Review of Parametric Active Contour Models in Image Processing

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Review of Parametric Active Contour Models in Image Processing

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Abstract

The parametric active contour models are widely used in computer vision and image processing. This paper reviews the research status of parametric active contour models in the past decade. The parametric active contour models have gained wide attention for a long time. First, the traditional snake and some typical modified models are introduced. Second, the parametric active contour models applications in the field of image processing are reviewed. Finally, the future work of parametric active contour models is pointed out.

Keywords: *Parametric Active Contour Models, Image Segmentation, Edge Detection, Target Object Tracking*

1. Introduction

According to the computer vision lamination theory of Marr, the contour extraction was regarded as an autonomous, from bottom, but on treating process. After the 1980s, however, the people thought that the integration of prior knowledge and image characteristic was the best way. Under this background, active contour model or snake [1] was presented by M. Kass et al. in 1987. A snake, which is an energy-minimizing curve, does not require any a prior knowledge about the image. The internal forces discourage the contours smoothness and tautness, and the external forces pull the contours toward features of interest, such as edges.

In general, there are two major types of active contour models: parametric active contour models [1] (PACMs) and geometric active contour models. This paper focuses on the PACMs. During the last decade, many improved models have been proposed. PACMs have been widely used in many applications of image processing. The purpose of this paper is to give complete view of the PACMs in image processing.

The remainder of this paper is organized as follows. Traditional snake and some modified models are introduced briefly in section 2. Section 3 describes the applications of the parametric active contour models in the field of image processing. The future trends are set out in section 4. Finally, section 5 provides the conclusions.

2. Parametric active contour models

Due to the potential of the PACM, it becomes an area of particular focus. Many modified models of the traditional snake have been proposed. Here the traditional snake model and some typical improved models are introduced. The traditional snake and some typical improved models are shown in Table 1.

Table 1. The traditional snake and some typical improved models

Models	Author	date
Traditional snake [1]	M. Kass et al.	1987
Distance potential force [17]	L.D. Cohen et al.	1993
Gradient vector flow (GVF) snake [8]	Xu et al.	1998
Vector field convolution (VFC) snake [9]	Li et al.	2007

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2.1. Traditional snake

A traditional active contour or a snake [1] is a curve $c(s) = [x(s), y(s)]$, $s \in [0,1]$. The snake model is an open or closed curve, it is closed when $c(0)=c(1)$. The contour is deformed by minimizing the following energy functional [1]:

$$E = \int_0^1 [\tfrac{1}{2}(\alpha|c'(s)|^2 + \beta|c''(s)|^2) + E_{ext}(c(s))]ds \tag{1}$$

where $E_{\text{int}}(c(s)) = \frac{1}{2}(\alpha|c'(s)|^2 + \beta|c''(s)|^2)$ represents the internal energy to smooth a snake, $c'(s)$ and $c''(s)$ denote the first and second derivatives of $c(s)$ with respect to parameter s. The weighting parameters α and β are used to control the strength of the contour's smoothness and tautness, respectively. In practice, α and β are often chosen to be constants. $E_{ext}(c(s))$ represents the external energy, its value is small at the feature of interest in an image, such as boundary. The typical external energies for a gray level image $I(x, y)$ are as follows:

$$E_{ext}^{(1)}(x, y) = -|\nabla I(x, y)|^2 \tag{2}$$

$$E_{\text{ext}}^{(2)}(x, y) = -|\nabla[G_\sigma(x, y) * I(x, y)]|^2 \tag{3}$$

where $G_\sigma(x, y)$ denotes 2-D Gaussian filter with standard deviation σ, * denotes linear convolution and ∇ denotes the gradient operator. If the image is a binary image (black on white), then the typical external energies are as follows:

$$E_{\text{ext}}^{(3)}(x, y) = I(x, y) \tag{4}$$

$$E_{ext}^{(4)}(x, y) = G_\sigma(x, y) * I(x, y) \tag{5}$$

In these equations, it is easy to see that a large σ may blur the feature of interest. However, a large σ is often necessary to increase the capture range of the active contours.

The contour should deform by minimizing the energy functional E, this problem can be solved by using the calculus of variations [2]. The Euler-Lagrange equation of (1) is

$$\alpha c'' - \beta c'''' - \nabla E_{ext}(c) = 0 \tag{6}$$

Eq. (6) can also be viewed as a force balance equation:

$$\boldsymbol{F}_{\text{int}}(c) + \boldsymbol{F}_{ext}(c) = 0 \tag{7}$$

where $\boldsymbol{F}_{int}(c) = \alpha\, c'' - \beta\, c''''$ is the internal force to discourage the contour smoothness and tautness, and $F_{ext}(c) = -\nabla E_{ext}(c)$ is the external force that pulls the contour toward the feature of interest. To solve the Euler-Lagrange equation, $c(s)$ is treated as a function of time t as well as s, i.e., $c(s, t)$. Then the partial derivative of $c(s)$ with respect to t is set equal to the left-hand side of (6) as follows:

$$c_t(s,t) = \alpha c''(s,t) - \beta c''''(s,t) - \nabla E_{ext}(s,t) \tag{8}$$

The solution of (6) is obtained when the steady state solution of (8) is reached from an initial contour $c(s, 0)$. A numerical solution to (8) can be found by solving a discretization of s iteratively using a finite difference approach [1, 17]. The continuous contour $c(s)$ is represented by a set of discrete points c_i, $i \in (0, 1, \ldots, M\text{-}1)$. The update procedure for the contour can be written in matrix form as:

$$(\boldsymbol{C}^{t+1} - \boldsymbol{C}^t)/\tau = \boldsymbol{AC}^{t+1} + \boldsymbol{F}^t \Rightarrow (\boldsymbol{I} + \tau\boldsymbol{A})\boldsymbol{C}^{t+1} = \boldsymbol{C}^t + \tau\boldsymbol{F}^t \tag{9}$$

where τ is the time step, I is the $M \times M$ identity matrix, $\boldsymbol{C}^t=[C_0^t, C_1^t, \ldots, C_{M\text{-}1}^t]^T$, $\boldsymbol{F}^t=[F_{ext}(C_0^t), F_{ext}(C_1^t), \ldots, F_{ext}(C_{M\text{-}1}^t)]^T$.

$$A = \begin{bmatrix} a & b & c & \cdots & c & b \\ b & a & b & c & \vdots & c \\ \vdots & \ddots & \ddots & \ddots & & \vdots \\ & & \ddots & \ddots & \ddots & \\ c & \vdots & c & b & a & b \\ b & c & \cdots & c & b & a \end{bmatrix}$$

where $a=2\alpha+6\beta$, $b=-(\alpha+4\beta)$, $c=\beta$. (10)

There are three key difficulties with the traditional snake. First, in the implementation of the traditional snake, the initial contour must, in general, be close to the desired target or else it will potentially converge to the wrong result, i.e., the capture range of the traditional snake is limited. Second, automatic initialization becomes impossible and user interaction is required. The third problem is that the traditional snake does not converge to the concave boundary.

2.2. Some improved models

The modified models have been presented for various purposes. For instance, balloons model [16] and distance potential force model [17] are proposed in order to increase the capture range of the active contour, some improved models have high convergence speed and low computational complexity without compromising contour accuracy [5, 11, 13-15, 18], gradient vector flow (GVF) snake model [8] and vector field convolution (VFC) snake model [9] have concavity converge property.

The factors affect the convergence include: the external forces, initialization and parameters. Generally, the researchers have improved the active contour models from these three aspects.

2.2.1. External forces

The choice of the external forces is a key problem because of the fact that different external forces lead to different results. Many researchers have presented different models via replacing the standard external force to improve the performance of snakes. The external forces can be classified as dynamic forces and static forces [8]. The dynamic forces are those that depend on the contour and change as the contours deform, the static forces are those that are calculated from the images, and remain unchanged as the contours deform. Here we list some typical models which have new external forces in the following.

a. Distance potential force model

The distance potential force model [17] was introduced by L. D. Cohen and I. Cohen (1993) for the first time. The distance potential force was defined based on the Euclidian distance. The external force of the distance potential force model is as follows:

$$\boldsymbol{F}(x,y) = -k\,(\nabla \mathrm{d}(x,y) / |\nabla d(x,y)|) \tag{11}$$

where k is a constant, d is the distance between a point (x, y) and the nearest edge points in the binary boundary map.

b. Gradient vector flow(*GVF*) snake

GVF (Xu and J. L. Prince, 1998) was introduced in [8] as a static external force to push the contour into object concavity. GVF is computed as a diffusion of the gradient vector of a gray-level or binary edge map of the image. GVF field is a vector field $\boldsymbol{V}(s)$ = [$u(s)$, $v(s)$] that minimizes the energy functional:

$$E_{GVF} = \iint \mu(\mathrm{u}_x^2 + u_y^2 + v_x^2 + v_y^2) + |\nabla f|^2 |\boldsymbol{V} - \nabla f|^2 \, dxdy \tag{12}$$

where f is an edge map of the image and μ is weighting parameter controlling the degree of smoothness of the GVF field, μ should be set according to the amount of noise present in the image (more noise, increase μ). ∇ is the gradient operator, u_x, u_y, v_x, v_y are the spatial derivatives of the field.

Using the calculus of variations [2], the GVF can be found by solving the following equations:

$$\mu \nabla^2 u - (u - f_x)(f_x^2 + f_y^2) = 0 \tag{13a}$$

$$\mu \nabla^2 v - (v - f_y)(f_x^2 + f_y^2) = 0 \tag{13b}$$

where ∇^2 is the Laplacian operator.

c. *Vector field convolution(*VFC*)snake*

A new static external force called VFC is introduced by Li and S. T. Acton in [6, 9]. This external force is calculated by convolving a vector field kernel with the edge map. The vector field kernel is defined $\boldsymbol{K}(x, y) = [u_k(x, y), v_k(x, y)]$ as follows:

$$\boldsymbol{K}(x, y) = m(x, y)\boldsymbol{n}(x, y) \tag{14}$$

where $\boldsymbol{n}(x, y)$ is the unit vector pointing to the kernel origin (0, 0) and $m(x, y)$ is the magnitude of the vector at (x, y).

$$\boldsymbol{n}(x, y) = [-x/r, -y/r] \tag{15}$$

except that $\boldsymbol{n}(0, 0) = [0, 0]$ at the origin, where $r = \sqrt{x^2 + y^2}$.

$$m_1(x, y) = (r + \varepsilon)^{-\gamma} \tag{16a}$$

$$m_2(x, y) = \exp(-r^2/\zeta^2) \tag{16b}$$

(16a) is inspired by Newton's law of universal gravitation in physics, where r is the distance from the origin, γ is positive parameter, ε is a small positive constant to prevent division by zero at the origin. (16b) is the Gaussian shape function, where ζ is positive parameter to control the decrease.

The VFC external force is:

$$\boldsymbol{F}_{VFC}(x, y) = f(x, y) * \boldsymbol{K}(x, y) \tag{17}$$

This external force pulls the contour toward the feature of interest instead of $\boldsymbol{F}_{ext}(c)$ in (7).

In a word, the distance potential force model has high capture range but it cannot extract concave object boundaries correctly. The GVF snake has a relative large capture range and can accommodate small concavities. Compared with the GVF snake, the VFC snake has larger capture range and better concavity converge property.

2.2.2. Initialization

The initialization is a crucial stage that affects the ultimate performance, because a poor initialized active contour model may become stuck at local energy minimum and fails to capture the features of interest. In the traditional snake, the initialization is performed manually by user. In order to simplify the manual initialization process, many researchers propose automatic initialization methods [4，6, 7, 10, 12] based on PACMs. For instance, Li et al. present the PIG automatic initialization method in [6]. An automatic initialization method via segmenting the external force field (FFS) is introduced by Li et al. in [10]. This method can't accommodate objects with broken edges. C. Tauber proposes a quasi-automated initialization method requiring only one user-defined point in [12], this method can be used to segment any closed or near closed shape. Here we mainly introduce the PIG automatic initialization approach.

The PIG initialization method [6] exploits a novel technique that essentially estimates the external energy field E from the external force field $\boldsymbol{F}_{ext}$ and determines the most likely initial segmentation.

$$E = \arg\min_E \iiint_\Omega \left| -\nabla E(x, y, z) - \boldsymbol{F}(x, y, z) \right|^2 dxdydz \tag{18}$$

with certain boundary conditions. To solve (18), E is the unique solution of the following Poisson's equation [19]

$$\Delta E(x, y, z) = -div\boldsymbol{F}(x, y, z)\, for(x, y, z) \in \Omega \tag{19}$$

where Δ is the Laplacian operator, and

$$\mathrm{div}\boldsymbol{F} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \tag{20}$$

is the divergence of $\boldsymbol{F} = [u, v, w]$. To solve (19), the boundary conditions must meet the Dirichlet boundary conditions [81]

$$E\big|_{\partial\Omega} = E_{\text{ext}}\big|_{\partial\Omega} = -f\big|_{\partial\Omega} \tag{21}$$

where f is the edge map derived from the image.

2.2.3. Parameters

The PACMs require some parameters to adjust the tradeoff between the internal forces and the external forces or to determine the forms of the external forces. The setting of the parameters is important to the final result. As with most parametric methods, the usual way of seeking the desired result is to run the algorithm several times for a set of different parameter values until a satisfactory performance is obtained. However, U. Ozertem presents a nonparametric method that exploits the underlying edge probability density in [13]. In this paper, the problem of seeking those unknown parameters is translated into the problem of seeking a good edge probability density estimate by the nonparametric formulation.

3. Applications in image processing

The PACMs are used extensively in many image processing areas, such as image segmentation. In order to introduce these contents more clearly, here the applications are divided into five subsections: image segmentation, edge detection, target object tracking, reconstruction and others.

3.1. Image segmentation

Image segmentation is one of the fundamental problems in image processing. The PACMs have been found to be effective in the domain of image segmentation and they are mainly applied in the image segmentation domain. Here we introduce two kinds of image segmentation: medical image segmentation and other image segmentation.

Medical image segmentation is an important issue in image processing applications. Some PACMs with fuzzy theory are applied in image segmentation. For example, in [24], H.C.V. Assen et al. present a 3-D active shape model (ASM) for semi-automatic segmentation of cardiac CT and MR volumes, a fuzzy c-means based fuzzy inference system was incorporated into the model. Y. Hata et al. propose a fuzzy segmentation and fuzzy maximum intensity projection (FMIP) approach [25] for the endorrhachis in magnetic resonance images, FMIP can project a 3D dataset onto a 2D plane for fuzzy segmented images.

There are many successful applications for PACMs in medical image segmentation [3-6, 20, 26-37]. For example, G. D. Giannogloua et al. propose a modified active contour model for fully automated segmentation of intravascular ultrasound (IVUS) images in [4]. This approach is fully automated and feasible, enabling accurate and rapid segmentation of IVUS images. A modified active contour segmentation method based on a faster gradient-vector-flow (GVF) calculation algorithm is proposed by Wu et al. in [5], this accelerated method is tested on multiple organs, including lung, right ventricle, kidney and prostate, and it reduces GVF calculation times to one-half or less without compromising contour accuracy. In [20], Tang presents a multi-direction gradient vector flow (GVF) snake-based scheme for the segmentation of skin cancer images. This approach is robust to noise and can effectively remove the hairs. In [26], a novel automatic approach to identify brain structures in magnetic resonance imaging (MRI) is presented for volumetric measurements. This approach combines the active contour model with support vector machine (SVM) classifier, it is effective for brain tissue segmentation. In [31], Wang et al. present fluid vector flow (FVF) active contour model for brain tumor segmentation. FVF has the ability to capture a large range and extract concave shapes. Shi proposes a new deformable model using both population-based and patient-specific shape statistics to segment lung fields from serial chest radiographs in [36].

Instead of medical image segmentation, many researchers pay attention to other image segmentation [7-14, 21-23, 38-44], such as complex image segmentation [11, 22, 38, 39, 44]. For example, in [22], Zhu et al. propose gradient and direction vector flow (G&DVF) snake model. WaterBalloons [41] is introduced by I. Dagher et al. The modified model combines both watershed segmentation and the balloon snake model, it provide the advantage of reducing watershed over-segmentation problems while preventing under-segmentation and ensure automatic initialization of traditional snake. F. Y. Shih et al. present an improved snake model [44] associated with new regional similarity energy and a gravitation force field for complex shape image segmentation.

Some improved models for image segmentation can capture the concave boundary image. For example, Li et al. propose the VFC snake model [9] and U. Ozertem presents nonparametric snakes [13] for the concave boundary image segmentation. In [21], Hou introduces the force field analysis snake model. This model can move parts of the snake into boundary concavities, it has a large capture range and a low computational cost.

3.2. Edge detection

The PACMs are also powerful tools for edge detection. Many research groups have proposed some methods of the PACMs for edge detection [46-60, 71-74]. Yuan et al. propose a multi-resolution method [54] for tagline detection. This method is rooted in frequency of the images, and incorporates a snake method for tagline recovery and indexing. It has a significant improvement in accuracy and robustness. A decoupled active contour is proposed by A. K. Mishra et al. in [58], this approach is robust to noise, can capture areas of the high curvature. In [59], dynamic directional gradient vector flow (DDGVF) snake, presented by Cheng et al, provides a much better performance.

Researchers attempt to combine the PACMs with other techniques to get a good effect. For instance, a modified time-adaptive self organizing map-based active contour model, introduced by M. H. Khosravi et al., is used to detect boundaries of the human eye sclera [46]. This method overcomes the original time-adaptive self organizing map algorithm's some weaknesses, and experimental results show a good performance. In [50], a method, which combines both Markov random field and active contours, for the boundary detection of human kidneys is proposed. Snakes perform poorly in many edge detection problems, J. C. Nascimento et al. present adaptive snake using the EM algorithm [55] can overcomes this difficulty.

3.3. Target object tracking

Besides image segmentation and edge detection, the PACMs are also widely used in the field of object tracking [18, 46, 61-65]. For instance, in [18], an improved active contour model based on the time-adaptive self organizing map is proposed by M. Izadi, this model has a high convergence speed and low computational complexity. Ref. [46] uses time-adaptive self organizing map (TASOM) based active contour models for tracking the eyes, this method overcomes some weaknesses of the original TASOM algorithm. These weaknesses include formation of undesired twists in the neuron chain and holes in the boundary, lengthy chain of neuron, and low speed of the algorithm. An energy functional that encodes the Lie group transform parameters is proposed by D. P. Mukherjee et al. [62]. Chen et al. present a parametric active contour model for object tracking based on matching degree image of object contour points [65].

3.4. Reconstruction

Some PACMs are proposed for reconstruction [45, 75-77] and shape recovery [78]. For example, in [75], X.M. Pardo et al. present Discriminant snakes for 3D reconstruction of anatomical organs. Feng et al. propose a multi-resolution statistical deformable model and the associated techniques for the reconstruction of soft-tissue organs such as livers [77]. This approach is effective to formulate the perceptual knowledge of the anatomies. A physically motivated deformable model, called charged-particle model (CPM), is introduced by A. C. Jalba et al. for shape recovery [78]. This model is easily extendable to 3D and copes well with noisy images.

3.5. Other applications

It is well known that the PACMs are famous for their abilities of image segmentation. In fact, the PACMs are also used in other applications [66-70, 79, 80]. For example, in [66], G. S. Muralidhar et al. present a modified algorithm termed snakules for the annotation of spicules on mammography, the design of this algorithm is guided by statistics of physical measurements of real spiculated masses on mammography. M. Jacob et al. introduce a 3-D parametric active contour algorithm for the shape estimation of DNA molecules from stereo cryo-electron micrographs [67]. An improved active contour model for automatic contour propagation is proposed by G. Hantvast et al. in cine cardiac magnetic resonance images [68]. In [69], a vector-to-image conflation approach to high resolution imagery is presented by Song et al., this technique uses a variety of algorithms, an improved snake algorithm moves intermediate road points toward the road image. A modified PACM is used in route navigation for a mobile robot with an omnidirectional image sensor [70]. G.L.T.F.Hautvast et al. propose an automatic cardiac contour propagation method based on active contour in [79]. This method is robust, accurate, and it also has lower processing time than manual contour delineation. In [80], a curved planar reformation method is presented for tubular structures.

4. Discussions and future trends

We introduced the traditional snake model. According to the drawbacks of the traditional snake model, some improved models have been proposed via various ways, including the external force, the initialization, and the parameters.

1) Some modified models have been presented by replacing the standard external force to improve the performance of snakes. Some typical models which have new external forces include the distance potential force model [17], GVF snake model [8], and VFC snake model [9]. The distance potential force model has higher capture range but it cannot extract concave object boundaries correctly. The GVF snake has a relative large capture range and can accommodate small concavities. Compared with the GVF snake, the VFC snake has larger capture range and better concavity converge property.

2) The initialization is a crucial stage that affects the ultimate performance. In the traditional snake, the initialization is performed manually by user. In order to simplify the manual initialization process, researchers presented some automatic initialization approaches based on PACMs. There were semi-automatic and fully-automatic initialization methods. We mainly introduced the PIG fully-automatic initialization approach [6]. It can initialize one or more active models automatically.

3) The setting of the parameters is important to the final result of the PACM. We would get good performance if we set proper parameters of the PACM. U. Ozertem [13] proposed a nonparametric method. This method translated the problem of seeking those unknown parameters into the problem of seeking a good edge probability density estimate by the nonparametric formulation.

The PACMs are used extensively in many image processing areas. Researchers applied the PACMs to image processing, and got good performance. In this paper, we review the applications in image processing of the PACMs: image segmentation, edge detection, target object tracking, reconstruction and others. We described the applications in image processing of the PACMs in detail.

However, it should be made clear that none of the methods provide the overall best performance. PACMs are still need of improvement. Further work is being carried out in several different directions. Here we would like to give some suggestions for researchers.

First, PACMs can be used in other areas, for example, it can extract calcification and mass in mammographic images. Second, the setting of parameters and the initialization are very important factors to the final results. The optimization algorithm of parameters setting should be further studied, and more suitable initialization methods for the parametric active contour models should be presented in future. Third, combining with other theories is a feasible and efficient way to make up for the deficiency of the PACMs, for example, combining the PACMs with the Pulse coupled neural network. Finally, how to reduce the processing time and get high accuracy at the same time is a big problem further work needed.

5. Conclusions

PACMs are effective tools for image processing. A critical up-to-date review is provided in this paper. First, the research status of the PACMs is described. Second, the traditional snake is introduced. Meanwhile, several improved models are reviewed due to some drawbacks of the traditional snake. Subsequently, applications in the field of image processing are expounded. Finally, some research directions in future are pointed out.

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