A Learning Automata Based Adaptive Uniform Fractional Guard Channel Algorithm

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Abstract. Uniform fractional policy (UFC) is a call admission policy that accepts new calls with a probability π. In order to find the optimal value of π, we need to know all traffic parameters or to estimate them. In this paper, we first propose a new adaptive algorithm based on learning automata for finding the optimal value of UFC parameter and then study its steady state behavior. It is shown that the given adaptive algorithm converges to an equilibrium point which is also optimal for UFC policy. In order to study the performance of the proposed call admission policy, the computer simulations are conducted. The simulation results show that the level of QoS is satisfied by the proposed algorithm and the performance of given algorithm is very close to the performance of uniform fractional guard channel policy which needs to know all parameters of input traffic.

Keywords: Cellular mobile networks, Guard channel policy, Uniform fractional guard channel policy, Learning automata, Adaptive uniform fractional guard channel policy.

1 Introduction

Micro cellular networks use channels efficiently but increase expected rate of handover per call. As a consequence, some network performance parameters such as *blocking probability of new calls* (B_n) and *dropping probability of hand-off calls* (B_h) are affected. In order to have these performance parameters at reasonable level, *call admission policies* are used. The call admission policy plays a very important role in the cellular networks because it directly controls B_n and B_h by putting some restrictions on allocation of channels to calls. Since the dropping probability of hand-off calls is more important than the blocking probability of new calls, call admission policies usually give the higher priority to hand-off calls. This priority is implemented through allocation of more resources (channels) to hand-off calls.

In last decades several call admission algorithms are proposed in the literature and some of them are given below. *Fractional guard channel policy* (FG), which is a general call admission policy, accepts new calls with a certain probability that depends on the current channel occupancy and accepts hand-off calls as long as channels are available [1]. Suppose that the given cell has C full duplex channels. The FG policy uses a vector $\Pi = \{\pi_0, \ldots, \pi_{C-1}\}$ to accept new calls, where $0 \leq \pi_i \leq 1$ (for $i = 0, 1, 2, \ldots, C-1$). This policy accepts new calls with probability π_k when k (for $0 \leq k < C$) channels are busy. Depending on the vector Π, we may have different call admission policies and some of which are reviewed below. *Guard channel policy* (GC) reserves a subset of channels allocated to a cell, called *guard channels*, for hand-off calls (say $C - T$ channels) [2]. Whenever the channel occupancy exceeds a certain threshold T, the guard channel policy rejects new calls until the channel occupancy goes below the threshold. The guard channel policy accepts hand-off calls as long as channels are available. Note that the GC policy can be obtained from FG policy by setting $\pi_k = 1$ (for $0 \leq k < T$) and $\pi_k = 0$ (for $T \leq k < C$). It has been shown that there is an optimal threshold T^* at which the blocking probability of new calls is minimized subject to the hard constraint on the dropping probability of hand-off calls and an algorithm for finding such optimal threshold is given in [3]. The GC policy reserves an integral number of guard channels for hand-off calls. If performance parameter B_h is considered, the guard channel policy gives very good performance, but performance parameter B_n is degraded to great extent. In order to have more control on blocking probability of new calls and dropping probability of hand-off calls, *limited fractional* *guard channel policy* (LFG) is introduced [1]. The LFG can be obtained from FG policy by setting $\pi_k = 1$ (for $0 \leq k < T$), $\pi_T = \pi$, and $\pi_k = 0$ (for $T < k < C$). It has been shown that there is an optimal threshold T^* and an optimal value of π^* for which blocking probability of new calls is minimized subject to the hard constraint on dropping probability of hand-off calls and an algorithm for finding these optimal parameters is given in [1]. *Uniform fractional guard channel policy* (UFC) is a restricted version of FG, which accepts new calls with probability of π independent of channel occupancy [4]. The UFC can be obtained from FG by setting $\pi_k = \pi$ (for $0 \leq k < C$). It has been shown that there is an optimal value of π^* for which blocking probability of new calls is minimized subject to the hard constraint on dropping probability of hand-off calls and an algorithm for finding the optimal value of π is given in [4]. It was shown that, the UFC policy performs better than GC policy under the low hand-off traffic conditions [4]. There are some call admission policies which allow either hand-off or new calls to be queued until free channels are obtained in the cell [5, 6].

All above mentioned call admission policies such as FG, GC, UFC, and LFG are static and assume that all parameters of traffic are known in advance. These policies are useful when input traffic is a stationary process with known parameters. Since the parameters of input traffic are unknown and possibly time varying, adaptive version of these policies must be used. In [7], a learning automata based algorithm is given that adjusts π for uniform fractional channel policy. In [8], a learning automata based algorithm is given which adjusts T and π for limited fractional guard channel policy. It was shown that the learning automaton finds the optimal values of T and π. In [9], a cellular learning automata based algorithm is given which adjusts T for guard channel policy. In [10], two learning automata based algorithms are given which adjust T for guard channel policy and their steady state behavior studied.

In this paper, we propose a learning automata based adaptive uniform fractional guard channel algorithm (AUFC). The proposed algorithm uses a learning automaton to accept/reject new calls and a pre-specified level of dropping probability of hand-off calls is used to penalize/reward the action selected by the learning automaton. This algorithm accepts new calls as long as the dropping probability of hand-off calls is below the pre-specified threshold. Then we study the steady state behavior of the proposed algorithm. It is shown that the given algorithm converges to an equilibrium point which is also optimal for UFC policy. The simulation results show that, the performance of the proposed algorithm is very close to the performance of the UFC policy, which needs to know all traffic parameters in high hand-off traffic conditions and maintains the level of QoS in the system.

The rest of this paper is organized as follows: The learning automata are given in section 2. Section 3 presents the UFC policy, its performance parameters and an algorithm for finding the optimal value of its parameter. The proposed algorithm for finding the optimal value of parameter π and its steady state behavior are given in section 4. The computer simulations is given in section 5 and section 6 concludes the paper.

2 Learning Automata

The automata approach to learning involves determination of an optimal action from a set of allowable actions. An automaton can be regarded as an abstract object that has finite number of actions. It selects an action from its finite set of actions and applies it to a random environment. The random environment evaluates the applied action and gives a grade to the applied action of automaton. The response from the environment (i.e. grade of action) is used by automaton to update its next action. By continuing this process, the automaton learns to select an action with the best grade. The learning algorithm used by automaton to determine the selection of next action from the response of environment. The interaction of an automaton with its environment is shown in figure 1.

An automaton acting in an unknown random environment and improves its performance in some specified manner, is referred to as *learning automaton* (LA). Learning automata can be classified into two main families: *fixed structure learning automata* and *variable structure learning automata* [11]. Variable structure learning automata are represented by triple $< \beta, \alpha, T >$, where β is a set of inputs, α is a set of actions, and T is learning algorithm. The learning algorithm is a recurrence relation and is used to modify action probabilities (p) of the automaton. It is evident that the crucial factor affecting the performance of the variable structure learning automaton, is learning algorithm for updating the action probabilities. Various learning algorithms have been reported in the literature. Let α_i be the action chosen at time k

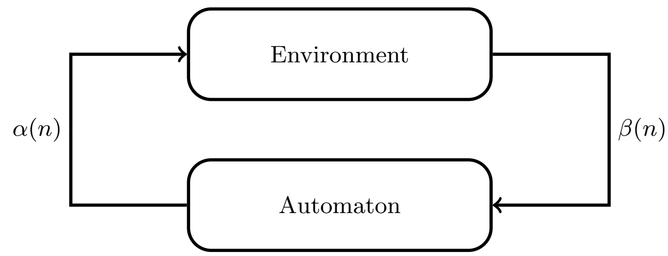

Fig. 1. The interaction of automata and its environment

as a sample realization from probability distribution $p(k)$. In what follows, two learning algorithms for updating the action probability vector are given. In linear reward-ϵpenalty algorithm ($L_{R-\epsilon P}$) scheme the recurrence equation for updating p is defined as

$$p_j(k+1) = \begin{cases} p_j(k) + a \times [1 - p_j(k)] \text{ if } & i = j \\ & \\ p_j(k) - a \times p_j(k) \qquad \text{if} & i \neq j \end{cases} \quad \text{if} \quad \beta(k) = 0 \tag{1}$$

$$p_j(k+1) = \begin{cases} p_j(k) \times (1-b) \qquad \text{if} & i = j \\ & \\ \frac{b}{r-1} + p_j(k)(1-b) \text{ if} & i \neq j \end{cases} \quad \text{if} \quad \beta(k) = 1 \tag{2}$$

where $p_j(k)$ is the probability of selecting action α_j, parameters $0 < b \ll a < 1$ represent *step lengths*, and r is the number of actions for learning automaton . Parameters a and b determine the amount of increase and decreases of the action probabilities, respectively. If parameter a equals to b the recurrence equations (1) and (2) is called *linear reward penalty*(L_{R-P}) algorithm.

Learning automata have been used successfully in many applications such as call admission control and channel assignment in cellular mobile networks [9, 10, 12–14], telephone and data network routing [15, 16], solving NP-Complete problems [17, 18], capacity assignment [19], and neural network engineering [20–23] to mention a few.

3 Uniform Fractional Guard Channel Policy

In this section, we first review UFC policy and then compute its blocking performance and finally give an algorithm for finding the optimal value of UFC parameter. The UFC policy uses admission probability π, which is independent of channel occupancy, to accept new calls and accepts hand-off calls as long as channels are available [4]. This policy can be obtained from FG policy by setting $\pi_k = \pi$ for $k = 0, 1, \ldots, C-1$, where C is the number of channels allocated to the cell. The UFC policy reserves non-integral number of guard channels for hand-off calls by rejecting new calls with probability $(1-\pi)$. The algorithmic description of UFC policy is given in algorithm 1.

In order to study the blocking performance of UFC policy, we assume that the following conditions are hold: (1) the network is homogeneous, (2) the arrival processes of new and hand-off calls are Poisson with rates λ_n and λ_h, respectively, (3) the channel holding time for both types of calls are exponentially distributed with mean μ^{-1}, and (4) the time interval between two calls from a mobile host is much greater than the mean call holding time. The above assumptions have been found to be reasonable as long as the number of mobile hosts in a cell is much greater than the number of channels allocated to that cell. Since the network is homogeneous, we can examine the performance of a single network cell in isolation. We define the state of a particular cell at time t to be the number of busy channels in that cell and is represented by $c(t)$. Process $\{c(t)|t \geq 0\}$ is a continuous-time Markov chain (birth-death process) with states $\{0, 1, \ldots, C\}$. The state transition rate diagram of a cell with C full duplex channels and UFC call admission policy is shown in figure 2.

At state n (for $0 \leq n < C$), new calls are accepted with probability $0 \leq \pi \leq 1$ and hand-off calls are accepted with probability 1. Both types of calls are blocked in state C. Thus, the state dependent arrival

Algorithm 1 Uniform fractional guard channel policy.

Require: *call* is the incoming call, C is the number of channels allocated to the cell, and π is the probability of accepting new calls.

```
procedure UFCCallAdmision(call, C, π)
    if call is a Hand-off call then
        if c(t) < C then
            Accept call
        else
            Reject call
        end if
    else                                                ▷ call is a new call.
        if (c(t) < C) and (rand(0,1) < π) then
            Accept call
        else
            Reject call
        end if
    end if
end procedure
```

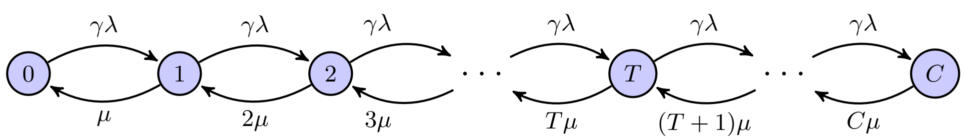

Fig. 2. Markov chain model of cell

rate in the birth-death process is equal to $[a + (1-a)\pi]\lambda$, where $\lambda = \lambda_n + \lambda_h$ and $\alpha = \lambda_h/\lambda$. Define the steady state probability

$$P_n = \lim_{t\to\infty} \text{Prob}[c(t) = n] \qquad\qquad n = 0, 1, \ldots, C. \tag{3}$$

The equilibrium equations for the steady-state probabilities P_n $(n = 0, 1, \ldots, C)$ is equal to

$$\gamma\lambda P_{n-1} = n\mu P_n,$$

where $\gamma = [a + (1-a)\pi]$. Solving the equilibrium equations, the following expression can be derived for P_n $(n = 0, 1, \ldots, C)$.

$$P_n = \frac{(\rho\gamma)^n}{n!} P_0, \tag{4}$$

where $\rho = \lambda/\mu$ and P_0 is the probability that all channels are free and obtained using equation $\sum_{n=0}^{C} P_n = 1$. The value of P_0 is calculated by the following expression.

$$P_0 = \left[\sum_{n=0}^{C} \frac{(\rho\gamma)^n}{n!}\right]^{-1} \tag{5}$$

Given the state probabilities, we can find the dropping probability of hand-off calls, $B_h(C, \pi)$, using the following expression.

$$\begin{aligned} B_h(C,\pi) &= P_C, \\ &= \frac{(\rho\gamma)^C}{C!} P_0. \end{aligned} \tag{6}$$

Similarly, the blocking probability of new calls, $B_n(C,\pi)$ equals to the following expression.

$$\begin{aligned} B_n(C,\pi) &= \sum_{n=0}^{C-1}(1-\pi)P_n + P_C, \\ &= 1-\pi\left[1-B_h(C,\pi)\right]. \end{aligned} \tag{7}$$

$B_n(C,\pi)$ and $B_h(C,\pi)$ have interesting properties, which some of them are listed below. Proofs of these properties can be found in [4].

Property 1. For any given value of π (for $0 \leq \pi \leq 1$) and C, we have $B_h(C,\pi) \leq B_n(C,\pi)$.

Property 2. For any given constraint $\rho/C < 1$, $B_h(C,\pi)$ is a monotonically increasing function of π.

Property 3. For any given constraint $C < \frac{\lambda_h}{\lambda p_h}$, $B_n(C,\pi)$ is a monotonically decreasing function of π.

In order to find the optimal value of π (π^*) that minimizes $B_n(C,\pi)$ subject to the hard constraint $B_h(C,\pi) \leq p_h$, algorithm 2 is proposed [4]. At the first, this algorithm considers the complete sharing case; i.e. all channels are shared between hand-off and new calls. If the complete sharing doesn't satisfy the level of QoS, then the algorithm considers the case when all channels are exclusively used for hand-off calls. If the exclusive use of channels for hand-off calls doesn't satisfy the level of QoS, then the number of allocated channels to the cell is not sufficient and the algorithm terminates; otherwise the algorithm searches for the optimal value of π^*. The search method used in this algorithm is binary search.

Algorithm 2 Algorithm for determination of π^* in UFC algorithm.

Require: C is the number of channels allocated to the cell, and p_h is the bound for dropping probability of hand-off calls.
Ensure: returns the optimal value of new call acceptance probability (π).

function FINDUFCPARAMETER(C, p_h)
  Set $upper \leftarrow 1$; $lower \leftarrow 0$
  if $B_h(C,1) \leq p_h$ **then**
    return 1
  else if $B_h(C,1) \geq p_h$ **then**
    return 0
  end if
  Set $k \leftarrow 0$
  while $k < 20$ **and** $(upper - lower) < 0.0001$ **do**
    Set $\pi \leftarrow (upper + lower)/2$
    if $B_h(C,\pi) > p_h$ **then**
      Set $upper \leftarrow \pi$
    else
      Set $lower \leftarrow \pi$
    end if
    Set $k \leftarrow k+1$
  end while
  return π
end function

4 Adaptive Uniform Fractional Guard Channel Algorithm

The UFC policy is static and assumes that all parameters of traffic are known in advance. Static policies are useful when input traffic is a stationary process with known parameters. Since the parameters of input traffic are unknown and possibly time varying, adaptive version of these policies must be used. In this section, we propose a learning automaton based adaptive uniform fractional guard channel algorithm (AUFC) shown in algorithm 3 and study its steady state behavior.

The proposed adaptive algorithm uses a learning automaton to determine the admission probability, π, when the parameters a and ρ (or equivalently λ_h, λ_n and μ) are unknown or probably time varying. The proposed algorithm adjusts parameter π as the network operates. This algorithm, which uses one reward-penalty type learning automaton with two actions in each cell, can be described as follows: The action set of this automaton corresponds to $\{ACCEPT, REJECT\}$. The automaton associated to each cell determines the probability of acceptance of new calls (π). Since initially the values of a and ρ are unknown, the probability of selecting these actions are set to 0.5. When a hand-off call arrives, it is accepted as long as there is a free channel. If there is no free channel, the hand-off call is dropped. When a new call arrives to a particular cell, the learning automaton associated to that cell chooses one of its actions. Let π be the probability of selecting action $ACCEPT$. Thus, the learning automaton accepts new calls with probability π as long as there is a free channel and rejects new calls with probability $1-\pi$. If action $ACCEPT$ is selected by automaton and the cell has at least one free channel, the incoming call is accepted and the selected action is rewarded. If there is no free channel to be allocated to the arrived new call, the call is blocked and action $ACCEPT$ is penalized. When the automaton selects action $REJECT$, the adaptive UFC computes an estimation of the dropping probability of hand-off calls ($\hat{B}_h$) and uses it to decide whether or not to accept the incoming new call. If the current estimate of dropping probability of hand-off calls is less than the given threshold p_h and there is a free channel, then the new call is accepted and the action $REJECT$ is penalized; otherwise, the new call is rejected and the action $REJECT$ is rewarded. The learning automaton, then uses this reinforcement signal to update the probability of accepting the new calls (π).

Algorithm 3 Adaptive uniform fractional guard channel policy.

Require: *call* is the incoming call, C is the number of channels allocated to the cell, and p_h is the bound for dropping probability of hand-off calls.

```
procedure AdaptiveUFCCallAdmision(call, C, p_h)
    if call is a new call then
        Learning automaton chooses an action.
        if Learning automaton choose the ACCEPT action then
            if c(t) < C then
                Accept call and reward the ACCEPT action
            else
                Reject call and penalize the ACCEPT action
            end if
        else                    ▷ Learning automaton chooses action REJECT action
            Reject call
            if (B̂_h < p_h) and (c(t) < C) then
                Accept call and penalize the REJECT action
            else
                Reject call and reward the REJECT action
            end if
        end if
    end if
end procedure
```

4.1 Behavior of the Adaptive UFC

In the rest of this section, we study the steady state behavior of the proposed algorithm. We show that, when the adaptive UFC algorithm uses the L_{R-P} reinforcement scheme, a unique value for π is found by the learning automaton, which is also optimal for the UFC algorithm. In order to study the behavior of the adaptive UFC algorithm, we first model environment for the learning automaton and then study the steady state behavior.

Lemma 1. *Let $p = (p_1, p_2)$ be the action probability vector of learning automata and $p_1 = \pi$ be the probability of accepting new calls. Then, the steady state behavior of the adaptive UFC algorithm can* *be shown by a triple* $< \underline{\alpha}, \underline{\beta}, \underline{C} >$*, where* $\underline{\alpha} = \{ACCEPT, REJECT\}$ *shows the set of actions of automaton,* $\underline{\beta} = \{0, 1\}$ *represents the set of inputs for automaton or the output of the environment and* $\underline{C}(p) = \{c_1(p), c_2(p)\}$ *is the set of penalty probabilities, where* $c_1(p)$ *and* $c_2(p)$ *are given by the following expressions.*

$$
\begin{aligned}
c_1(p) &= P_C && (8)\\
&= \frac{(\rho\gamma)^C}{C!} P_0 \\
c_2(p) &= \text{Prob}\left[\hat{B}_h < p_h\right] && (9)\\
&= \frac{1}{\sqrt{2\pi}\sigma_b} \int_{-\infty}^{p_h} e^{-\frac{1}{2}\left(\frac{x-\mu_b}{\sigma_b}\right)^2} dx
\end{aligned}
$$

where μ_b *and* σ_b^2 *are mean and variance of* $\hat{B}_h$*, respectively.*

Proof. Before we begin to prove the lemma, we introduce some definitions and notations. In order to count how many calls are arrived, we introduce concept of local time for each type of calls. The local time for each type of calls starts with 0 and incremented by 1 when a call of given type is arrived. Let us to define n^n and n^h as the local times for new and hand-off calls, respectively. Then, we define two sequences of random variables n_m^n $(n_1^n < n_2^n < \ldots)$ and n_m^h $(n_1^h < n_2^h < \ldots)$, where $n_m^n(n_m^h)$ is the global time when the mth new (hand-off) call is arrived.

The proof for penalty probability of $c_1(p)$ is trivial, because action ACCEPT is penalized when all allocated channels are busy. Since the probability of all channels being busy is equal to P_C, then $c_1(p)$ is equal to P_C. In order to find expression for $c_2(p)$, we define $X_n = I_b^h(e, n)$ as the indicator of blocking of a hand-off call at the hand-off local time n, where e is an event- either a new call arrival, a hand-off call arrival, or a call completion. Since in interval $[n, n+1]$, it is possible that $M \geq 0$ new calls to be accepted or $N \geq 0$ calls to be completed, then the state of the Markov chain describing cell at hand-off local time $n+1$ is independent of its state at the hand-off local time n if $N + M > 0$. Although there is an exception $N + M = 0$, which we ignore in our analysis. Therefore, $X_1, X_2, \ldots, X_n$ are independent identically distributed (i.i.d) random variables with the following first and second order statistics.

$$
\begin{aligned}
\text{E}\left[X_n\right] &= \sum_{k=0}^{C} k P_k, \\
&= \rho\gamma\left[1 - P_C\right]. && (10)\\
\text{Var}\left[X_n\right] &= \text{E}\left[X_k^2\right] - \left(\text{E}\left[X_k\right]\right)^2, \\
&= \rho\gamma\left[1 - P_C\right]\left[1 + \rho\gamma P_C\right] - (\rho\gamma)^2 P_{C-1}. && (11)
\end{aligned}
$$

Using the central limit theorem, we conclude that $\bar{X}_n = \hat{B}_h = \frac{1}{n}\sum_{k=0}^{n} X_k$ is a random variable with normal distribution ($\hat{B}_h \sim N(\mu_b, \sigma_b)$) with the following mean and variance [24].

$$
\begin{aligned}
\mu_b &= \text{E}\left[\hat{B}_h\right] = \rho\gamma\left[1 - P_C\right]. && (12)\\
\sigma_b &= \text{Var}\left[\hat{B}_h\right] = \frac{\text{Var}\left[X_n\right]}{n}, \\
&= \frac{\rho\gamma\left[1 - P_C\right]\left[1 + \rho\gamma P_C\right] - (\rho\gamma)^2 P_{C-1}}{n}. && (13)
\end{aligned}
$$

Thus, the value of penalty probability of $c_2(p)$ is equal to

$$
\begin{aligned}
c_2(p) &= \text{Prob}\left[\hat{B}_h < p_h\right], \\
&= \frac{1}{\sqrt{2\pi}\sigma_b} \int_{-\infty}^{p_h} e^{-\frac{1}{2}\left(\frac{x-\mu_b}{\sigma_b}\right)^2} dx
\end{aligned}
$$

which completes the proof of this lemma. Q.E.D.

The following lemma is concerned with the properties of the environment.

Lemma 2. *The environment corresponding to the adaptive UFC algorithm has the following characteristics when $\rho < C$. Let to write p for $p(n)$ and $c_i(p)$ for $c_i(n)$.*

1. *$c_i(p)$ (for $i = 1, 2$) are continuous functions in p.*
2. *$c_i(p)$ (for $i = 1, 2$) are continuously differentiable in all their arguments.*
3. *$c_i(p)$ and $\frac{\partial c_i(p)}{\partial p_i}$ (for $i = 1, 2$) are Lipschitz function of all their arguments.*
4. *The derivative of $c_i(p)$ (for $i = 1, 2$) have the following features.*

$$\frac{\partial c_i(p)}{\partial p_i} > 0, \tag{14}$$

$$\frac{\partial c_1(p)}{\partial p_2} \ll \frac{\partial c_2(p)}{\partial p_2}, \tag{15}$$

$$\frac{\partial c_2(p)}{\partial p_1} \ll \frac{\partial c_1(p)}{\partial p_1}. \tag{16}$$

Proof. Using equation (8), the proof of items 1 through 3 are trivial. Hence, in the rest of the proof we give only the proof of item 4. Using equation (8), we have

$$\frac{\partial c_1(p)}{\partial p_1} = (1-a)P_C\left[\frac{C}{\gamma} - \rho(1-P_C)\right],$$
$$\frac{\partial c_1(p)}{\partial p_2} = -(1-a)P_C\left[\frac{C}{\gamma} - \rho(1-P_C)\right].$$

Therefore, when $\rho < C$, we have

$$\frac{\partial c_1(p)}{\partial p_1} > 0, \tag{17}$$

and

$$\frac{\partial c_1(p)}{\partial p_2} < 0. \tag{18}$$

Using equation (8), we obtain

$$\frac{\partial c_2(p)}{\partial p_2} = \frac{1}{\sigma_b\sqrt{2\pi}}\left[e^{\frac{-1}{2}\left(\frac{p_h-\mu_b}{\sigma_b}\right)^2}\left\{\left(\frac{\mu-p_h}{\sigma_b}\right)\frac{\partial \sigma_b}{\partial p_2} - \frac{\partial \mu_b}{\partial p_2}\right\} - \frac{2}{\sigma_b}\frac{\partial \sigma_b}{\partial p_2}\int_{-\infty}^{p_h} e^{\frac{-1}{2}\left(\frac{x-\mu_b}{\sigma_b}\right)^2}dx\right],$$
$$\frac{\partial c_2(p)}{\partial p_1} = \frac{-1}{\sigma_b\sqrt{2\pi}}\left[e^{\frac{-1}{2}\left(\frac{p_h-\mu_b}{\sigma_b}\right)^2}\left\{\left(\frac{\mu-p_h}{\sigma_b}\right)\frac{\partial \sigma_b}{\partial p_1} - \frac{\partial \mu_b}{\partial p_1}\right\} - \frac{2}{\sigma_b}\frac{\partial \sigma_b}{\partial p_1}\int_{-\infty}^{p_h} e^{\frac{-1}{2}\left(\frac{x-\mu_b}{\sigma_b}\right)^2}dx\right].$$

Increasing p_2, decreases the probability of accepting new calls and hence the number of busy channels decreased. Therefore, the dropping probability of hand-off calls is decreased or $c_2(p) = \text{Prob}\left[\hat{B}_h < p_h\right]$ is increased. Thus

$$\frac{\partial c_2(p)}{\partial p_2} > 0 \tag{19}$$

and

$$\frac{\partial c_2(p)}{\partial p_1} < 0. \tag{20}$$

However, by choosing proper value for n, condition $\frac{\partial c_2(p)}{\partial p_2} > 0$ is also satisfied. From equations (17) and (19), equation (14) is concluded, from equations (17) and (20), equation (15) is concluded and from equations (18) and (19), equation (16) is concluded. This completes the proof of this lemma. Q.E.D.

The process $\{p(n)\}_{n\geq 0}$ defined by the adaptive UFC algorithm is a homogeneous Markov process. The following theorem is concerned with its convergence behavior.

Theorem 1. *The Markov process $\{p(n)\}_{n\geq 0}$ is ergodic and converges in distribution as $n \to \infty$ to a unique stationary probability $\bar{p}$ independent of the initial distribution of $\bar{p}$.*

Proof. The proof of this theorem is given in [25]. Q.E.D.

In what follows, the steady state behavior of the adaptive UFC algorithm is analyzed. Define the average penalty rate of action α_i as $f_i\left(p(n)\right) = c_i\left(p(n)\right) p_i(n)$, $p^* = (p_1^*, p_2^*)$ and $p_1^* + p_2^* = 1$. In the following lemma, it is shown that there is a unique p^* for which the average penalty rates for both actions become equal.

Lemma 3. *For the adaptive UFC algorithm, there exists a unique p^* such that*

$$\begin{aligned} f\left(p^*\right) &= f_2\left(p^*\right) - f_1\left(p^*\right), \\ &= 0. \end{aligned} \tag{21}$$

Proof. Consider $f(p)$ at its two end points

$$f(p) = \begin{cases} c_2(0,1) & p_1 = 0 \\ -c_1(1,0) & p_1 = 1. \end{cases} \tag{22}$$

Since $f(p)$ is a continuous function of p_1 and p_2, there exists at least a p^* such that $f(p^*) = 0$. In order to prove the uniqueness of p^*, the derivative of $f(p)$ with respect to p_1 is computed and using lemma 2 and equation (20), we obtain

$$\begin{aligned} \frac{\partial f(p)}{\partial p_1} &= \frac{\partial c_2(p)}{\partial p_1} - \left(1 + p_1\right)\left(c_1 + c_2\right), \\ &< 0. \end{aligned}$$

Since the derivative of $f(p)$ with respect to p_1 is negative, $f(p)$ is a strictly decreasing function of p_1. Thus there exists one and only one point p^* for which function $f(p)$ crosses the horizontal line and hence the lemma. Q.E.D.

Let $\Delta p_1(n) = p_1(n+1) - p_1(n)$, thus the conditional expectation and variance of Δp_1 are expressed as

$$\begin{aligned} \mathrm{E}\left[\frac{\Delta p_1(n)}{a} | p(n) = p\right] &= f_2(p) - f_1(p) \\ &= w(p) \end{aligned} \tag{23}$$

Let

$$S(p) = \mathrm{E}\left[\frac{\Delta p_1^2(n)}{a^2} | p(n) = p\right].$$

Thus the variance of Δp_1 is equal to

$$\tilde{S}(p) = S(p) - w^2(p).$$

Since $\{p(n)\}_{n\geq 0}$ is ergodic and converges in distribution to a unique stationary probability $\bar{p}$, thus in steady state, we obtain $\mathrm{E}\left[\Delta \bar{p}_i\right] = 0$ or $\mathrm{E}\left[w(\bar{p})\right] = 0$. The zero of $\mathrm{E}\left[w(\bar{p})\right]$ is p^* and, in general, $\mathrm{E}\left[w(\bar{p})\right] = 0$ need not yield p^*. However, if the learning parameter a is chosen to be sufficiently small, then the difference between these two values may be made small, as indicated by the following theorem.

Lemma 4. *Let $p(0)$ be the initial action probability vector of the adaptive UFC algorithm with stationary measure $\bar{p}$, then*

$$\begin{aligned}\mathrm{E}\left[p_i(n) - p_i^*\right]^2 &\leq Ka, \\ &= O(a),\end{aligned}$$

where $K > 0$ denotes a constant.

Proof. Define $p^2 = p^T p$ for vector p. Let $p = p_1$ and

$$g(p) = \begin{cases} \frac{w(p)}{p^* - p} & p \neq p^* \\ \\ -\left.\frac{\partial w(p)}{\partial p}\right|_{p=p^*} & p = p^* \end{cases} \tag{24}$$

Since $w(p) < 0$ when $p > p^*$ and $w(p) > 0$ when $p < p^*$, $g(p)$ is positive and continuous in interval $[0, 1]$. Hence, there exists a $R > 0$ such that $g(p) \geq R$. Thus, we have

$$\begin{aligned}\left[p^* - p(n)\right] w(p(n)) &= \left[p^* - p(n)\right]^2 g(p(n)), \\ &\geq R\left[p^* - p(n)\right]^2.\end{aligned} \tag{25}$$

for all probability p, then computing

$$\left[p(n+1) - p^*\right]^2 = \left[p(n) - p^*\right]^2 + 2\left[p(n) - p^*\right] \Delta p(n) + \Delta p^2(n)$$

and taking expectation on both sides, canceling $\mathrm{E}\left[p(n) - p^*\right]^2$ on left and right sides and dividing by $2a$, we obtain

$$0 = \mathrm{E}\left[\left\{p(n) - p^*\right\} \frac{\Delta p(n)}{a}\right] + \frac{a}{2}\mathrm{E}\left[\frac{\Delta p^2(n)}{a^2}\right],$$

or

$$0 = \mathrm{E}\left[\left\{p(n) - p^*\right\} w\left(p(n)\right)\right] + \frac{a}{2}\mathrm{E}\left[\tilde{S}\left(p(n)\right)\right].$$

Since, we have only bounded variables, $\tilde{S}\left(p(n)\right)$ is bounded; thus, there exists a $K > 0$ such that $\mathrm{E}\left[\tilde{S}\left(p(n)\right)\right] \leq K$. Hence we obtain

$$\begin{aligned}\mathrm{E}\left[\left\{p^* - p(n)\right\} w\left(p(n)\right)\right] &= \frac{a}{2}\mathrm{E}\left[\tilde{S}\left(p(n)\right)\right], \\ &\leq Ka.\end{aligned}$$

Using this equation and equation (25), we obtain

$$\begin{aligned}\mathrm{E}\left[p^* - p(n)\right]^2 &\leq K\mathrm{E}\left[\left\{p^* - p(n)\right\} w(p(n))\right], \\ &\leq Ka, \\ &= O(a).\end{aligned}$$

and hence the lemma. Q.E.D.

Comment 1 *Since* $\left(\mathrm{E}\left[p_i(n) - p^*\right]\right)^2 \leq \mathrm{E}\left[p_i(n) - p^*\right]^2$, $\mathrm{E}\left[p_i(n)\right] - p_i^* = O\left(\sqrt{a}\right)$ *and* $p_i(n) \to p_i^*$ *with probability 1 as* $a \to 0$. *In steady state,* $\mathrm{E}\left[\Delta \bar{p}_i\right] = 0$ *or* $\mathrm{E}\left[w\left(p(n)\right)\right] \to 0$ *as* $n \to \infty$. *This implies that* $\mathrm{E}\left[f_2(\bar{p})\right] = \mathrm{E}\left[f_1(\bar{p})\right]$. *Hence from lemma 3, we can write*

$$\mathrm{E}\left[f_2(\bar{p})\right] - f_2(p^*) = \mathrm{E}\left[f_1(\bar{p})\right] - f_1(p^*). \tag{26}$$

Since $f_i(.)$ is Lipschitz function with Lipschitz bound α, we have

$$\begin{aligned}|\mathrm{E}\left[f_i(\bar{p}) - f_i(p^*)\right]| &\leq \alpha \left|\mathrm{E}\left[\bar{p}_i - p_i^*\right]\right|, \\ &\leq \alpha \left|\mathrm{E}\left[\bar{p}_i\right] - p_i^*\right|, \\ &= O(\sqrt{a}).\end{aligned}$$

Thus we have $|\mathrm{E}\left[f_i(\bar{p}) - f_i(p^*)\right]| = O(\sqrt{a})$*. Hence for small values of the parameter* a*, it can be concluded that asymptotic behavior of the adaptive UFC algorithm can be approximated by* $f(p^*) = 0$*.*

Lemma 5. *Let* $p(0)$ *be the initial action probability vector of the adaptive UFC algorithm with stationary measure* $\bar{p}$ *and define* $z_i(n) = \frac{p_i(n) - p_i^*}{\sqrt{a}}$ *and* $z(n) = z_1(n)$*, then the following relations are held. These relations verifies the condition of lemma 2.1 (pp. 156) [26], whose application results in the normal approximation.*

$$\mathrm{E}\left[\Delta z(n) | z(n)\right] = a w'(p^*) z(n) + o(a), \tag{27}$$

$$\mathrm{E}\left[\Delta z^2(n) | z(n)\right] = a \tilde{S}(p^*) + o(a), \tag{28}$$

$$\mathrm{E}\left[|\Delta z(n)|^3 | z(n)\right] = o(a), \tag{29}$$

where $o(a)$ *denotes a random variable such that* $\mathrm{E}\left[\frac{o(a)}{a}\right] \to 0$ *as* $a \to 0$*.*

Proof. In order to prove equation (27), let us to define

$$\zeta = \frac{\mathrm{E}\left[\Delta z(n) | z(n)\right]}{\sqrt{a}} = \frac{\mathrm{E}\left[\Delta p(n) | z(n)\right]}{a} = w\left(p(n)\right) - w\left(p^*\right). \tag{30}$$

Since $w(.)$ is Lipshitz, we have

$$|w\left(p(n)\right) - w\left(p^*\right)| \leq K |p(n) - p^*|,$$

where $K > 0$ denotes a constant. Using this equation and equation (30), we obtain

$$\begin{aligned}|\zeta| &\leq K |p(n) - p^*|, \\ &\leq K \sqrt{a} |z(n)|.\end{aligned} \tag{31}$$

Let

$$h(\lambda) = w(x + \lambda(y - x))$$

where $\lambda \in [0, 1]$ and $w(.)$ is Lipschitz with bound β. It follows that

$$\begin{aligned}h'(\lambda) &= \frac{\partial h(\lambda)}{\partial \lambda}, \\ &= w'(x + \lambda(y - x))(y - x).\end{aligned} \tag{32}$$

Since $h'(.)$ is continuous, we have

$$\begin{aligned}w(y) - w(x) &= h(1) - h(0), \\ &= \int_0^1 w'(x + \lambda(y - x))[y - x] d.\lambda\end{aligned} \tag{33}$$

Subtracting $w'(x)(y-x)$ from both sides of the above equation, we obtain

$$w(y) - w(x) - w'(x)[y-x] = \int_0^1 \left[w'(x+\lambda(y-x)) - w'(x)\right][y-x]d\lambda$$

Since $w(.)$ is Lipschitz with bound β, we obtain

$$w(y) - w(x) - w'(x)(y-x) \leq \frac{\beta}{2}|y-x|^2.$$

Substituting y with $p(n)$ and x with p^* in the above equation, we obtain

$$\begin{aligned} w(p(n)) - w(p^*) - w'(p^*)(p(n)-p^*) &\leq K|p(n)-p^*|^2, \\ w(p(n)) - w(p^*) - \sqrt{a}w'(p^*)z(n) &\leq Ka|z(n)|^2. \end{aligned} \tag{34}$$

Using this equation, equations (30) and (31) and lemma 4, we obtain

$$\begin{aligned} |\zeta - \sqrt{a}w'(p^*)z(n)| &\leq Ka|z(n)|^2, \\ &\leq K|p(n)-p^*|^2, \\ &\leq Ka. \end{aligned} \tag{35}$$

Multiplying both sides of the above equation by $\sqrt{a}$, we obtain

$$\left|\sqrt{a}\zeta - aw'(p^*)z(n)\right| \leq Ka^{3/2},$$

or

$$|\mathrm{E}[\Delta z(n)|z(n)] - aw'(p^*)z(n)| \leq K\sqrt{a},$$

which implies equation (27). In order to prove equation (28), let us to define

$$\eta = \frac{\mathrm{E}\left[\Delta z^2(n)|z(n)\right]}{a} = S(p(n)) = \tilde{S}(p(n)) + \zeta^2.$$

By subtracting $\tilde{S}(p(n))$ from both sides of the above equation, we obtain

$$\begin{aligned} |\eta - \tilde{S}(p^*)| &= |\tilde{S}(p(n)) + \zeta^2 - \tilde{S}(p^*)| \\ &\leq |\tilde{S}(p(n)) - \tilde{S}(p^*)| + |\zeta^2|. \end{aligned} \tag{36}$$

Since $\tilde{S}(.)$ is Lipschitz, we have

$$|\tilde{S}(p(n)) - \tilde{S}(p^*)| \leq K|p(n)-p^*|.$$

Substituting this equation and equation (31) into equation (36), we obtain

$$|\eta - \tilde{S}(p^*)| \leq K|p(n)-p^*| + K|p(n)-p^*|^2$$

Using lemma 4, we have $\mathrm{E}\left[p(n)-p^*\right]^2 \leq Ka$ and $\mathrm{E}\left[p(n)-p^*\right] \leq K\sqrt{a}$. Thus we obtain $|\eta - \tilde{S}(p^*)| = o(a)$. Hence, as a consequence, we have $\mathrm{E}|\eta - \tilde{S}(p^*)| \to 0$ as $a \to 0$, which confirms equation (28). Equation (29) follows by observing that

$$\mathrm{E}\left[\left|\frac{\Delta p(n)}{a}\right|^3 \middle| p(n) = p\right] = \xi(p) < \xi < \infty$$

Substituting equation (29) into the above equation, we obtain

$$\begin{aligned} \mathrm{E}\left[\left|\frac{\Delta p(n)}{a}\right|^3 \middle| p(n)\right] &< \xi \\ \mathrm{E}\left[\left|\frac{|\Delta z(n)|^3}{a^{3/2}}\right| p(n)\right] &< \xi \\ \mathrm{E}\left[|\Delta z(n)|^3 \middle| p(n)\right] &< \xi a^{3/2} \end{aligned} \tag{37}$$

where $\xi a^{3/2} \to 0$ as $a \to 0$. This completes the proof of this lemma. Q.E.D.

Theorem 2. *Let $p(0)$ be the initial action probability vector of the adaptive UFC algorithm with stationary measure $\bar{p}$ and lemma 4 holds, then*

$$z_i(n) \sim N\left(0, \sigma^2\right),$$

where $\sigma^2 = \frac{\tilde{S}(p^*)}{2|w'(p^*)|}$ as $a \to 0$ and $na \to \infty$.

Proof. Let $h(u) = \mathrm{E}\left[e^{iuz(n)}\right]$ as the characteristic function of $z(n)$. Then using the third order taylor's expansion of e^{iu} for real u, we obtain

$$\mathrm{E}\left[e^{iuz(n)}\Big|\, z(n)\right] = 1 + iu\mathrm{E}[\Delta z(n)|z(n)] - \frac{u^2}{2}\mathrm{E}\left[\Delta z^2(n)\big|\, z(n)\right] + k|u|^3\mathrm{E}\left[|\Delta z(n)|^3\big|\, z(n)\right],$$

where $k \leq 1/6$; thus

$$\begin{aligned} h(u) &= \mathrm{E}\left[e^{iuz(n+1)}\right], \\ &= \mathrm{E}\left[e^{iuz(n)}\mathrm{E}\left(e^{iu\Delta z(n)}\Big|\, z(n)\right)\right], \\ &= h(u) + iu\mathrm{E}\left[e^{iuz(n)}\mathrm{E}\left\{\Delta z(n)|z(n)\right\}\right], \\ &- \frac{u^2}{2}\mathrm{E}\left[e^{iuz(n)}\mathrm{E}\left\{\Delta z^2(n)\big|\, z(n)\right\}\right] + k|u|^3\mathrm{E}\left[ke^{iuz(n)}\mathrm{E}\left\{|\Delta z(n)|^3\big|\, z(n)\right\}\right]. \end{aligned} \tag{38}$$

Cancelling $h(u)$ and dividing by u, results

$$0 = i\mathrm{E}\left[e^{iuz(n)}\mathrm{E}\left\{\Delta z(n)|z(n)\right\}\right] - \frac{u}{2}\mathrm{E}\left[e^{iuz(n)}\mathrm{E}\left\{\Delta z^2(n)\big|\, z(n)\right\}\right] + k|u|^2\mathrm{E}\left[ke^{iuz(n)}\mathrm{E}\left\{|\Delta z(n)|^3\big|\, z(n)\right\}\right].$$

Thus using estimates of lemma 5, we have

$$0 = iaw'(p^*)\mathrm{E}\left[e^{iuz(n)}z(n)\right] - \frac{u}{2}\tilde{S}(p^*)\mathrm{E}\left[e^{iuz(n)}\right] + \mathrm{E}\left[o(a)\right] + u\mathrm{E}\left[o(a)\right] + u^2\mathrm{E}\left[o(a)\right].$$

From equations (27) and (29) of lemma 5, it is evident that $\mathrm{E}[|z(n)|] < \infty$ when a is small or

$$0 = aw'(p^*)\frac{dh(u)}{du} - \frac{u}{2}\tilde{S}(p^*)h(u) + \mathrm{E}\left[o(a)\right] + u\mathrm{E}\left[o(a)\right] + u^2\mathrm{E}\left[o(a)\right].$$

Dividing the above equation by $aw'(p^*)$ and using fact $w'(p^*) < 0$, we obtain

$$\frac{dh(u)}{du} + u\frac{\tilde{S}(p^*)}{2\left|w'(p^*)\right|}h(u) + \epsilon(u) = 0,$$

where

$$\varphi = \sup_u \frac{|\epsilon(u)|}{1+u^2} \to 0,$$

as $a \to 0$. Since $h(0) = 1$, it follows that

$$h(u) = e^{-\frac{(u\sigma)^2}{2}}\left(1 - \int_0^u e^{\frac{(ux)^2}{2}}dx\right),$$

where $\sigma^2 = \frac{\tilde{S}(p^*)}{2|w'(p^*)|}$. But we have

$$\left|\int_0^{|u|} e^{\frac{(ux)^2}{2}}\epsilon(u)dx\right| \leq \varphi\int_0^{|u|} e^{\frac{(ux)^2}{2}}\left(1+x^2\right)dx \to 0,$$

as $a \to 0$; thus

$$h(u) \to e^{-\frac{(u\sigma)^2}{2}}.$$

Then using the facts that each characteristic function determines the distribution uniquely and $h(u)$ is characteristic function of $N(0, \sigma^2)$, thus we obtain

$$z(n) \sim N(0, \sigma^2),$$

and hence the theorem.

Theorem 3. *The equilibrium probability of learning automaton in the adaptive UFC algorithm, $p^* = (\pi^*, 1 - \pi^*)$, minimizes the blocking probability of new calls subject to the hard constraint on the dropping probability of hand-off calls ($B_h(C, \pi) \leq p_h$).*

Proof. In the equilibrium state, the average penalty rates for both actions are equal or $f_1(p^*) = f_2(p^*)$, which results $c_1\pi^* = c_2(1 - \pi^*)$. Thus we have

$$\pi^* = \frac{\delta}{\delta + P_C}, \tag{39}$$

where $\delta = \text{Prob}\left[\hat{B}_h < p_h\right]$. Thus average number of blocked new calls, $\bar{N}_n$, is equal to

$$\begin{aligned} \bar{N}_n &= \lambda_n \left[1 - \pi^*(1 - P_C)\right], \\ &= \lambda_n(1 + \delta)\frac{P_C}{P_C + \delta}. \end{aligned} \tag{40}$$

Computing derivative of $\bar{N}_n$ with respect to δ results

$$\begin{aligned} \frac{\partial \bar{N}_n}{\partial \delta} &= -\lambda_n \frac{P_C(1 - P_C)}{(P_C + \delta)^2}, \\ &< 0. \end{aligned} \tag{41}$$

Thus $\bar{N}_n$ is a strictly decreasing function of δ. Since the adaptive UFC algorithm gives the higher priority to the hand-off calls, it attempts to minimize the dropping probability of hand-off calls. Using this fact and equation (41), it is evident that $\bar{N}_n$ is minimized which results in minimization of the blocking probability of new calls and hence the theorem.

5 Simulation Results

In this section, through simulation we compare performance of the uniform fractional guard channel algorithm [4], an adaptive uniform fractional guard channel algorithm [7], and the proposed algorithm. To compare these algorithms, computer simulations are conducted. The results of simulations are summarized in figures fig:aufc-bh and . Simulation are conducted based on the single cell of homogeneous cellular network system. In such network, each cell has 8 full duplex channels ($C = 8$). In all simulations, new call arrival rate is fixed to 30 calls per minute ($\lambda_n = 30$), channel holding time is set to 6 seconds ($\mu^{-1} = 6$), and the hand-off call traffic is varied between 2 calls per minute to 20 calls per minute. The results shown in figures through are obtained by averaging 10 runs from $2,000,000$ seconds simulation of each algorithm. The level of QoS for the dropping probability of hand-off calls is set to 0.01. The optimal value of π of uniform fractional channel policy is obtained by algorithm 2 given [4]. Figure 3 shows the dropping probability of hand-off calls of the proposed algorithm and the algorithm given in [7] for different rates of hand-off traffic. As this figure shows, the ability of the proposed algorithm for maintaining the QoS of hand-off traffic for different hand-off load is higher than the algorithm given in [7], although there are some variation and the proposed algorithm can not maintain the hard constraint.

Figure 4 shows the blocking probability of new calls of the proposed algorithm and the algorithm given in [7] for different rates of hand-off traffic. As this figure shows, the blocking probability of new calls for the proposed algorithm is higher than the blocking probability of new calls the algorithm given in [7]. This is due to the higher priority given to hand-off calls.

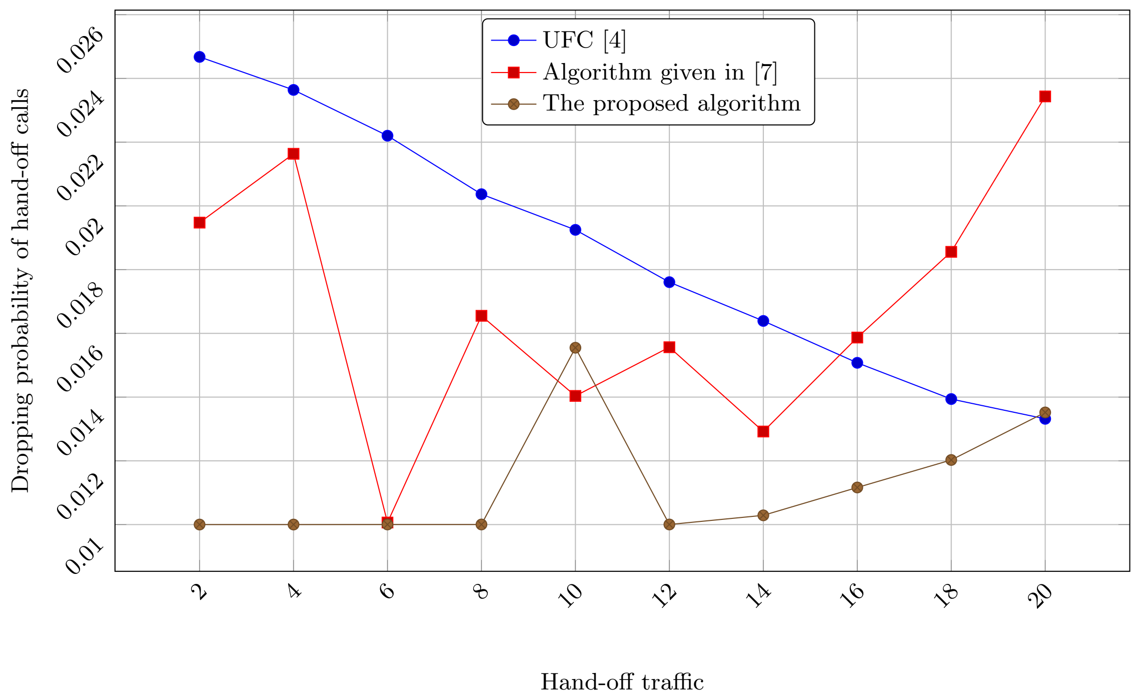

Fig. 3. Dropping probability of the proposed algorithm for different hand-off traffic.

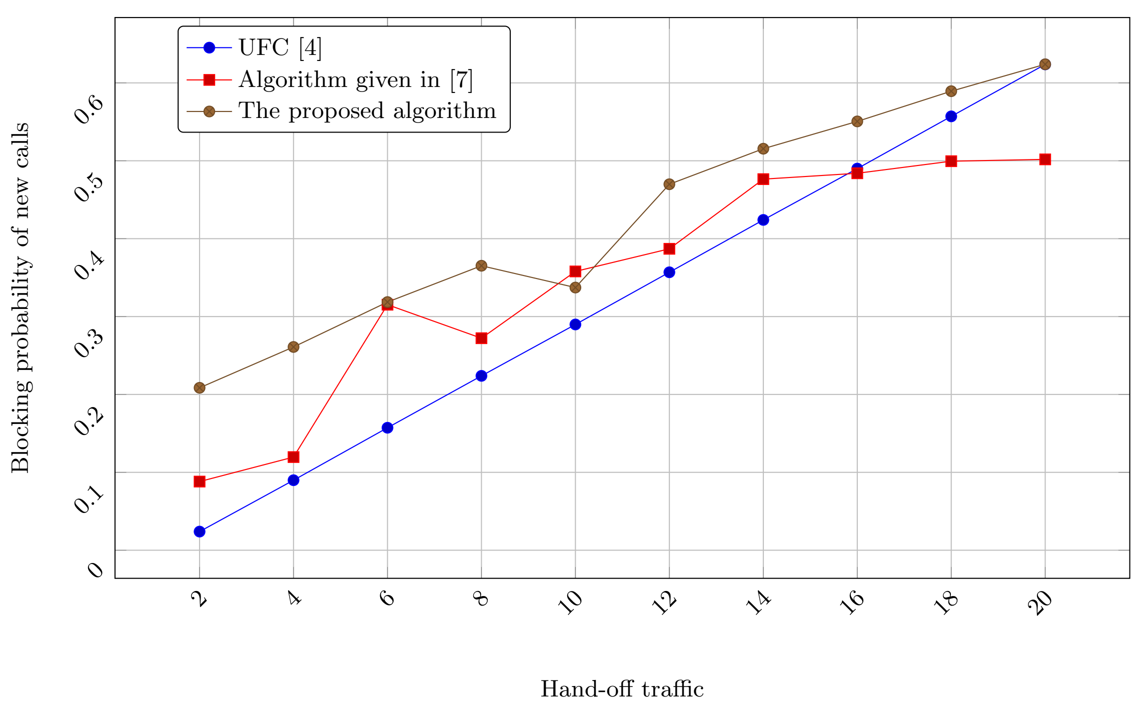

Fig. 4. Blocking probability of the proposed algorithm for different hand-off traffic.

By carefully inspecting figures 3 and 4, it is evident that for some range of input traffics, the performance of the proposed policy is close to the performance of the uniform fractional guard channel policy and performs better than the algorithm given in [7]. Since in the low hand-off traffic conditions, the UFC policy doesn't maintain the upper bound on the dropping probability of hand-off calls, the blocking probability of new calls for the proposed algorithm is greater than the blocking probability of new calls for UFC. When the hand-off traffic becomes high, the UFC policy maintains the upper bound on the dropping probability of hand-off calls and the performance of UFC policy and the proposed algorithm is very close. In such situations, the probability of accepting new calls converges to the optimal value found by the algorithm given in [4].

Figure 5 shows the new call acceptance probability of the above mentioned algorithms for different rates of hand-off traffic when the other parameters of the cell are fixed. As this figure shows, the proposed algorithm gives a smaller value of π. This is due to the higher priority given to hand-off calls.

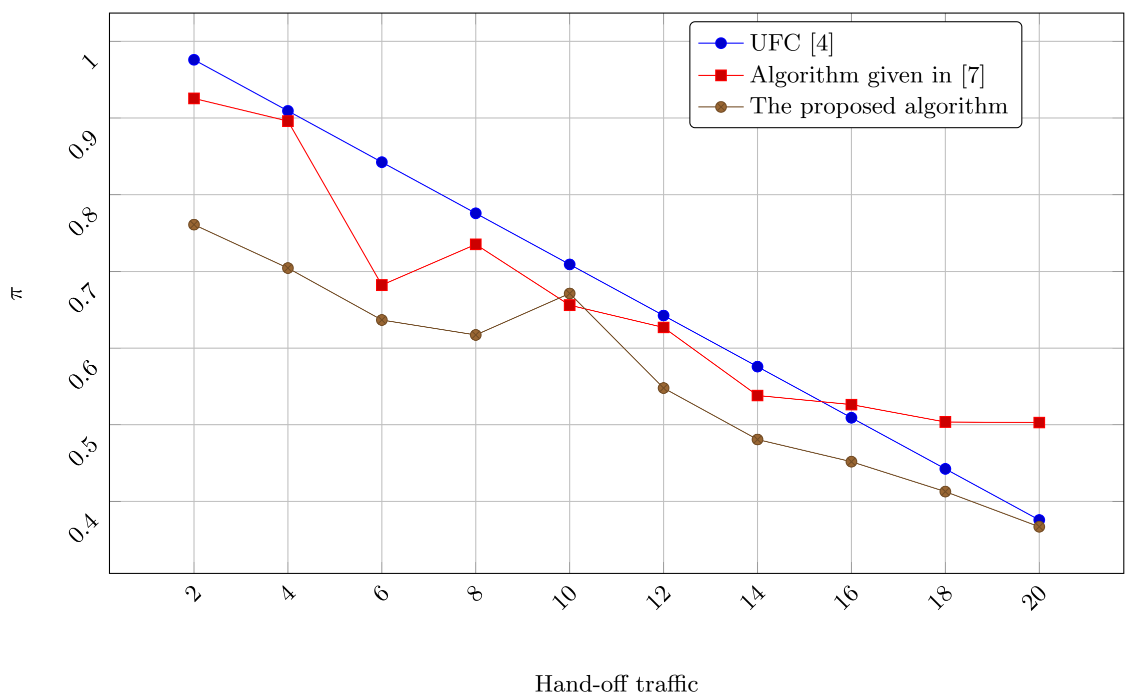

Fig. 5. New calls acceptance probability of algorithms for different hand-off traffic.

Figure 6 shows the blocking probability of new calls and dropping probability of hand-off calls of the proposed algorithm for different rates of hand-off traffic when the other parameters of the cell are fixed. As this figure shows, the proposed algorithm is able to maintain the QoS of hand-off traffic for different hand-off load. The blocking probability of new calls will be increased, because the number of channels allocated to the cell is fixed.

Figure 7 shows the evolution of the performance parameters for the adaptive uniform fractional guard channel policy. By carefully inspecting figure 7, it is evident that the admission probability, π, converges to a value, which is also optimal.

6 Conclusions

In this paper, we first propose a new learning automata based adaptive uniform fractional channel algorithm. The proposed algorithm uses a learning automaton to determine the admission probability, π, when the traffic parameters are unknown or probably time varying. This algorithm adjusts parameter π as network operates. Then we studied the steady state behavior of the proposed algorithm. It was shown

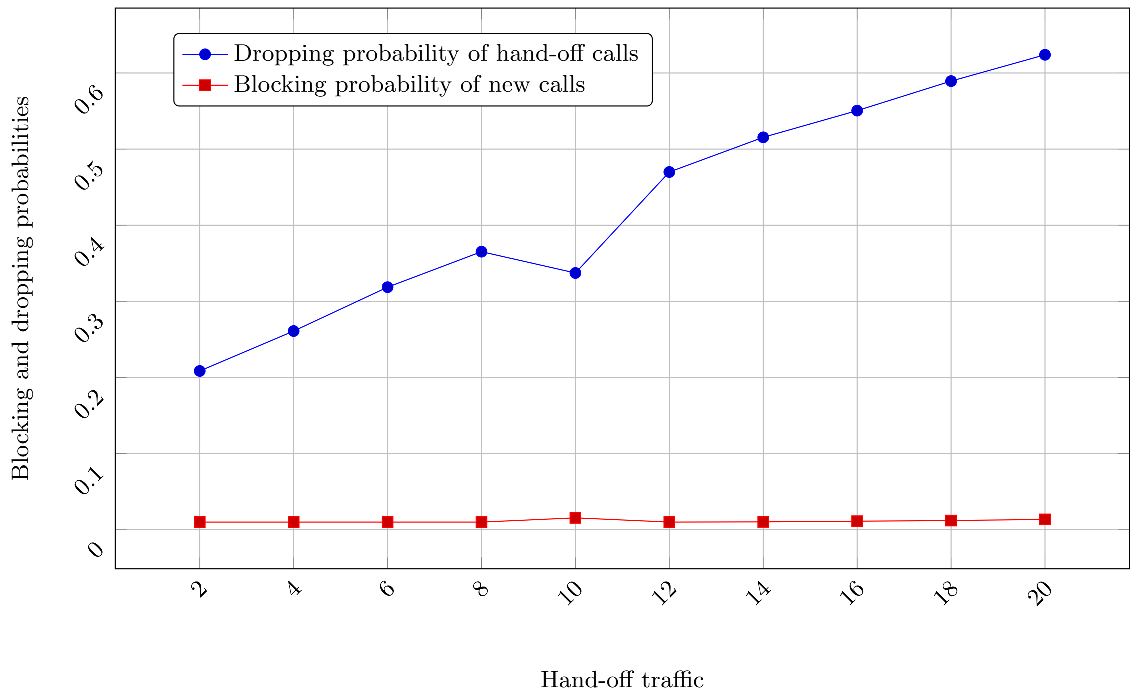

Fig. 6. Performance of the proposed algorithm for different hand-off traffic.

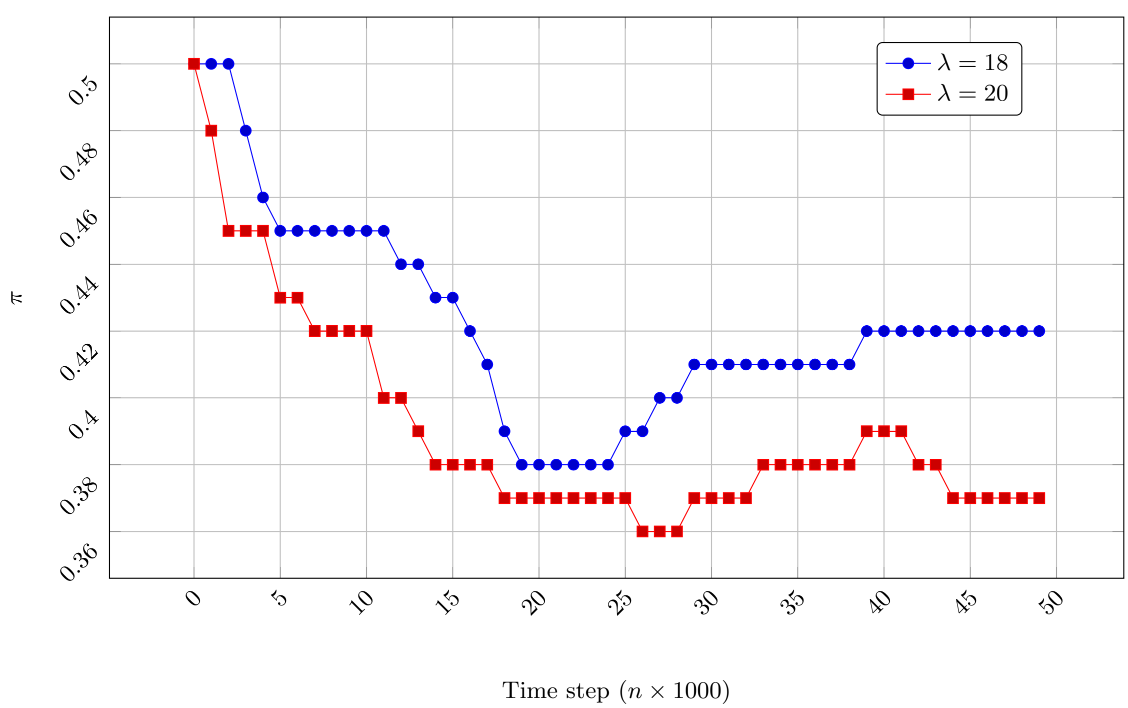

Fig. 7. Convergence of the proposed algorithm for different hand-off traffic.

that the proposed algorithm converges to an equilibrium point which is also optimal for UFC policy.The simulation results show that the level of QoS is satisfied by the proposed algorithm and the performance of given algorithm is very close to the performance of uniform fractional guard channel policy which needs to know all parameters of input traffic.

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