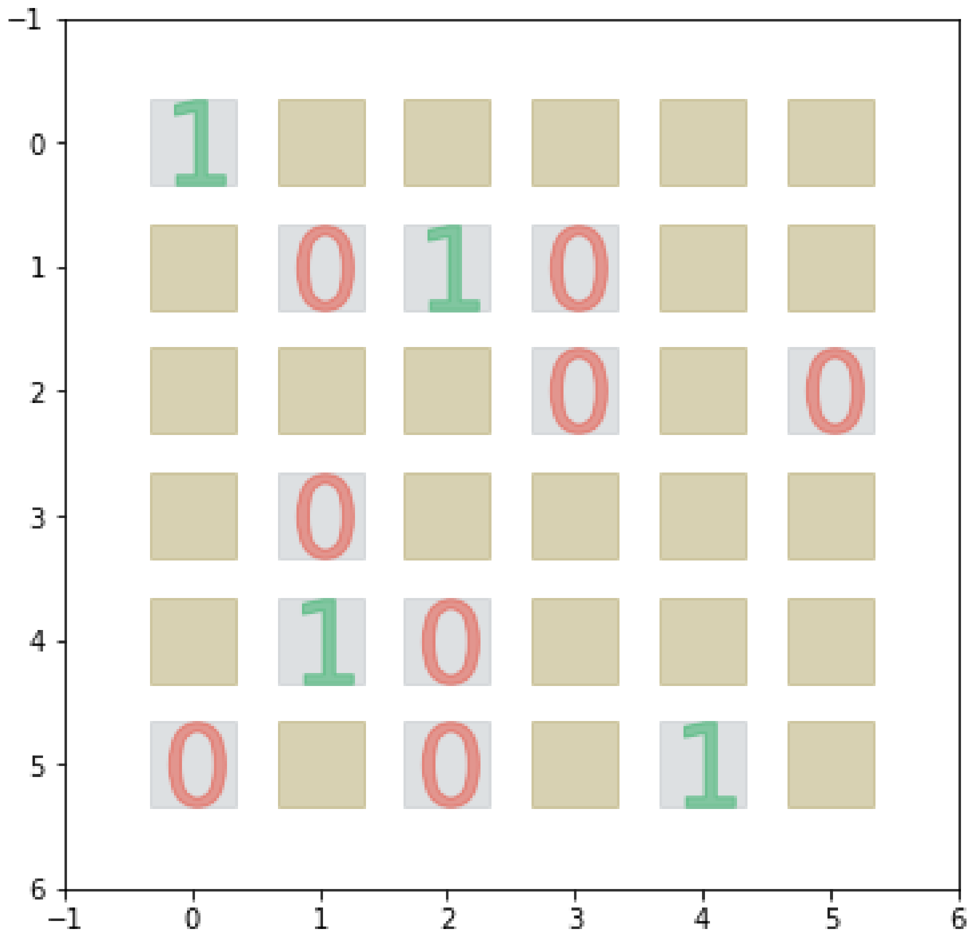
```python
    assignment = {}
    domains = {}
    for i in r:
        for j in r:
            variable = (i, j)

            if initial_map[variable] == 1:
                assignment[variable] = 1
                domains[variable] = [1]

            elif initial_map[variable] == 0:
                assignment[variable] = 0
                domains[variable] = [0]

            else:
                domains[variable] = [0, 1]

    config['domains']    = domains
    config['variables']  = domains.keys()
    config['assignment'] = assignment

# showing results
print('\n')
print('** A little hint: the yellowish color of the cell means it can be filled␣
 ↪with either 0 or 1')
print('**                the green     color of the cell means it can be filled␣
 ↪only with 1')
print('**                the red       color of the cell means it can be filled␣
 ↪only with 0')
print('**                some cells are already filled with 0 or 1\n\n')

for index, config in enumerate(configurations):
    print(f"Problem number {index} --------------------------------")
    pprint.pprint(config['map'])
    draw_config(config)
```

```
** A little hint: the yellowish color of the cell means it can be filled with
either 0 or 1
**                the green     color of the cell means it can be filled only
with 1
**                the red       color of the cell means it can be filled only
with 0
**                some cells are already filled with 0 or 1
```

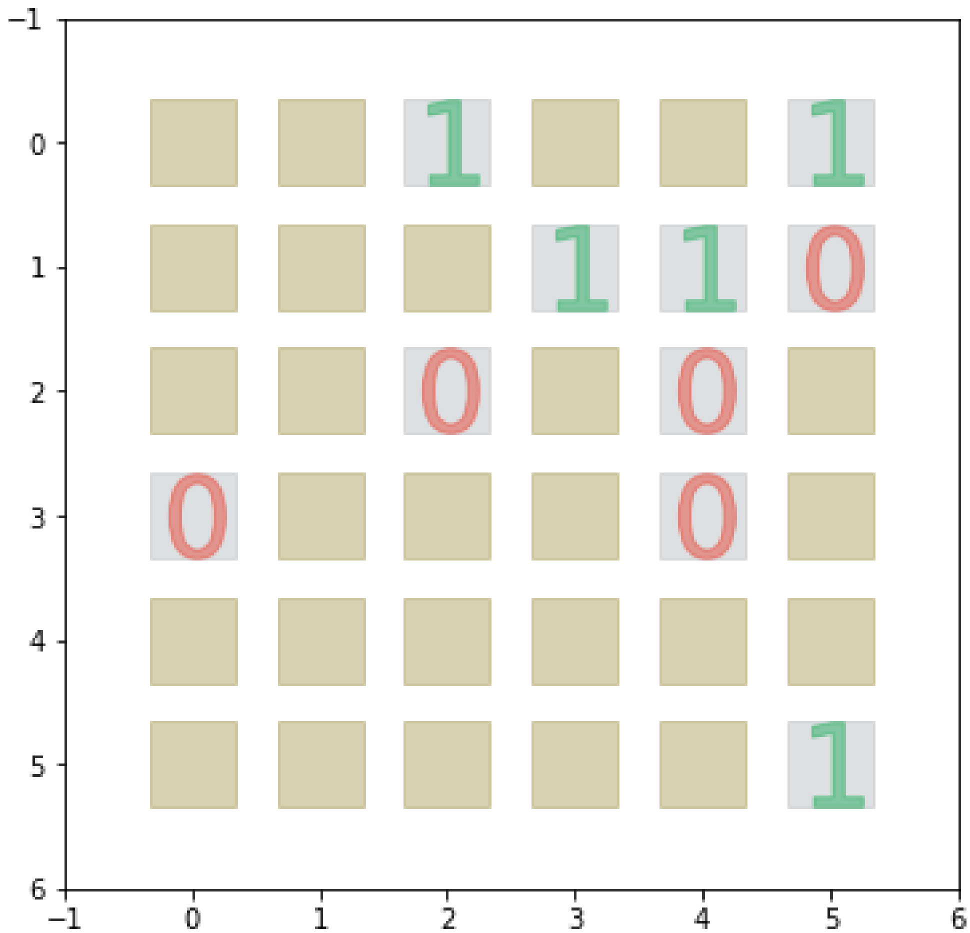
```
Problem number 0 --------------------------------
array([[ 1.,  0., nan, nan],
       [ 0., nan, nan,  1.],
       [nan,  0., nan,  1.],
       [nan,  1.,  1., nan]])
```



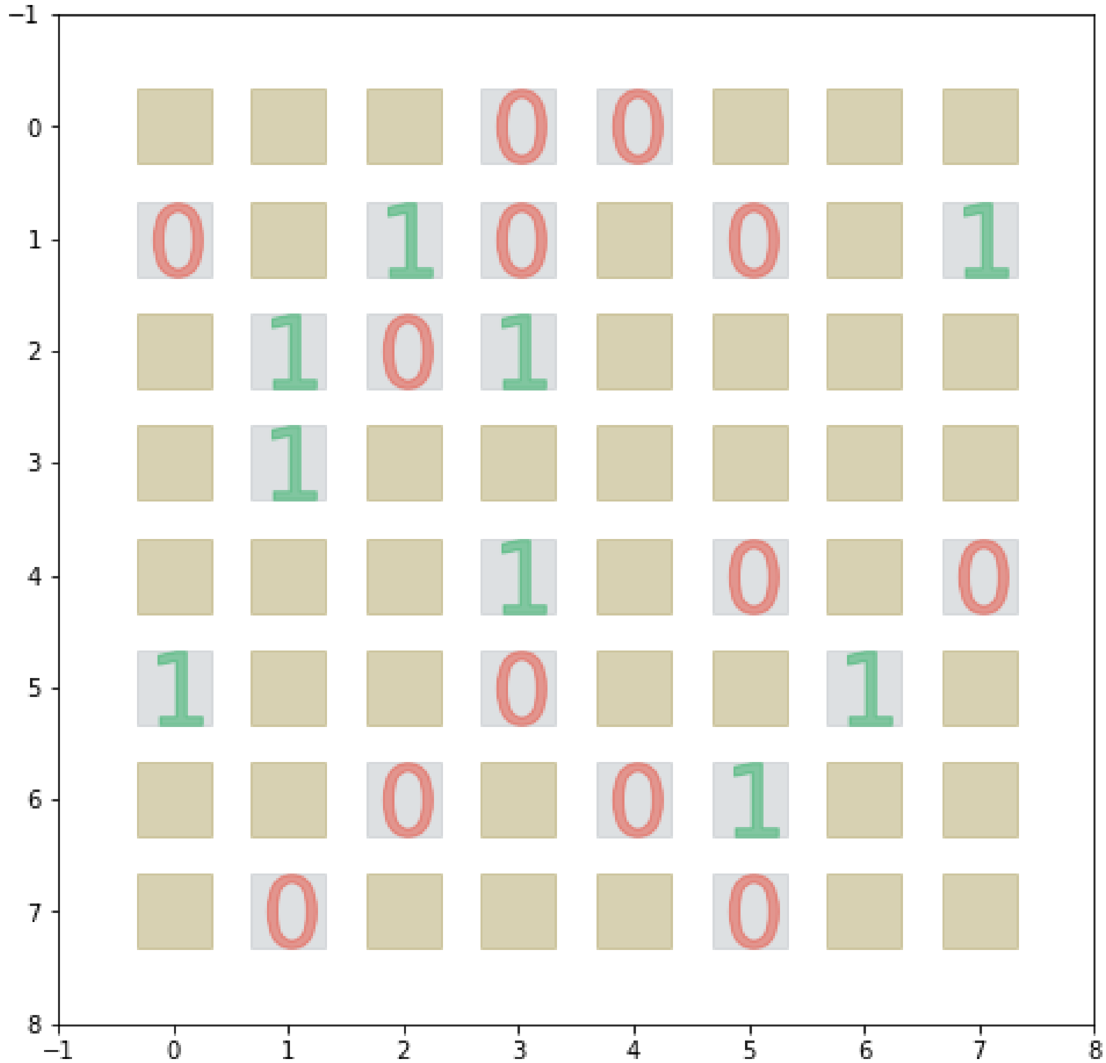

```
Problem number 1 --------------------------------
array([[ 1., nan, nan, nan, nan, nan],
       [nan,  0.,  1.,  0., nan, nan],
       [nan, nan, nan,  0., nan,  0.],
       [nan,  0., nan, nan, nan, nan],
       [nan,  1.,  0., nan, nan, nan],
       [ 0., nan,  0., nan,  1., nan]])
```



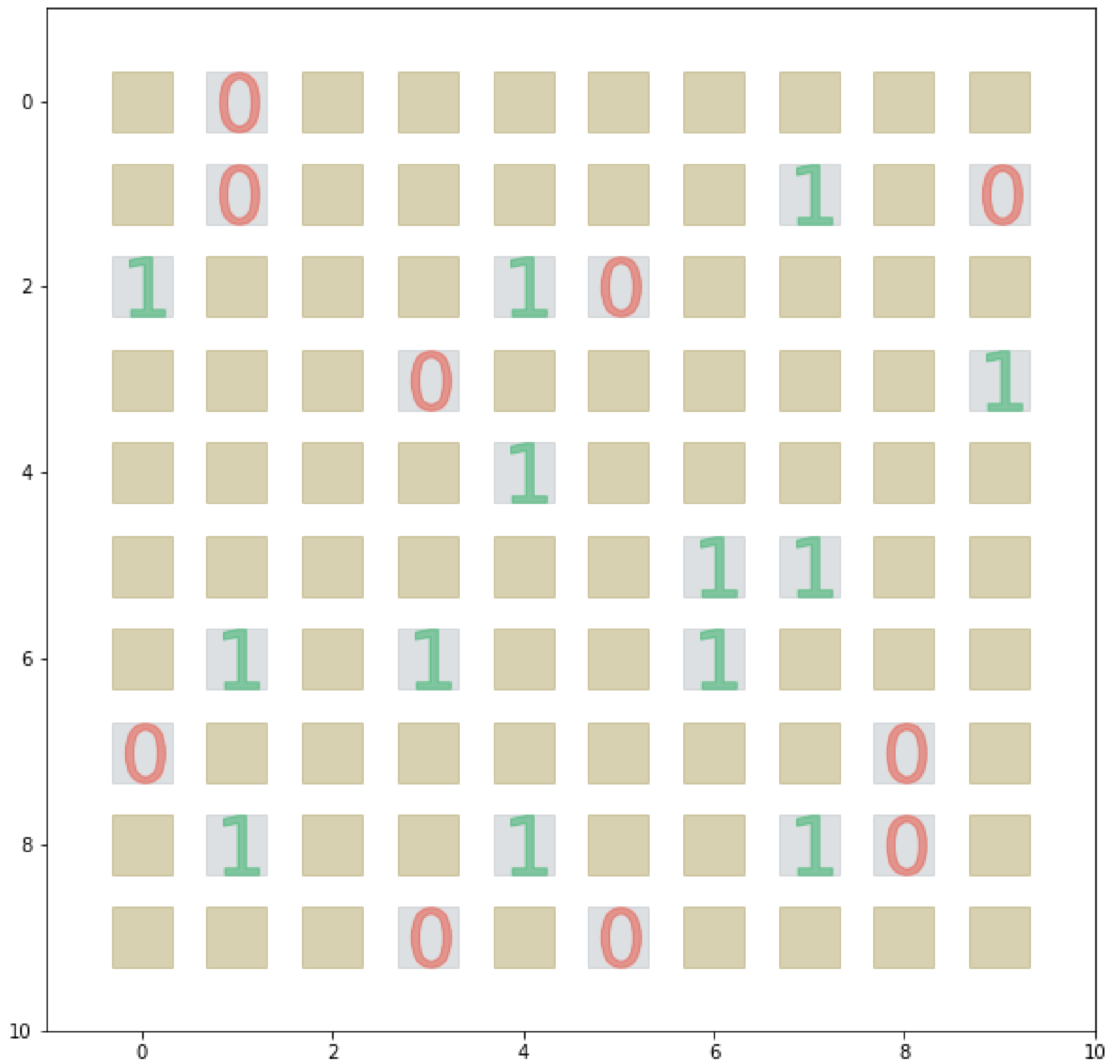

```
Problem number 2 ------------------------------
array([[nan, nan,  1., nan, nan,  1.],
       [nan, nan, nan,  1.,  1.,  0.],
       [nan, nan,  0., nan,  0., nan],
       [ 0., nan, nan, nan,  0., nan],
       [nan, nan, nan, nan, nan, nan],
       [nan, nan, nan, nan, nan,  1.]])
```



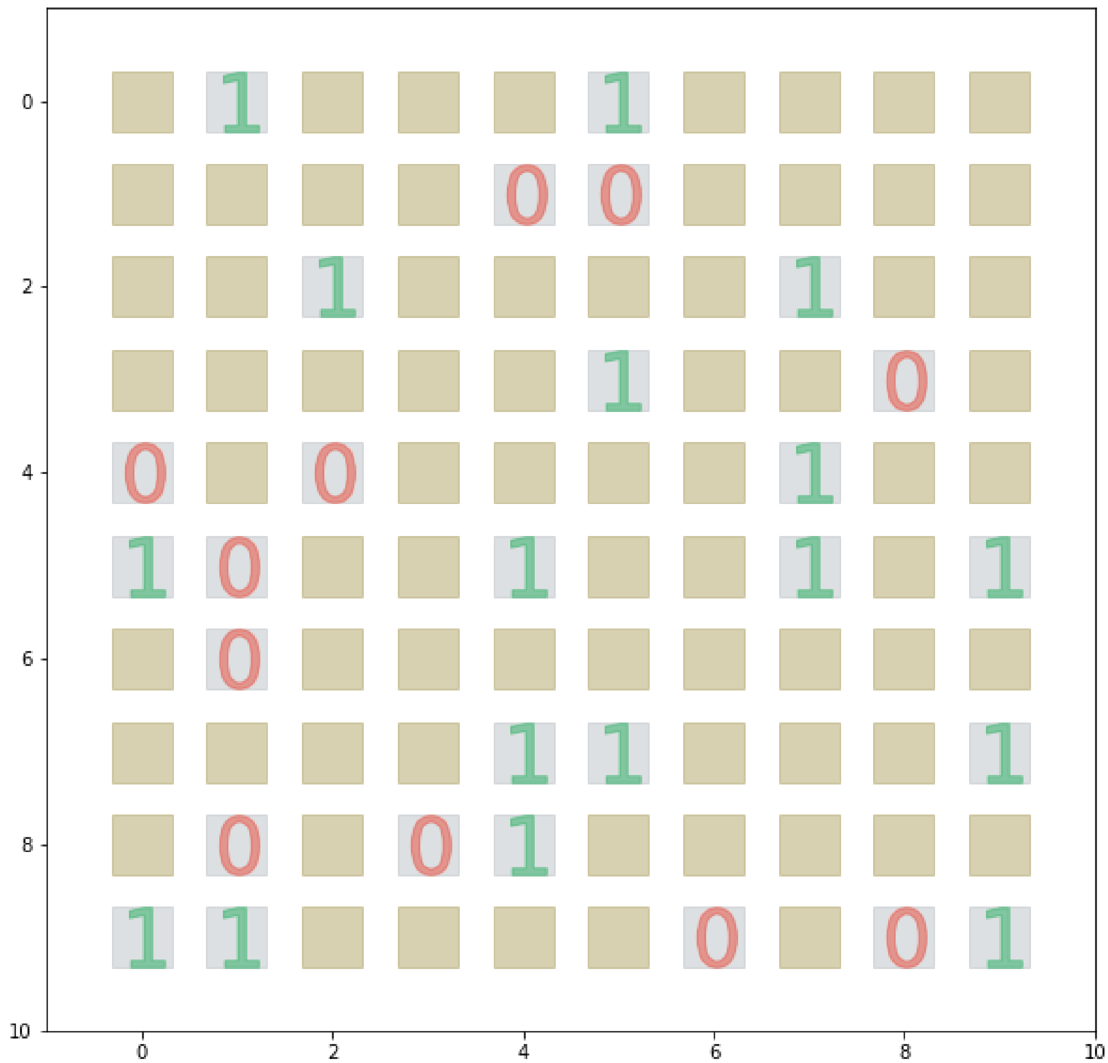

```
Problem number 3 --------------------------------
array([[nan, nan, nan,  0.,  0., nan, nan, nan],
       [ 0., nan,  1.,  0., nan,  0., nan,  1.],
       [nan,  1.,  0.,  1., nan, nan, nan, nan],
       [nan,  1., nan, nan, nan, nan, nan, nan],
       [nan, nan, nan,  1., nan,  0., nan,  0.],
       [ 1., nan, nan,  0., nan, nan,  1., nan],
       [nan, nan,  0., nan,  0.,  1., nan, nan],
       [nan,  0., nan, nan, nan,  0., nan, nan]])
```




```
Problem number 4 --------------------------------
array([[nan,  0., nan, nan, nan, nan, nan, nan, nan, nan],
       [nan,  0., nan, nan, nan, nan, nan,  1., nan,  0.],
       [ 1., nan, nan, nan,  1.,  0., nan, nan, nan, nan],
       [nan, nan, nan,  0., nan, nan, nan, nan, nan,  1.],
       [nan, nan, nan, nan,  1., nan, nan, nan, nan, nan],
       [nan, nan, nan, nan, nan, nan,  1.,  1., nan, nan],
       [nan,  1., nan,  1., nan, nan,  1., nan, nan, nan],
       [ 0., nan, nan, nan, nan, nan, nan, nan,  0., nan],
       [nan,  1., nan, nan,  1., nan, nan,  1.,  0., nan],
       [nan, nan, nan,  0., nan,  0., nan, nan, nan, nan]])
```




```
Problem number 5 --------------------------------
array([[nan,  1., nan, nan, nan,  1., nan, nan, nan, nan],
       [nan, nan, nan, nan,  0.,  0., nan, nan, nan, nan],
       [nan, nan,  1., nan, nan, nan, nan,  1., nan, nan],
       [nan, nan, nan, nan, nan,  1., nan, nan,  0., nan],
       [ 0., nan,  0., nan, nan, nan, nan,  1., nan, nan],
       [ 1.,  0., nan, nan,  1., nan, nan,  1., nan,  1.],
       [nan,  0., nan, nan, nan, nan, nan, nan, nan, nan],
       [nan, nan, nan, nan,  1.,  1., nan, nan, nan,  1.],
       [nan,  0., nan,  0.,  1., nan, nan, nan, nan, nan],
       [ 1.,  1., nan, nan, nan, nan,  0., nan,  0.,  1.]])
```



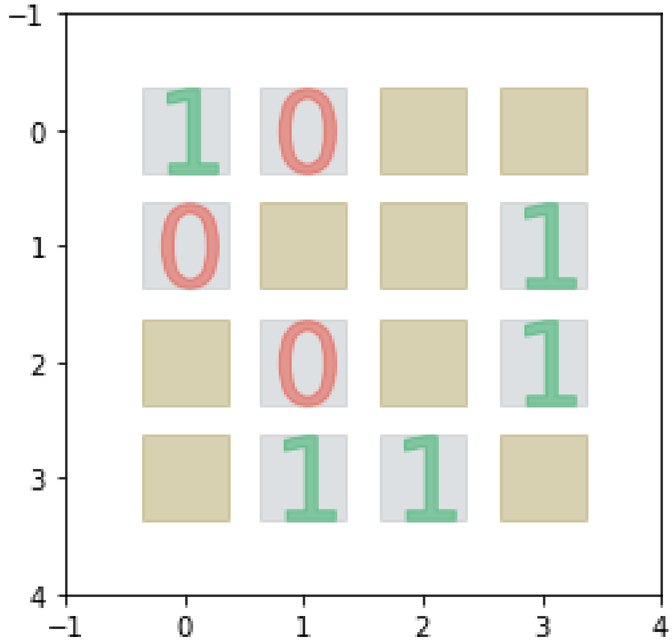
[19]:
```python
initial_ = [] # after first updating domains
results_ = [] # final results
history_ = [] # history of each assignment in solving procedure
for index, config in enumerate(configurations):

    result = {}
    domains:    Dict[Tuple, List[int]] = config['domains']
    variables:  List[Tuple]            = config['variables']
    assignment: Dict[Tuple, int]       = config['assignment']

    s = len(config['map'])

    csp: CSP[Tuple, int] = CSP(variables, domains)
    csp.add_constraint(FirstConstraint(variables, s))
```

```
    csp.add_constraint(SecondConstraint(variables, s))
    csp.add_constraint(ThirdConstraint(variables, s))

    start = time.time()
    initial_.append({
        'assignment': copy.deepcopy(assignment),
        'domains': copy.deepcopy(domains),
    })
    solution: Optional[Dict[Tuple, int]] = csp.
↪backtracking_search(assignment=assignment)
    end = time.time()

    history_.append(copy.deepcopy(csp.backtracking_history))
    if solution is None:
        print("No solution found!")
        result['final']      = None
        result['time']       = (end - start) - csp.history_gathering_time
        result['successful'] = False
        result['waste']      = csp.history_gathering_time
    else:
        print(f'elapsed time for problem {index}: {round(end - start, 5)}')
        mat = Matrix(s)
        mat.clear_populate(solution)
        m = mat.matrix

        assert np.where(np.isnan(m) == True)[0].size == 0 , "find np.nan in␣
↪result!"
        assert np.where(np.isnan(m) == True)[1].size == 0 , "find np.nan in␣
↪result!"

        result['final']      = m.astype(np.int8, copy=False)
        result['time']       = (end - start) - csp.history_gathering_time
        result['successful'] = True
        result['waste']      = csp.history_gathering_time

    results_.append({
        'result': result,
        'domains': csp.domains
    })
```

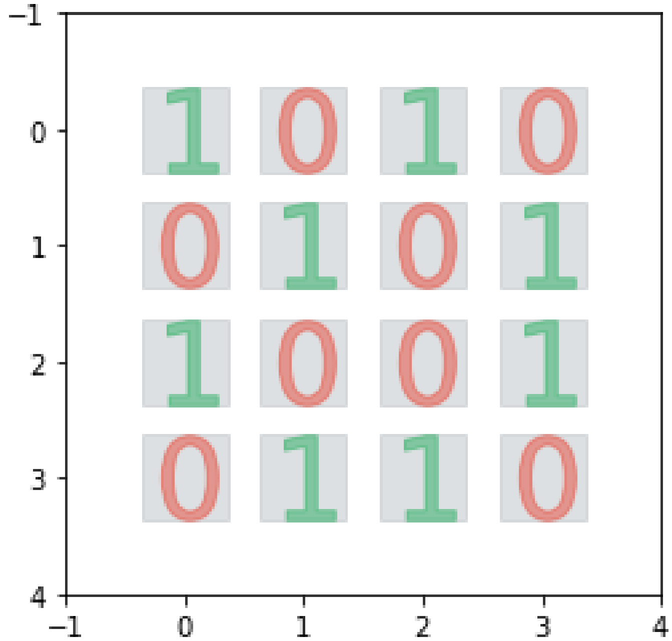
```
elapsed time for problem 0: 0.005
elapsed time for problem 1: 0.113
elapsed time for problem 2: 0.12623
elapsed time for problem 3: 0.22604
elapsed time for problem 4: 111.13367
No solution found!
```

[20]:
```
report(configurations, results_, initial_)
```

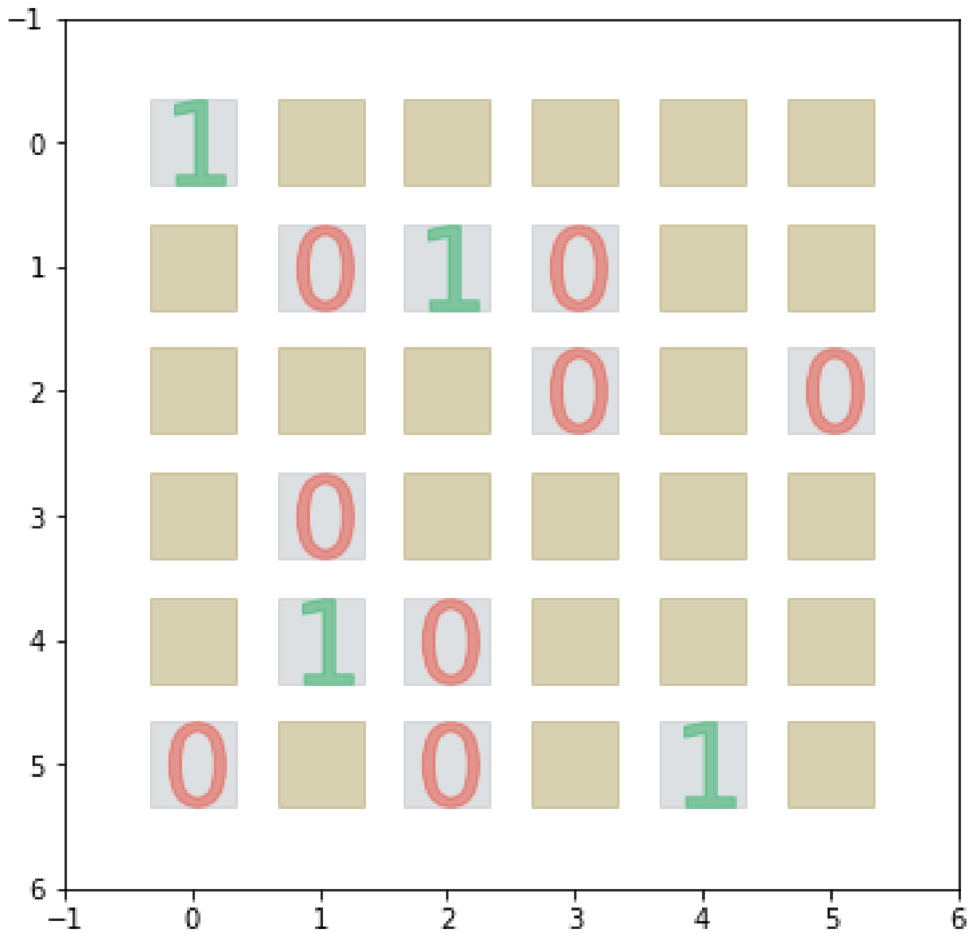
```
#problem number 0 ------------------------
@successful: True
@elapsed time: 0.001 seconds
@start state:
--graphical
```



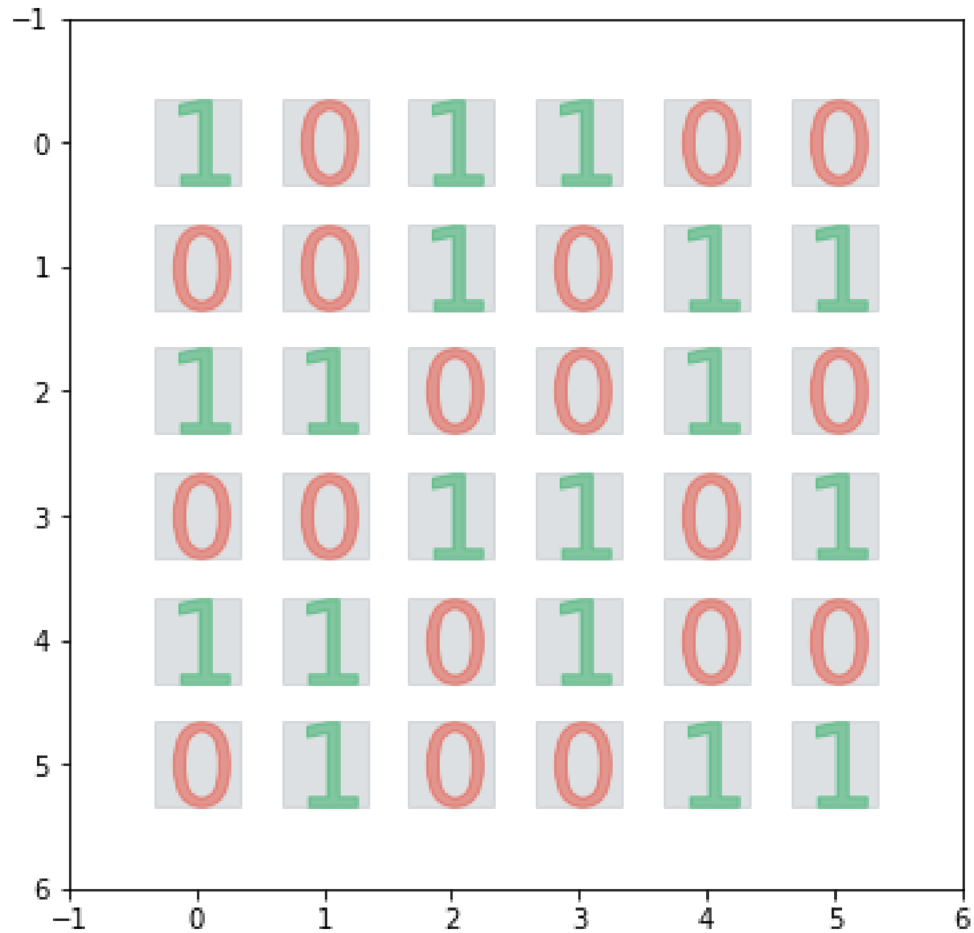

```
--textual
[[1.0, 0.0, nan, nan],
 [0.0, nan, nan, 1.0],
 [nan, 0.0, nan, 1.0],
 [nan, 1.0, 1.0, nan]]


@result:
--graphical
```



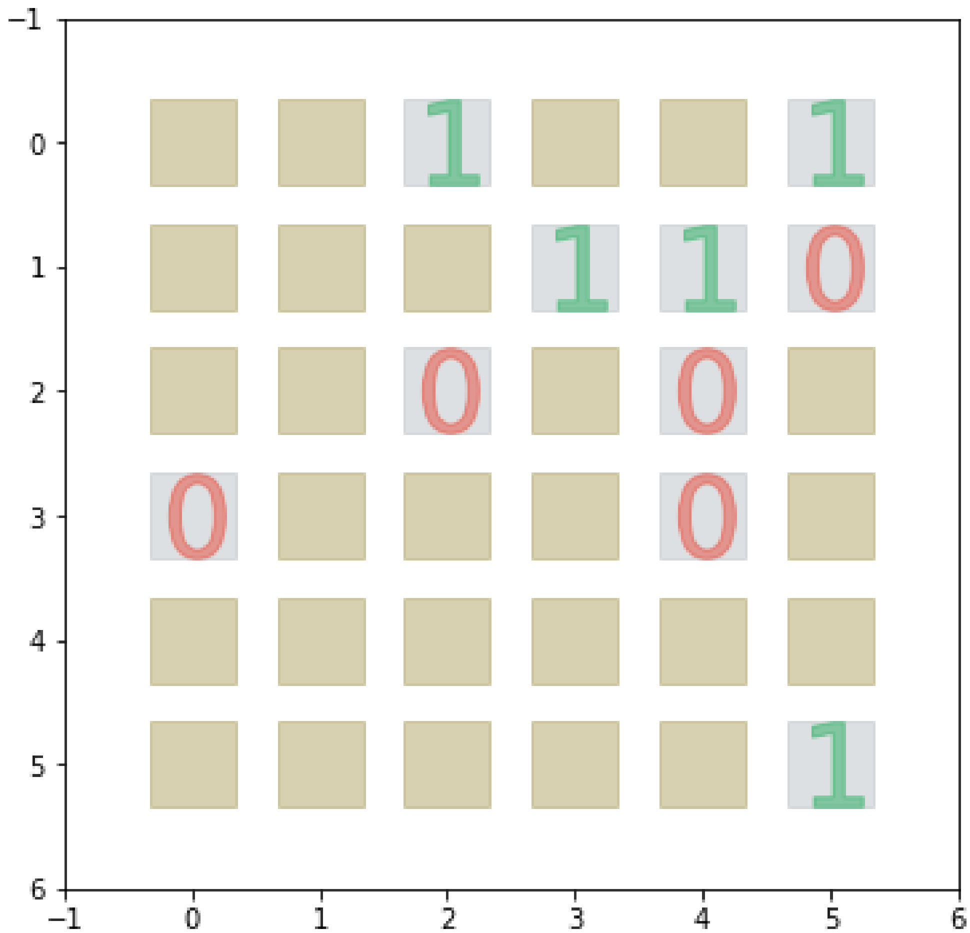

```
--textual
[[1, 0, 1, 0],
 [0, 1, 0, 1],
 [1, 0, 0, 1],
 [0, 1, 1, 0]]


#problem number 1 ------------------------
@successful: True
@elapsed time: 0.053 seconds
@start state:
--graphical
```



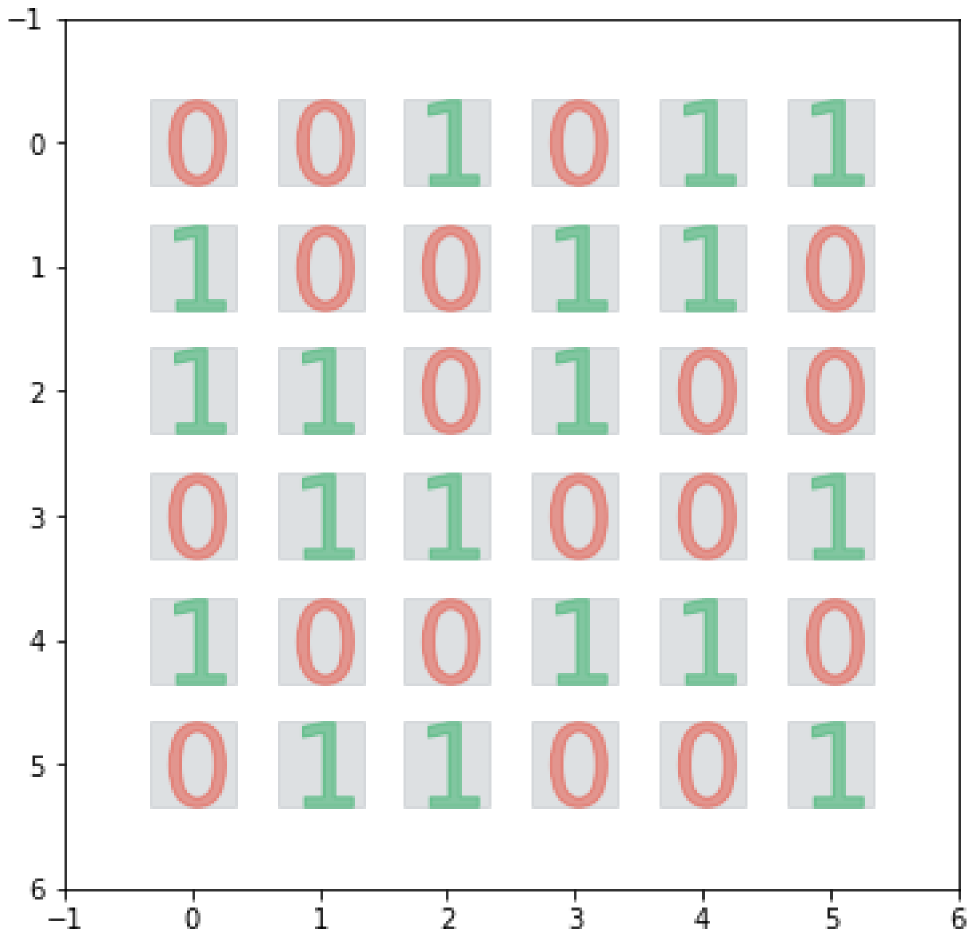

```
--textual
[[1.0, nan, nan, nan, nan, nan],
 [nan, 0.0, 1.0, 0.0, nan, nan],
 [nan, nan, nan, 0.0, nan, 0.0],
 [nan, 0.0, nan, nan, nan, nan],
 [nan, 1.0, 0.0, nan, nan, nan],
 [0.0, nan, 0.0, nan, 1.0, nan]]


@result:
--graphical
```



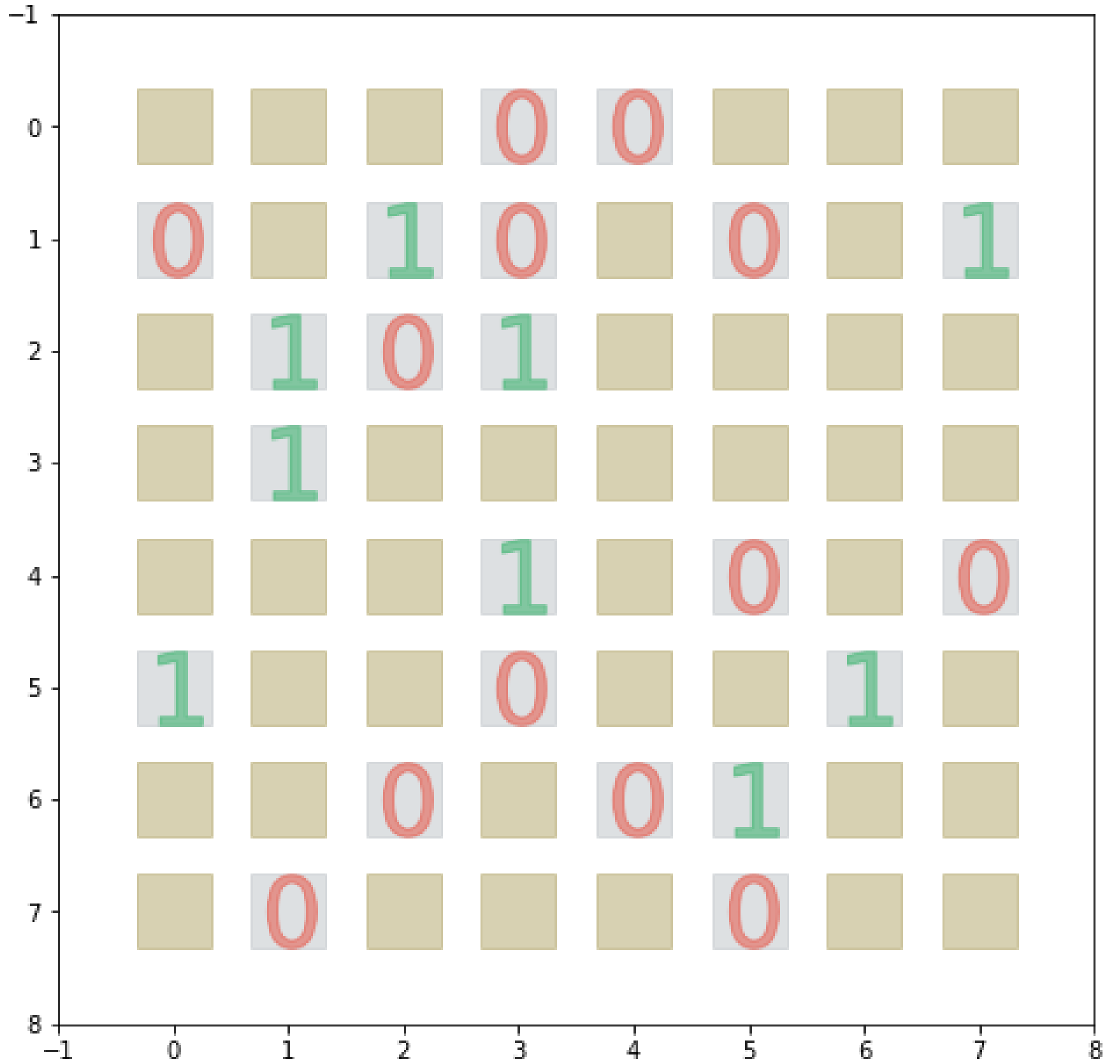

```
--textual
[[1, 0, 1, 1, 0, 0],
 [0, 0, 1, 0, 1, 1],
 [1, 1, 0, 0, 1, 0],
 [0, 0, 1, 1, 0, 1],
 [1, 1, 0, 1, 0, 0],
 [0, 1, 0, 0, 1, 1]]


#problem number 2 ------------------------
@successful: True
@elapsed time: 0.051 seconds
@start state:
--graphical
```



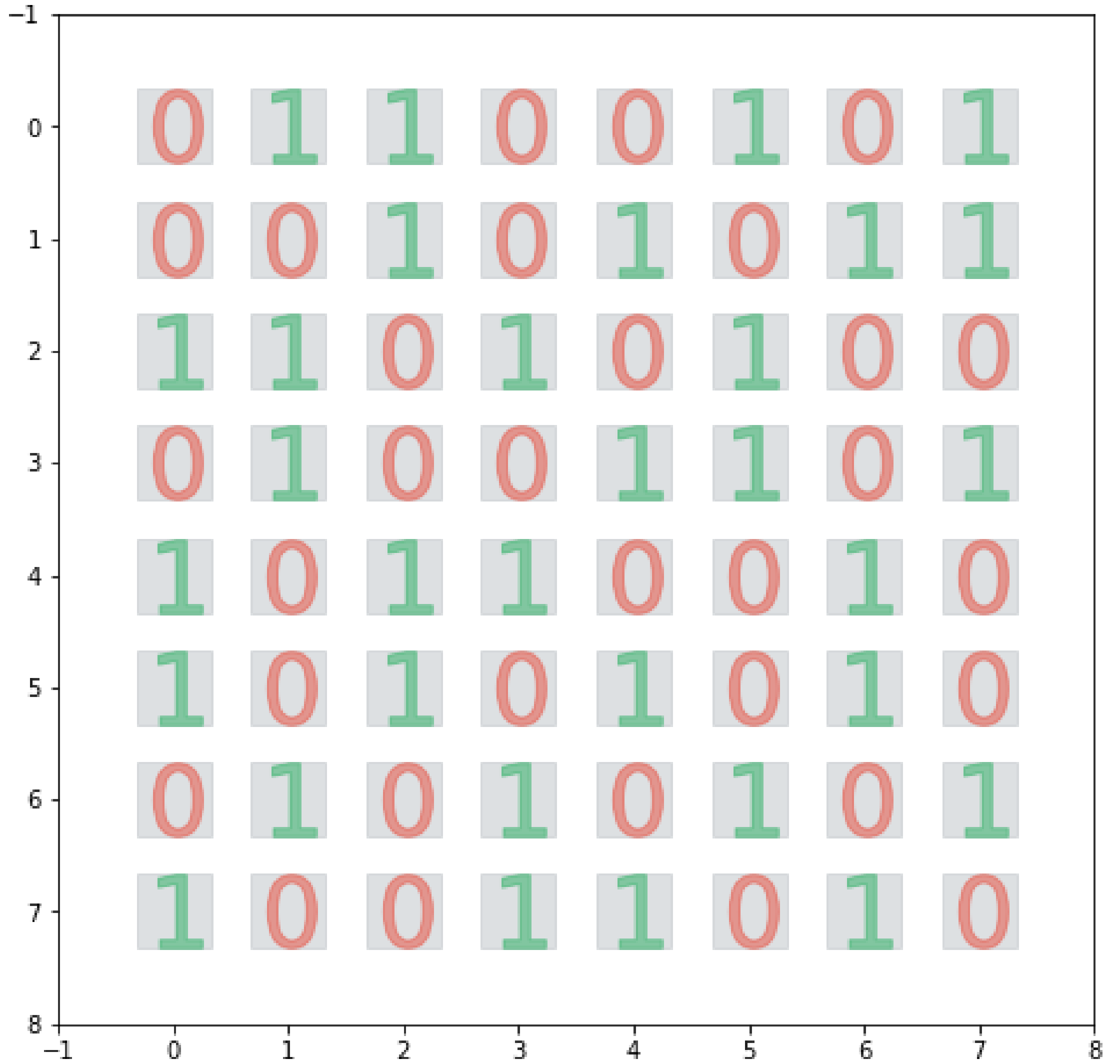

```
--textual
[[nan, nan, 1.0, nan, nan, 1.0],
 [nan, nan, nan, 1.0, 1.0, 0.0],
 [nan, nan, 0.0, nan, 0.0, nan],
 [0.0, nan, nan, nan, 0.0, nan],
 [nan, nan, nan, nan, nan, nan],
 [nan, nan, nan, nan, nan, 1.0]]


@result:
--graphical
```




```
--textual
[[0, 0, 1, 0, 1, 1],
 [1, 0, 0, 1, 1, 0],
 [1, 1, 0, 1, 0, 0],
 [0, 1, 1, 0, 0, 1],
 [1, 0, 0, 1, 1, 0],
 [0, 1, 1, 0, 0, 1]]


#problem number 3 ------------------------
@successful: True
@elapsed time: 0.105 seconds
@start state:
--graphical
```

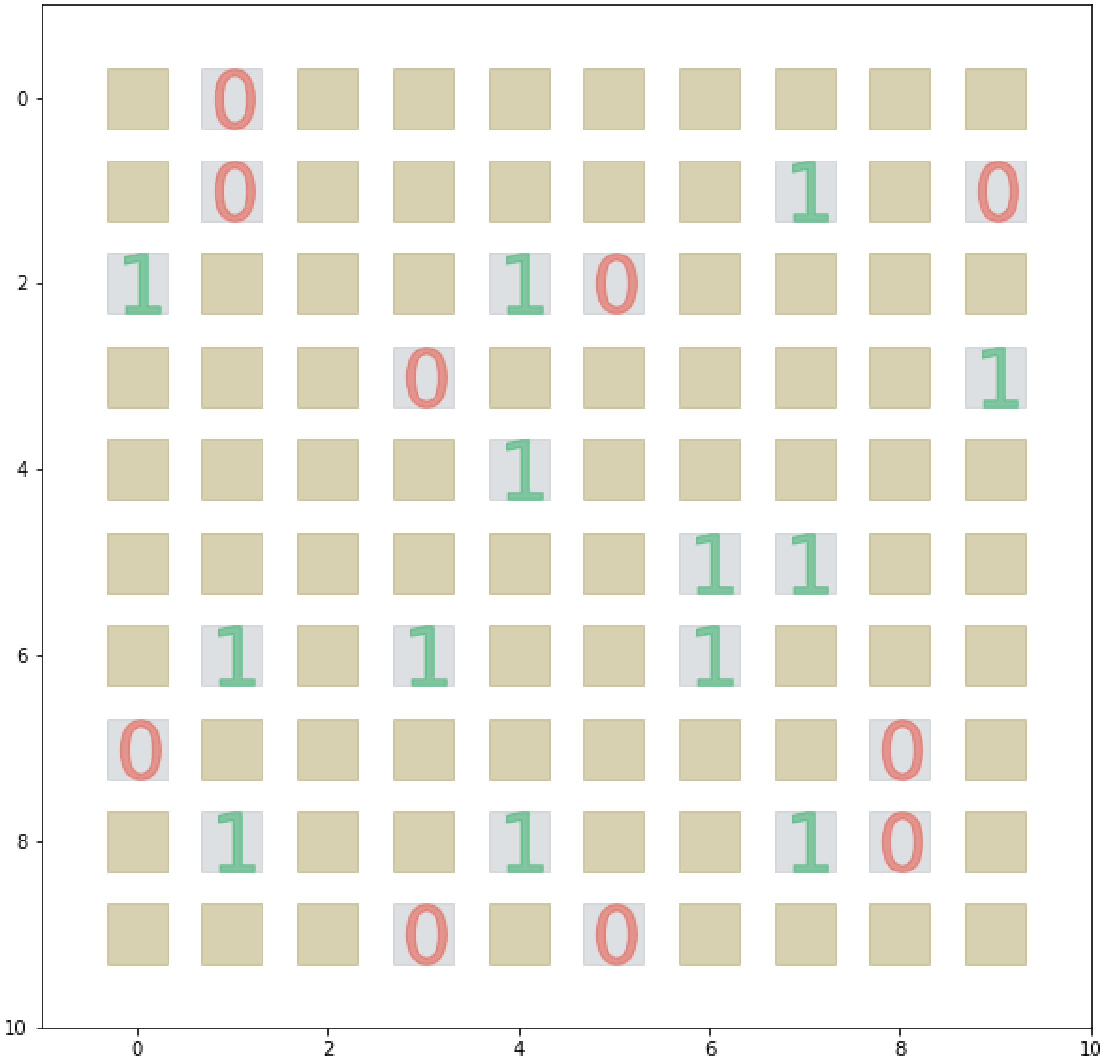



```
--textual
[[nan, nan, nan, 0.0, 0.0, nan, nan, nan],
 [0.0, nan, 1.0, 0.0, nan, 0.0, nan, 1.0],
 [nan, 1.0, 0.0, 1.0, nan, nan, nan, nan],
 [nan, 1.0, nan, nan, nan, nan, nan, nan],
 [nan, nan, nan, 1.0, nan, 0.0, nan, 0.0],
 [1.0, nan, nan, 0.0, nan, nan, 1.0, nan],
 [nan, nan, 0.0, nan, 0.0, 1.0, nan, nan],
 [nan, 0.0, nan, nan, nan, 0.0, nan, nan]]


@result:
--graphical
```



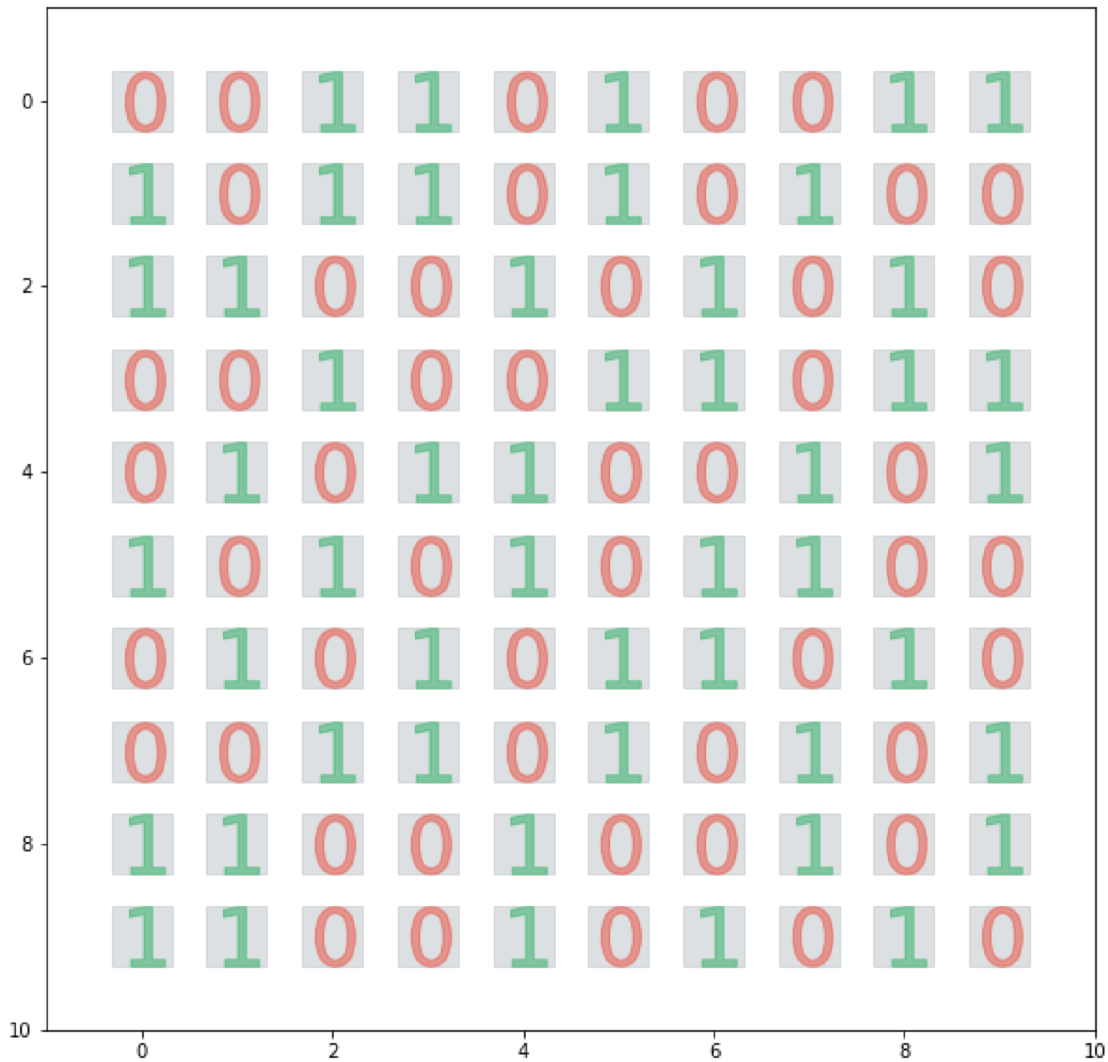
```
--textual
[[0, 1, 1, 0, 0, 1, 0, 1],
 [0, 0, 1, 0, 1, 0, 1, 1],
 [1, 1, 0, 1, 0, 1, 0, 0],
 [0, 1, 0, 0, 1, 1, 0, 1],
 [1, 0, 1, 1, 0, 0, 1, 0],
 [1, 0, 1, 0, 1, 0, 1, 0],
 [0, 1, 0, 1, 0, 1, 0, 1],
 [1, 0, 0, 1, 1, 0, 1, 0]]


#problem number 4 ------------------------
@successful: True
@elapsed time: 99.51 seconds
```

```
@start state:
--graphical
```



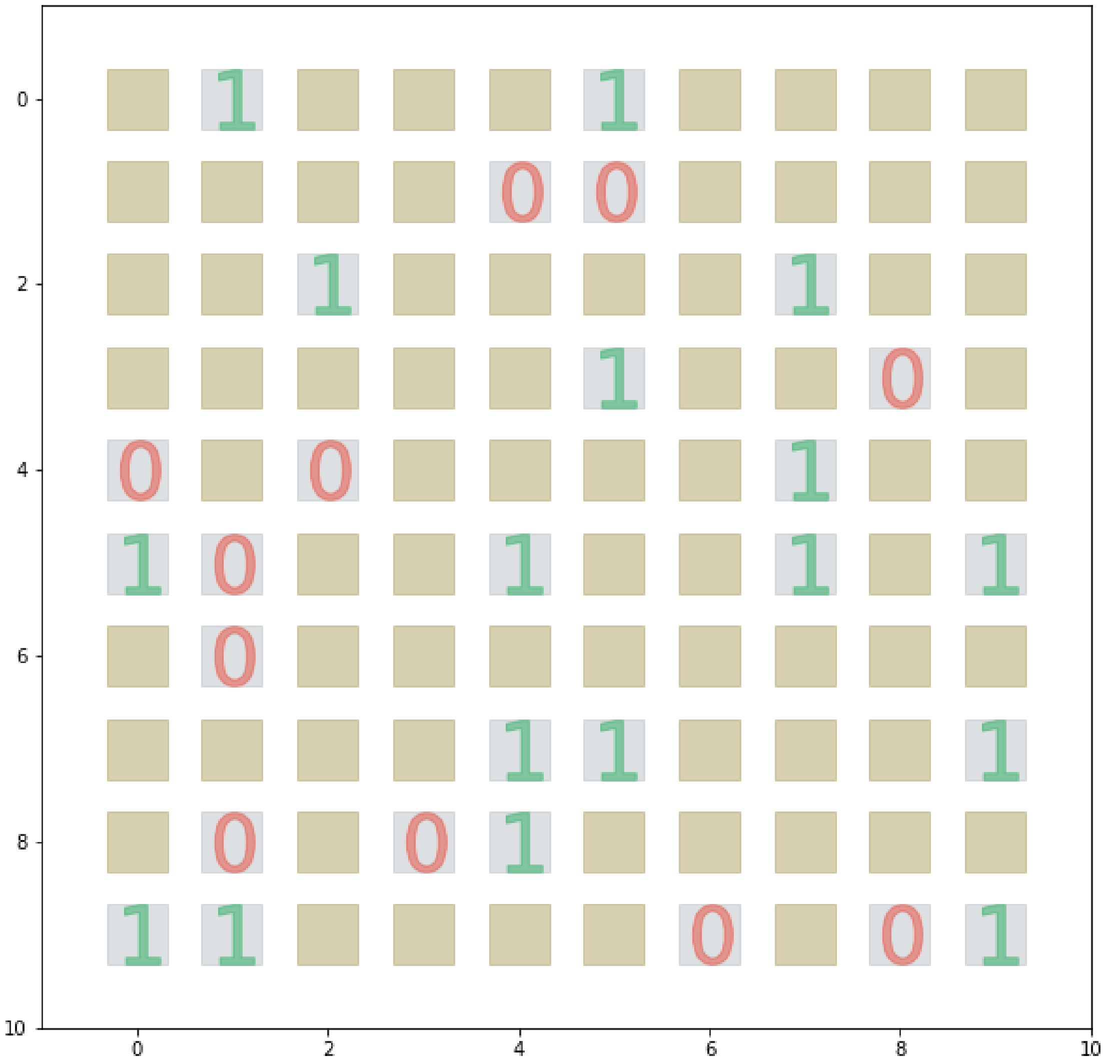
```
--textual
[[nan, 0.0, nan, nan, nan, nan, nan, nan, nan, nan],
 [nan, 0.0, nan, nan, nan, nan, nan, 1.0, nan, 0.0],
 [1.0, nan, nan, nan, 1.0, 0.0, nan, nan, nan, nan],
 [nan, nan, nan, 0.0, nan, nan, nan, nan, nan, 1.0],
 [nan, nan, nan, nan, 1.0, nan, nan, nan, nan, nan],
 [nan, nan, nan, nan, nan, nan, 1.0, 1.0, nan, nan],
 [nan, 1.0, nan, 1.0, nan, nan, 1.0, nan, nan, nan],
 [0.0, nan, nan, nan, nan, nan, nan, nan, 0.0, nan],
 [nan, 1.0, nan, nan, 1.0, nan, nan, 1.0, 0.0, nan],
 [nan, nan, nan, 0.0, nan, 0.0, nan, nan, nan, nan]]
```

```
@result:
--graphical
```




```
--textual
[[0, 0, 1, 1, 0, 1, 0, 0, 1, 1],
 [1, 0, 1, 1, 0, 1, 0, 1, 0, 0],
 [1, 1, 0, 0, 1, 0, 1, 0, 1, 0],
 [0, 0, 1, 0, 0, 1, 1, 0, 1, 1],
 [0, 1, 0, 1, 1, 0, 0, 1, 0, 1],
 [1, 0, 1, 0, 1, 0, 1, 1, 0, 0],
 [0, 1, 0, 1, 0, 1, 1, 0, 1, 0],
 [0, 0, 1, 1, 0, 1, 0, 1, 0, 1],
 [1, 1, 0, 0, 1, 0, 0, 1, 0, 1],
 [1, 1, 0, 0, 1, 0, 1, 0, 1, 0]]
```

```
#problem number 5 ------------------------
@successful: False
@elapsed time: 228.97 seconds
@start state:
--graphical
```




```
--textual
[[nan, 1.0, nan, nan, nan, 1.0, nan, nan, nan, nan],
 [nan, nan, nan, nan, 0.0, 0.0, nan, nan, nan, nan],
 [nan, nan, 1.0, nan, nan, nan, nan, 1.0, nan, nan],
 [nan, nan, nan, nan, nan, 1.0, nan, nan, 0.0, nan],
 [0.0, nan, 0.0, nan, nan, nan, nan, 1.0, nan, nan],
 [1.0, 0.0, nan, nan, 1.0, nan, nan, 1.0, nan, 1.0],
```

```
 [nan, 0.0, nan, nan, nan, nan, nan, nan, nan, nan],
 [nan, nan, nan, nan, 1.0, 1.0, nan, nan, nan, 1.0],
 [nan, 0.0, nan, 0.0, 1.0, nan, nan, nan, nan, nan],
 [1.0, 1.0, nan, nan, nan, nan, 0.0, nan, 0.0, 1.0]]


@result:
NO solution found…
```

[21]:

```
initial_FC = [] # after first updating domains
results_FC = [] # final results
history_FC = [] # history of each assignment in solving procedure
for index, config in enumerate(configurations):

    result = {}
    domains:    Dict[Tuple, List[int]] = config['domains']
    variables:  List[Tuple]            = config['variables']
    assignment: Dict[Tuple, int]       = config['assignment']

    s = len(config['map'])

    csp: CSP[Tuple, int] = CSP(variables, domains)
    csp.add_constraint(FirstConstraint(variables, s))
    csp.add_constraint(SecondConstraint(variables, s))
    csp.add_constraint(ThirdConstraint(variables, s))

    start = time.time()
    for a in assignment:
        domains = csp.forward_checking(domains, assignment, a)

    initial_FC.append({
        'assignment': copy.deepcopy(assignment),
        'domains': copy.deepcopy(domains),
    })

    solution: Optional[Dict[Tuple, int]] = csp.
↪backtracking_search_with_FC(assignment=assignment, domains=domains)
    end = time.time()

    history_FC.append(copy.deepcopy(csp.backtracking_FC_history))

    if solution is None:
        print("No solution found!")
        result['final']      = None
        result['time']       = (end - start) - csp.history_gathering_time
        result['successful'] = False
        result['waste']      = csp.history_gathering_time
```

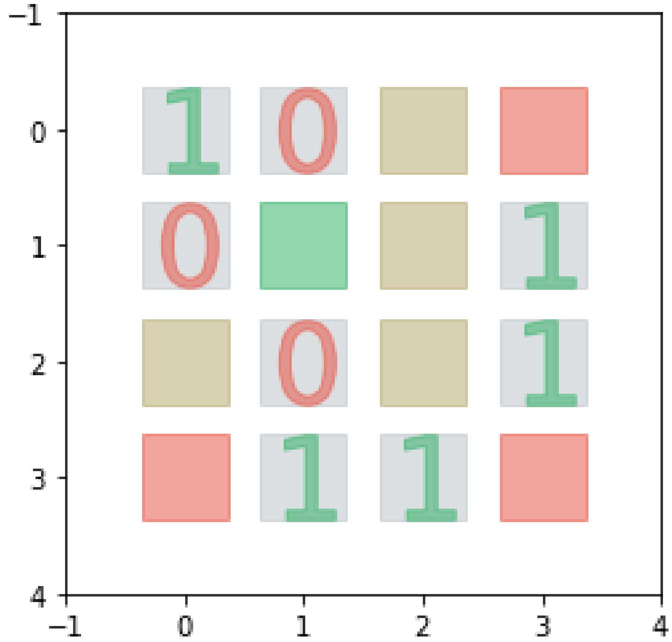
```
    else:
        print(f'elapsed time for problem {index}: {round(end - start, 5)}')
        mat = Matrix(s)
        mat.clear_populate(solution)
        m = mat.matrix

        assert np.where(np.isnan(m) == True)[0].size == 0 , "find np.nan in␣
↪result!"
        assert np.where(np.isnan(m) == True)[1].size == 0 , "find np.nan in␣
↪result!"

        result['final']      = m.astype(np.int8, copy=False)
        result['time']       = (end - start) - csp.history_gathering_time
        result['successful'] = True
        result['waste']      = csp.history_gathering_time

    results_FC.append({
        'result': result,
        'domains': csp.domains
    })
```

```
elapsed time for problem 0: 0.00703
elapsed time for problem 1: 0.03917
elapsed time for problem 2: 0.04899
elapsed time for problem 3: 0.18359
elapsed time for problem 4: 28.48573
No solution found!
```

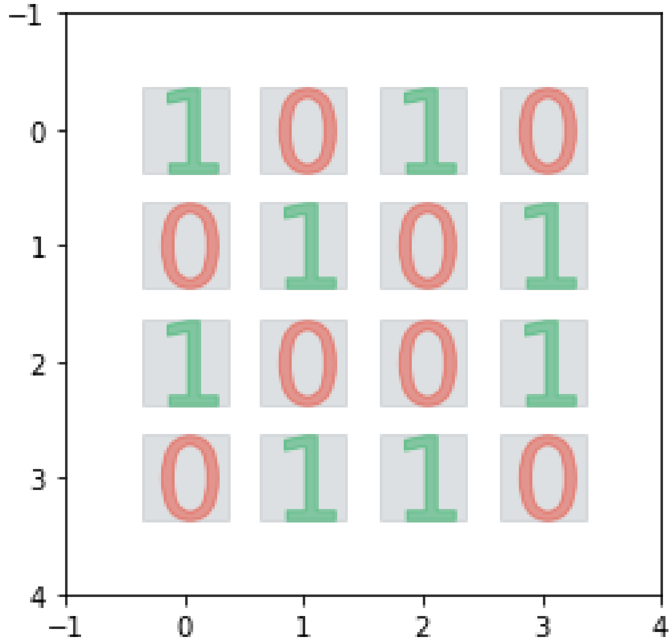
[22]:

```
report(configurations, results_FC, initial_FC)
```

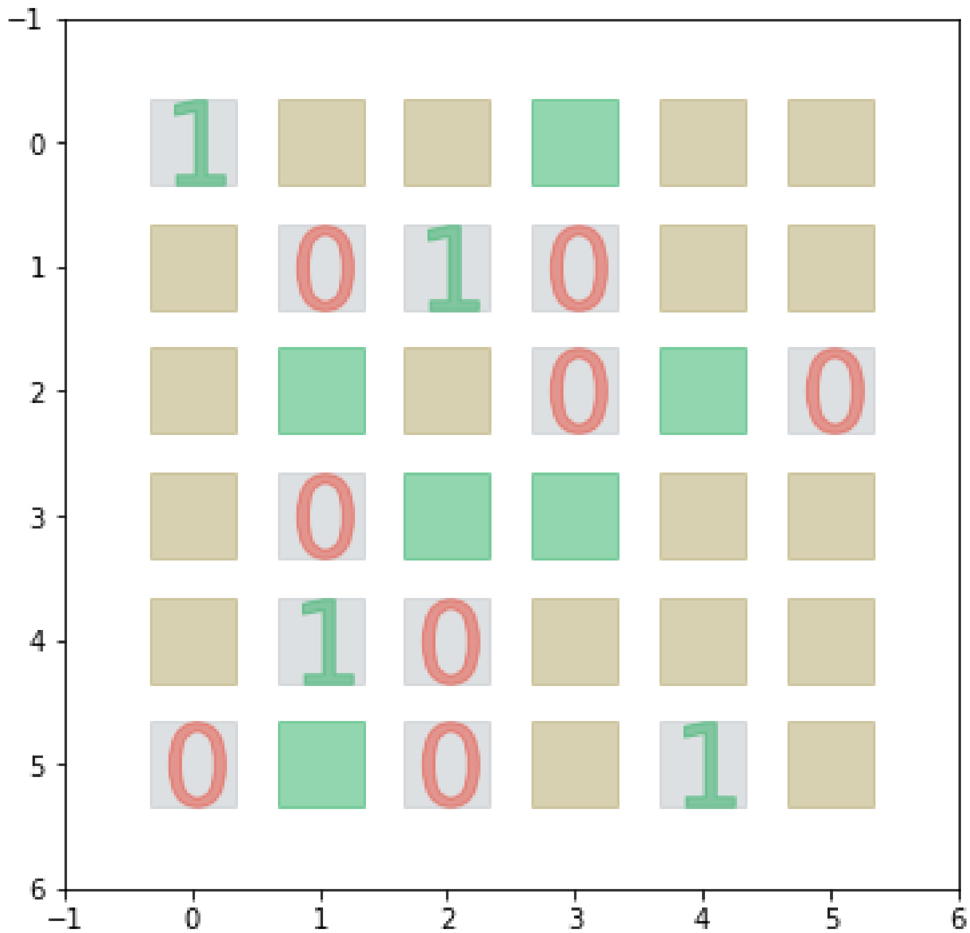
```
#problem number 0 ------------------------
@successful: True
@elapsed time: 0.005 seconds
@start state:
--graphical
```



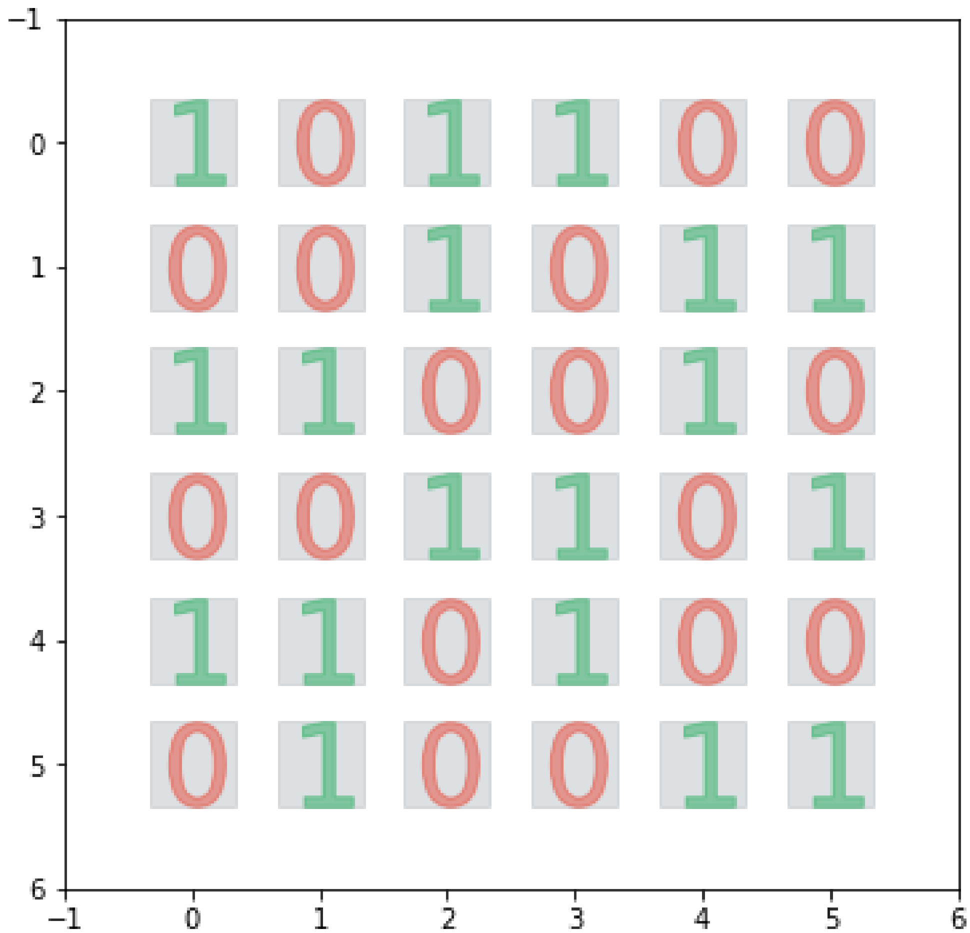

```
--textual
[[1.0, 0.0, nan, nan],
 [0.0, nan, nan, 1.0],
 [nan, 0.0, nan, 1.0],
 [nan, 1.0, 1.0, nan]]


@result:
--graphical
```



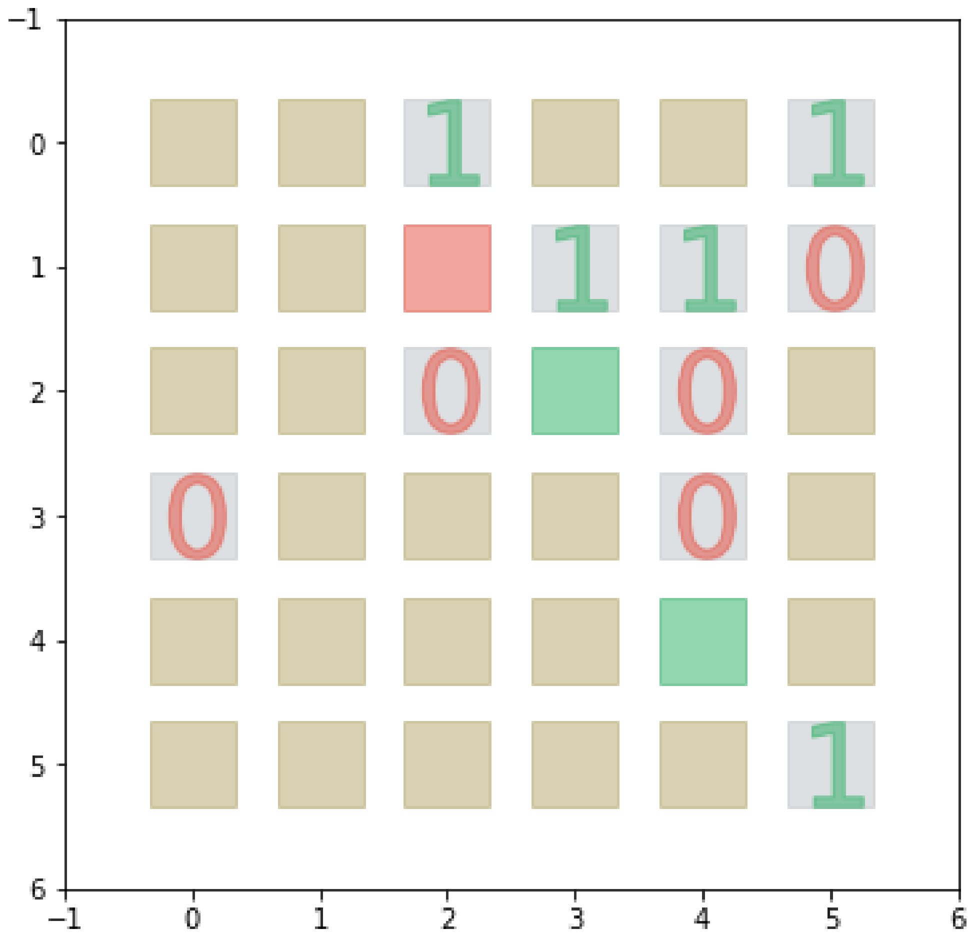

```
--textual
[[1, 0, 1, 0],
 [0, 1, 0, 1],
 [1, 0, 0, 1],
 [0, 1, 1, 0]]


#problem number 1 ------------------------
@successful: True
@elapsed time: 0.026 seconds
@start state:
--graphical
```



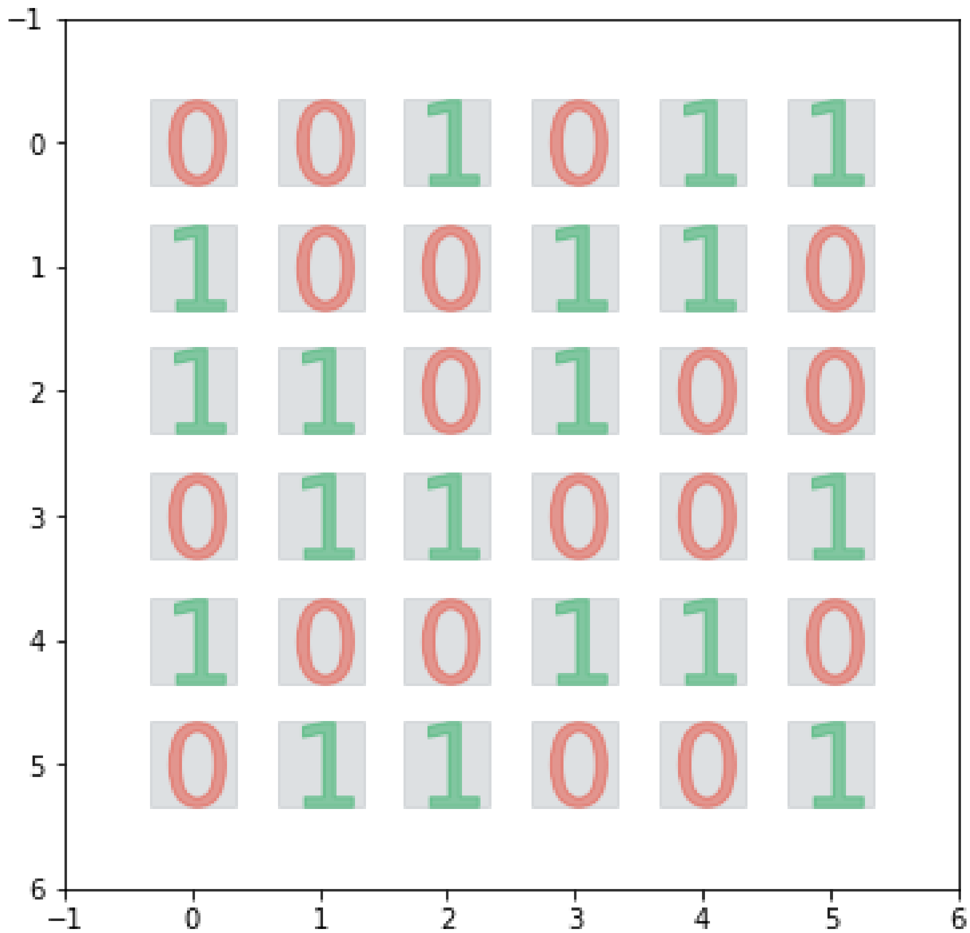

```
--textual
[[1.0, nan, nan, nan, nan, nan],
 [nan, 0.0, 1.0, 0.0, nan, nan],
 [nan, nan, nan, 0.0, nan, 0.0],
 [nan, 0.0, nan, nan, nan, nan],
 [nan, 1.0, 0.0, nan, nan, nan],
 [0.0, nan, 0.0, nan, 1.0, nan]]


@result:
--graphical
```



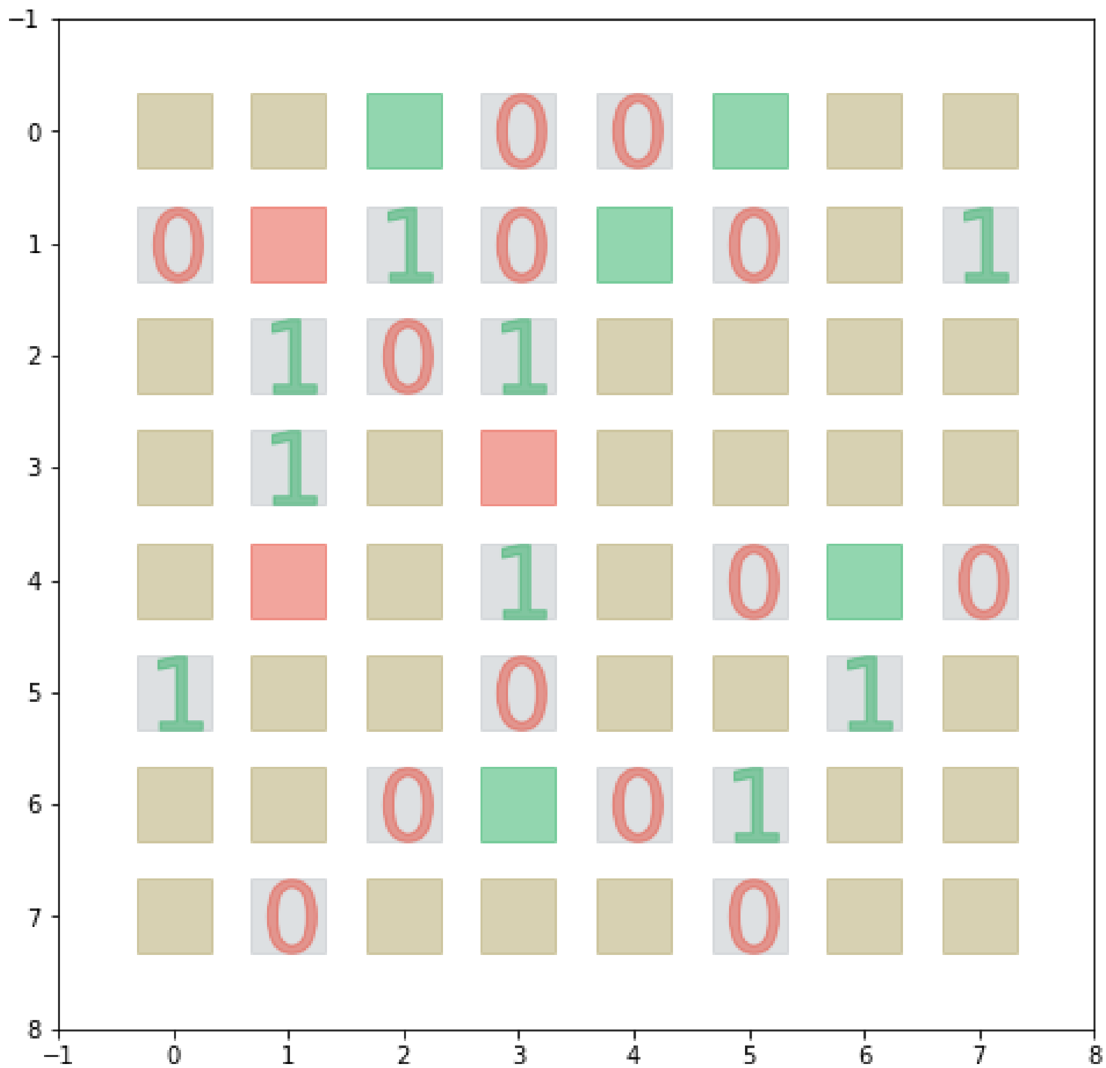

```
--textual
[[1, 0, 1, 1, 0, 0],
 [0, 0, 1, 0, 1, 1],
 [1, 1, 0, 0, 1, 0],
 [0, 0, 1, 1, 0, 1],
 [1, 1, 0, 1, 0, 0],
 [0, 1, 0, 0, 1, 1]]


#problem number 2 ------------------------
@successful: True
@elapsed time: 0.041 seconds
@start state:
--graphical
```



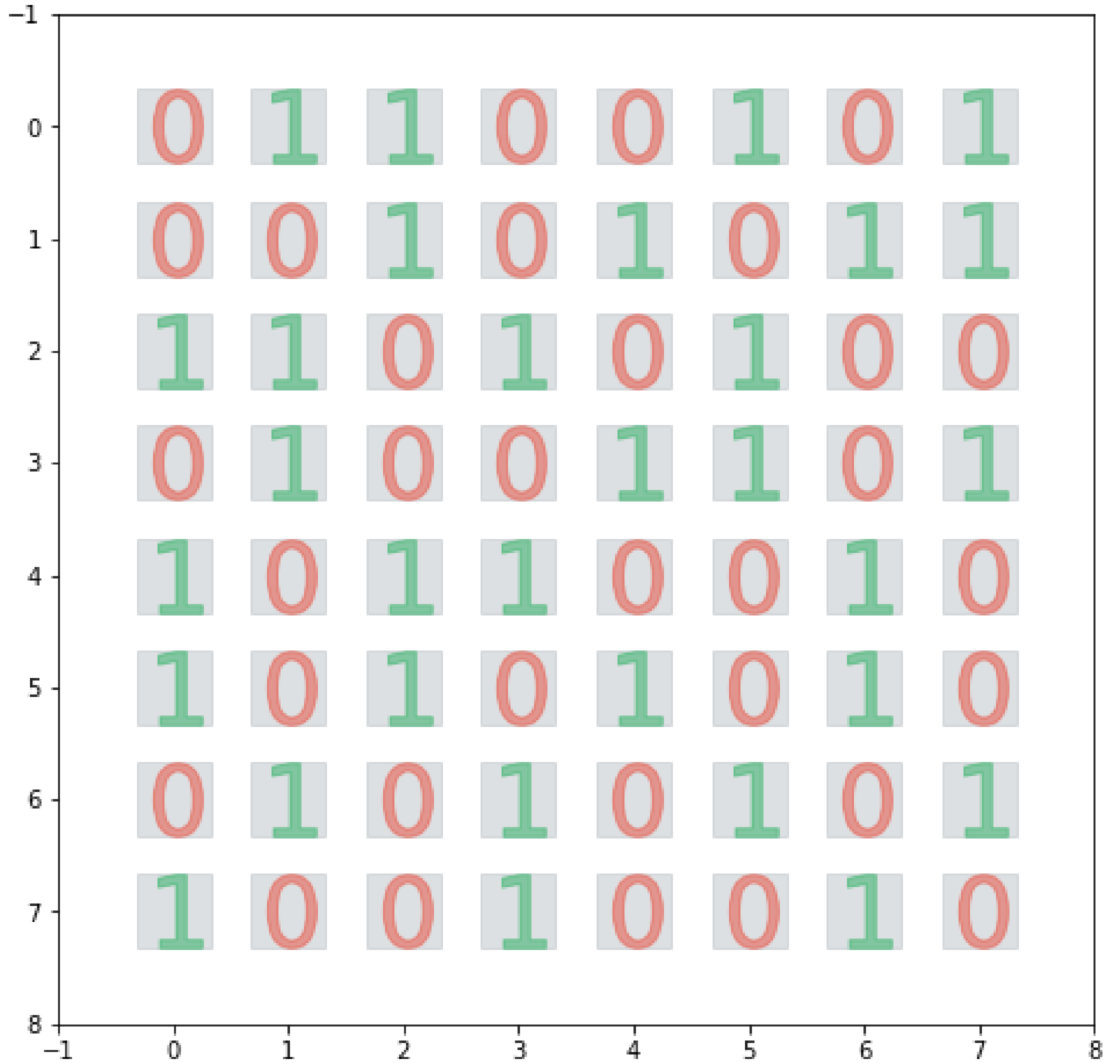

```
--textual
[[nan, nan, 1.0, nan, nan, 1.0],
 [nan, nan, nan, 1.0, 1.0, 0.0],
 [nan, nan, 0.0, nan, 0.0, nan],
 [0.0, nan, nan, nan, 0.0, nan],
 [nan, nan, nan, nan, nan, nan],
 [nan, nan, nan, nan, nan, 1.0]]


@result:
--graphical
```



```
--textual
[[0, 0, 1, 0, 1, 1],
 [1, 0, 0, 1, 1, 0],
 [1, 1, 0, 1, 0, 0],
 [0, 1, 1, 0, 0, 1],
 [1, 0, 0, 1, 1, 0],
 [0, 1, 1, 0, 0, 1]]


#problem number 3 ------------------------
@successful: True
@elapsed time: 0.15 seconds
@start state:
--graphical
```

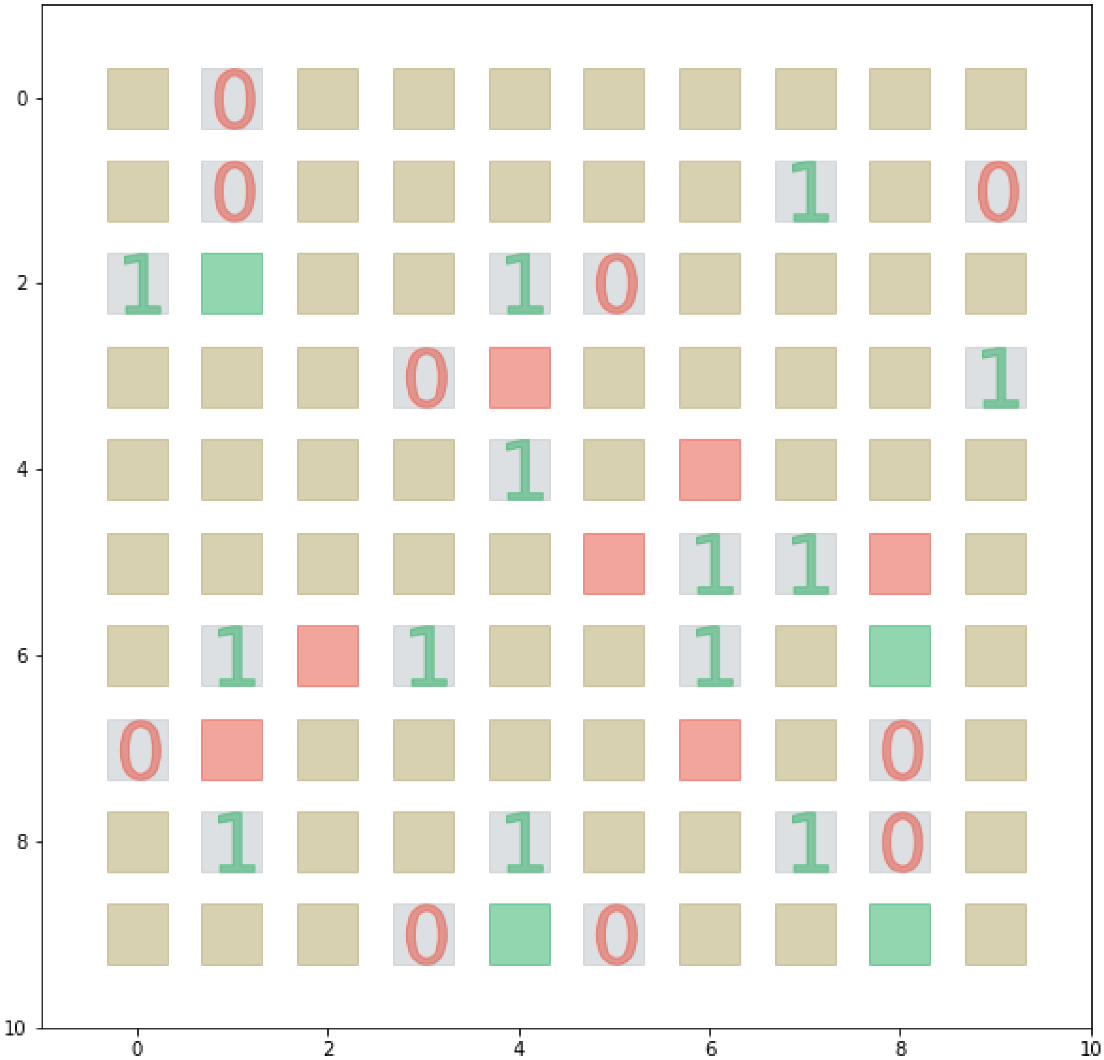



```
--textual
[[nan, nan, nan, 0.0, 0.0, nan, nan, nan],
 [0.0, nan, 1.0, 0.0, nan, 0.0, nan, 1.0],
 [nan, 1.0, 0.0, 1.0, nan, nan, nan, nan],
 [nan, 1.0, nan, nan, nan, nan, nan, nan],
 [nan, nan, nan, 1.0, nan, 0.0, nan, 0.0],
 [1.0, nan, nan, 0.0, nan, nan, 1.0, nan],
 [nan, nan, 0.0, nan, 0.0, 1.0, nan, nan],
 [nan, 0.0, nan, nan, nan, 0.0, nan, nan]]


@result:
--graphical
```



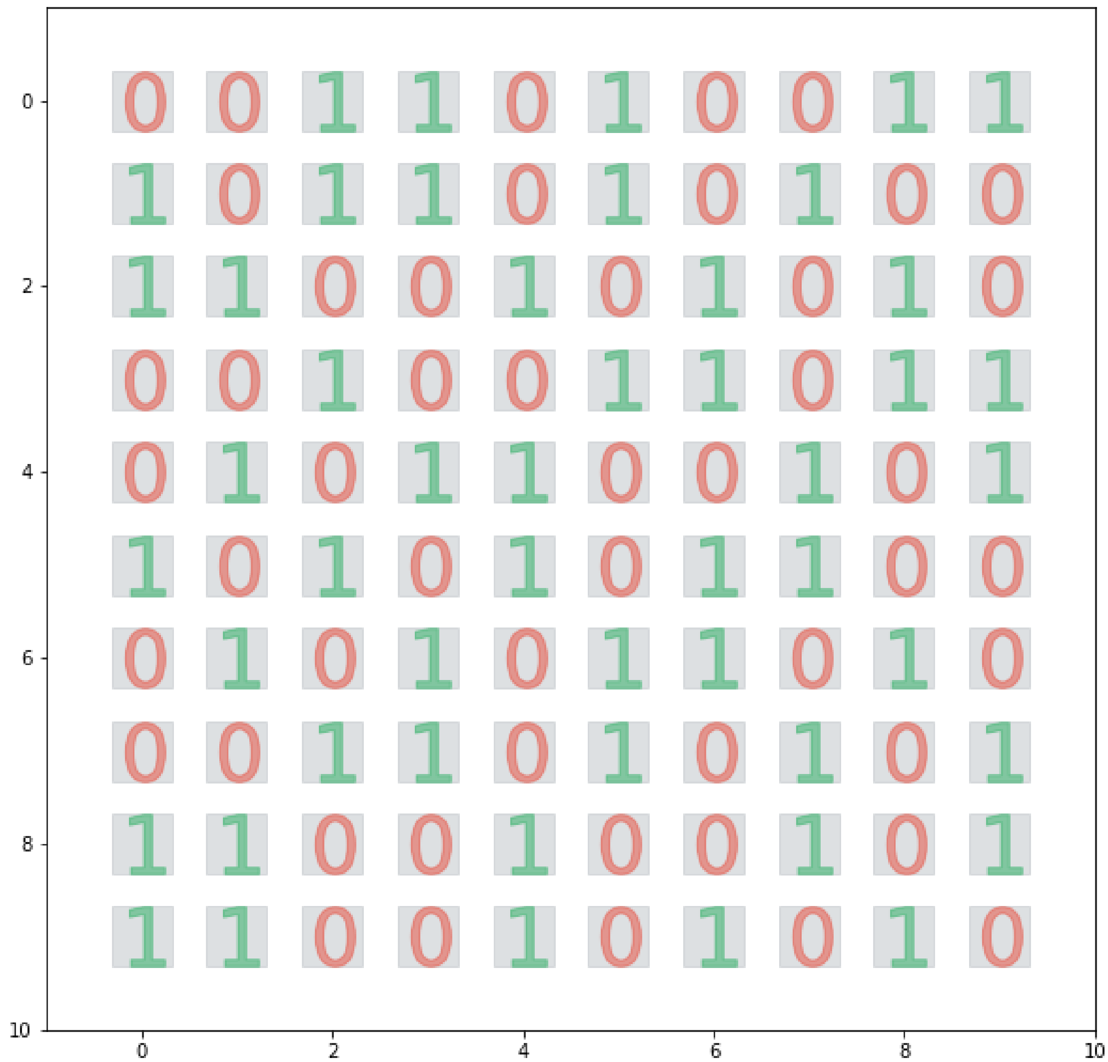

```
--textual
[[0, 1, 1, 0, 0, 1, 0, 1],
 [0, 0, 1, 0, 1, 0, 1, 1],
 [1, 1, 0, 1, 0, 1, 0, 0],
 [0, 1, 0, 0, 1, 1, 0, 1],
 [1, 0, 1, 1, 0, 0, 1, 0],
 [1, 0, 1, 0, 1, 0, 1, 0],
 [0, 1, 0, 1, 0, 1, 0, 1],
 [1, 0, 0, 1, 0, 0, 1, 0]]


#problem number 4 -----------------------
@successful: True
@elapsed time: 21.872 seconds
```

```
@start state:
--graphical
```



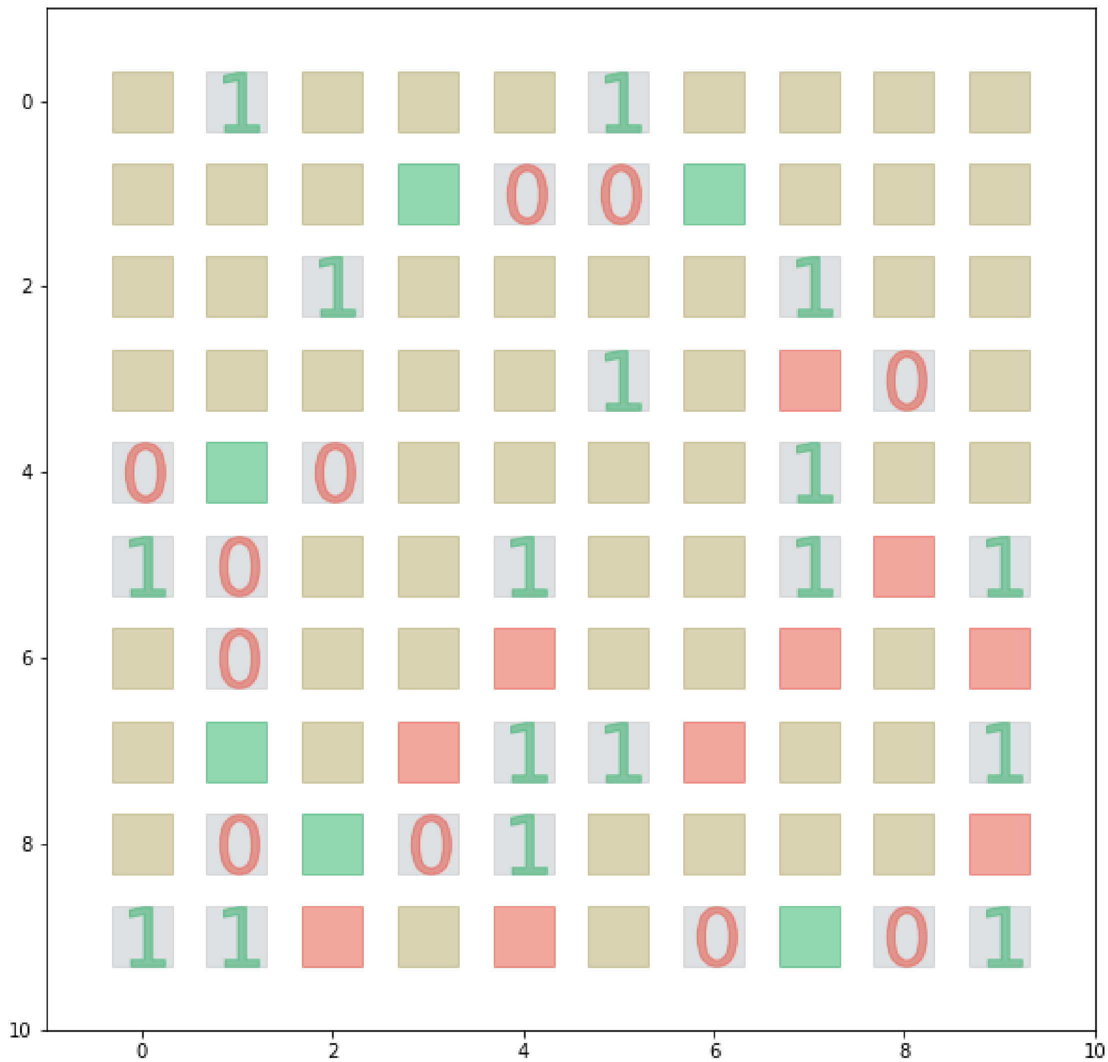

```
--textual
[[nan, 0.0, nan, nan, nan, nan, nan, nan, nan, nan],
 [nan, 0.0, nan, nan, nan, nan, nan, 1.0, nan, 0.0],
 [1.0, nan, nan, nan, 1.0, 0.0, nan, nan, nan, nan],
 [nan, nan, nan, 0.0, nan, nan, nan, nan, nan, 1.0],
 [nan, nan, nan, nan, 1.0, nan, nan, nan, nan, nan],
 [nan, nan, nan, nan, nan, nan, 1.0, 1.0, nan, nan],
 [nan, 1.0, nan, 1.0, nan, nan, 1.0, nan, nan, nan],
 [0.0, nan, nan, nan, nan, nan, nan, nan, 0.0, nan],
 [nan, 1.0, nan, nan, 1.0, nan, nan, 1.0, 0.0, nan],
 [nan, nan, nan, 0.0, nan, 0.0, nan, nan, nan, nan]]
```

```
@result:
--graphical
```



```
--textual
[[0, 0, 1, 1, 0, 1, 0, 0, 1, 1],
 [1, 0, 1, 1, 0, 1, 0, 1, 0, 0],
 [1, 1, 0, 0, 1, 0, 1, 0, 1, 0],
 [0, 0, 1, 0, 0, 1, 1, 0, 1, 1],
 [0, 1, 0, 1, 1, 0, 0, 1, 0, 1],
 [1, 0, 1, 0, 1, 0, 1, 1, 0, 0],
 [0, 1, 0, 1, 0, 1, 1, 0, 1, 0],
 [0, 0, 1, 1, 0, 1, 0, 1, 0, 1],
 [1, 1, 0, 0, 1, 0, 0, 1, 0, 1],
 [1, 1, 0, 0, 1, 0, 1, 0, 1, 0]]
```

```
#problem number 5 ------------------------
@successful: False
@elapsed time: 0.115 seconds
@start state:
--graphical
```




```
--textual
[[nan, 1.0, nan, nan, nan, 1.0, nan, nan, nan, nan],
 [nan, nan, nan, nan, 0.0, 0.0, nan, nan, nan, nan],
 [nan, nan, 1.0, nan, nan, nan, nan, 1.0, nan, nan],
 [nan, nan, nan, nan, nan, 1.0, nan, nan, 0.0, nan],
 [0.0, nan, 0.0, nan, nan, nan, nan, 1.0, nan, nan],
 [1.0, 0.0, nan, nan, 1.0, nan, nan, 1.0, nan, 1.0],
```

```
[nan, 0.0, nan, nan, nan, nan, nan, nan, nan, nan],
[nan, nan, nan, nan, 1.0, 1.0, nan, nan, nan, 1.0],
[nan, 0.0, nan, 0.0, 1.0, nan, nan, nan, nan, nan],
[1.0, 1.0, nan, nan, nan, nan, 0.0, nan, 0.0, 1.0]]
```

```
@result:
NO solution found…
```

[23]:
```python
initial_MAC = [] # after first updating domains
results_MAC = [] # final results
history_MAC = [] # history of each assignment in solving procedure
for index, config in enumerate(configurations):

    result = {}
    domains:    Dict[Tuple, List[int]] = config['domains']
    variables:  List[Tuple]            = config['variables']
    assignment: Dict[Tuple, int]       = config['assignment']

    s = len(config['map'])
    csp: CSP[Tuple, int] = CSP(variables, domains)

    csp.add_constraint(FirstConstraint(variables, s))
    csp.add_constraint(SecondConstraint(variables, s))
    csp.add_constraint(ThirdConstraint(variables, s))

    start = time.time()
    for a in assignment:
        domains = csp.MAC(domains, assignment, a)

    initial_MAC.append({
        'assignment': copy.deepcopy(assignment),
        'domains': copy.deepcopy(domains),
    })

    solution: Optional[Dict[Tuple, int]] = csp.
↪backtracking_search_with_MAC(assignment=assignment, domains=domains)
    end = time.time()

    history_MAC.append(copy.deepcopy(csp.backtracking_MAC_history))
    if solution is None:
        print("No solution found!")
        result['final']      = None
        result['time']       = (end - start) - csp.history_gathering_time
        result['successful'] = False
        result['waste']      = csp.history_gathering_time
    else:
```

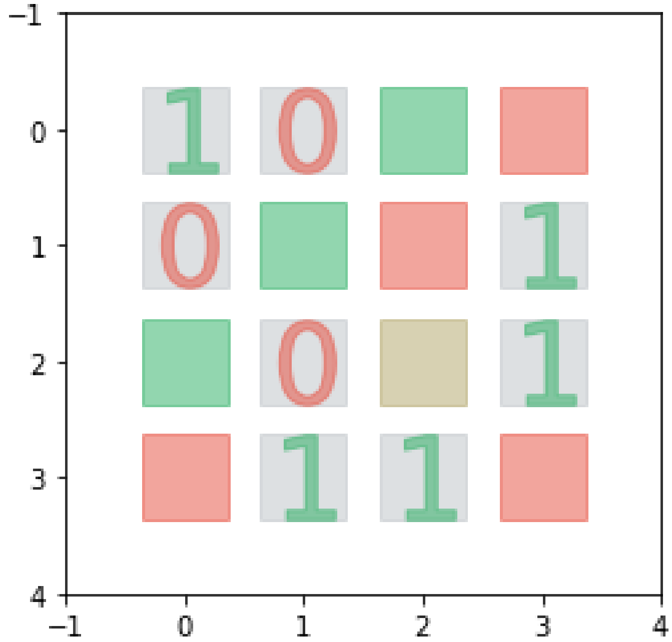
```
        print(f'elapsed time for problem {index}: {round(end - start, 5)}')
        mat = Matrix(s)
        mat.clear_populate(solution)
        m = mat.matrix

        assert np.where(np.isnan(m) == True)[0].size == 0 , "find np.nan in␣
↪result!"
        assert np.where(np.isnan(m) == True)[1].size == 0 , "find np.nan in␣
↪result!"

        result['final']      = m.astype(np.int8, copy=False)
        result['time']       = (end - start) - csp.history_gathering_time
        result['successful'] = True
        result['waste']      = csp.history_gathering_time

    results_MAC.append({
        'result': result,
        'domains': csp.domains
    })
```

```
elapsed time for problem 0: 0.01106
elapsed time for problem 1: 0.06098
elapsed time for problem 2: 0.08978
elapsed time for problem 3: 0.34405
elapsed time for problem 4: 64.11356
No solution found!
```

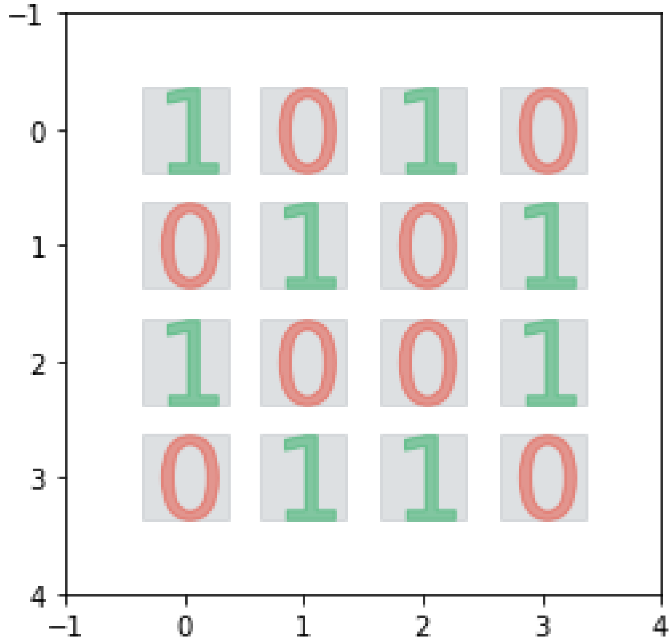
[ ]:
```
print(fuck you)
```

[25]:
```
report(configurations, results_MAC, initial_MAC)
```

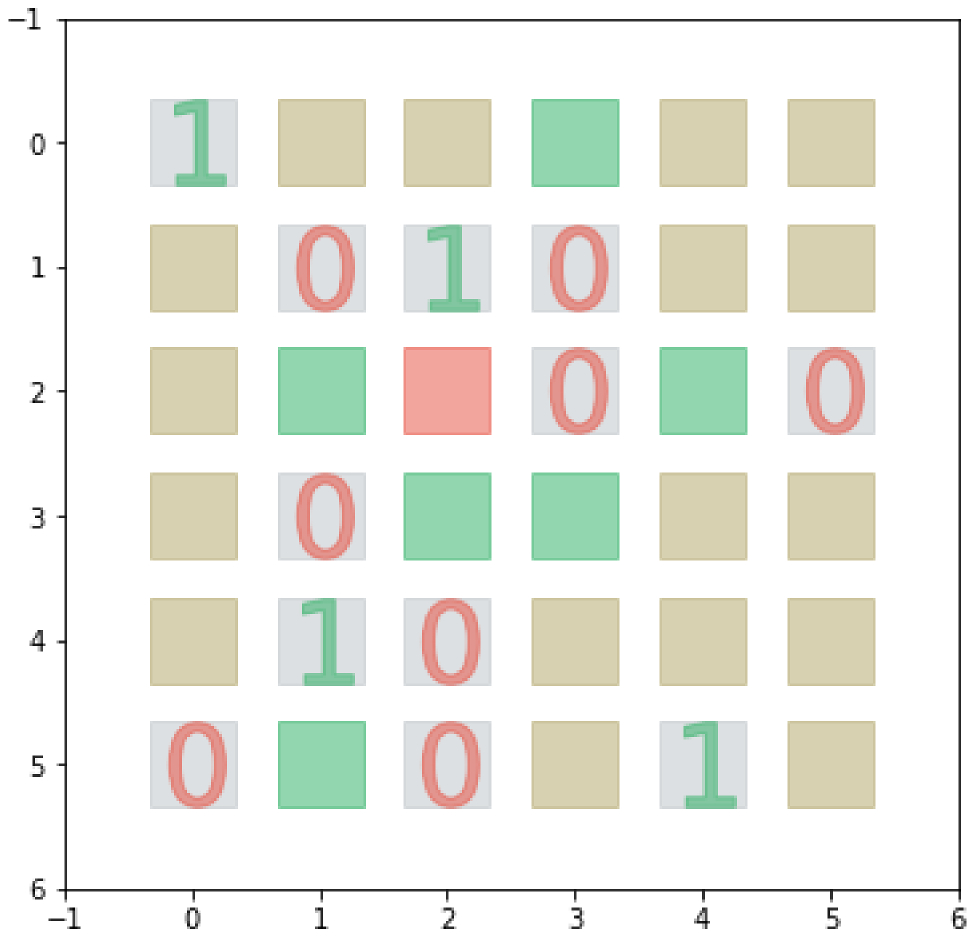
```
#problem number 0 ------------------------
@successful: True
@elapsed time: 0.01 seconds
@start state:
--graphical
```



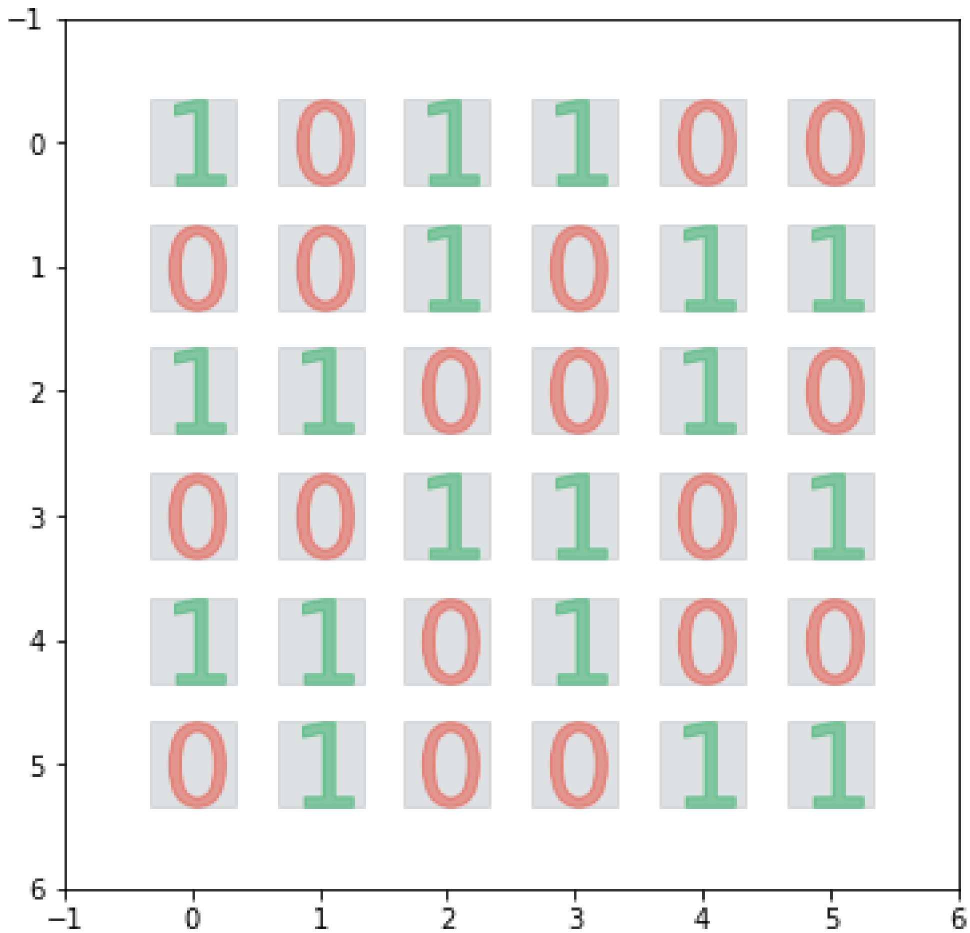

```
--textual
[[1.0, 0.0, nan, nan],
 [0.0, nan, nan, 1.0],
 [nan, 0.0, nan, 1.0],
 [nan, 1.0, 1.0, nan]]


@result:
--graphical
```



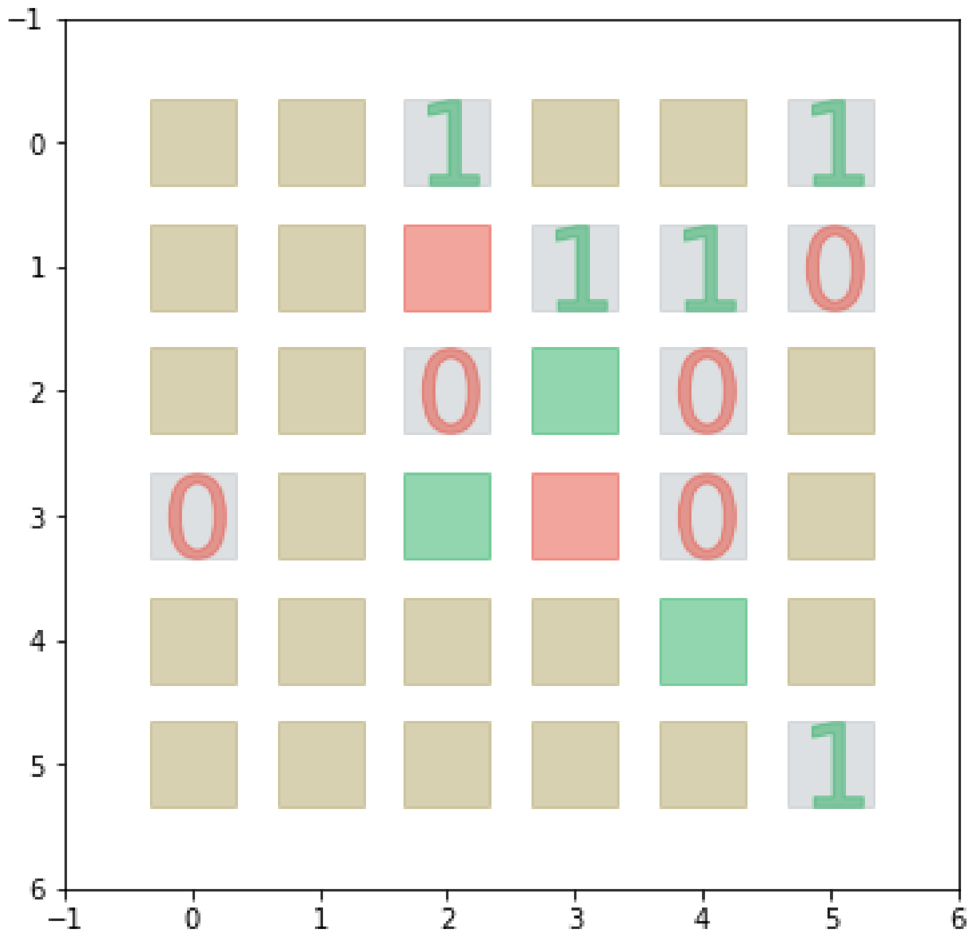

```
--textual
[[1, 0, 1, 0],
 [0, 1, 0, 1],
 [1, 0, 0, 1],
 [0, 1, 1, 0]]


#problem number 1 ------------------------
@successful: True
@elapsed time: 0.058 seconds
@start state:
--graphical
```



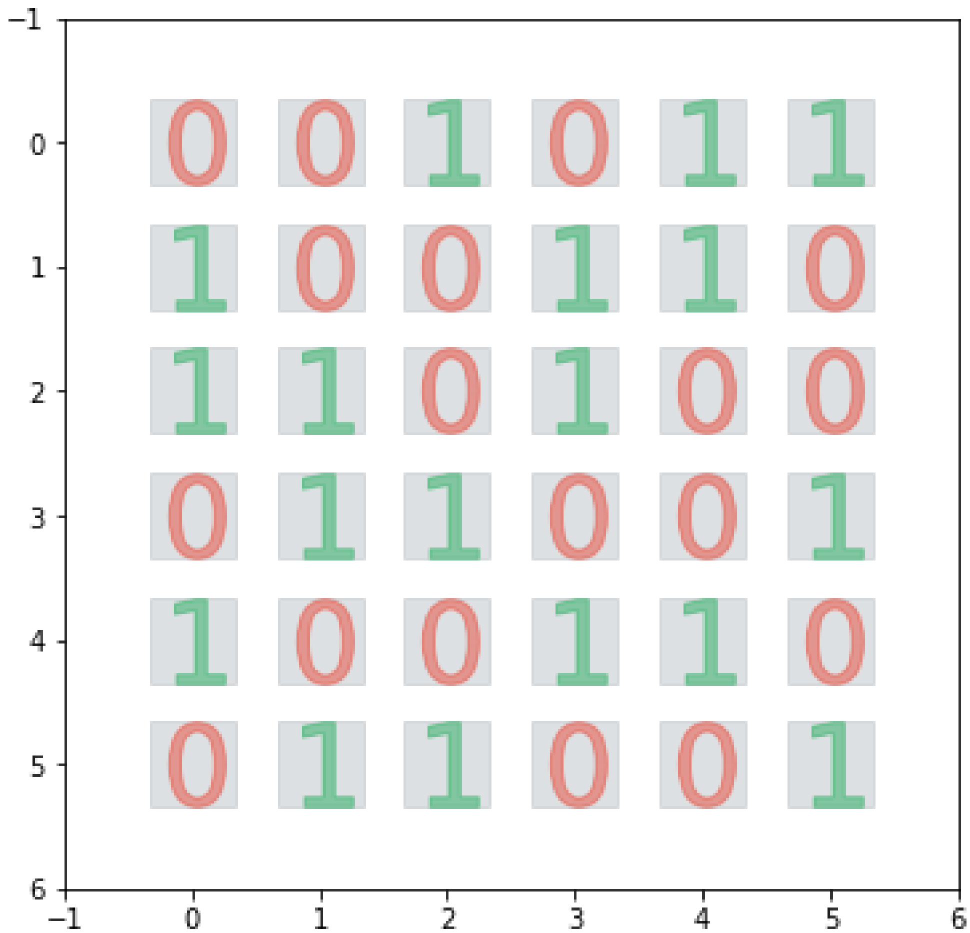

```
--textual
[[1.0, nan, nan, nan, nan, nan],
 [nan, 0.0, 1.0, 0.0, nan, nan],
 [nan, nan, nan, 0.0, nan, 0.0],
 [nan, 0.0, nan, nan, nan, nan],
 [nan, 1.0, 0.0, nan, nan, nan],
 [0.0, nan, 0.0, nan, 1.0, nan]]


@result:
--graphical
```



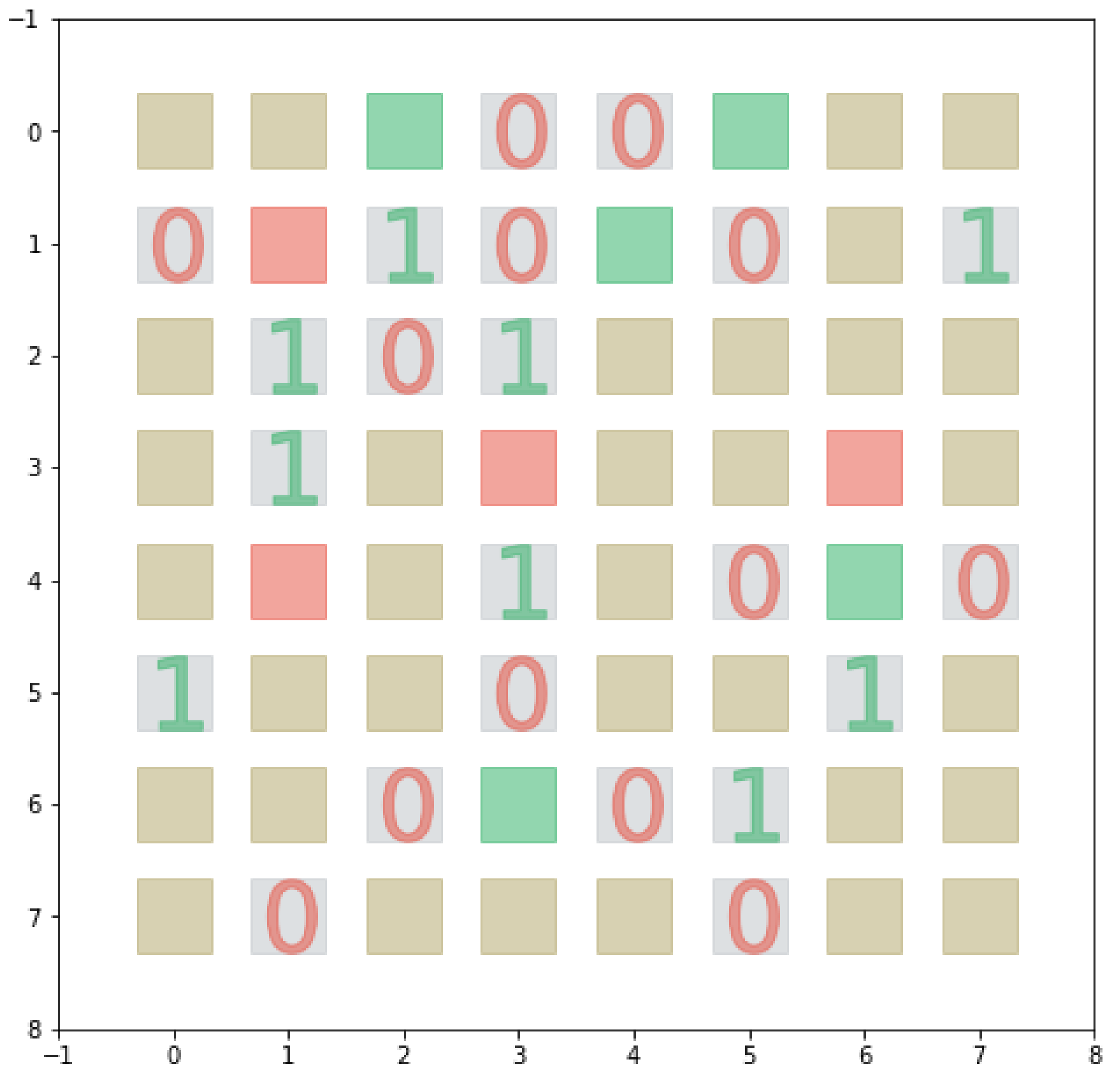

```
--textual
[[1, 0, 1, 1, 0, 0],
 [0, 0, 1, 0, 1, 1],
 [1, 1, 0, 0, 1, 0],
 [0, 0, 1, 1, 0, 1],
 [1, 1, 0, 1, 0, 0],
 [0, 1, 0, 0, 1, 1]]


#problem number 2 ------------------------
@successful: True
@elapsed time: 0.082 seconds
@start state:
--graphical
```



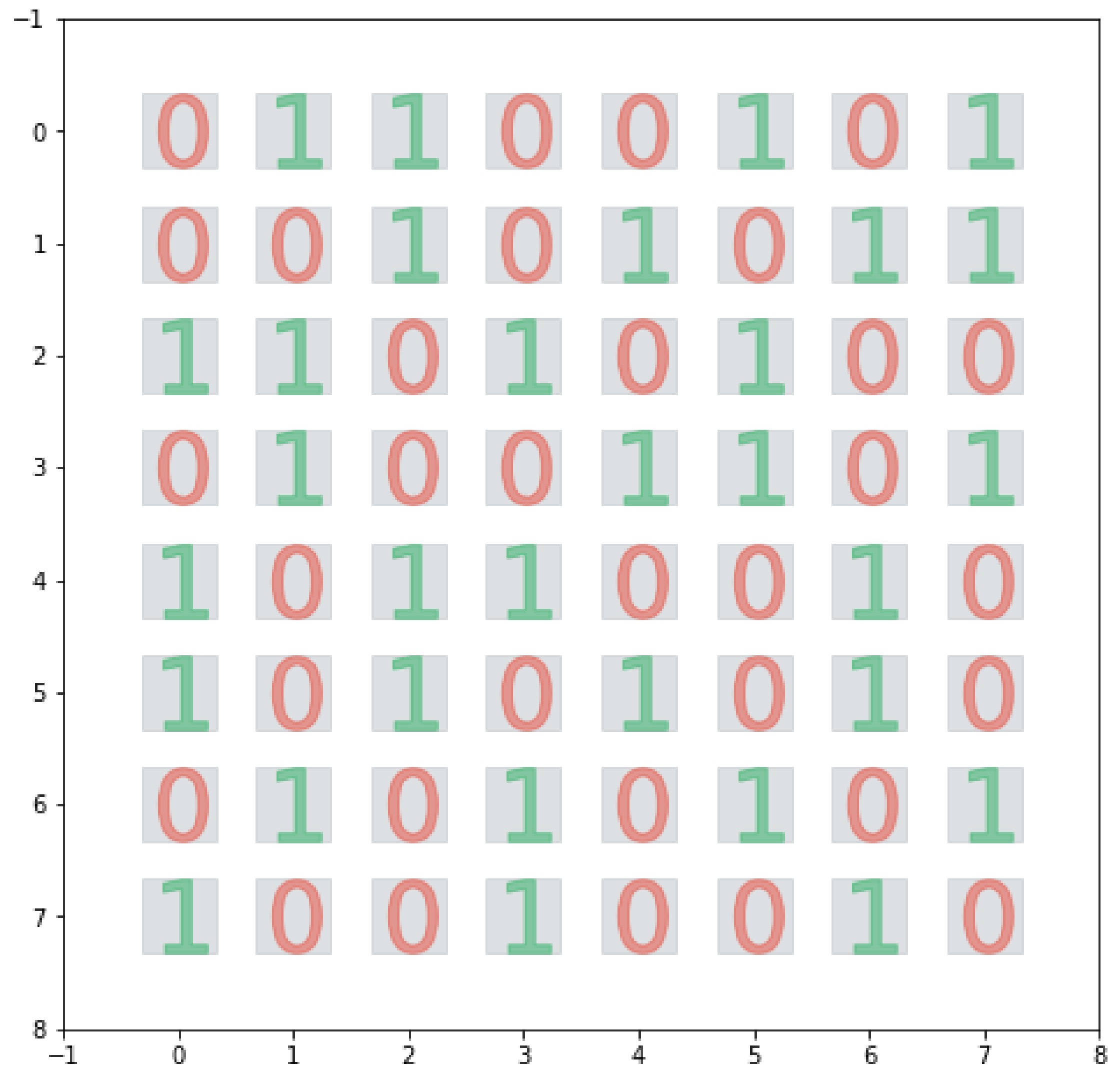

```
--textual
[[nan, nan, 1.0, nan, nan, 1.0],
 [nan, nan, nan, 1.0, 1.0, 0.0],
 [nan, nan, 0.0, nan, 0.0, nan],
 [0.0, nan, nan, nan, 0.0, nan],
 [nan, nan, nan, nan, nan, nan],
 [nan, nan, nan, nan, nan, 1.0]]


@result:
--graphical
```




```
--textual
[[0, 0, 1, 0, 1, 1],
 [1, 0, 0, 1, 1, 0],
 [1, 1, 0, 1, 0, 0],
 [0, 1, 1, 0, 0, 1],
 [1, 0, 0, 1, 1, 0],
 [0, 1, 1, 0, 0, 1]]


#problem number 3 ------------------------
@successful: True
@elapsed time: 0.308 seconds
@start state:
--graphical
```




```
--textual
[[nan, nan, nan, 0.0, 0.0, nan, nan, nan],
 [0.0, nan, 1.0, 0.0, nan, 0.0, nan, 1.0],
 [nan, 1.0, 0.0, 1.0, nan, nan, nan, nan],
 [nan, 1.0, nan, nan, nan, nan, nan, nan],
 [nan, nan, nan, 1.0, nan, 0.0, nan, 0.0],
 [1.0, nan, nan, 0.0, nan, nan, 1.0, nan],
 [nan, nan, 0.0, nan, 0.0, 1.0, nan, nan],
 [nan, 0.0, nan, nan, nan, 0.0, nan, nan]]


@result:
--graphical
```

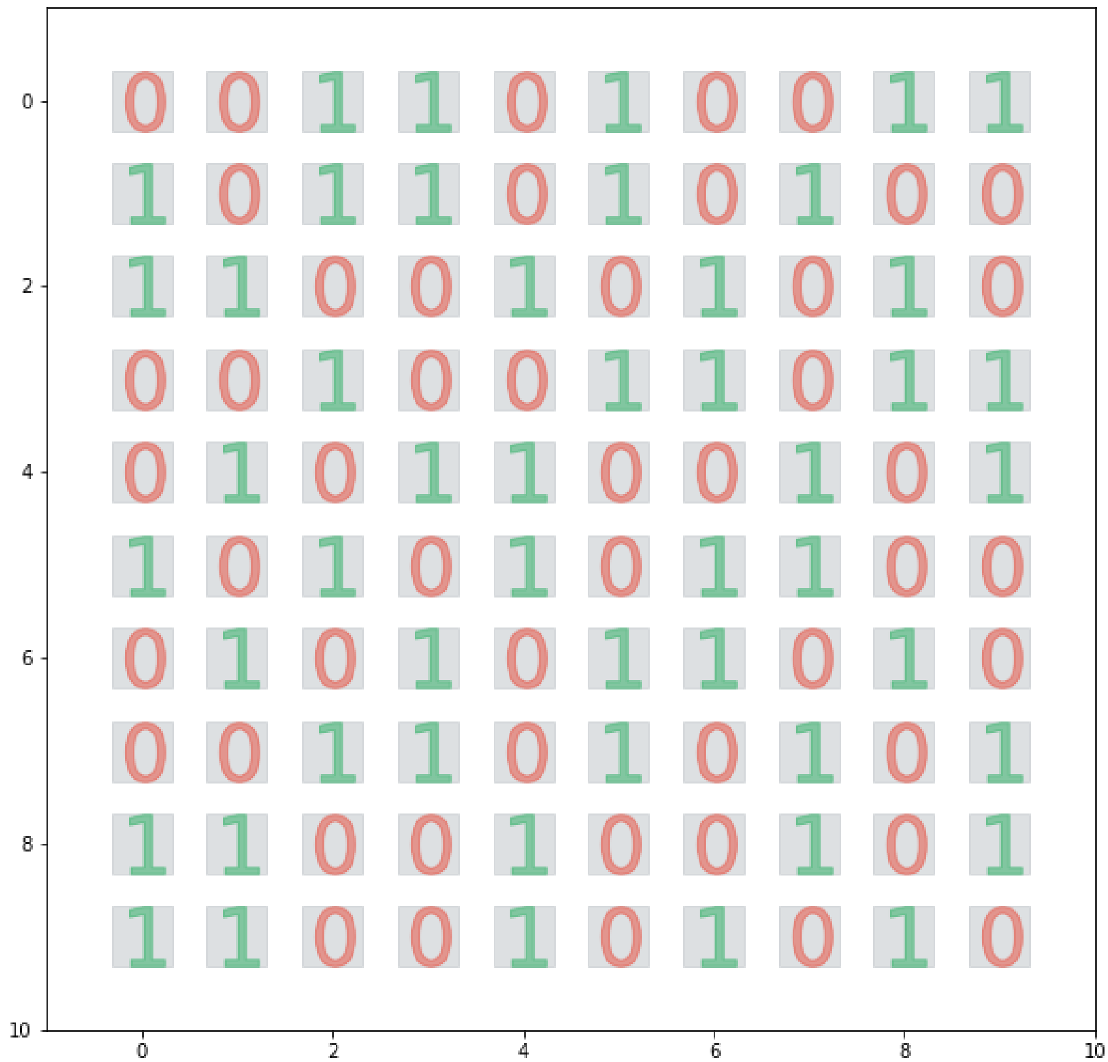



```
--textual
[[0, 1, 1, 0, 0, 1, 0, 1],
 [0, 0, 1, 0, 1, 0, 1, 1],
 [1, 1, 0, 1, 0, 1, 0, 0],
 [0, 1, 0, 0, 1, 1, 0, 1],
 [1, 0, 1, 1, 0, 0, 1, 0],
 [1, 0, 1, 0, 1, 0, 1, 0],
 [0, 1, 0, 1, 0, 1, 0, 1],
 [1, 0, 0, 1, 0, 0, 1, 0]]


#problem number 4 -----------------------
@successful: True
@elapsed time: 57.387 seconds
```

```
@start state:
--graphical
```

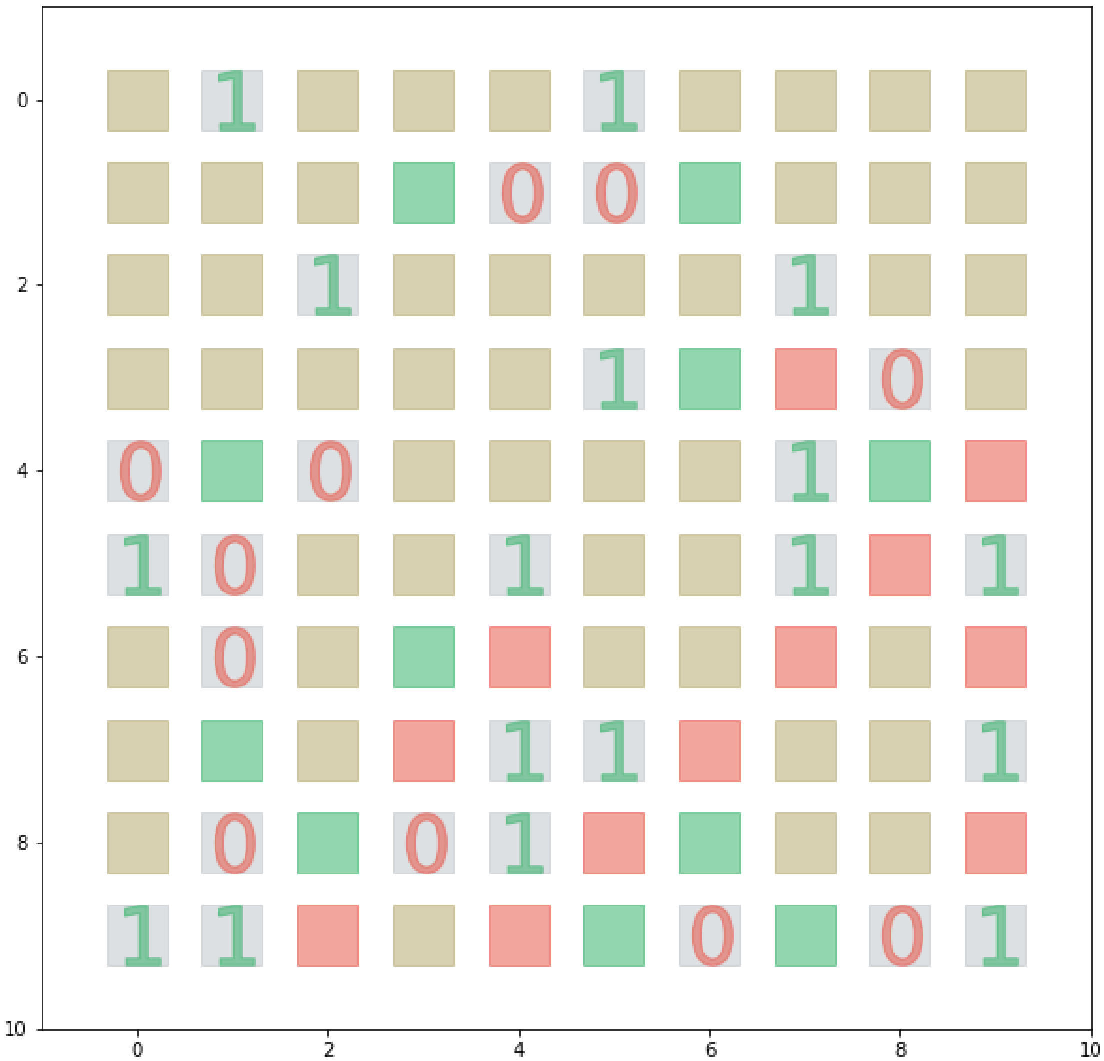

```
--textual
[[nan, 0.0, nan, nan, nan, nan, nan, nan, nan, nan],
 [nan, 0.0, nan, nan, nan, nan, nan, 1.0, nan, 0.0],
 [1.0, nan, nan, nan, 1.0, 0.0, nan, nan, nan, nan],
 [nan, nan, nan, 0.0, nan, nan, nan, nan, nan, 1.0],
 [nan, nan, nan, nan, 1.0, nan, nan, nan, nan, nan],
 [nan, nan, nan, nan, nan, nan, 1.0, 1.0, nan, nan],
 [nan, 1.0, nan, 1.0, nan, nan, 1.0, nan, nan, nan],
 [0.0, nan, nan, nan, nan, nan, nan, nan, 0.0, nan],
 [nan, 1.0, nan, nan, 1.0, nan, nan, 1.0, 0.0, nan],
 [nan, nan, nan, 0.0, nan, 0.0, nan, nan, nan, nan]]
```

```
@result:
--graphical
```




```
--textual
[[0, 0, 1, 1, 0, 1, 0, 0, 1, 1],
 [1, 0, 1, 1, 0, 1, 0, 1, 0, 0],
 [1, 1, 0, 0, 1, 0, 1, 0, 1, 0],
 [0, 0, 1, 0, 0, 1, 1, 0, 1, 1],
 [0, 1, 0, 1, 1, 0, 0, 1, 0, 1],
 [1, 0, 1, 0, 1, 0, 1, 1, 0, 0],
 [0, 1, 0, 1, 0, 1, 1, 0, 1, 0],
 [0, 0, 1, 1, 0, 1, 0, 1, 0, 1],
 [1, 1, 0, 0, 1, 0, 0, 1, 0, 1],
 [1, 1, 0, 0, 1, 0, 1, 0, 1, 0]]
```

```
#problem number 5 ------------------------
@successful: False
@elapsed time: 0.266 seconds
@start state:
--graphical
```




```
--textual
[[nan, 1.0, nan, nan, nan, 1.0, nan, nan, nan, nan],
 [nan, nan, nan, nan, 0.0, 0.0, nan, nan, nan, nan],
 [nan, nan, 1.0, nan, nan, nan, nan, 1.0, nan, nan],
 [nan, nan, nan, nan, nan, 1.0, nan, nan, 0.0, nan],
 [0.0, nan, 0.0, nan, nan, nan, nan, 1.0, nan, nan],
 [1.0, 0.0, nan, nan, 1.0, nan, nan, 1.0, nan, 1.0],
```

```
 [nan, 0.0, nan, nan, nan, nan, nan, nan, nan, nan],
 [nan, nan, nan, nan, 1.0, 1.0, nan, nan, nan, 1.0],
 [nan, 0.0, nan, 0.0, 1.0, nan, nan, nan, nan, nan],
 [1.0, 1.0, nan, nan, nan, nan, 0.0, nan, 0.0, 1.0]]


@result:
NO solution found…
```

[28]:
```
export_images(history_, '../images/vanilla')
```

```
exporting images in ../images/vanilla
exporting images for problem 0 ,elapsed time: 0.7695629596710205, images: 15
exporting images for problem 1 ,elapsed time: 12.853966236114502, images: 174
exporting images for problem 2 ,elapsed time: 15.172691106796265, images: 199
exporting images for problem 3 ,elapsed time: 23.192261695861816, images: 235
exporting images for problem 4, too many images [144541] => aborted
exporting images for problem 5, too many images [345895] => aborted
```

[29]:
```
export_images(history_FC, '../images/FC')
```

```
exporting images in ../images/FC
exporting images for problem 0 ,elapsed time: 0.42652058601379395, images: 8
exporting images for problem 1 ,elapsed time: 1.5740976333618164, images: 25
exporting images for problem 2 ,elapsed time: 2.3200955390930176, images: 37
exporting images for problem 3 ,elapsed time: 5.530399560928345, images: 74
exporting images for problem 4 ,elapsed time: 625.7136776447296, images: 7609
exporting images for problem 5 ,elapsed time: 2.106293201446533, images: 26
```

[30]:
```
export_images(history_MAC, '../images/MAC')
```

```
exporting images in ../images/MAC
exporting images for problem 0 ,elapsed time: 0.435619592666626, images: 8
exporting images for problem 1 ,elapsed time: 1.528517723083496, images: 25
exporting images for problem 2 ,elapsed time: 2.331545114517212, images: 37
exporting images for problem 3 ,elapsed time: 5.54382061958313, images: 71
exporting images for problem 4 ,elapsed time: 644.7141005992889, images: 7415
exporting images for problem 5 ,elapsed time: 2.223836898803711, images: 26
```

[34]:
```
frame_rates = []
for index, history in enumerate(history_):
    if isinstance(history, list):
        if len(history) < 10:
            frame_rates.append(1)
        elif 10 <= len(history) < 1000 :
            frame_rates.append(5)
        elif 1000 <= len(history):
            frame_rates.append(15)
```

```
    else:
        frame_rates.append(None)

make_animation(frame_rates, 'vanilla')
```

```
making animation for problem 0
making animation for problem 1
making animation for problem 2
making animation for problem 3
```

[35]:
```
frame_rates = []
for index, history in enumerate(history_FC):
    if isinstance(history, list):
        if len(history) < 10:
            frame_rates.append(1)
        elif 10 <= len(history) < 1000 :
            frame_rates.append(5)
        elif 1000 <= len(history):
            frame_rates.append(15)
    else:
        frame_rates.append(None)

make_animation(frame_rates, 'FC')
```

```
making animation for problem 0
making animation for problem 1
making animation for problem 2
making animation for problem 3
making animation for problem 4
making animation for problem 5
```

[36]:
```
frame_rates = []
for index, history in enumerate(history_MAC):
    if isinstance(history, list):
        if len(history) < 10:
            frame_rates.append(1)
        elif 10 <= len(history) < 1000 :
            frame_rates.append(5)
        elif 1000 <= len(history):
            frame_rates.append(15)
    else:
        frame_rates.append(None)

make_animation(frame_rates, 'MAC')
```

```
making animation for problem 0
making animation for problem 1
making animation for problem 2
```

```
making animation for problem 3
making animation for problem 4
making animation for problem 5
```

1.4 Conclusion

1.4.1 as we see in FC we have less computation time (especially on problem 4) but in MAC we have less wrong assignments. (the amount of this differences, heavily depends on the type of problem)

[37]:

```python
# headers = ['', 'simple BT', 'BT with FC', 'BT with MAC']
headers = ['', 'time', 'branchs', 'successful']
records = []
records_ = []
records_FC = []
records_MAC = []
import tabulate

for index, config in enumerate(configurations):
    records_.append((f'. puzzle {index}',
                     f"{round(results_[index]['result']['time'], 4):.4f}",
                     len(history_[index]) if isinstance(history_[index], list)␣
 ↪else history_[index],
                     results_[index]['result']['successful']))
# records_.append((f'# puzzle {index + 1}', f'very long!', 'many!', False))

for index, config in enumerate(configurations):
    records_FC.append((f'. puzzle {index}',
                     f"{round(results_FC[index]['result']['time'], 4):.4f}",
                     len(history_FC[index]),
                     results_FC[index]['result']['successful']))


for index, config in enumerate(configurations):
    records_MAC.append((f'. puzzle {index}',
                     f"{round(results_MAC[index]['result']['time'], 4):.4f}",
                     len(history_MAC[index]),
                     results_MAC[index]['result']['successful']))

records.append(('# simple BT', None, None, None))
records.extend(records_)
records.append((None, None, None, None))
records.append(('# BT with FC', None, None, None))
records.extend(records_FC)
records.append((None, None, None, None))
records.append(('# BT with MAC', None, None, None))
records.extend(records_MAC)
```

```python
print(tabulate.tabulate(records, headers=headers, tablefmt='psql'))
```

```
+---------------+----------+-----------+--------------+
|               |     time |   branchs | successful   |
|---------------+----------+-----------+--------------|
| # simple BT   |          |           |              |
| . puzzle 0    |   0.001  |        15 | True         |
| . puzzle 1    |   0.0531 |       174 | True         |
| . puzzle 2    |   0.0507 |       199 | True         |
| . puzzle 3    |   0.1049 |       235 | True         |
| . puzzle 4    |  99.5105 |    144541 | True         |
| . puzzle 5    | 228.97   |    345895 | False        |
|               |          |           |              |
| # BT with FC  |          |           |              |
| . puzzle 0    |   0.005  |         8 | True         |
| . puzzle 1    |   0.0261 |        25 | True         |
| . puzzle 2    |   0.041  |        37 | True         |
| . puzzle 3    |   0.1502 |        74 | True         |
| . puzzle 4    |  21.8716 |      7609 | True         |
| . puzzle 5    |   0.1146 |        26 | False        |
|               |          |           |              |
| # BT with MAC |          |           |              |
| . puzzle 0    |   0.0101 |         8 | True         |
| . puzzle 1    |   0.0579 |        25 | True         |
| . puzzle 2    |   0.0817 |        37 | True         |
| . puzzle 3    |   0.3081 |        71 | True         |
| . puzzle 4    |  57.3866 |      7415 | True         |
| . puzzle 5    |   0.266  |        26 | False        |
+---------------+----------+-----------+--------------+
```