An Adaptive Fuzzy Recommender System based on Learning Automata

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Abstract: *Incorporating trust and distrust information into collaborative recommender systems alleviates data sparsity and cold start problems. Since trust and distrust are a gradual phenomenon, they can be stated more naturally by fuzzy logic. Finding the most appropriate fuzzy sets which cover the domains of trust and distrust is not an easy task. Existing research on fuzzy modelling of trust and distrust has not considered the optimization of membership functions. In this paper, we address this issue and propose a continuous action-set learning automata (CALA)-based method to adjust membership functions of fuzzy trust and distrust during the lifetime of recommender system in terms of recommendation error. By assigning a CALA to the centre parameter of each triangular membership function, the proposed method optimizes the number and the position of fuzzy sets. To the best of our knowledge, this is the first effort in this direction. The experimental results indicate that using the proposed method in fuzzy recommender systems improves the recommendation accuracy.*

Keywords: Social networks, Recommender systems, Trust, Distrust, Fuzzy linguistic modelling, Learning automata

1. Introduction

Recommender systems (RSs) have emerged as a key tool to overcome the information overload problem and help users find relevant information based on their preferences. The user preferences are specified by the ratings provided by users on a subset of items they have visited before. Collaborative filtering (CF) is a most widely used technique in recommender systems (Adomavicius and Tuzhilin, 2005). A CF-based recommender system predicts the user rating for a new item based on past ratings of users with similar preferences. Using trust between users in these recommenders can improve the quality of recommendations (Yin et al., 2012; Tang et al., 2013b; Shambour and Lu, 2015; Moradi and Ahmadian, 2015; Moradi et al., 2015; Alahmadi and Zeng, 2015; Wu et al., 2016). The idea in trust-enhanced recommender systems is that users usually be influenced by their trusted friends and ask them for recommendations. A lot of social applications provide a web of trust for users to express their trust on other users, such as FilmTrust (Golbeck and Hendler, 2006a) and Epinions (Epinions.com). While there has been a great deal of research focusing on trust information, only few research works have investigated the incorporation of distrust in addition to trust into the recommendation process (Ma et al., 2009; Victor et al., 2011b; Victor et al., 2013; Kant and Bharadwaj, 2013). Researchers found that besides the trust relationships, the distrust relationships between users are also unavoidable and recommender systems can benefit from distrust information in social networks.

Each time trust and distrust values are not explicitly available, they are usually evaluated based on the ratings data of users (Bharadwaj and Al-Shamri, 2009; Yan et al., 2011). Various computational models for trust and distrust have been proposed so far (Bharadwaj and Al-Shamri, 2009; Kant and Bharadwaj, 2013; Hao et al., 2014). Since, in real life, people tend to express their trust in others in linguistic expressions, researchers have presented several fuzzy models of trust and distrust in literature (Bharadwaj and Al-Shamri, 2009; Kant and Bharadwaj, 2013; Hao et al., 2014; Ayadi et al., 2015). Due to the difficulties in designing fuzzy membership functions, the triangular membership function is most commonly used for representing various linguistic terms (Zadeh, 1996). To the best of our knowledge, in all works done on fuzzy modelling of trust and distrust, the number of membership functions for each of these fuzzy variables is determined based on expert opinion as a constant value and membership functions are often uniformly and symmetrically distributed on the values axis in terms of this fixed value. Using uniformly and symmetrically distributed linguistic terms the same discrimination levels will exist on both sides of the values axis. However, there are some cases in which trust and distrust variables need to be assessed with unbalanced linguistic terms, i.e., linguistic terms that are not uniformly and symmetrically distributed. For example, a recommender system usually looks for the trusted users of a target user to make recommendations. That is, it usually uses the opinions of users trusted by using linguistic trust statements on the right of the values axis of trust. Thus, a more accurate recommendation can be achieved if a higher number of discrimination levels on the right of axis is assumed. In these cases, where expert needs to assess a number of terms in a side of the values axis higher than in the other one, and in general, it is extremely hard for an expert to determine the appropriate number and position of fuzzy sets.

There exist several research works proposed in literature to optimize fuzzy membership functions (Simon, 2005; Kaya and Alhajj, 2006; Zhao and Li, 2007; Omizegba and Adebayo, 2009; Permana and Hashim, 2010; Acilar and Arslan, 2011; Huynh et al., 2012). However, to the best of our knowledge, none of these approaches try to optimize the number of membership functions, but only adjust the position of them on the values axis.

In this paper, we address the above problem and propose a method based on continuous action-set learning automata (CALA) for simultaneously optimizing the number and position of membership functions for fuzzy trust and distrust such that a recommender system can achieve the highest precision in the user rating estimation. The learning automata has shown to be a very useful tool in solving the optimization problems (Beigy and Meybodi, 2006; Akbari Torkestani and Meybodi, 2012; Rezvanian and Meybodi, 2015). In order to investigate the effect of membership functions optimization on the performance of recommender systems, we employ the proposed method in a fuzzy trust-distrust enhanced recommender system proposed in (Kant and Bharadwaj, 2013) and test its performance over well-known datasets. The experimental results indicate that the proposed method by providing fuzzy membership functions optimized with respect to used dataset improves the accuracy of recommendations in fuzzy recommender systems.

The rest of this paper is structured as follows. In section 2, we discuss the preliminaries and related work. Section 3 presents a brief background of learning automata and continuous action-set learning automata (CALA). The proposed method for tuning fuzzy membership functions is described in Section 4. The experimental evaluation and performance results are reported in Section 5. Section 6 is the conclusion.

2. Preliminaries and Related Work

Recommender systems (RSs) (Lü et al., 2012; Bobadilla et al., 2013) use information on the preferences of users for a set of items to provide recommendations to them. The information can be obtained either explicitly (typically by collecting users' ratings) or implicitly (typically by monitoring users' behaviour). Recommender systems can be classified into three main categories: content-based (CB), collaborative filtering (CF), and hybrid methods (Lü et al., 2012). In content-based RSs, Recommended items are those with content similar to the content of previously preferred items of a given user (Lops et al., 2011; Martinez-Romo and Araujo, 2012; Pera and Ng, 2013). In contrast, CF makes recommendations to each user based on information provided by those users that have the most in common with her (Bobadilla et al., 2011; Bobadilla et al., 2012; Altingovde et al., 2013; Formoso et al., 2013; Ortega et al., 2013; Lian et al., 2013; Halvey et al., 2014; Ramezani et al., 2014; Choi et al., 2016; Wang et al., 2016). The hybrid approaches combine

CF with CB methods or with different variants of other collaborative methods to gain advantages and avoid certain limitations of each method (Barragáns-Martínez et al., 2010; Kayaalp et al., 2010; Kim et al., 2010; Porcel et al., 2012; Zhang et al., 2012; Forsati et al., 2013; Zhang et al., 2014).

CF algorithm has been shown to be the most successful approach to recommender systems (Adomavicius and Tuzhilin, 2005). CF attempts to automate the process of "word-of-mouth": Recommended items are selected on the basis of past evaluations of like-minded users (Shardanand and Maes, 1995). CF has a major advantage over CB in that it does not require an explicit content description of the items, but it is based only on the ratings data. However, CF approaches often suffer from two problems: data sparsity and cold start (Adomavicius and Tuzhilin, 2005). In contrast to the huge number of available items, each user normally rates only a few items. Thus, two users don't have the enough number of items rated in common required by similarity metrics. Therefore, it is difficult for a recommender system to accurately measure user similarities based on that limited number of ratings. In addition, even for a system that is not particularly sparse, when a new user enter the system, there is usually insufficient information to accurately interpret this user's preference. This problem is referred as the cold start problem.

In order to overcome these two challenges, trust-enhanced recommender systems have been proposed in literature (Tang et al., 2013b; Shambour and Lu, 2015; Moradi and Ahmadian, 2015; Moradi et al., 2015; Alahmadi and Zeng, 2015; Wu et al., 2016). Trust-enhanced recommender systems consider trust relations and recommend items to each user from her trusted users. These RSs assume that a trust relation is formed when two users have similar opinions to similar items (Tang et al., 2013a). With this assumption, it is no longer necessary to measure the rating similarity for finding similar users. Thus, the data sparsity problem is alleviated. For the cold start problem, even if a user has no past ratings, recommender system can still make recommendations to the user based on the preferences of her trusted users.

Many online websites use a web of trust that allows users to express trust in other users. For example, FilmTrust (Golbeck and Hendler, 2006a), as a movie-focused social networking site, asks its users to assign a trust ranking to each person they add as a friend. Another example is the Epinions site (Epinions.com) in which users can specify users that they trust. Research on recommender systems shows that using measures of trust can improve the accuracy and quality of recommendations (Victor et al., 2011a). In contrast to trust for which there has been a great deal of research, the potential benefits of explicitly incorporating distrust in recommendation process is almost unexplored. Recently, few works have considered distrust in recommender systems and shown that the usage of distrust can increase the performance of these systems (Ma et al., 2009; Victor et al., 2011b; Victor et al., 2013; Kant and Bharadwaj, 2013).

In the absence of explicit trust statements, different information sources have been utilized in literature to assess trust values (Torkzadeh and Dhillon, 2002; Yan et al., 2011; Thorleuchter and Van den Poel, 2012). Recommender systems usually use users' past ratings on items to derive trust connectivity and trust statement (Bharadwaj and Al-Shamri, 2009). Existing trust models can be categorized into two groups (Victor et al., 2011a): probabilistic and gradual approaches. Probabilistic models assume that a user can either be trusted or not trusted and compute the probability that the user can be trusted (Jøsang et al., 2006a; Jøsang et al., 2006b; Gutscher, 2009). In contrast, a gradual model estimates trust value to some extent rather than in a black or white fashion (Golbeck, 2005; Ziegler and Lausen, 2005; Golbeck and Hendler, 2006b; Ziegler and Golbeck, 2007). In real life, too, trust is often considered as a gradual phenomenon, people would like to reason about others in terms of "trusting very much or less” rather than “trusting” or “not trusting”. Fuzzy logic (Zadeh, 1996) is very well-suited to represent these linguistic expressions and handle the ambiguity of the trust concept.

Ludwig et al. (2009) developed a fuzzy-based trust approach based on three requirements. Their proposed approach uses different trust sources to evaluate the overall trust of a service. Work in (Lesani and Montazeri, 2009) presented a fuzzy modelling of trust in virtual social networks. In this paper, trust values are represented as fuzzy sets with triangular membership functions defined in the trust range. Authors in (Kant and Bharadwaj, 2011) also considered trust as a gradual notion which can be expressed by linguistic terms. They quantified fuzzy trust into seven triangular membership functions and proposed a fuzzy trust propagation method for to alleviate sparsity problem of CF-based recommender systems. Work in (Capuruço and Capretz, 2012) proposed an inference mechanism of fuzzy trust based on homophily and separation factors in social networks. Authors then developed a trust-based recommendation strategy which incorporates this fuzzy trust model into the CF-based recommender systems. Hao et al. (2014) proposed a fuzzy trust inference mechanism, called MobiFuzzyTrust, for inferring trust semantically from one mobile user to another in mobile social networks (MSNs). MobiFuzzyTrust supports the linguistic expressions as trust ranking between users in MSNs and defines triangular membership functions for these expressions. Work in (Ayadi et al., 2015) presented a fuzzy collaborative assessment methodology for evaluating partner trust in horizontal collaborative networks. In this paper, information-sharing attributes considered in the literature are represented as unbalanced linguistic term sets and incorporated in the semantic fuzzy partitioning method for the collaborative trust assessment. Using the fuzzy linguistic modelling, Martinez-Cruz et al. (2015) developed an ontology to efficiently characterize the trust between users. Authors incorporated this ontology in a recommender system to improve the representation of user profiles. Work in (Kant and Bharadwaj, 2013) presented fuzzy computational models for both the trust and distrust concepts based on similarity and knowledge factors. This paper analysed various recommendation strategies utilizing proposed fuzzy computational models of trust and distrust.

In the fuzzy sets approach, due to the difficulties in designing fuzzy membership functions, various linguistic terms are most commonly represented by the triangular membership function (Zadeh, 1996). To the best of our knowledge, in all works done on fuzzy modelling of trust and distrust, each of these linguistic variables trust and distrust takes a predefined number of linguistic terms that are often uniformly and symmetrically distributed along the values axis. That is, the number and positions of linguistic terms on the values axis are determined not based on the requirement of problem, but based on expert opinion. However, it is unrealistic that experts can always provide such informations. Considering the above problem, this paper propose a method based on continuous action-set learning automata (CALA) for optimizing membership functions representing linguistic terms of fuzzy trust and distrust.

3. Learning Automata

The learning automata are shown to be a powerful tool in solving the optimization problems (Thathachar and Sastry, 1987; Najim and Poznyak, 1996). A stochastic learning automaton (Narendra and Thathachar, 2012) may be considered as a simple model for adaptive decision making in unknown random environments. The automation learns the optimal action using its repeated interaction with the environment and thus it improves its performance. At each instant t, the automaton performs the chosen action $\alpha(t)$ on the random environment. The action selection is done randomly based on the probability distribution kept over the action-set. The environment responds the chosen action $\alpha(t)$ in turn with either a reward or a penalty (denoted by $\beta(t)$). The automaton updates its action probability distribution based on the reinforcement signal $\beta(t)$ from the environment. The objective of a learning automaton is to minimize the average penalty received from the environment.

The random environment of a learning automaton can be described by a triple $\langle\alpha, \beta, c\rangle$, where $\alpha = \{\alpha_1, \alpha_2, \dots, \alpha_r\}$ represents the finite set of inputs (i.e. the possible actions for the automaton), $\beta = \{\beta_1, \beta_2, \dots, \beta_m\}$ denotes the set of values taken by the reinforcement signal, and $c = \{c_1, c_2, \dots, c_r\}$ is the set of penalty probabilities, where c_i corresponds to the given action α_i. The relationship between the learning automaton and its random environment is depicted in Fig. 1.

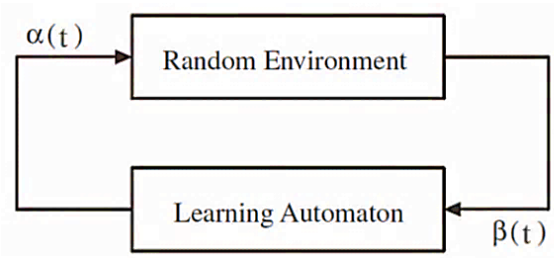


Fig.1 The relationship between the learning automaton and the random environment

Stochastic learning automata can be categorized into two main classes namely, finite action-set learning automata (FALA) and continuous action-set learning automata (CALA) (Thathachar and Sastry, 2011). The learning automaton is called FALA if it has a finite set of actions and it is called CALA otherwise. For an FALA with r actions, the action probability distribution is an r-dimensional probability vector. In many applications there is need to learn a real-valued parameter. In this situation, the actions of the automaton can be possible values of that parameter. To use an FALA for such an optimization problem, we have to discretize the value space of parameter to obtain a finite number of actions. However, a fine discretization increases the number of actions which in turn decreases the convergence speed of the automaton. A natural solution to this problem would be to employ an automaton with a continuous space of actions. In the next subsection, we describe one such model of LA called continuous action-set learning automata (CALA).

3.1 Continuous Action-Set Learning Automata (CALA)

A continuous action-set learning automata (CALA) (Thathachar and Sastry, 2011) is an automaton whose action-set is the real line and its action probability distribution is considered to be a normal distribution with mean $\mu(t)$ and standard deviation $\sigma(t)$. At each time instant t, the CALA chooses a real number α at random based on the current action probability distribution $N(\mu(t), \sigma(t))$. The two actions $\alpha(t)$ and $\mu(t)$ are served as the inputs to the random environment. The CALA receives the reinforcement signals $\beta_{\alpha(t)}$and $\beta_{\mu(t)}$ from the environment for both actions. At last, $\mu(t)$ and $\sigma(t)$ are updated as

$$\mu(t+1) = \mu(t) + \lambda \frac{\left(\beta_{\alpha(t)} - \beta_{\mu(t)}\right)}{\phi\big(\sigma(t)\big)} \frac{\big(\alpha(t) - \mu(t)\big)}{\phi\big(\sigma(t)\big)} \tag{1}$$

$$\sigma(t+1) = \sigma(t) + \lambda \frac{\left(\beta_{\alpha(t)} - \beta_{\mu(t)}\right)}{\phi\big(\sigma(t)\big)} \left[\left(\frac{\big(\alpha(t) - \mu(t)\big)}{\phi\big(\sigma(t)\big)}\right)^2 - 1\right] - \lambda K(\sigma(t) - \sigma_L) \tag{2}$$

Where,

$$\phi\big(\sigma(t)\big) = \begin{cases} \sigma_L & for\ \sigma(t) \le \sigma_L \\ \sigma(t) & for\ \sigma(t) > \sigma_L \end{cases}$$

and $0 < \lambda < 1$ denotes the learning parameter, $K > 0$ is a large positive constant controlling the shrinking of $\sigma(t)$ and σ_L is a sufficiently small lower bound on $\sigma(t)$. Since the updating given for $\sigma(t)$ does not automatically ensure that $\sigma(t) \ge \sigma_L$, the function ϕ provides a projected version of $\sigma(t)$, denoted by $\phi\big(\sigma(t)\big)$.The interaction with the environment continue until $\mu(t)$ does not change noticeably and $\sigma(t)$ converges close to σ_L.

The objective of CALA is to learn value of α for which $E\left[\beta_{\alpha(t)}\right]$ attains a minimum. That is, the objective is to make $N\big(\mu(t), \sigma(t)\big)$ converge to $N(\alpha^*, 0)$, where α^* is a minimum of $E[\beta_{\alpha(t)}]$. However, we cannot let $\sigma(t)$ converge to zero, since we want the asymptotic behaviour of the algorithm to be analytically tractable. Hence, the lower bound $\sigma_L > 0$ is used and the objective of learning is kept as $\sigma(t)$ converging to σ_L and $\mu(t)$ converging to α^*. By choosing σ_L and λ sufficiently small and K sufficiently large, $\mu(t)$ of the CALA algorithm will be close to a minimum of $E[\beta_{\alpha(t)}]$ with probability close to unity after a long enough time (Thathachar and Sastry, 2011).

4. Utilizing Learning Automata for Adaptation of Fuzzy Membership Functions

In this section, we propose a method based on continuous action-set learning automata (CALA), called *CALA-OptMF*, to optimize membership functions (MFs) in fuzzy modelling of trust and distrust in recommender systems. For simplicity, we describe our proposed method *CALA-OptMF* with the goal of tuning membership functions of fuzzy trust during the lifetime of a fuzzy trust-enhanced recommender system. The same approach can be also utilized to tune membership functions of fuzzy distrust. Having this in mind, *CALA-OptMF* aims to adjust the number and position of membership functions of trust linguistic terms. For this purpose, a set of n linguistic terms denoted as $\mathcal{L} = \{L_1, L_2, \dots, L_n\}$, such as {high, medium, low}, are considered to describe the various levels of trust. Membership functions ($\mathcal{F} = \{F_1, F_2, \dots, F_n\}$) for these linguistic terms are assumed to be in triangular shape, where each triangular membership function F_i is represented by a triple (ℓ_i, c_i, r_i). We also assume that the value of trust is in interval $[\tau_{min}, \tau_{max}]$. Except for the first and the last membership functions F_1 and F_n whose centres are fixed on τ_{min} and τ_{max} respectively, the centres of the other membership functions are the parameters of our optimization problem. As shown in Fig. 2, the interval of values for each centre parameter c_i is $[\ell_i, r_i]$.

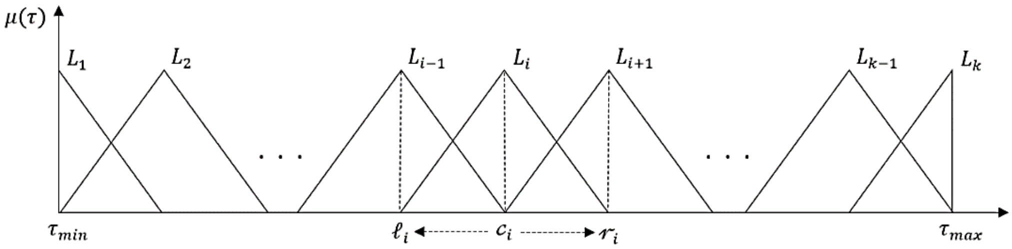


Fig.2 Membership functions and parameters of fuzzy trust

In the proposed method, at first a $CALA_i$ is assigned to the centre parameter c_i of each of the remaining $(n-2)$ membership functions (i.e. $F_2, F_3, \dots, F_{n-1}$) to learn the optimal value of that parameter, and the parameters $\mu(0)$ and $\sigma(0)$ of the action probability distributions for all CALAs are initialized. Then, *CALA-OptMF* iteratively adjusts the membership functions of fuzzy trust according to the actions selected by CALAs and updates the action probability distributions of CALAs based on the performance of trust-enhanced RS until it finds the most appropriate membership functions. The pseudo code for *CALA-OptMF* is given in Fig.3.

The iteration t of the proposed method *CALA-OptMF* is described in the three following steps:

Step 1. MFs adjustment

This step adjusts the membership functions of fussy trust. For this purpose, all CALAs select an action in parallel based on their action probability distributions. The chosen action by each $CALA_i$, denoted as α_i, determines the value of its corresponding centre parameter c_i on the values axis of trust, i.e. $F_i = (\ell_i, \alpha_i, r_i)$. Let $\acute{\mathcal{F}}$ be the set of membership functions truly adjusted which forms the output of this step. $\acute{\mathcal{F}}$ initially includes the first membership function F_1 of $\mathcal{F}$, namely $\acute{\mathcal{F}} = \{(\ell_1, c_1, r_1)\}$. For each other membership function F_i $(1 < i < n)$ belonging to $\mathcal{F}$, the following condition is checked:

$$max(\ell_i, \acute{c}_k) \le c_i \le r_i$$

Algorithm *CALA-OptMF*

01 **Input** Number of fuzzy sets n, Minimum value τ_{min}, Maximum value τ_{max}, Merge threshold ε, Lower bound σ_L
02 **Output** Set of optimized membership functions $\acute{\mathcal{F}}$
03 **Assumption**
04 Let each MF F_i be in triangular shape represented by a triple (ℓ_i, c_i, r_i)
05 Assign an automaton $CALA_i$ to the centre parameter c_i of each MF F_i $(i = 2, \ldots, n-1)$
06 **Begin**
07 Let $\acute{F}_k$ denotes the kth membership function added to $\acute{\mathcal{F}}$, represented by a triple $(\acute{\ell}_k, \acute{c}_k, \acute{r}_k)$
08 Initialize the parameters $\mu(0)$ and $\sigma(0)$ for all CALAs
09 $t = 0$
10 **Repeat**
11 All CALAs choose an action in parallel based on their action probability distributions
12 Set each centre parameter c_i to action α_i chosen by $CALA_i$
13 $\acute{\mathcal{F}} \leftarrow \{(\ell_1, c_1, r_1)\}$; $k \leftarrow 1$
14 **For** each MF F_i $(i = 2, \ldots, n-1)$ **do**
15 **If** $max(\ell_i, \acute{c}_k) \leq c_i \leq r_i$ **then**
16 **If** $(c_i - \acute{c}_k) > \varepsilon$ **then**
17 $\acute{\mathcal{F}} \leftarrow \acute{\mathcal{F}} \cup (\ell_i, c_i, r_i)$; $k \leftarrow k+1$
18 **Else**
19 $\acute{\mathcal{F}} \leftarrow \acute{\mathcal{F}} \backslash (\acute{\ell}_k, \acute{c}_k, \acute{r}_k) \cup (\acute{\ell}_k, Avg(c_i, \acute{c}_k), r_i)$
20 **End**
21 **End**
22 **End**
23 **If** $(c_n - \acute{c}_k) > \varepsilon$ **then**
24 $\acute{\mathcal{F}} \leftarrow \acute{\mathcal{F}} \cup (\ell_n, c_n, r_n)$; $k \leftarrow k+1$
25 **Else**
26 $\acute{\mathcal{F}} \leftarrow \acute{\mathcal{F}} \backslash (\acute{\ell}_k, \acute{c}_k, \acute{r}_k) \cup (\acute{\ell}_k, c_n, r_n)$
27 **End**
28 $\beta_{\alpha(t)} \leftarrow$ Compute evaluation metric f_{err} for RS using $\acute{\mathcal{F}}$
29 Set each centre parameter c_i to mean parameter μ_i of $CALA_i$
30 Repeat lines from 13 to 27 and construct $\acute{\mathcal{F}}$ again, this time in terms of mean parameters
31 $\beta_{\mu(t)} \leftarrow$ Compute evaluation metric f_{err} for RS using $\acute{\mathcal{F}}$
32 Update parameters $\mu(t)$ and $\sigma(t)$ of CALAs respectively based on equations 1 and 2
33 $t = t + 1$
34 **Until** (for all CALAs: $\mu(t)$ does not change noticeably and $\sigma(t)$ converges close to σ_L)
35 **End Algorithm**

Fig.3 The pseudo code of *CALA-OptMF* algorithm

where k is the size of $\acute{\mathcal{F}}$ and $\acute{c}_k$ refers to the centre parameter of the kth membership function $\acute{F}_k$ added to the set $\acute{\mathcal{F}}$. The above condition investigates whether the centre parameter of the membership function F_i is within its specified interval ($c_i \in [\ell_i, r_i]$) and after the centre parameter of the latest membership function in $\acute{\mathcal{F}}$. That is, this condition investigates whether the membership function F_i has been truly adjusted to be added to $\acute{\mathcal{F}}$ or not. Each membership function F_i that satisfies this condition will be added to $\acute{\mathcal{F}}$ as a new member if the distance between the centre parameters of F_i and $\acute{F}_k$ is larger than the merge threshold ε, i.e. $(c_i - \acute{c}_k) > \varepsilon$, and otherwise it will be merged with $\acute{F}_k$ resulting in the membership function $(\acute{\ell}_k, Avg(c_i, \acute{c}_k), r_i)$, i.e. $\acute{\mathcal{F}} \leftarrow \acute{\mathcal{F}} \backslash (\acute{\ell}_k, \acute{c}_k, \acute{r}_k) \cup (\acute{\ell}_k, Avg(c_i, \acute{c}_k), r_i)$ where the symbol \ denotes the set of all elements in $\acute{\mathcal{F}}$ except $\acute{F}_k = (\acute{\ell}_k, \acute{c}_k, \acute{r}_k)$.

Finally, if $(c_n - \acute{c}_k) \leq \varepsilon$ then $(\acute{\ell}_k, c_n, r_n)$ and otherwise the membership function F_n will be added to $\acute{\mathcal{F}}$.

Step 2. Performance evaluation

In this step, the set $\acute{\mathcal{F}}$ constructed in the previous step is employed in a trust-enhanced RS and the performance of this RS is measured in terms of an evaluation metric, such as the mean absolute error (MAE). We refer to this evaluation metric as f_{err} and use it for updating the parameters of the action probability distributions of CALAs. That is, the function f_{err} is considered as $\beta_{\alpha(t)}$ in the learning algorithm (for more details, see subsection 3.1). $\beta_{\mu(t)}$ is computed similar to $\beta_{\alpha(t)}$, difference is that in the step 1 the centre parameter of each membership function F_i $(1 < i < n)$ in $\mathcal{F}$ is set to the mean μ_i of the action probability distribution of its corresponding $CALA_i$ (rather than the action α_i chosen by $CALA_i$), i.e. $F_i = (\ell_i, \mu_i, r_i)$. The set $\acute{\mathcal{F}}$ is constructed again, this time in terms of mean parameters, and f_{err} of the RS using $\acute{\mathcal{F}}$ is considered as $\beta_{\mu(t)}$.

Step 3. Stop condition

The MFs adjustment process and evaluating the performance of RS continue until for all CALAs, $\mu(t)$ does not change noticeably and $\sigma(t)$ converges close to the Lower bound σ_L. In this situation, the set $\acute{\mathcal{F}}$ constitutes the output of the proposed method.

4.1 Complexity analysis

In this subsection, we analyse the time complexity of a recommender system using our proposed method *CALA-OptMF*. As mentioned before, the membership function optimization based on *CALA-OptMF* can be done online during the lifetime of the recommender system. That is, at the beginning of each time period the *CALA-OptMF* method adjusts membership functions serving as inputs to the recommender system. Then, RS utilizes these membership functions to make recommendations for users. Finally, the recommendation error at the end of this period will be used by *CALA-OptMF* to readjust the membership functions in the next period, and so on.

Based on the above paragraph, the time complexity of a recommender system using our proposed method *CALA-OptMF* in each time period is computed as follows. In the MFs adjustment step, at first all CALAs are activated in parallel to choose an action based on their action probability distributions which takes $O(1)$ time. The chosen action by each CALA determines the value of its corresponding centre parameter. Then, the membership functions that satisfy some predefined conditions will be added to the output set in $O(n)$ time, where n is the number of fuzzy sets. This step is repeated again, this time with considering the mean parameter of each CALA as its corresponding centre parameter. Hence, the time taken by the MF adjustment step is $O(n)$.

In the performance evaluation step, RS predicts the rating for each target user twice considering both output sets of membership functions. Note that recommendations for the target user is made based on ratings predicted in terms of the output MF set obtained from CALAs' actions. Therefore, the target user suffers no extra delay in receiving his recommendations. With the assumption that the rating prediction by the RS takes $O(x)$ time, the time required for making recommendations for k users in this period is $O(kx)$. Based on the recommendation error, *CALA-OptMF* updates the parameters of the action probability distributions for all CALAs in parallel in $O(1)$ time. Hence, the performance evaluation step has the time complexity of $O(kx)$. Since n is obviously much smaller than k (i.e. $n \ll k$), the total time taken by the recommender system using *CALA-OptMF* is $O(kx)$ which is equal to the time complexity of the recommender system without using *CALA-OptMF*. Thus, the membership function optimization based on *CALA-OptMF* does not influence the time complexity of any recommender system in which it is utilized.

4.2 Fuzzy trust-distrust enhanced Recommender System: A Case Study

In this subsection, we briefly describe a recommender system which we use in this paper to show the efficiency of our proposed method for the adaptation of fuzzy membership functions.

Authors in (Kant and Bharadwaj, 2013) proposed a recommender system exploiting both the trust and distrust concepts to enhance the quality of recommendations. They developed fuzzy computational models for trust and distrust using similarity and knowledge factors based on rating data. The similarity factor refers to the mechanism in which trust and distrust are computed based on social similarities and dissimilarities such as interests on common items (Zucker, 1986). There is a positive and significant correlation between trust and interest similarity which means, the more similar two users, the greater the value of trust between them (Ziegler and Golbeck, 2007; Golbeck, 2009). The knowledge factor refers to a mechanism where users get to know each other through repeated interactions and predict others' future behaviours based on the information obtained from these interactions (Lu et al., 2010). People usually tend to trust others that they are familiar with through interactions (Gefen et al., 2003). That is, the better one user knows another, the better he can trust what the other will do in most situations. The knowledge factor provides the most relevant and reliable information for measuring social trust (Abdul-Rahman and Hailes, 2000).

Based on their proposed computational models, the total trust and distrust values from user u_i to user u_j are computed by the following formulae:

$$Trust_{u_i}(u_j) = \frac{w_1 * Trust^1_{u_i}(u_j) + w_2 * Trust^2_{u_i}(u_j)}{w_1 + w_2} \tag{3}$$

$$Distrust_{u_i}(u_j) = \frac{w_1 * Distrust^1_{u_i}(u_j) + w_2 * Distrust^2_{u_i}(u_j)}{w_1 + w_2} \tag{4}$$

where w_1 and w_2 are weight coefficients, $Trust^1_{u_i}(u_j)$ and $Distrust^1_{u_i}(u_j)$ are the trust and distrust values from u_i to u_j computed on the basis of similarity factor, and $Trust^2_{u_i}(u_j)$ and $Distrust^2_{u_i}(u_j)$ are the trust and distrust values obtained on the basis of knowledge factor. The computed values of trust and distrust are fuzzified into seven triangular membership functions uniformly and symmetrically distributed along the values axis, as shown in Figure 4. In Figure $4(a)$, seven fuzzy sets are: no trust (NT), very low trust (VLT), low trust (LT), average trust (AT), high trust (HT), very high trust (VHT), and complete trust (CT), and in Figure $4(b)$, seven fuzzy sets represent: no distrust (ND), very low distrust (VLD), low distrust (LD), average distrust (AD), high distrust (HD), very high distrust (VHD), and complete distrust (CD).

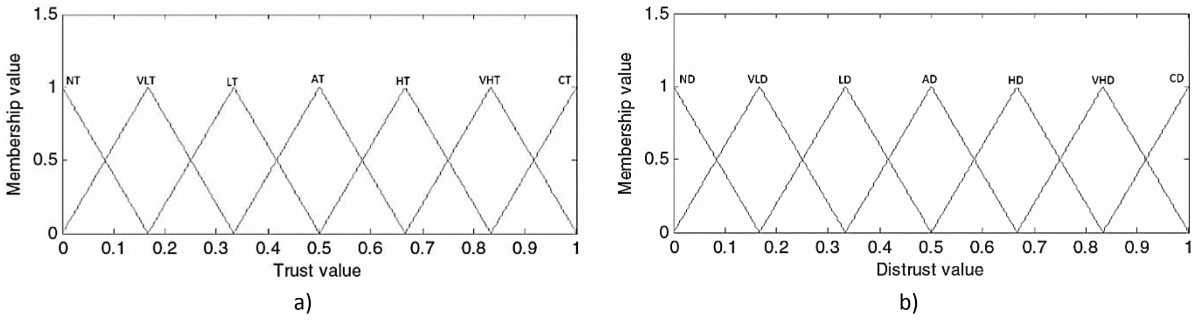


Fig.4 Membership functions for fuzzy a) trust, and b) distrust

Finally, a recommendation strategy is utilized to provide enhanced recommendations. Authors evaluated several recommendation strategies. Since based on their experimental results, the recommendation strategy called *hybrid fuzzy trust–distrust CF* performs the best, we therefore consider this strategy in this paper. The strategy *hybrid fuzzy trust–distrust CF* combines two other proposed recommendation strategies including *modified Pearson CF* and scheme *8-CF*, and computes the final prediction of rating of user u_i on item I_k as

$$Pred12(u_i, I_k) = \begin{cases} 0 & if\ P^{td}_{u_i,I_k} = 0\ and\ P^{cf}_{u_i,I_k} = 0 \\ P^{cf}_{u_i,I_k} & if\ P^{td}_{u_i,I_k} = 0 \\ P^{td}_{u_i,I_k} & if\ P^{cf}_{u_i,I_k} = 0 \\ \dfrac{2 * P^{cf}_{u_i,I_k} * P^{td}_{u_i,I_k}}{\left(P^{cf}_{u_i,I_k} + P^{td}_{u_i,I_k}\right)} & otherwise \end{cases} \tag{5}$$

where $P^{cf}_{u_i,I_k}$ is the predicted rating of user u_i on item I_k by the scheme *modified Pearson CF*, and $P^{td}_{u_i,I_k}$ is the predicted rating according to the scheme *8-CF*. the scheme *modified Pearson CF* uses the most similar top-k neighbours of the target user u_i for generating recommendations, where the similarity between users is computed based on their ratings. This strategy predicts the rating of an active user u_i for an item I_k as follows:

$$P^{cf}_{u_i,I_k} = \bar{r}_{u_i} + \frac{\sum_{u \in N} Sim(u, u_i) * (r_{u,I_k} - \bar{r}_u)}{\sum_{u \in N} |Sim(u, u_i)|} \tag{6}$$

where $Sim(u, u_i)$ denotes the similarity between the active user u_i and the user u belonging to the set N of the most similar top-k neighbours, $\bar{r}_{u_i}$ represents the mean rating of u_i, and r_{u,I_k} is the rating for the item I_k provided by the user u.

In contrast, scheme *8-CF* utilizes fuzzy trust and distrust informations in the recommendation process. In this strategy, the rating prediction of an item I_k for an active user u_i is done based on the following equation:

$$P^{td}_{u_i,I_k} = \bar{r}_{u_i} + \frac{\sum_{u\in(S-D)} w_u * (r_{u,I_k} - \bar{r}_u)}{\sum_{u\in(S-D)} w_u} \tag{7}$$

where D denotes the set of outliers in the trust-distrust network, where an outlier occurs when a user A trusts highly on a user B who highly trusts a user C, but A distrusts C completely. In this equation, S represents the set of trusted neighbours who are above the low trustworthy users, i.e. average trustworthy, high trustworthy, very high trustworthy, and complete trustworthy neighbours. Weights (w_u) associated with these neighbours are determined based on the core of corresponding triangular fuzzy sets.

In order to examine the efficiency of our proposed method *CALA-OptMF*, we employ it in the fuzzy recommender system described above. Note that the proposed method can be utilized in any other fuzzy recommender system with no needed changes.

5. Experimental Evaluation

In this section, we investigate the performance of the proposed method *CALA-OptMF* for improving the recommendation accuracy in fuzzy recommender systems. For this purpose, we consider the trust-distrust enhanced recommender system presented in (Kant and Bharadwaj, 2013) and employ our proposed method in this system to adjust the number and position of fuzzy membership functions for both trust and distrust. We test the recommender system using our proposed method over the well-known datasets MovieLens and Flixster.

5.1 Datasets

MovieLens. MovieLens is a CF-based recommender system and virtual community website created by (GroupLens.org) that recommends movies to users based on their film preferences. The MovieLens dataset contains 100,000 ratings provided by 943 users for 1682 movies, such that each user has rated at least 20 movies. The ratings are on the numerical five-star scale, i.e. 1: bad, 2: average, 3: good, 4: very good, and 5: excellent.

Flixster. Flixster is a social movie website that allows users to share movie ratings, discover new movies and meet others with similar movie taste. Flixster dataset (Jamali, 2010) contains movie ratings from the Flixster website. This dataset has 786936 users, 48794 items and 8196077 ratings, where each movie rating is a discrete value in the range [0.5, 5] with a step size of 0.5.

5.2 Experimental Setup

In the training phase, the proposed method *CALA-OptMF* operates as follows in order to adapt the membership functions of fuzzy trust and distrust. At each iteration, *CALA-OptMF* selects 50 users randomly from the used dataset and evaluates the accuracy of the recommender system using the fuzzy sets adjusted by CALAs for both trust and distrust for predicting ratings of these users. Evaluation is done using the leave-one-out method, where one rating is taken out and then compared with the rating predicted using the rest of user ratings. After that, the parameters of action probability distributions of CALAs are updated based on the mean absolute error (MAE) measure (as the evaluation metric f_{err}) which is computed as the deviation of predicted ratings from the true ratings provided by users. This process is repeated until the stop conditions are satisfied for all CALAs for both trust and distrust.

In the test phase, a mini dataset constructed by randomly selecting 50 users from the used dataset will be utilized. To prevent overlapping, the construction of this mini dataset is done before the training phase is started and the remaining of dataset will be used for training. Finally, the recommender system using the optimized membership functions of fuzzy trust and distrust is tested on the mini dataset.

5.3 Experimental Results

In order to demonstrate the performance of the proposed method *CALA-OptMF*, we conduct several experiments on the well-known datasets described in subsection 5.1. In the proposed method, we set the merge threshold ε to 0.01, the lower bound σ_L to 0.001, and the minimum value τ_{min} and the maximum value τ_{max} of trust/distrust respectively to 0 and 1. The number of fuzzy sets for both trust and distrust is assumed to be 7, as considered in (Kant and Bharadwaj, 2013). Therefore, *CALA-OptMF* needs five CALAs for each of the fuzzy variables trust and distrust to learn the optimal values of the centre parameters of five membership functions F_2 to F_6 (see Section 4 for more details). In the experiments, the average results of 30 independent runs are reported. In each run, we utilize a mini dataset including 50 users randomly selected from the used dataset to evaluate the performance of the fuzzy recommender system. It is ensured that the recommender system with and without using the proposed method *CALA-OptMF* uses the same mini dataset.

5.3.1 Experiment I

The first experiment is conducted to study the impact of different strategies for adjusting the learning parameter λ on the performance of the proposed method *CALA-OptMF*. The learning parameter λ introduces a trade-off between the convergence rate and the steady-state error of the estimated parameter. We consider four following strategies and for each strategy, we report MAE of the recommender system using *CALA-OptMF* on used datasets in Table 1. Note that for all strategies all the CALAs use the same learning rate.

Strategy 1. In this strategy, the learning rate is considered to be a fixed value. The experiment is conducted for values ranging from 0.0001 to 0.0009.

Strategy 2. In this strategy, the learning parameter λ is varies according to the following equation:

$$\lambda(t) = a - b\left\lfloor\frac{t}{c}\right\rfloor \tag{8}$$

where a and b are real values in interval $[0,1]$, $c > 0$ is an integer value and t is the iteration number. Based on the above equation, the learning parameter λ takes an initial value a and it is decremented by b on each c iterations. In this experiment, we test various settings for these three parameters a, b and c.

Strategy 3. In this strategy, the learning parameter λ at iteration t is computed as

$$\lambda(t) = a\, f_{err}(t) - b \tag{9}$$

where a and b take real values in interval $[0,1]$ and $f_{err}(t)$ is the prediction error for the recommender system at time t. In this experiment, we examine different values for a and b.

Strategy 4. This strategy is similar to strategy 3. The only difference is that the square of $f_{err}(t)$ is used for updating the parameter λ at iteration t.

$$\lambda(t) = a\, f_{err}^2(t) - b \tag{10}$$

In this experiment, the initial values of the mean parameters of the action probability distributions for $CALA_2$ to $CALA_6$ related to the trust variable are respectively set to 0.1667 to 0.8333 with step 0.1667 (this setting is adopted based on the values of the centre parameters of the fuzzy sets in Figure 3). The same setting is used for the CALAs related to the distrust variable. The variance parameter for all the CALAs is initialized to 0.05. From results in Table 1, we can see that varying the parameter λ along the learning phase decreases the MAE error on both datasets. Among three strategies that update λ at each iteration, strategy 3 with $a = 0.0006$ and $b = 0.0001$ provides the best results.

Table 1. MAE for different strategies

Strategy		Dataset	
		MovieLens	Flixster
Strategy 1	λ=0.0001	0.8421	1.0473
	λ=0.0003	0.8419	1.0467
	λ=0.0006	0.8418	1.0456
	λ=0.0009	0.8452	1.0478
Strategy 2	a=0.0003 b=0.0001 c=1000	0.8417	1.0441
	a=0.0006 b=0.0001 c=500	0.8417	1.0419
	a=0.0009 b=0.0001 c=100	0.8418	1.0534
Strategy 3	a=0.0002 b=0.00003	0.8416	1.0419
	a=0.0005 b=0.00004	0.8417	1.0415
	a=0.0006 b=0.0001	**0.8415**	**1.0346**
Strategy 4	a=0.0002 b=0.00003	0.8417	1.0372
	a=0.0005 b=0.00004	0.8417	1.0413
	a=0.0006 b=0.0001	0.8418	1.0419

5.3.2 Experiment II

In this experiment, we investigate the impact of different initial values for the mean and variance parameters of the action probability distributions of CALAs on the performance of the proposed method when it uses strategy 3 with $a = 0.0006$ and $b = 0.0001$ for updating the parameter λ (the best setting for λ as reported in the previous experiment). Since each centre parameter c_i $(i = 2, \dots, 6)$ can take the values in the interval $[(i-2)/6, i/6]$, initial values for the mean parameter μ_i are generated randomly in this interval. We also generate initial values for each variance parameter σ_i randomly in interval $[0.01, 0.05]$. Different initial assignments for the mean and variance parameters of CALAs related to the trust variable have been shown in Table 2. The same initial assignment is utilized for the mean and variance parameters of CALAs related to the distrust variable. Note that case 1 is the initial assignment considered in the previous experiment.

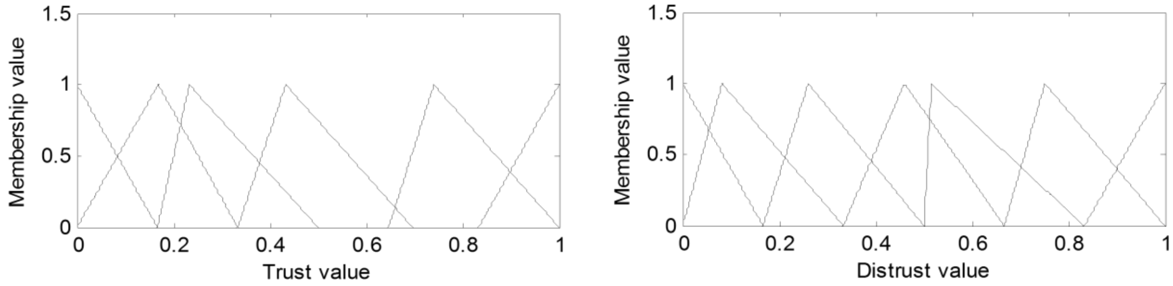
Table 2. Different initial assignments for mean and variance parameters

Case	Initial Values									
	Mean					Variance				
	$\mu_2(0)$	$\mu_3(0)$	$\mu_4(0)$	$\mu_5(0)$	$\mu_6(0)$	$\sigma_2(0)$	$\sigma_3(0)$	$\sigma_4(0)$	$\sigma_5(0)$	$\sigma_6(0)$
1	0.1667	0.3333	0.5000	0.6667	0.8333	0.0500	0.0500	0.0500	0.0500	0.0500
2	0.1780	0.2626	0.4639	0.5096	0.7422	0.0299	0.0401	0.0484	0.0436	0.0240
3	0.0691	0.3199	0.4120	0.7197	0.8603	0.0236	0.0302	0.0155	0.0426	0.0200
4	0.1620	0.2264	0.5118	0.7003	0.9008	0.0334	0.0380	0.0160	0.0197	0.0346
5	0.2154	0.3974	0.5703	0.6552	0.8387	0.0190	0.0456	0.0203	0.0472	0.0289

For each dataset, CALA-OptMF is run by using different initial assignments and its output membership functions are utilized in the recommender system. MAE of the recommender system for different datasets is reported in Table 3. As shown, the best result is obtained for MovieLens by using case 2 and for Flixster by using case 5. Figures 5 and 6 plot the optimized membership functions of fuzzy trust and distrust respectively for MovieLens (case 2) and Flixster (case 5).

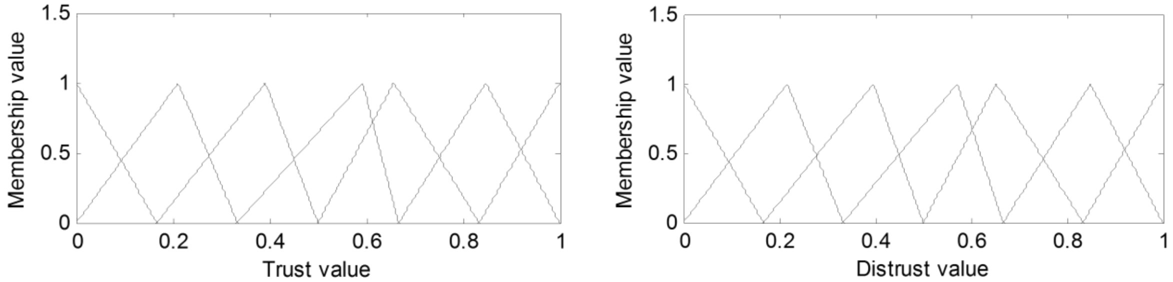
Table 3. MAE for different initial assignments

Case	Dataset	
	MovieLens	Flixster
1	0.8415	1.0346
2	**0.8302**	1.0576
3	0.8416	1.0419
4	0.8415	1.0296
5	0.8636	**1.0274**



a) b)

Fig.5 Optimized membership functions of fuzzy a) trust, and b) distrust for MovieLens



a) b)

Fig.6 Optimized membership functions of fuzzy a) trust, and b) distrust for Flixster

5.3.3 Experiment III

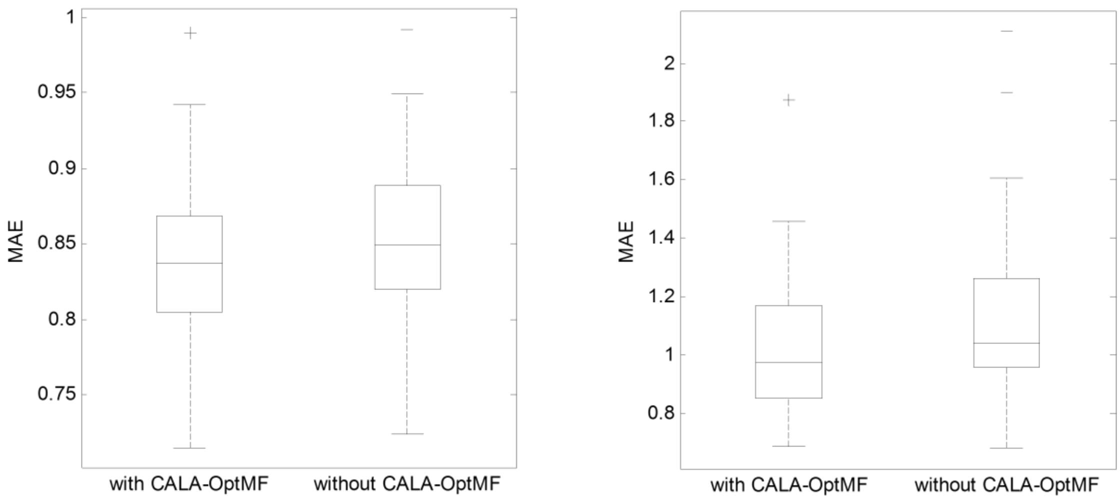
In this experiment, we compare the proposed method *CALA-OptMF* with the method reported in (Kant and Bharadwaj, 2013) in which the fuzzy membership functions are not varying during the lifetime of the recommender system. *CALA-OptMF* uses the best settings of the parameters obtained from the previous experiments. MAE and coverage of the recommender system with and without using *CALA-OptMF* (using the predefined membership functions given in Figure 3) are shown in Table 4. Coverage measures the percentage of items for which a RS is able to generate predictions. According to the results, *CALA-OptMF* by providing the membership functions optimized in terms of the used dataset improves the accuracy of recommendations. However, using *CALA-OptMF* does not change the recommendation coverage. This is because the coverage metric depends on the algorithm of the recommender system and the *CALA-OptMF* method does not make any changes in this system, but only optimizes the input fuzzy sets.

The number of iterations needed by *CALA-OptMF* to optimize the fuzzy sets of trust and distrust for each of two used datasets is also given in Table 4. As noted in subsection 4.1, the membership function optimization based on the proposed method is done online during the lifetime of the recommender system and it does not influence the time complexity of the system.

Figure 7 summarizes the performance of the recommender system with and without using the proposed method *CALA-OptM* over the used datasets in terms of the MAE error. The distribution of the MAE errors of 30 independent runs is represented in box plot format, which visualizes the median, upper quartile, lower quartile, outside value for each distribution of the experimental data. From this figure, we can see that on both datasets the recommender system using *CALA-OptM* has less MAE error.

Table 4. MAE of recommender system with and without using *CALA-OptMF*

Dataset	Without *CALA-OptMF*		With *CALA-OptMF*		
	MAE	Coverage	MAE	Coverage	Num. Iterations
MovieLens	0.8544	0.9446	**0.8302**	0.9446	2235
Flixster	1.1522	0.5728	**1.0274**	0.5728	1103



a) b)

Fig.7 Box plot for the MAE error on a) MovieLens, and b) Flixster

In order to investigate the statistical significance of the obtained results, this experiment uses t-test for comparing MAE of the recommender system with and without using the proposed method CALA-OptMF. The test results for the confidence level 0.95 are summarized in Table 5.

In this table, the first column includes the list of the used datasets. The test results are shown in the second column. We refer to the recommender system with and without CALA-OptMF as x and y respectively. x statistically performs better than y if the mean MAE $(x - y)$ (and thus $T - value$) is a negative value and $P - value$ is smaller than 0.05. Symbols "√" and "×" appeared in the column labelled as "Performance" indicate whether x outperforms y or not, respectively. According to the results, the recommender system obtains the higher accuracy on both datasets by using the proposed method CALA-OptMF.

Table 5. The results of statistical test in terms of MAE

Dataset	Test Result			
	Mean MAE $(x - y)$	T-Value	Difference Significance (P-Value)	Performance
MovieLens	-0.0132	-14.2468	4.9920E-17	√
Flixter	-0.1248	-33.1408	1.3067E-35	√

6. Conclusions

In this paper, we addressed the problem of supplying the most appropriate membership functions for fuzzy trust and distrust in recommender systems and proposed a method based on continuous action-set learning automata (CALA), called CALA-OptMF, to simultaneously optimize the number and the position of membership functions on the values axis. In this method, a CALA is assigned to the centre parameter of each triangular membership function to learn the optimal value of that parameter. CALA-OptMF adjusts fuzzy sets online during lifetime of a recommender system in terms of recommendation error. Therefore, the trust relationship changes and preference changes over time are considered in the optimization process.

The proposed method can be used in any fuzzy trust-distrust enhanced recommender system without any needed changes or other requirements. In order to investigate the effect of membership functions optimization on the performance of recommender systems, we utilized CALA-OptMF in a fuzzy trust-distrust enhanced recommender system presented in literature and test its performance over well-known datasets. The experimental results indicated that the proposed method by providing fuzzy membership functions optimized with respect to used dataset improves the accuracy of recommendations in fuzzy recommender systems.

As future work, authors intend to extend the method CALA-OptMF to be able to optimize nontriangular membership functions representing fuzzy trust and distrust.

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