On Channel Assignment and Multicast Routing in Multi-Channel Multi-Radio Wireless Mesh Networks

Abstract

Multicast is a key networking service, enabling one-shot delivery of information from a source to multiple destinations and is considered underlying for collaborating multimedia applications such as video conferencing, distance learning and other forms of content distributing over Multi-Channel Multi-Radio Wireless Mesh Networks (MCMR WMNs). Multicast protocol is designed in these networks, however, is tightly coupled with the specifics of the nodes' channel-radio associations to realize minimum interference communication. The mainstream of research in WMN multicasting is oriented towards heuristic or meta-heuristic strategies which basically take on a sequential approach to solve the channel assignment and the multicast routing as two disjoint sub-problems. The network resultings' configuration would be sub-optimal in this case. It is given that the cross-interaction between the two sub-problems is in effect of problem's specification. In this paper, firstly, we propose a cross-layer mathematical formulation of joint channel assignment and multicast tree construction in MCMR WMNs, which, opposed to the existing schemes and guarantees optimal solution. Simulation results demonstrate that our cross-layer design outperforms the LCA, MCM, the Genetic algorithm-, Simulated Annealing-, Tabu search-based methods, proposed by Zeng et al. (2010) and Cheng et al.(2011) respectively, in the terms of inter-channel interference. Secondly since, joint optimization modeling has been relatively demanding in terms of complexity, we relax the optimality requirement and alternatively exploring the option of a layered formulation in which to ensure an optimal solution for each sub-problem. Our alternative design, is proved superior to the prior art in view of the interference minimization too. We conduct an extensive series of simulations to analyze the optimality and complexity of our two design strategies. The overall result of the interference, is our optimality measurement. Also, complexity is evaluated in terms of the memory consumption as well as the required time to solve the multicast problem.

Keyword

Wireless mesh network, Multi-channel, Multi-radio, Channel assignment, Multicast tree construction, Cross-layer design, Binary integer programming.

1. Introduction

Wireless mesh network (WMN) has been envisioned as the economically viable networking paradigm for scalable QoS-aware delivery of heterogeneous traffic over broadband and large-scale wireless commodity networks [1, 2]. WMNs consist of mesh routers and mesh clients, where mesh routers, as opposed to nodes in a mobile ad hoc network (MANET), have minimal mobility and form the backbone of WMNs. They also provide network access for both mesh and conventional clients [1]. Compared to wireless sensor networks (WSNs), mesh nodes are not energy constrained. However, the main objective in these networks is physical layer capacity maximization through the interference of minimization, which is typically achieved by having each node transceive through multiples radios tuned on multiple channels [3 and 4], thus being a participant to a number of parallel communications [6].

Of the key applications for Multi-Channel Multi-Radio WMNs (MCMR WMNs) are multicast-based systems such as video conferencing, online games, webcast and distance learning, etc. However, while wireless communication is intrinsically apt for performing multicast routing due to the broadcast nature of the air medium, the inter-channel interference in WMNs plays a key factor in determining the actual data rate achievable for a multicast service. The wireless interference occurs when two links within two-hop distance of each other are assigned to the same channel. The interference caused by channel conflict diminishes the performance of the wireless communication dramatically. Therefore, for multicast routing, each link on the multicast tree should be assigned to a channel that its minimal interference is provided. As clearly stipulated in the Protocol model [6,17,43], which is also the interference model underlying the formulation presented in this paper, minimizing the number of interfering links in a wireless mesh network would have a direct impact on throughput maximization, as also corroborated in [31,32,38].

Clearly, multicast routing in WMNs not only the best routing tree's establishment requires but also we assign proper channels to its links. In MCMR WMNs, the problem of the interference minimization is normally tackled with by devising channel assignment schemes in the MAC layer. Efficient channel-radio associations often come at the expense of satisfying more complex connectivity constraints; in particular, for two nodes to be connected, not only should they be within transmission radius of one another, but also that the same channel be assigned to their radios. Multicast routing, under this additional constraint, is even more complicated, since multicast tree now has to be constructed for the channel-radio association which also minimizes the inter-channel interference.

Despite its vast number of applications and practical importance, few works have specifically been targeted at multicast performance optimization in MCMR WMNs. The mainstream of research in this area has considered the channel assignment and multicast routing as two disjoint sub-problems to be solved in sequence [31, 32, 36-38 and 40]; as envisaged in [33-35 and 39], it might even be the cast that the solution for either of these two sub-problems is assumed to be preparatively calculated and given as input to the other. The downside associated with these schemes, however, is that the cross-interaction between the two sub-problems would not be accounted for and that their reliance on heuristic or meta-heuristic initiatives does not come up with the optimal solution.

Networking problems are ideally formulated in mathematical terms so that the resulting configurations and performance tuning parameters can be optimized with ultimate guarantee. However, to date, we are unaware of any published study that has mathematically approached the cross-optimization of channel assignment and multicast tree constructions problems for MCMR WMNs.

In this paper, we propose a cross-layer design for joint optimization of channel assignment and multicast tree construction problems. As opposed to the prior art, the two sub-problems would be solved conjointly and their interplay is thoroughly accounted for. The mathematical formulation adopted in this article is based on a binary integer programming (BIP) model which, compared to the previous heuristic or meta-heuristic-based models, guarantees an optimal solution. We argue that linear programming-based formulations suit the typically low-density and limited scale of WMN settings [20, 21]. Such formulations also come with the advantage of easy extensibility in the sense that their constraint-based problem definition can simply be tailored to fit a new design context. The application of BIP, especially in our cross-optimization setting, yields a channel assignment scheme which fully exhausts the pool of available resources (i.e. channels and radios) as to construct the optimum (viz. minimum interference) multicast routing structure. Our modeling essentially provides a frame of reference to evaluate comparable centralized and/or distributed schemes by given the optimality of the outcome of a BIP formulation.

Simulation results reveal that our proposed cross-layer design outperforms the two heuristic-based schemes discussed in Zeng et al. (2010) [31,32], namely Level Channel Assignment (LCA) and multi-channel multicast (MCM), as well as the Genetic Algorithm- (GA-) , Simulated Annealing- (SA-), and Tabu Search- (TS-) based methods are proposed in [36-38] in terms of interference suffering by the network. Furthermore, since joint optimization modeling is relatively demanding in terms of complexity, we relax the optimality requirement and alternatively explore the option of a layered formulation to ensure an optimal solution for each sub-problem instead. As evidenced by the outcome of the simulations, the interference associated with the layered approach, though marginally higher than that of the cross-layer design, is still significantly smaller compared to those of LCA's, MCM's, GA-, SA-, and TS-based methods. The efficiency of our framework is further investigated by contrasting the characteristics of our layered and cross-layer designs in terms of both optimality and complexity. The metrics used to measure the optimality of the two proposed approaches includes the interference suffered by the network. On the other hand, complexity is measured in terms of the memory consumption as well as the required time to solve the multicast problem. In addition, time complexity of two proposed methods is analyzed.

We also explicitly address the *hidden channel problem* [40] which occurs when two nodes, within a two-hop distance and tuned on the same channel, attempt to send or receive data simultaneously.

The reminder of this paper is organized as follows: In section 2, we briefly review the existing multicast routing methods in MCMR WMNs, highlighting their strengths and shortcomings. Details of the mathematical formulations for the proposed layered and cross-layered schemes are presented in section 3. In section 4, the issues of connectivity and loop occurrence are investigated for our framework. Section 5 is dedicated to a comparative simulation study together with the analysis of the optimality and complexity of our solutions. In section 6, two proposed designs are further investigated with respect to time complexity and memory demands. We conclude the paper in 7.

2. Related Works

As also pointed out in the Introduction, in order to achieve high performance in WMNs, interference should be reduced. Of the most significant interference reduction techniques is channel assignment that specifies which channel should be assigned to a radio in a way with the least contribution to the overall interference. Channel assignment for unicast routing in MCMR WMNs has been addressed extensively in the literature (e.g. [7-17]). The mainstream of research in this area can be classified into two categories. The former is disjoint in which channel assignment is performed on a given routing topology [9, 12 and15] or routing is accomplished over a given channel assignment scheme [8, 10, 11 and 14]. The latter is joint methods [13 and 17].

Obviously, unicast-based implementations are not readily applicable or at least scalable enough to be employed in the one-to-many paradigm of a typical multicast communications setting. Moreover, given the bandwidth-constrained operation of wireless networks, the existing wireline multicast solutions cannot be ported to mesh systems without fundamentally changing their behavior to reduce overhead. Multicasting in MANETs and WSNs also address route recovery and energy concerns respectively which are characteristically different from the pivotal issues of throughput and interference raised in the middle-layer of MCMR WMNs. Routing in these networks is further complicated, given that the multiple radios on each node may dynamically switch on different channels. WMN-based multicasting has been discussed in [18, 19, 22-27 and 41], albeit for single-channel single-radio scenarios. The work in [28], on the other hand, targets at multi-channel single-radio settings, which characterize significantly different network configurations; also, the major emphasis in [28] is placed on throughput maximization of which multicast tree construction is not a necessity, as opposed to the problem of interest in this paper which essentially narrows in joint channel assignment and multicast tree construction with minimal interference.

In [30], multicast throughput optimization in MCMR WMNs is modeled in terms of an integer linear programming (ILP) formulation. To achieve the optimal result, the interplay between channel assignment in the MAC layer and multicast routing in the network layer should be accounted [27] that it has been neglected in [30]. Moreover, unlike the case in our design, multicast tree construction does not form the mainstay of [30].

In our brief review of the relevant literature, we specifically focus on studies addressing both channel assignment and multicast tree construction in MCMR WMNs. Existing schemes may be roughly classified into the following three categories; Methods taking channel assignment for granted, thus treating multicast tree construction as the main problem [39]. Methods assuming a given multicast tree, thus solving for channel assignment as the core problem [33-35] and finally, methods sequentially solving for both multicast tree construction and channel assignment [31, 32, 36-38 and 40].

In [31] and its extended version in [32], two methods for multicast tree construction and channel assignment in MCMR WMNs have been proposed. In the first method, mesh nodes are initially visited by conducting a BFS starting from the multicast source. This way, nodes are placed at different levels from source to multicast group members. Forwarding nodes in the multicast tree, on the other hand, are specified by taking on a bottom-up approach: if each receiver node v has several parents and one of their parents is on the multicast tree, this receiver is connected to that parent (f_v). Otherwise, one of the parent nodes is selected randomly and one link is established to that parent (f_v). The algorithm for node f_v would continue recursively. After constructing the multicast tree, a so-called 'LCA' (Level Channel Assignment) algorithm is used which assigns channels to nodes depending on to what level of BFS traversal tree they belong. In particular, channel *i* is assigned to nodes located at level *i* of the tree. While the main advantage of this scheme lies in its simplicity, it also comes with the following shortcomings; When presented with multiple candidates, a multicast receiver has to be content with a parent chosen at random, which may not always work out to be the most promising choice. Also, there is no discussion as to which receiver node should initiate tree construction, which is despite the implications it might have on the specifics of the resultant tree structure. As for channel assignment, if the number of channels is more than the number of levels, the pool of available channels will be left underutilized.

The second method, namely 'MCM' (Multi-Channel Multicast) also starts with placing the tree nodes at different levels using BFS. The minimum number of relay nodes (RNs), which form the multicast tree, would be determined according to the following approximation algorithm; Parents can be chosen as RN if one of their children has a fewer number of parents. When presented with a number of RNs, the node with the largest number of children is selected. Then, the elected RN together with its children is removed from the tree and the previous two steps would be repeated until all nodes at level *i+1* are removed.
Channel assignment for 'MCM', on the other hand, can be performed by either of the following two algorithms reported in [32]: The top-down 'Ascending Channel Assignment'(ACA) assigns channels to levels starting from zero. When running out of channels, the algorithm re-assigns channel zero to the nodes of the next level and this process repeats. The shortcomings associated with ACA are as follows; *under-utilization* may occur in case the number of sibling nodes is not equal in the whole tree. Also, ACA is prone to *Hidden channel problem* [40]. Furthermore, for almost diagonal trees, ACA is subject to the same shortcomings as those of LCA's [32]. The second channel assignment scheme, referred to as 'Heuristic Channel Assignment' (HCA) draws on the channel separating conception which indicates the disparity between two channel numbers. For example, the separation between channels 2 and 5 is 3. In this method when a channel is assigned to node *u*, it should minimize the sum of squares of interference factor [44] between node *u* and all nodes v in its neighborhood. However, since it has only account single-hop neighbors, the *hidden channel problem* may occur.

The *hidden channel problem* of [32] has motivated the work in [33] extended later in [34]. In order to reduce interference, a fitness function has been proposed to evaluate assignment of channel *c* to node *v*. The *hidden channel problem* is dealt with factoring the channel information of nodes within two hops into the objective function. However, it is assumed in [33 and 34] that the multicast tree is readily available. It also relies on heavy broadcast message exchanges.

Another evaluation function is utilized in [35]; in particular, the assignment of channel c to node v is evaluated based on the probability of packet transmission by neighboring nodes of v on channel c. However, the specifics of the computation of this probability are not elaborated in [35].

In [36-38] three methods based on Genetic algorithm, Simulated Annealing, and Tabu search techniques have been proposed. In these methods, each multicast tree is represented by a two dimensional array (chromosome) the rows of which determines a path from multicast source S to the receiver R_i. Basically, a chromosome has K rows for K receivers. The number of columns in the chromosome is equal to the maximum path length from S to the multicast group members. The representation of the chromosomes is based on the IDs of the nodes lying along the path from sender S to the multicast group members. In this method, chromosomes are created as follows: The algorithm begins from the multicast source node S. A one-hop neighbor of S is randomly selected and its ID is inserted into the chromosome. This process continues till it reaches a receiver R_i. Thus, a row of chromosomes is made. The same process also runs for the next receiver (within the next row). After constructing the multicast tree for each row of the chromosome, which is essentially a path from S to R_i, the channels are sequentially assigned to the edges. The following disadvantages can be pointed out for these methods; first, assignment of the channels to each row of the chromosome is subject to similar shortcomings as those of the LCA algorithm's; i.e. the *hidden channel problem* is not accounted. Second, the nodes within the same level in the multicast tree are prone to interference. Finally, if the number of channels is more than the number of multicast tree levels, some channels will not be used at all.

The methods proposed in [31-38] either construct a multicast tree or have assumed the tree is already constructed and then channels are assigned. In [39], however, the channel assignment is supposed to be done a priori and instead, the multicast tree should be constructed. More specifically, a centralized multicast tree algorithm, namely 'Minimum Number of Transmission' (MCMNT), has been proposed which aims to minimize the number of packets copied on to different channels in each node. The objective is to construct a tree with minimum cost using either the Dijkstra or the Bellman_Ford algorithm.

In [40], a distributing bottom-up approach has been proposed to construct a multicast tree and to establish channel-radio associations. Initially, an approximation algorithm is used to generate the minimum RN set and to construct the multicast tree; Each node identifies its two-hop neighbors with the least number of parents. Then, some of the parents of two-hop neighbors with the fewer children are candidate. Candidates with the best link quality will be selected. Afterwards, the selected nodes and all of their children are removed from the single-hop and two-hop nodes' list. This algorithm continues until both single-hop and two-hop neighbors' list would be empty. However, it is subject to both *hidden channel problem* and heavy broadcast message exchanges for placing nodes across levels.

Table 1 recapitulates our review of the existing methods with respect to their underlying ideas for both multicast tree construction and channel assignment. In sum, the major drawback with the existing methods is that they have considered channel assignment and multicast tree construction in the form of two independent issues and thus have taken on an essentially sequential approach to a joint problem. In particular, the cross-interaction between multicasting at the network and channel assignment at the MAC layer is not accounted for in [31, 36-38 and 40], and even in some methods, it is assumed that the solution for these two sub-problems is pre-calculated (e.g., [33-35 and 39]). This is while cross-layer design forms an integral part of a successful WMN-based implementation, that it has been extensively and methodically argued in [27]. Moreover, all the reviewed schemes, except of course for [40], are built around a centralized perspective, and their heuristic mentality is essentially incapable of providing the optimal solution.

Table 1: Existing multicast designs for multi-channel multi-radio wireless mesh networks

Ref.	Multicast tree construction	Channel assignment
[31,32]	A centralized heuristic bottom-up algorithm which utilizes BFS.	LCA: An ascending method.
	MCM: A centralized approximate top-down algorithm.	An ascending method.
		A heuristic method based on channel separation which only considers single-hop interference.
[33,34]	Multicast tree is assumed to be constructed a priori.	M4: An improved version of [32], which also considers two-hop interference.
[35]	Multicast tree is assumed to be constructed a priori.	A greedy channel assignment that utilizes BFS.
[36-38]	A centralized scheme based on GA, SA, and TS techniques.	An ascending method.
[39]	MCMNT: A heuristic method for computing edge cost, coupled with minimum cost tree construction using either the Dijkstra or the Bellman-Ford algorithms.	Channel assignment is assumed to be performed a priori.
[40]	An improved version of MCM which also factors in the link quality.	A heuristic algorithm.

Considering the above weaknesses, in what follows, we mainly focus on achieving the optimal solution for joint channel assignment and multicast tree construction problem in MCMR WMNs. For this, we present two mathematical BIP-based formulations which are inherently centralized. The former solves the problem via a cross-layer design and the latter is a layered protocol which provides the optimal solution for each sub-problem.

3. Mathematical Framework

The mathematical framework presented in this article is basically a BIP formulation, which opposed to the existing heuristic- or meta-heuristic-based schemes, guarantees the optimality of the solution. Given a problem definition in terms of a set of linear equality/ inequality constraints, a BIP model, as a variant of linear programming, determines a way to obtain the best feasible solution. From the geometric viewpoint, a feasible region, in the form of a convex polyhedron, is defined with the linear constraints. Within this region, provided that a feasible solution exists and also under the condition that the linear objective function is bounded, the optimum result is always achievable on the boundary of optimal level-set by the maximum/minimum principle for convex/concave functions [45]. Given the wholeness and properness of the set of constraints formulating the problem of interest, BIP guarantees global optimality of the resulting solution.

3.1 System Model and Assumptions

In this section, we present the assumptions of our mathematical formulations for the problem of multicast tree construction with minimal interference. We have formulated our BIP models to find the MCT-based multicast tree. As for interference computation, like [31-40] we have adopted the protocol model described in [41 and 43]. In this model, a given transmission from a node *Src* to a node *Des* is said to be successful only if: (1) the nodes' distance is less than the transmission range, and: (2) no third node, located within the interference range of the receiving node *Des*, is transmitting. The adopted model can further be refined to comply with IEEE 802.11-style MAC protocols; i.e. the sending node *Src* is also required to be free of interference as it needs to receive the link layer acknowledgement from the receiving node *Des*. Specifically, any node *Temp*, which is within the interference range of *Src* or *Des*, should not be transmitting.

Other assumptions include: all mesh routers are distributed randomly on a plane. Each router is equipped with multiple radio interfaces, and the number of radios is not more than that of the available non-overlapping channels. All radio interfaces on wireless routers make use of omni-directional antennas, and have identical transmission / interference ranges.

3.2 Proposed Methods

As previously mentioned, in the existing methods the cross-interaction between multicast at the network and channel assignment at the MAC layer has not been considered. Therefore, in this section we present a BIP based formulation in which both multicast tree construction and channel assignment problems are solved conjointly. We called this method "cross-layer" design. Afterwards, we will propose another formulation, namely "Sequential" design in which the optimal solution for each sub–problem is provided.

Before we present the formulations, common definitions for both methods should be described. Tables 2 and 3, show the common defined sets and parameters for both proposed Sequential and cross-layer methods. Necessary descriptions are included in the tables too.

Table 2: Common defined sets for both Sequential and cross-layer methods

$Nodes = \{N_1, N_2, N_3, .., N_n\}$	*mesh routers*
$ChannelList = \{C_1, C_2, C_3, \dots, C_c\}$	*channels*
$MulticastSource = \{N_i\}, N_i \in Nodes$	*multicast sources*
$MulticastGroup = \{T_1, T_2, T_3, .., T_t\}, T_i \in Nodes$	*multicast group*
$Radio = \{R_1, R_2, R_3, .., R_R\}$	*mesh routers' radios*
$R(N_i)$, $N_i \in Nodes$	*node N_i's Radios*

Table 3: Common predefined parameters for both Sequential and cross-layer methods

$G = (V, E)$ /* *A directed graph representing a multi-channel multi-radio wireless mesh network**/
R_T /* *Transmission range* */
$R_I = q \times R_T$, q≥ 1 /* *Interference range* */
$d_{Src,Des}$, $Src, Des \in Nodes$ /* *Euclidean distance between mesh node Src and Des* */
$UDG(Src, Des)$, $Src, Des \in Nodes$ /* *A binary neighborhood matrix named unit disk graph which is computed using R_T and $d_{Src,Des}$ and specifies whether there is a link between Src and Des or not* */
$\|R(Src)\|, Src \in Nodes$ /* *Number of radios for node Src* */
$IsNeigh(Src, Des, Temp1, Temp2)$, $Src, Des, Temp1, Temp2 \in Nodes$ /* *A binary parameter which determines whether Temp1 or Temp2 is in interference range of Src or Des or not* */

A. Proposed Cross-layered method

In this section the required variables used within the BIP cross-layer model followed by the defined constraints are described.

A.1 Defined variables

Before presenting the BIP cross-layer model for the joint multicast tree construction and channel assignment problem, necessary variables should be introduced. In what follows, the variables are discussed.

A binary variable $F_{C_{i,SR,Src}}$ where $Src \in Nodes, SR \in Radios, and\ C_i \in ChannelList$ is defined to determine whether or not the channel C_i is assigned to radio SR of node Src.

A binary variable $Link(Src, SR, Des, DR, C_i)$ such that $Src, Des \in Nodes$, $SR, DR \in Radios, and\ C_i \in ChannelList$ is defined to determine whether or not a link between radio SR of node Src and radio DR of node Des is established on channel C_i.

Two non-negative variables $InputLink(Src)$ and $OutputLink(Src)$ with the following definitions are required to determine the number of incoming / outgoing links to/from mesh router Src, respectively.

$$InputLink(Src) = \sum_{C_i} \sum_{SR} \sum_{TR} \sum_{Temp} Link(Temp, TR, Src, SR, C_i) \quad (1)$$
$$OutputLink(Src) = \sum_{C_i} \sum_{SR} \sum_{TR} \sum_{Temp} Link(Src, SR, Temp, TR, C_i) \quad (2)$$

where $Src, Temp \in Nodes,\ TR, SR \in Radios,\ C_i \in ChannelList$

Also, a non-negative variable $Interference(Src, SR, Des, DR, C_i)$ such that $Src, Des \in Nodes, SR, DR \in Radios, C_i \in ChannelList$ is required to determine the number of interfering links with $link\ (Src, SR, Des, DR, Ch)$.

SIL is another non-negative variable which stands for Sum of Interfering Links and computes the number of links within the interference range of *Src* or *Des* on channel C_i . This variable is mainly introduced to account for *hidden channel* problem:

$$SIL(Src, SR, Des, DR, C_i) = \sum_{Temp2} \sum_{TR2} \sum_{Temp1} \sum_{TR1} [IsNeigh(Src, Des, Temp1, Temp2) \times Link(Temp1, TR1, Temp2, TR2, C_i) - Link(Src, SR, Des, DR, C_i)] \quad (3)$$

where $Src, Des, Temp1, Temp2 \in Nodes, SR, DR, TR1, TR2 \in Radios, C_i \in ChannelList$.

$TotalInterference$ is a non-negative variable which determines total interference across the multicast tree:

$$TotalInterference = \sum_{Des} \sum_{DR} \sum_{Src} \sum_{SR} \sum_{C_i} Interference(Src, SR, Des, DR, C_i) \quad (4)$$

where $Src, Des \in Nodes,\ SR, DR \in Radios, C_i \in ChannelList$.

Finally we define a non-negative variable which determines the number of links forming the multicast tree:

$$TotalLinks = \sum_{Des} \sum_{DR} \sum_{Src} \sum_{SR} \sum_{C_i} Link(Src, SR, Des, DR, C_i) \quad (5)$$

where $Src, Des \in Nodes,\ SR, DR \in Radios, C_i \in ChannelList$

A.2 Defined constraints

In what follows the required constraints in cross-layer design are introduced;

From the multicast tree definition the number of all incoming links to a node except for the multicast source and multicast target should be at most 1. Constraint (6) satisfies this property.

$$InputLink(Src) \leq 1 \qquad , Src \in Nodes \backslash (MulticastSource, MulticastGroup) \quad (6)$$

As it is shown in constraint (7), in a multicast tree with multi-radio nodes, the number of all incoming links to and outgoing links from a node except for the multicast source and multicast target should be at most equal to the number of radios per each node.

$$InputLink(Src) + OutputLink(Src) \leq |R(Src)|, Src \in Nodes \backslash (MulticastSource, MulticastGroup) \quad (7)$$

It is obvious that there are no incoming links to a multicast source. Constraint (8) ensures this feature.

$$InputLink(Src) = 0 \qquad , Src \in MulticastSource \quad (8)$$

In a multicast tree, as shown in constraint (9), the number of outgoing links from the multicast source should be at most equal to the number of multicast targets.

$$OutputLink(Src) \leq |MulticastTarget| \qquad , Src \in MulticastSource \quad (9)$$

The multicast source must have at least one outgoing link. Constraint (10) guarantees this property.

$$OutputLink(Src) \geq 1 \qquad , Src \in MulticastSource \quad (10)$$

Also, all multicast targets should have just one incoming link. Equality constraint (11) ensures this characteristic.

$$InputLink(Src) = 1 \;, Src \in MulticastTarget \quad (11)$$

Similarly, the number of outgoing links from a multicast target should be exactly zero (constraint 12).

$$OutputLink(Src) = 0 \;, Src \in MulticastTarget \; (12)$$

In MCMR WMNs, every radio of a node should be used at most once for either of input or output purposes. Constraint (13) implements this feature.

$$\sum_{Des}\sum_{DR}\sum_{C_i} Link(Src, SR, Des, DR, C_i) + \sum_{Des}\sum_{DR}\sum_{C_i} Link(Des, DR, Src, SR, C_i) \leq 1$$
$$Src, Des \in Nodes\,, \qquad SR, DR \in Radios, C_i \in ChannelList \; (13)$$

Inequalities (14) and (15), together, define an "if and only if" constraint; i.e. if a node has an incoming edge, then it definitely has an outgoing edge as well, and vice versa.

$$OutputLink(Src) \geq InputLink(Src) \quad , Src \in Nodes \; (14)$$

$$\frac{OutputLink(Src)}{|R(Src)|} \leq InputLink(Src), Src \in Nodes \; (15)$$

In MCMR WMNs, every channel C_i should be assigned to radios of a node at most once. Inequality constraint (16) guarantees this property.

$$\sum_{Des}\sum_{DR}\sum_{SR} Link(Src, SR, Des, DR, C_i) + \sum_{Des}\sum_{DR}\sum_{SR} Link(Des, DR, Src, SR, C_i) \leq 1 \quad (16)$$
$$Src, Des \in Nodes\,, \qquad SR, DR \in Radios, C_i \in ChannelList$$

Aforementioned variable $Interference(Src, SR, Des, DR, C_i)$ is computed as Eq. 17.

$$Interference(Src, SR, Des, DR, C_i) = Link(Src, SR, Des, DR, C_i) \times SIL(Src, SR, Des, DR, C_i) \quad (17)$$

That is, if $Link(Src, SR, Des, DR, C_i))$ is established, then its interference is equal to variable *SIL*; otherwise, it is zero. Constraints (18), (19), and (20) correspond to the multiplication of the two variables *Link* and *SIL* in LP terms.

$$Interference(Src, SR, Des, DR, C_i) \leq (|Nodes| \times (|Nodes| - 1)/2 \times Link(Src, SR, Des, DR, C_i) \quad (18)$$

$$Interference(Src, SR, Des, DR, C_i) \leq SIL(Src, SR, Des, DR, C_i) \qquad (19)$$

$$Interference(Src, SR, Des, DR, C_i) \geq SIL(Src, SR, Des, DR, C_i) \text{-} (|Nodes| \times \frac{|Nodes|-1}{2}) \times \big(1 - Link(Src, SR, Des, DR, C_i)\big) \quad (20)$$
$$Src, Des \in Nodes\,, \qquad SR,, DR \in Radios, C_i \in ChannelList$$

Constraints (21) and (22) stipulate the link establishment conditions between two nodes *Src* and *Des*: 1) they are located within the transmission range of each other , and: 2) a common channel is assigned to both radio *SR* of *Src* and radio *DR* of *Des*.

$$Link(Src, SR, Des, DR, C_i) \times d_{Src,Des} \leq Link(Src, SR, Des, DR, C_i) \times R_T \quad (21)$$
$$Link(Src, SR, Des, DR, C_i) \times F_{C_{i,SR,Src}} = Link(Src, SR, Des, DR, C_i) \times F_{C_{i,DR,Des}} \quad (22)$$

where $Src, Des \in Nodes\,, SR, DR \in Radios, C_i \in ChannelList.$

Finally, since the interference is computed from the established links, the objective of this model can be defined:

$$Minimize\ (TotalLinks\ + TotalInterference)$$

Altogether, the BIP formulation for cross-layer design is modeled as follows:

$Minimize\ (TotalLinks\ + TotalInterference)$

Subject to:

$InputLink(Src) \leq 1$	$, Src \in Nodes \backslash (MulticastSource, MulticastGroup)$
$InputLink(Src) + OutputLink(Src) \leq \lvert R(Src) \rvert$	$, Src \in Nodes \backslash (MulticastSource, MulticastGroup)$
$InputLink(Src) = 0$	$, Src \in MulticastSource$
$OutputLink(Src) \leq \lvert MulticastSource \rvert$	$, Src \in MulticastSource$
$OutputLink(Src) \geq 1$	$, Src \in MulticastSource$
$InputLink(Src) = 1$	$, Src \in MulticastTarget$
$OutputLink(Src) = 0$	$, Src \in MulticastTarget$
$\sum_{Des}\sum_{DR}\sum_{C_i} Link(Src, SR, Des, DR, C_i) + \sum_{Des}\sum_{DR}\sum_{C_i} Link(Des, DR, Src, SR, C_i) \leq 1$	$Src, Des \in Nodes\ ,\ SR, DR \in Radios, C_i \in ChannelList$
$OutputLink(Src) \geq InputLink(Src)$	
$\frac{OutputLink(Src)}{\lvert R(Src) \rvert} \leq InputLink(Src)$	$, Src \in Nodes$
$\sum_{Des}\sum_{DR}\sum_{SR} Link(Src, SR, Des, DR, C_i) + \sum_{Des}\sum_{DR}\sum_{SR} Link(Des, DR, Src, SR, C_i) \leq 1$	$Src, Des \in Nodes\ ,\ SR, DR \in Radios, C_i \in ChannelList$
$Interference(Src, SR, Des, DR, C_i) \leq (\lvert Nodes \rvert \times (\lvert Nodes \rvert - 1)/2) \times Link(Src, SR, Des, DR, C_i)$	
$Interference(Src, SR, Des, DR, C_i) \leq SIL(Src, SR, Des, DR, C_i)$	
$Interference(Src, SR, Des, DR, C_i) \geq SIL(Src, SR, Des, DR, C_i) - (\lvert Nodes \rvert \times (\lvert Nodes \rvert - 1)/2) \times (1 - Link(Src, SR, Des, DR, C_i))$	$Src, Des \in Nodes\ ,\ SR, DR \in Radios, C_i \in ChannelList$
$Link(Src, SR, Des, DR, C_i) \times d_{Src,Des} \leq Link(Src, SR, Des, DR, C_i) \times R_T$	
$Link(Src, SR, Des, DR, C_i) \times F_{C_{i,SR,Src}} = Link(Src, SR, Des, DR, C_i) \times F_{C_{i,DR,Des}}$	$Src, Des \in Nodes\ ,\ SR, DR \in Radios, C_i \in ChannelList$

B. Proposed Layered Method

In this design, say Sequential method, we model the overall problem as two separated sub-problems which are solved in sequence. Clearly, in the first phase of the Sequential method MCT based optimal multicast tree is constructed. Afterwards, the resultant tree constructed from the first phase is fed to the channel assignment phase as input. Hereafter, we refer to multicast tree construction model as MT_Model. Similarly, channel assignment model is referred to as CA_Model. The algorithm of the layered design is shown in Fig. 1. In two sub-sections B.1 and B.2 two sub-problems of this approach are discussed in details.

> *1. Preprocessing Phase*
> *1.2 Defining the required Sets presented in table 2*
> *1.3 Defining the required Parameters presented in table 3*
> *2. Solving Phase*
> *2.1 Solve MT_Model*
> *2.2 Compute parameter Link_(Src,Des) using output of MT_Model and consider it as input to CA_Model*
> *2.3 Solve CA_Model*

Fig 1: Algorithm of the proposed Sequential method

B.1 Multicast Tree Construction Sub-problem

B.1.1 Defined variables

In this section required variables within the MT_Model of the proposed Sequential method are introduced as follows:

A binary variable is needed to indicate whether there is a link between radio *SR* of node *Src* and radio *DR* of node *Des* or not. We define this variable as:

$$Link(Src, SR, Des, DR), \quad Src, Des \in Nodes, \quad DR, SR \in Radios$$

Also, a non negative variable which determines the numbers of incoming links to every mesh router is required. For this, we define the variable $InputLink(Src)$ with the Eq. (23).

$$InputLink(Src) = \sum_{SR} \sum_{Des} \sum_{DR} Link(Des, DR, Src, SR)\ , Src, Des \in Nodes,\ DR, SR \in Radios \quad (23)$$

Similarly, we need another non negative variable which determines the number of outgoing links from every mesh router. Thus, we define the variable OutputLink(Src) with definition (24).

$$OutputLink(Src) = \sum_{SR} \sum_{Des} \sum_{DR} Link(Src, SR, Des, DR)\ , Src, Des \in Nodes,\ DR, SR \in Radios \quad (24)$$

Finally, the non negative variable $Total\_Link$ is required to determine total number of links forming the multicast tree:

$$Total\_Link = \sum_{Src} \sum_{SR} \sum_{Des} \sum_{DR} Link(Src, SR, Des, DR)\ , Src, Des \in Nodes,\ DR, SR \in Radios \quad (25)$$

B.1.2 Defined Constraints

In this sub-section, the required constraints for multicast tree construction sub-problem of the proposed Sequential method are discussed;

In case multicast tree, the number of all input links to a node except multicast source and multicast target should be at most 1. Constraint (26) ensures this property.

$$InputLink(Src) <= 1 \quad , Src \in Nodes \backslash (MulticastSource, MulticastTarget) \quad (26)$$

In MCMR WMNs, sum of all incoming links to and outgoing links from a node except multicast source and multicast target should be at most equals to the number of radios. Constraint (27) guarantees this feature.

$$InputLink(Src) + OutPutLink(Src) <= |R(Src)| \quad , Src \in Nodes \backslash (MulticastSource, MulticastTarget) \quad (27)$$

In MCMR WMNs, if two nodes are located in the transmission range of each other, then one link can be established between them (constraint (28)).

$$Link(Src, SR, Des, DR) <= UDG\ (Src, Des) \quad , Src, Des \in Nodes, \quad DR, SR \in Radios \quad (28)$$

Form the multicast tree definition, multicast source node has not any incoming links. The equality constraint (29) satisfies this property.

$$InputLink(Src) = 0 \quad , Src \in MulticastSource \quad (29)$$

Similarly, the number of outgoing links from multicast source node should be at most equal to the number of multicast targets. This feature is ensured by constraint (30).

$$OutputLink(Src) <= |MulticastTarget| \quad , \ Src \in MulticastSource \quad (30)$$

Furthermore, the number of outgoing links from multicast source node should be at least one (constraint 31).

$$OutputLink(Src) >= 1 \quad , \ Src \in MulticastSource \quad (31)$$

The number of incoming links to every multicast target should be exactly one:

$$InputLink(Src) = 1 \quad , Src \in MulticastTarget \quad (32)$$

Equality constraint (33) ensures the multicast targets have not any outgoing links.

$$OutputLink(Src) = 0 \qquad ,Src \in MulticastTarget \quad (33)$$

The proposed formulation should occur guarantees of one-hop loop in multicast tree. The constraint (34) ensures this inherent feature of the constructed tree.

$$UDG(Src, Temp) \times (\sum_{SR}\sum_{TR} Link(Src, SR, Temp, TR) + \sum_{SR}\sum_{TR} Link(Temp, TR, Src, SR) \leq 1 \times UDG(Src, Temp)$$
$$Src, Temp \in Nodes, \;\; TR, SR \in Radios \;\; (34)$$

In MCMR WMNs, every radio of a node has to form at most one link:

$$UDG(Src, Des) \times \left(\sum_{Des}\sum_{DR} Link(Src, SR, Des, DR) + \sum_{Des}\sum_{DR} Link(Des, DR, Src, SR)\right) \leq 1 \times UDG(Src, Des) \qquad (35)$$

Constraints (36) and (37) explain a 'if and only if' condition. That is, if a node has one or more outgoing links, then it should have an incoming link and also if it has an incoming link, then it will have at least one outgoing link:

$$OutPutLink(Src) >= InputLink(Src), Src \in Nodes \backslash (MulticastSource, MulticastGroup) \quad (36)$$

$$\frac{OutPutLink(Src)}{|radios|} <= InputLink(Src), Src \in Nodes \backslash (MulticastSource, MulticastGroup) \quad (37)$$

And finally, the objective of multicast tree construction sub-problem is:

$$Minimize\ Total\_Link$$

In sum, the BIP model for this sub-problem of the proposed layered design is summarized as follows:

$Minimize\ Total\_Link$
$Subject\ to$:

$InputLink(Src) <= 1$	$,Src \in Nodes \backslash (MulticastSource, MulticastGroup)$
$(InputLink(Src) + OutPutLink(Src)) <= \lvert R(Src) \rvert$	$, Src \in Nodes \backslash (MulticastSource, MulticastGroup)$
$Link(Src, SR, Des, DR) <= UDG\ (Src, Des)$	$,Src, Des \in Nodes,\ DR, SR \in Radios$
$InputLink(Src) = 0$	$, Src \in MulticastSource$
$OutputLink(Src) <= \lvert MulticastTarget \rvert$	$, Src \in MulticastSource$
$OutputLink(Src) >= 1$	$, Src \in MulticastSource$
$InputLink(Src) = 1$	$, Src \in MulticastTarget$
$OutputLink(Src) = 0$	$, Src \in MulticastTarget$
$UDG(Src, Temp) \times (\sum_{SR}\sum_{TR} Link(Src, SR, Temp, TR) + \sum_{SR}\sum_{TR} Link(Temp, TR, Src, SR) \leq 1 \times UDG(Src, Temp)$	$, Src, Temp \in Nodes,\ TR, SR \in Radios$
$UDG(Src, Des) \times \left(\sum_{Des}\sum_{DR} Link(Src, SR, Des, DR) + \sum_{Des}\sum_{DR} Link(Des, DR, Src, SR)\right) \leq 1 \times UDG(Src, Des)$	
$OutPutLink(Src) >= InputLink(Src)$	$,Src \in Nodes \backslash (MulticastSource, MulticastGroup)$
$\frac{OutPutLink(Src)}{\lvert radios \rvert} <= InputLink(Src)$	$,Src \in Nodes \backslash (MulticastSource, MulticastGroup)$

B.2 Channel Assignment Sub-problem

After solving the MT_Model, CA_Model, here, it is necessary to define a binary parameter $Link\_$ which is computed using the binary variable $Link\ (Src, Des)$ as output of the first phase (MT_Model). Parameter $Link\_$ is set to one if $Link\ (Src, Des)$ has been established and zero otherwise.

B.2.1 Defined variables

We define four variables within CA_Model of the proposed Sequential design as follows:

A non negative variable *SIL* (Sum of Interfering Links) is needed to compute the number of links within the interference range of source or destination of a link:

$$SIL(Src,Des,C_i) = Link\_(Src,Des) \times (\sum_{Temp1} \sum_{Temp2} (IsNeigh(Src,Des,Temp1,Temp2) \times Link\_(Temp1,Temp2) \times Assign(Temp1,Temp2,C_i)) - AssignSrc,Des,Ci) \quad (38)$$

where $Src, Des, Temp1, Temp2 \in Nodes,\ C_i \in ChannelList$

This variable considers *hidden channel* problem as well.

A binary variable $Assign(Src, Des, C_i)$ such as $Src, Des \in Nodes\ and\ C_i \in ChannelList$, is needed to determine whether C_i is assigned to the established link between two nodes *Src*, *Des* or not.

The non-negative variable $Interference(Src, Des, C_i)$, denotes the number of interfering links with link (Src,Des,C_i), and is computed as Eq. 39.

$$\mathrm{Interference(Src, Des, C_i)} = \mathrm{Assign(Src, Des, C_i)} \times \mathrm{SIL(Src, Des, C_i)} \quad (39)$$

Finally, the non negative variable *Total_Interference* is defined to compute the overall interference in the multicast tree:

$$Total\_Interference = \sum_{Src} \sum_{Des} \sum_{C_i} Interference(Src,Des,C_i), Src,Des \in Nodes,\ C_i \in ChannelList \quad (40)$$

B.2.2 Defined Constraints

The required constraints for CA_Model of the proposed Sequential method are discussed as follows:

Just one channel should be assigned to every link in MCMR WMNs (constraint (41)).

$$\sum_{C_i} Assign(Src,Des,C_i) = Link\_(Src,Des) \qquad ,Src,Des \in Nodes,\ C_i \in ChannelList \quad (41)$$

If a link is established and channel C_i is assigned to it, then its interference is equal to variable *SIL*; otherwise, it is zero. Constraints (42), (43), and (44), correspond to the multiplication of the two variables *Link* and *SIL* in LP terms.

$$UDG(Src,Des) \times Link_{Src,Des} \times Interference(Src,Des,C_i) \leq \left(\left(\sum_{temp1} UDG(Src,temp1) \times \left(\sum_{temp1} UDG(Src,temp1) - 1\right)\right) + \left(\sum_{temp2} UDG(Des,temp2) \times \left(\sum_{temp2} UDG(Des,temp2) - 1\right)\right)\right) \times Assign(Src,Des,C_i) \times UDG(Src,Des) \times Link_{Src,Des} \quad (42)$$

$$UDG(Src,Des) \times Link_{Src,Des} \times Interference(Src,Des,C_i) \leq SIL(Src,Des,C_i) \times UDG(Src,Des) \times Link_{Src,Des}, \quad Src,Des \in Nodes, \quad C_i \in ChannelList \quad (43)$$

$$\begin{aligned} UDG(Src,Des) \times Link\_(Src,Des) \times Interference(Src,Des,C_i) \\ \geq (SIL(Src,Des,C_i) - (\sum_{temp1} UDG(Src,temp1) \times (\sum_{temp1} UDG(Src,temp1) - 1)) + (\sum_{temp2} UDG(Des,temp2) \\ \times (\sum_{temp2} UDG(Des,temp2) - 1))) \times (1 - Assign(Src,Des,C_i))) \times UDG(Src,Des) \times Link\_(Src,Des) \\ Src,Des,temp1,temp2 \in Nodes,\ C_i \in ChannelList \quad (44) \end{aligned}$$

And finally, the objective of CA_Model is:

$$Minimize\ \ Total\_Interference$$

Therefore, BIP model for channel assignment sub-problem of the layered scheme is represented as follows:

Minimize Total_Interference
Subject to:

$$\sum_{C_i} Assign(Src, Des, C_i) = Link\_(Src, Des) \quad , Src, Des \in Nodes,\ C_i \in ChannelList$$

$$UDG(Src, Des) \times Link\_(Src, Des) \times Interference(Src, Des, C_i) \leq ((\sum_{temp1} UDG(Src, temp1) \times (\sum_{temp1} UDG(Src, temp1) - 1)) + (\sum_{temp2} UDG(Des, temp2) \times (\sum_{temp2} UDG(Des, temp2) - 1))) \times Assign(Src, Des, C_i) \times UDG(Src, Des) \times Link\_(Src, Des)$$

$$UDG(Src, Des) \times Link\_(Src, Des) \times Interference(Src, Des, C_i) \leq SIL(Src, Des, C_i) \times UDG(Src, Des) \times Link\_(Src, Des)$$
$$Src, Des \in Nodes\ ,\ C_i \in ChannelList$$

$$UDG(Src, Des) \times Link\_(Src, Des) \times Interference(Src, Des, C_i) \geq (SIL(Src, Des, C_i) - (\sum_{temp1} UDG(Src, temp1) \times (\sum_{temp1} UDG(Src, temp1) - 1)) + (\sum_{temp2} UDG(Des, temp2) \times (\sum_{temp2} UDG(Des, temp2) - 1))) \times (1 - Assign(Src, Des, C_i))) \times UDG(Src, Des) \times Link\_(Src, Des)$$
$$, Src, Des, temp1, temp2 \in Nodes,\ C_i \in ChannelList$$

4. Connectivity and Loop Considerations

We tend to demonstrate the immunity of our formulation to loop formations as well as to ensure connectivity that is preserved with the resulting channel-radio assignments. Our essential reason for the worst case is also evidently applicable to the average scenario.

4.1Connectivity

We examine the multicast tree connectivity in the proposed cross-layer method. A similar line of reasoning is also applicable to the case of our Sequential design.

Definition 1. Connectivity is satisfied only if there exists a path between the multicast source to all multicast group members.

Property 1. The BIP formulation, given in section 3, guarantees connectivity across the multicast tree.

Proof: We demonstrate the notion of connectivity using the *unit disk graph* depicted in Fig. 2-(a) without loss of generality. In this graph, each link between two nodes indicates that they are located within the transmission range of each other. Two inter linked nodes would be able to communicate if only identical channel numbers get assigned to one of their radios. MS denotes the multicast source and MT stands for the multicast target.

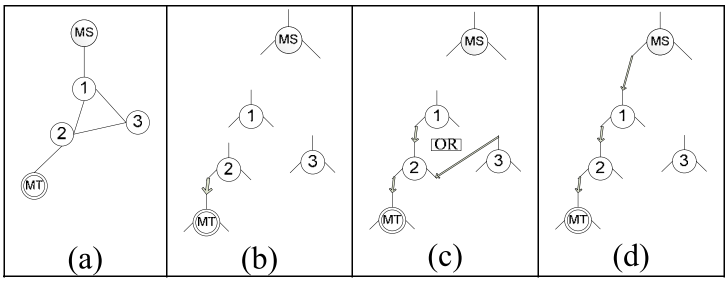


Fig. 2: Multicast tree connectivity

Constraint (11) ensures that MT has exactly one incoming link, as illustrated in Fig. 2-(b). Constraints (14) and (15) stipulate that in case a mesh node has an outgoing link then it ought to have an incoming link as well. Inequality (6) requires that only one of the links (1, 2) or (3, 2) be established as demonstrated in Fig. 2-(c). On the other hand, constraint (10) warrants that MS has at least one outgoing link. Therefore, mesh node 1 has to be necessarily associated with an output, and the outgoing link from node 3 should be removed (Fig. 2-(d)). Hence, there exists a path from MS to MT.■

4.2 Loop Occurrence

We investigate the correctness of our proposed formulation for the proposed cross-layer method in terms of the loop formation issue through the following property. Here again we have considered the worst case scenario. A similar analysis applies to the Sequential scheme.

Property 2. The multicast tree associated with the BIP formulation, given in section 3, is guaranteed to be loop-free.

Proof: Given the *unit disk graph* depicted in Fig. 2-(a), assume that there exists a loop, say (1, 2, 3, 1) within the multicast tree Fig. 3-(a). Constraints (9) and (10) ensure that MS has at least one outgoing link Fig. 3-(b). Inequality (6) requires that link (3, 1) be removed from the configuration (Fig. 3-(c)). Also, constraints (14) and (15) warrant that node 3 have no incoming link (Fig. 3-(d)). Finally, Constraint (11) stipulates that MT have exactly one incoming link (Fig. 3-(e)). ■

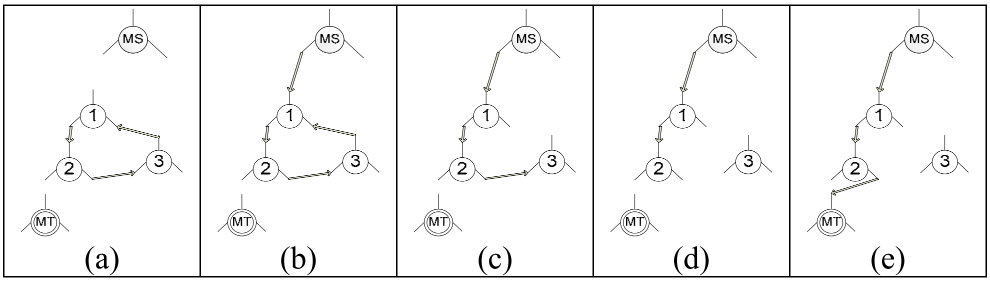


Fig. 3: Multicast tree loop prevention

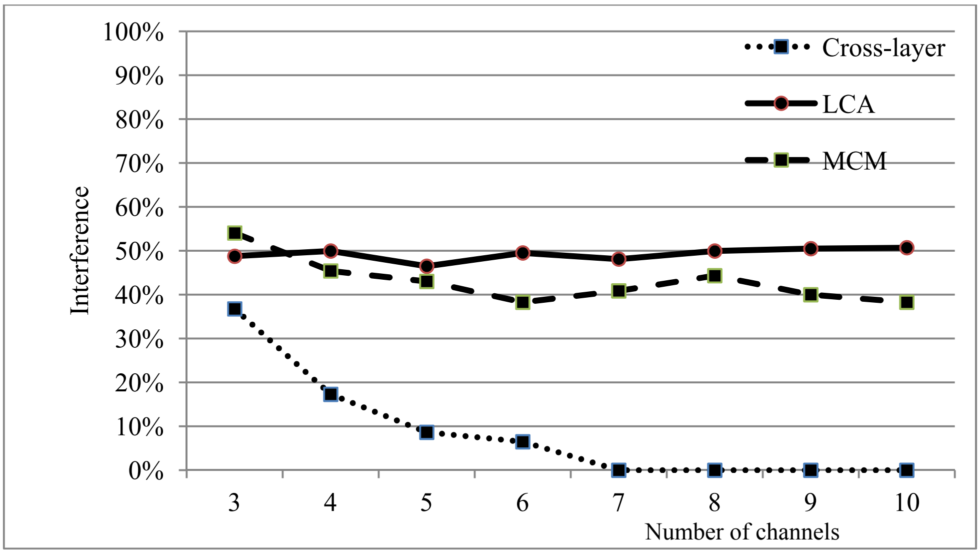
5. Simulation Results

We implemented our BIP framework using CPLEX 11 (for more information about CPLEX, see [46]). Also, to evaluate the performance of the proposed methods several simulations have been conducted. Simulation 1, was conducted to observe the impact of channel number variations on the overall interference in our cross-layer design as compared with LCA, MCM [31,32] and Genetic Algorithm (GA)-, Simulated Annealing (SA)-, and Tabu search-based [36-38] methods. In simulation 2, proposed cross-layer and Sequential methods are evaluated in terms of optimality, say overall interference, in one hand and complexity, say required time to solve the multicast problem and memory consumption, on the other hand. Our Sequential method is compared with LCA, MCM, GA-, SA-, and TS-based in the simulation 3. Simulation 4 is dedicated to compare resultant interference for different methods considering radio number variations. All simulations were conducted on a dense random generated *unit disk graph* in which the number of nodes and the size of multicast receivers set are 30 and 13 respectively. Simulations are described as follows.

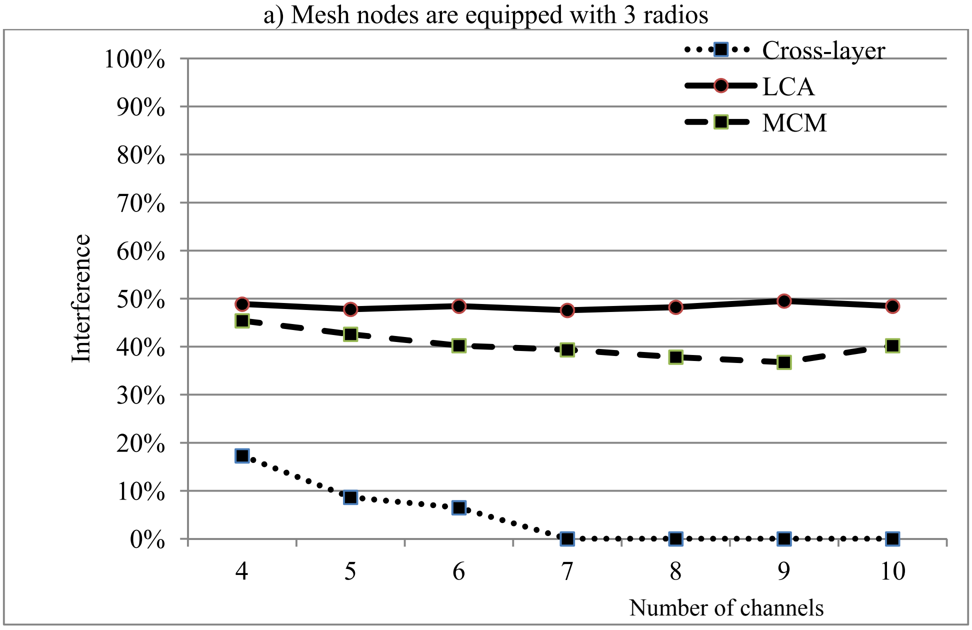
Simulation 1:

In this simulation we study the resultant interference in the proposed cross-layer design as compared with LCA and MCM as well as GA- , SA-, and TS-based methods. Recall from section 1 the number of channels in MCMR WMNs should be more than or equal to the number of radios. Therefore, for a network with *n-radio* nodes a set of simulations for n through m channels should be conducted ($m \geq n$), where m is a point in which the interference reaches to zero. Figures 4-(a) through 4-(f) show the resultant interference in different methods when the mesh nodes are equipped with different number of radios. In figures 4-(a) to 4-(e), every curve is the results of a simulation set not just one simulation. In other words, every data point shows the result of a simulation in a simulation set. Each data point associated to LCA, MCM, GA-, SA-, and TS-based methods in this simulation is the average of 20 times simulations. Since utilizing the GA-, SA-, and TS-based methods in a 3-

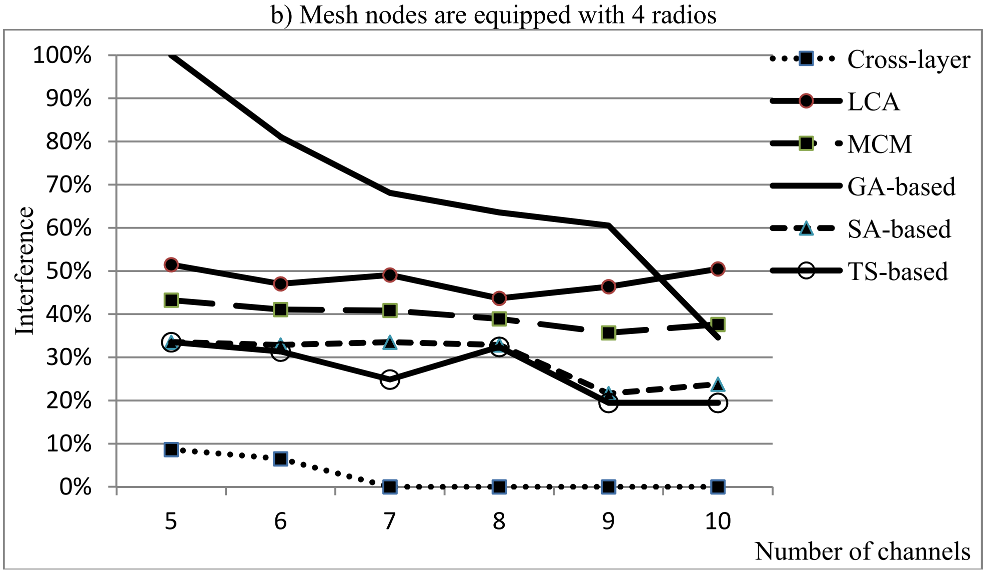
and 4-radios network leads the wireless mesh network to be disconnected, their corresponding results have not been shown in figures 4-(a) and 4-(b).



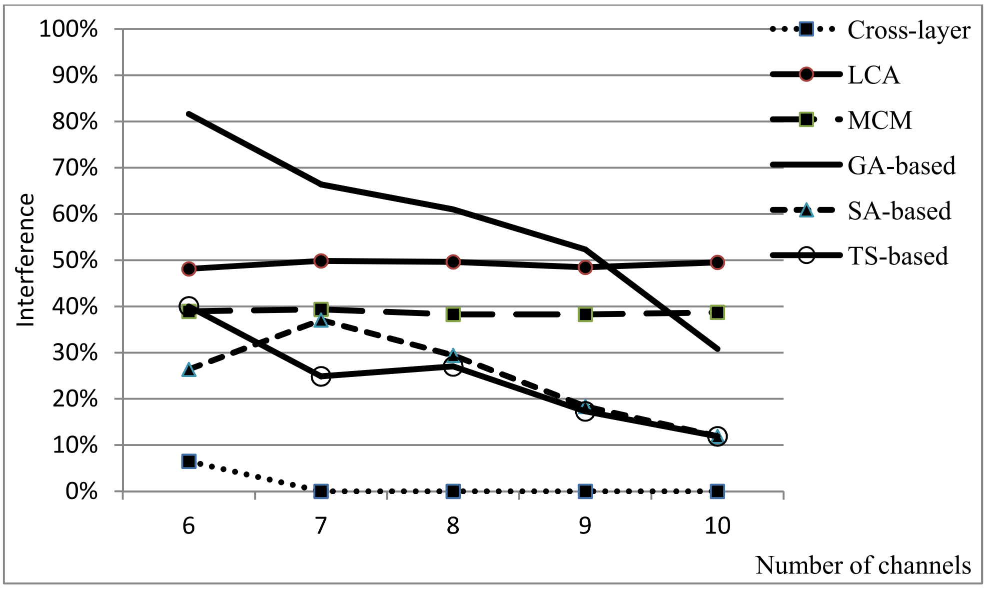
a) Mesh nodes are equipped with 3 radios



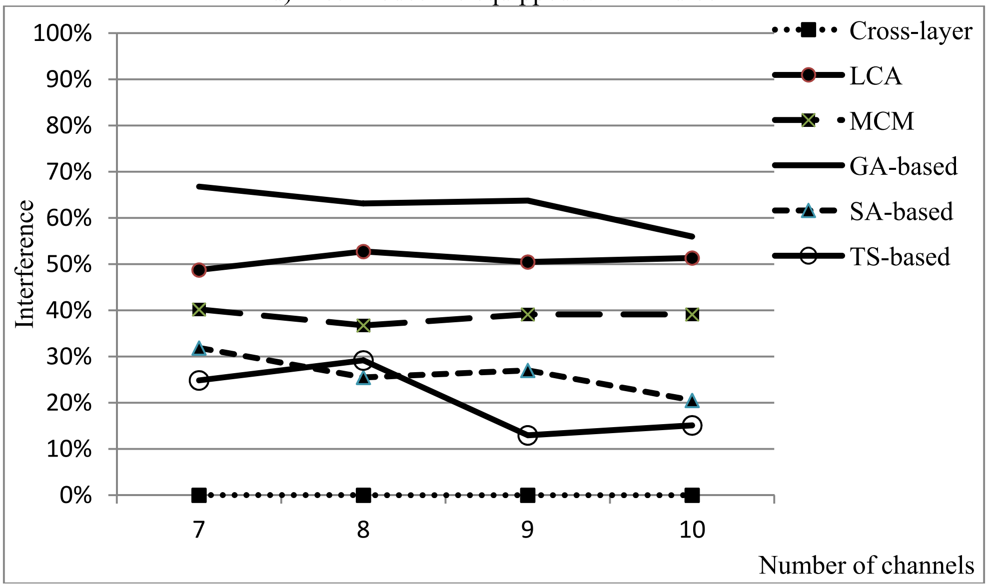
b) Mesh nodes are equipped with 4 radios



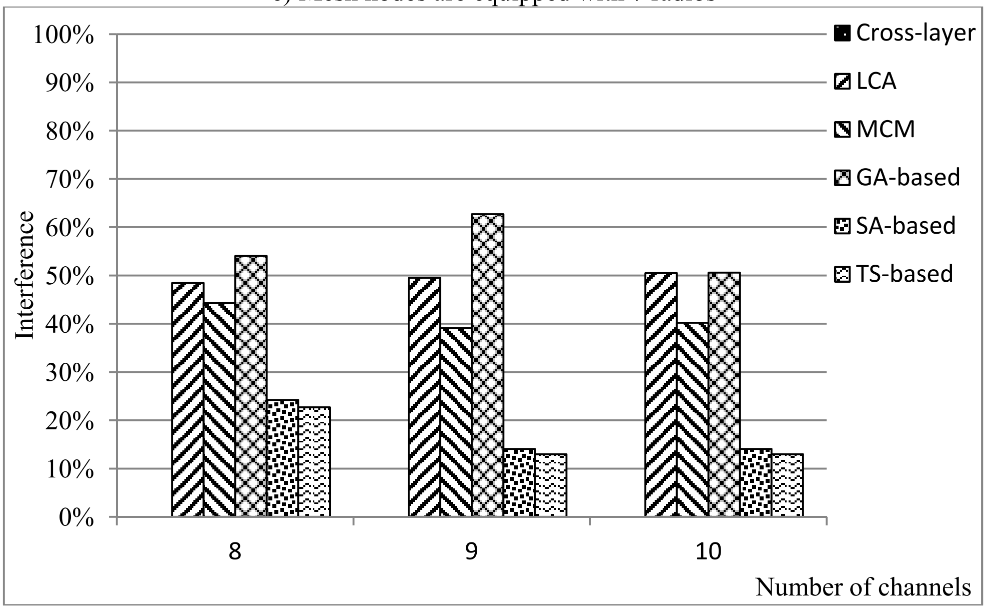
c) Mesh nodes are equipped with 5 radios



d) Mesh nodes are equipped with 6 radios



e) Mesh nodes are equipped with 7 radios



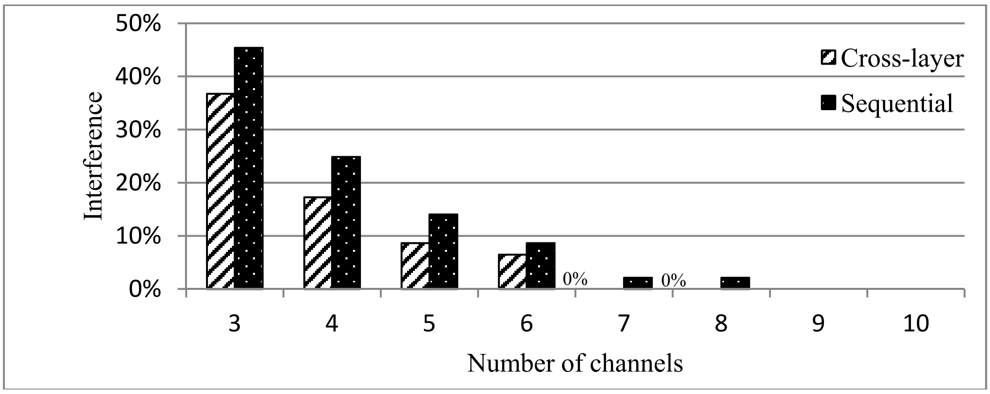
f) Mesh nodes are equipped with 8 radios

Fig. 4: Impact of channel number variations on the overall interference

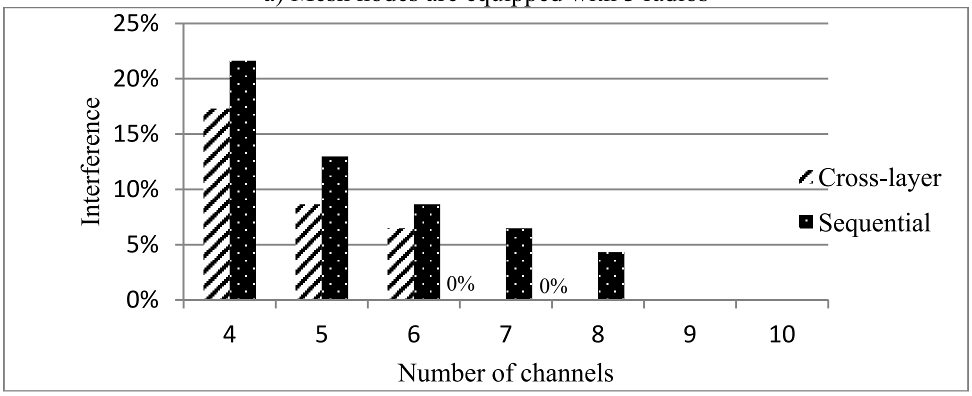
The results demonstrate that in a network with the same number of radios, increasing the number of channels leads to interference reduction in the proposed cross-layer design. From the results shown in all charts of figure 4, as it is expected, our method achieves to minimal interference. The reason is that our cross-layer design is based on BIP formulation in which two sub-problems are solved conjointly. In the other words, the proposed cross-layer design considers the impact of the routing and MAC layers on each other and hence, the optimal solution for the overall problem is provided. From results, the overall interference from running the LCA and MCM has not any specific trend. The reason is that, in these algorithms there is no channel / radio number considerations during multicast tree construction and channel assignment phases. The results obtained from this simulation show that the TS-based method outperforms other heuristic and meta-heuristic based methods in terms of interference. The reason is that using Tabu search technique will result in escaping from the local optima and searching the whole solution space intelligently. Figure 4-(f) shows that applying the cross-layer design in a MCMR WMN with 8-radio nodes, while there is 8, 9, and 10 available channels the interference gets to zero. Other Bar charts in the following simulations can be interpreted analogously.

Simulation 2:

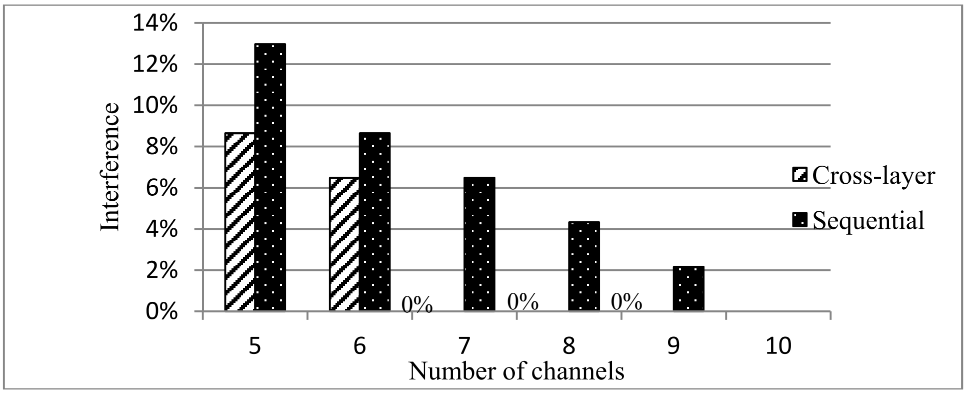
This simulation is conducted to compare the optimality of our proposed cross-layer and Sequential designs in terms of overall interference. The simulations' configuration is the same as simulation 1. As illustrated by Figure 5, the cross-layer method marginally outperforms the Sequential method in terms of interference in all network configurations. The reason is that solving both sub-problems sequentially and achieving the optimal solution for each one does not guarantee the optimal solution of the overall problem.



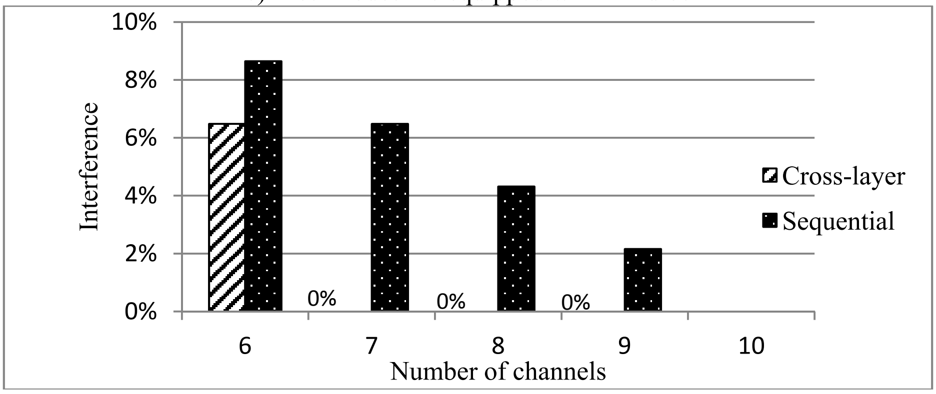
a) Mesh nodes are equipped with 3 radios



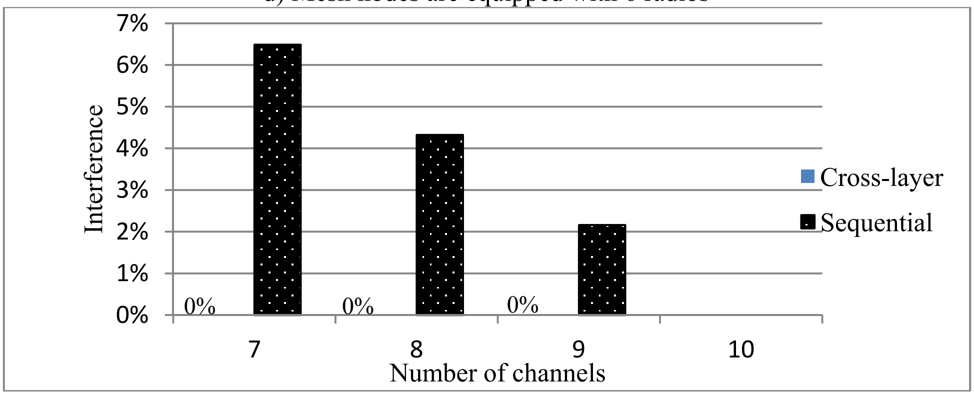
b) Mesh nodes are equipped with 4 radios



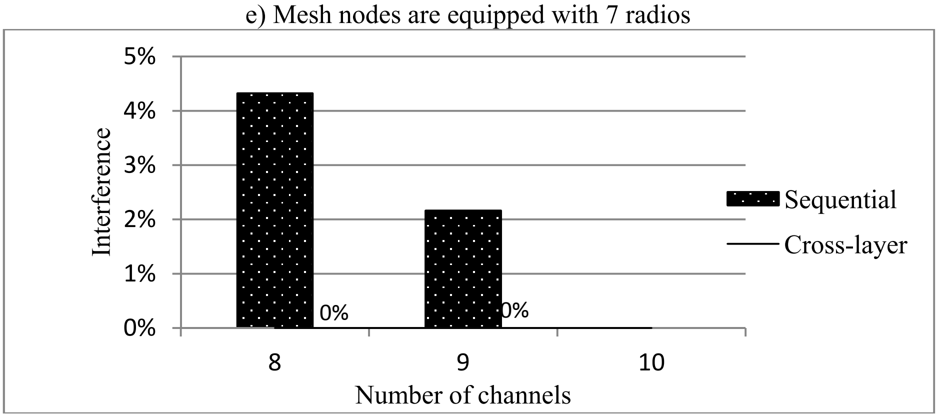
c) Mesh nodes are equipped with 5 radios



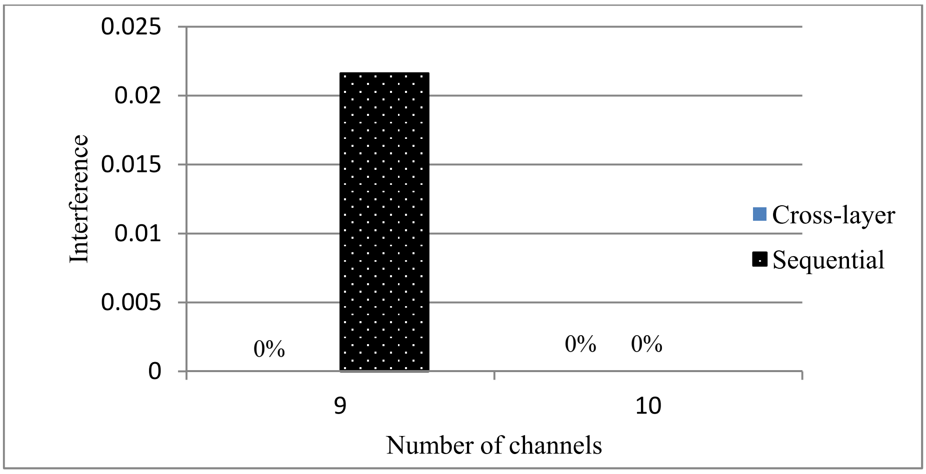
d) Mesh nodes are equipped with 6 radios



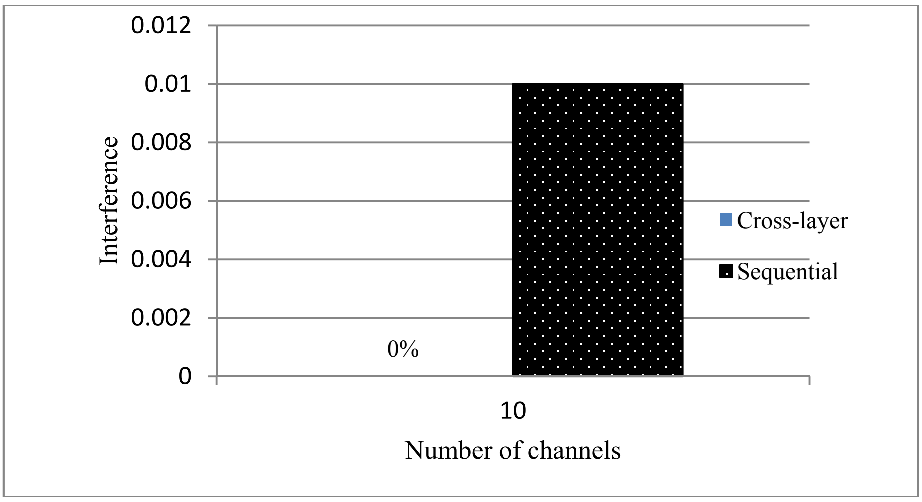
e) Mesh nodes are equipped with 7 radios



f) Mesh nodes are equipped with 8 radios



g) Mesh nodes are equipped with 9 radios



h) Mesh nodes are equipped with 10 radios

Fig. 5: Impact of channel number variations on interference for proposed cross-layer and Sequential methods

In this simulation we compared the complexity of two proposed methods as well. Table 4 shows the percentage of the constraints in both proposed methods. The more number of constraints in a given formulation is, the higher number of comparison operations and hence, the longer it takes to solve the problem. As can be seen, in all network configurations, constraints percentage in Sequential design is less than that of Cross-layer design, resulting in smaller multicast problem solving time.

In this simulation we also recorded the number of required variables to solve the problem. The more variables leads to more memory demands to solve the problem. The results are in figure 6 in which the legend *CnS* denotes that corresponding curve is the results of a simulation set obtained from running the proposed Sequential method where the number of channels is fixed to *n*. Also legends *CnCL* are associated with the results of a simulation set obtained from running the proposed cross-layer design. Our observation is that, the number of variables in Sequential method as compared with cross-layer method is dramatically decreased. Besides, memory management in the cross-layer method makes the processor to be busy and hence, again leads to higher solving time.

Table 4: percentage of the constraints in both proposed methods

		Number of Constraints	
Ch#	R#	Sequential Method	Cross-Layer Method
3	3	3.937%	3.992%
4	3	5.181%	5.302%
	4	8.991%	9.122%
5	3	6.421%	6.613%
	4	11.183%	11.387%
	5	17.320%	17.523%
6	3	7.665%	7.923%
	4	13.382%	13.651%
	5	20.741%	20.893%
	6	29.735%	30.012%
7	3	8.907%	9.234%
	4	15.577%	15.916%
	5	24.162%	24.358%
	6	34.635%	34.945%
	7	47.059%	47.260%
8	3	10.150%	10.545%
	4	17.772%	18.012%
	5	27.583%	27.827%
	6	39.574%	39.884%
	7	53.749%	54.033%
	8	70.106%	70.391%
9	3	11.393%	11.832%
	4	19.967%	20.255%
	5	31.004%	31.424%
	6	44.494%	44.908%
	7	60.440%	60.787%
	8	78.841%	79.141%
	9	99.694%	100%

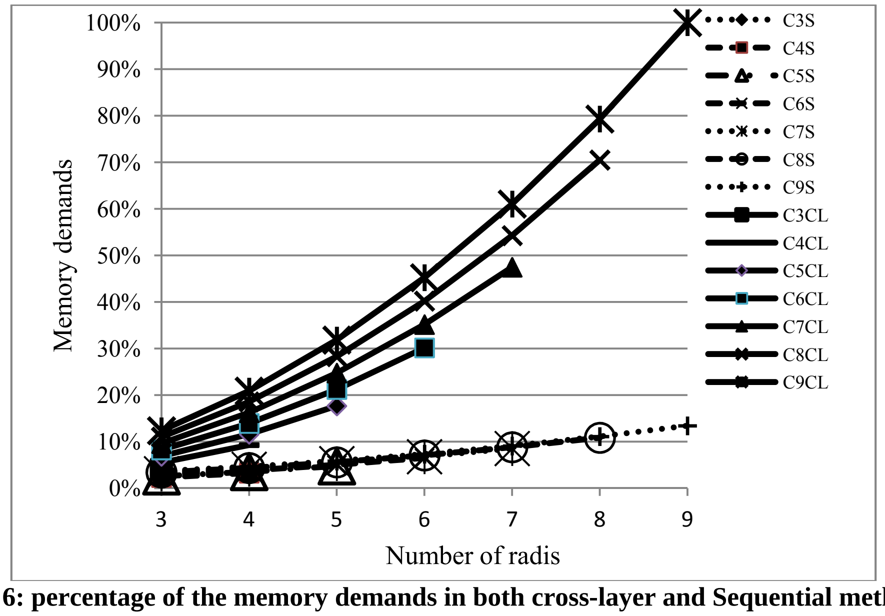
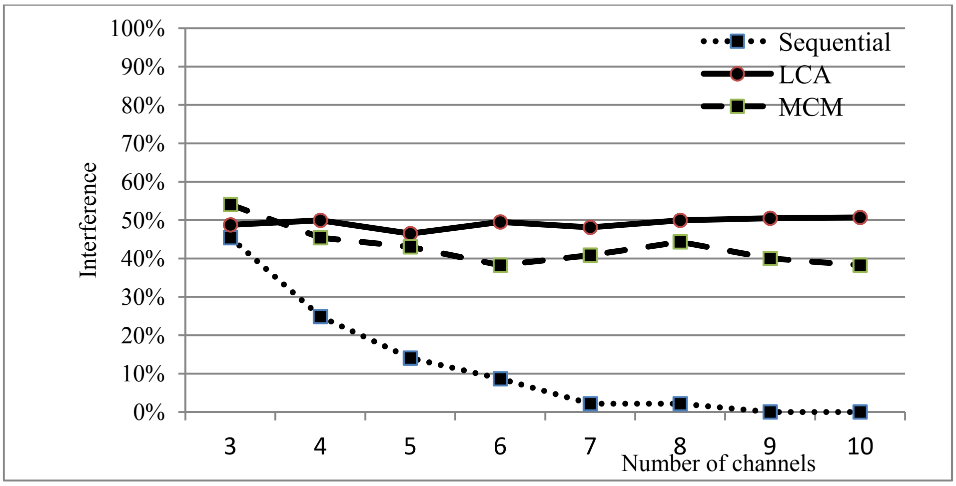


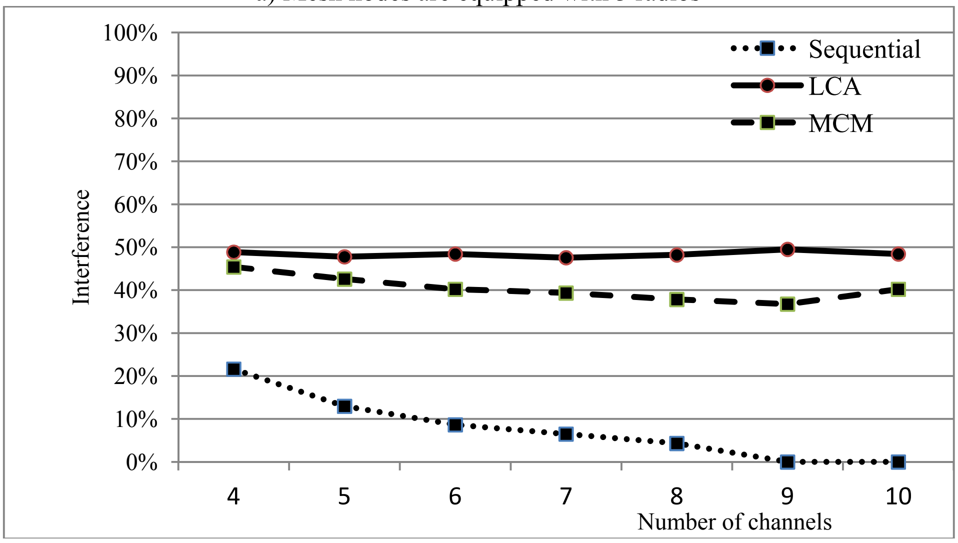
Fig. 6: percentage of the memory demands in both cross-layer and Sequential methods

Simulation 3:

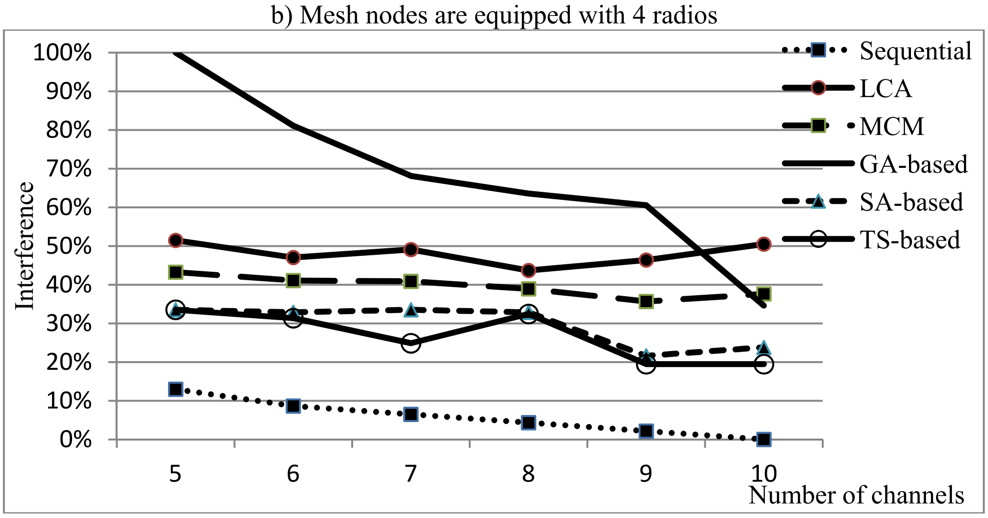
This simulation is the same as simulation 1 and was conducted to evaluate the resultant interference of our proposed Sequential design as compared with LCA, MCM, GA-, SA-, and TS-based methods. The results are in figure 7. As mentioned in simulation 1, applying GA-, SA-, TS-based methods leads the network with 3 and 4 radios per each node to be disconnected and hence, their associated curves are not shown in figures 7-(a) and (b).The results demonstrate that in a network with the same number of radios, increasing the number of channels leads to interference reduction in proposed Sequential method. As it is seen in this figure proposed Sequential method outperforms other methods in terms of interference.



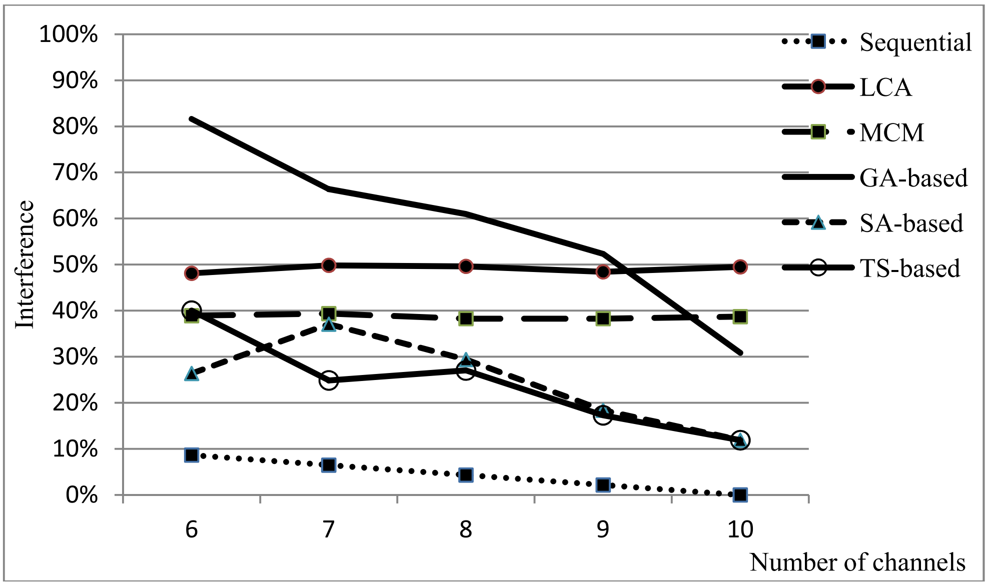
a) Mesh nodes are equipped with 3 radios



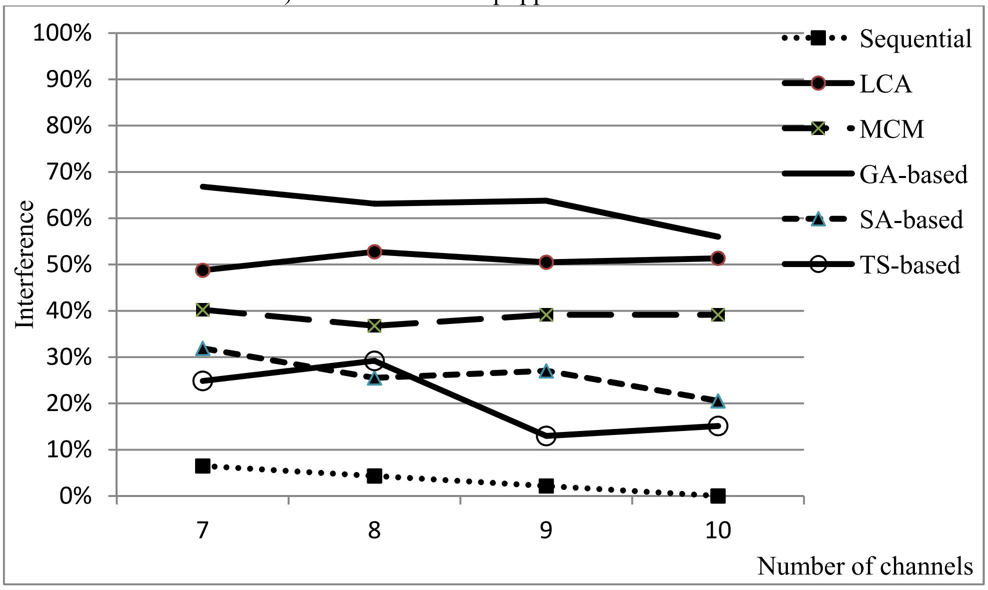
b) Mesh nodes are equipped with 4 radios



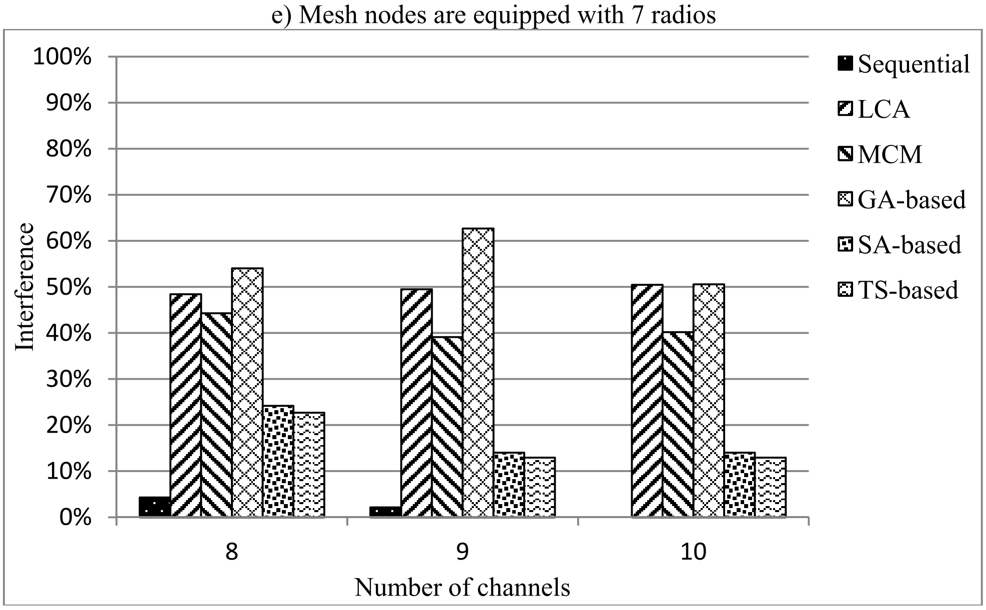
c) Mesh nodes are equipped with 5 radios



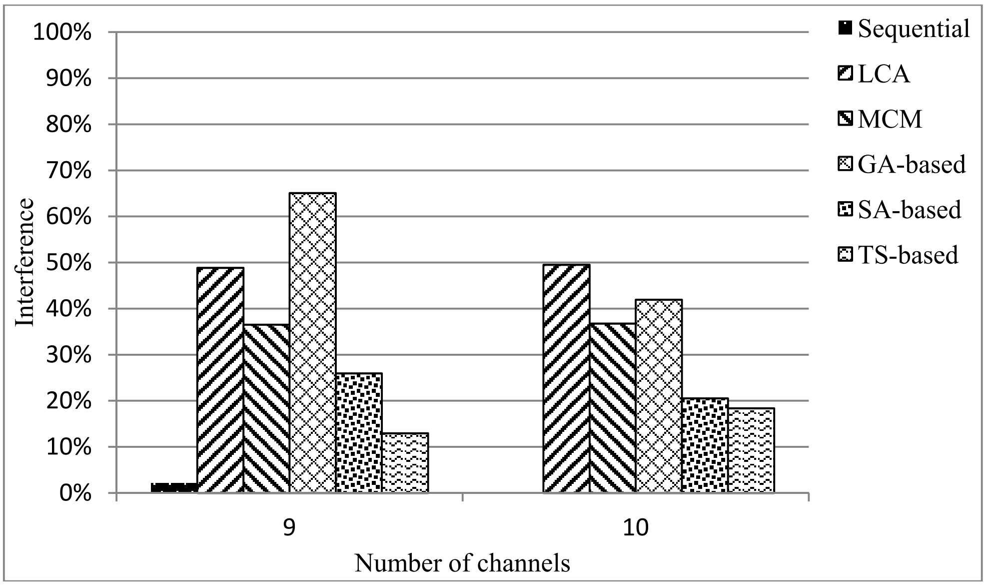
d) Mesh nodes are equipped with 6 radios



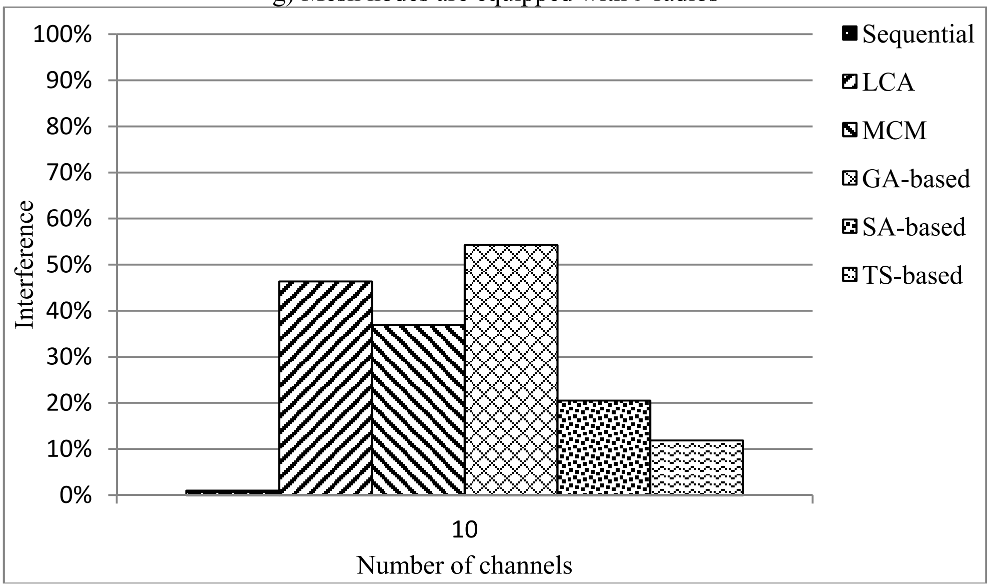
e) Mesh nodes are equipped with 7 radios



f) Mesh nodes are equipped with 8 radios



g) Mesh nodes are equipped with 9 radios



h) Mesh nodes are equipped with 10 radios

Fig. 7: Impact of channel number variations on interference for different method

Simulation 4:

Unlike simulations 1 through 3, this simulation was conducted to study the impact of radio number variations on the resultant interference for different methods. Recall from section 1 the number of radios is not more than the number of channels. Therefore, in a network with *n* available channels every node can have 2 to *n* radios. In all methods using two radios leads the given network to be disconnected. Hence, the number of radios in the simulations was considered from 3 to the number of channels. Percentage of interference obtained from applying different methods are summarized in Table 5. The columns *Ch#* and *R#* denote the number of channels and radios, respectively. Utilizing GA-, SA-, and TS-based methods in 3- and 4-radio networks leads the network to be disconnected. These situations are shown with 'DC' in Table 5. We ran the LCA, MCM, GA-, SA-, and TS-based algorithms 20 times and averaged the results. This table demonstrates that sequential design with marginally more interference as compared with cross-layer design stands in the second place of optimality.

From the results, obtained interference in Sequential method with 8 channels and 3 radios is around 2%. But increasing the number of radios to 4 or more leads the interference comes to almost 4.32%. The reason is that, if the number of radios is increased, then the Sequential design has more chance to find the optimal MCT multicast tree. In this case, the optimal solution for the multicast tree construction is provided, but the resultant tree may not be suitable input to channel assignment phase such the overall problem solution that is optimized. This is the weakness of all sequential methods.

Table 5. Percentage of obtained interference in different methods regarding radio number variations

Ch#	R#	LCA	MCM	GA-based method	SA-based method	TS-based method	Proposed Sequential method	Proposed Cross-layer method
3	3	48.75676	54.05405	DC	DC	DC	45.40541	36.75676
4	3	49.94595	45.40541	DC	DC	DC	24.86486	17.2973
	4	48.86486	45.40541	DC	DC	DC	21.62162	17.2973
5	3	46.48649	43.02703	DC	DC	DC	14.05405	8.648649
	4	47.78378	42.59459	DC	DC	DC	12.97297	8.648649
	5	51.45946	43.24324	100	33.51351	33.5135	12.97297	8.648649
6	3	49.51351	38.27027	DC	DC	DC	8.648649	6.486486
	4	48.43243	40.21622	DC	DC	DC	12.97297	6.486486
	5	47.02703	41.08108	81.08108	32.86486	31.3514	8.648649	6.486486
	6	48.10811	38.91892	81.62162	26.37838	40	8.648649	6.486486
7	3	48.10811	40.86486	DC	DC	DC	2.162162	0
	4	47.56757	39.35135	DC	DC	DC	6.486486	0
	5	49.08108	40.86486	68.10811	33.51351	24.8649	6.486486	0
	6	49.83784	39.35135	66.37838	37.05946	24.8649	6.486486	0
	7	48.75676	40.21622	66.81081	31.89189	24.8649	6.486486	0
8	3	49.94595	44.32432	DC	DC	DC	2.162162	0
	4	48.21622	37.83784	DC	DC	DC	4.324324	0
	5	43.67568	38.91892	63.56757	32.86486	32.4324	4.324324	0
	6	49.62162	38.27027	60.97297	29.40541	27.027	4.324324	0
	7	52.75676	36.75676	63.13514	25.51351	29.1892	4.324324	0
	8	48.43243	44.32432	54.05405	24.21622	22.7027	4.324324	0
9	3	50.48649	40	DC	DC	DC	0	0
	4	49.51351	36.75676	DC	DC	DC	0	0
	5	46.37838	35.67568	60.54054	21.62162	19.4595	2.162162	0
	6	48.43243	38.27027	52.32432	18.37838	17.2973	2.162162	0
	7	50.48649	39.13514	63.78378	27.02703	12.973	2.162162	0
	8	49.51351	39.13514	62.7027	14.05405	12.973	2.162162	0
	9	48.86486	36.54054	65.08108	25.94595	12.973	2.162162	0
10	3	50.7027	38.27027	DC	DC	DC	0	0
	4	48.43243	40.21622	DC	DC	DC	0	0
	5	50.48649	37.62162	34.59459	23.78378	19.4595	0	0
	6	49.51351	38.7027	30.81081	11.89189	11.8919	0	0
	7	51.35135	39.13514	56	20.54054	15.1351	0	0
	8	50.48649	40.21622	50.59459	14.05405	12.973	0	0
	9	49.51351	36.75676	41.94595	20.54054	18.3784	0	0
	10	46.37838	36.97297	54.27027	20.54054	11.8919	0	0

It is also observed that increasing the number of radios in Sequential method will be useful to a point. For example, in a network with 5 available channels, if the number of radios varies from 3 to 4, a multicast tree with interference almost 12.97 is constructed. But as it can be observed, increasing the number of radios has no longer impact on resulting multicast tree and interference reduction. This threshold is dependent on the network topology.

In sum, according to conducted simulations, the resultant interference is primarily driven by the number of channels rather than the number of radios. Extensive simulation results demonstrate that the overall interference computed by the proposed methods as compared with LCA, MCM, GA-, SA-, and TS-based methods are comprehensively low and hence outperform them. Among the two proposed approaches, cross-layer method gives the optimal solution for the joint channel assignment and multicast tree construction problem. But cross-layer design benefits always come together with difficulties. That means with increasing the size of problem the cross-layer method loses its efficiency due to more computational complexity and memory demands. In this case, proposed Sequential method solves the problem in an efficient way.

6. Performance Analysis

In this section, efficiency of two proposed designs regarding to two metrics memory demands and time complexity are investigated analytically.

A. Memory demands

Memory requirements for each method can be defined as a function of the generated variables during the implementation. The number of all generated variables depends on the number of different combinations of the variables' input sets. For example, the number of generated variables for variable $link(Node, Radios, Node, Radio, Channel)$ is $|Node|^2 \times |Radios|^2 \times |ChannelList\,|$. Therefore, the number of Integer and Binary variables for cross-layer design is computed as (45) and (46), respectively.

$$2 \times (|Node|^2 + |Node| + |Radios|^2 + |ChannelList\,| + 1) \qquad (45)$$

$$|Node|^2 \times |Radios|^2 \times |ChannelList\,| + |Node| \times |Radios| \times |ChannelList\,| \qquad (46)$$

Table 6 also shows the number of Integer and Binary variables for each sub-problem of the proposed Sequential design.

Table 6. The number of Integer / Binary variables within Sequential design

Sub-problem	Number of Binary variables	Number of Integer variables
Multicast tree construction	$\|Node\|^2 \times \|Radios\|^2$	$2 \times \|Node\| + 1$
Channel assignment	$\|Node\|^2 \times \|ChannelList\,\|$	$2 \times \|Node\|^2 \times \|ChannelList\,\| + 1$

From the above analysis, it can be concluded that the number of Binary variables in cross-layer design is larger than the sum of Binary variables of both sub-problems in Sequential design. Similarly, the number of Integer variables in cross-layer design is larger than the sum of the Integer variables specified in both sub-problems of Sequential method. That is, the number of all generated variables in cross-layer design is more than that of layered design which leads to more memory demands. Besides, due to existing of the larger solution space in cross-layer design, it is expected that solving the cross-layer formulation as compared with Sequential design needs more time.

B. Time Complexity

Every linear Integer programming problem with bounded variables can be converted into a linear Binary programming problem [47]. That is, our problem can be converted to strict Binary programming. Besides, coefficient matrix of our network based formulation is totally uni-modular [48]. This feature helps us to model continues variables in form of $x_{ij} \geq 0$

instead of Binary variables. In this case, it is assured that the optimal solution of new problem is Binary and it can be solved in $O(m^2 n)$, in which m and n are the number of constraints and variables, respectively [48].

In order to analyze time complexity of proposed cross-layer design, we initially compute the number of constraints of this model. Table 7 lists the number of generated constraints for each corresponding constraint within cross-layer model. In summary, the number of constraints in cross-layer design is $8|Nodes|^2|Radios|^2|ChannelList| + 4|Nodes| + 1$.

In what follows, the number of Binary variables within this model is computed; Every Integer variable with maximum value of k, can be modeled with at most $log(k)$ Binary variables. Therefore, we can model the Integer variables of the cross-layer design with the Binary variables. For example, the Eq. (1) is defined for each node of networks. The value of each one is at most $(|Nodes| - 1)$ that can be modeled with at most $log(|Nodes| - 1)$ Binary variables. That is, the whole number of Binary variables for Eq. (1) is defined as $|Nodes| log(|Nodes| - 1)$. Therefore, the whole number of converted Binary variables in cross-layer design is:

$$\frac{1}{2}(|Nodes|^4|Radios|^2|ChannelList| - |Nodes|^3|Radios|^6|ChannelList|) + |Nodes|(|Nodes| - 1) + 2|Nodes|log(|Nodes| - 1)$$

We had also the number of $(|Nodes|^2|Radios|^2|ChannelList| + |Nodes||Radios||ChannelList|)$ Binary variables in the original model. From above, time complexity of the cross-layer design is $O(|Nodes|^8|Radios|^6|ChannelList|^3)$.

Table 7. The number of generated constraints for each constraint of formulation.

Constraint in cross-layer model	**Number of constraints during implementation**
(6)	$\lvert Nodes\rvert - \lvert MulticastTarget\rvert - 1$
(7)	$\lvert Nodes\rvert - \lvert MulticastTarget\rvert - 1$
(8)	1
(9)	1
(10)	1
(11)	$\lvert MulticastTarget\rvert$
(12)	$\lvert MulticastTarget\rvert$
(13)	$\lvert Nodes\rvert^2 \times \lvert Radios\rvert^2 \times \lvert ChannelList\rvert$
(14)	$\lvert Nodes\rvert$
(15)	$\lvert Nodes\rvert$
(16)	$\lvert Nodes\rvert^2 \times \lvert Radios\rvert^2 \times \lvert ChannelList\rvert$
(17)	$\lvert Nodes\rvert^2 \times \lvert Radios\rvert^2 \times \lvert ChannelList\rvert$
(18)	$\lvert Nodes\rvert^2 \times \lvert Radios\rvert^2 \times \lvert ChannelList\rvert$
(19)	$\lvert Nodes\rvert^2 \times \lvert Radios\rvert^2 \times \lvert ChannelList\rvert$
(20)	$\lvert Nodes\rvert^2 \times \lvert Radios\rvert^2 \times \lvert ChannelList\rvert$
(21)	$\lvert Nodes\rvert^2 \times \lvert Radios\rvert^2 \times \lvert ChannelList\rvert$
(22)	$\lvert Nodes\rvert^2 \times \lvert Radios\rvert^2 \times \lvert ChannelList\rvert$

Table 8. The number of generated constraints for multicast tree construction sub-problem.

Constraint in multicast tree construction sub-problem	**Number of constraints during implementation**
(26)	$\lvert Nodes\rvert - \lvert MulticastTarget\rvert - 1$
(27)	$\lvert Nodes\rvert - \lvert MulticastTarget\rvert - 1$
(28)	$\lvert Nodes\rvert^2 \times \lvert Radios\rvert^2$
(29)	1
(30)	1
(31)	1
(32)	$\lvert MulticastTarget\rvert$
(33)	$\lvert MulticastTarget\rvert$
(34)	$\lvert Nodes\rvert^2 \times \lvert Radios\rvert^2$
(35)	$\lvert Nodes\rvert^2 \times \lvert Radios\rvert^2$
(36)	$\lvert Nodes\rvert - \lvert MulticastTarget\rvert - 1$
(37)	$\lvert Nodes\rvert - \lvert MulticastTarget\rvert - 1$

Table 9. The number of generated constraints for channel assignment sub-problem.

Constraint in channel assignment sub-problem	Number of constraints during implementation
(42)	$\lvert Nodes\rvert^2 \times \lvert ChannelList\rvert$
(43)	$\lvert Nodes\rvert^2 \times \lvert ChannelList\rvert$
(44)	$\lvert Nodes\rvert^2 \times \lvert ChannelList\rvert$

Similarly, the number of generated constraints for both multicast tree construction and channel assignment sub-problems of Sequential method are listed in Tables 8 and 9. In short, time complexity of multicast tree construction and channel assignment sub-problems are $O(\lvert Nodes\rvert^6 \log(n^2)\ \lvert ChannelList\rvert^3)andO(\lvert Nodes\rvert^6 \lvert Radios\rvert^6)$, respectively. Therefore, it is obvious that time complexity of the layered design is less than that of cross-layer design.

7. Conclusion

In this article, for the first time, a comprehensive mathematical optimization framework for the joint problem of multicast routing and channel assignment in MCMR WMNs has been proposed. Unlike the prior sub-optimal methods which are substantially based on heuristic- and meta-heuristic-based initiatives, we have come up with a rigorous formulation to achieve the optimal solution. In particular, we have proposed two binary integer programming (BIP)-based methods to achieve the optimal multicast solution. The first design solves the two underlying sub-problems (i.e. channel assignment and tree construction) conjointly and hence yields an optimal configuration. Nevertheless, the cross-layer approach is, relatively demanding in terms of computational complexity and hence, we proposed a second approach in which the two sub-problems are solved sequentially instead. As evidenced by simulation results, however, the resultant interference is marginally worse than that of our cross-layer formulation, but its mathematical foundation is still a remarkable superiority over comparable designs from prior art. Time complexity and memory demands of two proposed designs have been investigated as well. Analytical results demonstrate that cross-layer method needs more memory requirements and also its time complexity is more than that of Sequential design.

We have also briefly surveyed the existing proposals for multicast routing in MCMR WMNs, highlighting their advantages and disadvantages. Furthermore, the correctness of our designs has been demonstrated in terms of both connectivity and loopless-ness. Evaluation results derived from our simulation simulations reveal that the proposed methods outperform the LCA, MCM, GA-, SA-, TS-based methods in terms of interference. Among the proposed methods cross-layer method gives the optimal solution for the overall problem at the price of higher complexity. The interference associated with the layered approach, is only slightly higher than that of the cross-layer design, but significantly compared to those of LCA's, MCM's, GA-, SA-, and TS-based methods.

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