An Improved Firefly Algorithm with Directed Movement

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Abstract— **Firefly Algorithm is one of the evolutionary computing models that is inspired by fireflies behavior in nature. Each firefly's movement is based on absorption of the other one. In this paper for directing firefly's movement towards global best, it is proposed a new firefly's movement that guides fireflies to global best in each iteration, if there are no any better local optima in their neighborhood. It is proposed Gaussian distribution to draw out a better randomized position for next iteration. Proposed algorithm was tested on five standard functions that have ever used for testing the optimization algorithms. Experimental results show better performance and more accuracy than standard Firefly algorithm.**

Keywords-Firefly Algorithm;optimization; Global search; Local search.

I. INTRODUCTION

The meaning of optimization is finding a parameter in a function that makes a better solution. All of suitable values are possible solutions and the best value is optimum solution [1]. Often to solve optimization problems, optimization algorithms are used. Classification of optimization algorithm can be carried out in many ways. A simple way is looking at the nature of the algorithms, and this divides the algorithms into two categories: deterministic algorithm, and stochastic algorithms. Deterministic algorithms follow a rigorous procedure, and its path and values of both design variables and the functions are repeatable. For stochastic algorithms, in general we have two types: heuristic and metaheuristic. Nature-inspired metaheuristic algorithms are becoming powerful in solving modern global optimization problems. All metaheuristic algorithms use certain tradeoff a randomization and local search [2], [3], and [4].

Stochastic algorithms often have a deterministic component and a random component. The stochastic component can take many forms such as simple randomization by randomly sampling the search space or by random walks. Randomization provides a good way to move away from local search to the search on global scale. Most stochastic algorithms can be considered as metaheuristic and good examples are GA[1] [5] [6]. Many modern metaheuristic algorithms were developed based on the swarm intelligence in nature like PSO[2] and AFSA [3][7], [8], and [9].

For example, Firefly algorithm developed by the author shows its superiority over some traditional algorithms [10] [11]. Firefly algorithm is inspired by fireflies in nature. Fireflies in nature are capable of producing light thanks to special photogenic organs situated very close to the body surface behind a window of translucent cuticle [12].

This paper aims to formulate a new Firefly algorithm and to provide the comparison study of the new-firefly with standard Firefly algorithm. The rest of this paper is organized as follows: it outlines the Firefly algorithm in section II, and then describes new Firefly algorithm in section III. Experimental settings and results are presented in section IV. Section V concludes the paper.

II. FIREFLY ALGORITHM

The Firefly algorithm was developed by the author [3] [10] and it is based on idealized behavior of the flashing characteristics of fireflies. For simplicity, we can summarize these flashing characteristics as the following three rules:

- All fireflies are unisex, so that one firefly is attracted to other fireflies regardless of their sex.
- Attractiveness is proportional to their brightness, thus for any two flashing fireflies, the less bright one will move towards the brighter one. The attractiveness is proportional to the brightness and they both decrease as their distance increases. If no one is brighter than a particular firefly, it moves randomly.
- The brightness of a firefly is affected or determined by the landscape of the objective function to be optimised [13] [14].

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[1] Genetic Algorithm
[2] Particle Swarm Optimization
[3] Artificial Fish Swarm Algorithm

Assume continuous optimization problem where the task is to minimize cost function *f(x)* for $x \in S \subset R^n$ i.e. find x^* such as:

$$f(x^*) = min_{x \in S} f(x). \quad (1)$$

For solving an optimization problem by Firefly algorithm iteratively, there is a swarm of m agents (fireflies) and x_i represents a solution for firefly i in whereas $f(x_i)$ denotes its cost.

Initially all fireflies are dislocated in S (randomly or employing some deterministic strategy). $S_k(k = 1, \dots, d)$ In the d dimensions should be determined by the actual scales of the problem of interest. For simplicity we can assume that the attractiveness of a firefly is determined by its brightness or light intensity which in turn is associated with the encoded objective function. In the simplest case for an optimization problem, the brightness *I* of a firefly at a particular position *x* can be chosen as$I(x) \alpha f(x)$. However, the attractiveness β is relative, it should vary with the distance r_{ij} between firefly *i* and firefly *j*. As light intensity decreases with the distance from its source and light is also absorbed in the media, so we should allow the attractiveness to vary with degree of absorption [13] [14].

The light intensity $I(r)$ varies with distance *r* monotonically and exponentially. That is

$$I = I_0 e^{-\gamma r}, \quad (2)$$

Where I_0 the original light intensity and γ is the light absorption coefficient. As firefly attractiveness is proportional to the light intensity seen by adjacent fireflies, we can now define the attractiveness β of a firefly by Eq (3) [13] [14].

$$\beta = \beta_0 e^{-\gamma r^2} \quad (3)$$

1. Objective function f(x), x = (x1, ..., xd)T
2. initialize a population of fireflies $x_i(i = 1,2, .., n)$
3. Define light absorption coefficient γ
4. While (t<MaxGeneration)
5. $for\ i = 1{:}n$ (all n fireflies)
6. $for\ j = 1{:}i$
7. Light intensity $I_i\ at\ x_i$ is determined by $f(x_i)$
8. If $(I_i > I_j)$
1. Move firefly i towards j in all d dimensions (Apply Eq (5))
9. End if
10. Attractiveness varies with distance r via xp$[-\gamma r]$ $(\beta = \beta_0 e^{-\gamma r_{ij}^2})$
11. Evaluate new solutions and update light intensity
12. End for *j*
13. End for *i*
14. Rank the fireflies and find the current best.
15. End while
16. Postprocess results and visualization.

Pseudo code 1. Standard FireFly Algorithm

Where *r* is the distance between each two fireflies and β_0 is their attractiveness at $r = 0$ i.e. when two fireflies are found at the same point of search space S [12] [13]. In general $\beta_0 \in [0,1]$ should be used and two limiting cases can be defined: when $\beta_0 = 0$, that is only non-cooperative distributed random search is applied and when $\beta_0 = 1$ which is equivalent to the scheme of cooperative local search with the brightest firefly strongly determining other fireflies positions, especially in its neighborhood[3].

The value of γ determines the variation of attractiveness with increasing distance from communicated firefly. Using γ=0 corresponds to no variation or constant attractiveness and conversely setting γ→∞ results in attractiveness being close to zero which again is equivalent to the complete random search. In general $\gamma \in [0,10]$ could be suggested [3].

It is worth pointing out that the exponent γr can be replaced by other functions such as γr^m when $m > 0$. The distance between any two fireflies i and j at x_i and x_j can be Cartesian distance in Eq (4).

$$r_{ij} = \left\|x_i - x_j\right\|_2 = \sqrt{\sum_{k=1}^{d} (x_{i,k} - x_{j,k})^2} \quad (4)$$

The firefly *i* movement is attracted to another more attractive (brighter) firefly *j* is determined by:

$$x_i = x_i + \beta_0 e^{-\gamma r_{ij}^2} (x_j - x_i) + \alpha \varepsilon_i, \quad (5)$$

Where the second term is due to the attraction, while the third term is randomization with the vector of random variable ε_i being drawn from a Gaussian distribution and α ∈ [0,1] [11] [13]. In [14] uses an Lévy distribution instead of Gaussian one. Schematically, the Firefly algorithm can be summarized as the pseudo code in pseudo code 1.

III. PROPOSED ALGORITHM

A. Adaptive step lenght

In standard Firefly algorithm, firefly movement step length is a fixed value. So all the fireflies move with a fixed length in all iterations. Due to the fixed step length, the algorithm will miss better local search capabilities. In proposed algorithm, it is defined a coefficient for α that depends on iteration and it always produce a value less than one. This coefficient is determined by:

$$W_{\mathrm{itr}} = \mathrm{X} + \frac{(\mathrm{itr_{max}} - \mathrm{itr})^{\mathrm{n}}}{(\mathrm{itr_{max}})^{\mathrm{n}}} + (\mathrm{Y} - \mathrm{X}), \quad (6)$$

Where n>=1. In Eq (6), weight of W_{itr} is defined based on current iteration number and the last iteration number. The values that produce by this equation is between X and Y, and reduces by the time. Because α ∈ [0, 1], so X=0 and Y=1. n could be a linear or non-linear coefficient and $\mathrm{itr_{max}}$ is maximum number of iteration and itr is iteration i [15]. This causes that α fixed coefficient changes by the time and step length shrinks and the algorithm can get better result in local search.

B. Directed movement

In addition in standard Firefly algorithm, firefly movement is based on light intensity and comparing it between each two fireflies. Thus for any two fireflies, the less bright one will move towards the brighter one. If no one is brighter than a particular firefly, it will move randomly. In proposed algorithm this random movement is directed, and that firefly moves towards best solution with better cost in that iteration. The firefly i movement is attracted to best solution that is more attractive (brighter). This causes that if there was no local best in each firefly's neighborhood; they move towards best solution and make better position for each firefly for next iteration and they get more near to global best. Fireflies movement in this model is exactly similar to Eq (5) in standard Firefly algorithm.

C. Social behavior

In proposed algorithm, at the end of each iteration, it is introduced normal Gaussian distribution that is shown in Eq (7).

$$p = f(x|\mu, \delta) = \frac{1}{\delta\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\delta^2}} \quad (7).$$

Where x is an error between best solution cost and current solution cost, μ is mean and δ is standard deviation. Because of using standard normal distribution, it is set to μ=0 and δ=1. Then a random number will be drawn from Gaussian distribution that is related to each firefly probability. In standard FireFly algorithm α coefficient is introduced for fireflies movement and in proposed algorithm we add it to have fixed step length. Firefly i movement is determined by Eq (8).

$$x_i = x_i + \alpha * (1 - p) * rand(). \quad (8)$$

But firefly i new position causes better cost, it will move to that new position. New firefly algorithm can be summarized as the pseudo code is shown in pseudo code 2. This strategy makes a social behavior for all fireflies and they move towards global best.

2. Objective function f(x), x = (x1, ..., xd)T
3. initialize a population of fireflies $x_i (i = 1,2,..,n)$
4. Define light absorption coefficient γ
5. While (t<MaxGeneration)
6. $for\ i = 1{:}n$ (all n fireflies)
7. $for\ j = 1{:}i$
8. Light intensity I_i at x_i is determined by $f(x_i)$
9. If $(I_i > I_j)$
10. Move firefly i towards j in all d dimensions(Apply Eq (5))
11. Else
12. Move firefly i towards best solution in that iteration
13. End if
14. Attractiveness varies with distance r via xp $[-\gamma r]$ $(\beta = \beta_0 e^{-\gamma r_{ij}^2})$
15. End for j
16. End for i
17. Rank the fireflies and find the current best
18. Define normal distribution
19. $for\ k = 1{:}n$ (all n fireflies)
20. Draw a random number from defined distribution And apply Eq. 8.
21. Evaluate new solution(new_cost(k))
22. If((new_cost(k)<cost(i))&&(new_cost(k)<last_cost_iteration(k)))
23. Move firefly i towards current best
24. End if
25. End for k
26. End while
27. Postprocess results and visualization

Pseudo code 2. New firefly algorithm

IV. EXPERIMENTAL RESULTS

Proposed algorithm was tested on five standard functions that have ever used to evaluate optimization algorithm in continuous static problems. Standard functions are Sphere, Griewank, Ackley, Rosenbrock and Rastrigin that is shown in table I. The optimum values of all these functions are zero.

All of the results are in 10, 20 and 30 dimensions. Population size is 30 and the results are the average and best value for 30 iterations. In proposed algorithm γ=1, β_0=1same as [13] [14] and α=0.7 by experience. The value of n in Eq (6) is 0.01and $itr_{max} = 1000$.

The results of table II show that proposed algorithm performance is better than standard Firefly algorithm. Because of non-linear reduced step length, algorithm works with higher precision than usual and there is a good balance between problem landscape and the space that algorithm can get better solution. Proposed algorithm augment the local search and in final iteration we have a better local search than beginning. Also this model navigates firefly movement to go towards best solution. Social behavior of firefly makes better position for each firefly for next iteration.

Simulation results show that proposed algorithm has better performance than standard Firefly algorithm because firefly movement happens just when their movement has a better result in cost.

TABLE I. STANDARD TEST FUNCTIONS

Function	Range
Sphere= $\sum_{i=1}^{n} x_i^2$	±100
Ackley= $20 + e - 20 * e^{-0.2\sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2}} - e^{\frac{1}{n}\sum_{i=1}^{n}\cos(2\pi x_i)}$	±32
Rastrigin= $\sum_{i=1}^{n}(x_i^2 - 10\cos(2\pi x_i) + 10)$	±5.12
Griewank= $\sum_{i=1}^{n}\left(\frac{x_i^2}{4000}\right) - \prod_{i=1}^{n}\cos\left(\frac{x_i}{\sqrt{i}}\right) + 1$	±600
Rosenbrock= $\sum_{i=1}^{n}\left(\frac{x_i^2}{4000}\right) - \prod_{i=1}^{n}\cos\left(\frac{x_i}{\sqrt{i}}\right) + 1$	±50

TABLE II. COMPARISON OF BEST, AVERAGE AND STANDARD DEVIATION OF STANDARD FIREFLY ALGORITHM AND PROPOSED ALGORITHM FOR 30 ITERATIONS.

Function	Dimension	Standard-Firefly	Standard-Firefly	New-Firefly	New-Firefly

		(Average and Std)	(Best)	(Average and Std)	(Best)
Sphere	10	1.4501±0.3903	0.7339	3.59929e-056±1.9678-055	2.4120e-086
	20	4.2271±0.5187	2.6534	3.9693e-035±2.6363e-035	7.9599e-036
	30	7.0817±0.6191	5.8830	7.0469e-033±7.9851e-033	1.8008e-033
Ackley	10	3.1378±0.2836	2.4826	8.9410e-015±3.0850e-015	4.4409e-015
	20	3.3234±0.1347	3.0106	3.1086e-014±6.7762e-015	2.2204e-014
	30	3.4841±0.0952	3.3180	1.3204e-014±5.2557e-015	7.9936e-015
Rosenbrock	10	27.1940±8.6789	14.5519	5.5584±.9869	2.7266
	20	126.1294±37.8203	70.6155	18.3347±1.0603	15.6302
	30	288.8591±57.9833	152.0813	29.6222±1.7629	26.8140
Griewank	10	0.161±0.0326	0.1130	6.227e-008±9.9378e-008	1.1328e-009
	20	0.2445±0.0386	0.0386	1.7199e-007±1.9440e-007	1.5924e-008
	30	0.2961±0.0454	0.2182	1.5784e-006±1.4680e-006	2.3625e-007
Rastrigin	10	37.5260±8.3267	22.3223	3.3829±2.6077	0
	20	105.5577±17.7213	72.3500	5.8703±4.2962	1.7764e-015
	30	208.6290±24.3623	156.5912	10.3807±7.1255	1.7764e-014

V. CONCLUSION

In this paper, three approaches are presented for improving standard Firefly algorithm. In first approach, initial value of the step length of movement is assumed to be big that this causes increasing speed of movement towards global optimum and prevent to trap into local optimum. After some iteration this parameters shrink that causes focus on global optimum. Also this improves accuracy and reducing free movement

In proposed algorithm, if a firefly can't find any better fireflies with low cost in its neighborhood, it will move towards global best that all fireflies have a directed movement so they can get near to optimum solution at the end of iteration. Moving fireflies by a Gaussian distribution as a social behavior causes a better position for each of them for next iteration. Simulation results show a better performance than standard Firefly algorithm.

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