Assignment 03
Scaled Partial Pivoting

Mayank Pathania
204103314

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1 Algorithms

Algorithm 1 Scaled Partial Pivoting

procedure SCALED_PARTIAL_PIVOTING($A[][], B[], n$)
▷ A is system matrix
▷ B is RHS vector
▷ n is size of vector
    position ▷ initialize vector of size n
    S ▷ initialize vector of size n
    for $i \leftarrow 1 : 1 : n$ **do** ▷ calculating max element of each row
        $max\_element \leftarrow |A[i][1]|$
        for $j \leftarrow 1 : 1 : n$ **do**
            $max\_element \leftarrow max(max\_element, |A[i][j]|)$
        end for
        $S[i] \leftarrow max\_element$
        $position[i] \leftarrow i$
    end for

    for $i \leftarrow 1 : 1 : n$ **do**
        $max\_row \leftarrow i$
        $max\_ratio \leftarrow |\frac{A[position[i]][i]}{S[i]}|$
        for $j \leftarrow i + 1 : 1 : n - 1$ **do** ▷ Chose row with max ratio
            $ratio \leftarrow |\frac{A[position[i]][i]}{S[i]}|$
            if $ratio > max\_ratio$ **then**
                $max\_ratio \leftarrow ratio$
                $max\_row \leftarrow j$
            end if
        end for

        $swap(postition[i], position[max\_row])$

```
        for k ← i + 1 : 1 : n do                                   ▷ Performing Elimination
            ratio ← A[position[k]][i] / A[position[i]][i]
            for j ← i + 1 : 1 : n do
                A[position[k]][j] ← A[position[k]][j] − (ratio × A[position[i]][j])
            end for
            B[position[k]] ← B[position[k]] − (ratio × B[position[i]])
        end for
    end for
    return back_substitution(A, B, position)    ▷ Call back substitution to get solution
end procedure
```

Algorithm 2 Back Substitution

```
procedure back_substitution(A[][],B[],n,position[])
    x                                                  ▷ initialize vector of size n
    x[n] ← B[position[n]] / A[position[n]][n]
    for i ← n − 1 : −1; 1 do
        total ← 0
        for j ← i + 1 : 1 : n do
            total ← total + (A[position[i]][j] ∗ x[j])
        end for
        x[i] ← (B[position[i]] − total) / A[position[i]][i]
    end for
    return x
end procedure
```

2 Elimination Steps

Orignal Problem

$$\text{A} = \begin{bmatrix} 0.4096 & 0.1234 & 0.3678 & 0.2943 \\ 0.2246 & 0.3872 & 0.4015 & 0.1129 \\ 0.3645 & 0.192 & 0.3781 & 0.0643 \\ 0.1784 & 0.4022 & 0.2786 & 0.3927 \end{bmatrix} \quad \text{B} = \begin{Bmatrix} 0.4043 \\ 0.155 \\ 0.424 \\ 0.2557 \end{Bmatrix}$$

1. Rows Swapped 1 <===> 1

$$\text{A} = \begin{bmatrix} 0.4096 & 0.1234 & 0.3678 & 0.2943 \\ 0 & 0.3195 & 0.1998 & -0.04847 \\ 0 & 0.0821 & 0.05079 & -0.1975 \\ 0 & 0.3484 & 0.1184 & 0.2645 \end{bmatrix} \quad \text{B} = \begin{Bmatrix} 0.4043 \\ -0.0666 \\ 0.0642 \\ 0.0796 \end{Bmatrix}$$

2. Rows Swapped 2 <===> 4

$$A = \begin{bmatrix} 0.4096 & 0.1234 & 0.3678 & 0.2943 \\ 0 & 0.3484 & 0.118406 & 0.264519 \\ 0 & 0 & 0.02287 & -0.259985 \\ 0 & 0 & 0.0912414 & -0.291042 \end{bmatrix} B = \begin{Bmatrix} 0.4043 \\ -0.1396 \\ 0.0454 \\ 0.0796 \end{Bmatrix}$$

3. Rows Swapped 3 <===> 4

$$A = \begin{bmatrix} 0.4096 & 0.1234 & 0.3678 & 0.2943 \\ 0 & 0.3484 & 0.1184 & 0.2645 \\ 0 & 0 & 0.0912 & -0.2910 \\ 0 & 0 & 0 & -0.1870 \end{bmatrix} B = \begin{Bmatrix} 0.4043 \\ -0.1396 \\ 0.0804 \\ 0.0796 \end{Bmatrix}$$

3 Solution

After perfoming elimination by Gauss elimination with scaled partial pivoting the solution can be obtained by back subsitution.

Solution

$$x = \begin{Bmatrix} 3.43863 \\ 1.54152 \\ -2.90318 \\ -0.43016 \end{Bmatrix}$$

4 Remarks

- In algorithm the elements are not actually made zero as these elements will not be accessed during back substitution it is not necessary to perform elimination on them.
- The rows are not swapped in the algorithm but vector "position" is used to store the order of the rows and any swap is done by swaping values of "position" vector.