ćwiczenia 1

October 27, 2021

1. (a) $7 + 4i + 3(5 - 9i) = 7 + 4i + 15 - 27i = 22 - 23i$

   (b) $Im(3 + 2i)(4 - 2i) - Re(5 + 3i)(1 - 4i) = 2(4 + 2i) - 5(1 - 4i) = 8 - 4i - 5 - 20 = 3 + 16i$

   (c) $(3 - 2i)(3 + 4i) = 9 + 12i - 6i - 8i^2 = 17 + 6i$

   (d) $(4 - 5i)(4 + 5i) = 4^2 - (5i)^2 = 16 + 25 = 41$

   (e) $\frac{-11+7i}{2+6i} = \frac{(-11+7i)(2-6i)}{(2+6i)(2-6i)} = \frac{-22+66i+14i-42i^2}{4+36} = \frac{20+80i}{40} = \frac{1}{2} + 2i$

   (f) $\frac{-1+2i}{5-3i} = \frac{(-1+2i)(5+3i)}{(5-3i)(5+3i)} = \frac{-5-3i+10i+6i^2}{25+9} = \frac{-11+7i}{31} = -\frac{11}{31} + \frac{7i}{31}$

   (g) $\frac{3+i}{2-5i} = \frac{(3+i)(2+5i)}{(2-5i)(2+5i)} = \frac{6+15i+2i+5i^2}{4+25} = \frac{1+17i}{29} = \frac{1}{29} + \frac{17i}{29}$

   (h) $\frac{5+3i}{8-2i} = \frac{(5+3i)(8+2i)}{(8-2i)(8+2i)} = \frac{40+10i+24i+6i^2}{64+4} = \frac{34+34i}{68} = \frac{1}{2} + \frac{1}{2}i$

   (i) $\frac{3+2i}{-1+3i} = \frac{(3+2i)(-1-3i)}{(-1+3i)(-1-3i)} = \frac{-3-9i-2i-6i^2}{1+3} = \frac{3-11i}{4} = \frac{3}{4} - \frac{11i}{4}$

2. (a)

$$\begin{aligned} 5a - b &= 5 \\ -b - a &= 4 \\ a + b &= -4 \\ b &= -4 - a \end{aligned}$$

$$\begin{aligned} 2z + (3 - i)\bar{z} &= 5 + 4i \\ 2(a + bi) + (3 - i)(a - bi) &= 5 + 4i \\ 2a + 2bi + 3a - 3bi - ai + bi^2 &= 5 + 4i \\ 5a - bi - ai - b &= 5 + 4i \\ 5a - b + i(-b - a) &= 5 + 4i \end{aligned}$$

$$\begin{aligned} 5a - (-4 - a) &= 5 \\ 5a + 4 + a &= 5 \\ 6a &= 1 \\ a &= \frac{1}{6} \end{aligned}$$

$$\begin{aligned} b &= -4 - \frac{1}{6} \\ b &= -\frac{25}{6} \end{aligned}$$

$$z = \frac{1}{6} - \frac{25}{6}i$$

   (b)