

# History-Driven Particle Swarm Optimization in dynamic and uncertain environments



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## ABSTRACT

Due to dynamic and uncertain nature of many optimization problems in real-world, an algorithm for applying to this environment must be able to track the changing optima over the time continuously. In this paper, we report a novel multi-population particle swarm optimization, which improved its performance by employing an external memory. This algorithm, namely History-Driven Particle Swarm Optimization (HdPSO), uses a BSP tree to store the important information about the landscape during the optimization process. Utilizing this memory, the algorithm can approximate the fitness landscape before actual fitness evaluation for some unsuitable solutions. Furthermore, some new mechanisms are introduced for exclusion and change discovery, which are two of the most important mechanisms for each multi-population optimization algorithm in dynamic environments. The performance of the proposed approach is evaluated on Moving Peaks Benchmark (MPB) and a modified version of it, called MPB with pendulum motion (PMPB). The experimental results and statistical test prove that HdPSO outperforms most of the algorithms in both benchmarks and in different scenarios.

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## 1. Introduction

Over the past decade, there has been a growing interest in applying swarm intelligence and evolutionary algorithm for optimization in dynamic and uncertain environments [1]. The most important reason for this growing can be related to the dynamic nature of many optimization problems in real-world. World Wide Web (WWW) is an outstanding example of such an environment with multiple dynamic problems. Web page clustering, web personalization and web community identification are some of the most well-known sample problems from this dynamic environment.

Generally, the goals, challenges, performance measures and the benchmarks are completely different from the static to dynamic environments. In static optimization problems, the goal is only finding the global optima. However, in dynamic optimization problems, detecting change and tracking global optima as closely as possible are the other goals for optimization. Also, in many cases in real-world optimization problems, the old and new environments are somehow correlated to each other. Therefore, a good optimization algorithm also must be able to learn from the previous environments to work better [2].

From the challenges exist for optimization in static environments, we can mention to premature convergence, trapping in local optima and trade-off between exploration and exploitation. Although, optimization in dynamic environments has some new challenges such as Detecting change in environment, transforming a local optima to a global optima and vice versa, losing diversity after change, outdated memory after change, chasing one peak by more than one sub-population in multi-modal environment, unknown severity of change in environment, changing part of environment instead of the entire environment, unknown time dependency between changes in environment.

Up to now, multiple benchmarks and performance measures are defined in literature for evaluation the optimization algorithms in dynamic environments, which are completely different from static ones. Offline error, Offline performance [2] and best-before-change error [3] are some of the most commonly used measures in literature. Furthermore, MPB [4], DF1 [5], XOR dynamic problem generator [6]

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are some of the most commonly used benchmarks in state of the art. In this paper, we use the MPB and a modified version of it, called PMPB as the benchmark problems and offline error as the performance measure for evaluation of the proposed algorithm.

Over the past two decades, many optimization algorithms are proposed based on swarm intelligence and evolutionary algorithm for multimodal, time-varying and dynamic environments. The most widely used algorithms are PSO [7–13], GA [6,14–16], ACO [17–19], DE [20–30] and ABC [31–36].

We can divide the proposed approaches for optimization in dynamic environments into six different groups: (1) introducing diversity when changes occur, (2) maintaining diversity during the change, (3) memory-based approaches, (4) prediction approaches, (5) self-adaptive approaches and (6) multi-population approaches. In this paper, we use a combination of second, third and sixth approaches to take advantages of the whole of them.

In many dynamic optimization problems in real-world, the current state is often similar or related to the previously seen states. Using the past information may help the algorithm to be more adaptive to the changes in the environment and to perform better over time. One way to maintain the past information is the use of memory in implicit or explicit type. Implicit memory stores the past information as part of an individual, whereas explicit one stores information separate from the population. Explicit memory has been much more widely studied and has been produced much better performance on dynamic optimization problem than implicit one.

Explicit memory can be divided into direct and associative memory. In most cases, the information in direct memories are the previous good solutions [37,38]. However, associative memory can be included various type of information such as the probability vector that created the best solutions [37], the probability of the occurrence of good solutions in the landscape [39].

In [40,41] a non-revisiting genetic algorithm (NrGA) is proposed, which memorize all the solutions by a binary space partitioning (BSP) tree structure. This scheme uses the BSP tree as an explicit memory in a static optimization problem to prevent the solutions evaluates more than one time during the run. Also, [42] utilized the BSP tree in continuous search space to guide the search by doing some adaptation on the mutation operator. In [43,44], the BSP tree is utilized for creating an adaptive parameter control system. It automatically adjusts the parameters based on entire search historical information in a parameter-less manner. In this paper, the authors used the BSP tree for different aims, which include improving the exploration capability of the finder swarm, improving the exploitation capability of fine-tuning mechanism and proposing different mechanism for exclusion.

Unlike many other direct memory approaches, the proposed approach stores all the past individuals evaluated during each environment. At the first glance, it seems that it needs a large amount of memory and it is not an economical method. However, in many optimization problems in real-world, the fitness evaluation cost is very higher than individual generation cost. For such problems, the total number of fitness evaluation is limited and it is a mistake to throw away any information achieved from the past fitness evaluation [42].

In this paper, we report a novel swarm intelligence algorithm, namely History-Driven Particle Swarm Optimization (HdPSO) for optimization in dynamic environments. It is shown that using explicit memory during the run can help the algorithm to guide the local search and global search toward the promising area in the search space and remember the previous environments if they happen again. Moreover, we incorporate the BSP tree to approximate the fitness for the individuals before actual fitness evaluation and in this way, we decrease the number of wasted fitness evaluation for some inappropriate individuals and improve the performance of the algorithm significantly.

It is worth mentioning that the proposed approach can be seen as a framework for applying optimization algorithms on dynamic environments. Most of the techniques applied on standard particle swarm optimization in this paper, can be also applied on more powerful optimization algorithms such as CoBiDE [45], CoDE [46], BSA [47] and DSA [48] which are introduced recently.

The remainder of this paper is organized as follows: Section 2 presents the fundamental of the proposed HdPSO, and the structure of the explicit memories used in this work in detail. Section 3 reports the experimental results on moving peaks benchmark and the new modified version of it with various configurations. Finally, Section 4 presents some concluding remarks.

## 2. Proposed algorithm

In the following subsections, we first describe how an explicit memory-based approach work generally. Then, we describe the overall scheme of the proposed HdPSO algorithm in detail.

### 2.1. Explicit memory-based approach

In this paper, an explicit memory-based approach is proposed for storing the past useful information during the computation. This explicit memory has two parts, including long-term and short-term memories.

The long-term memory is responsible for storing the most important information about the past environments. This information includes the position and fitness value of the all discovered local and global optima in the past environments. Taking advantage of such a memory, an algorithm has the ability to remember the previous environments, if they re-appeared in the future exactly or with some noise. The information in the long-term memory is persistent and does not disappear by the happening changes in environment. However, it becomes more complete and more powerful by happening each change in environment. Fig. 1 shows the structure of this long-term memory in more detail.

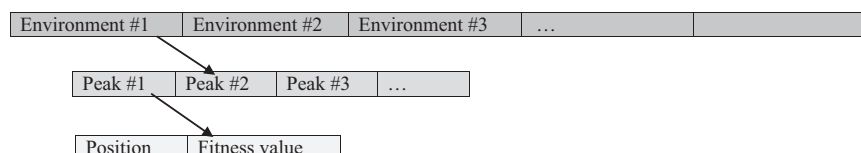


Fig. 1. The structure of the long-term memory in detail.

The process of remembering the previous environments during the computation is as follows. At the beginning of each iteration, the similarity between current and previous environments is calculated using the modified Hausdorff Distance [49]

$$d(A, B) = \frac{1}{N(A)} \sum_{a \in A} d(a, B) \quad (1)$$

$$d(a, B) = \min_{b \in B} d(a, b) \quad (2)$$

$$d(a, b) = \|a - b\| \quad (3)$$

where  $d(A, B)$  is directed distance between two point sets of Environment  $A = \{a_1, \dots, a_{N(A)}\}$  and  $B = \{b_1, \dots, b_{N(B)}\}$ ,  $d(a, B)$  is distance between a point  $a$  and a set of points in Environment  $B = \{b_1, \dots, b_{N(B)}\}$  and  $d(a, b)$  is the Euclidean distance between two points  $a$  and  $b$ . It is important to mention that each point include the peak's position and its fitness as a dimension as well. If the distance between current and  $i$ th environment decreased sequentially more than  $M$  times ( $M$  is equal to 3 in this paper), we can say maybe a previous environments is re-appeared. Then the remembering process will be started by moving the GbestPosition of each peak in current environment  $GbestPos_{Peak(i)}^{Env(cur)}$  toward the GbestPosition of the nearest swarm of the most similar environment  $GbestPos_{Peak(j)}^{Env(MSE)}$  using Eq. (4). In this algorithm,  $r_f$  is a parameter for adjusting the speed of remembering and in this paper we consider 0.5 for it

$$GbestPos_{Peak(i)}^{Env(cur)} = GbestPos_{Peak(i)}^{Env(cur)} + r_f * (GbestPos_{Peak(j)}^{Env(MSE)} - GbestPos_{Peak(i)}^{Env(cur)}) \quad (4)$$

The short-term memory is responsible for storing the all evaluated solution in current environment. It uses the Binary Space Partitioning (BSP) tree structure for storing the valuable information. In this structure, the whole search space partition based on the distribution of the individuals in the environment. Each node in this tree represents a unique hyper-rectangular box in the search space and contains a representative solution for this sub-area. Suppose a parent node has two child nodes. The sub-areas represented by the child nodes are disjoint and their union is the sub-area of the parent. Fig. 2 shows the structure of the short-term memory which is used in this paper.

#### Algorithm 1: ShortTermMem\_insertNode()

---

```

Input:  $[x, f(x)]$ 
1  begin
2    set  $cur\_node$  to root node
3    while ( $cur\_node$  has two child nodes:  $a$  and  $b$ )
4      Set Dimension  $j = \arg \max |a(k) - b(k)|$  where  $k \in [1, D]$ 
5      if ( $|a(j) - x(j)| \leq |b(j) - x(j)|$ ) then
6         $cur\_node = a$ 
7      else
8         $cur\_node = b$ 
9      end if
10   end while
11   add two child nodes  $[x_{cur\_node}, F(x_{cur\_node})]$  and  $[x, F(x)]$  to the parent node  $cur\_node$ 
12   convert the  $cur\_node$  to a non-terminal node by removing  $[x_{cur\_node}, F(x_{cur\_node})]$ 
13 end

```

---

The process of adding a new solution to the short-term memory is shown in Algorithm 2. After each fitness evaluation, first, the short-term memory – BSP tree – must be traversed to find the sub-area which the new solution belongs to ( $cur\_node$ ). Then, the new solution must be compared with the representative solution of that sub-area ( $x_{cur\_node}$ ) to find a cut-off point. The middle of the largest difference between the all dimensions of these two solutions ( $x$  and  $x_{cur\_node}$ ) will be select as a cut-off point for creating two new child nodes. For more detail on the BSP tree construction as well as a numerical example, please refer to [42].

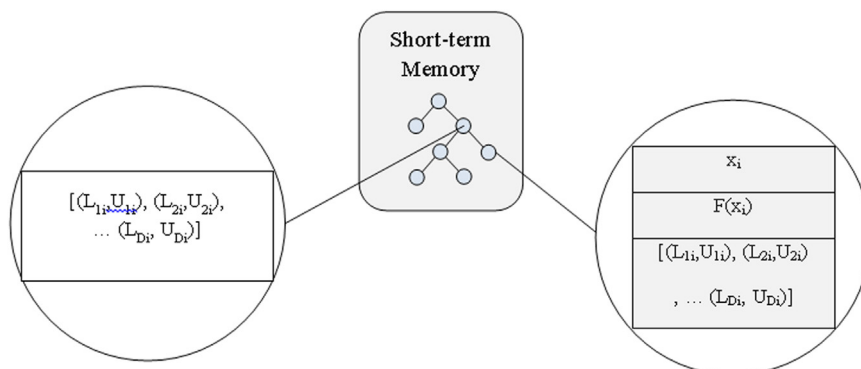


Fig. 2. The structure of the each terminal and non-terminal node in short-term memory.

HdPSO utilizes the short-term memory during the run (before each fitness evaluation) to decrease number of fitness evaluations by predicting the fitness value of the solution using [Algorithm 2](#).

**Algorithm 2: ShortTermMem\_PredictSolution()**

---

```

Input: [x]
1  begin
2    set cur_node to root node
3    while(cur_node has two child nodes: a and b)
4      Set Dimension  $j = \arg \max |a(k) - b(k)|$  where  $k \in [1, D]$ 
5      if ( $|a(j) - x(j)| \leq |b(j) - x(j)|$ ) then
6        cur_node = a
7      else
8        cur_node = b
9      end if
10   end while
11   if the short-term memory is matured sufficiently then
12     return  $f_{\text{cur\_node}}$  as a predicted value for x
13   else
14     return  $\infty$ 
15   end if
16 end
17 Output: [ $f_{\text{cur\_node}}$ ]

```

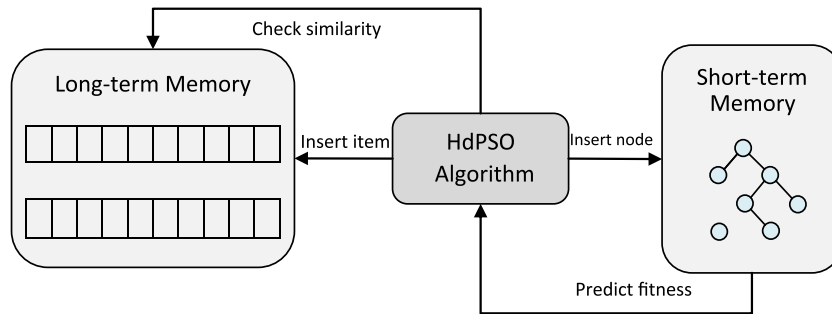
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As we can see in this algorithm, the preliminary steps are completely similar to the [Algorithm 1](#). The only difference is from line 11 to 15. If the short-term memory is matured sufficiently, the algorithm will return the fitness value of the representative for the area which *x* is belong to as a predicted value for solution *x*. The short-term memory is matured if the percentage of correct prediction is more than a pre-defined value (*maturity\_threshold*). This percentage can be calculated after each fitness evaluation.

It is obvious that using this function for predicting the solutions in early stages of the run and before reaching enough maturity for the short-term memory can be harmful for the algorithm and in the opposite direction with the aim of using this function. That is why; one threshold parameter – *maturity\_threshold* – is defined for HdPSO to check the maturity of the short-term memory before using the predicted value.

## 2.2. History-Driven Particle Swarm Optimization (HdPSO)

In this section, a novel algorithm named History-Driven Particle Swarm Optimization (HdPSO) is proposed for optimization in dynamic environments. This algorithm utilizes the landscape historical information to improve its convergence speed, accuracy, and also remembering the previous environments if they re-appeared again in the future. [Fig. 3](#) shows the block diagram of HdPSO and its relationship with the explicit memories.



**Fig. 3.** The block diagram of HdPSO with explicit memories.

**Algorithm 3. Proposed Algorithm(HdPSO)**


---

```

1  begin
2  call Initialization()
3  repeat
4      /*-----remembering previous environments process start-----*/
5      Find the minimum distance ( $md$ ) between the current environment and the other
6      environments in long-term memory based on Eq.1
7      if  $md$  decreased more than  $M$  times sequentially for environment  $k$  then
8          Call LongTermMem_rememberEnv( $k$ )
9      end if
10     /*-----remembering previous environments process end-----*/
11     call change_discovery()
12     /*-----finder swarm process start-----*/
13     if finder swarm is active then
14         call Swarm_Moving(Finder)
15         call Exclusion(Finder)
16         call Finder_Convergence_Checking()
17     end if
18     /*-----finder swarm process end-----*/
19     /*-----tracker swarm process start -----*/
20     foreach active swarm  $s_i$  in tracker swarm  $S$  do
21         call Swarm_Moving( $s_i$ )
22     end
23     call Swarm_Freezing()
24     /*-----tracker swarm process end-----*/
25     call fine_Tuning()
26 until stopping criterion is met
27 end

```

---

History-Driven Particle Swarm Optimization is a real coded swarm intelligence algorithm. For a  $D$ -dimensional landscape  $S \subset R^D$ , a solution  $x$  of HdPSO is a 1 by  $D$  real valued vector, i.e.,  $x \in R^D$ . This algorithm employs two different population – finder and tracker swarm – and also two different explicit memories with vary structures to overcome the challenges which exist in dynamic environments. Among them, we can mention to happening change in environment, transforming a local optima to a global one after a change in environment, losing the position of the all optima after change, losing diversity in environment after change, outdated memory, converging two sub-swarms to one local optima and short interval of time between two change in environment. This work tries to introduce some mechanisms to overcome these challenges in dynamic optimization problem.

The proposed algorithm starts with initializing the finder and tracker swarms and also, short-term and long-term memories. Afterwards, change discovery, finder and tracker swarm processing and also fine tuning process are repeated until the number of fitness evaluations exceeds a pre-determined value. Algorithm 3 shows the block diagram of the HdPSO. In the following, each step in this algorithm is described in detail.

**Algorithm 4:. Initialization()**


---

```

1  begin
2  Initialize the finder swarm  $S_{finder} = [x_i]$  for  $i=1, 2, \dots, \text{Finder\_number}$ .
3  Initialize the tracker swarm  $S_{tracker} = [x_i]$  for  $i=1, 2, \dots, \text{Tracker\_number}$ .
4  Initialize the short-term memory STM to consist of a root node only.
5  Initialize the long-term memory LTM.
6  Initialize the  $x_{test-point}$  and calculate  $f(x_{test-point})$ .
7  Activate the finder swarm.
8  foreach Particle $_i$  in finder swarm  $F$  do
9       $f(x_i) = \text{calculateFitness}(x_i)$ 
10     ShortTermMem_insertNode( $[x_i, f(x_i)]$ )
11 end
12 end

```

---

The initialization process is shown in Algorithm 4. The algorithm consists of two different types of swarms including finder and tracker swarms. The finder swarm is configured somehow to explore the new promising areas in the search space effectively and efficiently. Once the finder swarm converged to a local optima, the  $k$ -best particles in it should be considered as a new tracker swarm, and the finder swarm should be re-initialized. The tracker swarm is responsible for chasing the discovered optima in the environment and does not have

any particle initially. Also, the short-term memory initialized by adding a single node which is the root node of the BSP tree. However, the long-term memory has not any entry at the first.

At the first step in the main loop of the algorithm, the previous environment remembering process will happen. The detail of this step is provided in the previous section completely.

As noted in the introduction part, there are two methods for discovering change in dynamic environment, including re-evaluating a pre-specified test solution, and monitoring behavioral change in algorithm during the run. The current work presents a hybrid approach based on two mentioned methods for discovering change in environment as shown in Algorithm 5. In this algorithm first, the trend of a performance KPI should be checked during  $k$ -latest fitness evaluations. This KPI is the mean of all fitness evaluations after the latest change in environment. If the trend of this KPI is increasing, the algorithm needs to re-evaluate the pre-specified test solution for ensuring about happening change in environment. This hybrid approach can help the algorithm to decrease the number of wasted fitness evaluation for discovering change in environment. In this work, we used 4 for the  $k$  value.

**Algorithm 5. Change\_discovery()**

---

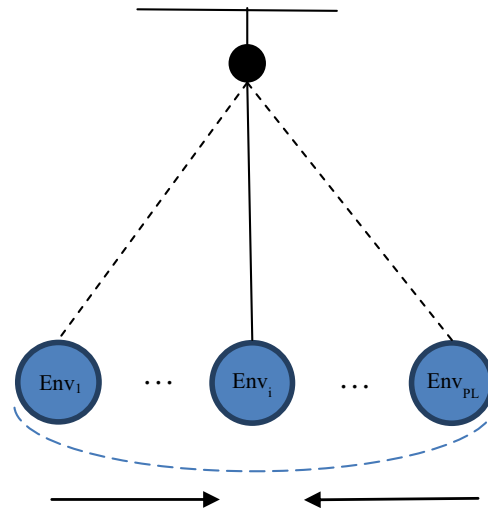
```

1  begin
2      /* ----- checking behavioral change in the algorithm ----- */
3      if mean of all fitness evaluations after the change in environment increases ...
        more than k times sequentially then
4          /* ----- re-evaluation a test solution ----- */
5           $f(x_{test\ solution})^{new} = \text{calculate\_fitness}(x_{test\ solution})$ 
6          if  $f(x_{test\ solution})^{new}$  is different from  $f(x_{test\ solution})^{old}$  then
7              /* ----- change happened ----- */
8              Add an entry to long-term memory including the position and fitness ...
9              values of all discovered optima
10             Re-initialize short-term memory
11             Re-evaluate the pre-specified test solution
12             Diversify all the particles in each tracker swarm by adding a random value...
13             to the GBestPosition of each swarm between  $[-Vlength*P, Vlength*P]$ 
14         end if
15     end if
16 end if
17 end

```

---

After the change is discovered in the environment, two actions should be done in environment. First, all discovered optima should be added to the long-term memory and then all the particles in each tracker swarm should be diversified. This diversification can be done by adding a random value in range of  $[-Vlength*P, Vlength*P]$  to the GbestPosition of each tracker swarm.



**Fig. 4.** Pendulum motion of environments.

**Algorithm 6. Swarm\_Moving()**


---

```

1  Input:[swarm SW]
2  Begin
3    foreach particle i in SW
4      initialize  $x_i, v_i$ 
5       $PBest_i = x_i$ 
6    end for
7     $GBest = \arg \max f(PBest_i)$ 
8    repeat
9      foreach particle i in SW
10       update velocity  $v_i$ 
11       check the velocity boundaries.
12       set position  $x_i^{new}$ 
13        $p(x_i^{new}) = \text{ShortTermMem\_PredictSolution}(x_i^{new})$ 
14       if  $p(x_i^{new}) \leq Pbest(x_i^{old})$  then
15          $x_i = x_i^{new}$ 
16          $f(x_i) = \text{calculate\_fitness}(x_i^{new})$ 
17         if  $f(x_i) < f(Pbest_i)$ 
18            $Pbest_i = x_i$ 
19         end
20         if  $f(x_i) < f(Gbest_i)$ 
21            $Gbest_i = x_i$ 
22         end
23       end
24     end for
25   until stopping criteria is met
26 end

```

---

After the change discovery, the algorithm will start a finder swarm process if this swarm is active in environment. The finder swarm process includes three actions including swarm moving, exclusion and convergence checking.

The swarm moving, as shown in Algorithm 6, is very similar to the standard particle swarm optimization with some minor changes as below:

- 1) The fitness value of the each particle should be predicted by the BSP tree before actual fitness evaluation. If the predicted value for this new position is equal or better than its Pbest fitness value, the actual fitness evaluation will be happen on new position of the particle.
- 2) The position of each particle will change if the fitness value of the new position is better than current position.

Also, the exclusion and convergence checking algorithms are shown in Algorithm 7,8. In Exclusion algorithm, the finder swarm will be re-initialized if the distance between the  $Gbest$  of each swarm and finder swarm or any other tracker swarm is less than  $r_{excl}$ .

**Algorithm 7. exclusion()**


---

```

1  begin
2    foreach particlei in  $S_{particle}$  do
3      if  $\|x_i, G_f\| < r_{excl}$  then
4        re-initialize finder swarm F
5      end
6    end
7  end

```

---

**Algorithm 8. finderConvergenceChecking()**


---

```

1  begin
2    if (  $f(G_f)^{t-2} - f(G_f)^t < r_{conv}$  ) or (  $\|G_f^{t-2}, G_f^t\| < r_{conv}/5$  ) then
3      add m-best particles in the finder swarm to the tracker swarm
4      re-initialize the finder swarm
5    end
6  end

```

---



Based on Algorithm 8, the finder swarm is converged if the difference between the fitness value of the best global particle during the last two iterations is less than  $r_{conv}$  or the distance between the best global particle during the last two iterations is less than  $r_{conv}/5$ . After the convergence of finder swarm, the  $m$  best particle in it will move to the tracker swarm and the finder swarm will be re-initialized.

After the finder swarm process, the tracker swarm will start by swarm moving, swarm freezing and fine-tuning process. The swarm moving for the tracker swarm uses the same algorithm as finder swarm (Algorithm 6).

The swarm freezing, as shown in Algorithm 9, is an action which used for freezing the tracker swarms which have not any influence in improvement of the algorithm's result. In this way, the algorithm does not waste the fitness evaluations to improve some particles which have not any influence on performance of the whole algorithm. A tracker swarm will be freeze if its Gbestposition or fitness did not change considerably during their last three iterations.

#### Algorithm 9. Swarm\_freezing()

---

```

1  begin
2  foreach swarmi in tracker swarm
3  if ( f(Gbesti)t-3 - f(Gbesti)t ) < rconv or ( ||(Gbesti)t-3, (Gbesti)t || < rconv/5 ) then
4  freeze swarmi
5  end
6  end for
7  end

```

---

#### Algorithm 10. Fine-tuning()

---

```

1  Input: [particle Gbest_Particle]
2  begin
3  for counter = 1 to try_number
4  set xnew based on Equation (5)
5  p(xnew) = ShortTermMem_PredictSolution(xnew)
6  if p(xnew) ≤ Pbest(Gbest_Particle) then
7  f(xnew) = calculate_fitness(xnew)
8  if f(xnew) < Pbest(Gbest_Particle) then
9  Gbest_Particle = xnew
10 Pbest(Gbest_Particle) = f(xnew)
11 end
12 end
13 update rcloud based on Equation(6)
end

```

---

The aim of fine-tuning algorithm (Algorithm 10) is to improve the overall result of the algorithm as much as possible. That is why this algorithm tries to enhance the quality of global best particle in environment by doing some local search around it. Predicting new solutions after each movement can help the algorithm to reduce the number of wasted fitness evaluation during this phase. New position can be achieved by using the following equation:

$$x^{new} = Gbest\_position + (r_{cloud} \times R_j) \quad (5)$$

where  $R_j$  is a random number with uniform distribution in  $[-1, 1]$ . Hence, the  $x^{new}$  is located in the radius of  $r_{cloud}$  from  $Gbest$ . Also,  $r_{cloud}$  will decrease based on Eq. (6)

$$r_{cloud} = r_{cloud} \times (w_{min} + (rand \times (w_{max} - w_{min}))) \quad (6)$$

where  $w_{min}$ ,  $w_{max}$  are the lower and the upper bounds for multiplying in  $r_{cloud}$ .

### 3. Experimental study

#### 3.1. Benchmark problems

Due to time consuming evaluations of fitness functions in the real-world problems, the computation cost of an algorithm in most applications is expressed in terms of the number of evaluations; as is done in this paper. The first benchmark used in this paper is the Moving Peaks Benchmark (MPB) [4,50], which is the most commonly used benchmark for evaluating algorithms in dynamic environments [51]. The reason for this popularity amongst the researchers is related to the large number of adjustable parameters in this benchmark for generating a wide range of dynamic environments. Using this benchmark, we can study algorithms from a number of perspectives.

In MPB, there are a number of peaks in a D-dimensional space which their widths, heights and positions will change by a fixed amount  $s$  (shift severity), every  $\alpha$  fitness evaluation. The goal of the problem is to locate and track the global peak (optima) in environment during the all changes which happen to environment over the time. More detail about MPB, can be fined in [4].

The second benchmark used in this paper, is a modified version of MPB with pendulum motion characteristic (Fig. 4), namely Pendulum-MPB (PMPB). In fact, most of the dynamic optimization algorithms that enhanced with memory are expected to work better on



environments that re-appear in the future. In this benchmark, pendulum length ( $PL$ ) parameter is introduced to control the linear pendulum motion accurately and correctly.

**Algorithm 11. Environment\_selector()**

---

```

Input : Environment  $i$ , Direction  $dir$ , Pendulum
1  length  $PL$ 
2  Output: next environment
3  begin
4    if  $dir == right$ 
5      if  $i < PL$  then
6         $i = i + 1$ 
7      else if  $i == PL$  then
8         $dir = left$ 
9         $i = i - 1$ 
10     end if
11   else if  $dir == left$  then
12     if  $i > 1$  then
13        $i = i - 1$ 
14     else if  $i == 1$  then
15        $dir = right$ 
16        $i = i + 1$ 
17     end if
18   end if
19   return  $i$  //the index of the next environment
20 end

```

---

The implementation for PMPB is as follows. For the first  $PL$  changes in environments, the whole peak positions should be stored in an array memory for the future use. Afterwards, for every change in the environment, the position of the peaks should be achieved from the next environment returned from Algorithm 11. It should be mentioned that the first value for the *direction* parameter should be set to *right*.

### 3.2. Performance metric

There are several performance metrics used in the literature to measure the efficiency of optimization algorithm in dynamic environments [52]. In order to make our results comparable with other state-of-the-art algorithms, the metric selected in this paper is the *offline error* ( $OE$ ) which is the average of the differences between the values found so far by the algorithm and the global optimum value [4,50].

$$OE = \frac{1}{FE} \sum_{t=1}^{FE} (f(gbest(t)) - f(globalOptimum(t))) \quad (7)$$

where  $FE$ s is the total number of fitness evaluations, and  $globalOptimum(t)$  and  $gbest(t)$  are the global optimum value and the best value found by the algorithm at the  $t$ -th fitness evaluation, respectively. It is worth mentioning that in [8,9], offline error is calculated using different approach, in which instead of using the average of current errors in all fitness evaluations, average of current errors before each environment change is utilized. In this paper, we calculate offline error based on Eq. (7) for comparing the algorithms' results.

**Table 1**  
MPB parameters setting.

Parameter	Value
Number of peaks, $M$	Variable between 1 and 200
Change frequency, $\alpha$	500, 1000, 2500, 5000, 10000
Height change	7.0
Width change	1.0
Peaks shape	Cone
Basic function	No
Change severity, $s$	1, 2, 3, 5
Number of dimensions, $D$	5
Correlation Coefficient, $\lambda$	0
Peaks location range	[0–100]
Peak height	[30.0–70.0]
Peak width	[1–12]
Initial value of peaks	50.0

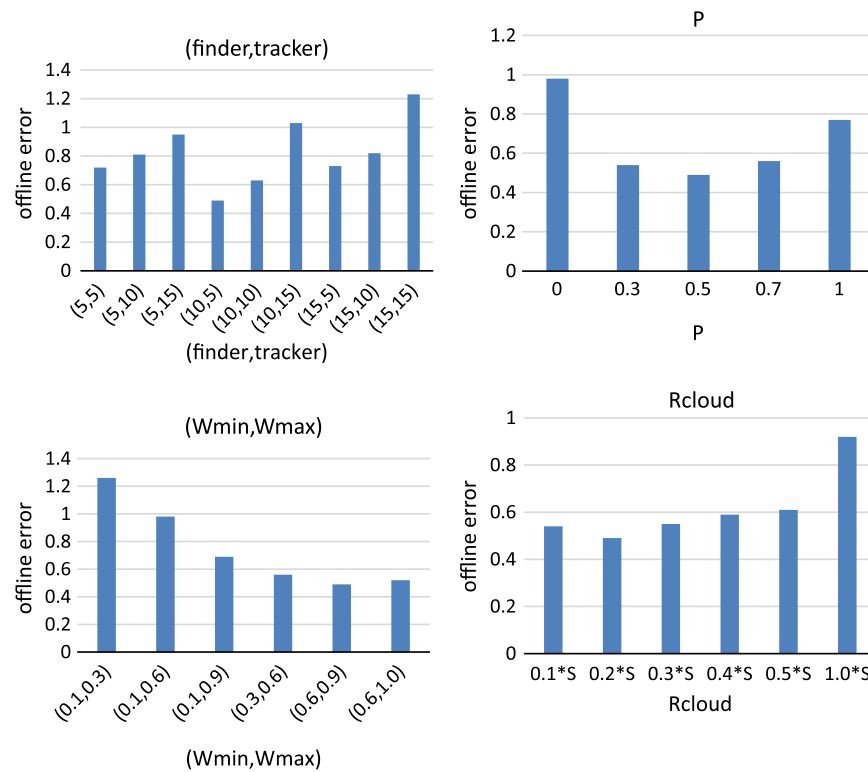


Fig. 5. Preliminary result for different parameters setting.

Table 2  
HdPSO parameters setting.

Parameter	Value
Number of particles in finder swarm	10
Number of particles in tracker swarm	5
$P$	0.5
$W_{min}$	0.6
$W_{max}$	0.9
$r_{cloud}$	0.2*s (change severity)

### 3.3. Parameters setting

In this section, the required parameters setting for the algorithm is provided. For the MPB, the parameters are set according to scenario 2 of this benchmark [4] in Table 1. However, in order to study the various aspects of the algorithm for some important parameters such as number of peaks, change frequency, dimensions, and change severity, several values are specified in this table.

In addition, for the PMPB problem, the PL parameter is set to 5, 10 and 20. It means that the environments will re-appear in pendulum manner after 5, 10 and 20 changes in the environment.

Also, based on the default configuration of MPB ( $M=10$ ,  $\alpha=5000$ ,  $s=1$ ), some preliminary experiments are conducted to achieve the best values for the parameters of HdPSO. The obtained preliminary results are presented in Fig. 5.

So, the initial configuration of the proposed algorithm is achieved based on the best combination of the parameters as follows (Table 2). Also, the  $r_{excl}$  and  $r_{conv}$  parameters are considered according to [12].

### 3.4. Comparison between HdPSO and other state of art methods

In this part, the efficiency and performance of HdPSO is tested on different MPB configurations and is compared with several state-of-the-art algorithms. The experimental results are obtained by the average of 50 executions of algorithm. Also, in each of them, the initial position of the particles and the peaks are determined randomly and with different random seeds. Termination condition of the all algorithms is 100 changes in environment which is equal to  $100 * \alpha$  fitness evaluation. Some of the presented results of the state-of-art algorithms are obtained by implementing the methods and some of them are extracted from the related references. In Tables 3–7, the efficiency of sixteen related algorithms including: mQSO [12], AmQSO [53], mPSO [54], HmPSO [55], APSO [56], FTMPPO [57], CESO [58], mNAFSA [59], PSO-AQ [60], CDEPSO [61], CellularDE [21], SFA [62], DynPopDE [25], MLDE-S [20], CbDE-wCA [23] and DPSABC [34] is compared with that of HdPSO. All simulations are performed on a PC with 2.8 GHz CPU and 8 GB memory. Also, the proposed algorithm is implemented in MATLAB language version 2012. Furthermore, for each environment,  $t$ -test with a significance level of 0.05 has been applied and the result of the best performing algorithms is printed in bold. When the results for the best performing algorithms are not significantly different, all are printed in bold.

**Table 3**

The comparison of offline error (standard error) on MPB problem with different number of peaks, dimension=5, change frequency=500 and shift severity=1.

Algorithm	Change freq.	Number of peaks							
		1	5	10	20	30	50	100	200
mQSO(5,5q)	500	33.67(3.42)	11.91(0.76)	9.62(0.34)	9.07(0.25)	8.80(0.21)	8.72(0.20)	8.54(0.16)	<b>8.19(0.17)</b>
AmQSO		3.02(0.32)	5.77(0.56)	5.37(0.42)	6.82(0.34)	7.10(0.39)	7.57(0.32)	7.34(0.31)	<b>7.48(0.19)</b>
mPSO		8.71(0.48)	6.69(0.26)	7.19(0.23)	8.01(0.19)	8.43(0.17)	8.76(0.18)	8.91(0.17)	<b>8.88(0.14)</b>
HmPSO		8.53(0.49)	7.40(0.31)	7.56(0.27)	7.81(0.20)	8.33(0.18)	8.83(0.17)	8.85(0.16)	<b>8.85(0.16)</b>
APSO		4.81(0.14)	4.95(0.11)	5.16(0.11)	5.81(0.08)	6.03(0.07)	5.95(0.06)	6.08(0.06)	<b>6.20(0.04)</b>
FTMPSO		1.76(0.09)	2.93(0.18)	3.91(0.19)	4.83(0.19)	5.05(0.21)	4.98(0.15)	5.31(0.11)	<b>5.52(0.21)</b>
SFA		4.72(0.12)	4.88(0.12)	5.11(0.14)	5.72(0.13)	5.97(0.12)	5.94(0.15)	6.15(0.08)	<b>6.18(0.11)</b>
DPSABC		2.77(0.00)	–	<b>3.42(0.00)</b>	<b>3.12(0.00)</b>	<b>3.69(0.00)</b>	<b>3.22(0.00)</b>	<b>3.01(0.00)</b>	<b>3.16(0.00)</b>
HdPSO		<b>1.50(0.05)</b>	<b>2.58(0.03)</b>	<b>3.41(0.03)</b>	<b>4.39(0.01)</b>	<b>5.09(0.03)</b>	<b>5.89(0.03)</b>	<b>6.50(0.02)</b>	<b>6.32(0.04)</b>

**Table 4**

The comparison of offline error (standard error) on MPB problem with different number of peaks, dimension=5, change frequency=1000 and shift severity=1.

Algorithm	Change freq.	Number of peaks							
		1	5	10	20	30	50	100	200
mQSO(5,5q)	1000	18.60(1.63)	6.56(0.38)	5.71(0.22)	5.85(0.15)	5.81(0.15)	5.87(0.13)	5.83(0.13)	<b>5.54(0.11)</b>
AmQSO		2.33(0.31)	2.90(0.32)	4.56(0.40)	5.36(0.47)	5.20(0.38)	6.06(0.14)	4.77(0.45)	<b>5.75(0.26)</b>
mPSO		4.44(0.24)	3.93(0.16)	4.57(0.18)	4.97(0.13)	5.15(0.12)	5.33(0.10)	5.60(0.09)	<b>5.78(0.09)</b>
HmPSO		4.46(0.26)	4.27(0.08)	4.61(0.07)	4.66(0.12)	4.83(0.09)	4.96(0.03)	5.14(0.08)	<b>5.25(0.08)</b>
APSO		2.72(0.04)	2.99(0.09)	3.87(0.08)	4.13(0.06)	4.12(0.04)	4.11(0.03)	4.26(0.04)	<b>4.21(0.02)</b>
FTMPSO		0.89(0.05)	1.70(0.10)	2.36(0.09)	3.01(0.12)	3.06(0.10)	3.29(0.10)	3.63(0.09)	<b>3.74(0.09)</b>
SFA		2.45(0.12)	2.71(0.06)	3.64(0.04)	4.01(0.07)	4.02(0.08)	4.12(0.07)	4.40(0.07)	<b>4.43(0.07)</b>
DPSABC		1.68(0.00)	–	3.23(0.00)	3.40(0.00)	3.28(0.00)	<b>2.67(0.00)</b>	<b>3.08(0.00)</b>	<b>3.01(0.00)</b>
HdPSO		<b>0.82(0.02)</b>	<b>1.20(0.02)</b>	<b>1.71(0.03)</b>	<b>2.38(0.02)</b>	<b>2.88(0.02)</b>	<b>3.01(0.02)</b>	<b>3.39(0.02)</b>	<b>3.57(0.01)</b>

**Table 5**

The comparison of offline error (standard error) on MPB problem with different number of peaks, dimension=5, change frequency=2500 and shift severity=1.

Algorithm	Change freq.	Number of peaks							
		1	5	10	20	30	50	100	200
mQSO(5,5q)	2500	7.64(0.64)	3.26(0.21)	3.12(0.14)	3.58(0.13)	3.63(0.10)	3.63(0.10)	3.58(0.08)	<b>3.30(0.06)</b>
AmQSO		0.87(0.11)	2.16(0.19)	2.49(0.10)	2.73(0.11)	3.24(0.18)	3.68(0.15)	3.53(0.14)	<b>3.07(0.12)</b>
mPSO		1.79(0.10)	2.04(0.12)	2.66(0.16)	3.07(0.11)	3.15(0.08)	3.26(0.07)	3.31(0.05)	<b>3.36(0.05)</b>
HmPSO		1.75(0.10)	1.92(0.11)	2.39(0.16)	2.46(0.09)	2.57(0.05)	2.65(0.05)	2.72(0.04)	<b>2.81(0.04)</b>
APSO		1.06(0.03)	1.55(0.05)	2.17(0.07)	2.51(0.05)	2.61(0.02)	2.66(0.02)	2.62(0.02)	<b>2.64(0.01)</b>
FTMPSO		0.39(0.02)	0.91(0.08)	1.21(0.06)	1.66(0.05)	1.87(0.05)	2.09(0.07)	2.22(0.06)	<b>2.22(0.07)</b>
SFA		0.85(0.06)	1.49(0.07)	1.95(0.04)	2.28(0.05)	2.46(0.05)	2.57(0.05)	2.74(0.04)	<b>2.76(0.04)</b>
DPSABC		1.20(0.02)	–	1.76(0.01)	1.91(0.01)	2.23(0.00)	3.12(0.00)	3.02(0.00)	<b>3.14(0.00)</b>
HdPSO		<b>0.28(0.01)</b>	<b>0.52(0.01)</b>	<b>0.75(0.01)</b>	<b>1.21(0.01)</b>	<b>1.42(0.01)</b>	<b>1.51(0.01)</b>	<b>1.80(0.01)</b>	<b>1.85(0.01)</b>

As can be seen from Tables 3–7, the efficiency of the HdPSO, in most of the cases outperform that of other sixteen state-of-the-art algorithms. Utilizing memory for fitness prediction before actual fitness evaluation and new change discovery and exclusion methods are the most prominent reasons for superiority of the proposed approach compared with other state-of-the-art algorithms. In most of them, they used a test solution to discover the change in environment. When the time interval between two consecutive changes in environment is extremely high, re-evaluating a test solution in each iteration can waste a lot of fitness evaluations during the run. E.g. if the mean number of fitness evaluation in each iteration and the change frequency are 100 and 10,000 respectively, then at least  $(10,000/100 - 1 = 99)$  fitness evaluation will be wasted in each environment. By multiplying 99 by 100 (number of change in environment), 9900 fitness evaluation will be wasted during the 100 times change in environment.

In Table 8, the efficiency of the proposed algorithm on MPB with 10 peaks, shift severities of 1, 2, 3, 5, dimension of 5 and a change frequency of 5000 is compared with that of other ten algorithms including: mQSO [12], AmQSO [53], mCPSO [12], SPSO [63], rSPSO [64], PSO-CP [65], FTMPSO [57], SFA [62], MLDE-S [20], CbDE-wCA [23]. By increasing the value of shift severity, the process of tracking peaks becomes more complex, since the peaks move to further distances after each change in environment. As can be seen in Table 8, the efficiency of all algorithms decreases by increasing the shift severity. However, less degradation is involved in HdPSO, CbDE-wCA and FTMPSO by increasing shift severity, compared to other algorithms. Also, for shift severity above 2, FTMPSO and CbDE-wCA will outperform the HdPSO.

Table 9 shows the rank of algorithms on MPB problem based on different change frequency. Each value is calculated as follows. First, rank all the offline errors in Tables 3–7 in column order for different algorithms. Then, add the previous ranks in each row and in each table for different number of peaks. Finally, rank the values of achieved column for each table (change frequency). As can be seen, the rank of proposed algorithm is 1 at the total but it is not 1 for change frequency of 500. So, the proposed algorithm does not work well on the environments which change happen very rapidly (less or equal to 500 fitness evaluation). Also, FTMPSO and SFA, the two other algorithms which proposed by the authors are

**Table 6**

The comparison of offline error (standard error) on MPB problem with different number of peaks, dimension=5, change frequency=5000 and shift severity=1.

Algorithm	Change freq.	Number of peaks							
		1	5	10	20	30	50	100	200
mQSO(5,5q)	5000	2.24(0.05)	1.82(0.08)	1.85(0.08)	2.48(0.09)	2.51(0.10)	2.53(0.08)	2.35(0.06)	<b>2.24(0.05)</b>
AmQSO		2.62(0.10)	1.01(0.09)	1.51(0.10)	2.00(0.15)	2.19(0.17)	2.43(0.13)	2.68(0.12)	<b>2.62(0.10)</b>
mPSO		2.24(0.05)	1.82(0.08)	1.85(0.08)	2.48(0.09)	2.51(0.10)	2.53(0.08)	2.35(0.06)	<b>2.24(0.05)</b>
HmPSO		2.62(0.10)	1.01(0.09)	1.51(0.10)	2.00(0.15)	2.19(0.17)	2.43(0.13)	2.68(0.12)	<b>2.62(0.10)</b>
APSO		0.53(0.01)	1.05(0.06)	1.31(0.03)	1.69(0.05)	1.78(0.02)	1.95(0.02)	1.95(0.01)	<b>1.90(0.01)</b>
FTMPSO		0.18(0.01)	0.47(0.05)	0.67(0.04)	0.93(0.04)	1.14(0.04)	1.32(0.04)	1.61(0.03)	<b>1.67(0.03)</b>
SFA		0.42(0.03)	0.89(0.07)	1.05(0.04)	1.48(0.05)	1.56(0.06)	1.87(0.05)	2.01(0.04)	<b>1.99(0.06)</b>
CESO		1.04(0.00)	–	1.38(0.02)	1.72(0.02)	1.24(0.01)	1.45(0.01)	1.28(0.02)	–
mNAFSA		0.59(0.06)	0.66(0.05)	0.94(0.04)	1.29(0.05)	1.60(0.06)	1.81(0.06)	1.92(0.05)	<b>1.97(0.05)</b>
PSO-AQ		0.34(0.02)	0.80(0.12)	0.89(0.03)	1.45(0.06)	1.52(0.04)	1.77(0.05)	1.95(0.05)	<b>1.96(0.04)</b>
CDEPSO		0.41(0.00)	0.97(0.01)	1.22(0.01)	1.54(0.01)	2.62(0.01)	2.20(0.01)	1.54(0.01)	<b>2.11(0.01)</b>
CellularDE		1.53(0.07)	1.50(0.04)	1.64(0.03)	2.46(0.05)	2.62(0.05)	2.75(0.05)	2.73(0.03)	<b>2.61(0.02)</b>
DynPopDE		–	1.03(0.13)	1.39(0.07)	–	–	2.10(0.06)	2.34(0.05)	<b>2.44(0.05)</b>
MLDE-S		1.11(0.07)	–	1.35(0.07)	–	–	–	1.65(0.08)	–
CbDE-wCA		<b>0.14(0.03)</b>	<b>0.30(0.02)</b>	0.86(0.08)	0.98(0.05)	1.34(0.04)	1.31(0.04)	1.35(0.03)	<b>1.29(0.02)</b>
DPSABC		2.25(0.00)	–	2.13(0.00)	2.07(0.00)	1.88(0.00)	1.91(0.00)	1.89(0.00)	<b>1.87(0.00)</b>
HdPSO		<b>0.16(0.01)</b>	<b>0.34(0.01)</b>	<b>0.49(0.01)</b>	<b>0.68(0.01)</b>	<b>0.83(0.01)</b>	<b>1.08(0.01)</b>	<b>1.15(0.01)</b>	<b>1.18(0.01)</b>

**Table 7**

The comparison of offline error (standard error) on MPB problem with different number of peaks, dimension=5, change frequency=10,000 and shift severity=1.

Algorithm	Change freq.	Number of peaks							
		1	5	10	20	30	50	100	200
mQSO(5,5q)	10,000	1.90(0.18)	1.03(0.06)	1.10(0.07)	1.84(0.08)	2.00(0.09)	1.99(0.07)	1.85(0.05)	<b>1.71(0.04)</b>
AmQSO		0.19(0.02)	0.45(0.04)	0.76(0.06)	1.28(0.12)	1.78(0.09)	1.55(0.08)	1.89(0.14)	<b>2.52(0.10)</b>
mPSO		0.27(0.02)	0.70(0.10)	0.97(0.04)	1.34(0.08)	1.43(0.05)	1.47(0.04)	1.50(0.03)	<b>1.48(0.02)</b>
APSO		0.25(0.01)	0.57(0.03)	0.82(0.02)	1.23(0.02)	1.39(0.02)	1.46(0.01)	1.38(0.01)	<b>1.36(0.01)</b>
FTMPSO		0.09(0.00)	0.31(0.04)	0.43(0.03)	0.56(0.01)	0.69(0.09)	0.86(0.02)	1.08(0.03)	<b>1.13(0.04)</b>
SFA		0.26(0.03)	0.53(0.04)	0.72(0.02)	0.91(0.03)	0.99(0.04)	1.19(0.04)	1.44(0.04)	<b>1.52(0.03)</b>
DPSABC		<b>2.67(0.00)</b>	–	9.00(0.01)	6.60(0.01)	7.70(0.01)	8.10(0.01)	8.34(0.01)	<b>8.52(0.01)</b>
HdPSO		<b>0.06(0.00)</b>	<b>0.25(0.01)</b>	<b>0.29(0.01)</b>	<b>0.38(0.00)</b>	<b>0.57(0.00)</b>	<b>0.73(0.00)</b>	<b>0.84(0.00)</b>	<b>0.97(0.01)</b>

**Table 8**

The comparison of offline error (standard error) on MPB problem with different shift severities, dimension=5, peak number=10, change frequency=5000.

Algorithm	Shift severity			
	1	2	3	5
mQSO(5,5q)	1.85(0.08)	2.40(0.06)	3.00(0.06)	<b>4.24(0.10)</b>
AmQSO	1.51(0.10)	2.09(0.08)	2.72(0.09)	<b>3.71(0.11)</b>
mCPSO	4.89(0.11)	3.57(0.08)	2.80(0.07)	<b>2.08(0.07)</b>
SPSO	2.51(0.09)	3.78(0.09)	4.96(0.12)	<b>6.76(0.15)</b>
rSPSO	1.50(0.08)	1.87(0.05)	2.40(0.08)	<b>3.25(0.09)</b>
PSO-CP	1.31(0.06)	1.98(0.06)	2.21(0.06)	<b>3.20(0.13)</b>
FTMPSO	0.67(0.04)	1.20(0.06)	1.40(0.09)	<b>1.69(0.07)</b>
SFA	1.05(0.04)	1.44(0.06)	2.06(0.07)	<b>2.89(0.13)</b>
MLDE-S	1.35(0.07)	1.91(0.08)	2.34(0.07)	<b>2.68(0.11)</b>
CbDE-wCA	0.86(0.08)	0.88(0.08)	<b>0.98(0.09)</b>	<b>1.54(0.7)</b>
HdPSO	<b>0.49(0.01)</b>	<b>0.78(0.01)</b>	1.48(0.03)	<b>1.98(0.02)</b>

in the second and third place in this ranking which proposed by the authors recently. It is worth mentioning that CbDE-wCA does not included in this table because the result in its paper is only available for change frequency of 5000. However, from [Tables 6 and 8](#), it seems that this algorithm can outperform the proposed approach on some specific configuration such as high shift severity.

### 3.5. Comparison of HDPSO on PMPB problem

In this section, the performance of the proposed algorithm is evaluated on Pendulum-MPB (PMPB) problem. This benchmark is defined for the first time in this paper and is described in [Section 3.1](#) completely. As we said before, the PL parameter is set to 5, 10 and 20. It means that the environments will re-appear in pendulum manner after 5, 10 and 20 changes in an environment. [Table 9](#) shows the performance of HdPSO on MPB and PMPB. As we can see in [Table 10](#), the more the pendulum length parameter increases, the more the efficiency of the algorithm decreases.

**Table 9**

The rank of algorithms on MPB problem based on different change frequenc.

Algorithm	Change frequency					
	500	1000	2500	5000	10,000	Overall ranking
mQSO(5,5q)	9	8	9	8	7	9
AmQSO	6	5	7	7	5	6
mPSO	8	7	8	8	6	8
HmPSO	7	6	6	7	8	7
APSO	5	4	5	5	4	5
FTMPSO	2	2	3	3	2	2
SFA	4	3	4	4	3	3
DPSABC	1	2	2	6	9	4
HdPSO	3	1	1	1	1	1

**Table 10**

comparison of offline error (standard error) on PMPB and MPB problems with different number of peaks, dimension=5, change frequency=5000 and shift severity=1.

Algorithm	Problem	Number of peaks							
		1	5	10	20	30	50	100	200
HdPSO	MPB	0.14(0.00)	0.30(0.01)	0.51(0.01)	0.70(0.01)	0.88(0.01)	1.17(0.01)	1.23(0.01)	1.26(0.01)
HdPSO	PMPB l=5	0.05(0.00)	0.10(0.01)	0.20(0.01)	0.26(0.01)	0.32(0.01)	0.37(0.01)	0.41(0.01)	0.43(0.01)
HdPSO	PMPB l=10	0.07(0.00)	0.14(0.01)	0.25(0.01)	0.36(0.01)	0.42(0.01)	0.53(0.01)	0.63(0.01)	0.63(0.01)
HdPSO	PMPB l=20	0.10(0.00)	0.23(0.01)	0.33(0.01)	0.53(0.01)	0.60(0.01)	0.70(0.01)	0.80(0.01)	0.88(0.01)

#### 4. Conclusion

In this paper, a novel optimization algorithm with and explicit memory was proposed for working on dynamic environments. The proposed algorithm could quickly find the peaks in the problem space and follow them after an environment change. Also, it could utilize the previous history of the search space to guide the search to the promising area and avoid wasting fitness area. In the proposed algorithm, swarms in the problem space were categorized into finder and tracker swarms which were configured somehow to complete each other during the search process.

Efficiency of the proposed algorithm has been evaluated on moving peaks benchmark, which is the most well-known benchmark in this domain, and a modified version of it, called PMPB. The results were compared with those of the state-of-the-art algorithms. The experimental results and comparative studies showed the superiority of the proposed algorithm. Diverse experiments showed that the proposed algorithm involved a high convergence speed which is significantly important in designing any optimization algorithm in dynamic environments. Each swarm has been equipped with some mechanisms, based on its corresponding function, to conquer its particular challenges. As a result, the functions of swarms were appropriately performed.

We would like to apply the proposed methods as a boosting technique to some powerful optimization algorithms such as CoBiDE [45], CoDE [46], BSA [47] and DSA [48] instead of PSO. Also, we would like to use the proposed algorithm on different dynamic benchmark problems and also real applications. Currently, we are working on its applications to the clustering in web, and to the personalization of web pages dynamically. Adapting the algorithm to dynamic multi-objective optimization is one of the goals that will be pursued in near future. Also, we are working to make adaptive the important parameters in HdPSO for easier to use.

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