

# Linear programming notes

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## Abstract

This document contains notes about Linear programming.

## 1 Supply chain model

We define a simplified supply chain model as a system that produces products with a chain of product transformations performed by suppliers.

Let it be

$A = a_1 \dots a_n$  the set of supplier types

$B = b_1 \dots b_m$  the set of product types.

The supplier can perform only a single transformation for the its duration. Each product can be produced by a single supplier type.

Let it be

$S = s_i \in A$  the supplier of product  $i \in B$

$N = n_i$  the number of suppliers of type  $i \in A$ .

$V = v_i$  the value of product  $i \in B$

$Q = q_i$  the quantity of produced product  $i \in B$

$T = t_i$  the production interval for product  $i \in B$

$D = d_{ij}$  the quantity of product  $j \in B$  consumed to produce the product  $i \in B$ .

## 1.1 Production

By now we do not consider the constraint on the availability of consuming products. It will be considered later.

Let be

$\theta_{ij}$  The matrix that map product  $j \in B$  to the supplier  $i \in A$

$\nu_i$  The number of supplier of product  $i \in B$

such that

$$\begin{aligned}\theta_{ij} &= 1, s_j = i \\ \theta_{ij} &= 0, s_j \neq i \\ \nu_i &= n_{s_j}\end{aligned}$$

When a supplier of type  $i \in A$  is ready to produce we assign a production slot for the product  $j \in B$  such that  $s_j = i$  and the production time is  $t_j w_j$  where

$W = w_i$  is the slot weight for product  $i \in B$

such that

$$w_i \geq 0 \quad (1)$$

Moreover we may put the supplier  $i \in A$  in idle state for a while

$Z = z_i$  is the idle time during a whole production cycle

such that

$$z_i \geq 0 \quad (2)$$

The total time of production cycle for supplier  $i$  is

$$\begin{aligned}U_i(w_j, z_i) &= \sum_{j \in B | s_j = i} t_j w_j + z_i \\ &= \sum_{j \in B} \theta_{ij} t_j w_j + z_i \\ &= \sum_{j, k \in B} \theta_{ik} T_{kj} w_j + z_i \\ &= UW + Z\end{aligned}$$

such that

$$U = u_{ij} = \sum_{k \in B} \theta_{ik} T_{kj} = \Theta T$$

Let normalize the total time production to 1

$$U_i(w_j, z_i) = 1 \quad (3)$$

During this interval the suppliers produce the product  $i \in A$  at a slot rate

$$R_i(w_i) = n_{s_i} \frac{w_i}{y_{s_i}} = n_{s_i} w_i = \sum_{j \in B} N_{ij} w_j = NW \quad (4)$$

The production rate of product  $i \in B$  is

$$P_i(w_j) = R_i(w_j) q_i = Q_{ik} R_k(w_j) = QNW = PW \quad (5)$$

such that

$$P = QN$$

## 1.2 Consumption

The (4) expresses the slot rate of a product for specific suppliers. Therefore we can calculate the consumption rate of a product  $i \in B$  as the sum of consumptions among to produced products

$$C_i(w_j) = \sum_{k \in B} R_k(w_j) d_{ki} = \sum_{k \in B} d'_{ik} R_k(w_j) = D'NW = CW \quad (6)$$

such that

$$C = D'N$$

The effective production rate is

$$F_i(w_j) = P_i(w_j) - C_i(w_j) = (P - C)W = FW$$

such that

$$F = f_{ij} = P - C = (Q - D')N$$

If

$$F_i(w_j) \geq 0$$

the product is produced at a higher rate then it is consumed creating a surplus that can be sold.

On the other hane we cannot have

$$F_i(w_j) < 0$$

because it cannot consume more product than the produced.

So the system must satisfy the constraint

$$F_i(w_j) \geq 0 \quad (7)$$

## 1.3 Profit rate

The profit rate for the whole supply chain are

$$G(w_i) = \sum_{j \in B} v_j F_j(w_i) = V'FW = G'W \quad (8)$$

such that

$$G = g_i = (V'F)'F'V = N'(Q' - D)V$$

The problem is to find the optimal production configuration given by the values of  $w_i$  and  $z_i$  that maximize the value rate  $G(w_i)$ . This is defined by the linear system composed by (1) (2), (3), (7), (8)

$$\begin{cases} \max_{(w_i, z_j)} G(w_i) & , i \in B, j \in A \\ U_i(w_j, z_i) = 1 & , i \in A, j \in B \\ F_i \geq 0 & , i \in B \\ w_i \geq 0 & , i \in B \\ z_i \geq 0 & , i \in A \end{cases} \quad (9)$$

## 1.4 Supplier configuration

Una volta risolto il sistema (9) ci troviamo con le distribuzioni degli slot di produzione dei produttori. Per l'uso pratico dobbiamo trasformare i pesi di distribuzione degli slot in una configurazione di produzione reale.

Il primo passo per far questo è trasformare le  $w_i$  e  $z_i$  nel numero di produttori per prodotto.

Il numero di produttori che producono il prodotto  $i \in B$  è dato dal tempo totale di produzione dei suppliers per ogni singolo prodotto rispetto al totale  $U_i(w_i, z_j) = 1$

$$\pi_i = n_{s_i} w_i t_i$$

$$\Pi = NTW$$

La parte intera di  $\Pi$  ci da il numero di produttori fissi per prodotto

$$F = |\Pi|$$

Il numero totale di produttori fissi è quindi

$$tot_i = \sum_{j \in B} \theta_{ij} f_j$$

$$Tot = \Theta F$$

I rimanenti produttori

$$Var = N - Tot$$

sono distribuiti secondo la parte frazionaria  $\Pi - F$ .