A Cross-Layer Optimization Framework for Joint Channel Assignment and Multicast Routing in Multi-Channel Multi-Radio Wireless Mesh Networks

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Abstract

Existing literature on multicast routing protocols in wireless mesh networks (WMNs) from the view point of the links involved in routing are divided into two categories: schemes aiming at the NP-hard problem of minimal interference multicast tree construction. In contrast, other methods develop network coding-based solutions with the main objective of throughput maximization which can effectively reduce the complexity of finding the optimal routing solution from exponential to polynomial time. The proposed framework in this paper is placed in the second category. In Multi-Channel Multi-Radio WMNs (MCMR WMNs), each node is equipped with multiple radios, each tuned on a different channel. In this paper, for the first time, we propose a cross-layer convex optimization framework for joint channel assignment and multicast throughput maximization in MCMR WMNs. The proposed method is composed of two phases; in the first phase, using Cellular Learning Automata, channels are assigned to the links established between the radios of the nodes in a distributed fashion such that the minimal interference coefficient for each link is provided. Then, the resultant channel assignment scheme is utilized in the second phase for throughput maximization within an iterative optimization framework based on Lagrange relaxation and primal problem decomposition. We have conducted many experiments to contrast the performance of our solution against many representative approaches.

Keyword

Wireless Mesh Network, Multi-Channel, Multi-Radio, Channel Assignment, Multicast Routing, Primal Decomposition

1. Introduction

1.1 Background

Wireless mesh network (WMN) is an emerging wireless networking paradigm which enables a broader array of applications compared to the existing wireless ad-hoc networks, sensor networks, and cellular networks [37, 64]. The WMN architecture is typically composed of three layers; the highest layer includes one or more gateways (mesh portals) which are mainly responsible for interconnecting the WMN with the wired Internet. The middle layer including mesh routers forms the WMN infrastructure for traffic flow distribution throughout the WMN. Contrary to the case in Mobile Ad-hoc Networks (MANETs), routers are usually stationary, meanwhile, unlike sensor nodes, there is no limitation on their power consumption, which

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essentially relieves the constraints concerning the unpredictable network dynamics in protocol design. In order to increase the physical layer capacity, each node in a multi-channel multi-radio WMN (MCMR WMN) is equipped with a limited number of radios usually less than or equal to a standard-specified number of available channels [5, 66]. Each radio is then able to transmit and receive data on different channels simultaneously [17, 22]. Finally, the lowest layer entails network users usually with limited capability which may be organized into several WLANs or simply consists of groups of cell phone users to name a few.

In WMNs, the inherent interference of shared wireless medium is a primary factor to determine the data rate of the links. Therefore, the interference reduction is of great importance to these networks. As also mentioned in [7-9], the interference can be classified into either protocol models or physical models; the latter is being more realistic in the context of wireless networks. In MCMR WMNs, the problem of interference minimization is normally tackled with by devising channel assignment schemes in the MAC layer. A review of the relevant literature, albeit for unicast routing will result in the identification of the following three categories of channel assignment schemes [27]:

- Channel assignment with regard to a presumed routing structure [16, 32, 34, 38, 48].
- Routing over a given channel-radio association [36, 46, 56, 57].
- Coupled cross-layer design of channel assignment and routing[4, 31].

In this paper, we narrow in on multicasting [11, 19], which is an underlying service for a multitude of collaborative applications in MCMR WMNs such as video conferencing, online games, webcast and distance learning. In MCMR WMNs two constraints are necessary for communication of adjacent nodes: first, two intended nodes are located in the communication range of each other and second, a common channel is assigned to their radios.

Due to the lack of broadcasting nature in wire-line networks in which, a single transmission can be received by all neighbors of the sender node, the reported multicast routing protocols for the traditional wired networks are not suitable for the wireless networks. Also, the conventional multicasting schemes specialized for ad-hoc networks (e.g. [50, 55]), MANETs, and sensor networks address characteristically different concerns such as node mobility and energy consumption. Multicast protocol designs for single-channel single-radio WMNs have been reported in [30, 45, 49, 51, 52, 54, 68, 71], which classify significantly different network configurations. The work in [29], on the other hand, targets at throughput maximization for multi-channel single-radio WMNs. The multicast problem in MCMR WMNs has been dealt with a variety of heuristic- and meta-heuristic-based schemes in [12-14, 24-26, 33, 41-43, 67, 69, 70], which are primarily centered on the NP-hard problem of minimal interference multicast tree construction.

1.2 Contributions

Rather than adopting a scheme based on the NP-hard problem of multicast tree construction, we can work up a solution for multicast routing in polynomial–time by taking on a network-coding approach [2]. In [20], a network coding based formulation for multicast throughput maximization problem has been proposed. However, power control in physical layer and the Signal to Interference and Noise Ratio (SINR) computation have not been accounted for. As extensively discussed in [3], cross-layer design is a vital part of a successful WMN-based implementation, which has been addressed in some works [25, 26, 65]. The ideal multicast

solution in MCMR WMNs is essentially a remedy to three sub-problems: channel assignment at the MAC layer, routing at the network layer, and power control at the physical layer. The channel assignment sub-problem determines which channel should be assigned to a specific link. Using this information in the power control sub-problem, the SINR of the link is computed which in turn determines the capacity of the link. The routing sub-problem, on the other hand, is aimed at the maximization of multicast flows so as to fully utilize the available link capacities. Therefore, in this paper, for the first time, we propose a holistic cross-layer convex optimization framework for the multicast throughput maximization problem in MCMR WMNs. Decomposition has several advantages such as providing a unifying view and methodology to examine performance in protocol layering and the mathematical language to generate analytic basics in designing modularized and distributed control of networks. In addition, it leads to reduction of computational complexity. Considering strengths of decomposing idea mentioned above and the point that there is no priority between the constraints of our sub-problems we adopt the Network Utility Maximization (NUM) framework [44] to formulate the multicast throughput maximization problem across two layers. The routing and power control sub-problems can be modeled in LP terms along with an additional coupling LP constraint, hence satisfying the convexity requirement necessary for cross-optimization problem decomposition. The channel assignment sub-problem, however, is inherently of an NP Binary Integer Programming (BIP) form, essentially making the overall formulation non-convex [53]. Therefore, we methodically extract the channel assignment sub-problem from our NUM-based formulation, and alternatively propose a meta-heuristic algorithm based upon the notion of Cellular Learning Automata (CLA) [8] to obtain a sub-optimal solution in polynomial time.

Clearly, the proposed framework in this article is organized into two phases; initially, using the CLA-based algorithm, channels are assigned to the radios of the nodes in a distributed fashion so that the minimal interference coefficient for each link is provided. The resultant channel-radio associations then serve as a backdrop to perform throughput maximization in the second phase as a joint problem. The iterative optimization algorithm in this phase is formulated according to Lagrange relaxation and primal problem decomposition to jointly optimize the solution to the two sub-problems of multicast routing at the network and power control at the physical layers.

From several aspects, the proposed framework is different from the proposed idea named LAMR in [24]; In LAMR, two sub-problems channel assignment and routing are solved conjointly. But, here channel assignment is firstly solved and then, two sub-problems power control and routing are solved in a cross-layer fashion. As another difference, in LAMR, residing LAs (Learning Automata) in DNICs of multicast source are activated in parallel. Then, they decide to appropriately route RReq messages to the next hop using a channel according to their action probability vectors (APV). The LAs residing on the next hope go on with the same process. This process continues until the RReq messages reach the one of receivers. After this step, receivers send RRep back to the source node. The early arriving RRep are considered as minimal-delay routes. Now, multicast source computes the overall tree contention. If the value is less than the previous one, it sends a Reward message to all nodes forming newly generated tree. In contrast, here, channel assignment is conducted using ICLA (Irregular Cellular Learning Automata) and, routing is not taken into account in this step. All LAs residing on all DNICs are activated asynchronously. Then, they choose an action (a channel) according to their APVs. Afterwards, thanks to the ICLA theory they receive feedback regarding the selected channels by the LAs residing on the neighboring cells (mesh nodes). In this step, the LAs are penalized or

rewarded. This procedure continues until it provides all cells with a stable set. After this phase, next phase, throughput maximization phase is initiated in a cross-layer paradigm. Another significant difference between this work and LAMR is that multicasting in LAMR is based on a tree-based paradigm whereas, here is based on network-coding based design.

The efficiency of our proposed formulations is demonstrated through extensive simulations. We also compare the performance of our solution with the analogous works in this area [12-14, 29, 69, 70]

1.3 Paper structure

The rest of this paper is organized as follows: In section 2, the existing multicast routing methods for WMNs are briefly surveyed. Our cross-layer framework for the multicast optimization problem is presented in section 3. Performance measurement studies are reported in section 4. In section 5, time complexity of the proposed algorithm is discussed. Section 6 concludes the paper and presents several directions for future work in this area.

2. Related works

Ref [30] discusses multicast routing in single-channel single-radio WMNs where no channel assignment is considered for multicast tree construction. The edge costs were intended for construction of The Shortest Path Tree (SPT). However, the properties of its multicast routing mechanism have not been adequately explained yet.

Ref [54] examines a hybrid method for multicast routing where a multicast proactive mechanism operates for inter-router communication at the backbone when a multicast reactive mechanism is established between clients and access points. The assessment of the proposed method has not been performed yet; however it does not consider the vital role of channel assignment in the whole problem. Ref [68] encompasses an approach of throughput maximization with an NUM-theoretic formulation of multicast routing at the network layer and power control at the physical layer. Miscellaneous studies in the context of multicast routing for single-channel single-radio WMNs can be found in the references [49, 51, 52, 54, 68, 71].

In [29] an NUM-based formulation for multicast throughput maximization poses in multi-channel single-radio WMNs. Assuming that the cross-optimization constraints include a logarithmic function in conjunction with the product of two continuous variables, the convexity requirement is not met. A Breadth-First Search (BFS) heuristic algorithm was suggested as an alternative but it is not able to dynamically tune the links' capacities for interfering links in the WMN.

In [20], an Integer Linear Programming (ILP) problem as a model of multicast throughput optimization in MCMR WMNs is introduced, and to scale, LP relaxation has been adopted. Employing a BFS algorithm in [20], channels are assigned to radios in a greedy manner as the first proposed scheme. Next, the concluded channel-radio associations are allocated to an LP model to optimize the network throughput. The substitute model in [20], runs an iterative solving process to rectify the consequence of the BFS-driven channel assignment using an LP model. The defect concerned with this approach is that it relies upon the simplistic assumption of operating under non-overlapped channels. That is, the basic role of Signal to Interference and Noise Ratio (SINR) of the links, which depends upon the power control problem in the physical layer, has not been intended. Indeed, fixed channel capacity is not an actual assumption in wireless networks, where the link status is related to interference from adjacent transmissions.

Ref [15], proposes a call admission control mechanism for multicasting in multi-radio multi-channel WMNs that also considers a different setup in comparison with that introduced in this article.

The efforts investigated in [12-14, 33, 41-43, 67, 69, 70] basically concentrate on joint channel assignment and multicast tree construction in MCMR WMNs with the major objective of reaching a minimum interference between multicast links. According to their underlying assumptions, three categories can be listed as follows:

- Multicast tree construction schemes assuming a prior channel-radio association [42].
- Channel assignment schemes with a pre-constructed multicast tree assumption [32,34, 41, 43, 67].
- Methods successively solving the two multicast tree construction and channel assignment as a cross-layer paradigm [12-14, 33, 42, 65, 69, 70].
- Several papers the same as [24, 26, 65] present a cross-layer design that attempt to collectively solve the sub-problems.

Following, the methods concentrated on multicast tree construction are elaborated:

Ref. [69] and [70] introduced two methods for multicast tree construction and channel assignment in MCMR WMNs. In the first one, mesh nodes are visited by conducting a BFS originating from the multicast source in the beginning. Afterwards, to designate the forwarding nodes in the multicast tree, a bottom-up approach is employed. If each receiver node v owns several parents and one of the parents exists on the multicast tree, this receiver is linked to that parent (f_v). If not, one of the parent nodes is chosen randomly and one link is created for that parent (f_v). The algorithm for node f_v would continue recursively. The 'LCA' (Level Channel Assignment) algorithm is used upon constructing the multicast tree. How to assign channels to nodes in LCA depends on the level of BFS traversal tree they belong to. Specifically, channel i is assigned to nodes resided at level i of the tree. This method has the following imperfections: The random choose of multiple candidates for each receiver. This will result in different multicast trees. In addition, starting the algorithm from different multicast receivers is the cause of different multicast trees. Because for channel assignment, the nodes resided at the same level are allowed to interference. Besides, if the number of available channels in WMN is greater than the number of levels, designated by BFS, the pool of available channels will be left underutilized. The next introduced method, called 'MCM' (Multi-Channel Multicast), employing BFS again, puts the nodes across different levels. Then, the edges created between the nodes resided at identical level are removed. Next, the least number of relay nodes (RNs) constituting the multicast tree, would be identified based on the algorithm below:

1) Parents can be selected as RN in case that one of their children owns a fewer number of parents.
2) When presented with a number of RNs, the node with the greatest number of children is chosen.
3) The selected RN and its children are eliminated from the tree and the last two steps would be iterated until all nodes at level $i+1$ are omitted.

The main defect concerned with this method is its overhead because of message passing. Furthermore, the elimination of edges between the nodes resided at similar level can decrease the opportunity of finding a better solution.

Moreover, channel assignment for 'MCM', can be performed by either of the two algorithms below: The top-down 'Ascending Channel Assignment' (ACA) assigns channels to consecutive sub-trees in ascending order beginning from channel zero. When channels are terminated, the same situation happens to the next levels. Defects concerned with ACA are as follows; under-utilization may happen in the event that the number of sibling nodes is not the same in the entire tree. Meanwhile, ACA is permitted to *hidden channel problem* [33]. Furthermore, for almost-diagonal trees, ACA has similar shortcomings as those of LCA's. The second channel assignment scheme, named 'Heuristic Channel Assignment' (HCA) utilizes the *channel separation* notion which represents the distance between the numbers of two channels. Using this method, in case that a channel is assigned to node *u*, the sum of squares of interference factor [18] between node *u* and all nodes *v* in its neighborhood, should reach a minimum level. However, since it only applies to single-hop neighbors, the problem of hidden channel may happen.

The hidden channel problem of [70] has been a motivation for the efforts in [43] extended later in [41]. To decrease interference, a different function for assessing the assignment of channel *c* to node *v* has been introduced. The hidden channel problem is investigated with regard to the channel information of nodes within two hops into the objective function. However, it is presumed in [41, 43] that the multicast tree is already available. It also depends upon heavy broadcast message delivery.

Another assessment function is also used in [67] assuming that the multicast tree is already generated; especially, the assignment of channel *c* to node *v* is assessed according to the probability of transmitting packet by neighboring nodes of *v* on channel *c*. However, the features of the generation of this probability are not fully explained in [67]. In addition, despite the greedy nature of channel assignment, the reality that nodes should be visited starting from the first BFS level for collecting information from all the edges, causes too much overhead. In [12-14], three methods based upon genetic algorithm, simulated annealing, and tabu search techniques have been introduced. The idea for multicast tree construction and channel assignment in all the three is basically similar. It means that the multicast tree is indicated by a two-dimensional array (chromosome) the rows of which designate a path from multicast source S to the receiver R_i. Fundamentally, a chromosome has K rows for K receivers. The number of columns in the chromosome equals the maximum path length from S to the members of multicast group. The chromosomes representation is according to the IDs of the nodes locating along the path from sender S to the members multicast group. Using this method, chromosomes are generated as follows: the algorithm starts from the multicast source node S. A one-hop neighbor of S is randomly chosen and its ID is inserted into the chromosome. This process will be going on until it ends to a receiver R_i. The similar process also applies to the next receiver (within the next row). Upon building each row of the chromosome, which is basically a path from S to R_i, the channels are assigned to the edges in order. The following imperfections can be mentioned for these methods; Firstly, assigning the channels to each row of the chromosome leads to similar defects as those of the LCA; i.e. the hidden channel problem is not considered. Secondly, the nodes within similar level in the multicast tree are allowed to interference. Finally, if the number of channels is greater than the number of multicast tree levels, some channels will be underused.

The methods introduced in [12-14, 41, 43, 67, 69, 70] either generate a multicast tree or have supposed the tree is already generated and then channels were assigned. In [42], however, the channel is supposed to be assigned before and now the multicast tree should be built. Particularly, a centralized multicast tree algorithm, called 'Multi Channel Minimum Number of

Transmission' (MCMNT), has been introduced which is targeted to minimize the number of packets copied on to different channels per node.

Ref [33], proposes a distributed bottom-up approach to build a multicast tree and to create channel-radio associations. In the beginning, an approximate algorithm is employed to compute the minimal RN set and to build the multicast tree:

1. Each node designates its two-hop neighbors with the minimum number of parents.
2. The parents of two-hop neighbors, who own fewer children, are candidate. Candidates having the best link quality will be chosen.
3. The chosen nodes and all of their children are omitted from the single-hop and two-hop nodes' list.
4. The algorithm will go on until both single-hop and two-hop neighbors' lists become empty.

Ref [33] also presents a mechanism for joining and leaving the multicast session for residing nodes across levels.

A Learning Automata (LA) based approach for MCMR WMNs which is designed in [24] , unlike the [12-14, 33, 41-43, 67, 69, 70] solves two aforementioned sub-problems collectively and thus, outperforms them in terms of achieved throughput, end-to-end delay, average packet delivery ratio, and multicast tree total cost. Also, with regard to Cross-Layer and Load Oriented design paradigm, application demands, multicast routing, and channel assignment are considered jointly by CLLO [65] during the shaping of a channel-allocated multicast tree in a top-down mode. In this work, WMN is mapped into a graph traversing BFS procedure to place the nodes in different levels. Authors used a concept named crossing link as a link that can connect a node to a partial tree T with another node which does not belong to T. A feasible crossing link is a crossing link that can be allocated with at least one non-interference channel. Algorithm iteratively adds the feasible cross links until the receiver set is included in T.

In contrast to [12-14, 24, 33, 41-43, 67, 69, 70] which lead to achieve the sub-optimal solution, in [26] the multicast routing problem for MCMR WMNs has been modeled as a cross-optimization framework based on binary integer programming (BIP) formulation which will result in optimal solution. Also, trade-off between the scalability and optimality has been extensively considered in [25].

Table 1 compiles the major prior art in the area of multicast tree construction and channel assignment for wireless mesh settings. In short, the major drawback with the methods discussed in [12-14, 24-26, 32-34, 41-43, 67, 69, 70] lies in the rationale of approaching the issue of multicast routing from the perspective of the inherently NP multicast tree construction problem [59]. Armed with this understanding, we take the network-coding-based initiative and come up with an NUM-theoretic formulation for cross-layer multicast throughput maximization. A similar notion has also been adopted in [68], albeit for the characteristically different configuration of single-channel single-radio mesh systems.

Our research is also distinguished from [29] in several ways; first, in contrast to [29], we formulate the WMN multicast problem specifically for multi-radio multi-channel configurations. Second, channel assignment in [29] is carried out on the basis of a centralized BFS algorithm, while we propose a novel distributed scheme based on the notion of CLA which aims at the provision of minimal interference coefficient for all links. Third, contrary to [29], we work out a proper approximation to render the inherently non-convex formulation of the problem into an alternative form with the desirable feature of convexity, which provides our underlying NUM

iterative process with guaranteed convergence throughout the solution development. Finally, the work in [20] relies on the simplifying assumption of operating under non-overlapping channels. That is, the significant role of SINR, determined by the power control module at the physical layer, has been overlooked, which is however needed, in turn, for dynamic computation of the link capacities. In fact, link capacities are assumed to be fixed, an assumption not particularly realistic in the context of wireless networks.

Table 1: Existing multicast methods for multi-channel multi-radio WMNs

Reference	Idea for multicast tree construction	Idea for channel assignment
[12-14]	Routing links are constructed using centralized protocols based on genetic algorithm, Tabu search, and simulated annealing.	Channels are assigned using an ascending method.
[24]	Routing links are constructed via a meta-heuristic approach	channel assignment is conducted via a meta-heuristic approach
[26]	Routing links are constructed via BIP-based formulation	channel assignment is conducted via BIP-based formulation
[25]	Routing links are constructed via BIP-based formulation	channel assignment is conducted via BIP-based formulation
[32]	It is assumed that multicast tree is already constructed	Channels are assigned using a utility-based LP formulation
[33]	Routing links are constructed using an enhanced MCM which takes the link quality into account.	Channels are assigned using a heuristic algorithm.
[34]	It is assumed that multicast tree is already constructed	Channels are assigned using a weight-awareness heuristic method
[42]	Routing links are constructed using a heuristic based on minimum cost tree metric named MCMNT	It is assumed that channel-radio association is already performed.
[41, 43]	It is assumed that multicast tree is already constructed	Channels are assigned using a heuristic method named M4 which is based on channel separation.
[67]	It is assumed that multicast tree is already constructed	Channels are assigned using a greedy channel assignment algorithm which utilizes BFS.
[69, 70]	Routing links are constructed using two different techniques: 1) a centralized heuristic bottom up algorithm which utilizes BFS. 2) a centralized approximate top down algorithm named MCM	Channels are assigned using 3 different techniques: 1): using LCA algorithm 2) An ascending method. 3) A heuristic method based on channel separation.
[65]	Routing links are constructed using a load aware heuristic	Channels are assigned using a heuristic method based on minimal interference.

3. Multicast routing cross-optimization framework

The objective of this study is to propose a cross-layer optimization framework for the multicast throughput maximization problem in MCMR WMNs. As also mentioned earlier in the Introduction, the proposed framework is composed of two phases; the first phase which takes on a CLA-based [8] approach to channel assignment yields a set of established links between the radios of the individual nodes, each of which tuned to the one of the available channels such that the minimal interference coefficient for each link is achieved. The second phase works on the resultant channel-radio association to maximize multicast routing throughput. In this phase, we adopt the Network Utility Maximization framework [44] to formulate an iterative optimization solution based on Lagrange relaxation and primal problem decomposition in which two disjoint sub-problems are optimized in parallel and hence, the overall solution globally converges to the

optimal point under convexity conditions. In what follows, we explain the details of the proposed framework.

3.1 Preliminaries and assumptions

In this section, a general description of the concepts of Cellular Automata [9], Learning Automata [40, 58], and Cellular Learning Automata [6, 9, 23] will be presented followed by the associated assumptions required for the proposed framework.

Cellular Automata (CA), which mathematically model dynamic systems, are composed of similar elements which locally interact to provide complex behavior efficiently and robustly. In fact, CA form a definite-dimensional lattice of cells in which the number of states are limited. In this model both the states and times are discrete. Defined rules determine the next state of each cell with regard to the states of neighboring cells at previous and current time instances. A finite dimension d leads to a d-dimensional lattice [9, 23].

A learning automaton (LA) is an adaptive agent that improves its behavior with respect to the feedbacks received from a stochastic environment. That is, every taken action by agent is evaluated and a reinforcement signal is responded by the environment. Afterwards, LA updates its action probability vector considering both taken action and gained reinforcement signal. This procedure will continue until a stop condition is met. Fig. 1 shows the interaction between LA and the environment.
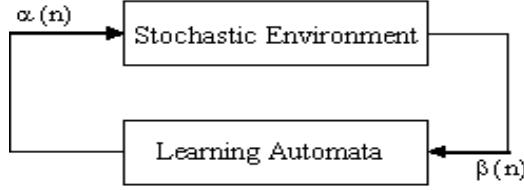
Fig. 1: Interaction between learning automata and environment

Every environment is denoted by $E = \{\alpha, \beta, c\}$, in which sets $\alpha = \{\alpha_1, \alpha_2, ..., \alpha_r\}$, $\beta = \{\beta_1, \beta_2, ..., \beta_r\}$, and $c = \{c_1, c_2, ..., c_r\}$ represent inputs, outputs, and penalty probabilities, respectively. Reinforcement signal β, categorizes the environments into P-model, Q-model, and S-model. If the set β has just two elements, the environment is named p-model. In p-model environment, $\beta_1 = 1$ and $\beta_2 = 0$ correspond to penalty and reward signals, respectively. Similarly, the β takes discrete values between 0 and 1 in Q-model environment. In the S-model environment, the number of elements of the β is infinite. At last, the penalty of the probability of each taken action α_i is represented by c_i.

The structure of LA is divided into fixed and variable. The variable-structure LA is shown by $\{\alpha, \beta, p, T\}$, in which sets $\alpha = \{\alpha_1, \alpha_2, \cdots, \alpha_r\}$, $\beta = \{\beta_1, \beta_2, \cdots, \beta_r\}$, and $p = \{p_1, p_2, \cdots, p_r\}$ in turn denote actions, inputs, and action probability vector. Also, $p(n+1) = T[\alpha(n), \beta(n), p(n)]$ represents its learning algorithm. The LA chooses an action according to its action probability vector (P_i).

Assuming the taken action is α_i, the process of updating action probability vector is accomplished by formula (1) and (2) for reward and penalty reinforcement signals, respectively.

a and *b* correspond to reward and penalty parameters, respectively. When *a* and *b* have equal values, the algorithm is called L_{R-P}. If $b \ll a$, the algorithm is named $L_{R \varepsilon P}$. Finally, if b has zero value, then the algorithm is known as L_{R-I}.

$$p_i(n+1) = p_i(n) + a.(1 - p_i(n))$$
$$p_j(n+1) = p_j(n) - a.p_j(n) \qquad \forall j \; j \neq i \quad (1)$$

$$p_i(n+1) = (1-b).p_i(n)$$
$$p_j(n+1) = \frac{b}{r-1} + (1-b)p_j(n) \qquad \forall j \; j \neq i \quad (2)$$

A Cellular Learning Automaton (CLA) is a sub-class of CA, where each cell is equipped with an LA (or multiple LAs) [6, 8, 9, 23]. The LA resided in each cell, determines its state according to its action probability vector. The same as CA, the CLA operates in a rule-based fashion. The reinforcement signal for a given LA residing in each cell depends on CLA rule and the state of the adjacent cells. In fact, they can be interpreted as the environment for that particular cell. CLA works based on the following procedure: Initially, LA residing in each cell determines the state of its associated cell with regard to its action probability vector. The probability of choosing each of *r* action is given by $\frac{1}{r}$ at the beginning. Next, the predefined rule of CLA identifies the reinforcement signal for each LA. Each LA then updates its action probability vector considering provided reinforcement signal and its chosen action. This procedure goes on until a favorable state is met. The operation mode of CLA can be either s*ynchronous* or *asynchronous*. In synchronous CLA, a global clock synchronizes all cells and activated them simultaneously. Some of CLA applications are listed as follows: target tracking in wireless sensor networks [23], channel assignment in cellular networks [7], multicast routing [61, 63] and clustering in ad hoc networks [60], graph coloring [62] to name a few.

The limitation of the rectangular grid structure in traditional CLA can be eliminated by Irregular Cellular Learning Automaton (ICLA) that is a class of CLA. ICLA is specially preferred within the context of wireless mesh and sensor networks, immune network systems, graph-related applications, etc. which cannot be effectively modeled with rectangular grids. An ICLA is mapped to an undirected graph in which, each vertex corresponds to a cell. Since the action probability vectors of the adjacent LAs change during the evolution of the ICLA, the local environment of a cell is non-stationary [23]. Similarly to [67], the following assumptions are the foundation of our study: all mesh routers in the WMN are randomly distributed on a plane. In addition, the number of radio interfaces in each router does not exceed the number of available channels. All radio interfaces on wireless routers operate in the same transmission / interference ranges.The next sub-section describes our channel assignment scheme which underlies the initial phase of the proposed optimization framework.

3.2 Channel assignment

In this sub-section, the proposed channel assignment method is discussed. In this research all channels including overlapping and orthogonal channels are utilized. The wireless mesh network is mapped to an asynchronous ICLA, isomorphic to its topology. Each cell of the ICLA corresponds to a mesh node M_i in the network. Two cells *a* and *b* are considered adjacent if their

corresponding nodes are located within the interference range of each other. Each cell is equipped with n learning automata, each of which is associated to one of the n radios of the mesh node. This multiple-LA-in-each-cell configuration has first been proffered in [8], which is shown to be particularly apt for multivariate decision making scenarios.

In [47], it is argued that to prevent the ripple effect of channel changing in MCMR WMNs, the radios of the mesh nodes should be divided into downlink and uplink radios while each node is only responsible to assign channels to its downlink radios. Accordingly, in our model, the i-th LA residing at the i-th radio of each mesh node j, referred to as LA_{ij}, is responsible to decide on which channel the radio R_{ij} should send data to R_{ik} , where k is a member of the node j's one-hop downlink neighbors. The goal of each LA is to minimize the channel correlation coefficient for its associated downlink which, in turn, depends on the inter-channel separation. The correlation coefficient between two channels is known for all possible channel separations, as depicted in Fig. 2 [29]. For instance, the correlation coefficient between channels 1 and 2 equals 0.7906 for the thirteen-channel IEEE 802.11.

Associated with each LA is a $\Gamma_{N \times C}$ matrix, where N is the cardinality of the set of a node's one-hop downlink neighbors, and C denotes the number of available channels. Each row n of this matrix forms the LA's action probability vector (APV) for connecting to neighboring node n. Each element γ_{nc} is initialized with $1/C$ and represents the probability of picking the action α_{nc} (i.e. assigning channel c for connection to neighboring node n).
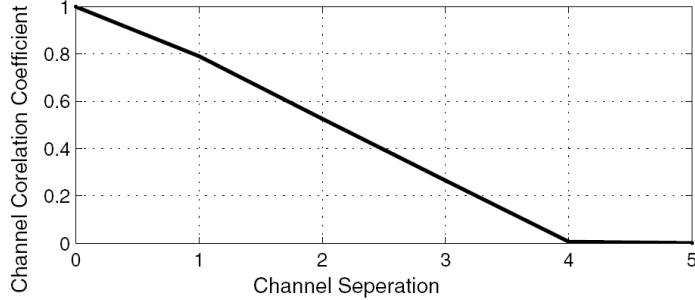
Fig. 2: Channel correlation coefficient versus channel separation for IEEE 802.11 $[29]$.

The operation of the proposed channel assignment algorithm is organized into a number of rounds. In each round, the learning automata residing at the radios of each mesh node are activated in ascending order. An activated learning automaton LA_{ij} starts with the node's first downlink neighbor, picking an action according to its first-row APV (i.e. the APV associated with this neighbor in its $\Gamma_{N \times C}$ matrix). As its feedback mechanism, then the LA_{ij}, computes the sum of interference coefficients within the node's neighborhood. Depending on the resultant interference, the action chosen by the LA will be punished or rewarded. That is, if the newly calculated interference, after taken the action α_{1c} by LA_{ij}, will be less than its previous value, α_{1c} will be rewarded according to L_{R-I} algorithm. It should be noted that in the case that resultant interference aggravates, α_{1c} is penalized by getting no reward. Then, LA_{ij}, repeats the aforementioned steps for all its subsequent one-hop downlink neighbors. Once LA_{ij} ends up $LA_{(i+1)j}$ starts up with the same process. The channel assignment algorithm iteratively runs through every node until it provides all the ICLA cells with a stable set. Since the rule used in our CLA-based channel assignment is commutative, as mathematically proved in [6, 9] the CLA

converges to a globally stable state. Assuming the learning parameter takes sufficiently a small value.

3.3. Throughput maximization

In this section, we propose an NUM-based formulation for multicast throughput optimization problem in the second phase. There are two inter-dependencies: first, the maximum achievable throughput is characterized by per link aggregated flow. The second is that the link capacities are constrained by bounds on maximum power at the physical layer. The solution to the power control sub-problem guarantees that the maximal capacity for each link is provided, while the routing sub-problem warrantees that the link capacity is fully utilized. Tables 2 and 3 list the parameters and variables used in formulating the sub-problems, respectively.

Table 2: Used parameters

$Nodes = \{N_1, N_2, N_3, .., N_n\}$	Set of mesh routers
$ChannelList = \{C_1, C_2, C_3, ..., C_c\}$	Set of channels
$MulticastSource = \{N_i\}, N_i \in Nodes$	Set of multicast sources
$MulticastGroup = \{T_1, T_2, T_3, .., T_t\}, T_i \in Nodes$	Set of multicast group
$Radio = \{R_1, R_2, R_3, .., R_r\}$	Set of mesh routers' radios
$p_{max}(Src)$	Maximum power of node Src
AC(Src,SR,Des,DR)	Assigned Channel to the links established between radios SR of node Src and radios DR of node Des
IC(AC(Src,SR,Des,DR), AC(Src',SR',Des',DR'))	Interference Coefficient between two links (Src,SR,Des,DR) and (Src',SR',Des',DR') that already have been assigned to the available channels.
$G(Src', SR', Des', DR', Src, SR, Des, DR)$	Gain between two links (Src,SR,Des,DR) and (Src',SR',Des',DR')

Table 3: Used variables

$G(Src', Sr', Des', Dr', Src, Sr, Des, Dr)$	Gain of the links
p(Src,SR,Des,DR)	Required power for communication between radios SR of node Src and radios DR of node Des
$\alpha(Src, SR, Des, DR)$	Dual variable
$c(Src, SR, Des, DR)$	Capacity of the link(Src, Sr, Des, Dr)
$flow(Src, SR, Des, DR)$	Actual flow rate on a link
$e(Src, SR, Des, DR, t)$	Conceptual flow
$SINR(Src, SR, Des, DR)$	Signal to interference and noise ratio of a link
$link(Src, Sr, Des, Dr)$	Link established between radio SR of node Src and radio DR of node Des
r	Multicast throughput

We consider a concave utility function for the multicast throughput optimization problem [68]:

$$U(r) = log(1 + r)$$

Now the cross-optimization problem can be formulated as:

$$Maximize\ U(r) \qquad (3)$$
$$Subject\ to:$$
$$Routing\ layer\ constraints$$
$$Physical\ layer\ constraints$$

It is worthy of noting that the routing and physical layers are coupled with each other via the following inequality constraint:

$$flow \leq capacity \qquad , \forall Link \qquad (4)$$

That is, the aggregated flow rate at each link cannot exceed the link capacity. It is also noted that when the objective function is concave and the constraints in each sub-problem are linear, the overall optimization problem is convex and can be solved in a polynomial time order. The link capacity constraint (4) is relaxed when computing the Lagrange function for problem (3) and therefore, a dual variable α emerges in the objective function as follows:

$$L = U(r) + \sum_{\forall all\ link} \alpha * (capacity - flow) \qquad (5)$$

Now, the overall optimized problem (5) can be decomposed into two disjoint sets of variables at both network and physical layers, sub-problems (6) and (7), with the dual variable α playing the role of coordinator between the two. The higher the value of α, the more the capacity required to be provided by the physical layer and the less the flow rate achievable at the network layer.

$$Maximize\ U(r) - \sum_{\forall Link}(\alpha.flow) \qquad (6)$$
Subject to:
$\qquad$ *Routing layer constraints*

and

$$Maximize\ \sum_{\forall Link}(\alpha.capacity) \qquad (7)$$
Subject to:
$\qquad$ *Physical layer constraints*

The key to decomposing the optimization problem into routing and power control sub-problems is its underlying convexity, as has been justified in [68]. Furthermore, as strong duality [53] holds for our formulation[2], the optimization problem (3) can be effectively solved using its dual instead. Fig. 3 represents the proposed primal-dual algorithm for the overall framework. The algorithm continues until successive updates of dual variables converge.

Initialize:
$\qquad$ *Iteration=0;*
$\qquad$ $\alpha(Src, SR, Des, DR) = 0$ $\qquad\qquad\qquad$ $\forall Src, Des\ \in Nodes,\ SR, DR \in Radios$

Until successive updates of dual variables converge do steps 1 and 2 sequentially // continues until the values of dual variables no longer change
$\qquad$ Begin
$\qquad\quad$ *Step 1 (Primal domain):*
$\qquad\qquad$ *Solve the network and physical layers' sub-problems*
$\qquad\quad$ *Step 2 (Dual problem):* $\qquad\qquad$ // dual variables for all links are updated.

$$\alpha(Src, SR, Des, DR) = max\left(0, \alpha(Src, SR, Des, DR) + \frac{1}{k.Iteration} * \big(c(Src, SR, Des, DR) - flow(Src, SR, Des, DR)\big)\right)$$
$\qquad\qquad\qquad$ $\forall Src, Des\ \in Nodes,\ SR, DR \in Radios, k\ is\ a\ constant$

$\qquad\quad$ *Iteration ++;*
$\qquad$ *End*

Fig. 3: Primal-Dual algorithm for multicast throughput maximization

3.4. Sub-problems characterization

One of the most crucial advantages of the primal-dual framework is modularity. That is, each sub-problem can be solved independently at its corresponding layer, enabling parallel optimization of two or more sub-problems across layers. In this section, we show how two modules of power control at the physical and routing at the network layer can be efficiently cross-optimized.

A. Network layer module

Our network-coding-based formulation of the flow control sub-problem is as follows:

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[2] The strong duality theorem states that if the primal is bounded and feasible, then the value of the primal LP equals the value of the dual LP.

$$Maximize \; U(r) - \sum_{Src \in Nodes} \sum_{SR \in Radios} \sum_{Des \in Nodes} \sum_{DR \in Radios} \alpha(Src, SR, Des, DR) * flow(Src, SR, Des, DR) \quad (8)$$

Subject to:

$$r \leq \sum_{Src \in Nodes} \sum_{SR \in Radios} \sum_{DR \in Radios} e(Src, SR, Des, DR, Des) \qquad , \forall Des \in MulticastGroup \qquad (9)$$

$$e(Src, SR, Des, DR, Des) \leq flow(Src, SR, Des, DR) \qquad , \forall Src, Des \in Nodes, \; \forall SR, DR \in Radios \qquad (10)$$

$$\sum_{Src \in Nodes} \sum_{SR \in Radios} \sum_{DR \in Radios} e(Src, SR, Intermediate, DR, s) = $$
$$\sum_{SR \in Radios} \sum_{Des \in Nodes} \sum_{DR \in Radios} e(Intermediate, SR, Des, DR, s)$$
$$\forall Intermediate \in Nodes \backslash (MulticastSource, MulticastGroup), s \in MulticastSource \qquad (11)$$

$$flow(Src, SR, Des, DR) \geq 0, \quad e(Src, SR, Des, DR, t) \geq 0, \quad r \geq 0 \qquad (12)$$

Inequality constraint (9) requires that the multicast throughput is at most equal to the sum of the conceptual flow rates from the source to each destination. Finally, equality (11) characterizes the flow conservation law for conceptual flows.

In the above formulation, the objective function (8) is concave and all constraints are linear. In this case, the network sub-problem is one of convex optimization which can be solved in polynomial time order [53].

It is worth mentioning that there are two aspects of a general network-coding-based multicast solution: one is modeling flow control in terms of a linear programming formulation and the second is the specification of coding/decoding operations across mesh nodes for determining the content of individual links. The coding assignment is complementary to our work and thus, remains outside the scope of this paper.

B. Physical layer module
In this section, we describe the formulation details for the power control optimization sub-problem at the physical layer:

$$Maximize \; \sum_{Src \in Nodes} \sum_{SR \in Radio} \sum_{Des \in Nodes} \sum_{DR \in Radio} \alpha(Src, SR, Des, DR) * c(Src, SR, Des, DR) \qquad (13)$$

$$subject \; to:$$

$$c(Src, SR, Des, DR) = b * log_2 \big(1 + SINR(Src, SR, Des, DR)\big), \quad \forall Src, Des \in Nodes, \forall SR, DR \in Radio \qquad (14)$$

$$SINR(Src, SR, Des, DR) = \frac{G(Src, SR, Des, DR, Src, SR, Des, DR) * p(Src, SR, Des, DR)}{Denominator(Src, SR, Des, DR)} \qquad (15)$$

$$Denominator(Src, SR, Des, DR) = \sum_{Src' \in Nodes} \sum_{SR' \in Radio} \sum_{Des' \in Radio} \sum_{DR' \in Nodes} IC\big(AC(Src', SR', Des', DR'), AC(Src, SR, Des, DR)\big) *$$
$$p(Src', SR', Des', DR') * G(Src', SR', Des', DR', Src, SR, Des, DR) + Noise \qquad (15')$$

$$\sum_{SR \in Radio} \sum_{Des \in Nodes} \sum_{DR \in Radio} p(Src, SR, Des, DR) \leq p_{max}(Src), \quad \forall Src \in Nodes \qquad (16)$$

Equality (14) computes the capacity of the link(Src, Sr, Des, Dr). Furthermore, constraint (15) characterizes the signal to interference and noise ratio of a link. The last constraint (16) guarantees that the sum of powers radiated by a node for different links is bounded by the pre-specified maximum power. It should be noted that the parameter AC(.), corresponding to an assigned channel, in constraint (15') is supposed to be already computed as the outcome of the first phase of the overall framework using CLA such that the value of parameter IC (interference coefficient) for each link is minimized. We also model the gain of the links using the Friis free space model [39], as follows :

$$G(Src', SR', Des', DR', Src, SR, Des, DR) = \frac{\gamma}{Distance(Src', Src)^{\beta}} \qquad (17)$$

where γ, is a constant which depends on the antenna gains and signal wavelengths of the transmitter and receiver. Also, β denotes the path loss exponent.

The physical layer sub-problem is not convex since its constraint set includes the product of two continuous variables $SINR(Src, SR, Des, DR)$ and $p(Src', SR', Des', DR')$ in equality constraint (15)). Also, the appearance of a logarithmic function in equality constraint (14), makes it non-linear. In sub-section 3.5, using λ − formulation [10], we approximate these two non-linear constraints in LP terms so as to render the physical layer sub-problem convex and thus solvable in polynomial time order.

3.5 Approximations of the non-linear terms

This section describes the specifics of linearization the constraints (14) and (15) which comes at the cost of approximation. As elaborated in [10], the product of two continuous variables can be converted into a separable form; in particular, in order for the troublesome product of $SINR(Src, SR, Des, DR)$ and $p(Src', SR', Des', DR')$ to be linearized, two auxiliary continuous variables v_1 and v_2 are introduced and set as follows:

$$v_1 = \frac{1}{2}[SINR(Src, SR, Des, DR) + p(Src', SR', Des', DR')]$$
$$v_2 = \frac{1}{2}[SINR(Src, SR, Des, DR) - p(Src', SR', Des', DR')]$$

Supposing that $l_1 \leq SINR(Src, SR, Des, DR) \leq u_1$ and $l_2 \leq p(Src', SR', Des', DR') \leq u_2$, the boundary of the new variables v_1 and v_2 will be as follows:

$$\frac{(l_1 + l_2)}{2} \leq v_1 \leq \frac{(u_1 + u_2)}{2}$$

$$\frac{(l_1 - u_2)}{2} \leq v_2 \leq \frac{(u_1 - l_2)}{2}$$

In this case, the term $(SINR(Src, SR, Des, DR) * p(Src', SR', Des', DR'))$ can be replaced by the following separable function:

$$v_1{}^2 - v_2{}^2$$

Now, each of the resulting square functions can be approximated using the piecewise linear approximation. Fig. 4 depicts the curve associated with a hypothetical square function which is divided into three pieces approximated by straight lines. The points at which the slope of the piecewise linear function changes are referred to as breakpoints. This approximation can be expressed mathematically in several ways. We use a method known as λ-formulation which is described below.
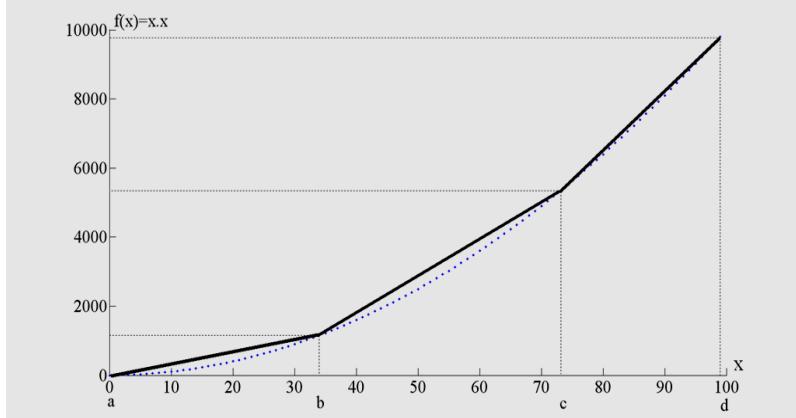
Fig. 4: Piecewise linear approximation of $f(x) = x^2$

It is assumed that a, b, c and d denote four breakpoints along the x-axis in Fig. 4, while $f(a), f(b), f(c)$ and $f(d)$ denote their corresponding function values. Also, supposing that λ_a, λ_b, λ_c, λ_d denote four nonnegative weights, the piecewise linear approximation of $f(x)$ can be written as the following set of LP terms:

$$\lambda_a f(a) + \lambda_b f(b) + \lambda_c f(c) + \lambda_d f(d) = \widetilde{f(x)}$$
$$\lambda_a a + \lambda_b b + \lambda_c c + \lambda_d d = x$$
$$\lambda_a + \lambda_b + \lambda_c + \lambda_d = 1$$

According to the arguments in [10], an additional constraint should be satisfied as well, requiring that at most two consecutive λ's be greater than zero. λ-formulation with this technique guarantees that the points $(x, \widetilde{f(x)})$ always lie on the approximating line segments. The non-linearity within our formulation can effectively be eliminated for using a similar linearization initiative.

Convergence analysis

Theorem 1. The Primal-Dual algorithm presented in Fig. 3, always converges to the global optimum of the overall network optimization problem (5) as long as the routing and power control sub-problems are convex and the step size k is aptly chosen.

Proof: Since constraint (4) is linear, the objective function is strictly concave, and the constraint set of each sub-problem is linear as well, the overall multicast throughput maximization problem turns out to be convex. Moreover, as long as the step size k is chosen properly, successive updates of the dual variable ultimately converge to an optimal solution. The strong duality also guarantees that the equilibrium point of the convergence corresponds to the global optimum of the problem.

4. Experimental results

In what follows, we present the results of four conducted experiments. Experiments 1was basically conducted to observe the convergence behavior of the proposed framework via simulations. Experiment 2 was dedicated to compare the most relevant research in the line of network-coding based cross-layer multicast throughput optimization [29] with the proposed approach. Finally, the performance of the proposed approach compared to LCA and MCM [69, 70] as well as genetic algorithm-, simulated annealing-, and tabu-search-based methods [12-14] was studied.

It is noted that there exists two sets of parameters: the first one is associated with network configuration such as the maximum power, the number of channels/interfaces, and the number of nodes which are tuned manually before staring the simulation. The second is learning rate of the LAs which is tuned in such a way that the best result is obtained.

Experiment 1:

The proposed solution for the convex problem of multicast throughput maximization was solved by *cvx*, a package for specifying and solving convex problems [21]. Throughout this experiment as well as experiment 2, each mesh node is assumed to use channels consistent with IEEE 802.11g standard as depicted in Fig. 5 [28].
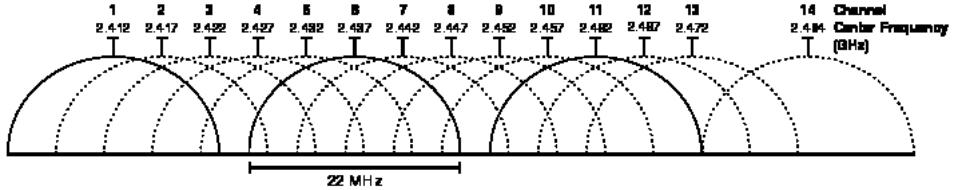
Fig. 5: Graphical representation of the available channels in IEEE 802.11g [28]

Here, the focus is on analyzing the convergence behavior of all links' capacities, aggregated flows, and dual variables as well as network throughput. For this reason, we choose a 4-node network to observe the situation of all links simultaneously, since for a large-node network is not possible to effectively represent aforementioned parameters. Clearly, this experiment tries to show correctness of the proposed framework. Instead, in experiment 3 we will compare our results with different methods in different networks sizes. In the network shown in Fig. 6, each mesh node is supposed to be equipped with two radios, with the added simplifying assumption in which the links are only established between radios of the same number for every pair of mesh routers. Fig. 7 depicts the achieved throughput during the execution of the simulation.
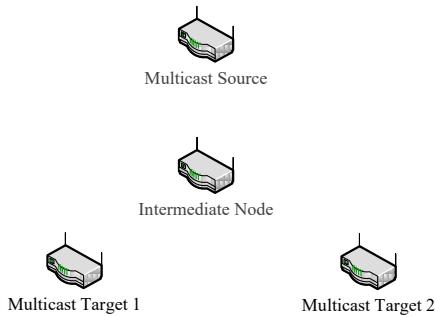
Fig. 6: Hypothetical network configuration for experiment 1
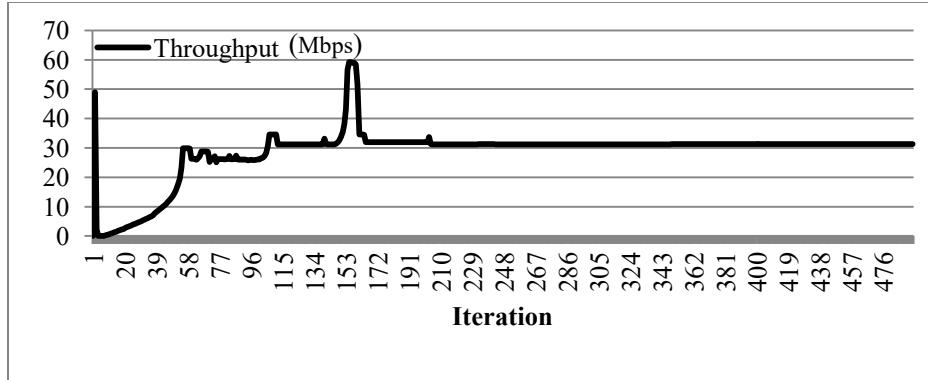
Fig. 7: The resulted throughput using the proposed framework for the given network shown in Fig. 6

Also, Figs. 8 throughout 13 depict the provided capacities of links, together with their actual flows. As can be seen, the mathematical solution tries to optimally utilize the provided capacity in physical layer such that the maximal throughput is achieved.
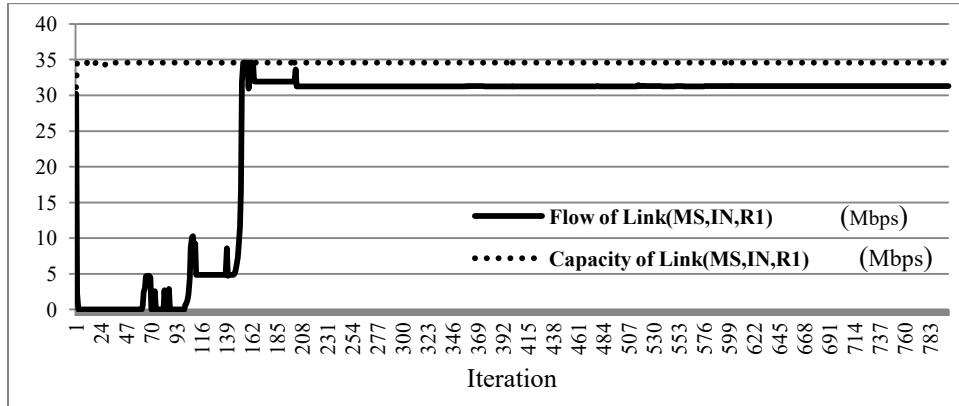

Fig. 8: Provided capacity and the associated flow for the link established between radio 1 of the multicast source node and radio 1 of the intermediate node for the given network shown in Fig. 6
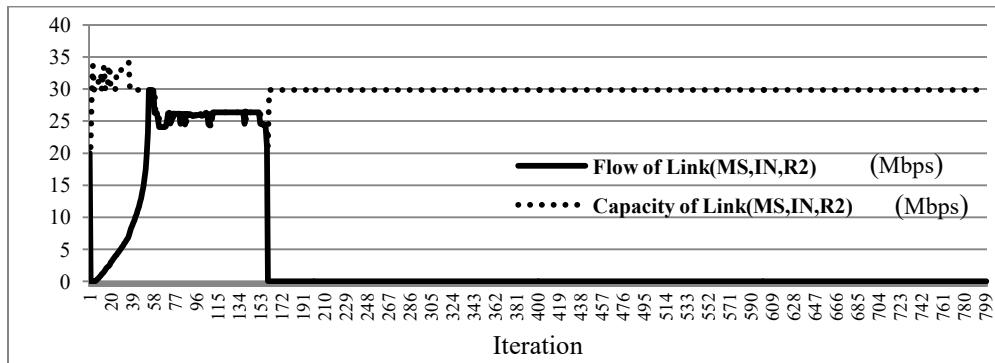
Fig. 9: Provided capacity and the associated flow for the link established between radio 2 of the multicast source node and radio 2 of the intermediate node for the given network shown in Fig. 6

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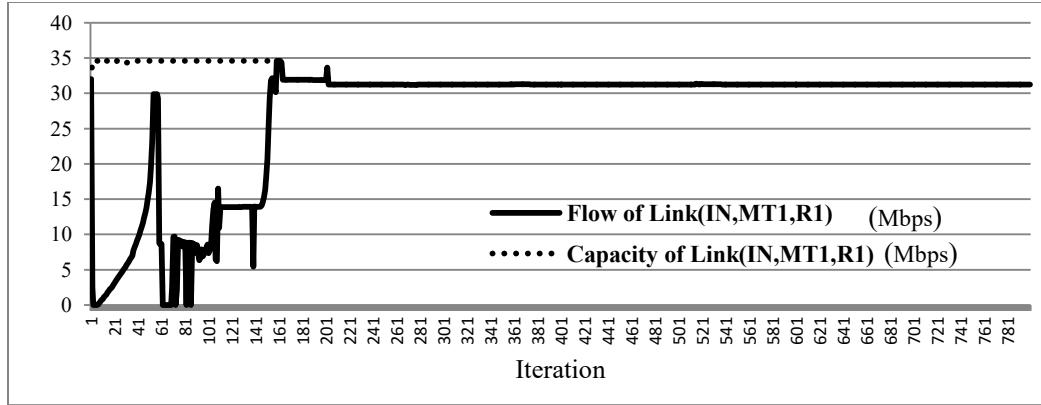
Fig. 10: Provided capacity and the associated flow for the link established between radio 1 of the intermediate node and radio 1 of the multicast target 1 for the given network shown in Fig. 6
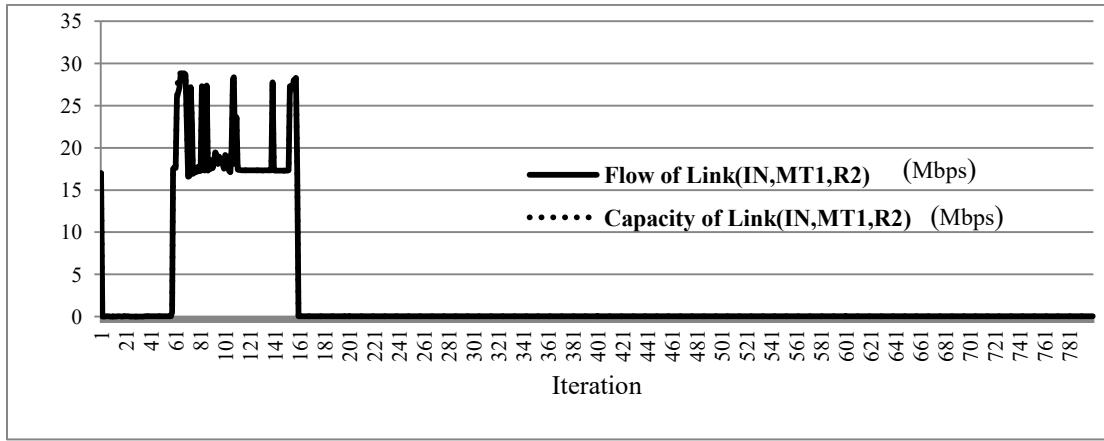

Fig. 11: Provided capacity and the associated flow for the link established between radio 2 of the intermediate node and radio 2 of the multicast target 1 for the given network shown in Fig. 6
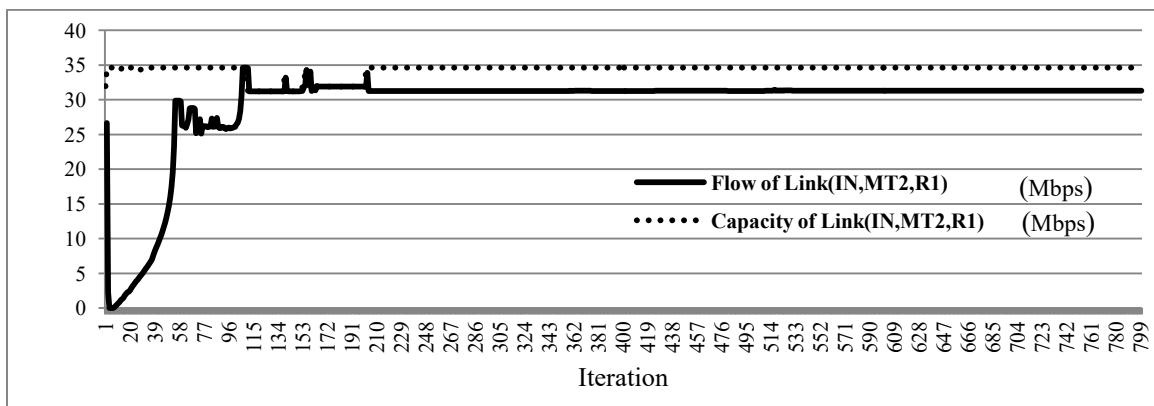
Fig. 12: Provided capacity and the associated flow for the link established between radio 1 of the intermediate node and radio 1 of the multicast target 2 for the given network shown in Fig. 6

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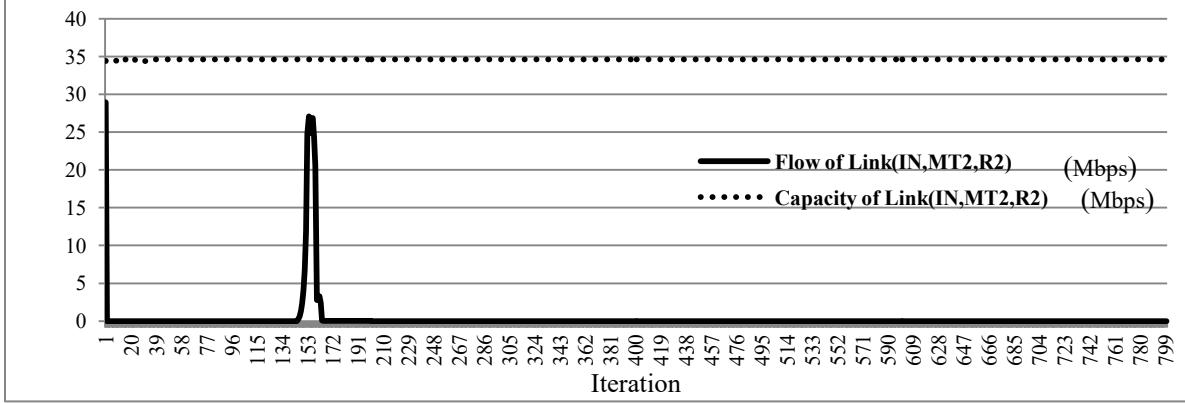
Fig. 13: Provided capacity and the associated flow for the link established between radio 2 of the intermediate node and radio 2 of the multicast target 2 for the given network shown in Fig. 6

In this experiment, the trend of the dual variable α (shadow price) for each link has been recorded during the simulation. Fig. 14 shows the convergence behavior of the dual variable α for different links.
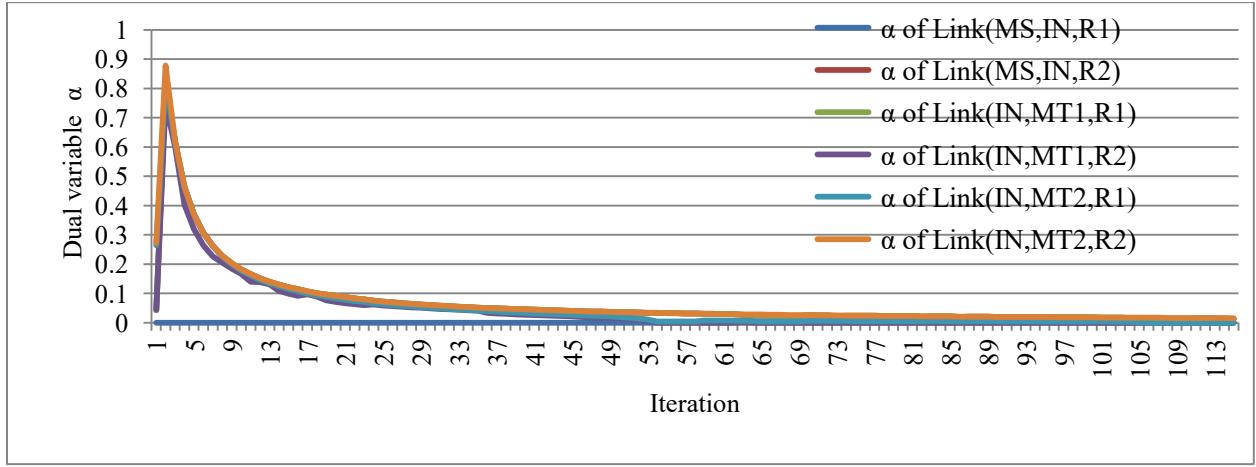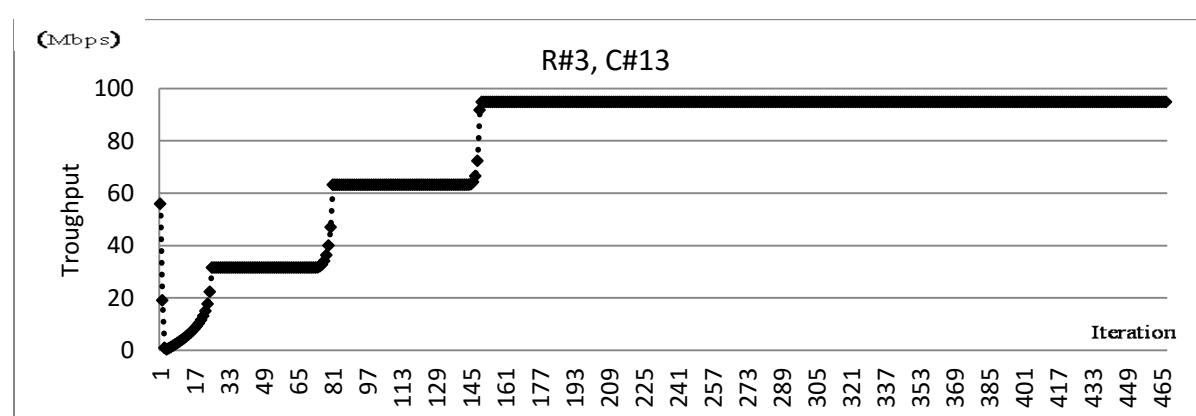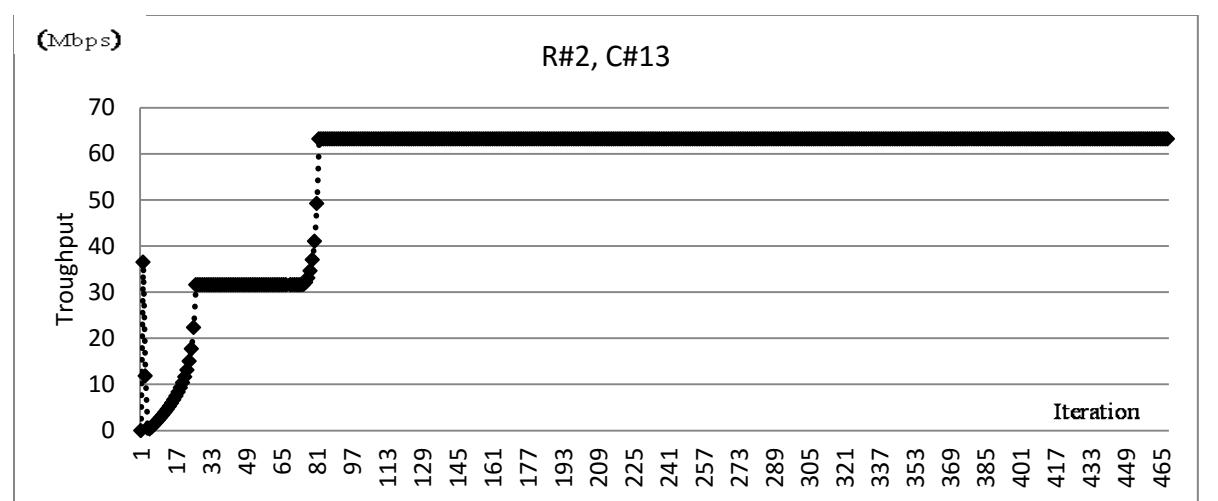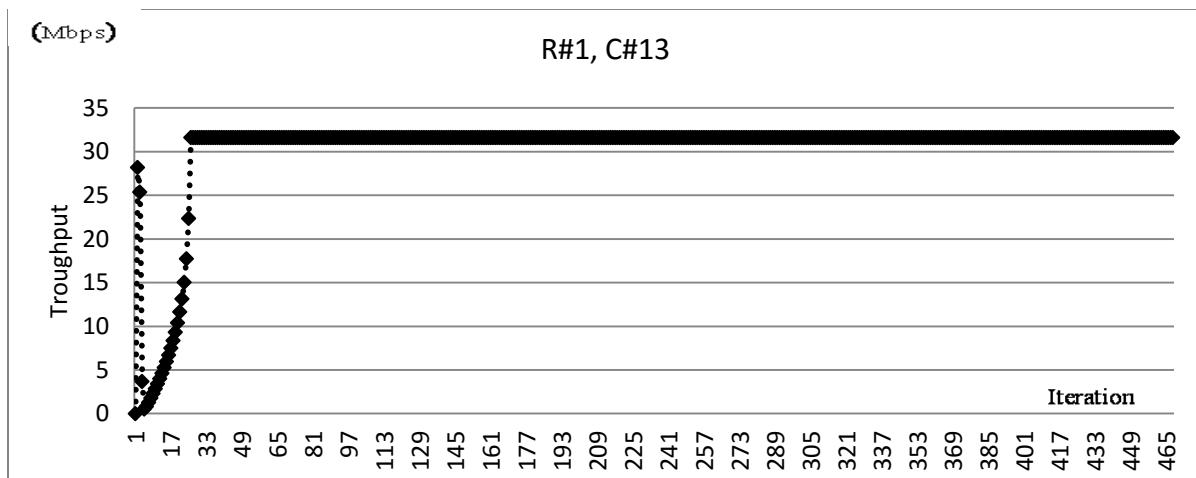

Fig. 14: Convergence behavior of the dual variable for individual links of the given network shown in Fig. 6

We further observe the impact of the number of radios on the achievable multicast throughput; in particular, given a *unit disk graph*, the number of channels, and the size of multicast group, we increase the number of radios from 1 to 5 and report the resulting network throughput levels for each configuration (Fig. 15.a to 15.e). As expected, in all scenarios the achieved throughput, thanks to the proposed NUM-based framework converged. That is, during the iterations the link prices are properly adjusted using the algorithm. It can be also observed in Figs 15.a to 15.e utilizing more radios leads to higher throughput values.

Fig. 15.a: The impact of the number of radios on the achieved throughput while there is just one radio and the number of channels is 13


Fig. 15.b: The impact of the number of radios on the achieved throughput while there are two radios and the number of channels is 13


Fig. 15.c: The impact of the number of radios on the achieved throughput while there are three radios and the number of channels is 13

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Fig. 15.d: The impact of the number of radios on the achieved throughput while there are four radios and the number of channels is 13
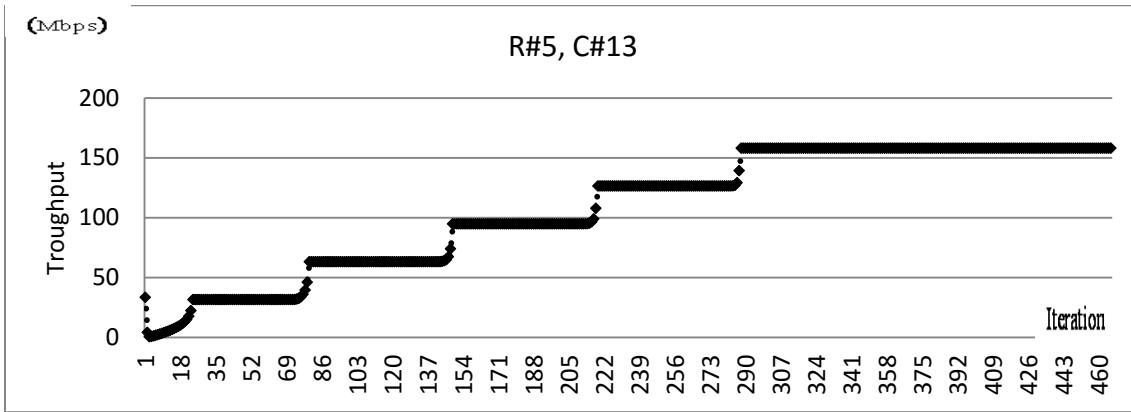

Fig. 15.e: The impact of the number of radios on the achieved throughput while there are five radios and the number of channels is 13

Experiment 2:

To the best of our knowledge, the most relevant research in the line of network-coding based cross-layer multicast throughput optimization, is the study carried out in [29], which is primarily proposed for multi-channel single-radio configurations. Therefore, we apply our formulation to a network with one interface per node for fairly comparison of our solution with [29].

In this experiment and the next, we use a 15-mesh node network consisting of one multicast source and 5 multicast targets. Fig. 16 depicts the achieved throughput for both methods. As it is expected from theorem 1 and strong duality concept, the achieved throughput in our design converges to the global optimum of the overall network optimization problem. From obtained results, the proposed method in the steady state phase can achieve around 51% throughput improvement, which can be attributed to more realistic dynamical tuning of link capacities featured in our framework compared to the physical-layer-agnostic scheme in [29].

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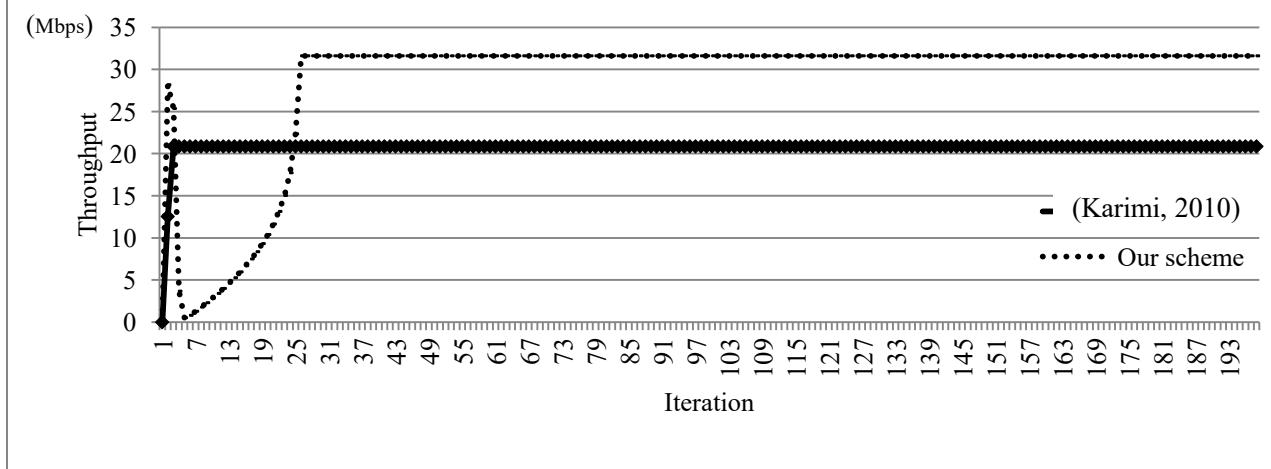
Fig. 16: Multicast throughput in the proposed method and [29]

Experiment 3:
To further study the efficiency of the proposed method, cross-layer-optimization-based multicast routing (CMR), compared to LCA and MCM [69, 70] as well as genetic algorithm-, simulated annealing-, and tabu-search-based methods [12-14] several simulations have been conducted in OMNET++ which is a discrete event network simulation framework [1]. Conducted simulations in OMNET++ are organized in three scenarios. In the first scenario, we varied the number of receivers in a multicast group to assess end-to-end delay, average throughput, average packet delivery ratio, and total multicasting cost in different methods. In the second scenario, the number of radio interfaces per nodes was changed and aforementioned criteria were investigated. Finally, the scalability of the CRM was studied while the number of mesh nodes increased. In all simulations of this experiment we use IEEE 802.11b/g with 13 available channels. Also, simulation time is set to 300 seconds. In addition, maximum power (Pmax) per node is assumed to be 28.92 mw. It is noted that each data point reported here has been averaged over the results of ten different runs. Definitions of the aforementioned measures are presented as follows:

- **Average packet delivery ratio (APDR)**
  Packet delivery ratio is defined as the number of packets received at multicast receivers to the number of packets sent by the multicast source averaged on all multicast receivers. This criterion specifies the number of packets delivered to the multicast receivers to the number of packets expected to be received by the multicast receivers.

- **Average end-to-end delay**
  End-to-end delay is defined as the average time elapsed between sending the packets by the multicast source and receiving at all the multicast receivers. This criterion is averaged on all multicast receivers.

- **Average throughput**
  Throughput is defined as the number of packets received by the receiver over the required time to deliver this number of packets averaged on all multicast receivers.

$$\text{Average throughput} = \frac{1}{|MRS| \times RT} \sum_{i=1}^{|MRS|} NRP(MR_i)$$

  where $NRP(MR_i)$ denotes the number of received packet at i-th multicast receiver. Also $|MRS|$ specifes cardinality of multicast receiver set and RT is the required time to deliver the number of packets.

- **Total cost**
  Total cost is defined as the number of links involved in multicast routing.

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In what follows, the simulation results are presented for three different scenarios:

Scenario #1: this scenario is mainly considered to evaluate CMR compared to other representative methods while the number of multicast group members is raised from 2 to 8 in a 64-node MRMC WMN. It is noted that in this scenario the number of radio interfaces is set to 8. Also, each set of performed simulations is associated with the assessment of one of the aforementioned measures. In the first set of simulations, the achieved throughput for different methods was investigated. As it can be seen in Fig. 17, the representative methods don't utilize all link capacities and potential links. In contrast, in the proposed framework, first, the maximum achievable throughput is characterized by per link aggregated flow. Next, the link capacities are bounded by maximum power provided in the physical layer. The solution to the power control sub-problem ensures that the maximal capacity for each link is exploited, while the routing sub-problem fully utilizes the link capacity. That is why, our results are better than those of in other methods. It should be noted that less multicast receivers, less the forming multicasting links and hence, less overall interference. When the number of multicast receivers gradually goes up the results are changed. Because by increasing the number of multicast receiver in a network with the constant number of nodes and limited channel pool, the multicasting links will be increased and therefore, interference will be more. In this case the achieved throughput in our approach gets closer to that of in other methods.
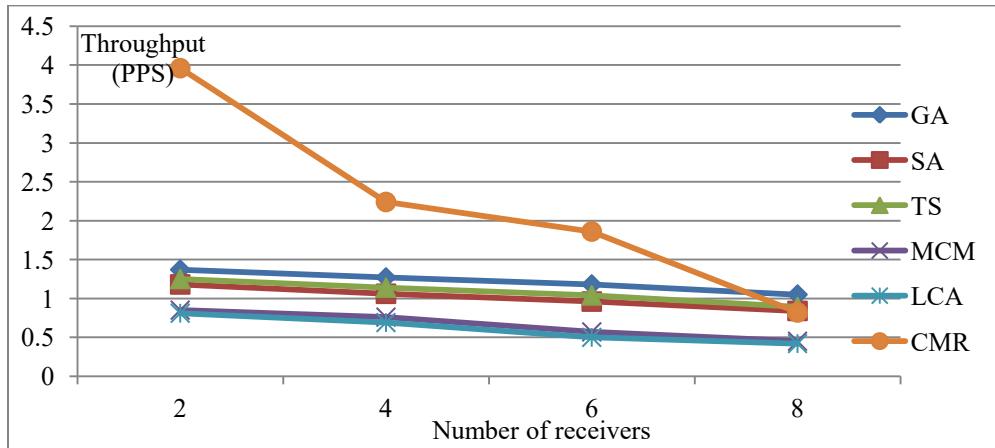

Figure 17: Average throughput versus the number of receivers for different methods

The second set of simulations in this scenario is dedicated to study resultant average end-to-end delay. As it is shown in Fig. 18, employing CMR, leads to least end-to-end delay among the other representative methods. The reason is that in tree-based multicast routing protocols, the multicast receivers are leaves of multicast tree. For example, if there is a minimum-delay path to receiver *i*, it can not be used to relay the traffic to the neighboring receivers of receiver *i* and hence, other paths should be constructed from multicast source to other receivers with more end-to-end delay. But in the mesh-based multicast routing protocols, a situation may occur in which one or many receivers relay the traffic to the other neighboring receivers reducing the end to end delay.
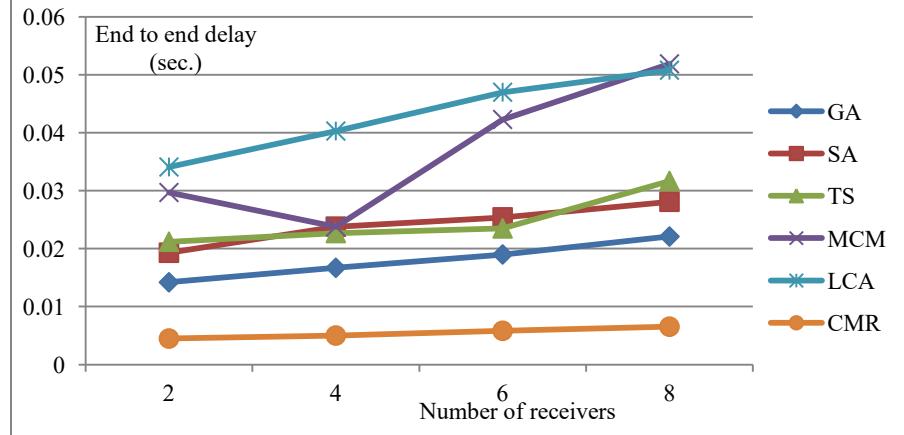
Figure 18: The average end-to-end delay versus the number of receivers for different methods

In this scenario, the APDR is also investigated. From the results reported in Fig. 19, CMR yields to the most APDR as compared with other methods. The reason is that, unlike the other representative methods in which channels are assigned to the links in an ascending order, CMR effectively employs all available channels using CLA so that the minimal interference for every link is provided and hence, the APDR rises. It should be noted that the reason for reduction of APDR is that the increase in the number of receivers with constant network size, will result in raising multicasting links. In such case, due to limitation of channel pool the overall interference becomes more and consequently, APDR is decreased.
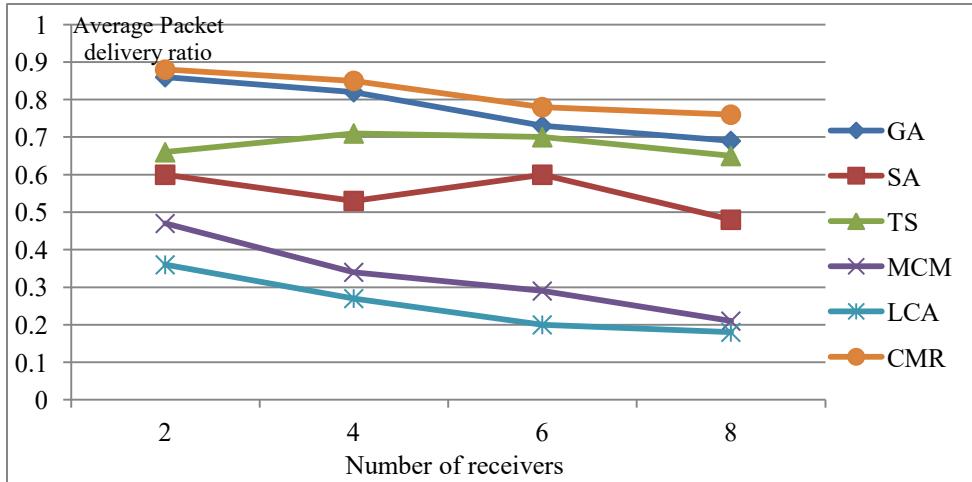

Figure 19: The average packet delivery ratio versus the number of receivers for different methods

The objective of this set of simulations is to measure the cost of multicasting in terms of total number of links forming the overlay network. It is noted that from the view point of the links involved in routing, all multicast routing protocols fall into two main categories; mesh-based approaches and tree-based approaches. CMR falls in mesh based category. As compared with tree based methods, mesh based approaches thanks to existence of alternative paths are fault tolerant and robust to link or node failures. This implies that the overall cost in tree based approaches is less than that of mesh-based approaches. That is why in Fig. 20 the obtained cost in CMR is more than other approaches.
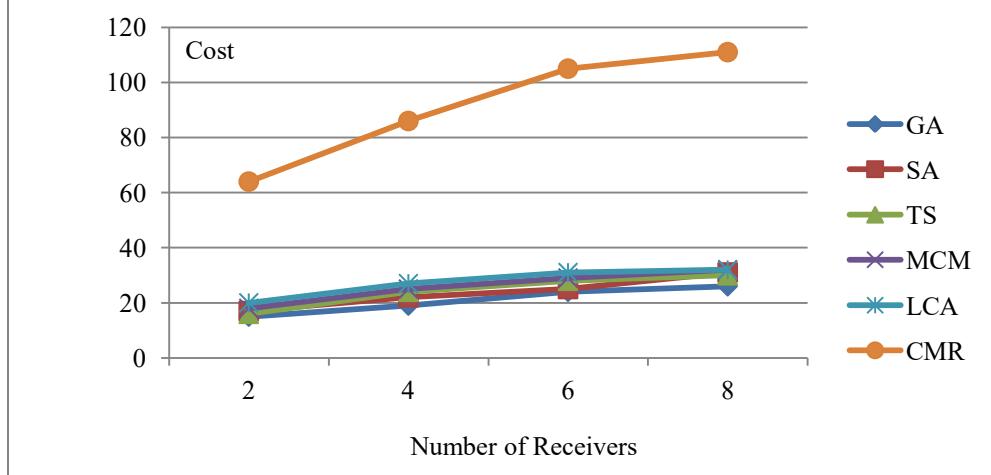
Figure 20: Multicast routing cost versus the number of receivers for different methods

Scenario #2: here, we used a 64-node MRMC WMN in a grid style deployment. The number of multicast receivers is the square root of the number of network nodes. In this scenario, the number of radios per node is changed to evaluate resultant average throughput, end to end delay, APDR, and cost in CMR compared to the other methods. For each metric a set of simulations were conducted. From results reported in Fig. 21, as it is expected, the use of CMR will result in higher throughput when the number of radio interfaces is set to 2, 4, 6, and 8.
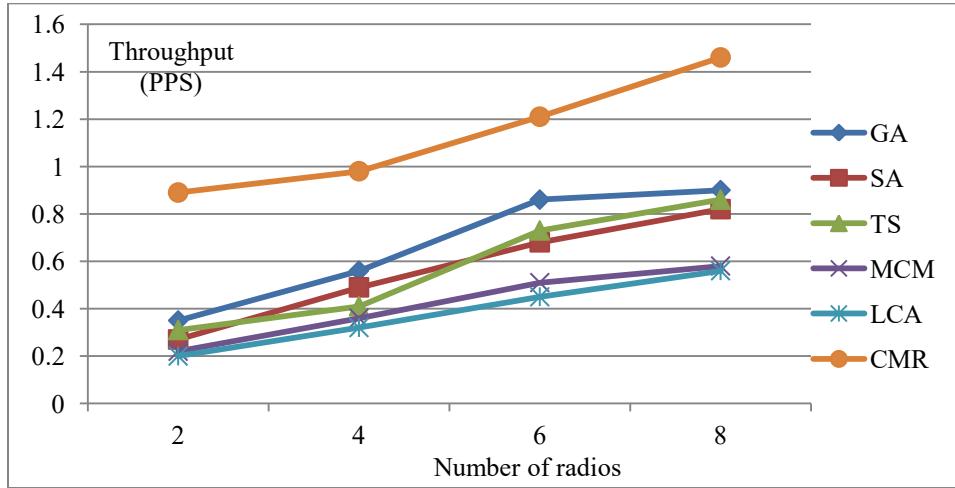

Figure 21: The average throughput versus the number of receivers for different methods

To assess the average end-to-end delay for different multicast routing protocols, we have conducted four sets of simulations associated to 2, 4, 6, and 8 radio interfaces per nodes. Fig. 22 demonstrates that CRM significantly outperforms other representative methods in terms of end to end delay. In fact, when the number of radios is increased, the number of alternative paths will be increased. In this case, the paths with less end-to-end delay can be constructed.

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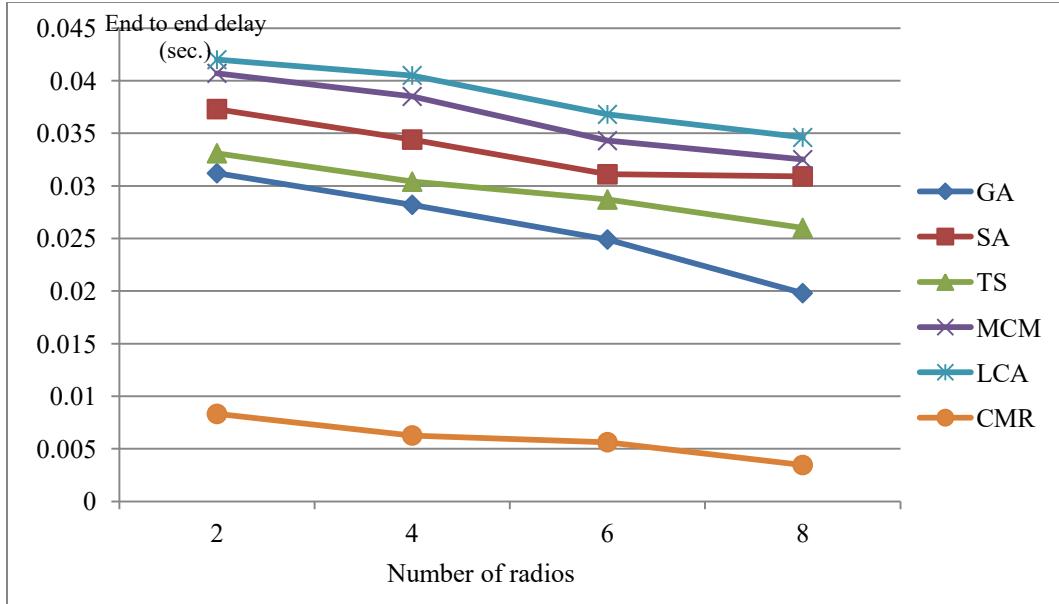
Figure 22: The average end to end delay versus the number of radios per node for different methods

As discussed earlier, in the proposed framework, throughout iterations residing LA on each mesh router learns to assign proper channels to the links in such a way that minimal interference for each one is achieved. If channel correlation is decreased, then APDR will be increased. Obtained results shown in Fig. 23 for different configurations demonstrate efficiency of CRM. Here, it is obvious that the channel pool is not properly exploited in GA, SA, TS, LCA, and MCM methods.
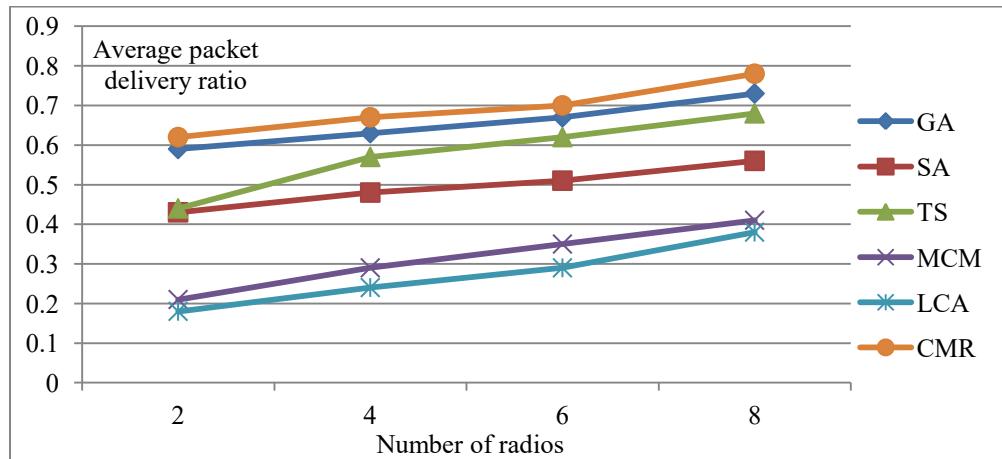

Figure 23: The average packet delivery ratio versus the number of radios for different methods

The next set of simulations is dedicated to study overall cost of multicasting. From Fig. 24, as it is expected again CMR as a mesh-perspective based framework improves throughput, the end to end delay, the APDR at higher cost.
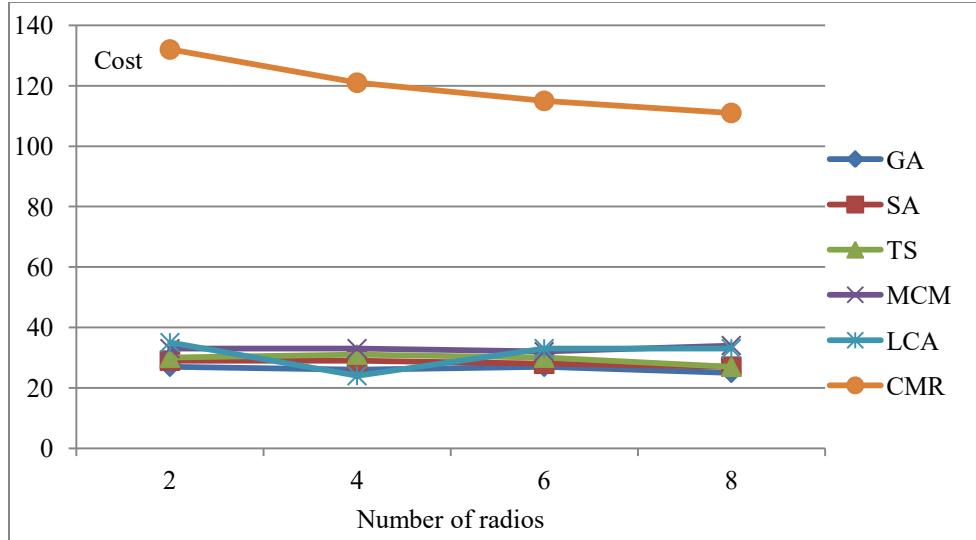
Figure 24: The multicasting cost versus the number of radios per node for different methods

Scenario #3: in this scenario, our aim is to observe aforementioned metrics for different methods while the number of nodes in the given network rises from 16 to 64. It is noted that here, the number of radios per node is set to 8. Fig. 25 shows that CRM outperforms other methods in terms of resultant throughput when the number of nodes is set to 16, 25, 36, 49, and 64. Once the number of nodes is small, there is no need to utilize all the radios and hence, the number of multicasting links is low. In this case, the number of channels is sufficient to route the packets with no interference. However, raising the network size from 16 to 64 naturally will result in more involving radios which in turn, leads to more established links. In this case, since the number of channels is not sufficient, some channels are assigned to more than one link and consequently, throughput is affected.
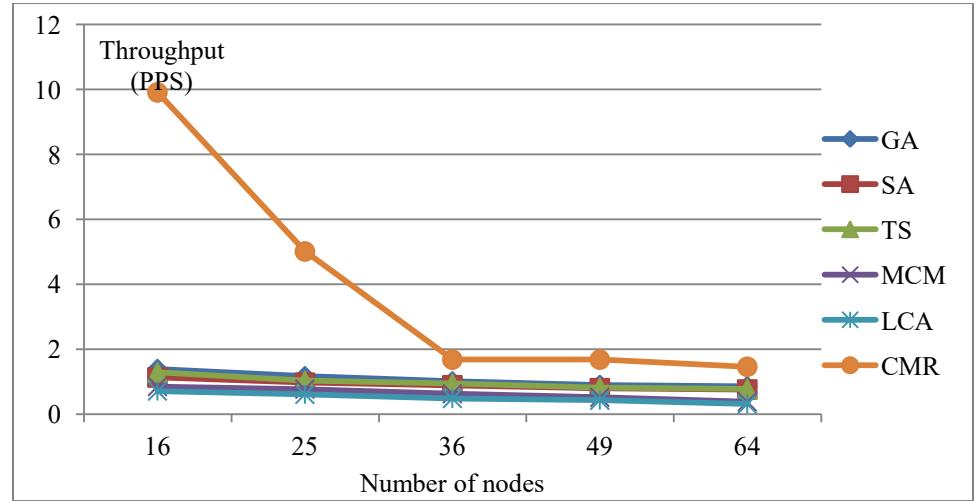

Figure 25: The average throughput versus the number of node in WMN for different algorithms

From results reported in Fig. 26, average end-to-end delay in CMR gently rises when the size of the network is increased from 16 to 64, whereas the attained results from other methods show that end to end delay sharply increases versus the number of nodes.
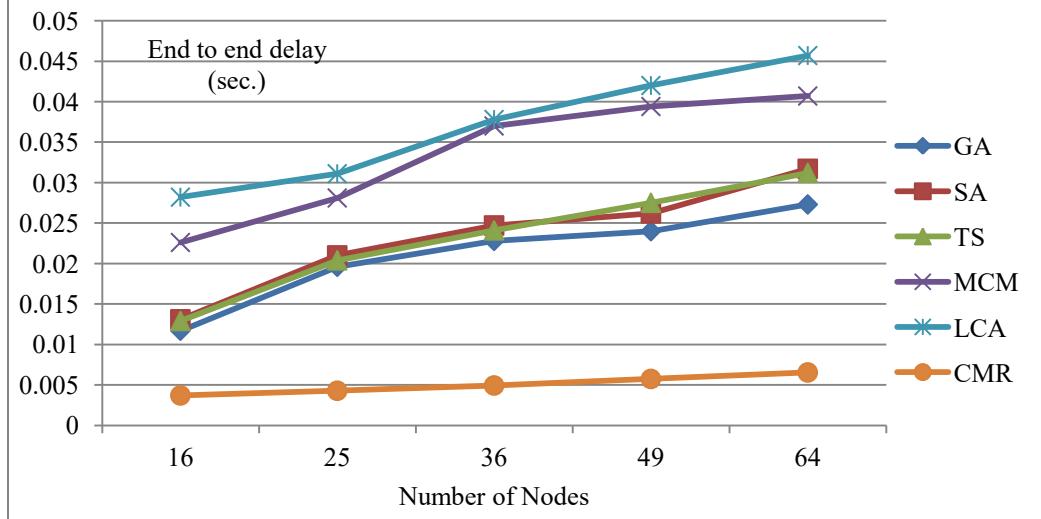
Figure 26: The average end to end delay versus the number of nodes in WMN for different algorithms

Fig. 27 presents the APDR for different approaches. As it is expected the proposed approach, CMR, attains the highest APDR. As discussed earlier, in the first phase of the proposed framework, the LAs resident on different links learn to assign minimal correlation channels to their associated links. In the second phase, the precise power control is provided to guarantee more APDR.
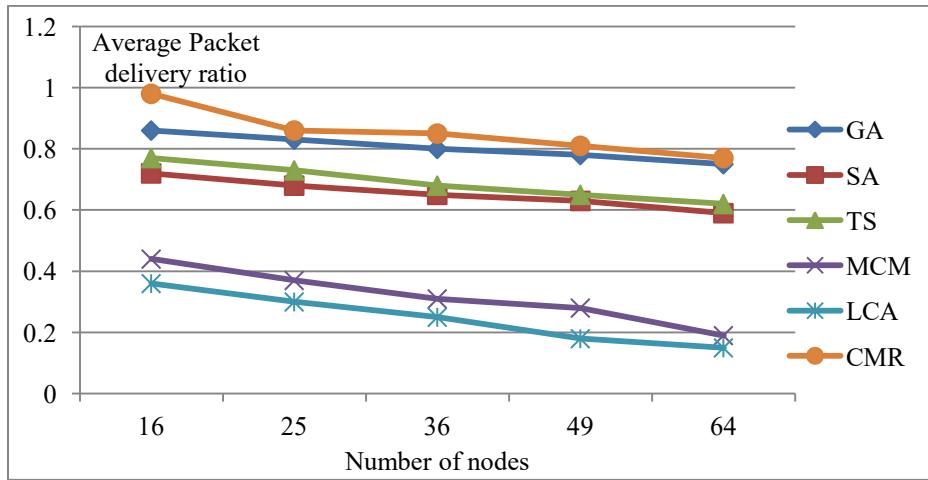
Figure 27: Average packet delivery ratio versus the number of nodes in WMN for different methods

5. Discussion of time complexity

In what follows, we analyze time complexity of the given algorithm to classically investigate it independent of used hardware / software configuration of a machine. Therefore, to compute time complexity of the proposed throughput maximization cross layer optimization, we will study the number of variables and constraints in sub-sections 5.1 and 5.2, respectively.

5.1 Number of generated variables

Memory demands for each algorithm can be assessed as a function of the generated variables during the implementation. The number of all generated variables depends on the number of different combinations of the variables input sets. For example, the number of generated variables for variable $capacity(Src, SR, Des, DR)$, where $Src, Des \in Nodes$ and $SR, DR \in Radios$ is $|Nodes|^2 \times |Radios|^2$. Therefore, the number of variables in network layer module and the number of associated generated variables during implementation can be observed in Table 4:

Table 4: Number of generated variables in network layer

Name of variables	Number of generated variables
$\alpha(.)$	$|Nodes|^2 \times |Radios|^2$
$e(.)$	$|Nodes|^2 \times |Radios|^2 \times |MulticastSources|$
$flow(.)$	$|Nodes|^2 \times |Radios|^2$
r	1
Order of total generated variables	$O(|Nodes|^2 \times |Radios|^2 \times |MulticastSources|)$

Similarly, the number of variables in physical layer module and corresponding generated variables can be found in Table 5:

Table 5: Number of generated variables in physical layer

Name of variables	Number of generated variables
$\alpha(.)$	$|Nodes|^2 \times |Radios|^2$
$c(.)$	$|Nodes|^2 \times |Radios|^2$
$SINR(.)$	$|Nodes|^2 \times |Radios|^2$
$p(.)$	$|Nodes|^2 \times |Radios|^2$
Order of total generated variables	$O(|Nodes|^2 \times |Radios|^2)$

In addition, we have C_1 additional variables in the linear approximation phase which is constant and negligible. Therefore, the order of all variables is in throughput maximization cross-layer optimization is the maximum order of variables in both sub-problems:

$$O(|Nodes|^2 \times |Radios|^2 \times |MulticastSources|)$$

5.2 Number of generated constraints

In what follows, we study the number of constraints. The number of generated constrains in network layer module is listed in Table 6 for each modeled constraint, separately.

Table 6: Number of generated constraints in network layer

Constraint number	Number of generated constraints
9	$|Nodes|^2 \times |Radios|^2 \times |MulticastSources|$
10	$|Nodes|^4 \times |Radios|^4 \times |MulticastSources|$
11	$|Nodes|^4 \times |Radios|^4 \times |MulticastSources|^2$
12	$|Nodes|^2 \times |Radios|^2$
Total generated constraints	$O(|Nodes|^4 \times |Radios|^4 \times |MulticastSources|^2)$

Also, the number of constraints generated to solve physical layer sub-problem is shown in Table 7.

Table 7: Number of generated constraints in physical layer

Constraint number	Number of generated constraints
14	$|Nodes|^2 \times |Radios|^2$
15	$|Nodes|^2 \times |Radios|^2$
16	$|Nodes|^2 \times |Radios|^2$
Total generated constraints	$O(|Nodes|^2 \times |Radios|^2)$

We have also C_2 additional constraint for linear approximation which is constant and negligible. Finally, the order of overall constraints is maximum order of constrains in two sub-problems:

$$O(|Nodes|^4 \times |Radios|^4 \times |MulticastSources|^2).$$

According to [35], the average empirical time complexity of the Simplex method which is mostly used in linear programming problems is bounded from above by $O(M^2N)$, where M and N denote the number of constraints and variables, respectively. Therefore, time complexity of the proposed cross-layer optimization is:

$$O(|Nodes|^{10} \times |Radios|^{10} \times |MulticastSources|^5).$$

6. Conclusion and future works

In this paper, the cross-layer problem of multicast routing in the context of multi-channel multi-radio wireless mesh networks has been dealt with a rigorous network-coding-based formulation. A novel aspect to our holistic framework is that the channel assignment sub-problem is separately tackled with a heuristic algorithm based on the notion of Cellular Learning Automata targeted at the provision of minimal interference coefficient for all links. Routing and physical layer modules, on the other hand, are jointly optimized using an NUM-theoretic solution concept, and we have methodically proved the convergence of the underlying cross-optimization algorithm. Extensive simulation experiments have also been conducted to evaluate the efficiency of our proposed framework and to contrast its performance against many successful representative research works.

As a part of our plan for future work and primarily to enhance the practical contribution of the present article, we intend to incorporate the fading and time varying characteristics of the wireless channel into our formulation. Yet another promising direction of research is to extend our single-gateway design, which is only capable of supporting one active multicast session at a time, with the assumption of a multi-gateway networking configuration; this way, provisions can made for not only enabling multi-session applications, but also presenting with the added bonus of load balancing in single session scenarios. Therefore, it is typically the case that most multi-media networking applications are designed with a multi-resolution data transmission initiative. It would thus be desirable to maximize the achievable rate at each layer in such a way that the ultimate nominal rate, associated with that layer, is reached by the optimization algorithm.

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