Newton Form Polynomial

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1 Algorithms

1.1 Algorithm to get coefficients

Algorithm 1 Newton Form

```
procedure NEWTON_FORM_COEFFICIENT(x_values, y_values, data_points)
    coefficients[data_points]                          ▷ Initialize array of size data_points
    for (i=0;i<data_points;i++) do
        denominator = 1
        nominator = 0
        for (j=0;j<i;j++) do
            product = 1
            for (k=0;k<j;k++) do
                product *= (x_values[i] - y_values[k])
            nominator += coefficients[j]*product;
            denominator *= (x_value[i] - x_value[j])
        coefficients[i] = (y_values[i] - nominator)/denominator
    return coefficients
```

1.2 Algorithm to evaluate polynomial at given point

coefficients are evaluated by Algorithm 1 and x_values are the values used in Algorithm 1.

Algorithm 2 To evaluate polynomial at given point

```
procedure EVALUATE(coefficients,x_values,point,degree)
    value = coefficients[degree]
    while (degree–) do
        value *= (point - x_value[degree])
        value += coefficients[degree]
    return value
```

2 Calculations

2.1 Parameters

start_point : 1
end_point : 6
points : 11

2.2 Data Points

x	1	1.5	2	2.5	3	3.5	4	4.5	5	5.5	6
y=arctan(x)	0.785	0.982	1.107	1.190	1.249	1.292	1.325	1.352	1.373	1.390	1.405

2.3 Coefficients

a	a_0	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}
values	0.785	0.394	-0.146	0.042	-0.009	0.001	-0.003	3.59e-5	-2.27e-6	-2.85e-7	1.37e-7

3 Evaluation

Since in newtons form the addition of new point do no alter the previously calculated coefficients we can write the polynomials of degree 2, 4 and 10 by only evaluating coefficients for degree 10 polynomial and degree 2 and degree 4 polynomials can be obtained from it.

3.1 Parameters

start_point : 0
end_point : 8
Points : 33

3.1.1 Coefficients and x values

x	x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
values	1	1.5	2	2.5	3	3.5	4	4.5	5	5.5	6
a	a_0	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}
values	0.78	0.39	-0.14	0.04	-0.009	0.001	-0.003	3.59e-5	-2.27e-6	-2.85e-7	1.37e-7

3.2 2^{Nd} Degree Polynomial

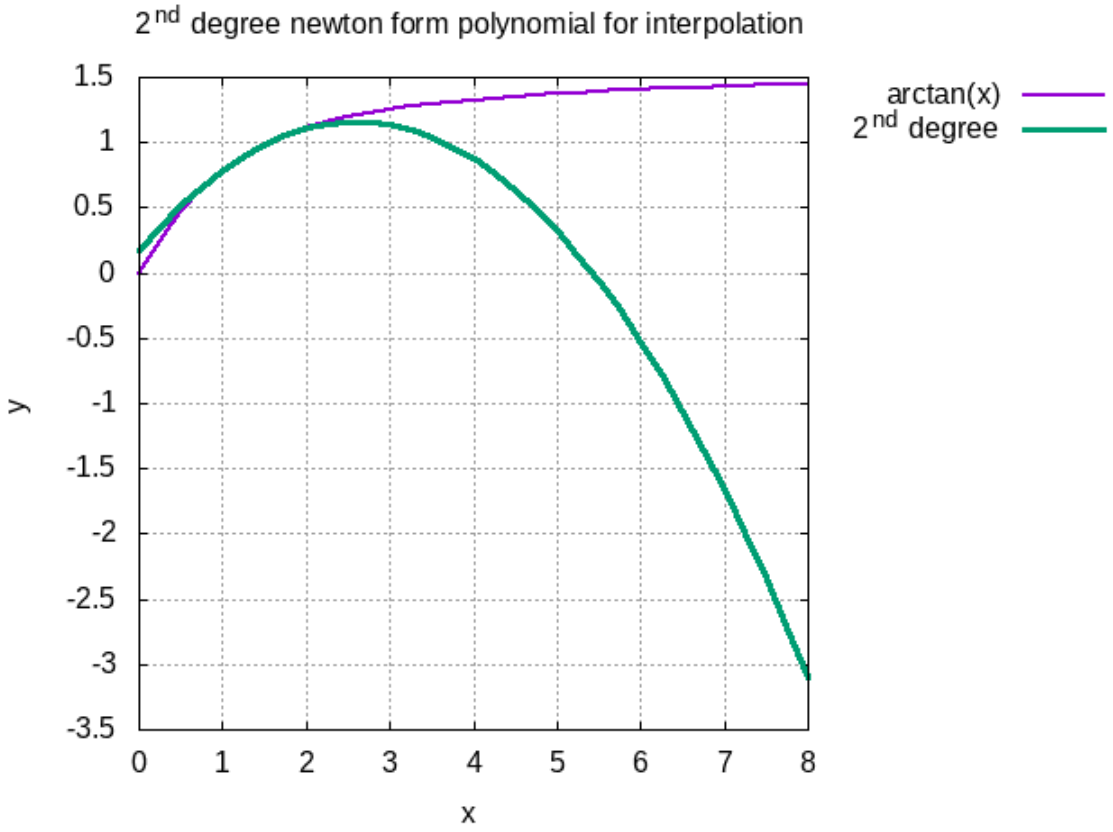
3.2.1 Equation

$$y = 0.785 + (x - x_0)(0.394 + (x - x_1) * (-0.146))$$

3.2.2 Coefficients

a	a_0	a_1	a_2
values	0.785	0.394	-0.146
x	x_0	x_1	x_2
values	1	1.5	2

3.2.3 Plot


2nd degree newton form polynomial for interpolation
arctan(x)
2nd degree
y
x
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
-3
-3.5
0
1
2
3
4
5
6
7
8

3.2.4 Table 2^{Nd} Degree Polynomial

x	arctan(x)	obtained_value	(arctan(x) - obtained_value)
0	0	0.171485	-0.171485
0.25	0.244979	0.352354	-0.107375
0.5	0.463648	0.514962	-0.0513144
0.75	0.643501	0.65931	-0.0158091
1	0.785398	0.785398	0
1.25	0.896055	0.893226	0.00282937
1.5	0.982794	0.982794	0
1.75	1.05165	1.0541	-0.00245108
2	1.10715	1.10715	0
2.25	1.15257	1.14194	0.010636
2.5	1.19029	1.15846	0.0318268
2.75	1.22203	1.15673	0.0652952
3	1.24905	1.13674	0.112309
3.25	1.2723	1.09848	0.173814
3.5	1.2925	1.04197	0.250526
3.75	1.31019	0.967197	0.342997
4	1.32582	0.874163	0.451655
4.25	1.33971	0.762869	0.576836
4.5	1.35213	0.633315	0.718812
4.75	1.3633	0.485501	0.877799
5	1.3734	0.319427	1.05397
5.25	1.38257	0.135092	1.24748
5.5	1.39094	-0.0675021	1.45844
5.75	1.39861	-0.288357	1.68696
6	1.40565	-0.527472	1.93312
6.25	1.41214	-0.784847	2.19699
6.5	1.41815	-1.06048	2.47863
6.75	1.42372	-1.35438	2.77809
7	1.4289	-1.66653	3.09543
7.25	1.43373	-1.99695	3.43068
7.5	1.43824	-2.34562	3.78387
7.75	1.44247	-2.71256	4.15503
8	1.44644	-3.09776	4.5442

3.3 4^{Th} Degree Polynomial

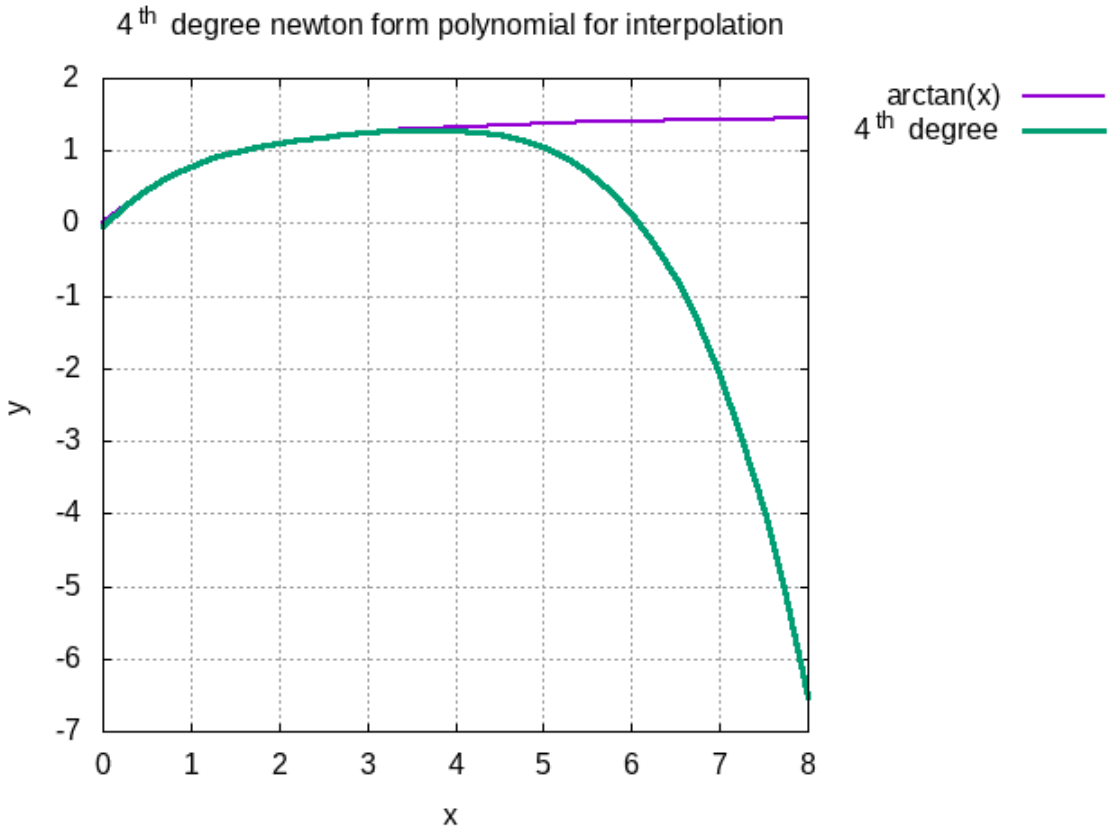
3.3.1 Equation

$$y = 0.785 + (x - x_0)(0.394 + (x - x_1)(-0.146 + (x - x_2)(0.042 + (x - x3) * (-0.009))))$$

3.3.2 Coefficients

a	a_0	a_1	a_2	a_3	a_4
values	0.785	0.394	-0.146	0.042	-0.009
x	x_0	x_1	x_2	x_3	x_4
values	1	1.5	2	2.5	3

3.3.3 Plot



3.3.4 Table 4^{Th} Degree Polynomial

x	arctan(x)	obtained_value	(arctan(x) - obtained_value)
0	0	-0.0308141	0.0308141
0.25	0.244979	0.245822	-0.000843724
0.5	0.463648	0.468137	-0.00448918
0.75	0.643501	0.645263	-0.00176205
1	0.785398	0.785398	0
1.25	0.896055	0.895801	0.000254319
1.5	0.982794	0.982794	0
1.75	1.05165	1.05176	-0.000110377
2	1.10715	1.10715	0
2.25	1.15257	1.15247	0.000104241
2.5	1.19029	1.19029	0
2.75	1.22203	1.22225	-0.000224819
3	1.24905	1.24905	0
3.25	1.2723	1.27044	0.00186052
3.5	1.2925	1.28525	0.00725058
3.75	1.31019	1.29136	0.0188353
4	1.32582	1.28572	0.0400953
4.25	1.33971	1.26435	0.075358
4.5	1.35213	1.22231	0.12982
4.75	1.3633	1.15374	0.209562
5	1.3734	1.05184	0.321564
5.25	1.38257	0.908864	0.473711
5.5	1.39094	0.716145	0.674798
5.75	1.39861	0.464064	0.934542
6	1.40565	0.14207	1.26358
6.25	1.41214	-0.261325	1.67347
6.5	1.41815	-0.758548	2.17669
6.75	1.42372	-1.36296	2.78668
7	1.4289	-2.08887	3.51777
7.25	1.43373	-2.95151	4.38524
7.5	1.43824	-3.96705	5.40529
7.75	1.44247	-5.15261	6.59509
8	1.44644	-6.52625	7.97269

3.4 10^{Th} Degree Polynomial

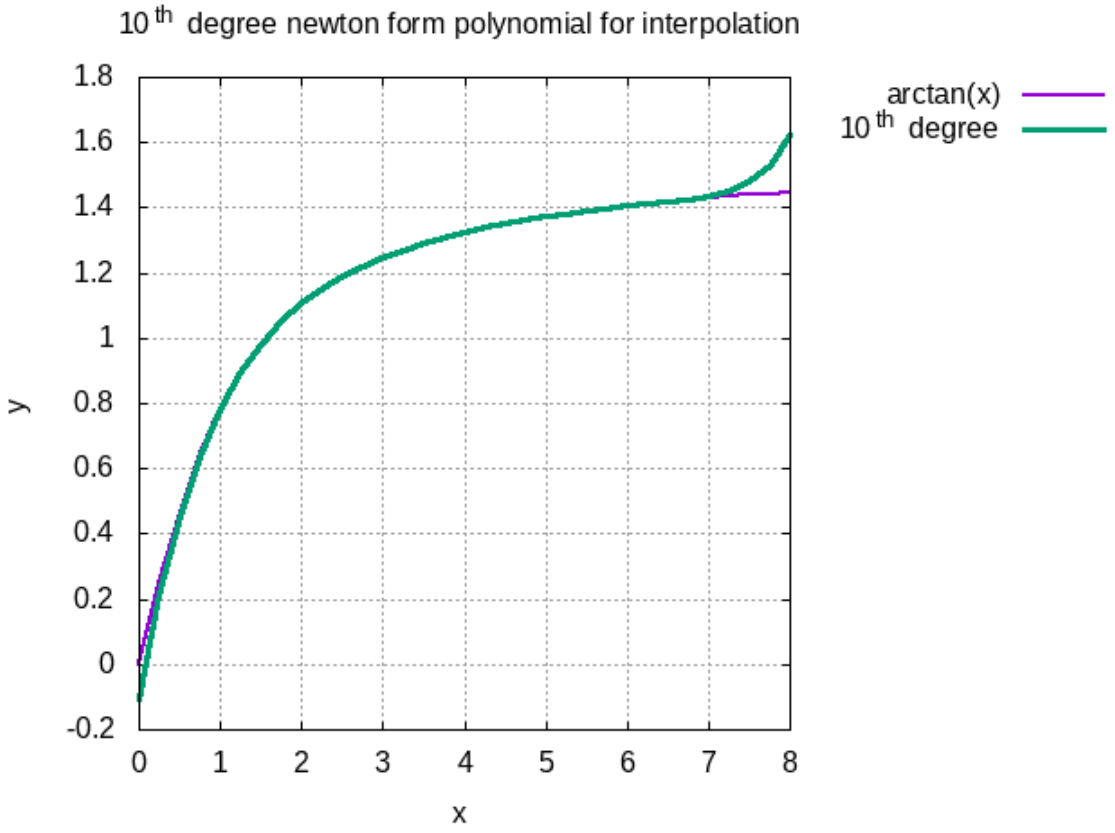
3.4.1 Equation

$$y = 0.785 + (x - x_0)(0.394 + (x - x_1)(-0.146 + (x - x_2)(0.042 + (x - x3)(-0.009 + (x - x_4)(0.00193 + (x - x_5)(-0.003 + (x - x_6)(3.59e-5 + (x - x_7)(-2.271e-6 + (x - x_8)(-2.85e-7 + (x - x_9) * (1.37e-7)))))))))$$

3.4.2 Coefficients

a	a_0	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}
values	0.785	0.394	-0.146	0.042	-0.009	0.001	-0.003	3.59e-5	-2.27e-6	-2.85e-7	1.37e-7
x	x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
values	1	1.5	2	2.5	3	3.5	4	4.5	5	5.5	6

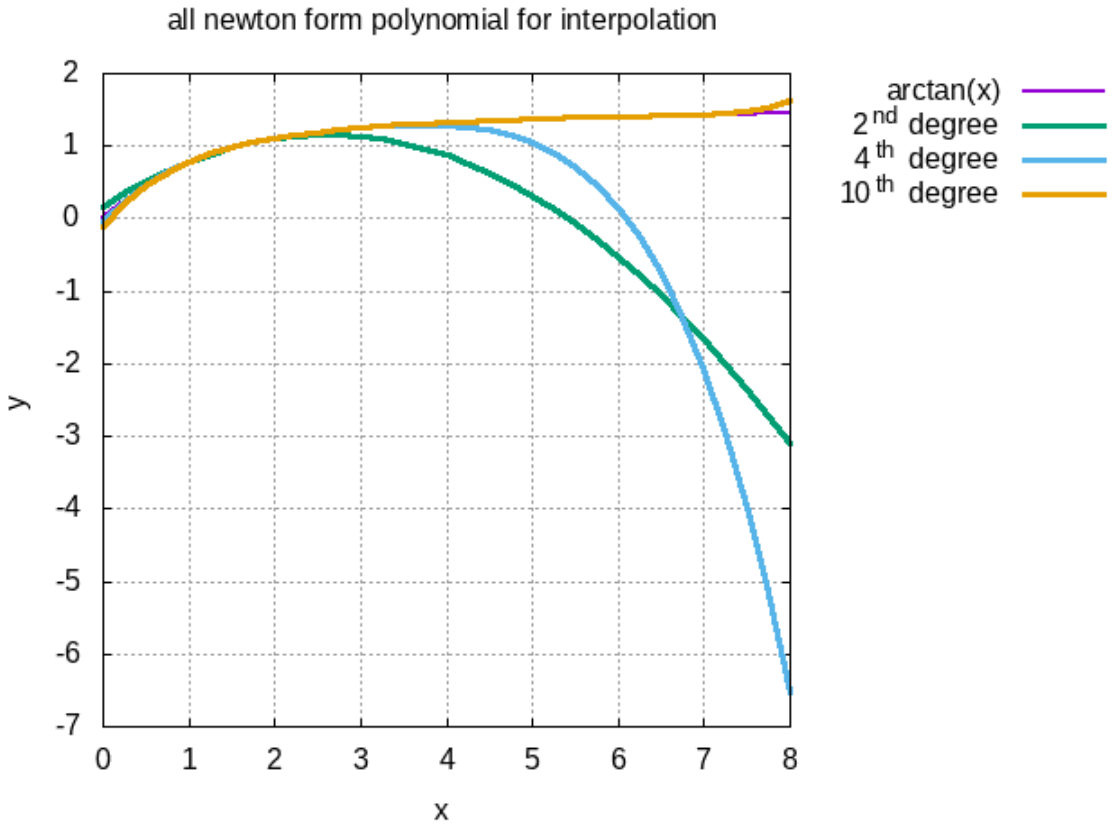
3.4.3 Plot



3.4.4 Table 10^{Th} Degree Polynomial

x	arctan(x)	obtained_value	(arctan(x) - obtained_value)
0	0	-0.105316	0.105316
0.25	0.244979	0.213081	0.031898
0.5	0.463648	0.456396	0.00725133
0.75	0.643501	0.642467	0.00103414
1	0.785398	0.785398	0
1.25	0.896055	0.89609	-3.45774e-05
1.5	0.982794	0.982794	0
1.75	1.05165	1.05165	3.94587e-06
2	1.10715	1.10715	0
2.25	1.15257	1.15257	-8.76923e-07
2.5	1.19029	1.19029	0
2.75	1.22203	1.22203	3.21874e-07
3	1.24905	1.24905	0
3.25	1.2723	1.2723	-1.8111e-07
3.5	1.2925	1.2925	2.22045e-16
3.75	1.31019	1.31019	1.51123e-07
4	1.32582	1.32582	0
4.25	1.33971	1.33971	-1.86048e-07
4.5	1.35213	1.35213	-2.22045e-16
4.75	1.3633	1.3633	3.45709e-07
5	1.3734	1.3734	-2.22045e-16
5.25	1.38257	1.38258	-1.03388e-06
5.5	1.39094	1.39094	0
5.75	1.39861	1.3986	5.82841e-06
6	1.40565	1.40565	-4.44089e-16
6.25	1.41214	1.41225	-0.000110048
6.5	1.41815	1.41877	-0.000622478
6.75	1.42372	1.42601	-0.00229534
7	1.4289	1.4357	-0.00679997
7.25	1.43373	1.45121	-0.0174749
7.5	1.43824	1.47876	-0.0405148
7.75	1.44247	1.52924	-0.0867665
8	1.44644	1.62079	-0.174352

3.5 Plots of all Polynomials



4 Conclusion/Observations

- Polynomial give exact value on the data points.
- Higher degree polynomial gave more accurate results and also for more range of points.
- The value of the new coefficients were lower as more data points were added.
- After adding some data points the value of coefficients will be low and adding more data points will not cause much change.