ARLIZ

A JOURNEY THROUGH ARRAYS

Mahdi

ARLIZ

ARRAYS • REASONING • LOGIC • IDENTITY • ZERO

"From ancient counting stones to quantum algorithms—every data structure tells the story of human ingenuity."

LIVING FIRST EDITION

Updated August 14, 2025

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A Living Architecture of Computing

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Repository: The complete source code (LaTeX, diagrams, examples) is available at: https://github.com/m-mdy-m/Arliz

Preferred Citation Format:
Mahdi. (2025). *Arliz*. Retrieved from https://github.com/m-mdy-m/Arliz

Version Control: This is a living document. Check the repository for the most current version and revision history.

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Educational Purpose: This work is intended for educational and research purposes. Practical implementation of algorithms and techniques should be thoroughly tested and validated for production use.

TECHNICAL SPECIFICATIONS

Typeset with: LaTeX using Charter and Palatino font families
Graphics: TikZ and custom illustrations
Standards: Follows academic publishing conventions
Encoding: UTF-8 with full Unicode support
Format: Available in PDF, and LaTeX source formats

License last updated: August 14, 2025

For questions about licensing, contact: bitsgenix@gmail.com

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Preface

Every book has its origin story, and this one is no exception. If I were to capture the essence of creating this book in a single word, that word would be **curiosity**—though *improvised* comes as a close second. What you hold in your hands (or view on your screen) is the result of years of persistent questioning, a journey that began with a simple yet profound realization: I didn't truly understand what an array was.

This might sound trivial to some. After all, arrays are fundamental to programming, covered in every computer science curriculum, explained in countless tutorials. Yet despite encountering terms like `array`, `stack`, `queue`, `linked list`, `hash table`, and `heap` repeatedly throughout my studies, I found myself increasingly frustrated by the superficial explanations typically offered. Most resources assumed you already knew what these structures fundamentally represented—their conceptual essence, their historical significance, their mathematical foundations.

But I wanted the *roots*. I needed to understand not just how to use an array, but what it truly meant, how it came to exist, and what hidden capacities it possessed. This led me to a decisive moment:

If I truly want to understand, I must start from zero.

And so began the journey that became Arliz.

The Name and Its Meaning

The name "Arliz" started as a somewhat arbitrary choice—I needed a title, and it sounded right. However, as the book evolved, I discovered a fitting expansion that captures its essence:

Arliz = Arrays, Reasoning, Logic, Identity, Zero

This backronym embodies the core pillars of our exploration:

- **Arrays:** The fundamental data structure we seek to understand from its origins
- **Reasoning:** The logical thinking behind systematic data organization
- **Logic:** The formal principles that govern how computers manipulate information
- **Identity:** The concept of distinguishing, indexing, and assigning meaning to elements within structures
- **Zero:** The philosophical and mathematical foundation from which all computation, counting, and indexing originate

You may pronounce it "Ar-liz," "Array-Liz," or however feels natural to you. I personally say "ar-liz," but the pronunciation matters less than the journey it represents.

What This Book Represents

Arliz is not merely a technical manual on data structures, nor is it a traditional computer science textbook. Instead, it represents something more personal and, I believe, more valuable: a comprehensive exploration of understanding itself. This book grows alongside my own learning, evolving as I discover better ways to explain concepts, uncover new connections, and develop deeper insights.

This living nature means that Arliz is, in many ways, a conversation—between past and present understanding, between theoretical foundations and practical applications, between the author and reader. As long as I continue learning, Arliz will continue growing.

The structure of this book reflects a fundamental belief: genuine understanding requires context. Rather than beginning with syntax and moving to application (the typical approach), we start with the conceptual and historical foundations that make modern data structures possible. We trace the evolution of human thought about organizing information, from ancient counting methods to contemporary computing paradigms.

This approach serves a specific purpose: when you understand the intellectual journey that led to arrays, you develop an intuitive grasp of their behavior, limitations, and potential that no amount of syntax memorization can provide.

My Approach and Principles

Throughout the writing process, I have maintained three core principles:

1. **Conceptual Clarity:** Every concept is presented in its simplest form while maintaining accuracy and depth. My goal is accessibility without superficiality.
2. **Visual Understanding:** Complex ideas are accompanied by diagrams, figures, and visual examples. I believe that concepts which can be visualized are concepts that can be truly understood and retained.
3. **Practical Implementation:** Nearly every topic includes working code and pseudocode that can be easily adapted to major programming languages. Theory without practice is incomplete; practice without theory is fragile.

An important disclosure: many of the algorithms and implementations in this book are my own constructions. Rather than copying optimized solutions from established sources, I have chosen to build understanding from first principles. This means some implementations may run slower than industry standards—or occasionally faster. For me, the process of understanding and constructing has been more valuable than simply achieving optimal performance.

This approach reflects the book's core philosophy: genuine mastery comes from understanding principles deeply enough to reconstruct solutions, not from memorizing existing ones.

About the Author

I am **Mahdi**, though you may know me by my online alias: *Genix*. At the time of writing, I am a Computer Engineering student, but more fundamentally, I am someone who grew up alongside computers—from simple games to terminal commands—always wondering what lies behind the screen of black and green text.

My relationship with computers has been one of continuous curiosity. I am someone who gives computers commands and, more importantly, learns from their responses. There is not much more you need to know about me personally, except that this book represents my attempt to understand the digital world I inhabit as completely as possible.

How to Use This Book

Arliz is freely available and open source. You can access the complete PDF, LaTeX source code, and related materials at:

https://github.com/m-mdy-m/Arliz

Each chapter includes carefully designed exercises and projects. Please do not skip these—they are not busy work but essential components of the learning process. True understanding comes only through active engagement with concepts, through solving problems and building solutions yourself.

I encourage you to approach this book as a collaborative effort. If you discover errors, have suggestions for improvement, or develop insights that could benefit other readers, please share them. This book improves through community engagement, and your contributions make it more valuable for everyone.

A Living Document

Finally, I want to be transparent about what you are engaging with. This is not a finished, polished product in the traditional sense. It is an evolving exploration of fundamental concepts, growing and improving as understanding deepens. You may encounter sections that could be clearer, examples that could be more intuitive, or explanations that could be more complete.

This is intentional. Arliz represents learning in progress, understanding in development. It invites you to participate in this process rather than simply consume its content.

I hope this book serves you well—whether you are beginning your journey with data structures, seeking to deepen existing knowledge, or simply satisfying intellectual curiosity. And if you learn something valuable, discover an error, or develop an insight worth sharing, I hope you will let me know.

After all, this book grows with all of us.

Acknowledgments

I would like to express my gratitude to everyone who supported me during the creation of this book. Special thanks to the open-source community for their invaluable resources and to all those who reviewed early drafts and provided feedback.

How to Read This Book

I understand what you might be thinking. You picked up a book called "Arliz" expecting to learn about arrays, and here I am about to take you on a journey through ancient civilizations and counting systems. You're probably wondering, "What does Mesopotamian mathematics have to do with `int[] myArray = new int[10]`?" That's not just a reasonable question—it's the *right* question to ask.

Let me address this directly: if you find this approach fundamentally misguided, you're free to close this book right now. But before you do, let me make my case for why this seemingly roundabout journey is actually the most direct path to genuine understanding.

Why This Book Exists

Every programming resource I've encountered follows the same pattern: "Here's an array. It stores elements. Here's the syntax. Moving on." This approach produces programmers who can use arrays functionally but lack deep understanding. They can write code that works, but when things break—and they inevitably will—they're left guessing rather than reasoning through solutions.

This book exists because I believe you deserve better than surface-level knowledge. When I began programming, I wasn't satisfied with "arrays are containers for data." I wanted to understand *why* they exist, *how* they actually work, and *what* principles govern their behavior at the most fundamental level.

The deeper I investigated, the more I realized that truly understanding arrays requires understanding the entire intellectual tradition that made them possible. Arrays aren't just programming constructs—they represent the culmination of humanity's longest-running intellectual project: the systematic organization of information.

Every time you write `arr[i]`, you're employing concepts developed by ancient mathematicians who first realized that *position* could carry meaning. When you work with multidimensional arrays, you're using geometric principles refined over millennia. When you optimize array operations, you're applying algorithmic thinking that emerged from centuries of mathematical tradition.

Understanding this heritage doesn't just provide context—it builds *intuition*. When you know why arrays work as they do, you can predict their behavior. When you understand the mathematical principles underlying their structure, you can optimize their usage effectively. When you grasp the conceptual frameworks that enabled their creation, you can extend and adapt them in ways that would otherwise be impossible.

The Journey Ahead

This book is structured as a systematic exploration through seven interconnected parts:

Part 1: Philosophical & Historical Foundations

We begin with the human journey from basic counting to systematic representation, exploring how different civilizations developed the conceptual tools that make modern computation possible. We examine the invention of positional notation, the development of the abacus, the emergence of algorithmic thinking, and the philosophical frameworks that enabled abstract mathematical representation.

This foundation matters because every array operation builds on concepts developed in this part. Array indexing directly descends from positional notation. Multidimensional arrays extend geometric thinking developed by ancient mathematicians. Algorithmic optimization applies systematic procedures that emerged from medieval mathematical traditions.

Part 2: Mathematical Fundamentals

Here we transform historical intuition into precise mathematical language. We develop set theory, explore functions and relations, examine discrete mathematics, and build the linear algebra foundations that directly enable array operations.

Without these mathematical tools, you'll remain mystified by why certain array operations are efficient while others are expensive, why some algorithms work better with particular data arrangements, and how to reason about the mathematical properties of your code.

Part 3: Data Representation

We explore how information is encoded in digital systems—number systems, binary representation, character encoding, and the various methods computers use to store and manipulate data. This is where abstract concepts become concrete implementations.

Understanding data representation is crucial because it determines how array elements are stored, how memory is allocated, and how operations are performed at the hardware level.

Part 4: Computer Architecture & Logic

We examine the hardware foundations of computation—logic gates, processor architecture, memory systems, and how the physical structure of computers influences data organization. This connects software concepts to hardware realities.

Arrays don't exist in isolation. They're implemented on real hardware with specific characteristics and constraints. Understanding this foundation is essential for writing efficient array-based code.

Part 5: Array Odyssey

Finally, we encounter arrays in their full complexity. By this point, they won't be mysterious constructs but the natural evolution of thousands of years of human thought about organizing information. We explore their implementation, behavior, and applications with unprecedented depth.

This is where everything converges. The historical foundations provide context, the mathematical frameworks provide analytical tools, the representation and architecture parts provide implementation understanding—and now we can explore arrays as sophisticated, well-understood mathematical objects.

Part 6: Data Structures & Algorithms

Having mastered arrays, we expand to explore the broader landscape of data structures. We see how other structures relate to and build upon array concepts, and how our deep understanding transfers to enable more sophisticated algorithmic thinking.

Part 7: Parallelism & Systems

We examine how data structures behave in complex, multi-threaded, and distributed systems. This explores the cutting edge of modern computation and shows how classical array concepts extend to contemporary challenges.

Reading Strategies for Different Audiences

The question remains: do you need to read all of this? The answer depends on your goals and current knowledge.

Complete Beginners

Read everything sequentially. The concepts build systematically, and skipping sections will create gaps that will limit your understanding later. This book is designed to take you from zero knowledge to deep, intuitive mastery.

Experienced Programmers

You could potentially begin with Part 5, but I strongly recommend at least reviewing Parts 1 and 2. You may be surprised how much the historical and mathematical context enriches concepts you thought you already understood. Parts 3 and 4 will fill in hardware and representation details that most programmers never learn properly.

Intermediate Learners

Parts 2, 3, and 4 might be your optimal starting point. You can always return to Part 1 for broader context and advance to Part 5 when you're ready for comprehensive array exploration.

Students and Educators

Different parts serve different pedagogical purposes. Part 1 provides motivation and historical context. Parts 2-4 build theoretical foundations. Parts 5-7 provide practical applications and advanced concepts. Use whatever combination serves your specific learning objectives.

Important Expectations

This is not a reference manual. It's not designed for quick lookups when you need to remember syntax. This book is about building deep, intuitive understanding—the kind that transforms how you think about programming and data structures.

Each part includes exercises, thought experiments, and projects. These are not optional supplements—they're carefully designed to help you internalize concepts and develop the mathematical intuition that distinguishes competent programmers from exceptional ones.

Don't expect this to be a quick read. Building genuine understanding requires time and sustained attention. The historical and mathematical foundations demand patience. The technical sections require careful study and practical application. This isn't a weekend book—it's a resource you'll work through over months, returning to sections as your understanding deepens and evolves.

A Living Exploration

This book grows and evolves as I learn better ways to explain concepts and discover new connections. You'll likely find areas that could be clearer, examples that could be more intuitive, or explanations that could be more complete. When you do, I encourage you to let me know. This book improves through community engagement, and your insights make it more valuable for everyone.

The Fundamental Promise

When you complete this journey, you won't just know how to declare and manipulate arrays. You'll understand them as mathematical objects with precise properties and predictable behaviors. You'll be able to anticipate their performance characteristics, optimize their usage intelligently, and extend their applications in innovative ways.

More importantly, you'll have developed a way of thinking about programming that transcends memorizing syntax and following patterns. You'll understand the deep principles that make computation possible, and you'll be equipped to apply those principles to solve novel problems that don't have cookbook solutions.

So if you're ready for this journey—if you're willing to invest the time and intellectual energy required to build genuine understanding—then let's begin together. We're going to start with humans counting on their fingers, and we're going to end with sophisticated data structures that process information in ways that would seem magical to our ancestors.

Welcome to Arliz. Let's explore the fascinating world of arrays—from the very beginning.

Part I

Philosophical & Historical Foundations

Introduction

> Every number is an echo of humanity's need to comprehend and order nature.

Before we jump into syntax and algorithms, consider this: each time you create an array, you join a practice that spans millennia. Ancient Mesopotamians etched symbols on clay tablets; Chinese scholars arranged numbers in grids; early Islamic thinkers devised systematic methods—all aiming to tame complexity through order.

In this part, we follow that journey from first counting attempts to the verge of mechanical computation. We'll see how the abacus foreshadowed array operations, how positional notation set the stage for indexing, and how mathematical reflection shaped our approach to structured data.

Why begin here? Because grasping the *why* behind arrays transforms your relationship with them. Rather than memorizing rules, you build intuition; concepts become natural rather than obstacles. When you recognize arrays as modern echoes of an ancient drive to organize information, they lose their mystery and reveal their elegance.

Imagine early humans under a silent sky, returning from a hunt or storing seeds, faced with a simple yet profound question: how to keep track of quantities? Could a few stones or marks on bone open a door to abstraction? This urge—to count and impose order—marks a pivotal shift in human consciousness.

In this chapter, we explore the philosophical and cognitive spark behind counting, survey the earliest archaeological hints, and examine how the Neolithic shift to settled life and record-keeping paved the way for symbols and sign systems. Ultimately, we trace how these ancient steps set the foundations for the abstract structures—like arrays—that power modern programming.

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Part II

Mathematical Fundamentals

Introduction

The historical journey in Part 1 showed us how humans developed systematic thinking about organized information. Now we need to translate those insights into the precise mathematical language that makes arrays work.

This isn't about learning math for math's sake. Every mathematical concept we explore here—from basic number properties to linear algebra—directly enables the array operations you'll use in programming. When you understand why multiplication is commutative, you'll understand why certain array optimizations work. When you grasp set theory, you'll see the logic behind array search algorithms. When you work with mathematical functions, you'll understand the elegant relationship between array indices and their values.

We'll build everything from first principles, assuming no advanced mathematical background. But we won't treat mathematics as a collection of arbitrary rules. Instead, we'll see how each concept emerged from the same human drive for systematic organization that we traced in Part 1.

Think of this part as building your mathematical toolkit. Every tool we create here will be used extensively in later parts. By the end, you'll have the mathematical foundation needed to truly understand not just how arrays work, but why they work the way they do.

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Reflections at the End

As you turn the final pages of *Arliz*, I invite you to pause—just for a moment—and look back. Think about the path you've taken through these chapters. Let yourself ask:

> "Wait... what just happened? What did I actually learn?"

I won't pretend to answer that for you. The truth is—**only you can**. Maybe it was a lot. Maybe it wasn't what you expected. But if you're here, reading this, something must have kept you going. That means something.

This book didn't start with a grand plan. It started with a simple itch: **What even is an array, really?** What began as a curiosity about a "data structure" became something much stranger and—hopefully—much richer. We wandered through history, philosophy, mathematics, logic gates, and machine internals. We stared at ancient stones and modern memory layouts and tried to see the invisible threads connecting them.

If that sounds like a weird journey, well—yeah. It was.

This is Not the End

Arliz isn't a closed book. It's a snapshot. A frame in motion. And maybe your understanding is the same. You'll return to these ideas later, years from now, and see new angles. You'll say, "Oh. That's what it meant." That's good. That's growth.

Everything you've read here is connected to something bigger—algorithms, networks, languages, systems, even the people who built them. There's no finish line. And that's beautiful.

From Me to You

If this book gave you something—an idea, a shift in thinking, a pause to wonder—then it has done its job. If it made you feel like maybe programming isn't just code and rules, but a window into something deeper—then that means everything to me.

Thank you for being here.
Thank you for not skipping the hard parts.
Thank you for choosing to think.

One More Thing

You're not alone in this.
The Arliz project lives on GitHub, and the conversations around it will (hopefully) continue. If you spot mistakes, have better explanations, or just want to say hi—come by. Teach me something. Teach someone else. That's the best way to say thanks.
Knowledge grows in community.
So share. Build. Break. Rebuild.
Ask better questions.
And always, always—stay curious.

Final Words

Arrays were just the excuse.
Thinking was the goal.
And if you've started to think more clearly, more deeply, or more historically about what you're doing when you write code—then this wasn't just a technical book.
It was a human one.
Welcome to the quiet, lifelong joy of understanding.

This completes the first living edition of Arliz.

Thank you for joining this journey from zero to arrays, from ancient counting to modern computation.
The exploration continues...

Author's Notes and Reflections

On Naming Conventions and Creative Processes

I should confess something about my naming process: I tend to pick names first and find meaningful justifications later. Very scientific, I know! The name "Arliz" started as a random choice that simply felt right phonetically. Only after committing to it did I discover the backronym that now defines its meaning. This probably says something about my creative process—intuition first, rationalization second.

This approach extends beyond naming. Many aspects of this book emerged organically from curiosity rather than systematic planning. What began as personal notes to understand arrays evolved into a comprehensive exploration of computational thinking itself.

On Perfectionism and Living Documents

You should know that many of the algorithms presented in this book are my own implementations, built from first principles rather than copied from optimized sources. This means you might encounter code that runs slower than industry standards—or occasionally faster, when serendipity strikes.

Some might view this as a weakness, but I consider it a feature. The goal isn't to provide the most optimized implementations but to demonstrate the thinking process that leads to understanding. When you can reconstruct a solution from fundamental principles, you've achieved something more valuable than memorizing an optimal algorithm.

On Academic Formality and Personal Voice

You might notice that this book alternates between formal academic language and more conversational tones. This is intentional. While I respect the precision that formal writing provides, I also believe that learning happens best in an atmosphere of intellectual friendship rather than academic intimidation.

When I suggest you could "use this book as a makeshift heating device" if you find the approach ridiculous, I'm not being flippant—I'm acknowledging that not every approach works for every learner. Intellectual honesty includes admitting when your methods might not suit your audience.

On Scope and Ambition

The scope of this book—from ancient counting to modern distributed systems—might seem overly ambitious. Some might argue that such breadth necessarily sacrifices depth. I disagree, but I understand the concern.

My experience suggests that understanding connections between disparate fields often provides insights that narrow specialization misses. When you see arrays as part of humanity's broader intellectual project, you understand them differently than when you see them as isolated programming constructs.

That said, if you find the historical sections tedious or irrelevant, you have my permission to skip ahead. The book is designed to be valuable even when read non-sequentially.

On Errors and Imperfection

I mentioned that you'll find errors in this book. This isn't false modesty—it's realistic acknowledgment. Complex explanations, mathematical derivations, and code implementations inevitably contain mistakes, especially in a work that grows and evolves over time.

Rather than viewing this as a flaw, I encourage you to see it as an opportunity for engagement. When you find an error, you're not just identifying a problem—you're participating in the process of building better understanding. The best learning often happens when we encounter and resolve contradictions.

On Time Investment and Expectations

When I suggest this book requires months rather than weekends to master, I'm not trying to inflate its importance. Complex concepts genuinely require time to internalize. Mathematical intuition develops gradually, through repeated exposure and active practice.

If you're looking for quick solutions to immediate programming problems, this book will frustrate you. If you're interested in developing the kind of deep understanding that serves you throughout your career, the time investment will prove worthwhile.

On Community and Collaboration

This book exists because of community—the open-source community that provides tools and resources, the academic community that develops and refines concepts, and the programming community that applies these ideas in practice.

Your engagement with this material makes you part of that community. Whether you find errors, suggest improvements, or simply work through the exercises thoughtfully, you're contributing to the collective understanding that makes books like this possible.

Final Reflection

Writing this book has been an exercise in understanding my own learning process. I've discovered that I learn best by building connections between disparate ideas, by understanding historical context, and by implementing concepts from scratch rather than accepting them as given.

Your learning process might be entirely different. Use what serves you from this book, adapt what needs adaptation, and don't hesitate to supplement with other resources when my explanations fall short.

The goal isn't for you to learn exactly as I did, but for you to develop your own path to deep understanding.

These notes reflect thoughts and observations that didn't fit elsewhere but seemed worth preserving. They represent the informal side of a formal exploration—the human element in what might otherwise seem like purely technical content.