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Critical Density in Adjustable-Orientation Directional Sensor Networks Using Continuum Percolation

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Abstract

In this paper, we consider adjustable-orientation directional sensor networks (ADODSNs) in which nodes are deployed based on Poisson Point Process and the orientation of sensor nodes is distributed between 0 and 2π independently and uniformly. Despite of other kinds of directional sensor networks, orientation of sensor nodes in ADODSNS could be adjusted after deployment by using an algorithm. We call this kind of sensor networks, adjustable-orientation directional sensor networks. We calculate the critical density of nodes for both sensing coverage and network connectivity in such networks using continuum percolation where field-of-view angles of sensors could be between 0 and π. Critical density is the infimum density of nodes to prepare barrier coverage. Also, extensive simulations have been conducted to represent the results. The findings of this research could be used for offline design of directional sensor networks and also its online algorithms such as scheduling, coverage and tracking.

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Keywords: Critical density, continuum percolation, sensing coverage, network connectivity, adjustable-orientation directional sensor networks.

1. Introduction

Sensing coverage is one of the main criterion of quality of service in wireless sensor networks. Moreover, network connectivity is a graph-based problem to help sensors to communicate each other and forward their data to the sink.

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Preparing both sensing coverage and network connectivity are crucial to sense the region of interest and send the gathered data to the sink. There are several reviews e.g. [3] on sensing coverage and network connectivity issues.

In the other hand, Percolation Theory has been considered recently as a mathematical model to investigate coverage in wireless networks. Because Percolation Theory investigate the behavior of connected clusters in random graphs, it has been used in wireless networks e.g. [4] and also in sensor networks too e.g. [6-9,16]. Due to Broadbent and Hammersley [17] percolation model gave birth as a model for disordered mediums. In general, percolation theory is divided to two categories: discrete percolation [18] and continuum percolation [19]. In discrete percolation the edges of the lattice could be close or open due to a probability p and may have different tessellation such as square, triangle, honeycomb and we are interested in finding critical probability (p_c) in which percolation occurs. In contrast, in continuum percolation, the positions of the points are randomly distributed and we are interested in finding the critical density (λ_c) at which a large clump of objects first appears that spans the entire network.

Considering the orientation properties of sensor nodes, directional sensor networks could be classified to three categories: 1) Aligned-orientation directional sensor networks (ALODSNs) [9] in which the orientation of all sensor nodes is the same and fixed. 2) Fixed-orientation directional sensor networks (FIODSNs) [16] in which orientation of nodes is distributed on $[0, 2\pi]$ independently and uniformly and is fixed. 3) Adjustable-orientation directional sensor networks (ADODSNs) in which the deployment is like FIODSNs but the orientation of sensing sectors could be adjusted after deployment by using an algorithm. In [9,16], we analytically proposed a general approach to calculate the critical density for sensing coverage and network connectivity in ALODSNs and FIODSNs, respectively. In this paper, we present an analytical method to calculate the critical density of nodes for both sensing coverage phase transition (SCPT) and network connectivity phase transition (NCPT) in ADODSNs for all field-of-view angles between 0 and π using continuum percolation. Based on percolation theory, the critical density is infimum density of nodes such that for densities above it sensing coverage and network connectivity almost surely occur.

The remainder of this paper is organized as follows: section 2 gives a review on related works in the literature, section 3 presents terminology of the paper, section 4 illustrates our approach to calculate the critical density for sensing coverage in ADODSNs, Section 5 discusses on integrated sensing coverage and network connectivity in ADODSNs, section 6 presents the simulation results and finally section 7 concludes the paper. [†]

2. Related works

Due to Gilbert [1], the main concept of continuum percolation is finding the critical density in a network of objects distributed based on a Poisson point process at which an unbound spanning component almost surely appears that spans the region. The Gilbert's model has been considered as the basis for studying continuum percolation in wireless networks e.g. [4] and also examining coverage and connectivity in sensor networks [6-9,10-11,15-16].

As the first research in sensor networks, Ammari et al. [10] considered the critical density for sensing coverage and network connectivity in sensor networks simultaneously. They used continuum percolation in two-dimensional sensor networks consist of homogenous sensor nodes with communication radius of R and sensing radius r. Also, they extended their research for three-dimensional homogeneous sensor network [11].

Xing et al. [15] used continuum percolation to find the time when first partition would be occurred in network due to lack of power in sensor nodes and showed that it must be between $log\,(\log n)$ and $(log\ n\,)^{(1/p)}$ which n is the network size and $\rho > 1$. The result provides a theoretical upper bound for time of redeployment. Liu et al. [7] considered exposure-path prevention in omnidirectional sensor networks. Exposure-path refers to the path that an intruder could traverse without being detected by sensor nodes. They mapped exposure path problem into a bond percolation model and derived the critical density for two-dimensional sensor networks. In their next research, they also studied the exposure-path prevention in directional sensor networks [20].

Yang et al. [2] considered minimum number of sensor nodes needed to prepare connected-k-coverage in a randomly-deployed sensor network where sensors are active with probability p. They applied the percolation theory to

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† This paper is the third part of a three-part research to find critical density in three different kinds of directional sensor networks which its first two parts has been published in [9] for ALODSNs and in [16] for FIODSNs. Therefore, there might be some similarities.

connected-k-coverage problem and derived its critical density for different proportion of sensing radius to communication radius of sensors. Also, Balister et al. [8] introduced Trap Coverage that scales better for sensor networks that are deployed in large regions. A sensor network provides Trap Coverage if guarantees detecting any moving object that moves more than a known displacement for any trajectory and speed. They derived some estimates for the required density to achieve trap coverage with a given diameter for randomly-deployed sensor networks.

3. Terminology

This section defines the terms we will use throughout the paper and the network and percolation model.

Definition 1 (spatial Poisson point process) [12]. Let X_λ be a random variable representing the number of points in an area A. The probability that there are k points inside A is computed as

$$P(X_\lambda(A) = k) = \frac{\lambda^k |A|^k}{k!} e^{-\lambda|A|} \tag{1}$$

for all $k \geq 0$, where $|A|$ is the area of A and λ is the density per unit area. In this paper, center of sensor nodes (ξ_i) is considered as points in Poisson point process.

Definition 2 (directional sensor nodes). Each sensor node i is denoted by a tuple $S_i(\xi_i, r, \varphi, \overrightarrow{\vartheta_\iota}, R)$ where ξ_i is the center, r is the sensing radius, φ is the field-of-view angle ($0 < \varphi \leq \pi$), $\overrightarrow{\vartheta_\iota}$ is the orientation vector and R is the transmission radius of the sensor.

Definition 3 (sensing range). The sensing range of a directional sensor node $S_i(\xi_i, r, \varphi, \overrightarrow{\vartheta_\iota}, R)$ is a sector which defined by

$$C_i(r, \varphi, \overrightarrow{\vartheta_\iota}) = \left\{\xi \in \mathbb{R}^2 : |\xi_i - \xi| \leq r \text{ } and \text{ } \theta(\overrightarrow{\vartheta_\iota}, \overrightarrow{\xi_\iota \xi}) \leq \frac{\varphi}{2}\right\}$$

where $|\xi_i - \xi|$ stands for the Euclidean distance between ξ_i and ξ, $\theta(\overrightarrow{\vartheta_\iota}, \overrightarrow{\xi_\iota \xi})$ stands for angle between orientation vector of sensor ($\overrightarrow{\vartheta_\iota}$) and vector from point ξ_i to ξ (see Fig. 1 (a)).

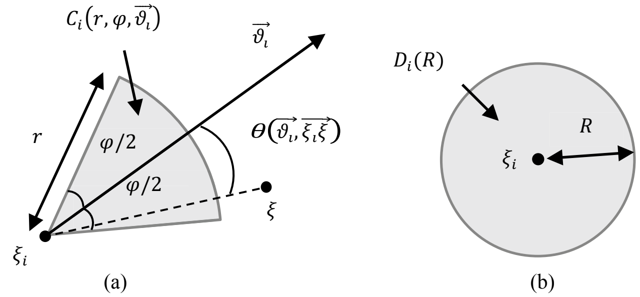

Fig. 1. (a) Sensing range (b) Transmission range.

Definition 4 (transmission range). The transmission range of sensor $S_i(\xi_i, r, \varphi, \overrightarrow{\vartheta_\iota}, R)$ is a disk of radius R which defined by

$$D_i(R) = \{\xi \in \mathbb{R}^2 : |\xi_i - \xi| \leq R\}$$

where $|\xi_i - \xi|$ stands for the Euclidean distance between ξ_i and ξ (see Fig. 1 (b)).

Definition 5 (Adjustable-Orientation Directional Sensor Network (ADODSN)). An ADODSN is a directional sensor network in which orientation of sensors is distributed on $[0,\ 2\pi]$ uniformly and independently. However, the orientation of sensors can be adjusted by using an algorithm after deployment.

Definition 6 (collaborating sensors). Two sensors S_i and S_j are said to be collaborating if and only if there exist some points belong to sensing range of both sensors (see Fig. 2 (b)).

$$Col(S_i) = \{S_j : \exists \xi \in \mathbb{R}^2, \xi \in C_i \text{ } and \text{ } \xi \in C_j\}$$

Definition 7 (communicating sensors). Two sensors S_i and S_j are said to be communicating if and only if the Euclidean distance between the centers of their transmission disks is less than R (see Fig. 2 (a)).

$$Com(S_i) = \{S_j : |\xi_i - \xi_j| \leq R\}$$

Definition 8 (filling factor) [13]. If each object has area equal to a, the filling factor of a homogenous Poisson point process with density equal to λ ($X_\lambda = \{\xi_i : i \geq 1\}$) is given by

$$\phi = 1 - e^{-\eta} \tag{2}$$

which is the mean fraction of region covered by the objects. In Eq. (2), η is the density of objects and is given by

$$\eta = \lambda a \tag{3}$$

where a is the area of objects and λ is the density of the Poisson point process.

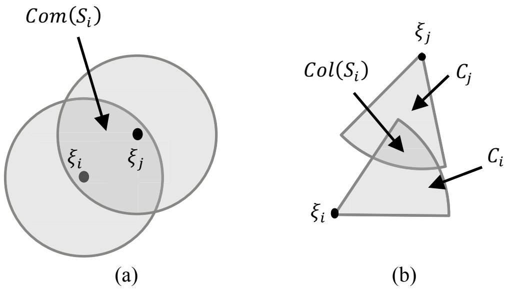

Fig. 2. (a) Communicating sensors (b) Collaborating sensors.

4. Critical density for sensing coverage in ADODSNs

Let $X_\lambda = \{\xi_i : i \geq 1\}$ be a two-dimensional homogeneous Poisson point process with density of λ (the number of sensors per unit area) where ξ_i represents the location of the sensor S_i.

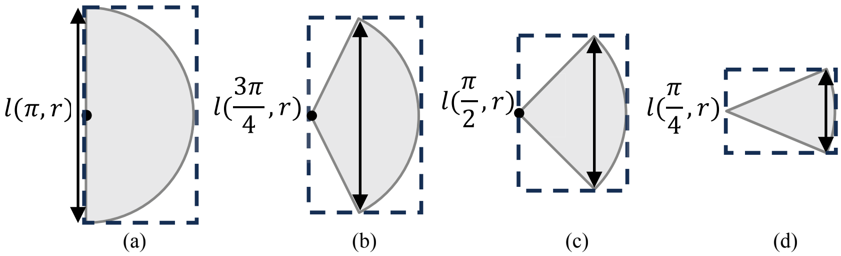

Fig. 3. $l(\varphi, r)$ for sensors with (a) $\varphi = \pi$, (b) $\varphi = 3\pi/4$, (c) $\varphi = \pi/2$, and (d) $\varphi = \pi/4$.

Table 1. Elongation in Circular Sectors for 8 Different φ.

φ	$l(\varphi, r)$ $(\times r)$	$l_{max}(\varphi, r)$ $(\times r)$
π	2.00000	2.0000
3π/4	1.84776	1.8478
π /2	1.41421	1.4142
π /4	0.76537	1.0000

4.1. Asymmetry of Sensing Range in Directional Sensors

Due to definition 3, sensing range of sensors in ADODSNs (in general, in directional sensors) is a circular sector. In this section we consider asymmetry of sensing ranges in directional sensor. In Fig. 3, $l(\varphi, r)$ is the length of front side of sensing sector and could be calculated by using law of cosine [15].

$$l(\varphi, r)^2 = r^2 + r^2 - 2r^2 \cos(\varphi)$$
$$l(\varphi, r) = \sqrt{2r^2(1 - \cos(\varphi))}$$
$$l(\varphi, r) = r\sqrt{2(1 - \cos(\varphi))} \tag{4}$$

where r is the radius of sensing sector and φ is the field-of-view angle.

As it is seen in Fig. 3, sectors have asymmetry and have vertical or horizontal elongation. To find this elongation, we assume that if we want to put the sector in a rectangle, which side will be the longest side of this rectangle. This will help us to find the length of biggest gap could be filled by a sensor in ADODSNs. As seen, the height of rectangle when $\varphi \in (\pi/3, \pi]$ is higher than r and therefore, is the longest side of rectangle. However, when $\varphi \in (0, \pi/3]$ the height of rectangle is less than or equal to r. Therefore, the longest side of rectangle is its width and its length is equal to r. Therefore, as a general equation, the longest side of the rectangle which represents the length of biggest gap could be filled by a sensor is given by

$$l_{max}(\varphi, r) = \begin{cases} r\sqrt{2(1-cos(\varphi))} & if \varphi \in \left(\frac{\pi}{3}, \pi\right] \\ r & if\ \varphi \in \left(0, \frac{\pi}{3}\right] \end{cases} \quad (5)$$

4.2. Critical Density for SCPT

In this section, we present two approaches to calculate number of points (which represents sensors in our article) in a Poisson point process and derive the critical density for SCPT in ADODSNs by equating them.

Lemma 1. In a Poisson point process, number of points could be found by following equation

$$\mathrm{N}^1 = \lambda|A| \quad (6)$$

where $|A|$ is the area of region of interest and λ is the density of Poisson point process.

Proof. Based on definition of Poisson Point Process [12], λ is the density of points per unit. This means in each unit there will exist λ point. Therefore, in a region with $|A|$ area, total number of point will be found by multiplication of these two

$$\mathrm{N}^1 = \lambda|A| \quad \blacksquare$$

Also, if we know the number of clusters and also number of points in each cluster, number of nodes could be found by another equation.

Lemma 2. In a Poisson point process, number of points could be found by following equation

$$\mathrm{N}^2 = \frac{|A|}{A_{cluster}} N_{cluster} \quad (7)$$

where $|A|$ is the area of the region, $A_{cluster}$ is the area of each cluster and $N_{cluster}$ is the number of points in each cluster.

Proof. If there is m cluster in a Poisson point process, number of nodes will be

$$\mathrm{N}^2 = m \times N_{cluster} \quad (8)$$

Also, if area of each cluster is equal to $A_{cluster}$, then number of clusters could derived by

$$m = \frac{|A|}{A_{cluster}}$$

Therefore, the Eq. (8) could be rewritten as

$$\mathrm{N}^2 = \frac{|A|}{A_{cluster}} N_{cluster} \quad \blacksquare$$

Theorem 1. In a Poisson point process, density of objects could be found by following equation

$$\eta = \frac{a}{A_{cluster}} N_{cluster} \quad (9)$$

where a is area of the objects, $A_{cluster}$ is the area of each cluster and $N_{cluster}$ is number of points in each cluster.

Proof. From Eq. (6) and (7), the number of points at critical percolation should verify the equality $\mathrm{N}^1 = \mathrm{N}^2$. Therefore, we have

$$\lambda|A| = \frac{|A|}{A_{cluster}} N_{cluster}$$

$$\lambda = \frac{1}{A_{cluster}} N_{cluster} \quad (10)$$

We simply multiply the sensing range area of sensors (a) in both sides of Eq. (10).

$$\lambda a = \frac{a}{A_{cluster}} N_{cluster} \tag{11}$$

By using definition 8, we can rewrite Eq. (11) as

$$\eta = \frac{a}{A_{cluster}} N_{cluster}$$ ■

Table 2. Critical density and filling factor for 8 different φ of table 1.

φ	η_c	ϕ_c
π	0.5000000	0.3934693
3π/4	0.4393398	0.3555383
π /2	0.5000000	0.3934693
π /4	0.5000000	0.3934693

Consider Fig. 4 as a directional sensor networks in which nodes have ability to rotate their orientations. By rotation, a node can adjust its situation to fill a gap in the network (in other word, to make collaboration with neighboring nodes). As it is seen, rotation of a directional node creates a circle called rotation circle (C). We are interested in finding the diameter of this circle. As it is shown in section 3.1, the length of biggest gap could be filled by a directional sensor (diameter of the rotation circle) is its elongation and could be found by using Eq. (5). Therefore, if there is (at least) one node in each circle with diameter found by Eq. (5), percolation occurs. In other word, if there exists (at least) one sensor ($N_{cluster}$) in each cluster (rotation circle) then sensors could fill the gaps between their neighbors by rotation and percolation occurs. Therefore, we have

$$\eta_c = \frac{a}{\pi {R_c}^2} N_{cluster} \tag{12}$$

where $N_{cluster} = 1$, $a = \frac{1}{2}\varphi r^2$ and R_c is the radius of rotation circle and is equal to half of corresponding elongation, $R_c = \frac{l_{max}(\varphi,r)}{2}$. By replacing values in Eq. (12), we reach to

$$\eta_c(\varphi,r) = \begin{cases} \dfrac{\frac{1}{2}\varphi r^2}{\pi\left(\dfrac{r\sqrt{2(1-cos(\varphi))}}{2}\right)^2} & if \varphi \in \left(\frac{\pi}{3},\pi\right] \\ \dfrac{\frac{1}{2}\varphi r^2}{\pi\left(\frac{r}{2}\right)^2} & if\ \varphi \in \left(0,\frac{\pi}{3}\right] \end{cases} \tag{13}$$

Finally, after simplifications on Eq. (13) we have

$$\eta_c(\varphi,r) = \begin{cases} \dfrac{\varphi}{\pi(1-cos(\varphi))} & if \varphi \in \left(\frac{\pi}{3},\pi\right] \\ \dfrac{2\varphi}{\pi} & if\ \varphi \in \left(0,\frac{\pi}{3}\right] \end{cases} \tag{14}$$

Table 2 shows the filling factor and density of nodes for typical field-of-view angles of table 1 by using Eq. (14).

5. Integrated sensing coverage and network connectivity in ADODSNs

Connectivity between two collaborating sensors depends on their distance. Due to randomly-deployment of sensors in ADODSNs, orientation of nodes could be unpredictable. Therefore, to prepare network connectivity, we must consider the worst case of collaborating sensors. Like FIODSNs [16], regardless of field-of-view angle of sensors, the maximum distance of two collaborating sensors in ADODSNs is equal to $2r$. Therefore, in an ADODSN with critical density of nodes for sensing coverage (SCPT) calculated by Eq. (14), when $R \geq 2r$ almost surely network connectivity (NCPT) percolates too.

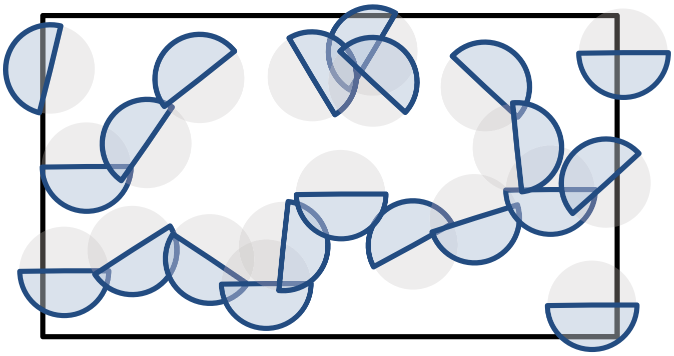

Fig. 4. Forming of spanning cluster in ADODSNs with $\varphi = \pi$.

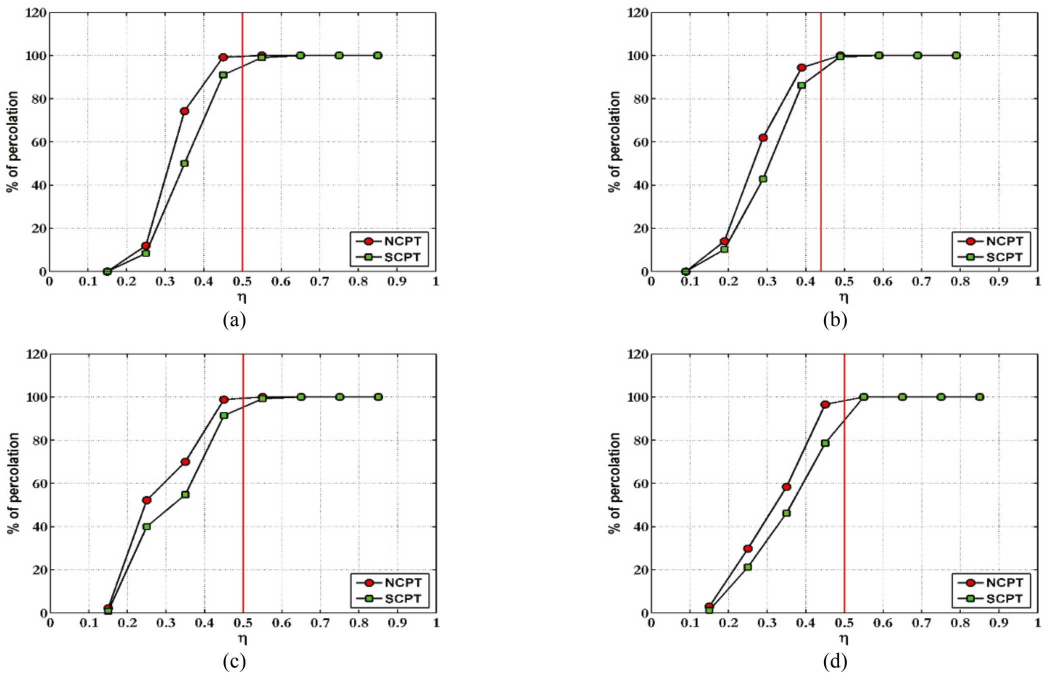

Fig. 5. Plot of simulation results for ADODSNs when a) $\varphi = \pi$, b) $\varphi = 3\pi/4$, c) $\varphi = \pi/2$ and d) $\varphi = \pi/4$.

6. Simulations

As [9,16], to examine the calculated critical densities, we simulated the percolation phase transition for all eight different field-of-view angles of table 3. As [9,16], we set the size of simulation environment to a square 50×50, sensing radius (r) to 5 and transmission radius to $R = 2r$. The density interval for simulation has been set to 0.1 and the simulation repeated 500 times for each step. The percolation charts of SCPT and NCPT have been shown in Fig. 5. In all charts, the vertical line shows the calculated critical density for corresponding φ by using Eq. (14).

There are several algorithms to choose the appropriate orientation of each sensor in a directional sensor networks and create the global covered component (the barrier line) [25]. To examine occurrence of the percolation (barrier line) in each iteration, we used the proposed algorithm by Dan Tao et al. [26].

As seen, in all charts, the percolation for sensing coverage almost surely occurs on or very close to calculated critical densities which are reasonable for an analytical and general equation. Also, as seen, network connectivity will be maintained almost surely for all different φ on or after calculated critical densities.

7. Conclusion

In this paper, we considered adjustable-orientation directional sensor networks (ADODSNs) in which nodes are deployed based on Poisson Point Process and the orientation of sensor nodes is distributed on $[0, 2\pi]$ independently and uniformly. But, the orientation of sensors could be adjusted by using an algorithm after deployment. We proposed a mathematical method to calculate the critical density for sensing coverage and network connectivity in ADODSNs by using continuum percolation while field-of-view angles could be between 0 and π. The simulations confirmed that percolation occurs on or very close to the estimated critical densities.

Possible future works include finding critical density for sensing coverage and network connectivity in directional sensor networks with directional transmission antenna instead of common disk model, heterogeneous directional sensor networks and also three-dimensional directional sensor networks.

References

1. E.N. Gilbert. (1961) “Random Plane Networks.” *J. SIAM* **9** (**4**) 533-543.
2. G. Yang, D. Qiao. (2008) “Critical conditions for connected-k-coverage in sensor networks.” *IEEE Communications Letters* **12** (**9**): 651-653.
3. C. Zhu, C. Zheng, L. Shu, G. Han. (2012) “A survey on coverage and connectivity issues in wireless sensor networks.” *Journal of Network and Computer Applications* **35** (**2**): 619-632.
4. I. Glauche, W. Krause, R. Sollacher, M. Greiner. (2003) “Continuum percolation of wireless ad hoc communication networks.” *Physica A* **325**: 577-600.
5. D. Tao, T.-Y. Wu. (2015) “A survey on barrier coverage problem in directional sensor networks.” *IEEE Sensors Journal*, **15** (**2**): 876-885.
6. D. Tao, S. Tang, H. Zhang, X. Mao, H. Ma. (2012) “Strong barrier coverage in directional sensor networks.” *Computer Communications* **35** (**8**): 895-905.
7. L. Liu, X. Zhang, H. Ma. (2013) “Percolation theory-based exposure-path prevention for wireless sensor networks coverage in internet of things.” *IEEE Sensors Journal* **13** (**10**): 3625-3636.
8. P. Balister, Z. Zheng, S. Kumar, P. Sinha. (2009) “Trap coverage: allowing coverage holes of bounded diameter in wireless sensor networks.” *IEEE INFOCOM*: 136-144.
9. M. Khanjary, M. Sabaei, M.R. Meybodi. (2014) “Critical density for coverage and connectivity in two-dimensional aligned-orientation directional sensor networks using continuum percolation.” *IEEE Sensors Journal* **14** (**8**): 2856-2863.
10. H. M. Ammari, S. K. Das. (2008) “Integrated coverage and connectivity in wireless sensor networks: A two-dimensional percolation problem.” IEEE Transactions on Computers 7 (10): 1423-1434.
11. H. M. Ammari, S. K. Das. (2009) “Critical density for coverage and connectivity in three-dimensional wireless sensor networks using continuum percolation.” IEEE Transactions on Parallel and Distributed Systems 20 (6): 872-885.
12. R.E. Miles. (1970) “On the homogeneous planar Poisson point process.” Mathematical Biosciences (6): 85-127.
13. P. Hall. (1988) “Introduction to the theory of coverage processes.” John Wiley & Sons.
14. S. Mertens, C. Moore. (2012) “Continuum percolation thresholds in two dimensions.” Phys. Rev. E 86: 061109.
15. F. Xing, W. Wang. (2008) “On the critical phase transition time of wireless multi-hop networks with random failures.” ACM MOBICOM: 175-186.
16. M. Khanjary, M. Sabaei, M. R. Meybodi. (2015) “Critical density for coverage and connectivity in two-dimensional fixed-orientation directional sensor networks using continuum percolation.” Journal of Network and Computer Applications 57: 169-181.
17. S. R. Broadbent, J. M. Hammersley. (1957) “Percolation processes: I. crystals and mazes.” Cambridge Philosophical Society 53: 629-641.
18. G. Grimmett. (1989) “Percolation.” Springer Verlag.
19. R. Meester, R. Roy. (1996) “Continuum percolation.” Cambridge University Press.
20. L. Liang, Z. Xi, M. Huadong. (2009) “Exposure-path prevention in directional sensor networks using sector model based percolation.” IEEE ICC: 1-5.