1

设总体X服从二项分布$b(m,p)$，其中m,p未知，$x_1,...,x_n$为来自总体X的样本。求参数m和p的矩估计量。

$$E[X]=mp$$

$$E[X^2]=\mathrm{Var(X)}+(E[X])^2=mp(1-p)+(mp)^2=mp(1-p+mp)=mp+m(m-1)p^2$$

样本一二阶矩分别为：

$$\mu_1=\frac{1}{n}\sum_{i=1}^{n}x_i$$

$$\mu_2=\frac{1}{n}\sum_{i=1}^{n}x_i^2$$

联立

$$\begin{cases}\frac{1}{n}\sum_{i=1}^{n}x_i=mp\\ \frac{1}{n}\sum_{i=1}^{n}x_i^2=mp+m(m-1)p^2\end{cases}$$

解得$p=\frac{\mu_1}{m}$，带入第二个方程得

$$\mu_2=\mu_1+\mu_1^2-\frac{1}{m}\mu_1^2$$

$$m=\frac{\mu_1^2}{\mu_1-\mu_2+\mu_1^2}$$

$$p=\frac{\mu_1-\mu_2+\mu_1^2}{\mu_1}$$

m和p的矩估计量分别为

$$\hat{m}=\left[\frac{\left(\frac{1}{n}\sum_{i=1}^{n}x_i\right)^2}{\left(\frac{1}{n}\sum_{i=1}^{n}x_i\right)-\left(\frac{1}{n}\sum_{i=1}^{n}x_i^2\right)+\left(\left(\frac{1}{n}\sum_{i=1}^{n}x_i\right)^2\right)}\right],\quad \text{中括号表示取整}$$

$$\hat{p}=\frac{\left(\frac{1}{n}\sum_{i=1}^{n}x_i\right)-\left(\frac{1}{n}\sum_{i=1}^{n}x_i^2\right)+\left(\left(\frac{1}{n}\sum_{i=1}^{n}x_i\right)^2\right)}{\frac{1}{n}\sum_{i=1}^{n}x_i}$$

2

螺纹面的方程为：

$$\begin{cases}x=r\cos\varphi\\ y=r\sin\varphi\,,\quad(0\le r\le a,0\le\varphi\le 2\pi)\\ z=b\varphi\end{cases}$$

求曲面面积

$$\boldsymbol{r}=(r\cos\varphi, r\sin\varphi, b\varphi)$$

$$\boldsymbol{r}_r=(\cos\varphi, \sin\varphi, 0)$$

$$\boldsymbol{r}_\varphi=(-r\sin\varphi, r\cos\varphi, b)$$

$$\boldsymbol{r}_r\times\boldsymbol{r}_\varphi=\begin{vmatrix}\boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k}\\ \cos\varphi & \sin\varphi & 0\\ -r\sin\varphi & r\cos\varphi & b\end{vmatrix}=(b\cos\varphi, -b\sin\varphi, r)$$

$$\|\boldsymbol{r}_r\times\boldsymbol{r}_\varphi\|=\sqrt{b^2+r^2}$$

$$S=\iint_D\sqrt{b^2+r^2}\,\mathrm{d}r\,\mathrm{d}\varphi,\quad D=\{(r,\varphi)|0\le r\le a, 0\le\varphi\le 2\pi\}$$

$$\begin{aligned}S&=\int_0^{2\pi}\mathrm{d}\varphi\int_0^a\sqrt{b^2+r^2}\,\mathrm{d}r=2\pi\left[\int_0^a\sqrt{b^2+r^2}\,\mathrm{d}r\right]\\&=2\pi\left[\frac{r}{2}\sqrt{r^2+b^2}+\frac{b^2}{2}\ln\left(r+\sqrt{r^2+b^2}\right)\right]_0^a\\&=2\pi\left[\frac{a}{2}\sqrt{a^2+b^2}+\frac{b^2}{2}\ln\left(a+\sqrt{a^2+b^2}\right)-\frac{b^2}{2}\ln b\right]\\&=\pi a\sqrt{a^2+b^2}+\pi b^2\ln\left(\frac{a+\sqrt{a^2+b^2}}{b}\right)\end{aligned}$$

3

计算$az=xy$包含在圆柱面$x^2+y^2=a^2$内部分的面积

$$S=\iint_D\sqrt{1+z_x^2+z_y^2}\,\mathrm{d}x\,\mathrm{d}y,\quad D=\{(x,y)|x^2+y^2\le a^2\}$$

$$\begin{aligned}&=\iint_D\sqrt{1+\left(\frac{y}{a}\right)^2+\left(\frac{x}{a}\right)^2}\,\mathrm{d}x\,\mathrm{d}y\\&=\iint_D\sqrt{\frac{1}{a^2}(x^2+y^2)+1}\,\mathrm{d}x\,\mathrm{d}y\\&=\frac{1}{a}\iint_D\sqrt{r^2+a^2}\,\mathrm{d}r\,\mathrm{d}\theta\\&=\frac{1}{a}\int_0^{2\pi}\mathrm{d}\theta\int_0^a\sqrt{r^2+a^2}\,\mathrm{d}r\\&=\frac{2\pi}{a}\left[\frac{r}{2}\sqrt{r^2+a^2}+\frac{a^2}{2}\ln\left(r+\sqrt{r^2+a^2}\right)\right]_0^a\\&=\frac{2\pi}{a}\left[\frac{a}{2}\sqrt{2a^2}+\frac{a^2}{2}\ln\left(a+\sqrt{2a^2}\right)-\frac{a^2}{2}\ln a\right]\\&=\pi\sqrt{2}a+\pi a\ln\left(a+\sqrt{2}a\right)-\pi a\ln a\\&=\pi\sqrt{2}a+\pi a\ln\left(1+\sqrt{2}\right)\end{aligned}$$