Critical Density for Coverage and Connectivity in Two-Dimensional Aligned-Orientation Directional Sensor Networks Using Continuum Percolation

Mohammad Khanjary, Masoud Sabaei, *Member, IEEE*, and Mohammad Reza Meybodi

Abstract—Sensing coverage is one of the fundamental design issues in wireless sensor networks, which reflects the surveillance quality provided by them. Moreover, network connectivity enables the gathered data by sensors to reach to the sink node. Given an initially uncovered field, and as more and more directional sensors are continuously added to the sensor network, the size of partial covered areas increases. At some point, the situation abruptly changes from small fragmented covered areas to a single large covered area. We call this abrupt change the sensing-coverage phase transition (SCPT). Likewise, given an originally disconnected sensor network, as more and more sensors are added, the number of connected components changes such that the sensor network suddenly becomes connected at some point. We call this sudden change the network connectivity phase transition (NCPT). The nature of such phase transitions is a central topic in the percolation theory. In this paper, we introduce aligned-orientation directional sensor networks in which nodes are deployed based on Poisson point process and the orientation of all sensor nodes is the same. Then, we propose an approach to compute density of nodes at critical percolation for both of the SCPT and NCPT problems in such networks, for all angles of field-of-view between 0 and π by using continuum percolation. Due to percolation theory, the critical density is infimum density that for densities above it SCPT and NCPT almost surely occur. In addition, we propose a model for percolation in directional sensor networks, which provides a basis for solving the SCPT and NCPT problems together.

Index Terms—Critical density, continuum percolation, excluded area, sensing coverage, network connectivity, aligned-orientation directional sensor networks, video sensor networks, ALODSN.

I. INTRODUCTION

SENSING coverage can be considered as one of the main criterion of quality of service in sensor networks. Extensive researches have been done to solve technical problems related to coverage in sensor networks and several types of coverage have been introduced by researchers so far. Network connectivity, on the other hand, is a graph-theoretic

concept that helps sensors communicate with each other for forwarding their data to a central gathering node, called the sink. To sense a region of interest sufficiently and receive the sensed data by the sink, it is necessary that both *sensing coverage* and *network connectivity* be maintained. Due to wide range of technical problems in *sensing coverage* and *network connectivity* and also their significance, a lot of researches have been focused on them. Some recent reviews on different issues of coverage and connectivity in general sensor networks could be found in [1]–[3] and in directional sensor networks could be found in [4] and [5]. Moreover, *Percolation Theory* has been considered as a rigorous mathematical model for assessing coverage and connectivity in wireless networks by researchers, recently.

In this paper, we introduce aligned-orientation directional sensor networks (ALODSNs) in which the orientation of all sensor nodes is the same (which could be done during or after deployment) and then propose an approach to compute the *critical density* for *sensing coverage* and *network connectivity* in such networks, for all field-of-view angles between 0 and π by using continuum percolation. We also propose a model for percolation in ALODSNs which provides a basis for solving the SCPT and NCPT problems together.

A. Problem Statement

Finding the minimum number of sensors required to achieve a certain degree of *sensing coverage* and *network connectivity* is one of the fundamental issues in design of sensor networks. This will be more important and complicated when we deal with directional sensor networks. As more and more directional sensors with aligned orientation are continuously deployed, the size of covered areas increases and, at some instant, the situation suddenly changes from small fragmented covered areas to a single large covered area which spans the entire field. We call this sudden change in the *sensing coverage* of a field the *sensing-coverage phase transition* (SCPT). The SCPT problem in an ALODSN can be stated as follows:

"Given a field that is initially uncovered, what is the probability of the first appearance of an infinite (or single large) covered component that spans the entire network?"

Likewise, the number of connected components increases when more and more sensors are continuously added to a sensor network that is originally disconnected. At some point, the situation abruptly changes from a disconnected network to

connected network. We call this abrupt change in the network topology the *network-connectivity phase transition* (NCPT). The NCPT problem in a directional sensor network can be expressed as follows:

"Given a network that is initially disconnected, what is the probability of the first appearance of an infinite (or single large) connected component that spans the entire network?"

Because these phase transitions occur at a given density (called *critical density*) and study of such phase transitions is an important topic in *percolation theory*, we are interested in finding the *critical density* for SCPT and NCPT and then finding *critical density* for both by using *percolation theory*. Due to *percolation theory*, the *critical density* is infimum density that for densities above it SCPT and NCPT *almost surely* occur. It is worth noting that similar research for omnidirectional sensor networks has been done by Ammari et al. [6], [7] and we are interested to do such research when the sensor network consists of nodes with only directional sensing ability (such as camera sensor networks and Doppler probes).

B. Related Works

The concept of *continuum percolation*, originally due to Gilbert [8], is to find the *critical density* of a Poisson point process at which an unbounded connected component *almost surely* appears so that the network can provide long-distance multihop communication. Since then, Gilbert's model has become the basis for studying continuum percolation in wireless networks e.g. [9]–[10]. Recently, *percolation theory* has been considered by researchers to be used to examine coverage and connectivity in sensor networks too [6], [7], and [11]–[16].

Ammari et al. [6] considered the *critical density* required to prepare sensing coverage and network connectivity in sensor networks simultaneously. They used *continuum percolation* in two-dimensional sensor networks consist of homogenous sensor nodes with communication radius of R and sensing radius r. They analytically proved that the value of *critical density* for these networks is equal to 0.575. Then, they extended their contribution to three-dimensional network by considering sphere model for sensor nodes [7].

Xing et al. [11] considered the connectivity of a large sensor network by using *continuum percolation*. They were interested in finding the proper time to redeployment of failed nodes to keep the network connected. So, they examined the time of first partition in network and found it must be between $\log(\log n)$ and $(\log n)^{1/p}$ which n is the network size and $\rho > 1$. This result provides a theoretical upper bound of the latest time that redeployment has to be carried out.

Liu et al. [12] and [13] considered exposure-path prevention. Exposure-path refers to the path in a sensor network that an intruder could traverse and not being detected by sensor nodes. They proposed a bond-percolation theory based scheme by mapping the exposure path problem into a bond percolation model. Using this model, they derived the *critical densities* for omnidirectional sensor networks under random sensor deployment where sensors are deployed according to a two-dimensional Poisson process. They also investigated the exposure-path prevention in directional sensor networks [14].

Yang et al. [15] considered following problem: *given a randomly deployed sensor network where sensors are active with probability p, how many sensors are needed to achieve connected-k-coverage?* Connected-k-coverage requires the monitored region to be k-covered by a connected component of active sensors, which is less demanding than traditional k-coverage and connectivity in which all active sensors participate in both coverage and connectivity simultaneously. They investigated the theoretical foundations about connected-k-coverage by applying the *percolation theory* and derived the *critical density* for connected-k-coverage under different ratio between sensing and communication radius of sensors.

Also, Balister et al. [16] introduced a new type of coverage called Trap Coverage that scales better with large deployment regions. A sensor network providing Trap Coverage guarantees that any moving object or phenomena can move at most a known displacement before to be detected by the network, for any trajectory and speed. In fact, Trap Coverage generalizes full coverage by allowing holes of a given maximum diameter. They derived reliable and explicit estimates for the density needed to achieve trap coverage with a given diameter when sensors are deployed randomly. Also, they proposed some polynomial-time algorithms to determine the level of trap coverage achieved once sensors are deployed on the ground.

In this paper, we consider the directional sensor networks in which sensor nodes have been deployed in the region of interest according to Poisson point process and have same orientation vectors. While most of researches in this field are based on Monte Carlo method [17], we are interested to find the *critical density* analytically and find out an equation for *critical density* based on angle of field-of-view of sensors. To the best of our knowledge, this is the first attempt to find the *critical density* for *sensing coverage* and *network connectivity* simultaneously in such networks.

The remainder of this paper is organized as follows: section II presents the model and definition of the paper, section III presents our method for calculation of *critical density* for *sensing coverage* and *network connectivity*, Section IV presents the simulation results for examining the calculated critical densities and finally section V concludes the paper and presents the possible future works.

II. MODEL AND TERMINOLOGY

A. Definitions

Let $X_\lambda = \{\xi_i : i \geq 1\}$ be a two-dimensional homogeneous Poisson point process of density λ (the number of sensors per unit area), where ξ_i represents the location of the sensor s_i.

Definition 1 (Spatial Poisson Point Process) [18]: Let X_λ be a random variable representing the number of points in an area A. The probability that there are k points inside A is computed as

$$P(X_\lambda(A) = k) = \frac{\lambda^k |A|^k}{k!} e^{-\lambda |A|} \tag{1}$$

for all $k \geq 0$, where $|A|$ is the area of A and λ is the density per unit area. In this paper, center of sensor nodes (ξ_i) is considered as points in Poisson point process.
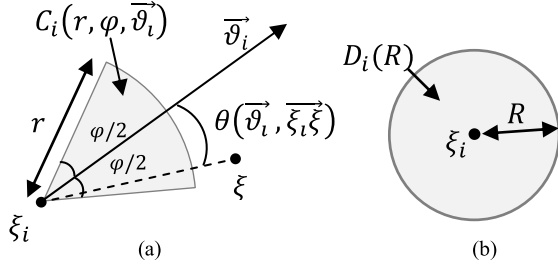
Fig. 1.   (a) Sensing range. (b) Transmission range.

Definition 2 (Directional Sensor Nodes): Each sensor node i is denoted by a tuple $S_i(\xi_i, r, \varphi, \overrightarrow{\vartheta_i}, R)$ where ξ_i is the center, r is the sensing radius, φ is the field-of-view angle ($0 < \varphi \leq \pi$), $\overrightarrow{\vartheta_i}$ is the orientation vector and R is the transmission radius of the sensor.

Definition 3 (Sensing Range): The sensing range of a directional sensor node $S_i(\xi_i, r, \varphi, \overrightarrow{\vartheta_i}, R)$ is a sector which defined by

$$C_i\left(r, \varphi, \overrightarrow{\vartheta_i}\right) = \{\xi \in \mathbb{R}^2 : |\xi_i - \xi| \leq r \text{ and } \theta(\overrightarrow{\vartheta_i}, \overrightarrow{\xi_i\xi}) \leq \varphi/2\}$$

where $|\xi_i - \xi|$ stands for the Euclidean distance between ξ_i and ξ, $\theta(\overrightarrow{\vartheta_i}, \overrightarrow{\xi_i\xi})$ stands for angle between orientation vector of sensor $(\overrightarrow{\vartheta_i})$ and vector from point ξ_i to ξ (see Fig. 1 (a)).

Definition 4 (Transmission Range): The transmission range of sensor $S_i(\xi_i, r, \varphi, \overrightarrow{\vartheta}_i, R)$ is a disk of radius R which defined by

$$D_i(R) = \{\xi \in \mathbb{R}^2 : |\xi_i - \xi| \leq R\}$$

where $|\xi_i - \xi|$ stands for the Euclidean distance between ξ_i and ξ (see Fig. 1 (b)).

Definition 5 (Aligned-Orientation Directional Sensor Network (ALODSN)): An ALODSN is a directional sensor network in which all sensors have same orientation vector (see Fig. 2). This alignment could be done during or after deployment. Due to the same orientation of sensors, each sensor node i of an ALODSN will be denoted by $S_i(\xi_i, r, \varphi, \overrightarrow{\vartheta}, R)$ where $\overrightarrow{\vartheta}$ is the orientation vector of network.

Definition 6 (Collaborating Sensors): Two sensors S_i and S_j are said to be collaborating if and only if there exist some points belong to sensing range of both sensors (see Fig. 3(b)).

$$Col(S_i) = \{S_j : \exists \xi \in \mathbb{R}^2, \xi \in C_i \text{ and } \xi \in C_j\}$$

Definition 7 (Communicating Sensors): Two sensors S_i and S_j are said to be communicating if and only if the Euclidean distance between the centers of their transmission disks is less than R (see Fig. 3 (a)).

$$Com(S_i) = \{S_j : |\xi_i - \xi_j| \leq R\}$$

Definition 8 (Filling Factor) [19]: If each object has area equal to a, the *filling factor* of a homogenous Poisson point process with density equal to λ ($X_\lambda = \{\xi_i : i \geq 1\}$) is given by
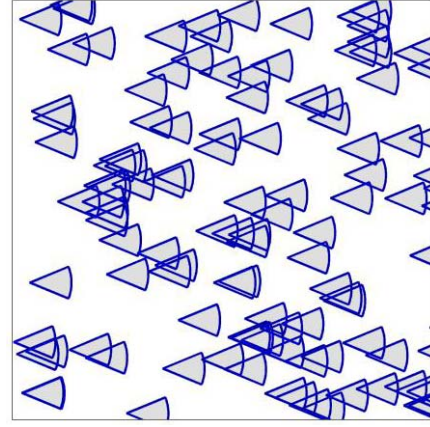
$$\phi = 1 - e^{-\eta} \qquad (2)$$

Fig. 2.   An aligned-orientation directional sensor network (ALODSN) in which all nodes have orientation vectors with angle equal to 0 and $\varphi = \pi/4$.
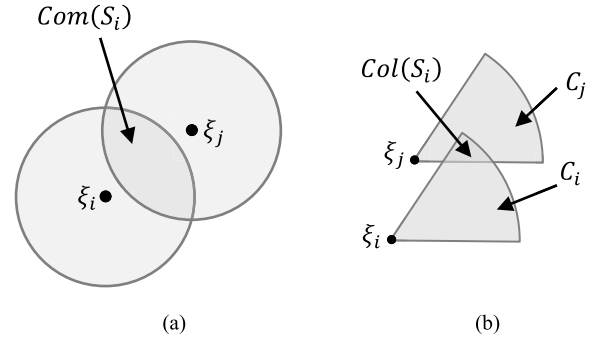

Fig. 3.   (a) Communicating sensors. (b) Collaborating sensors.

which is the mean fraction of region covered by the objects. In equation (2), η is the density of objects and is given by

$$\eta = \lambda a \qquad (3)$$

where a is the area of the objects and λ is the density of the Poisson point process.

In this paper, like other researches in percolation theory e.g. [6], [7], and [17], we are interested in finding η and ϕ in ALODSNs rather than λ, because they are independent of sensing radius (r) and communication radius (R) (in general the area of the objects) and also could be used to compare with other researches.

Definition 9 (Excluded Area and Total Excluded Area) [20]: The *excluded area* of an object is defined as the area around it into which the center of another similar object is not allowed to enter if overlapping of the two objects is to be avoided and denoted by $\langle a_{\mathrm{ex}} \rangle$. The *total excluded area* is excluded area multiplied by *critical density* (λ_c) and denoted by $\langle A_{\mathrm{ex}} \rangle$.

$$\langle A_{\mathrm{ex}} \rangle = \langle a_{\mathrm{ex}} \rangle * \lambda_c \qquad (4)$$

B. Percolation Theory

Due to Broadbent and Hammersley [21] percolation model gave birth as a model for disordered mediums. In general, percolation theory is divided to two models called *discrete percolation* [22] and *continuum percolation* [18].
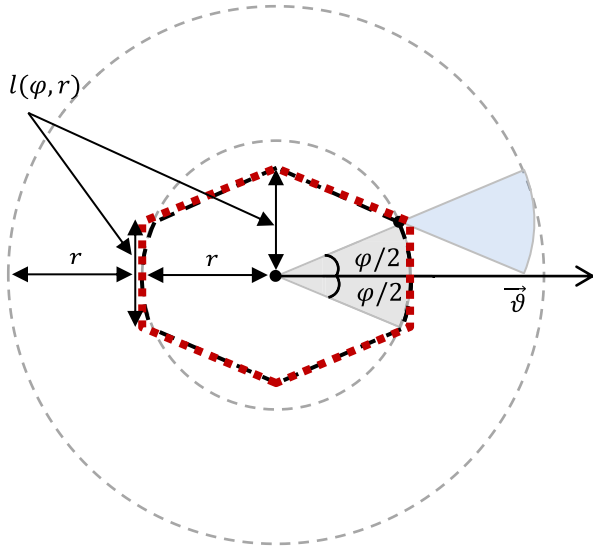
Fig. 4. Excluded area for field-of-view angle (φ) equal to $\pi/4$. The real area has been shown by dashed line and the approximation by dotted line.

Fig. 5. Excluded area for field-of-view angle (φ) equal to $\pi/2$. The real area has been shown by dashed line and the approximation by dotted line.

In *discrete percolation* (also called the lattice model), the sites of the lattice are close or open due to probability *p* and may have different tessellation such as square, triangle, honeycomb and etc. While in *continuum percolation*, the positions of the sites are randomly distributed and thus, there is no need to have a different analysis for each of these regular lattices. While in *discrete percolation* theory, we are interested in finding the *critical probability* denoted by p_c in which percolation occurs, in *continuum percolation* we are interested in finding the *critical density* denoted by λ_c at which an infinite or large clump of overlapping objects first appears that spans the entire network. The density λ_c is the critical value for the density λ such that there exists no such clump of overlapping objects *almost surely* when $\lambda < \lambda_c$ (the system is said to be in the *subcritical* phase), but it exists *almost surely* when $\lambda > \lambda_c$ (the system is said to be *supercritical*) and we say that *percolation* occurs.

In this paper, we consider a *continuum percolation* model which consists of homogeneous points (representing locations of the sensors) which are randomly distributed in $\mathbb{R}^2$ according to a spatial Poisson point process of density λ.

III. CRITICAL DENSITY IN ALODSNs

A. Critical Density for Sensing Coverage in ALODSNs

1) Excluded Area in ALODSNs

The *excluded area* is defined as the minimum area around an object into which the center of another similar object cannot enter in order to avoid the overlapping of the two objects. The *excluded area* of a shape can be obtained simply by moving one around the other and registering the center of the moving shape [20]. In fact, the *excluded area* of nodes in ALODSNs is the collaboration space between sensing sectors as defined in definition 6.

In this section, we discuss on how we could approximate the *excluded area* of sensing sectors based on their field-of-view angle (φ) and radius (r). Fig. 4 shows the excluded area for
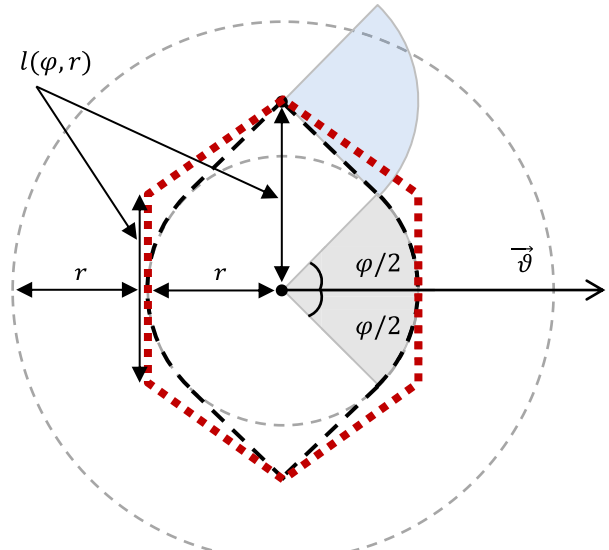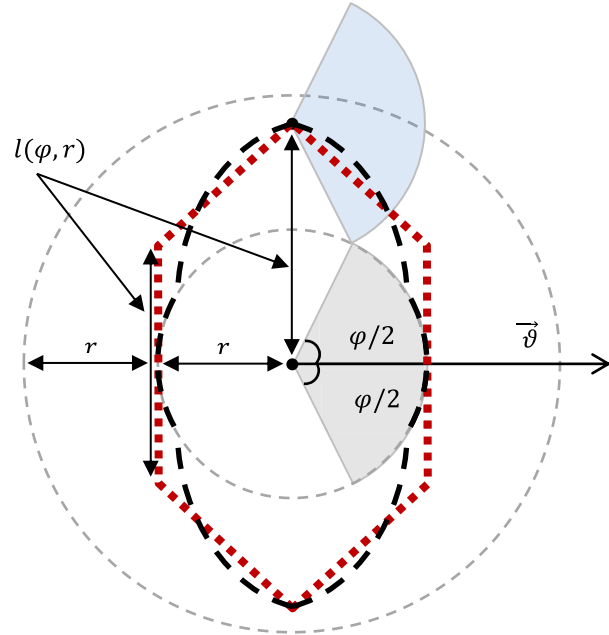

Fig. 6. Excluded area for field-of-view angle (φ) equal to $3\pi/4$. The real area has been shown by dashed line and the approximation by dotted line.

sectors having $\varphi = \pi/4$. The *excluded* area has been shown by dashed line. In this figure, $l(\varphi, r)$ is length of front side of sensing sector and could be calculated by using the law of cosines [23]

$$l(\varphi, r) = \sqrt{2r^2(1 - \cos(\varphi))} \tag{5}$$

Likewise, Fig 5, 6 and 7 show the *excluded area* for field-of-view angle $\pi/2$, $3\pi/4$ and π, respectively. Because we are looking for a general equation to find the *critical density* of different field-of-view angles belong to $[0, \pi]$ and the *excluded areas* show different shapes, we approximate the *excluded areas* to their closest hexagon. The hexagon has been shown by dotted line in Fig. 4 to Fig. 7. Considering the general
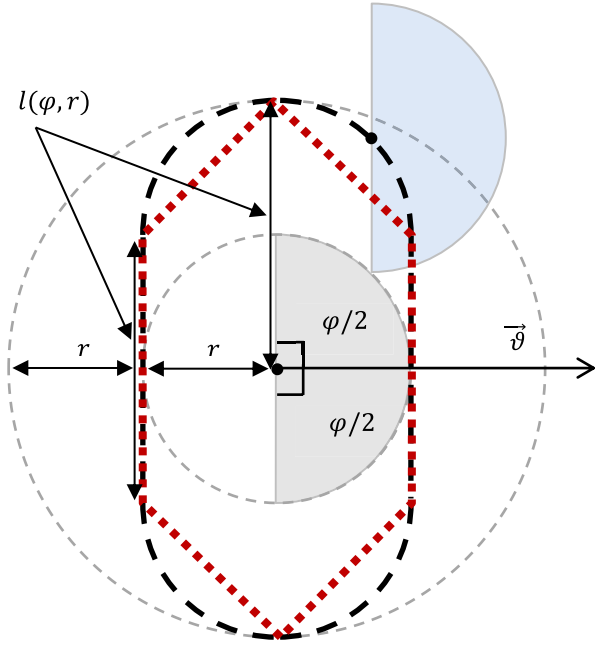
Fig. 7. Excluded area for field-of-view angle (φ) equal to π. The real area has been shown by dashed line and the approximation by dotted line.
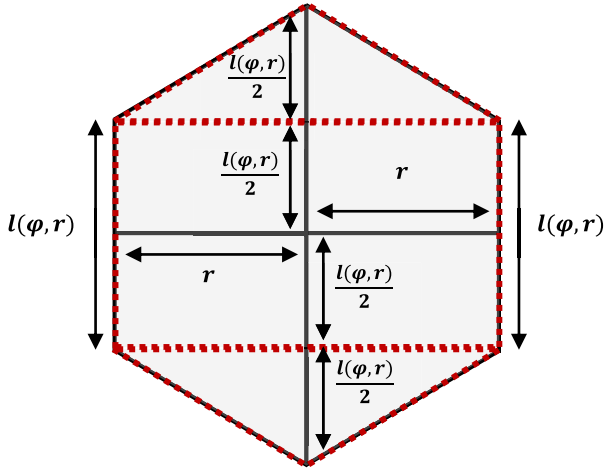

Fig. 8. The general scheme of excluded area approximated to a hexagon.

scheme of the hexagon (see Fig. 8), area of these hexagons simply could be found by

$$\langle \alpha_{\text{ex}}(\varphi, r) \rangle = l(\varphi, r) * (2 * r) + 2 * (\frac{1}{2} * \frac{l(\varphi, r)}{2} * 2 * r)$$
$$= 3 * l(\varphi, r) * r \qquad (6)$$

Table I shows the approximated excluded area for field-of-view angles of Fig. 4 to Fig.7.

2) Critical Density

In this section, we discuss on how we can obtain a general equation for finding infimum *critical density* in ALODSNs based on field-of-view angle which for densities above it SCPT *almost surely* occur.

Finding a general dimensional invariant total excluded area, $\langle A_{\text{ex}} \rangle$, for all different objects and different random orientations, is a fundamental issue which has been attracted researches for several years e.g. [20], [24]–[30]. So far, it is

TABLE I
ESTIMATED EXCLUDED AREA FOR FIELD-OF-VIEW ANGLES
OF FIG. 4 TO FIG. 7

φ	Approximated Excluded Area ($\times r^2$)
π	6.000000
$3\pi/4$	5.543277
$\pi/2$	4.242641
$\pi/4$	2.296101

believed that it must be between 3.2 and 4.5 [20], [27], [30]. But for aligned/parallel objects distributed in a region based on uniform random deployment such as Poisson point process, it is shown that it is equal 4.5 ± 0.1 [20], [24]–[27], [29].

Substituting $\langle A_{\text{ex}} \rangle$ by 4.5, equation (4) could be rewritten as

$$4.5 = \langle \alpha_{\text{ex}} \rangle * \lambda_c \qquad (7)$$

We replace the λ_c by η_c which is independent of radius of sensing sector as explained before. Replacing λ_c by using equation (3), the equation (7) will be changed to

$$4.5 = \langle \alpha_{\text{ex}} \rangle * \frac{\eta_c}{a} \qquad (8)$$

where a is the area of the sensing sector and could be calculated by following equation [31]

$$a = \frac{1}{2}\varphi r^2 \qquad (9)$$

in which r is the radius of sector and $\varphi \in [0, \pi]$ is its angle.

Replacing $\langle \alpha_{\text{ex}} \rangle$ by equation (6) and a by equation (9) and reducing, the equation (8) could be rewritten as

$$\eta_c = \frac{a}{\langle \alpha_{\text{ex}} \rangle} * 4.5$$
$$= \frac{\frac{1}{2}\varphi r^2}{3 * l(\varphi, r) * r} * 4.5$$
$$= \frac{\frac{1}{2}\varphi r^2}{3 * \sqrt{2r^2(1 - \cos(\varphi))} * r} * 4.5$$
$$= \frac{\frac{1}{2}\varphi r^2}{3 * \sqrt{2(1 - \cos(\varphi))} * r^2} * 4.5$$
$$= \frac{\varphi}{6 * \sqrt{2(1 - \cos(\varphi))}} * 4.5$$

Therefore the *critical density* for SCPT could be estimated by an equation independent of radius of sectors

$$\eta_c(\varphi) = \frac{\varphi}{6 * \sqrt{2(1 - \cos(\varphi))}} * 4.5 \qquad (10)$$

where φ denotes the angle of sensing sector.

Afterward, the *filling factor*, ϕ_c, could be found by using equation (2). Table II shows the *critical density* (η_c) and *filling factor* (ϕ_c) for angles of table I.

B. Critical Density for Network Connectivity in ALODSNs

Let $X_\lambda = \{\xi_i : i \geq 1\}$ be a two-dimensional homogeneous Poisson point process of density λ (the number of sensors per unit area), where ξ_i represents the center of the sensor S_i.

TABLE II
η_c AND ϕ_c FOR SCPT FOR ANGLES OF TABLE I

φ	η_c	ϕ_c
π	1.1780970	0.6921360
$3\pi/4$	0.9563725	0.6157156
$\pi/2$	0.8330406	0.5652745
$\pi/4$	0.7696291	0.5368152

The NCPT problem is: *Given a network that is initially disconnected, what is the probability of the first appearance of an infinite (or single large) connected component that spans the entire network?*

As defined in definition 7, The NCPT problem requires that transmission disks be at a distance of at most R from each other. Therefore, two sensors belong to the same communicated component, if R (the radii of the transmission disks) of the sensors are at least equal to greatest distance between nodes. Therefore, due to Fig. 4 to Fig. 7 it is dependent of ratio of R and r. Next, we study the SCPT and NCPT problems together using percolation theory.

C. Integrated Sensing Coverage and Network Connectivity in ALODSNs

To discuss on *critical density* in this section, we consider a correlated model of sensor nodes. In this model, each sensor is associated with a disk with radius R and a sector with radius r which are concentric. This kind of structure reveals a double behavior of the sensors that can be described by their *collaboration* and *communication*. The *collaboration* between sensors depends on the relationship between the radii of their sensing sectors and also the angle φ, whereas *communication* is related to the relationship between the radii of their transmission disks.

Xing et al. [32] proved that if an omnidirectional sensor network is configured to be covered and the radius R of the transmission disk of the sensors is at least double the radius r of their sensing disk, then the network is guaranteed to be connected. Also, in omnidirectional sensor networks, it is a reasonable assumption that the radius of the transmission disks of the sensors cannot be less than the radius of their sensing disks as indicated in [33, Tables 2 and 3] for a wide spectrum of sensor devices. Obviously, both of above-mentioned properties are applicable in ALODSNs too. But we are interested in finding tighter bound for transmission range. Therefore, we focus on $r \leq R < 2r$. In other way, we want to find the *critical density* when $R = \alpha r, 1 \leq \alpha < 2$.

As seen in Fig. 4 to Fig. 7, the maximum distance of centers of two sensors in *collaboration space* (in fact the real *excluded areas* which are shown by dashed lines) could be found by

$$l_{max}(\varphi, r) = \max \{l(\varphi, r), r\} \qquad (11)$$

where $l(\varphi, r)$ is the length of front side to angle φ calculated by equation (5) and r is the radius of sensing sector. Because for maintaining connectivity, the transmission radius (R) must be at least equal to maximum distance between center of
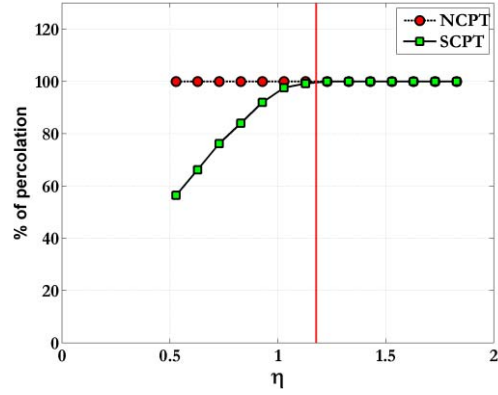

Fig. 9. Percolation chart for ALODSNs with $\varphi = \pi$ in which $\eta_c = 1.1780970$ due to equation (10) and $\phi_c = 0.6921360$ due to equation (2). The vertical line shows the estimated critical density (η_c).
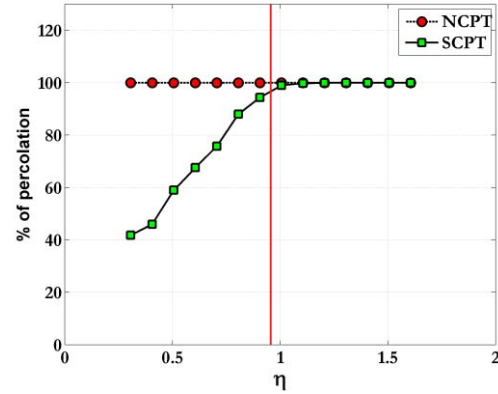

Fig. 10. Percolation chart for ALODSNs with $\varphi = 3\pi/4$ in which $\eta_c = 0.9563725$ due to equation (10) and $\phi_c = 0.6157156$ due to equation (2). The vertical line shows the estimated critical density (η_c).

nodes, we can simply find the coefficient α by

$$\alpha = \frac{l_{max}(\varphi, r)}{r} \qquad (12)$$

where α always belongs to $[1, 2]$, because the greatest value for $l_{max}(\varphi, r)$ is equal to $2r$ (when $\varphi = \pi$) and the lowest value for it will be r due to equation (11).

Therefore, in an ALODSN with *critical density* of nodes for *sensing coverage* as calculated by equation (10), when $R \geq \alpha r$ where α found from equation (12), *almost surely* network connectivity *percolates* too.

IV. NUMERICAL RESULTS

To examine the estimated critical densities, we simulate the percolation phase transition for all different angles of table II. We set the size of simulation environment to a square 50×50, sensing radius (r) to 5 and transmission radius to $R = \alpha r$ where α is given by equation (12). The density interval for simulation has been set to 0.1 and the simulation repeated 500 times for each step. The percolation charts of SCPT and NCPT have been shown in Fig. 9 to Fig. 12. In all charts, the vertical line shows the estimated *critical density* for corresponding φ by using equation (10).
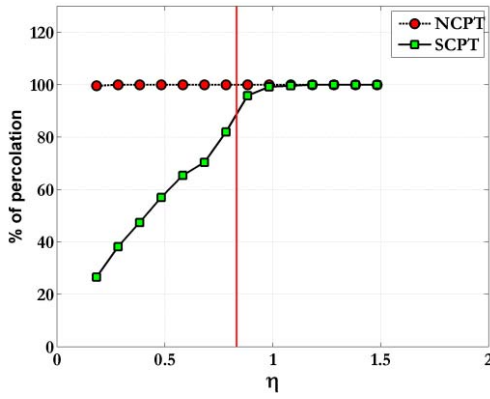
Fig. 11. Percolation chart for ALODSNs with $\varphi = \pi/2$ in which $\eta_c = 0.8330406$ due to equation (10) and $\phi_c = 0.5652745$ due to equation (2). The vertical line shows the estimated critical density (η_c).
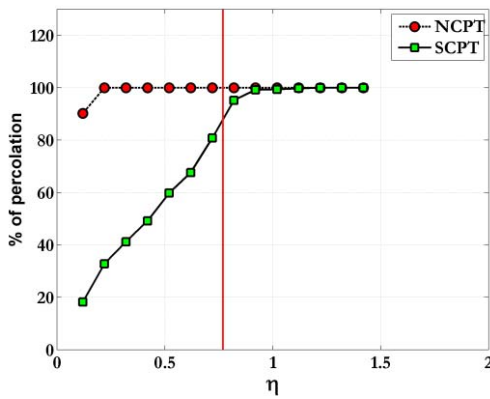

Fig. 12. Percolation chart for ALODSNs with $\varphi = \pi/4$ in which $\eta_c = 0.7696291$ due to equation (10) and $\phi_c = 0.5368152$ due to equation (2). The vertical line shows the estimated critical density (η_c).

As seen, in all charts, the percolation *almost surely* occurs on or very close to estimated *critical densities* which is reasonable for an analytical and general equation. Also, as seen, *network connectivity* will be maintained *almost surely* for all different φ on or after estimated *critical densities*.

V. CONCLUSION AND FUTURE WORKS

In this paper, we introduced aligned-orientation directional sensor networks (ALODSNs) in which the orientation of all sensor nodes is the same and then calculated the estimated critical density of nodes for sensing coverage and network connectivity in such networks for all angles of field-of-view between 0 and π by using continuum percolation. The simulation confirmed that percolation occurs on or very close to the estimated critical densities which are reasonable for a general equation and analytical approach. Manually measuring the excluded areas for different angles between 0 and π could lead to higher accuracy of critical densities.

We also proposed a model for percolation in directional sensor network which provides a basis for solving the problem of finding critical density of nodes for SCPT and NCPT together. We proposed a correlated concentric disk and sector model in order to study SCPT and NCPT problems in directional sensor

networks in an integrated way from a continuum percolation perspective. Precisely, we have considered the physical correlation between them, which is based on the ratio of the radius of the transmission disks of sensors to the radius of their sensing sectors. Thus, when an infinite covered component arises for the first time, an infinite connected component will almost surely appear based on the ratio $\alpha = R/r$.

As future works, we are working on finding critical density of nodes for fixed-orientation directional sensor networks (FIODSNs) in which orientation of nodes independently and uniformly distributed on $[0, 2\pi]$ and is fixed, and also adjustable-orientation directional sensor networks (ADODSNs) in which the deployment is such as FIODSNs but the orientation of sensing sectors could be adjusted after deployment by using a distributed algorithm. Other possible future works include directional sensor networks with directional antenna instead of common disk model e.g. [34], inhomogeneous directional sensor networks and three-dimensional directional sensor networks.

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Mohammad Khanjary received his B.Sc. degree in computer engineering from Amirkabir University of Technology, Tehran, Iran and his M.Sc. degree from Qazvin Branch, Islamic Azad University, Qazvin, Iran in 2004 and 2009, respectively. Currently, He is Ph.D. student of computer engineering at Science and Research Branch, Islamic Azad University, Tehran, Iran. His current research interest includes computer networks and mathematical modelling.

Masoud Sabaei received his B.Sc. degree from Esfahan University of Technology, Esfahan, Iran, and his M.Sc. and Ph.D. form Amirkabir University of Technology (Tehran Polytechnic), Tehran, Iran, all in the field of Computer Engineering in 1992, 1995 and 2000, respectively. Dr. Sabaei has been professor of Computer Engineering Department, Amirkabir University of Technology (Tehran Polytechnic), Tehran, Iran since 2002. His research interests are wireless networks, software defined networks, and telecommunication network management.

Mohammad Reza Meybodi received the BS and MS degrees in Economics from Shahid Beheshti University in Iran, in 1973 and 1977, respectively. He also received the MS and PhD degree from Oklahoma University, USA, in 1980 and 1983, respectively in Computer Science. Currently, he is a full professor in Computer Engineering Department, Amirkabir University of Technology, Tehran, Iran. Prior to his current position, he worked from 1983 to 1985 as an assistant professor at Western Michigan University, and from 1985 to 1991 as an associate professor at Ohio University, USA. His research interests include channel management in cellular networks, learning systems, parallel algorithms, soft computing and software development.