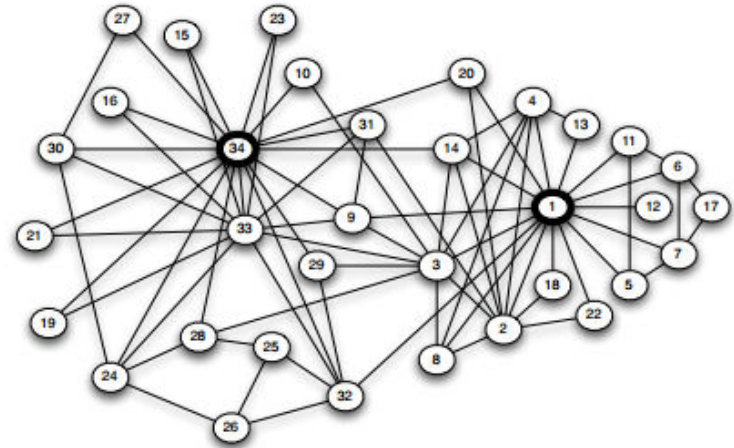


# Node Importance

Based on the structure of the network, which are the 5 most important nodes in the Karate Club friendship network?



Friendship network in a 34-person karate club  
[Zachary 1977]

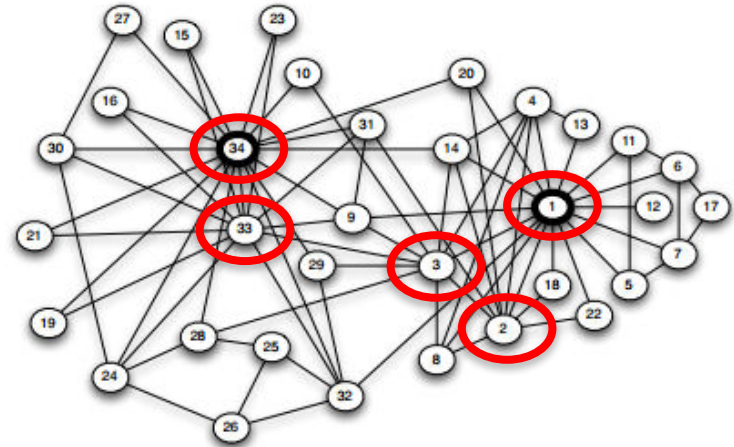
# Node Importance

Different ways of thinking about “importance”.

Ex. Degree: number of friends.

5 most important nodes are:

34, 1, 33, 3, 2



Friendship network in a 34-person karate club  
[Zachary 1977]

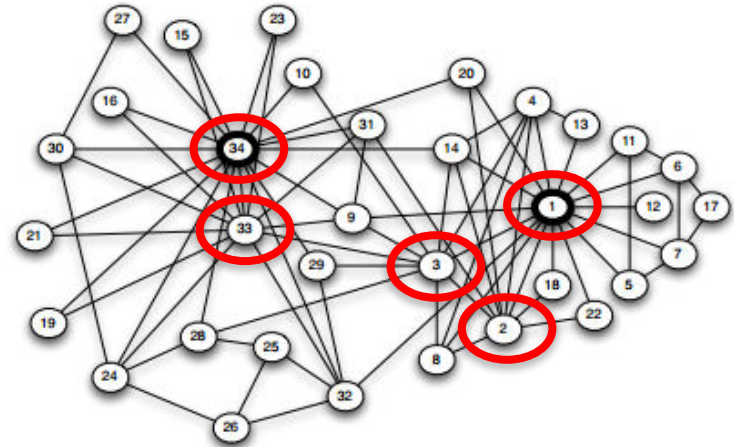
# Node Importance

Different ways of thinking about “importance”.

Ex. Average proximity to other nodes.

5 most important nodes are:

1, 3, 34, 32, 9



Friendship network in a 34-person karate club  
[Zachary 1977]

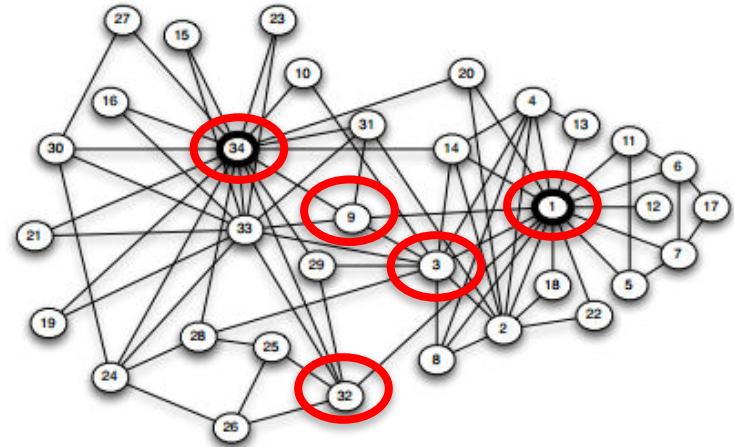
# Node Importance

Different ways of thinking about “importance”.

Ex. Fraction of shortest paths that pass through node.

5 most important nodes are:

1, 34, 33, 3, 32



Friendship network in a 34-person karate club  
[Zachary 1977]

# Network Centrality

Centrality measures identify the most important nodes in a network:

- Influential nodes in a social network.
- Nodes that disseminate information to many nodes or prevent epidemics.
- Hubs in a transportation network.
- Important pages on the Web.
- Nodes that prevent the network from breaking up.

# Centrality Measures

- **Degree centrality**
- **Closeness centrality**
- Betweenness centrality
- Load centrality
- Page Rank
- Katz centrality
- Percolation centrality

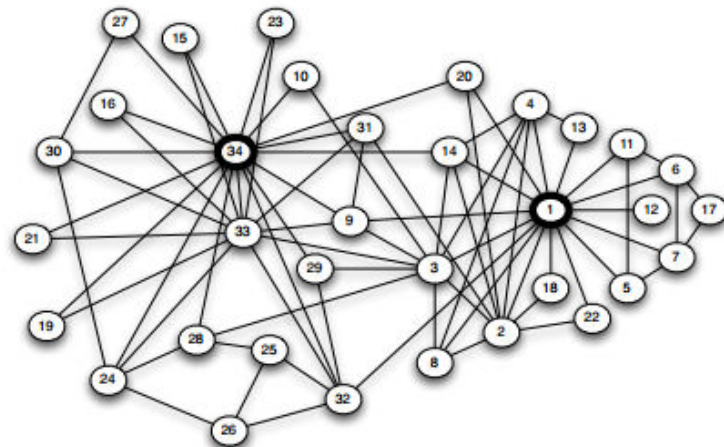
# Degree Centrality

**Assumption:** important nodes have many connections.

The most basic measure of centrality: number of neighbors.

Undirected networks: use degree

Directed networks: use in-degree or out-degree



Friendship network in a 34-person karate club  
[Zachary 1977]

# Degree Centrality – Undirected Networks

$C_{deg}(v) = \frac{d_v}{|N|-1}$ , where  $N$  is the set of nodes in the network and  $d_v$  is the degree of node  $v$ .

```
In: G = nx.karate_club_graph()
```

```
In: G =
```

```
nx.convert_node_labels_to_integers(G,first_label=1)
```

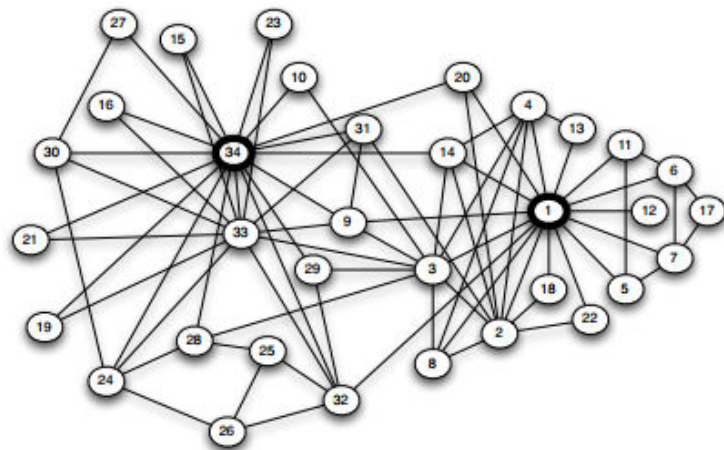
```
In: degCent = nx.degree_centrality(G)
```

```
In: degCent[34]
```

```
Out: 0.515 # 17/33
```

```
In: degCent[33]
```

```
Out: 0.182 # 6/33
```



Friendship network in a 34-person karate club  
[Zachary 1977]





# Degree Centrality – Directed Networks

$$C_{outdeg}(v) = \frac{d_v^{out}}{|N|-1}, \text{ where}$$

$N$  = set of nodes in the network,

$d_v^{out}$  = the out-degree of node  $v$ .

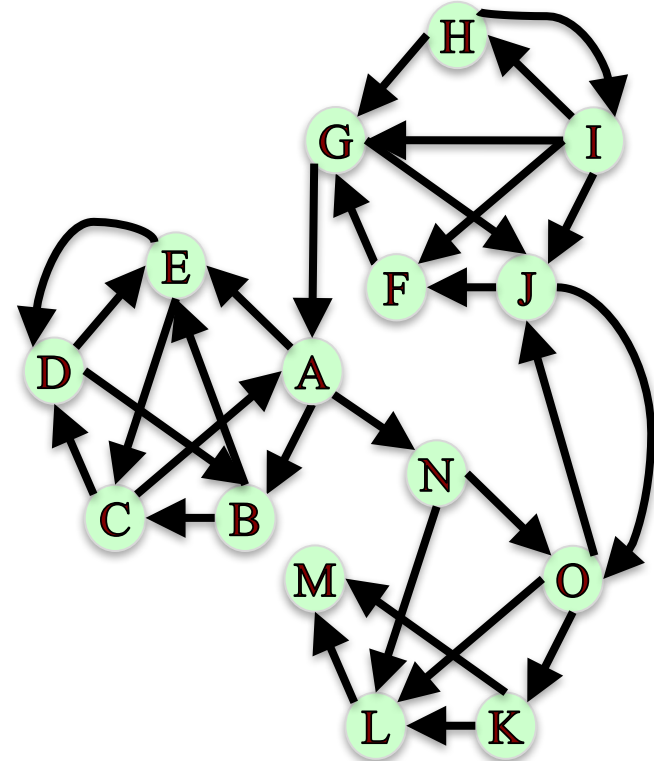
```
In: outdegCent = nx.out_degree_centrality(G)
```

```
In: outdegCent['A']
```

```
Out: 0.214 # 3/14
```

```
In: outdegCent['L']
```

```
Out: 0.071 # 1/14
```



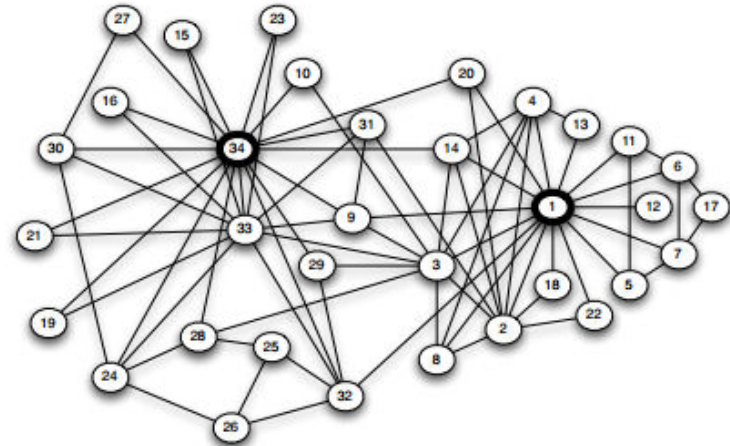
# Closeness Centrality

**Assumption:** important nodes are close to other nodes.

$$C_{close}(v) = \frac{|N|-1}{\sum_{u \in N \setminus \{v\}} d(v,u)}, \text{ where}$$

$N$  = set of nodes in the network,

$d(v,u)$  = length of shortest path from  $v$  to  $u$ .



Friendship network in a 34-person karate club  
[Zachary 1977]

# Closeness Centrality

**Assumption:** important nodes are close to other nodes.

```
In: closeCent = nx.closeness centrality(G)
```

```
In: closeCent[32]
```

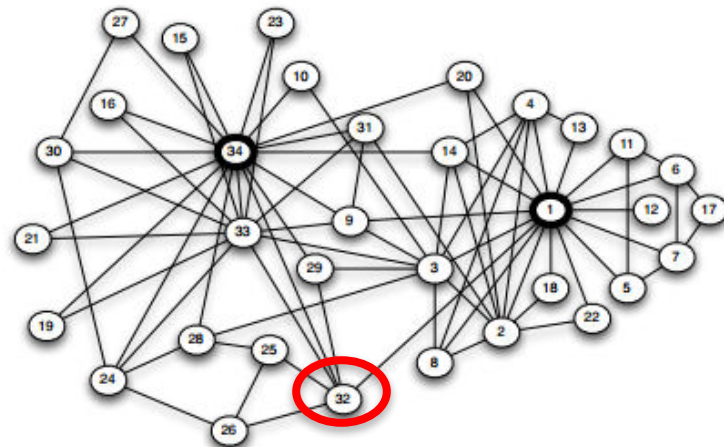
```
Out: 0.541
```

```
In: sum(nx.shortest_path_length(G,32).values())
```

```
Out: 61
```

```
In: (len(G.nodes())-1)/61.
```

```
Out: 0.541
```



Friendship network in a 34-person karate club  
[Zachary 1977]

# Disconnected Nodes

How to measure the closeness centrality of a node when it cannot reach all other nodes?

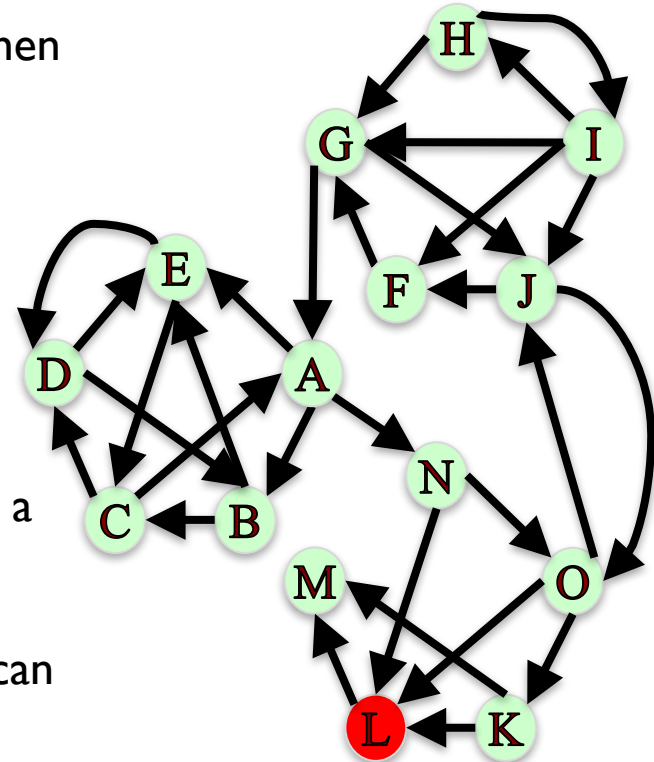
**What is the closeness centrality of node L?**

**Option 1:** Consider only nodes that L can reach:

$C_{close}(L) = \frac{|R(L)|}{\sum_{u \in R(L)} d(L, u)}$ , where  $R(L)$  is the set of nodes L can reach.

$C_{close}(L) = \frac{1}{1} = 1$ , since L can only reach M and it has a shortest path of length 1.

**Problem:** centrality of L is too high for a node that can only reach one other node!



# Disconnected Nodes

How to measure the closeness centrality of a node when it cannot reach all other nodes?

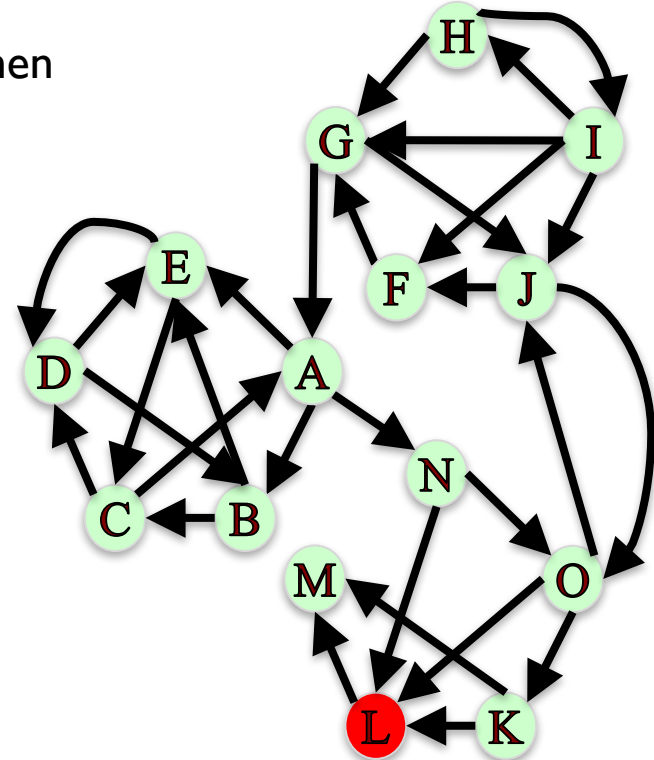
What is the closeness centrality of node L?

**Option 2:** Consider only nodes that L can reach and normalize by the fraction of nodes L can reach:

$$C_{close}(L) = \left[ \frac{|R(L)|}{|N-1|} \right] \frac{|R(L)|}{\sum_{u \in R(L)} d(L, u)}$$

$$C_{close}(L) = \left[ \frac{1}{14} \right] \frac{1}{1} = 0.071$$

Note that this definition matches our definition of closeness centrality when a graph is connected since  $R(L) = N - 1$



# Disconnected Nodes

How to measure the closeness centrality of a node when it cannot reach all other nodes?

What is the closeness centrality of node L?

```
In: closeCent = nx.closeness centrality(G, normalized = False)
```

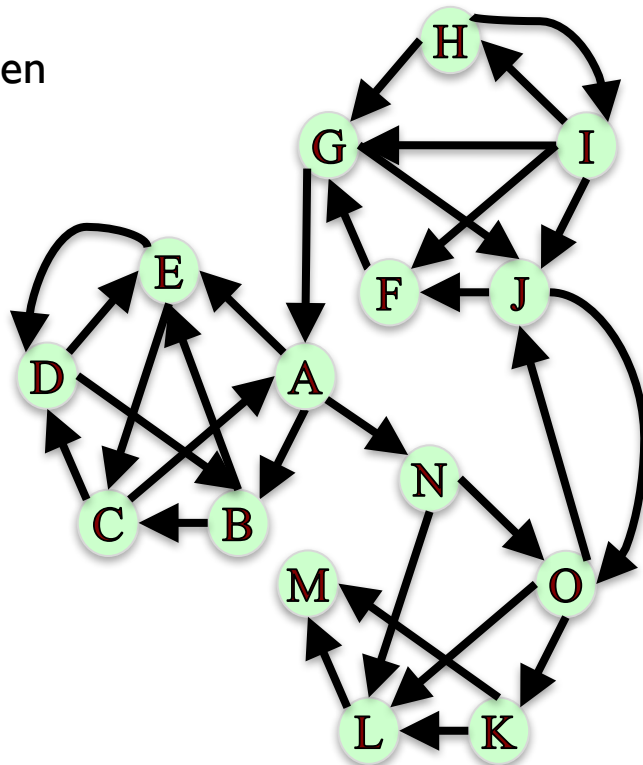
```
In: closeCent['L']
```

```
Out: 1
```

```
In: closeCent = nx.closeness centrality(G, normalized = True)
```

```
In: closeCent['L']
```

```
Out: 0.071
```



# Summary

Centrality measures identify the most important nodes in a network:

## Degree Centrality

**Assumption:** important nodes have many connections.

$$C_{deg}(v) = \frac{d_v}{|N| - 1}$$

`nx.degree_centrality(G)`

`nx.in_degree_centrality(G)`

`nx.out_degree_centrality(G)`

## Closeness Centrality

**Assumption:** important nodes are close to other nodes.

$$C_{close}(L) = \left[ \frac{|R(L)|}{|N|-1} \right] \frac{|R(L)|}{\sum_{u \in R(L)} d(L,u)}$$

`nx.closeness_centrality(G, normalized = True)`