

Distributed learning automata based algorithm for community detection in complex networks

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Community structure is an important and universal topological property of many complex networks such as social and information networks. The detection of communities of a network is a significant technique for understanding the structure and function of networks. In this paper, we propose an algorithm based on distributed learning automata for community detection (DLACD) in complex networks. In the proposed algorithm, each vertex of network is supposed to be isomorphic with learning automata. According to the cooperation among network of learning automata and updating action probabilities of each automaton, the algorithm interactively tries to identify high-density local communities. The performance of the proposed algorithm is investigated through a number of simulations on popular synthetic and real networks. Experimental results in comparison with popular community detection algorithms such as walk trap, Danon greedy optimization, Fuzzy community detection, Multi-resolution community detection and label propagation demonstrated the superiority of DLACD in terms of modularity, NMI, performance, min-max-cut and coverage.

Keywords: Complex networks; online social networks; social network analysis; community detection; distributed learning automata.

1. Introduction

Many complex phenomena in real-world systems are modeled and represented as networks such as ecological, biological, technological and information and social networks¹⁻³. These real world networks can be defined as a graph with a set of vertices (*e.g.*, users in an online social networks) and edge-sets (*e.g.*, a particular type of relationship between actors in an online social networks)⁴. It is shown that in variants of

real world networks, there are common universal characteristics such as small world phenomena⁵, small shortest path lengths in average manner⁶, power law degree distribution⁷ and specially existence of community structures in networks⁸. Community structure refers to a set of vertices whose edges inside are more densely than edges outside⁹. Finding community in a network known as a community detection problem (also called community structure identification or cluster finding). Community detection plays a significant role for studying and understanding the structure and function of real world networks including several various domains such as clustering web stations having similar functionality and are geographically near to each other may improve the performance of services provided on the World Wide Web¹⁰. Online social networks often consist of a set of communities based on common properties of users such as locations, hobbies, interests, activities and carriers¹¹. Citation networks form communities on the basis of similar research interest topics between authors¹². Communities (modules) in protein–protein interaction (PPI) networks may correspond to known or even unknown functional modules/protein complexes with significant functionality in cellular biological systems¹³. Therefore, the detection of the community structure in a network has important practical applications and can help researchers to understand the organization and function of complex network systems.

Detecting the communities in complex networks due to the wide spread of applications have been received a great attentions in literature by scholars^{14–17}. A good review for community detection consists of techniques and applications is the one presented by *Fortunato*⁸. This paper classified community detection methods into five categories including traditional algorithms, hierarchical algorithms, modularity-based methods, spectral algorithms and dynamic algorithms. Among the all types of community detection approaches, hierarchical clustering techniques are widely used techniques which put similar vertices into larger communities. Hierarchical clustering algorithms including two categories of divisive and agglomerative form the communities gradually in a hierarchical manner. Several scholars improved the hierarchical algorithms using some metrics to select a suitable partition or a proper set of partitions that satisfies particular metrics such as the number of desired communities, the maximum (or minimum) number of vertices in each community and optimize an objective function⁸. Divisive techniques try to find the edges that connect vertices of different communities and iteratively eliminate them, so that the communities separated from each other. *Girvan* and *Newman* have introduced a famous divisive method¹² which includes the removal of the edges based on their values of edge betweenness. An agglomerative method, however, tries to form communities in a bottom up manner. In general, the common notion of agglomerative methods is to partition vertices into communities iteratively starting from a partition in which communities are composed of a single vertex. The process of partitioning vertices continues until a single community consists of all vertices of input network is achieved¹⁸. Most of agglomerative algorithms select the best partition that maximizes a typical quality objective function. One of the best known quality function and the most widely used quality function is a *modularity* metric which is proposed by

*Newman et.al*¹⁸. Using modularity, *Clauset, et. al* proposed a fast greedy modularity optimization method¹⁹ which starting from a set of isolated vertices and then a pair of vertices iteratively connected to each other such that it achieves the maximum possible value of modularity at each step. Although using this measure by modularity optimization achieves many promising results for community detection, it is shown that it has some limitations such as resolution limit. For example in the extreme case, the modularity optimization algorithms are failed for a network with several cliques connected by a single edge²⁰.

Another direction of research for community detection algorithms is devoted to random walk based methods. Random walk based method consider not only the topological structure of network but also the local neighborhood properties of vertices. So it can be proper for community detection in some types of complex networks. *Hagen et. al*²¹ used random walks to detect cluster circuits in VLSI design. They defined a concept of cycle to find communities in circuit network. A new clustering method for complex network called CONCLUDE, is presented by *De Meo et al.*²² which combines the scalability of local algorithms and accuracy of global approaches. *Harel et. al*²³ introduced a new definition based on random walks for community. Then, they proposed two operators that change the weights of the edges and reduce the weights of inter-community edges. By enforcing them iteratively the inter-community edge weights are reduced until they are trivial. Finally, the community structure is revealed by omitting the near-zero weight edges. In²⁵, a divisive methods presented by employing edge centrality based on random walks. It first removes the connected edges with a leaf vertex, and then finds edge bridges between communities and removes them. Some methods try to enhance the performance of random walk based methods by combining random walks with well-known algorithms such as k -path edge-centrality²⁶. However, these hybrid methods have a high time complexity which is infeasible for real application of real world networks. Another efficient random walk based algorithm is walk trap²⁷ in which the distance between two vertices is defined in terms of random walk process. The basic idea is that if two vertices, i and j , are in the same community, the probability to get a third vertex k , in the same community through a random walk should not be very different for i and j .

In this paper, an algorithm based on distributed learning automata proposed for detecting communities in complex networks. Since the distributed learning automata serves in a stochastic environment equivalent with unknown communities in the environment of complex networks, according to the capabilities of learning automata for solving many complicated problems, it can be useful for finding community in complex networks. In the proposed algorithm, a learning automaton is assigned to each vertex of the network, during the execution of the proposed algorithm, a set of learning automata cooperate with each other to find proper communities in the given network. In order to study the performance of the proposed algorithm, several experiments conducted on the well-known complex network datasets. And then, the proposed algorithm is compared with several community detection algorithms such as walk trap²⁷, *Danon* greedy optimization²⁸, Fuzzy community detection²⁹, Multi-resolution community detection³⁰ and label propagation³¹. Experimental results show that the proposed algorithm outperforms the

other community detection algorithms in terms of modularity, NMI, performance, min-max-cut and coverage.

The rest of this paper is organized as follows. In section 2, the learning automata and distributed learning automata are introduced in brief. The proposed community detection algorithm based on distributed learning automata for complex networks is described in Section 3. In Section 4, the performance of the proposed algorithm is compared to other community detection algorithms on the well-known real and synthetic networks with respect to some well-known measures. Finally, section 5 concludes this paper.

2. Learning Automata

In this section, we briefly describe the learning automata and the interested reader may refer to Ref. ³² for more information about learning automata models, algorithms, application and theoretical aspects. A learning automaton (LA) ³² is an adaptive decision-making unit that improves its performance by learning how to choose the optimal action from a finite set of allowed actions through repeated interactions with a random environment. The action is chosen at random based on a probability distribution kept over the action-set and at each instant the given action is served as the input to the random environment. The environment responds the taken action in turn with a reinforcement signal. The action probability vector is updated based on the reinforcement feedback from the environment. The objective of a learning automaton is to find the optimal action from the action-set so that the average penalty received from the environment is minimized. The environment can be described by a triple $E=\{\alpha, \beta, c\}$ where $\alpha=\{\alpha_1, \alpha_2, \dots, \alpha_r\}$ represents the finite set of the inputs, $\beta=\{\beta_1, \beta_2, \dots, \beta_m\}$ denotes the set of values can be taken by the reinforcement signal, and $c=\{c_1, c_2, \dots, c_m\}$ denotes the set of the penalty probabilities, where the element c_i is associated with the given action α_i . If the penalty probabilities are constant, the random environment is said to be a stationary random environment, and if they vary with time, the environment is called a non-stationary environment. The environments depending on the nature of the reinforcement signal β can be classified into P-model, Q-model and S-model. The environments in which the reinforcement signal can only take two binary values 0 and 1 are referred to as P-model environments. Another class of the environment allows a finite number of the values in the interval [0, 1] can be taken by the reinforcement signal. Such an environment is referred to as Q-model environment. In S-model environments, the reinforcement signal lies in the interval [0, 1]. The relationship between the learning automaton and its random environment has been shown in figure 1.

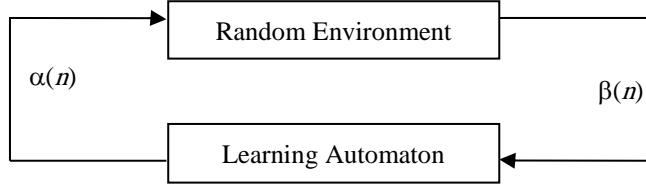


Fig. 1. The relationship between the learning automata and its random environment.

Learning automata can be classified into two main families: fixed structure learning automata and variable structure learning automata. Variable structure learning automata are represented by a triple $\langle \beta, \alpha, T \rangle$ where β is the set of inputs, α is the set of actions, and T is learning algorithm. The learning algorithm is a recurrence relation which is used to modify the action probability vector. Let $\alpha(k)$ denotes the action chosen at instant k and $p(k)$ being the corresponding action probability vector. Suppose $a(k)$ and $b(k)$ denote the reward and penalty parameters, respectively and let $\alpha_i(k)$ be the action chosen by the automata at instant k . When the taken action is rewarded by the environment (i.e., $\beta(k)=0$), the action probability vector, $p(k)$, is updated by the recurrence equation

$$p_j(k+1) = \begin{cases} p_j(k) + a(k)[1 - p_j(k)] & \text{if } i = j \\ p_j(k) - a(k)p_j(k) & \text{if } i \neq j \end{cases} \quad (1)$$

which is a linear learning algorithm. Similarly, in case of penalizing (i.e., $\beta(k)=1$), the update formula would be

$$p_j(k+1) = \begin{cases} p_j(k)(1 - b(k)) & \text{if } i = j \\ \frac{b}{r-1} + p_j(k)(1 - b(k)) & \text{if } i \neq j \end{cases} \quad (2)$$

In (1) and (2), r is the number of actions can be chosen by the automaton.

Based on the value of $a(k)$ and $b(k)$, three types of learning algorithm have been defined. If the $a(k)$ equals to $b(k)$ then recurrence equations (1) and (2) is called linear reward-penalty (L_{R-P}) algorithm, if $a(k) \gg b(k)$ the given equations are called linear reward- ϵ penalty ($L_{R-\epsilon P}$) and finally if $b(k) = 0$ they are called linear reward-Inaction (L_{R-I}). In the latter case, the action probability vectors remain unchanged when the taken action is penalized by the environment.

2.1. Variable action set learning automata

If the number of action set for a learning automata change over the time, it is called a variable action set learning automata. In variable action set a learning automata consist of finite set of n actions, $\alpha = \{\alpha_1, \alpha_2, \dots, \alpha_n\}$. Let $A = \{A_1, A_2, \dots, A_m\}$ be the set of actions which can be selected and $A(k) \subseteq \alpha$ be the subset of all actions can be selected by a specific learning automata, at each instance k . According to the probability distribution

$q(k) = \{q_1(k), q_2(k), \dots, q_m(k)\}$, an external agency chooses a particular action subset randomly, where, $q_i(k) = \text{prob}[A(k) = A_i / A_i \in A, 1 \leq i \leq 2^n - 1]$. Moreover, $\hat{p}_i(k)$ is defined as the probability of choosing action α_i , if the action subset $A(k)$ has already been selected and $\alpha_i \in A(k)$. The scaled probability $\hat{p}_i(k)$ is also defined as

$$\hat{p}_i(k) = \frac{p_i(k)}{K(k)} \quad (3)$$

where $K(k) = \sum_{\alpha_i \in A(k)} p_i(k)$ is the sum of the probabilities of the actions in subset $A(k)$, and $p_i(k) = \text{prob}[\alpha(k) = \alpha_i]$.

2.2. Distributed learning automata

In this section, we introduce a new model of interconnected automata calling distributed learning automata (DLA)³³. Distributed learning automata is a network of learning automata which collectively cooperate to solve a particular problem with many applications including: Vehicle Routing Problem³⁴, wireless networks,³⁵⁻³⁷, data mining³⁸, information retrieval³⁹, complex networks^{2,40} and grid computing⁴¹. A DLA can be modeled by a directed graph in which the set of vertices of graph constitutes the set of actions for corresponding automaton. When a learning automaton selects one of its actions, another learning automaton on the other end of edge corresponding to the selected action will be activated. Formally, a DLA with n learning automata can be defined by a graph (A, E) , where $A = \{A_1, A_2, \dots, A_n\}$ is the set of learning automata and $E \subset A \times A$ is the set of edges in the graph which an edge (i, j) corresponds to action α_i^j of automaton A_i . Let action probability vector for learning automaton A_j is represented by p_j where a component p_j^m of p_j denotes the probability of choosing action α_j^m that is the probability of choosing edge (j, m) . An example of DLA is given in figure 2, in which every automaton has two actions. If automatons A_1 select action α_1^2 then automaton A_2 will be activate. Activated automaton A_2 chooses one of its actions which in turn it activates one of the automata connected to A_2 . At any time only one automaton in the network will be activated.

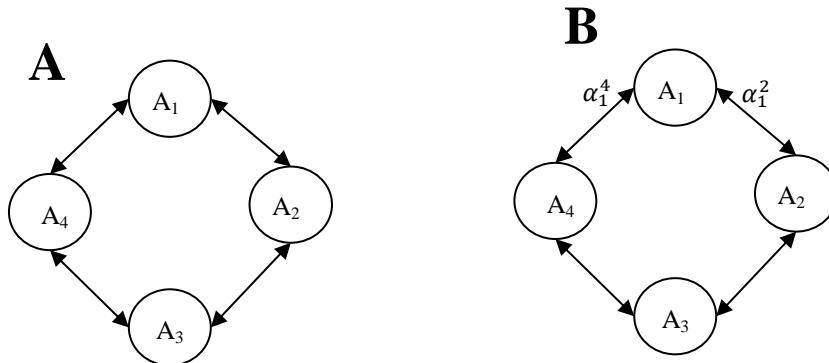


Fig. 2. Distributed learning automata.

3. Proposed Community Detection Algorithm Based on Distributed Learning Automaton

In this section, we describe the proposed algorithm based on distributed learning automaton for finding communities in complex networks. It is assumed that the input network $G=\langle V, E \rangle$ is an undirected and un-weighted network where $V=\{v_1, v_2, \dots, v_n\}$ is the set of vertices and $E \subseteq V \times V$ is the set of edges in the given network. The proposed distributed learning automaton algorithm including four steps tries to find iteratively a set of communities that are more densely connected internally to each other than to the rest of the network. After the initialization step is performed, by assigning a learning automaton to the vertices of input network, the proposed algorithm repeats community finding by doing a guided traversal in the network with the help of distributed learning automata, evaluates the set of found communities and updates their action probability vectors iteratively until stopping criteria are satisfied. We describe four steps of the proposed algorithm in the following subsections in details.

3.1. Initialization

In the first step, a distributed learning automata $\langle A, \alpha \rangle$ which is isomorphic to the input network is constructed. The resulting network can be defined by 2-tuple $\langle A, \alpha \rangle$ where $A=\{A_1, A_2, \dots, A_n\}$ is the set of learning automata corresponding to the set of vertices, $\alpha = \{\alpha_1, \alpha_2, \dots, \alpha_n\}$ denotes the set of actions in which $\alpha_i = \{\alpha_i^1, \alpha_i^2, \dots, \alpha_i^{r_i}\}$ are the set of actions that can be taken by learning automaton A_i and r_i is the number of actions that can be taken by learning automaton A_i . An action of a learning automaton A_i corresponds to choosing an adjacent vertex of the corresponding vertex v_i . Let $p(v_i) = \{p_i^1, p_i^2, \dots, p_i^{r_i}\}$ be the action probability vector of learning automaton A_i and $p_i^j = 1/r_i$ equally initialized for all j . At all iterations each learning automaton can be in either active or inactive mode. At the beginning of the proposed algorithm all learning automata initially are set in inactive mode. Let C_k be the set of vertices in the k^{th} community and initially is set to be empty, G' represent the set of unvisited vertices in the execution of algorithm and is initially equal to G and also π' is the path of visited vertices at the iteration t .

3.2. Finding communities

At t^{th} iteration of this step, the algorithm finds k communities in such a way that the proposed algorithm starts with randomly selecting vertex v_i among unvisited vertex set G' and the selected vertex v_i is inserted in the set of current community C_k and current path π' . Then, learning automaton A_i corresponds to the starting vertex v_i is activated and chooses one of adjacent vertex v_i according to its action probability vector. Let the chosen action by learning automaton A_i be vertex v_j . If the number of internal connections for union of selected vertex v_j and current community C_k is greater than the number of internal connections for current community C_k then v_j is inserted to the set of current community C_k , C_k is removed from set G' and also visited vertex v_j is updated in path π' .

The process of activating an automaton, choosing an action, checking the condition of inserting chosen vertex v_j in the current community C_k , inserting new vertex v_j to C_k , updating visited vertex v_j in path π' and removing C_k from set G' is repeated until total number of edges inside the current community C_k is more than the total number of edges outside the current community C_k or active learning automaton could not select any action. The process of finding new communities according to the above description and updating path of visited vertices at the current iteration π' is continued when the union of all vertex-set of found communities is equal to the input network G .

3.3. Computing objective function

Let $C^t = \{C_1, C_2, \dots, C_k\}$ be the set of k communities found at the iteration t . The quality of the set of communities found at the iteration t is evaluated via normalized cut as objective function⁴² by following equation

$$NC(C^t) = \frac{1}{k} \sum_{i=1}^k \frac{cut(C_i, \bar{C}_i)}{vol(C_i)} \quad (4)$$

where $cut(C_i, \bar{C}_i)$ denotes the number of edges between communities C_i and $\bar{C}_i = G \setminus C_i$, $vol(C_i)$ is the total degree of vertices that are the members of community C_i and also k is the number of communities. Since mentioned in Ref.⁴⁷, normalized cut with low complexity considers extracting the global impression of the network, instead of local features and measures both the total dissimilarity between the different communities as well as the total similarity within the communities. So, using the proposed algorithm can gradually decreased normalized cut which means that the algorithm gradually converges to the minimum normalized cut and approach to the proper set of communities.

3.4. Updating action probability vectors

In this step, the set of k communities found at the iteration t is evaluated via corresponding normalized cut and if the value of the normalize cut at current iteration $NC(C^t)$ is less than or equal to the value of normalized cut at previous iteration $NC(C^{t-1})$, then the chosen action along the path π' by all the activated learning automata are rewarded according to the learning algorithm described in section 2 and penalized otherwise.

3.5. Stopping criteria

The proposed algorithm iterates steps 2, 3 and 4 until the number of iterations exceeds a given threshold T or $P_t = \prod_{v_i \in C^t} \max_{v_j \in N(v_i)} (p_i^j)$ at iteration t becomes greater than a particular threshold τ where p_i^j is the probability of choosing neighboring vertex v_j by learning automaton A_i residing in vertex v_i and $N(v_i)$ is the set of neighboring vertex v_i . Figure 3 shows the pseudo-code for the proposed community detection algorithm for complex networks.

Algorithm 1. proposed algorithm for community detection in complex networks
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Input: A network  $G=(V, E)$ , Thresholds  $\tau, T$  //  $\tau$ : stopping threshold for product of probabilities,  $T$ :maximum iteration number
Output: Set of found communities  $C^*$ 
Assumptions
Assign an automaton  $A_i$  to each vertex  $v_i$ ;
Let  $k$  is the number of communities;
Let  $C_k$  is the set of  $k^{\text{th}}$  community and initially set to empty;
Let  $t$  is the iteration number of algorithm and initially set to 0;
Let  $NC(C^t)$  is the normalized cut value for set of communities found at iteration  $t$  and initially set to 0;
Let  $P_t$  is the product of maximum probabilities in probability vector of learning automata the vertices of a set of communities at iteration  $t$  and initially set to 0;
Let  $\pi^t$  is the path of visited vertices by the algorithm at iteration  $t$  and initially set to empty;
Begin
 $G' \leftarrow G$ ; // set of unvisited vertices
All learning automata initially are set inactive mode;
While ( $t < T$  or  $P_t < \tau$ )
    Repeat
         $t \leftarrow t + 1$ ;
         $k \leftarrow 1$ ;
        Vertex  $v_i$  is selected randomly from  $G'$ ;
         $C_k \leftarrow C_k \cup v_i$ ;
         $\pi^t \leftarrow \pi^t \cup v_i$ ;
        While ( $d_{in}(C_k) < d_{out}(C_k)$  AND  $|\alpha_i| \neq 0$ ) Do // Finding  $k^{\text{th}}$  community
            Automaton  $A_i$  is activated and then choose an action using its action probability vector;
            Let the chosen action by  $A_i$  be  $v_j$ ;
            If ( $d_{in}(C_k \cup v_j) > d_{in}(C_k)$  AND  $d_{out}(C_k \cup v_j) < d_{out}(C_k)$ ) then //checking internal and external connections
                 $C_k \leftarrow C_k \cup v_j$ ;
                 $\pi^t \leftarrow \pi^t \cup v_j$ ;
                 $v_i \leftarrow v_j$ ;
            End If
        End while
         $G' \leftarrow G' \setminus C_k$ ;
         $k \leftarrow k + 1$ ;
    Until ( $|G'| = 0$ )
    Compute  $NC(C^t)$  according to equation (4);
    If ( $NC(C^t) < NC(C^{t-1})$ )
        Reward the actions chosen along the path  $\pi_t$  by all the activated learning automata;
    Else
        Penalize the actions chosen along the path  $\pi_t$  by all the activated learning automata;
    End If
    Compute  $P_t = \prod_{v_i \in C^t} \max_{v_j \in N(v_i)} (p_i^j)$ ;
    Set all learning automata in inactive mode;
End while
 $C^* \leftarrow C^t$ 
End Algorithm

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Fig. 3. Pseudo-code of proposed algorithm for community detection in complex network.

4. Simulation Results

In order to study the performance of the proposed distributed learning automaton based algorithm for community detection (DLACD), we conducted a number of experiments on the well-known real and synthetic modular networks. Table 1 describes the set of real test networks that are used for experiments including the popular real networks: *Karate*⁴³, *Dolphins*⁴⁴, *Books*⁴⁵, *Football*¹², *Reactome*⁴¹ and *Les-miserables*⁴⁵ and also *LFRI* and *LFR2* as LFR benchmark networks⁴² for synthetic modular networks. To assess the performance of the proposed algorithm and other community detection algorithms, we applied several commonly used standard measures consist of modularity, coverage, Min-max-cut, performance and NMI. In the next subsection these measures for assessing finding communities will be described in brief.

Table 1. Description of the test networks used for the experiments.

Networks	Vertex	Edge	Description
<i>Karate</i>	34	78	Zachary karate club network.
<i>Dolphins</i>	62	159	The network of dolphins.
<i>Les-miserables</i>	77	254	Co-appearance network of characters in the novel Les Miserables.
<i>Books</i>	105	441	The network of American Politics Books.
<i>Football</i>	115	615	Network of American college football teams.
<i>Reactome</i>	6327	147547	Network of protein–protein interactions in Humans.
<i>LFRI</i>	5000	38160	Synthetic modular benchmark.
<i>LFR2</i>	5000	250000	Synthetic modular benchmark.

4.1. Evaluation Measures

In the following subsections, we will describe evaluation measures for assessing found communities by the algorithms.

4.1.1. Modularity

The first measure is *Modularity* Q ¹⁸ as the popular measure for evaluating set of communities in network found by the algorithm. This measure is defined as follows

$$Q = \frac{1}{2m} \sum_{C \in P} \sum_{v_i, v_j \in C} \left[A_{i,j} - \frac{k_i k_j}{2m} \right] \quad (5)$$

where A is the adjacency matrix that $A_{i,j}$ is equal to 1 if there is an edge between vertex v_i and vertex v_j and zero otherwise. $k_i = \sum_j A_{ij}$ is the degree of vertex v_i and m is the total number of edges in the network. The summation is over all pairs of vertices that member of the same community C of partitioning P .

4.1.2. Coverage

The *coverage* measure⁴⁴ defined as the fraction of edges inside a community compared with total number of edges in the whole network and defined as following

$$Coverage = \frac{\sum_{i=1}^k E_i}{E} \quad (6)$$

where k is the number of communities obtained from algorithm. In this fashion a good partitioning of the network could be maximized the value of coverage.

4.1.3. Min-Max-Cut

The *Min-Max-Cut*⁴⁵ is the fraction of between community edges compared to the number of edges associated to the different communities. The minimizing value of the Min-Max-Cut results in having a good partitioning of communities. The Min-Max-Cut is defined by

$$Min - Max - Cut = \sum_{i=1}^k \frac{E'_i}{E_i} \quad (7)$$

where k is the number of communities obtained by the algorithm, E_i is the number of edges of community C_i and E'_i is the number of edges between community C_i and the rest of communities.

4.1.4. Performance

The *performance*⁴⁴ evaluates the rate of edges that set on the basis of the partitioning of the network well. Performance is calculated as follows

$$performance = 1 - \frac{2(T + F)}{n(n - 1)} \quad (8)$$

where T is the number of non-adjacent pairs (v_i, v_j) in the same community and F is the number of edges between community. Hence, a good partitioning should be maximized the performance.

4.1.5. Normalized Mutual Information (NMI)

Normalized Mutual Information (NMI)⁴³ measures the similarity between known set of communities and set of communities found by the algorithm, where A and B are two partitions of the input network, and NMI is a value between [0, 1], the higher value indicates the partitions A and B are totally independent.

$$NMI(A, B) = \frac{-2 \sum_{a \in A} \sum_{b \in B} |a \cap b| \log \left(\frac{|a \cap b| n}{|a||b|} \right)}{\sum_{a \in A} |a| \log \left(\frac{|a|}{n} \right) + \sum_{b \in B} |b| \log \left(\frac{|b|}{n} \right)}. \quad (9)$$

This measure is useful for artificial networks with a prior knowledge about built-in communities.

4.2. Experimental setup

To investigate the performance of the proposed algorithm, we employed several simulations on the well-known real and synthetic networks. In all experiments presented in this paper, the learning scheme is L_{R-I} and the learning rate is set to be 0.02. The maximum threshold τ is set to 0.9 and maximum iteration T is set to $n \times 1000$ where n is the number of vertices of graph. For LFR benchmark parameters, N is the number of vertices, K is average degree, max_k is maximum degree of nodes, min_c is minimum size of communities, max_c is maximum size of communities and μ is mixing parameter. Each vertex shares a fraction of $1 - \mu$ connections with the other vertices in its community and a fraction μ with the other vertices of the network. For synthetic modular networks, we set LFR benchmark network parameters as $N=5000$, $K \in \{15, 100\}$, $max_k=50$, $\mu=\{0.1, 0.5\}$, $min_c=10$, and $max_c=50$.

4.3. Experimental results

4.3.1. Experiment I

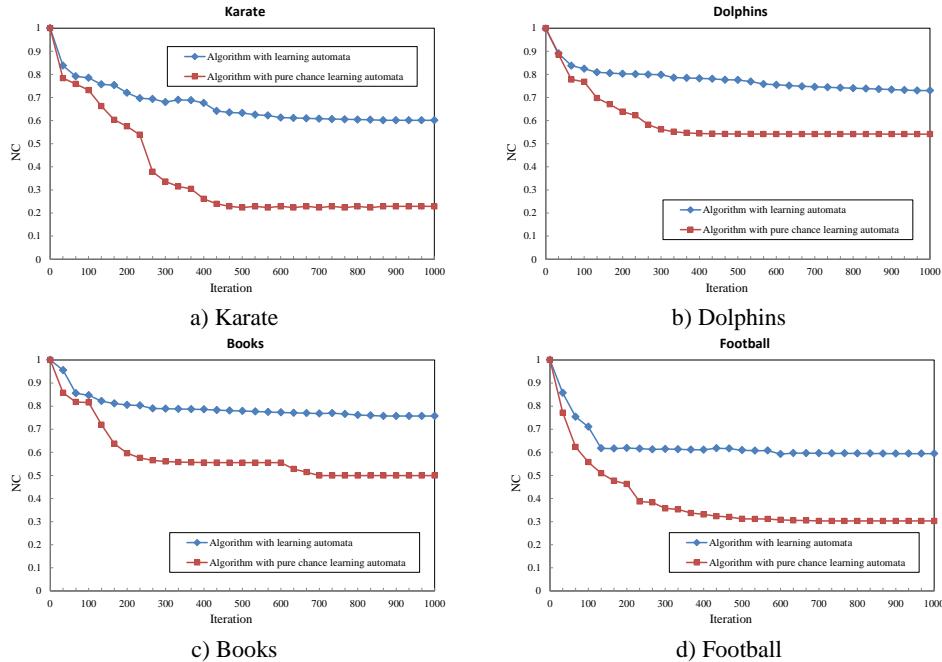
This experiment is carried out to investigate the impact of learning rate a on the performance of the proposed community detection algorithm. The normalized cut value for the real and synthetic networks for varying learning rates from 0.01 to 0.1 with step size 0.01 is given in Table 2. From the results of this experiment, we may conclude that the performance of the proposed community detection algorithm completely depends on the choice of the learning rate a . For Karate, Dolphins, Books, Les-miserable networks, small values of learning rate results in higher performance while large values results in lower performance. However, for Football, Reactome and LFR2 networks, larger values for learning rate results in higher performance. This may be because of their structure of these networks. It is noted that for the rest of experiments conducted in this paper, the learning rate is set to 0.02.

Table 2. Result of different learning rate in term of normalized cut.

Learning rate	Networks							
	Karate	Dolphins	Books	Les-miserables	Football	Reactome	LFR1	LFR2
0.01	0.4309	0.3877	0.4112	0.3680	0.4003	0.5643	0.5812	0.5858
0.02	0.3443	0.3622	0.3291	0.3273	0.4934	0.5642	0.5982	0.5925
0.03	0.4072	0.4082	0.3449	0.3796	0.5069	0.5647	0.5794	0.4635
0.04	0.3825	0.4635	0.3506	0.4643	0.3954	0.5353	0.5813	0.4423
0.05	0.4457	0.3094	0.3293	0.4032	0.4000	0.5217	0.5383	0.4024
0.06	0.4429	0.3765	0.4104	0.3720	0.4053	0.5225	0.5544	0.4310
0.07	0.4456	0.3863	0.3746	0.5375	0.3711	0.5141	0.5984	0.4211
0.08	0.4421	0.4171	0.3496	0.5877	0.3533	0.5111	0.5469	0.4214
0.09	0.4423	0.3654	0.4101	0.5119	0.3863	0.5082	0.5613	0.4199
0.10	0.3936	0.3783	0.3609	0.4348	0.4123	0.5025	0.5658	0.4214

4.3.2. Experiment II

This experiment is performed to study the behavior of the proposed algorithm during the process of finding the set of communities and the effect of learning in the proposed algorithm by replacing the learning automaton residing in each vertex by a pure chance automaton. In pure chance automaton, the initial probability of choosing actions is always remaining unchanged³². The comparison is made with respect to normalized cut $NC(C')$. Note that $NC(C')$ is the average of the normalized cut value for set of k communities found up to iteration t (equation (4)). The plot of $NC(C')$ versus iteration for different real networks demonstrated in figure 4 indicates the important role of learning automata in guiding the process of finding communities in real and synthetic networks. With the aid of learning automata, the process of finding communities from the network is done faster and more accurate as compared to the same case in absence of learning. Also, since, NC considers extracting the global impression of the network, instead of local features and measures both the total dissimilarity between the different communities as well as the total similarity within the communities. So, NC gradually decreases which means that the algorithm gradually converges to the minimum normalized cut and approach to the proper set of communities.



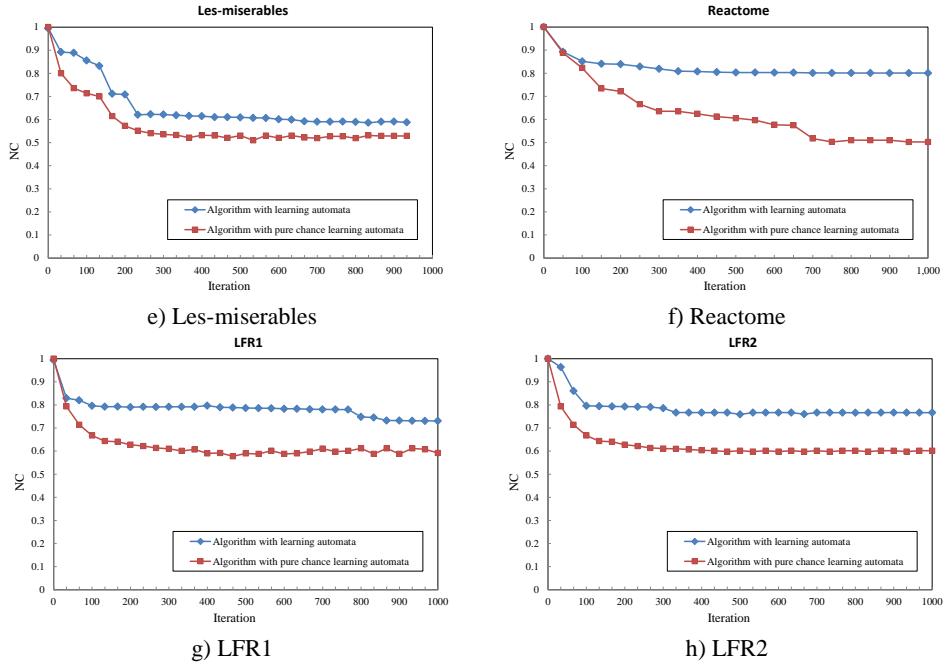


Fig. 4. Comparison of proposed community detection algorithm with proposed community detection algorithm in which learning automata are replaced with pure chance learning automata.

4.3.3. Experiment III

In this experiment, we investigate the impact of initial vertex selection of the proposed algorithm for finding communities in real and synthetic networks. For this purpose, we compared a high degree vertex and a random initial vertex as the initial selection in the proposed algorithm. The plot of $NC(C_t)$ for high degree initial vertex selection and random initial vertex selection versus iteration is presented in figure 5. From the results one can conclude that the proposed algorithm with high degree initial vertex selection has higher convergence speed than the proposed algorithm with random initial vertex selection for all networks. This may be because of process of initial formation of communities for each network affects the process of formation of rest of communities. In this regard, the vertex with highest degree may be member of the initial main community.

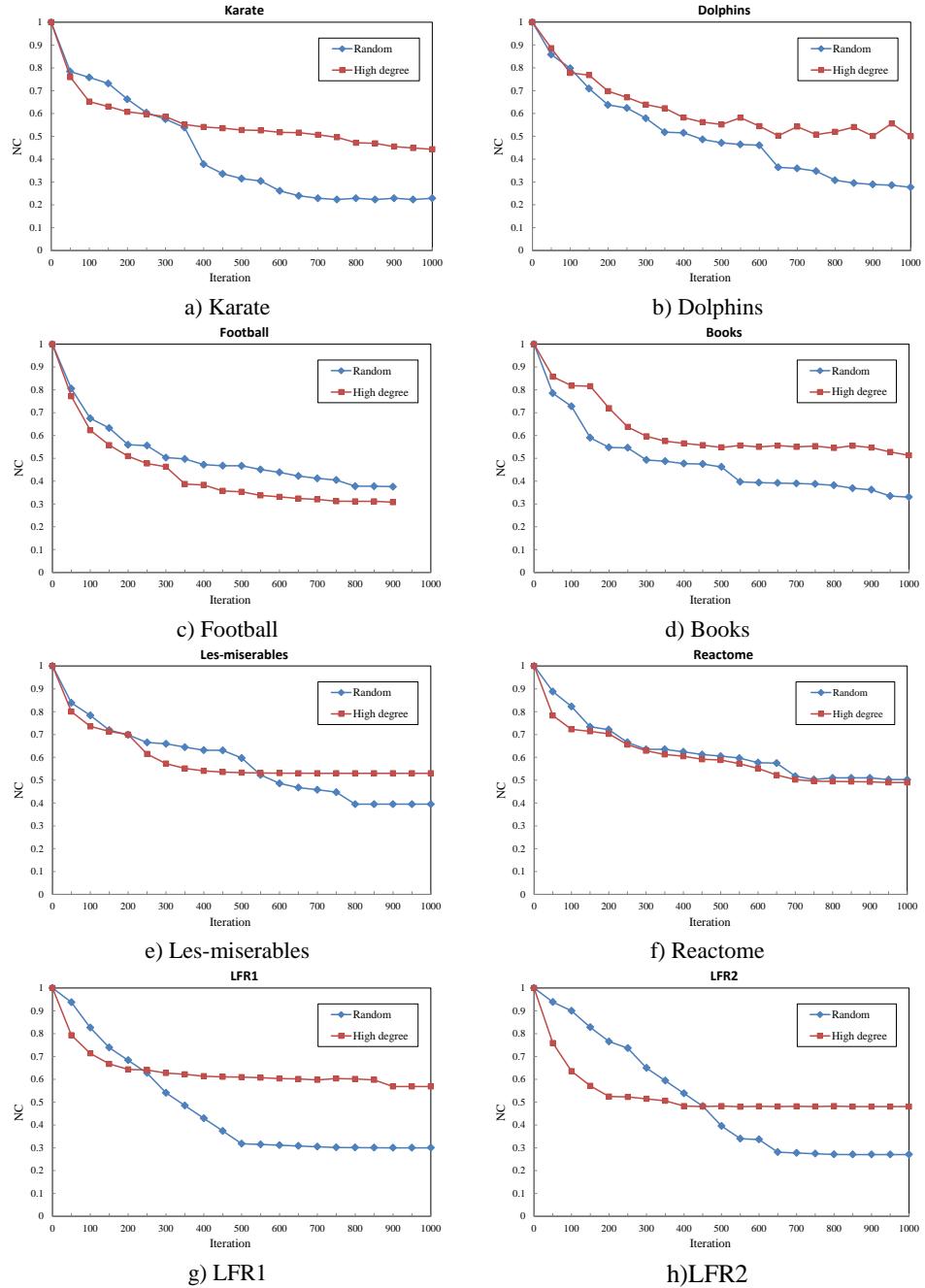


Fig. 5. Comparison of the proposed algorithm with random initial vertex selection versus the proposed algorithm with high degree initial vertex selection.

4.3.4. Experiment IV

In order to evaluate the performance of different community detection algorithms, we provide an experiment on the known synthetic modular networks based on LFR benchmark. In this experiment, the LFR benchmark network parameters are set as $N=5000$, $K=15$, $\max_k=50$, $\mu=\{0.1,0.5\}$, $\min_c=10$, $\max_c=50$ and the mixing parameter is changed from 0.05 to 0.5 with 0.05 increment. Several community detection algorithms including walk trap (WT)²⁷, label propagation (LP)⁴⁶, DANON greedy optimization (DGO)²⁸, Fuzzy community detection (FCD)²⁹ and Multi-resolution community detection (MRCD)³⁰ are conducted on mentioned synthetic modular networks and compared with respect to NMI results. As shown in figure 6, for the small size mixing parameter, all algorithms obtained almost same and high NMI values. As the complexity of the network is increased by the value of mixing parameter, the quality of each algorithm with respect to NMI is decreased. This figure also shows that the proposed community detection algorithm is better than MRCD, DGO and FCD in term of NMI.

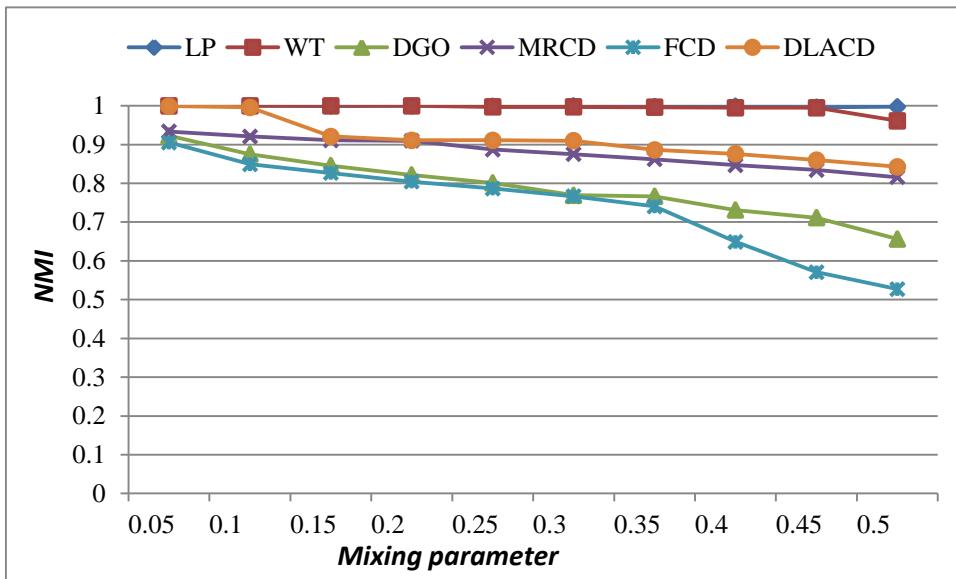


Figure 7. The comparison results for synthetic LFR benchmark with different mixing parameter μ

4.3.5. Experiment V

This experiment is done to study the performance of the proposed community detection algorithm in comparison with other community detection algorithms including walk trap (WT)²⁷, label propagation (LP)⁴⁶, DANON greedy optimization (DGO)²⁸, Fuzzy community detection (FCD)²⁹ and Multi-resolution community detection (MRCD)³⁰ in term of modularity, Min-max-cut, Coverage, Performance value for real world networks described in table 1. The results of this experiment are given in tables 3, 4, 5 and 6 for

modularity, min-max-cut, coverage and performance respectively. The best results for each network are highlighted in boldface. According to the results of table 3, for Books and Les-Miserables, the proposed community detection algorithm outperforms the other algorithms in terms of modularity. Also, for other networks, the modularity of proposed algorithm is approximately in the same range as of the other algorithms. It is noted that the DANON is designed for only modularity as objective function and thus it is biased toward high modularity's; therefore, it is not surprising that DANON reaches higher modularity in some networks, and this means that some more criteria must be considered to analyze the quality of algorithms accurately. The modularity of DLACD is low in some of the datasets, but it is still much higher than those of many metrics that shown in the following. In table 4, DLACD has the best value for Les-miserable real network and the proposed algorithm outperforms other algorithms in term of min-max-cut for other networks such as Karate, Dolphins and Books. According to Table 5, DLACD has highest possible value compared to the other algorithms for Dolphins and Les-miserable network meaning that the proposed algorithm could find an appropriate cut. The obtained results are negligible for other network datasets and for most of networks are in the same range. According to Table 6, the proposed algorithm reaches an acceptable value in set of optimization based algorithms. Note that the proposed community detection algorithm try to find optimal normalized cut value in the network and it is independent of some restrictions such as resolution limits.

Table 3. Comparison of the community detection algorithms in term of modularity.

Methods	Networks					
	Karate	Dolphins	Books	Les-miserable	Football	Reactome
MRCD	0.267	0.376	0.337	-	0.504	0.621
DGO	0.383	0.522	0.523	-	0.566	0.652
FCD	0.360	0.389	0.442	-	0.476	0.524
WT	0.407	0.519	0.52	0.521	0.602	0.733
LP	0.317	0.514	0.497	0.353	0.583	0.741
DLACD	0.384	0.349	0.526	0.524	0.584	0.651

Table 4. Comparison of the community detection algorithms of min-max-cut.

Methods	Networks					
	Karate	Dolphins	Books	Les-miserable	Football	Reactome
MRCD	1.094	1.012	1.227	-	0.707	1.475
DGO	0.452	0.303	0.179	-	0.338	1.842
FCD	0.149	0.063	0.067	-	0.197	1.756
WT	0.379	3.261	0.115	1.015	0.221	0.992
LP	0.379	3.170	0.067	0.426	0.445	0.845
DLACD	0.241	0.209	0.300	0.282	0.409	1.025

Table 5. Comparison of the community detection algorithms of Coverage.

Methods	Networks					
	Karate	Dolphins	Books	Les-miserable	Football	Reactome
MRCD	0.477	0.498	0.433	-	0.585	0.641
DGO	0.683	0.769	0.848	-	0.747	0.645
FCD	0.864	0.940	0.936	-	0.835	0.654
WT	0.865	0.371	0.864	0.496	0.735	0.892
LP	0.865	0.477	0.936	0.700	0.691	0.880
DLACD	0.800	0.826	0.768	0.779	0.709	0.761

Table 6. Comparison of the proposed algorithm in term of performance value.

Methods	Networks					
	Karate	Dolphins	Books	Les-miserables	Football	Reactome
MRCD	0.921	0.947	0.954	-	0.936	0.759
DGO	0.812	0.852	0.833	-	0.877	0.755
FCD	0.694	0.697	0.755	-	0.782	0.789
WT	0.815	0.889	0.832	0.887	0.922	0.890
LP	0.815	0.855	0.755	0.750	0.926	0.899
DLACD	0.750	0.760	0.815	0.869	0.906	0.844

5. Conclusion

Complex networks are composed by a large number of elements, organized into sub-communities. In this paper, an algorithm based on a distributed learning automaton was proposed for community detection in complex networks. In the proposed algorithm, a set of learning automata was assigned into vertices of the given network. Then based on cooperation between automata, the proposed algorithm tries to find optimal communities iteratively based on a learning mechanism. The performance of the proposed community detection algorithm was empirically investigated against the well-known community detection algorithms. According to the obtained results in several experiments, the proposed community detection algorithm outperforms the other algorithms in terms of the modularity, NMI, coverage, Min-Max-cut and performance in most cases.

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