Linear programming notes

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Abstract

This document contains notes about Linear programming.

1 Supply chain model

We define a simplified supply chain model as a system that produces products with a chain of product transformations performed by suppliers.

Let be

$A = a_1 \dots a_n$ the set of supplier types

$B = b_1 \dots b_m$ the set of product types.

$N_i, \ i \in A$ the number of suppliers of type i.

The supplier can perform only a single transformation for the its duration.

Let us define

N_i the number of suppliers $i \in A$

V_i the value of product $i \in B$

Q_{ij} the quantity of produced product $j \in B$ for supplier $i \in A, j$ in production interval

$T_{ij}, i \in A, j \in B$ the production interval for product $j \in B$ in supplier $i \in A, j$

C_{ijk} the quantity of product $k \in B$ consumed to produce the product $j \in Bj$ in production interval of supplier $i \in A$.

1.1 Production

By now we do not consider the constraint on the availability of consuming products. It will be considered later.

When a supplier of type i is ready to produce we assign a production slot for the product j such that

Y_{ij} is the weighted distribution of slots

The value Y_{ij} is constrained to

$$0 \leq Y_{ij} \leq 1 \tag{1}$$

Moreover we may put the supplier in idle state for a while

Z_{i0} is the idle time during a whole production cycle

with

$$Z_{i0} \geq 0 \tag{2}$$

The total time of production cycle for supplier i is

$$Z_i = \sum_{j \in B} Y_{ij} T_{ij} + Z_{i0}$$

Let normalize the total time production to 1

$$Z_i = \sum_{j \in B} Y_{ij} T_{ij} + Z_{i0} = 1 \tag{3}$$

During this interval all the suppliers i produce the product j at a slot rate

$$S_{ij} = N_i \frac{Y_{ij}}{Z_i} = N_i Y_{ij} \Big|_{i \in A, j \in B} \tag{4}$$

The production rate of product j for supplier i is

$$P_{ij} = Q_{ij} S_{ij} = N_i Q_{ij} Y_{ij}|_{i \in A, j \in B}$$

While the production rate of product j is

$$P_{j \in B} = \sum_{i \in A,} P_{ij}$$

1.2 Constraints

The (4) expresses the slot rate of a product for specific suppliers.

Therefore we can calculate the consumption rate of a product k to produce product j for suppliers i

$$E_{ijk} = C_{ijk} S_{ij}|_{i \in A, j, k \in B}$$

Then the total consumption rate for product k is

$$E_k = \sum_{i \in A, j \in B} E_{ijk}$$

If

$$E_i < P_i$$

the product s produced at a higher rate then it is consumped creating a surplus that can be sold.

On the other hane we cannot have

$$E_i > P_i$$

because it cannot consume more product than the produced.

So the system must satisfy the constraint

$$P_i - E_i \geq 0 \tag{5}$$

1.3 Value rate

We can compute the value rate for the whole supply chain

$$V = \sum_{i \in B} (P_i - E_i) V_i \tag{6}$$

The problem is to find the optimal production configuration that maximize the value rate. This is defined by the linear system composed by (1), (2), (3), (5), (6)

$$\begin{cases} \max_{(Y_{ij}, Z_{i0})}(V) & , i \in A, j \in B \\ \sum_{j \in B} Y_{ij} T_{ij} + Z_{i0} = 1 & , i \in A \\ Z_{i0} \geq 0 & , i \in A \\ Y_{ij} \geq 0 & , i \in A, j \in B \\ Y_{ij} \leq 1 & , i \in A, j \in B \\ P_i - E_i \geq 0 & , i \in B \end{cases}$$

1.4 Tensor form

Let us normalize the problem such that

$$\begin{cases} \min C^T X \\ A_1 X = B_1 \\ A_2 X <= B_2 \\ X >= 0 \end{cases}$$

so

$$\min(-V) = \min(-\sum_{i \in B} (P_i - E_i) V_i) =$$