1. $E(x) = 300$ $\quad P(x>a) \le \frac{Ef(x)}{b}$ - нерівність Маркова

а) $P(x>400) \le \frac{300}{400} = 0,75$

б) $P(x \le 500) \ge 1 - \frac{300}{500} = 0,4$

2. $n = 1600$ $\quad P(x>a) \le \frac{E(x^2)}{a^2}$

$p = 0,3$

$q = 0,7$

$E(x) = np = 1600 \cdot 0,3 = 480$

$D(x) = npq = 1600 \cdot 0,3 \cdot 0,7 = 336$

$P(|480 - x| < 50) \ge 1 - \frac{336}{50^2} = 1 - \frac{336}{2500} = 1 - 0,1344 = 0,8656$

3. $X: 9, 5, 7, 7, 4, 10$ $\quad \bar{x} - x_{\alpha/2}\frac{\sigma}{\sqrt{n}} \le \mu \le \bar{x} + x_{\alpha/2}\frac{\sigma}{\sqrt{n}}$

$\frac{\alpha}{2} = 0,495$

$x_{1/2} = 2,58$

$D = 1 = \sigma^2$

$\alpha = 0,99$

$n = 6$ $\quad E = \frac{9+5+7+7+4+10}{6} = 7 = \bar{x}$

$7 - 2,58 \cdot \frac{1}{\sqrt{6}} \le \mu \le 7 + 2,58\frac{1}{\sqrt{6}}$

$\mu \in (5,95; 8,05)$

4. $X: \quad N(\mu, \sigma^2)$

$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}}$

$L = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}} = \left(\frac{1}{\sqrt{2\pi\sigma^2}}\right)^n e^{-\frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i-\mu)^2}$

$\ln L = -\frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i-\mu)^2 + \ln\left(\frac{1}{\sqrt{2\pi\sigma^2}}\right)^n = -\frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i-\mu)^2 - \ln\left(\sqrt{2\pi}\right)^n - \frac{n}{2}\ln\sigma^2$

$(\ln L)'_{\mu} = \frac{2}{2\sigma^2}\sum_{i=1}^{n}(x_i-\mu) = 0 \qquad n\bar{x} - n\mu = 0 \qquad \mu = \bar{x}$

$(\ln L)'_{\sigma^2} = \frac{1}{2\sigma^4}\sum_{i=1}^{n}(x_i-\mu)^2 - \frac{n}{2\sigma^2} = 0 \qquad \sigma^2 = \frac{\sum_{i=1}^{n}(x_i-\mu)^2}{n}$

$\hat{\mu} = \bar{x} \qquad \hat{\sigma}^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})^2$