A Hybrid Method based on Gas Diffusion Model and Fuzzy Cellular Automata for Image Sharpening

F. Mahdavifard   M. R. Meybodi
Computer Engineering and Information Technology Department
Amirkabir University of Technology
Tehran Iran
mahdavifard@cs.aut.ac.ir meybodi@aut.ac.ir

Abstract: *Image sharpening is one of the most significant operations in image processing. In Gas Diffusion Model for image sharpening, the ambiguity of the image is modelled to the process of gas diffusion. In this method usually the value of parameter α is the same for all pixels in the image; therefore all parts of the image would be sharpened uniformly. If we consider a local value for parameter α for each pixel, the performance of method would be much better. This local value should be high in parts of the image with high frequency variations (edges) and should be low in other parts.In this paper a novel method for image sharpening is presented which is a hybrid of image sharpening based on Gas Diffusion Model and Fuzzy Cellular Automata. In this method Fuzzy Cellular Automaton calculates appropriate local value for parameter α in each pixel using fuzzy rules. The result obtained from implementation shows that the performance of this method is much better compared to other sharpening methods.*

Keywords: Cellular automata, fuzzy cellular automata, image processing, image sharpening, Gas Diffusion Model

1   Introduction

One of the most important operations in image processing is image sharpening. In contrast with image smoothing that weakens the effect of high frequency components, image sharpening strengthens them. The goal of image sharpening is to clarify the details of an image, especially the edges. Therefore image sharpening can be used as a tool for edge detection. There are a lot of methods for image sharpening, but each of them has some weaknesses.

In Gas Diffusion Model for image sharpening, the ambiguity of image is modelled to the process of gas diffusion. One of the challenges in this method is that how to find the appropriate value for parameter α. Usually the value of parameter α is the same for all pixels of the image; therefore all parts of the image would be sharpened uniformly. The performance of the method increases if we consider a local value for parameter α for each pixel. In parts of the image, with high frequency variations (edges), the value of parameter α should be higher than the value used for low frequency variation parts.

In this paper, we propose a novel method for image sharpening which is a hybrid of Gas Diffusion Model and Fuzzy Cellular Automaton. The Fuzzy Cellular Automaton uses fuzzy rules to calculate the appropriate value for parameter α for each pixel. Experimental results show that the performance of this method is better compared to other sharpening methods.

Section 2 introduces some preliminary definitions of Cellular Automata, Cellular Learning Automata, and Fuzzy Cellular Automata. It also explains the image sharpening method based on Gas Diffusion Model. Section 3 presents a brief history of the application of CA-based models in image processing. Section 4 explains the proposed algorithm in detail. Section 5 presents the experimental results and section 6 draws conclusion and talks about some suggestions for future research in this area.

2   Definitions

In this section some definitions are introduced which will be used throughout the paper.

2.1   Cellular Automata

Cellular Automata (CA) are discrete dynamic systems whose behaviour is completely based on local relations. A cellular automaton consists of a grid of cells; each of them is in one of the finite number of states. In CA, the time is also discrete, and the state of a cell is a function of the previous states of its neighbour cells. A uniform rule is applied to each cell and its neighbours and each time it is applied, the new states of cells are generated [17].

2.2   Cellular Learning Automata

The Cellular Learning Automaton (CLA) is a mathematical model for modelling dynamics of a complex system which consists of a large number of simple components. In fact CLA is a CA in which every cell is equipped with at least one learning automaton. The Learning Automaton (LA) has a finite set of actions and its goal is to learn which action in this set is the optimal action. Like CA, there is a uniform local rule applied to the cells and based on this rule the selected action gets a reward or a penalty. If the learning algorithm is chosen properly, the iterative process of interacting with the environment can result in selection of the optimal action in every cell [7].

2.3   Fuzzy Cellular Automata

Fuzzy Cellular Automaton (FCA) is a CA in which fuzzy logic is applied to the states of the cells and transition functions. There are some definitions for FCA [8,11]. The latest one was proposed by Meybodi and Anvarinejad [1,3].

In FCA, the states of cells are linguistic variables and the transition functions are fuzzy rules. The linguistic variables are determined based on our knowledge of the problem. The next state of each cell is determined by transition function. The current state of each cell and its neighbours are the input arguments of the transition function. The transition function is a uniform fuzzy function that takes the membership values of the neighbour cells and calculates the value of membership of the next state. The membership values of linguistic variables which represent the state of cells are used to present the evolution of FCA during the process. The set of neighbour cells is uniform and fixed during the process.

An FCA can be represented by a 4-tuple $\langle Z, S, r, f \rangle$, where Z is an n-dimensional regular grid of cells, S is the set of states, $r \in N$ is the radius of neighbourhood, and $f : s^{2r+1} \to s$ is the transition function. Throughout this paper, we use the definition of FCA as described.

2.4   Image Sharpening Using Gas Diffusion Model

Assume that $\{(x,y) \mid 1 \le x \le M, 1 \le y \le N\}$ is the original image and $f(x,y)$ is the blurred image. The blurring process of an image could be modelled to the process of Gas Diffusion. Suppose $f(x,y,t)$ is the blurred image at time t. Image blurring (Gas Diffusion) begins at time $t = 0$ and at time $t = \tau$ the blurred image is $f(x,y)$, so

$$f(x, y, \tau) = f(x, y)$$
(1)
$$f(x, y, 0) = g(x, y)$$
(2)
Also we have
$$\partial f / \partial t = K \nabla^2 f \qquad K > 0$$
(3)

If we derive Taylor series of $f(x,y,t)$ about the point $t = \tau$, and omit the terms of expansion with degrees more than one, the final result is presented in Equation (4).

$$f(x, y, 0) \approx f(x, y, \tau) - \tau \frac{\partial f(x, y, \tau)}{\partial t}$$
(4)

Combining Equations (1), (2), (3) and (4), results in Equation (5).

$$g(x, y) = f(x, y) - K\tau \nabla^2 f(x, y)$$
(5)

The $\nabla^2 f$, Laplacian of an image, is calculated by Equation (6).

$$\nabla^2 f(x, y) = f(x+1, y) + f(x, y+1) + f(x, y+1)$$
$$+ f(x-1, y) + f(x, y-1) - 4f(x, y)$$
(6)

Combining Equations (5) and (6) results in Equation (7); in Equation (7) we consider $\alpha = K\tau$.

$$g(x,y) = (1+4\alpha)f(x,y) - \alpha(f(x+1,y) + f(x,y+1) + f(x,y-1) + f(x-1,y))$$

(7)

Therefore, we should apply the template which is shown in Figure 1 to the image [6].

0	$-\alpha$	0
$-\alpha$	$1+4\alpha$	$-\alpha$
0	$-\alpha$	0

Figure 1: The template which is used in image sharpening through Gas Diffusion Model

One of the problems in this method is that parameter α usually takes a constant value for all pixels of an image. Therefore, all parts of the image will be sharpened uniformly.

3   Related Works

Speed of the operations is the key factor in real-time image processing. Thus, researchers try to design algorithms that could be run on several distributed processors simultaneously. Yang and Ye used CA as a parallel computational model for an edge detection algorithm [18]. Sahota, Daemi and Elliman used a CA model controlled genetically for solving some problems in image processing [15]. Also Hernandez and Herrmann used several CAs for elementary image enhancement. They also applied CA in image sharpening and image smoothing [10].
CLA is also used in the area of image processing. For example Marchini, Soleymani and Meybodi used CLA model for image sharpening [6]. Also Kharazmi and Meybodi used CLA for image restoration [5], for image segmentation [4] and for noise reduction, segmentation and feature extraction in noisy images [13]. Bohlool and Meybodi used CLA for edge detection purpose [9]. Mojaradi, Lucas and Varshosaz used CLA for classification of satellite images [14]. Marchini proposed several applications of CLA in image processing in [2].
The CA and CLA have been used widely in the area of image processing, but to our knowledge there is no application of FCA in image processing. In this paper we used FCA for image sharpening purpose.

4   A Hybrid Method based on Gas Diffusion Model and Fuzzy Cellular Automata for Image Sharpening

In hybrid method FCA is used in order to determine the value of parameter α for each pixel of the image. Fuzzy rules in each cell of FCA determine the appropriate value of parameter α for the pixel assigned to the cell. FCA decides that for which pixel a high value is suitable and for which one a low value. In parts of the image that there is a superior variation in frequency, high values of parameter α are fitted and in other parts, low values. In order to achieve this goal, the concept of derivative in images would be used and based on the value of derivative in each pixel and using some fuzzy rules, the fuzzy derivative in all pixels and all directions will be calculated. In fact we try to detect the edges of an image and apply a higher value for parameter α for those pixels.
For sharpening an $R \times C$ image, a 2-dimensional CA with R rows and C columns is used. Each pixel of the image is mapped to one of the cells in FCA. For example the pixel in row r and in column c is mapped to the cell of CA in row r and column c.
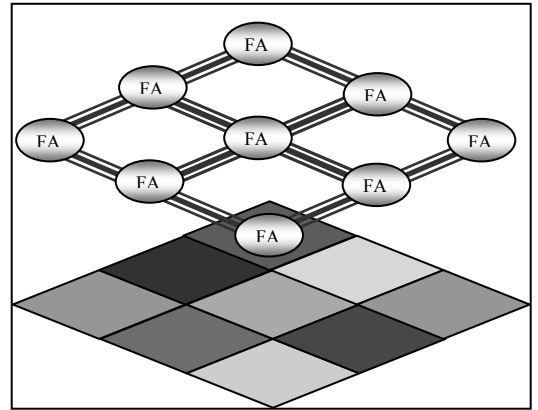

Figure 2: Each pixel of image is mapped to one cell of FCA

4.1   Derivative of Image

The simple derivative at pixel position (x,y) in direction D is defined as the difference between the value of pixel at (x,y) and its neighbour in direction D $(D \in dir = \{NW, W, SW, S, SE, E, NE, N\})$. The neighbours of pixel in eight different directions are shown in Figure 3. This derivative value is denoted by $\nabla_D(x,y)$, e.g.

$$\nabla_N(x,y) = I(x,y-1) - I(x,y)$$
$$\nabla_{NW}(x,y) = I(x-1,y-1) - I(x,y)$$

NW	N	NE
W	(x,y)	E
SW	S	SE

Figure 3: Neighborhood of a central pixel *(x,y)*

4.2 Fuzzy Derivative

The idea of fuzzy derivative is introduced by Van De Ville *et. al.* They used fuzzy derivative for filtering images corrupted with Gaussian noise [16]. In this paper we apply this concept for image sharpening using FCA.

The principle of fuzzy derivative is based on the following observation. Consider an edge passing through the neighbourhood of a pixel *(x,y)* in the SW-NE direction (the bold line in Figure 4), the derivative value $\nabla_{NW}(x,y)$ will be large but also derivative values of neighbouring pixels perpendicular to the edge's direction, *i.e.* $\nabla_{NW}(x,y)$, $\nabla_{NW}(x-1,y+1)$, $\nabla_{NW}(x+1,y-1)$, are expected to be large (arrows in Figure 4).

Therefore to calculate fuzzy derivative value in direction NW, the derivative values $\nabla_{NW}(x,y)$, $\nabla_{NW}(x-1,y+1)$ and $\nabla_{NW}(x+1,y-1)$ should be calculated first.
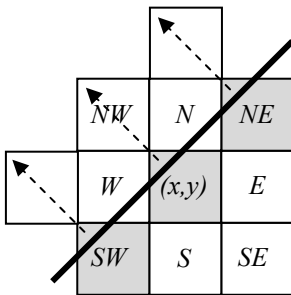

Figure 4: The bold line is an edge passing pixel *(x,y)*, therefore derivative values at this pixel and its two neighbours in NW direction are large (arrows)

The idea in fuzzy derivative is that if two out of three derivative values are small, it is assumed that no edge passes in that direction and if all three derivative values are large, it is assumed that an edge passes in that direction. This observation will be taken into account when

formulating fuzzy rules for calculating fuzzy derivative values. To compute the value that expresses the degree to which the fuzzy derivative in a certain direction is small or large, the fuzzy set *CD_small* and *CD_Large* is used (see Figure 5).
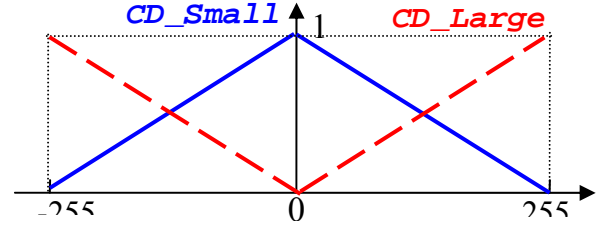

Figure 5: Membership function for *CD_small* and *CD_Large* fuzzy set

Therefore fuzzy derivative in pixel position *(x,y)* and in NW direction is calculated using these fuzzy rules:

If ($\nabla_{NW}(x,y)$ is *CD_Small* **and** $\nabla_{NW}(x-1,y+1)$ is *CD_Small*) **or**
($\nabla_{NW}(x,y)$ is *CD_Small* **and** $\nabla_{NW}(x+1,y-1)$ is *CD_Small*) **or**
($\nabla_{NW}(x-1,y+1)$ is *CD_Small* **and** $\nabla_{NW}(x+1,y-1)$ is *CD_Small*)
then $\nabla_{NW}^F(x,y)$ is *FD_Small*.

If ($\nabla_{NW}(x,y)$ is *CD_Large* **and** $\nabla_{NW}(x-1,y+1)$ is *CD_Large* **and** $\nabla_{NW}(x+1,y-1)$ is *CD_Large*)
then $\nabla_{NW}^F(x,y)$ is *FD_Large*

Eight rules similar to each of these rules are applied to each cell of FCA to compute the degree of membership of fuzzy derivatives $\nabla_D^F(x,y), D \in dir$ to *FD_small and FD_large* fuzzy sets. In this phase a defuzzification is not needed because the next phase of algorithm directly uses the degree of memberships.

4.3 Calculating the value of parameter *α*

In this phase each cell of FCA uses a fuzzy rule. The idea is that if an edge is present at pixel *(x,y)* in direction D, the value of fuzzy derivative at that pixel and at least in one of eight directions is high and if the value of fuzzy derivative at that pixel and in all eight directions is low, no edge passes that pixel. In the first case parameter *α* should take a high value for this pixel and in the second case a low value. The value of parameter

α is linguistic. To determine the numerical value of parameter α, two fuzzy set of *high* and *Low* have been used (see Figure 6).

FD_Small   FD_Large

1

0   1

(a)

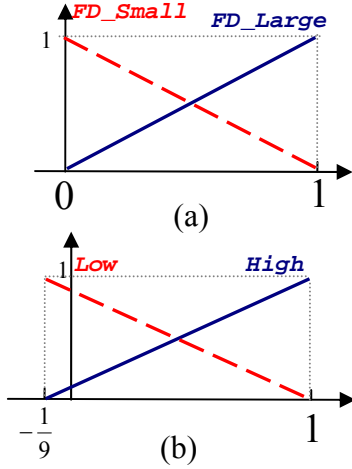Low   High

1

$-\frac{1}{9}$ (b)   1

Figure 6: Membership function for *big* fuzzy set

The parameter α for each pixel is calculated by using the following fuzzy rule at each cell of FCA:

If $((\nabla_N^F(x,y)$ is *Large*) **or** $(\nabla_{NE}^F(x,y)$ is *Large*)

    or $(\nabla_{NW}^F(x,y)$ is *Large*) **or** $(\nabla_W^F(x,y)$ is *Large*)

    or $(\nabla_{SW}^F(x,y)$ is *Large*) **or** $(\nabla_S^F(x,y)$ is *Large*)

    or $(\nabla_{SE}^F(x,y)$ is *Large*) **or** $(\nabla_E^F(x,y)$ is *Large*))

then $\alpha(x,y)$ is *High*.

If $((\nabla_N^F(x,y)$ is *FD_Small*) **and** $(\nabla_{NE}^F(x,y)$ is

and$(\nabla_{NW}^F(x,y)$ is *FD_Small*) **and** $(\nabla_W^F(x,y)$ is *FD_Small*)

and$(\nabla_{SW}^F(x,y)$ is *FD_Small*) **and** $(\nabla_S^F(x,y)$ is *FD_Small*)

and$(\nabla_{SE}^F(x,y)$ is *FD_Small*) **and** $(\nabla_E^F(x,y)$ is *FD_Small*))

then $\alpha(x,y)$ is *Low*.

Mamdani inference mechanism [12] is used as fuzzy inference mechanism and the defuzzification method is *smallest of maximum* (SOM).

5   Experimental Result

The proposed algorithm is applied for sharpening different images. The results obtained from the proposed algorithm are compared to the results obtained from using a constant value for parameter α, a low value and a high value. The proposed algorithm is compared with the similar algorithm using CLA [6] and also with image sharpening using Sobel Gradient. Figure 7 depicts the results obtained for Lena image. More results are available at www.ce.aut.ac.ir/~mahdavifard. The proposed algorithm clarifies the details of image better compared to other methods.

One of the most important characteristics of proposed algorithm is that it is a distributed method; therefore it is easy to run it in a parallel fashion.

6   Conclusion

In this paper we applied FCA to image sharpening using Gas Diffusion Model. The proposed algorithm is the enhanced version of image sharpening using Gas Diffusion Model in that FCA is applied to calculate the value of parameter α for each pixel of the image.

The experimental results showed that this method outperforms the traditional methods of image sharpening.

FCA has a lot of capabilities in the area of image processing. Through combining FCA with other models, *e.g.* CLA, we can make it more capable to tune the membership functions locally and using learning in every pixel.

References

[1] T. Anvarinejad, "Fuzzy Dynamic Channel Assignment in Cellular Mobile Networks", *M.Sc. thesis*, *Computer Engineering and Information Technology Department, Amirkabir University of Technology,* 2003.

[2] F. Marchini, "The Application of Cellular Learning Automata to Image Processing", *M.Sc. thesis*, *Computer Engineering and Information Technology Department, Amirkabir University of Technology*, 2003.

[3] T. Anvarinejad and M. R. Meybodi, "Fuzzy Cellular Automata", *Proceedings of The 5th Iranian Conference on Fuzzy Systems, Imam Hussein University, Tehran,* pp.57-65, Sept. 2004.

[4] M. R. Kharazmi and M. R. Meybodi, "Application of Cellular Learning Automata to Image Segmentation", *Proceedings of*

Tenth Conference on Electrical Engineering (10th ICEE), University of Tabriz, Vol. 1, pp. 298-306, May 2002.

[5] M. R. Kharazmi and M. R. Meybodi, "An Algorithm Based on Cellular Learning Automata for Image Restoration", *Proceedings of The First Iranian Conference on Machine Vision & Image Processing, University of Birjand,* pp. 244 –254, March 2001.

[6] F. Marchini, M. R. Meybodi and V. Soleymani, "A Hybrid Method for Image Sharpening (Gas Diffusion Model + Cellular Learning Automata)", *Proceedings of 11th Annual CSI Computer Conference of Iran, Fundamental Science Research Center (IPM), Computer Science Research Lab., Tehran, Iran,* pp. 715-720, January 24-26, 2006.

[7] M. R. Meybodi, H. Beigy and M. Taherkhani, "Cellular Learning Automata", *Proceedings of 6th Annual CSI Computer Conference, Computer Engineering Department, University of Isfahan,* pp. 153-163, 2001.

[8] M. Mraz, N. Zimic, I. Lapanja and I. Bajec. "Fuzzy Cellular Automata: From Theory to Application", *Proceedings of The 12th IEEE International Conference on Tools with Artificial Intelligence (ICTAI)*, pp. 320-323, 2000.

[9] M. Bohlool and M. R. Meybodi, "Edge Detection Using Asynchronous Cellular Learning Automata", *Technical Report, Computer Engineering and Information Technology Department, Amirkabir University of Technology*, 2006.

[10] G. Hernandez and H. J. Herrmann, "Cellular Automata for Elementary Image Enhancement", *Graphical Models and Image Processing,* Vol. 58, No.1, pp. 82-89, January, 1996.

[11] P. Maji, P. Chaudhuri. "Fuzzy Cellular Automata for Modeling Pattern Classifier". *IEICE Transaction on Information and Systems,* pp. 691-702, April 2005.

[12] E. H. Mamdani and S. Assilian, "An experiment in linguistic synthesis with a fuzzy logic controller", *International Journal of Man-Machine Studies*, Vol. 7, No. 1, pp. 1-13, 1975.

[13] M. R. Meybodi and M. R. Kharazmi, "Application of Cellular Learning Automata to Image Processing", *Journal of Amirkabir University of Technology,* Vol. 14, No. 56A, pp. 1101-1126, 2004.

[14] B. Mojaradi, C. Lucas and M. Varshosaz, "Using Learning Cellular Automata for Post Classification Satellite Imagery", *Geo-Imagery Bridging Continents, XXth ISPRS Congress, Istanbul, Turkey,* 12-23 July 2004.

[15] P. Sahota, M. F. Daemi and D. G. Elliman, "Training Genetically Evolving Cellular Automata for Image Processing", *International Symposium on Speech, Image Processing and Neural Networks,* Vol. 2, pp. 753-756, 1994.

[16] D. Van De Ville, M. Nachtegael, D. Van der Weken, E. Kerre, W. Philips, I. lemahieu. "Noise Reduction By Fuzzy Image Filtering", *IEEE Transaction on fuzzy systems*, VOL. 11 , NO. 4, pp. 429-436, AUGUST 2003.

[17] S. Wolfram, "Theory and Applications of Cellular Automata", *Advanced Series on Complex Systems, Singapore: World Scientific Publication*, 1986.

[18] C. Yang and H. Ye, "Cellular Automata Modeling in Edge Recognition", *Proceedings of the Seventh International Symposium on Artificial Life and Robotics,* pp 128-132, 2000.
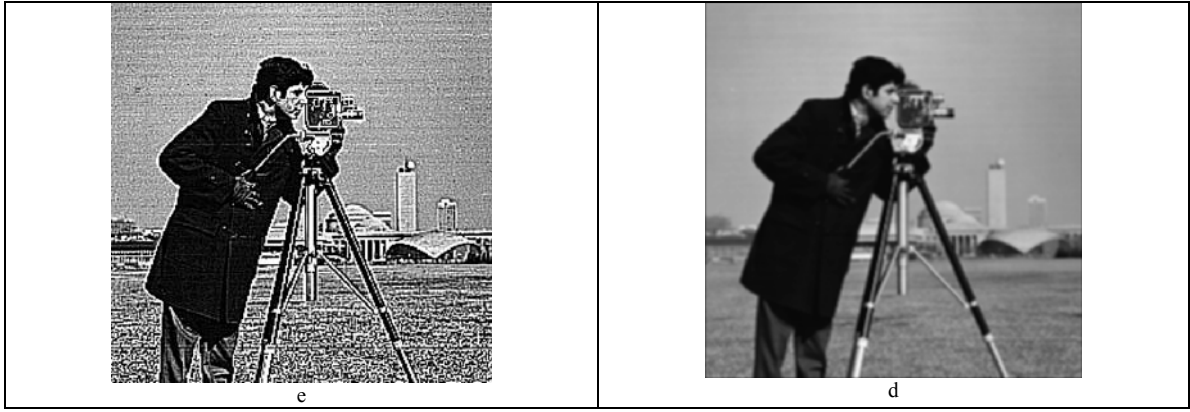
c

b

a

e

d

c

b

a
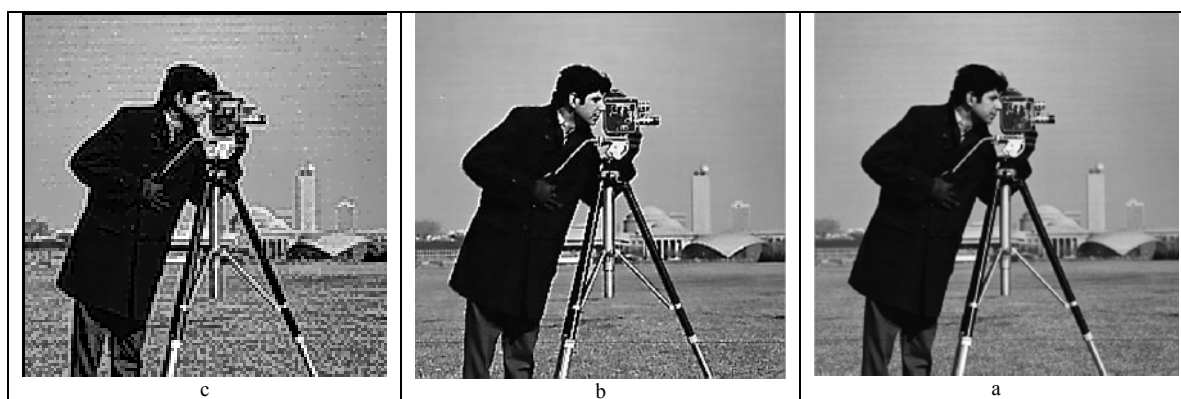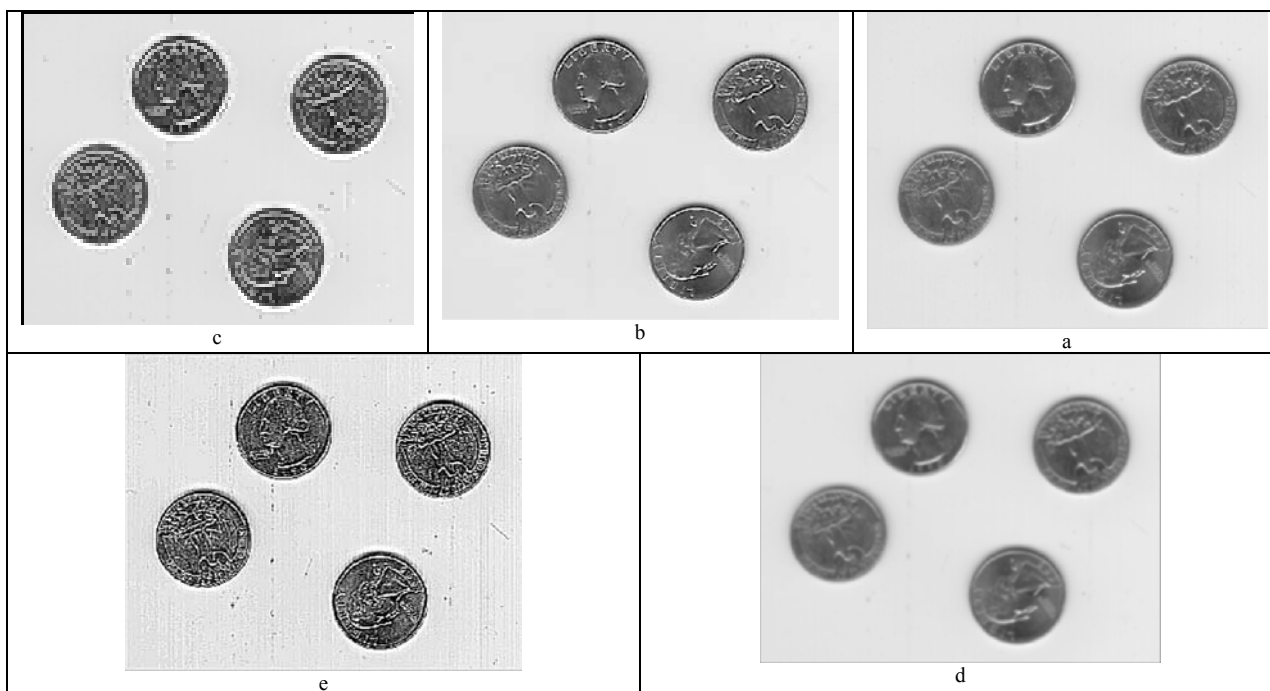
Figure 7: a) Original image. b) Image sharpened by proposed algorithm. c) Image sharpened by Gas Diffusion Model using low values for parameter α. d) Image sharpened by Gas Diffusion Model using high values for parameter α. e) Image sharpened by hybrid model of CLA and Gas Diffusion