

Statistical packages - report 3

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February 4, 2019

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1 Introduction

This report concerns periodic inspections and interval censored data. The first task is to prepare a generator for periodically inspected lightbulb. Then there would be presented a short analysis basing on this generator. Finally there are two approaches shown: first uses the naive estimator (without censored data) and the second one with censored data.

1.1 Task 1 – creating generator

We assume, that the life times of lightbulbs is a vector of iid random variables from exponential distribution with failure rate λ . The times between inspections is a vector of iid random variables from exponential distribution with rate v . Change of a failed lightbulb can occur only at the moment of inspection. The generator should take 3 parameters: two are mentioned above and additional T_0 which is the time horizon. The output of the generator is:

- interval censored lifetimes of lightbulbs,
- moments of inspection,
- moments of light failures.

The exemplary samples are shown on the figure ??

```
> v <- .7
> lambda <- 0.5
> T_0 <- 100
> inspection_times <- c(0, sort(runif(n = rpois(1, lambda = T_0 * v), min = 0, max = T_0)))
> lightbulb <- 1
> new_bulb_moments <- c(0)
> lightbulbTime <- rexp(1, rate = lambda)
> lightbulbLifetimes <- c()
> for (i in 1:(length(inspection_times) - 1)){
+   if ( inspection_times[i + 1] - new_bulb_moments[lightbulb] > lightbulbTime){
```

```

+     new_bulb_moments <- append(new_bulb_moments, inspection_times[i + 1])
+     lightbulb <- lightbulb + 1
+     lightbulbLifetimes <- append(lightbulbLifetimes, lightbulbTime)
+     lightbulbTime <- rexp(1, rate = lambda)
+   }
+ }
> df_inspection_times <- data.frame(number = 1:length(inspection_times), moments = inspection_times)

```

1.2 Task 2 – generator analysis

This section presents the answers for the question about generator – in our case the parameters of the generator are $v = 0.7$ $\lambda = 0.5$ $T_0 = 100$ and we made 100 Monte Carlo iterations (when it was needed):

1. What is the percentage of time when there is no light?
Answer: The share of time without light is at level 49.5%.
2. What is the average number of lightbulb replacements?
Answer: The average number of replacements is 29.75.
3. What is the average time without light?
Answer: The average time without light is 1.47.

```

> momentsOfFailure <- lightbulbLifetimes + new_bulb_moments[1:length(new_bulb_moments) - 1]
> percentage_of_time_withoutlight <- sum(new_bulb_moments[2:length(new_bulb_moments)]
+   - momentsOfFailure) / tail(new_bulb_moments, n = 1)
> N <- 100
> v <- .7
> lambda <- 0.5
> T_0 <- 100
> avg_no_of_replacements <- 0
> avg_time_without_light <- 0
> for (j in 1:N){
+   inspection_times <- c(0, sort(runif(n = rpois(1, lambda = T_0 * v), min = 0, max = T_0)))
+
+   lightbulb <- 1
+   new_bulb_moments <- c(0)
+   lightbulbTime <- rexp(1, rate = lambda)
+   lightbulbLifetimes <- c()
+   for (i in 1:(length(inspection_times) - 1)){
+     if ( inspection_times[i + 1] - new_bulb_moments[lightbulb] > lightbulbTime){
+       new_bulb_moments <- append(new_bulb_moments, inspection_times[i + 1])
+       lightbulb <- lightbulb + 1
+       lightbulbLifetimes <- append(lightbulbLifetimes, lightbulbTime)
+       lightbulbTime <- rexp(1, rate = lambda)
+     }
+   }
+   momentsOfFailure <- lightbulbLifetimes + new_bulb_moments[1:length(new_bulb_moments) - 1]
+   avg_time_without_light <- avg_time_without_light
+     + mean(new_bulb_moments[2:length(new_bulb_moments)]
+     - momentsOfFailure)
+   avg_no_of_replacements <- avg_no_of_replacements + lightbulb
+ }
> avg_no_of_replacements <- avg_no_of_replacements / N
> avg_time_without_light <- avg_time_without_light / N

```

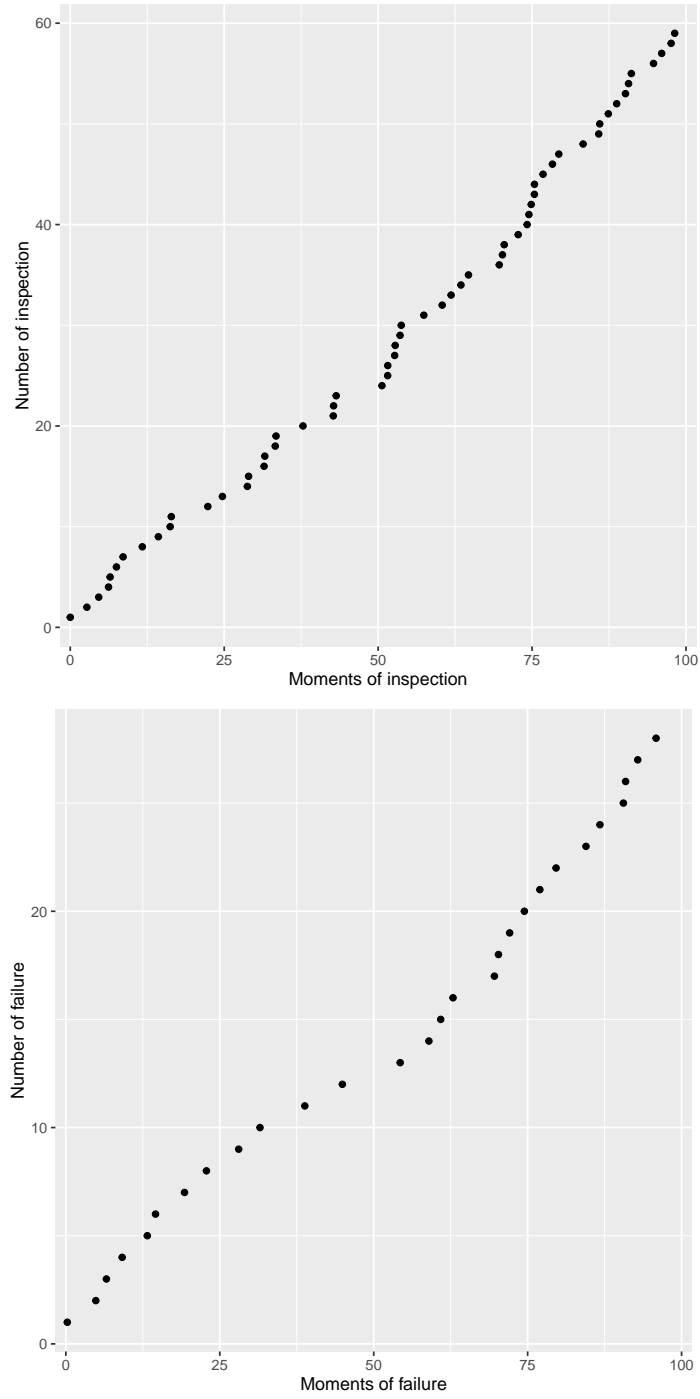


Figure 1: Samples produced by generator with $v = 0.7$, $\lambda = 0.5$, $T_0 = 100$

1.3 Task 3 – naive estimator

This approach assumes that failures occurred during inspections. We estimate the failure rate using an average of right sides of intervals.

```
> list_of_means <- c()
> list_of_vars <- c()
> list_of_biases <- c()
> list_of_mses <- c()
> N <- 100
> # for (T_0 in seq(50,250,10)){
> for (v in seq(0.5,3,0.1)){
+ # for (lambda in seq(0.1,2.5,0.1)){
+   # v <- .7
+   lambda <- 0.5
+   T_0 <- 100
+   avg_mean <- 0
+   avg_var <- 0
+   avg_bias <- 0
+   avg_mse <- 0
+   for (j in 1:N){
+     inspection_times <- c(0, sort(runif(n = rpois(1, lambda = T_0 * v), min = 0, max = T_0)))
+     lightbulb <- 1
+     new_bulb_moments <- c(0)
+     lightbulbTime <- rexp(1, rate = lambda)
+     lightbulbLifetimes <- c()
+     for (i in 1:(length(inspection_times) - 1)){
+       if ( inspection_times[i + 1] - new_bulb_moments[lightbulb] > lightbulbTime){
+         new_bulb_moments <- append(new_bulb_moments, inspection_times[i + 1])
+         lightbulb <- lightbulb + 1
+         lightbulbLifetimes <- append(lightbulbLifetimes, lightbulbTime)
+         lightbulbTime <- rexp(1, rate = lambda)
+       }
+     }
+     naive_lightbulb_lifetime <- new_bulb_moments[2:length(new_bulb_moments)] - new_bulb_moments[1:length(new_bulb_moments)]
+     avg_mean <- avg_mean + mean(naive_lightbulb_lifetime)
+     avg_var <- avg_var + var(naive_lightbulb_lifetime)
+     avg_bias <- lambda - mean(naive_lightbulb_lifetime)
+   }
+ }
>
```

The charts 2 and 3 presents the dependence between mean, variance bias and MSE and respectively failure rate and inspection rate. In both cases the mean and variance have downward trend. The bias and the MSE result in values near 0 because we have taken 100 Monte Carlo Steps.

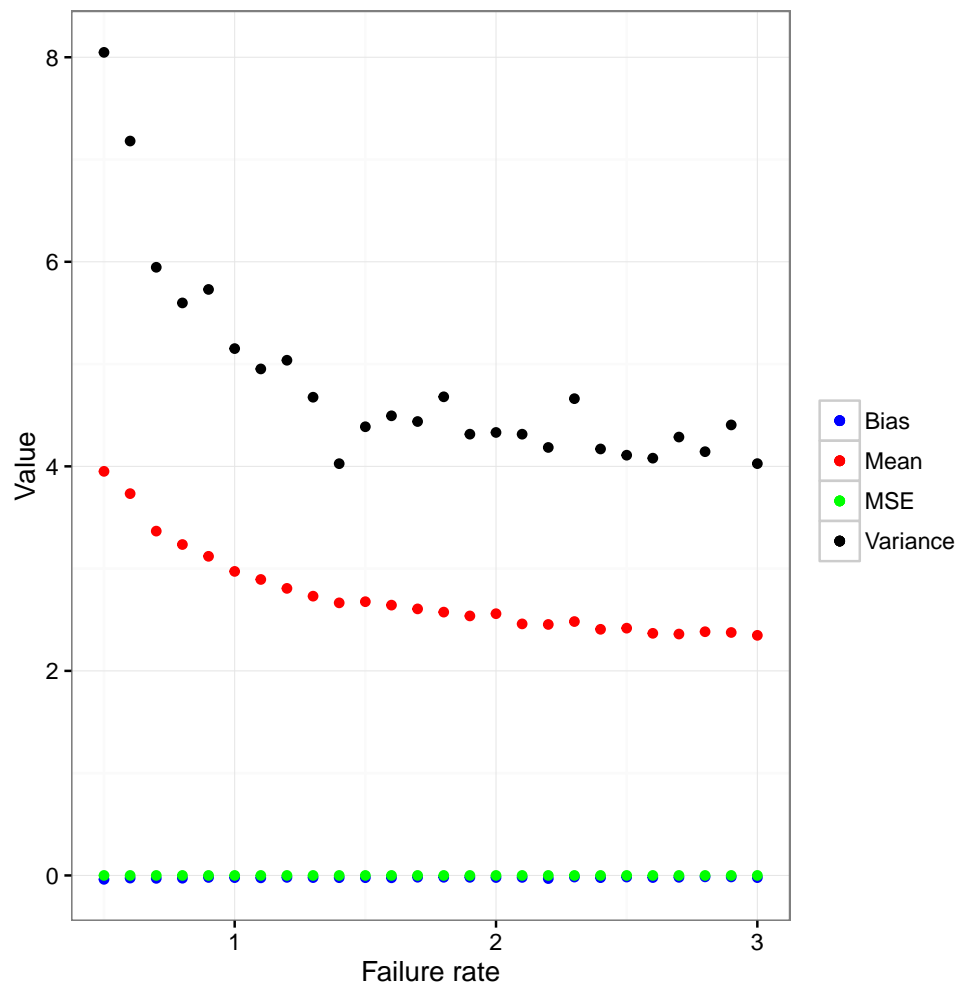


Figure 2: Characteristics in dependence of failure rate.

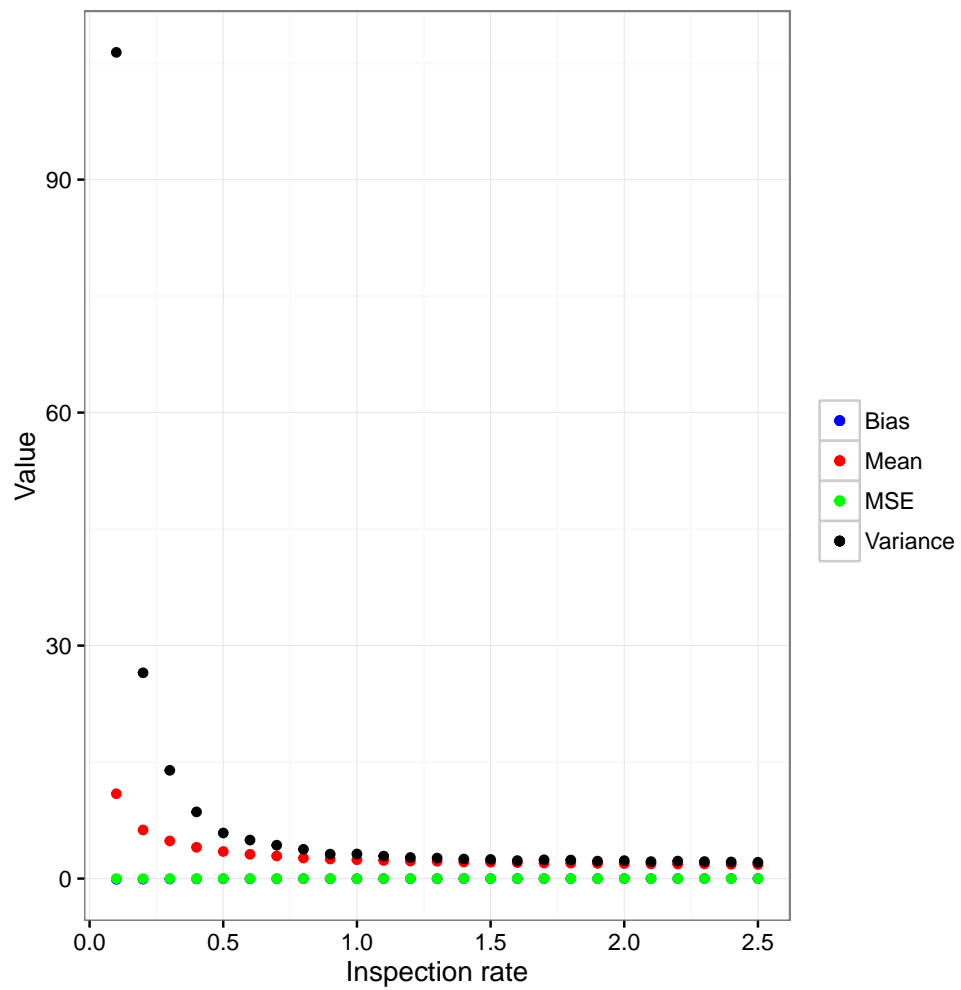


Figure 3: Characteristics in dependence of inspection rate.

2 Conclusions

Our analysis consist of only one of approach about censored data analysis. That one is a simple approach in which we assume that the failurs occures in the same time as inspections. In such case our analysis shows that about half of time there is no light. Taking into consideration the mean and the variance of the estimator we obtain the results that shows that with increasing inspection rate or falure rate both of this characteristics decrease. Another aproach is the Turnbull estimator which reject the asumption about the occurance of the failure in the inspection times. This way of analysing such data seems to be more correct.