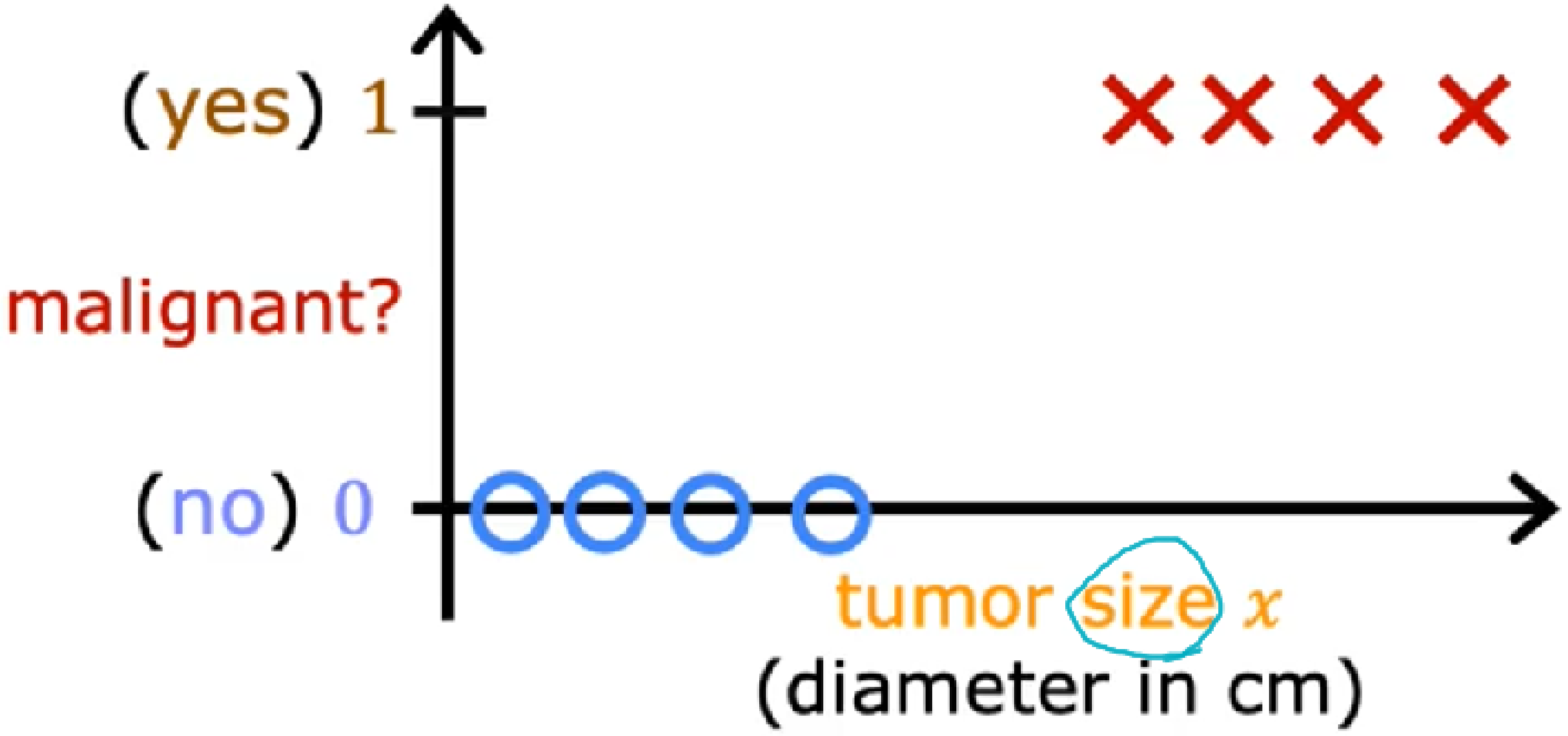
Classification

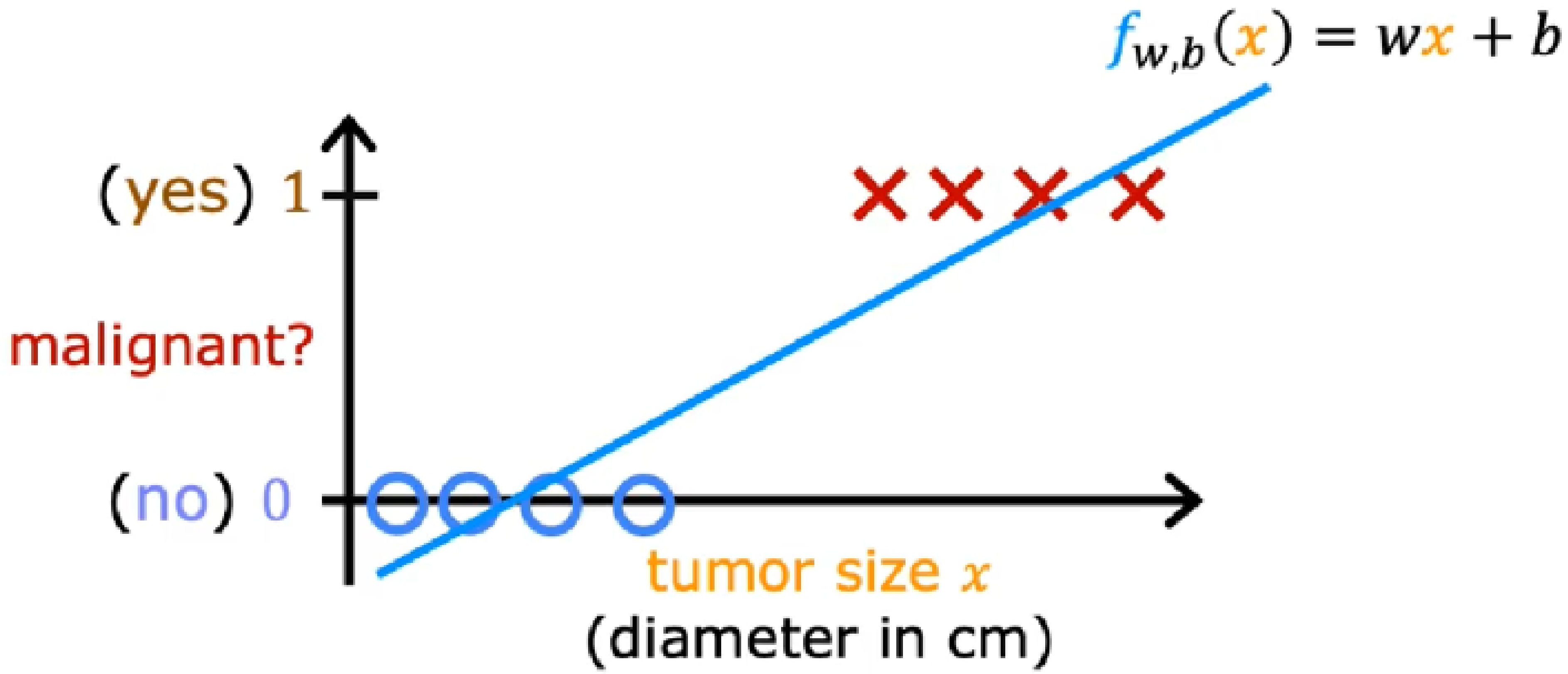
> logistic regression

binary classification

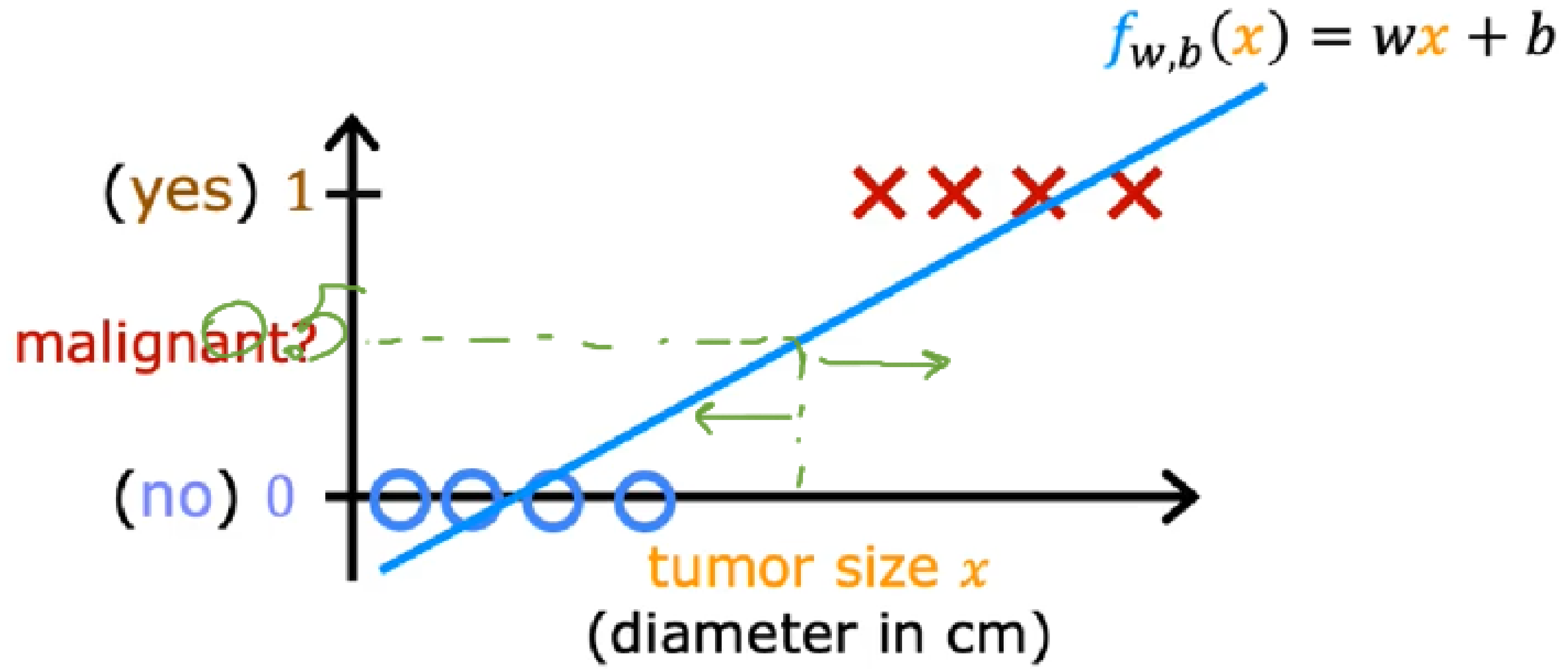
Question	Answer "y"	
Is this email spam?	no	yes
Is the transaction fraudulent?	no	yes
Is the tumor malignant?	no	yes
	false	true
	0	1


(yes) 1
malignant?
(no) 0
tumor size x
(diameter in cm)

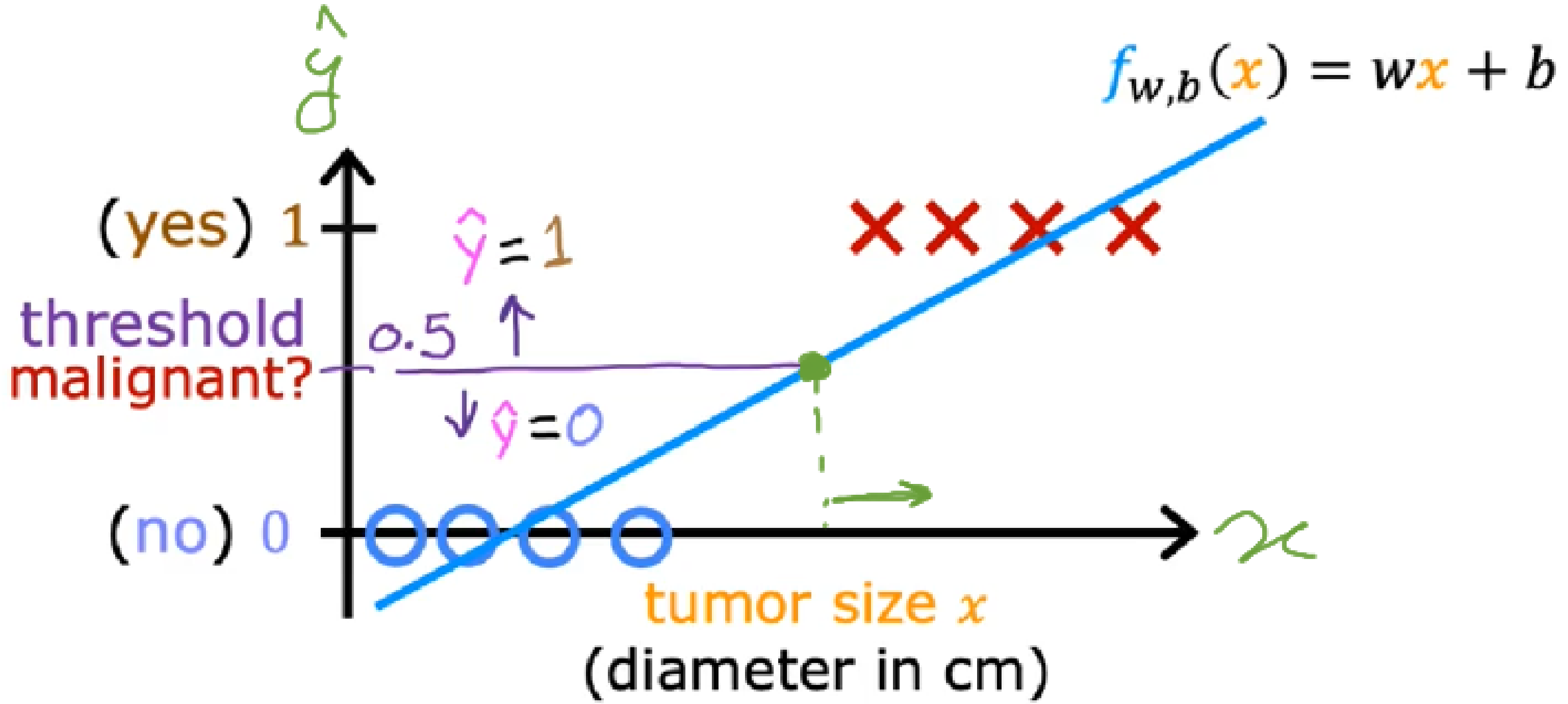
Linear Regression



Linear Regression
threshold



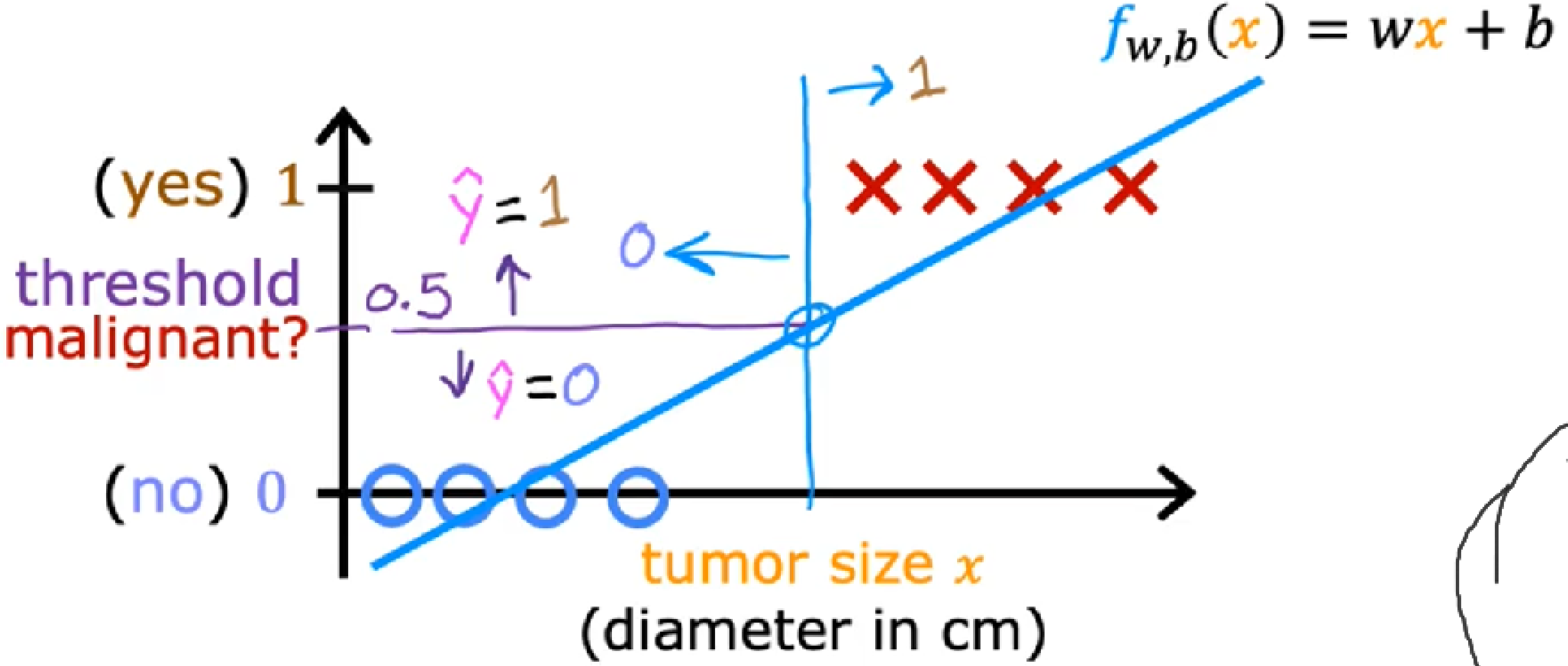
Linear Regression
threshold



if $f_{w,b}(x) < 0.5 \rightarrow \hat{y} = 0$

if $f_{w,b}(x) \geq 0.5 \rightarrow \hat{y} = 1$

Linear Regression
threshold



if $f_{w,b}(x) < 0.5 \rightarrow \hat{y} = 0$

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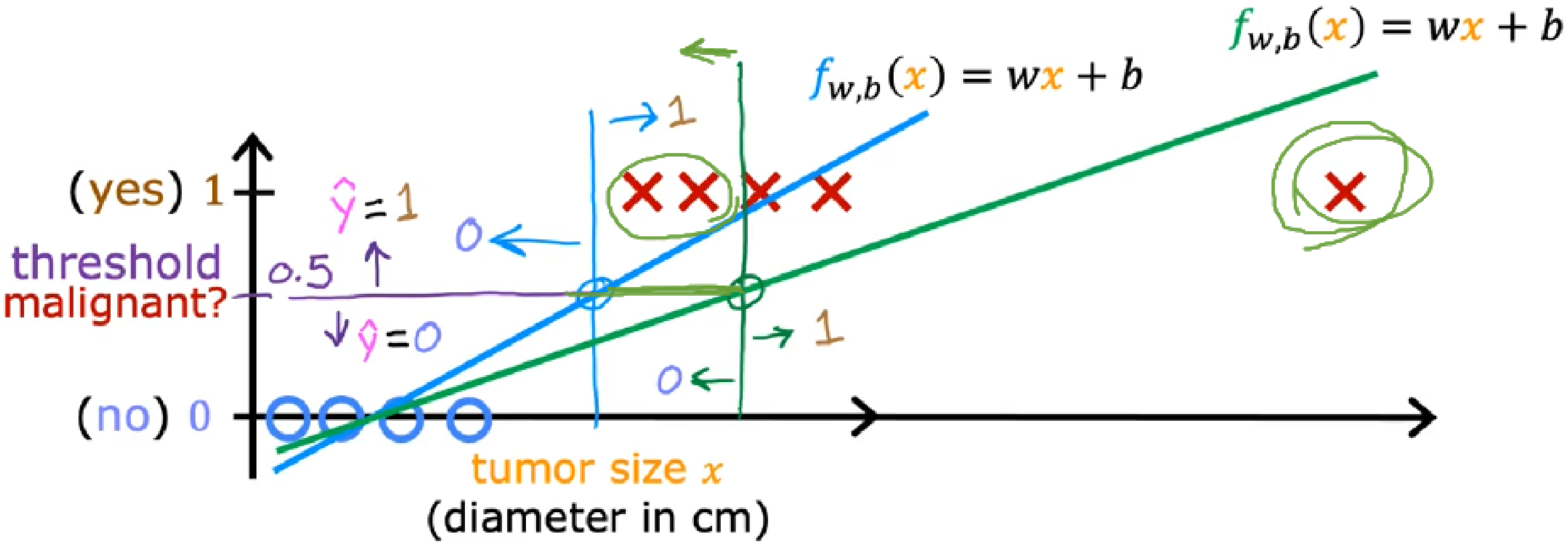
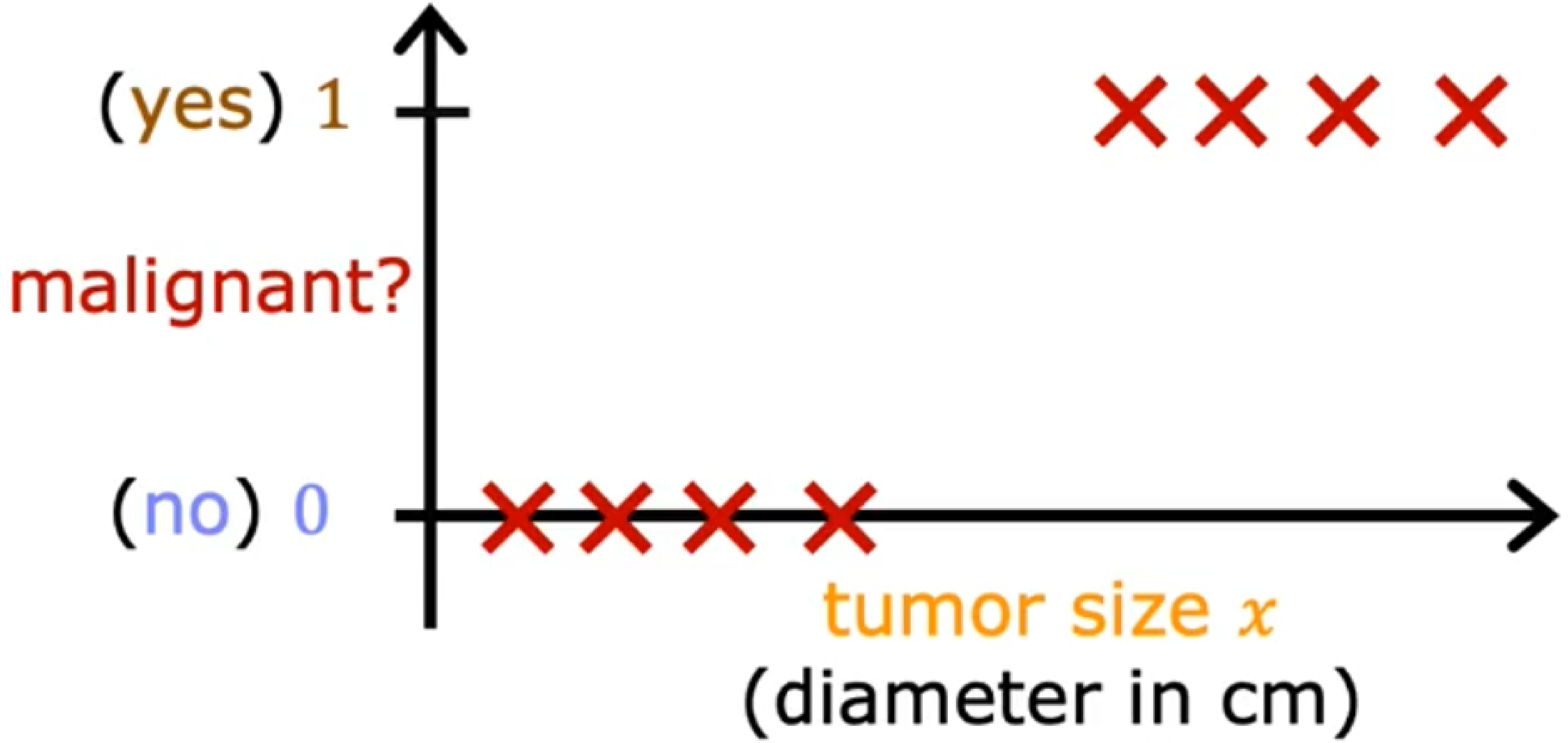
Linear Regression
threshold



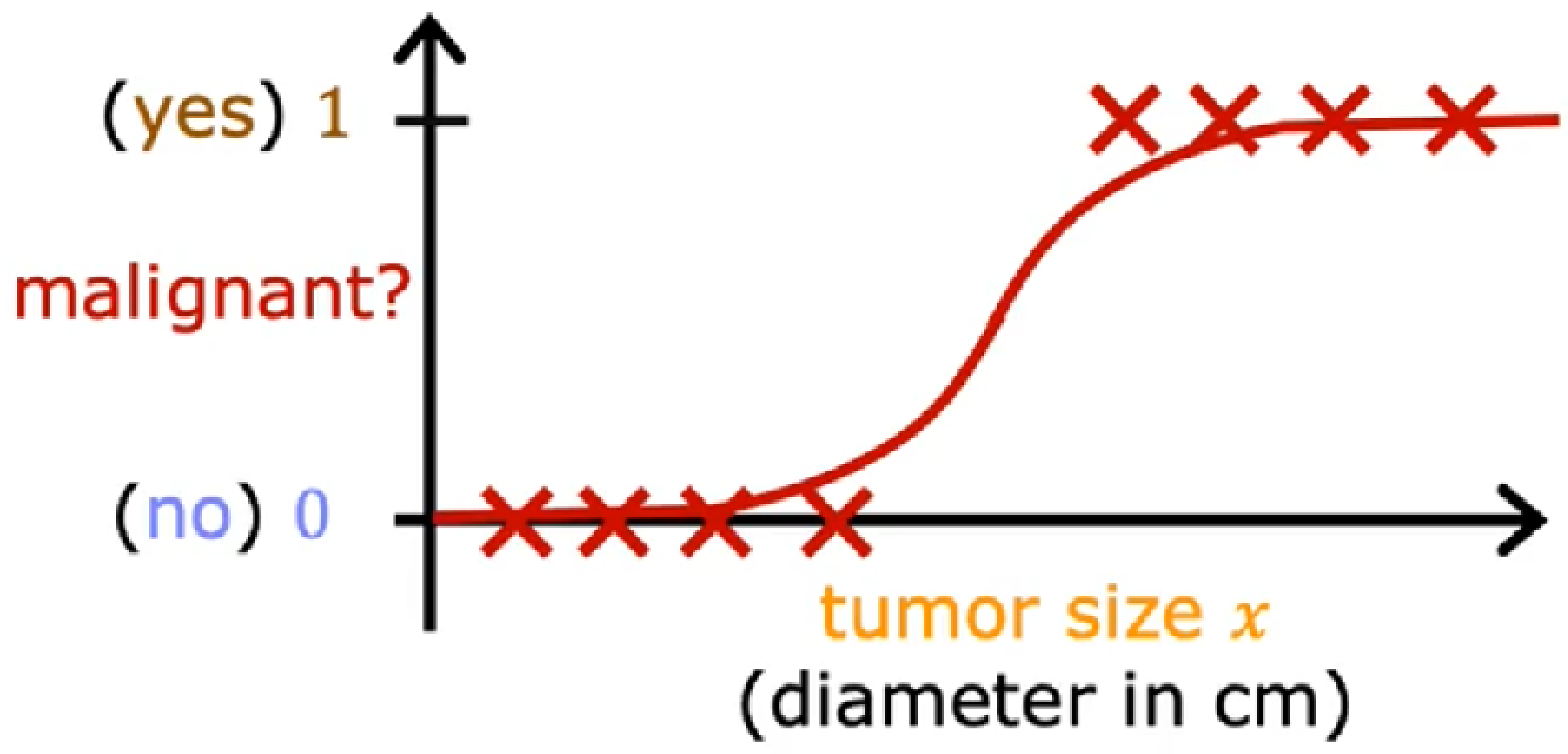
if $f_{w,b}(x) < 0.5 \rightarrow \hat{y} = 0$

if $f_{w,b}(x) \geq 0.5 \rightarrow \hat{y} = 1$

Our Data

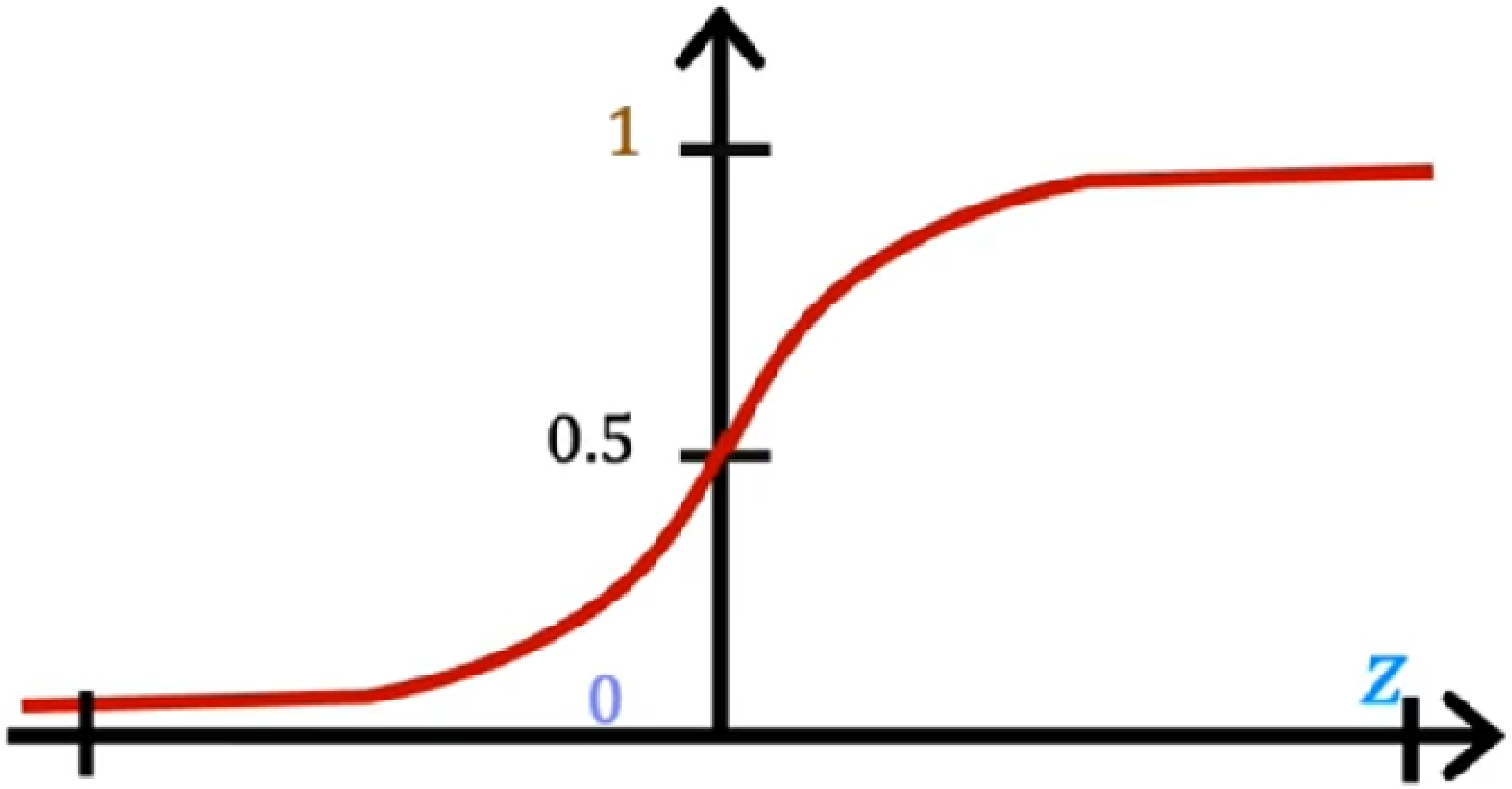


Logistic Regression > S-shapeed curve



sigmoid function (logistic function)

$$g(z) = \frac{1}{1+e^{-z}} \qquad 0 < g(z) < 1$$



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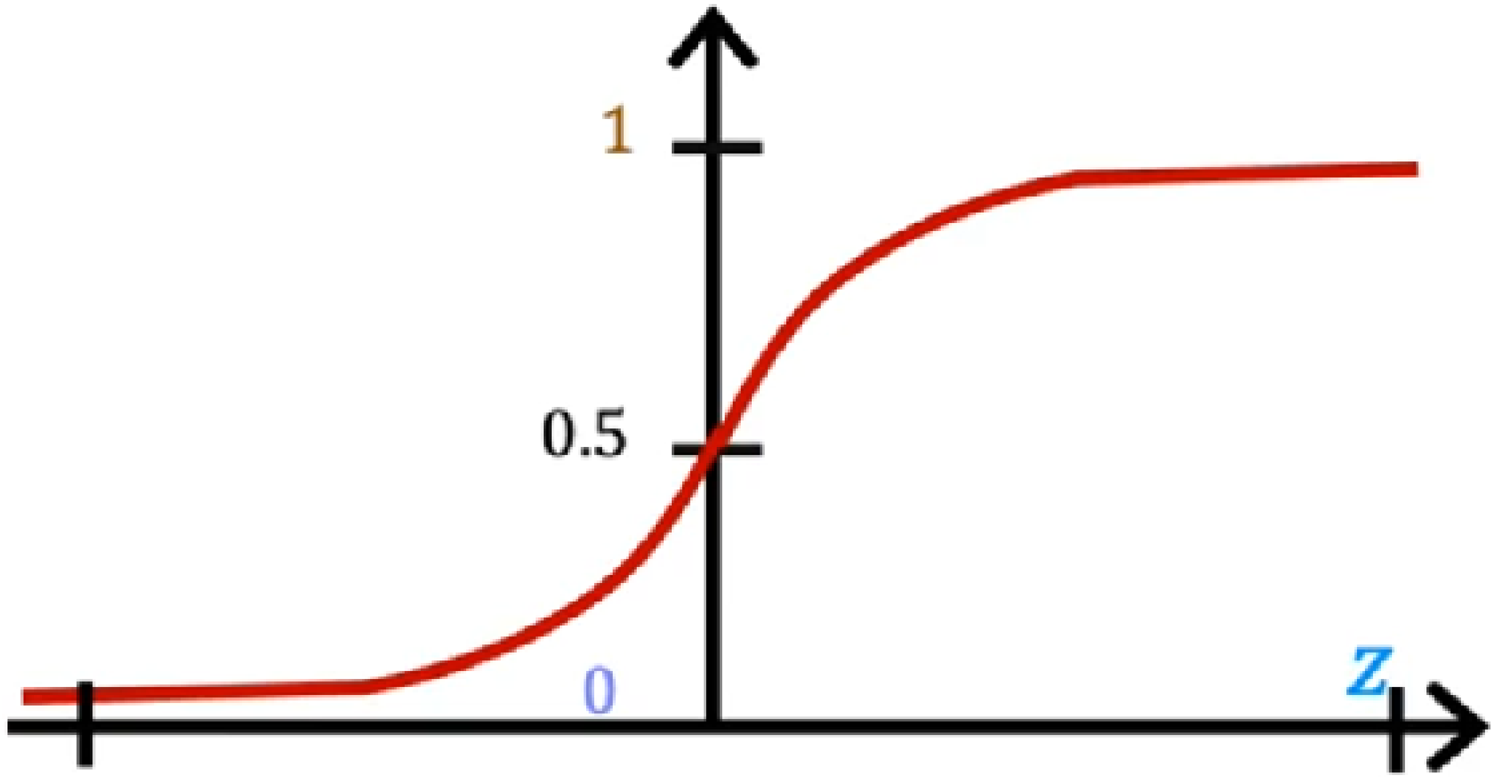


$$f_{\vec{w},b}(\vec{x})$$

$$\boxed{z = \vec{w} \cdot \vec{x} + b}$$

$$\downarrow$$

$$z$$

$$\downarrow$$

$$\boxed{g(z) = \frac{1}{1+e^{-z}}}$$

sigmoid function (logistic function)

$g(z) = \frac{1}{1+e^{-z}}$ $\quad 0 < g(z) < 1$

$\frac{1}{1+e^{-x}}$

$f_{\vec{w},b}(\vec{x})$

$\boxed{z = \vec{w} \cdot \vec{x} + b}$

$\downarrow$

z

$\downarrow$

$\boxed{g(z) = \frac{1}{1+e^{-z}}}$

$$f_{\vec{w},b}(\vec{x}) = g(\underbrace{\vec{w} \cdot \vec{x} + b}_{z}) = \frac{1}{1 + e^{-(\vec{w} \cdot \vec{x} + b)}}$$

"logistic regression"

logistic function

reg: $f = wx + b$

class: $f_{\vec{w},b}(\vec{x}) = \dfrac{1}{1 + e^{-(\vec{w}\cdot\vec{x}+b)}}$

"probability" that class is 1

$$\begin{cases} \text{if } f(x) >= 0.5 & \Rightarrow \quad 1 \\ \text{if } f(x) < 0.5 & \Rightarrow \quad 0 \end{cases}$$

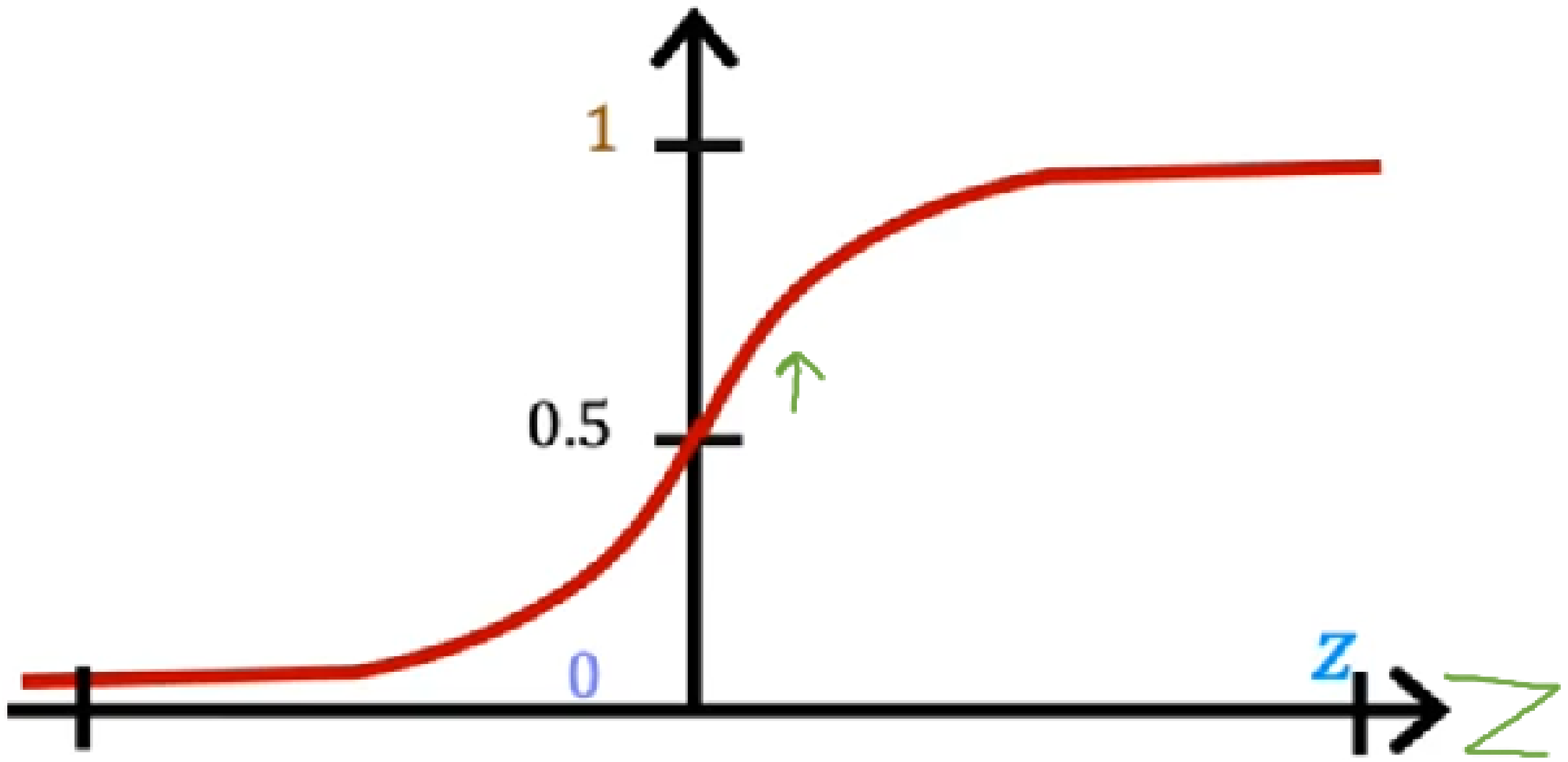
logistic function

$$f_{\vec{w},b}(\vec{x}) = \frac{1}{1+e^{-(\vec{w}\cdot\vec{x}+b)}}$$

→ 0.3 → 0

→ 0.8

"probability" that class is 1



if f(x) >= 0.5 => 1

if f(x) < 0.5 => 0

w.x + b >= 0 => 1

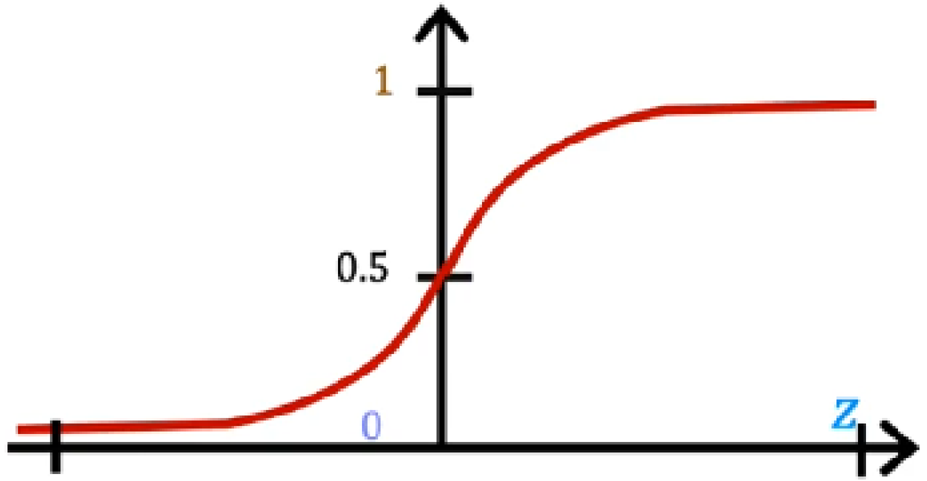
w.x + b < 0 => 0

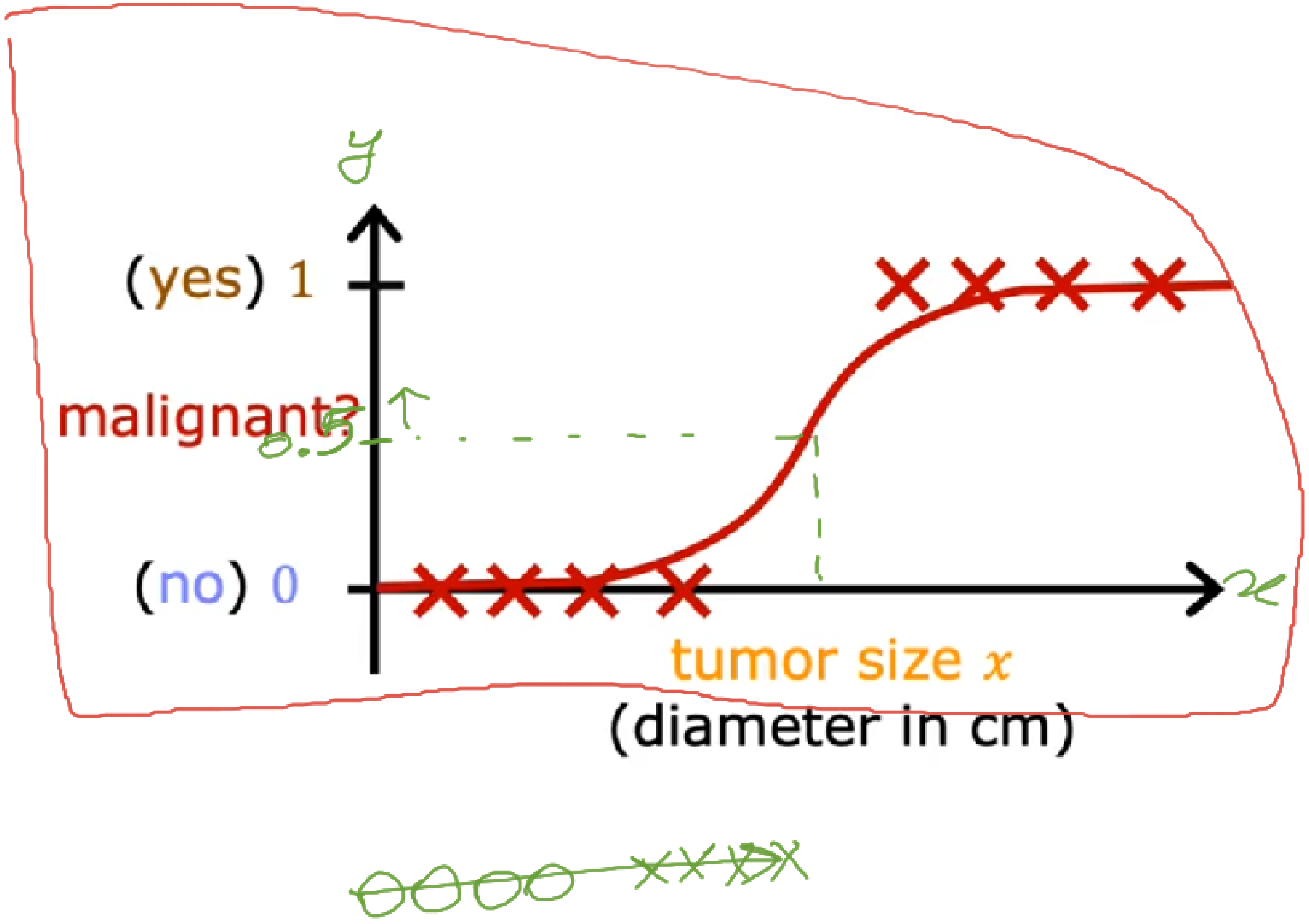
$$f_{\vec{w},b}(\vec{x}) = \frac{1}{1 + e^{-(\vec{w}\cdot\vec{x}+b)}}$$

"probability" that class is 1

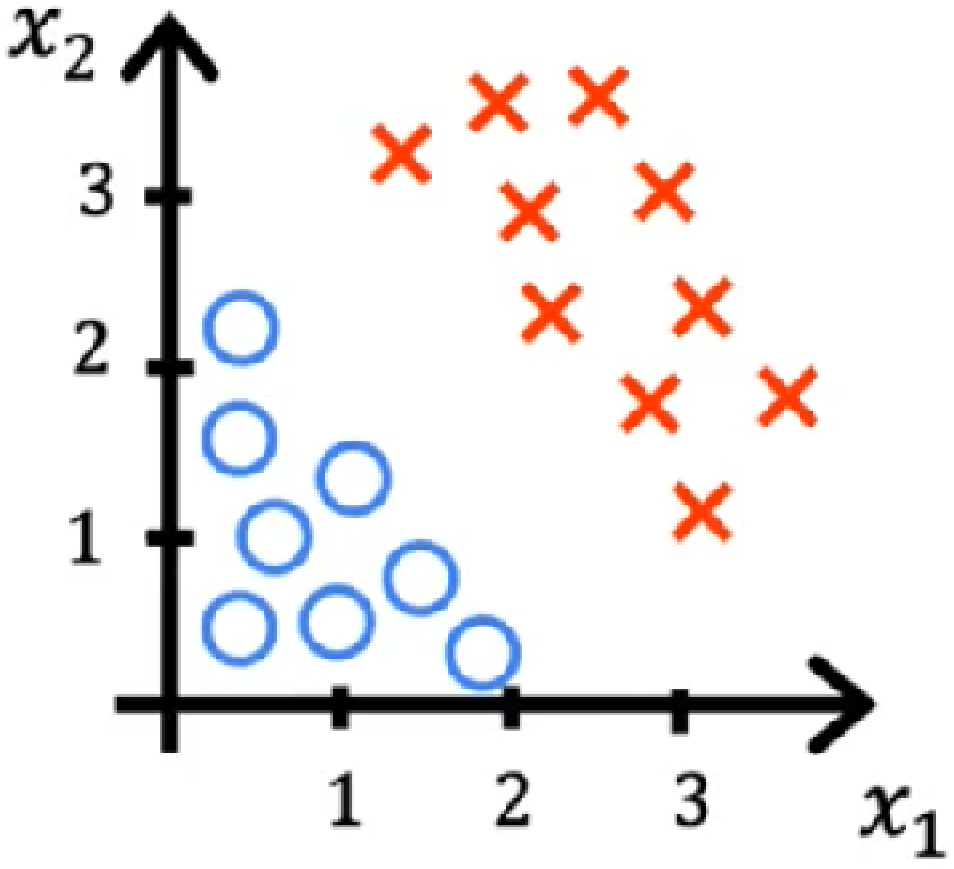
if f(x) >= 0.5   =>   1

if f(x) < 0.5   =>   0





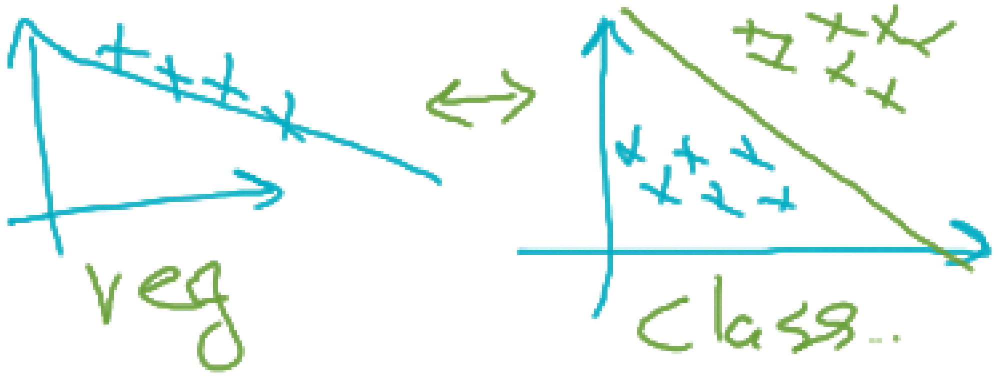
Decision boundary



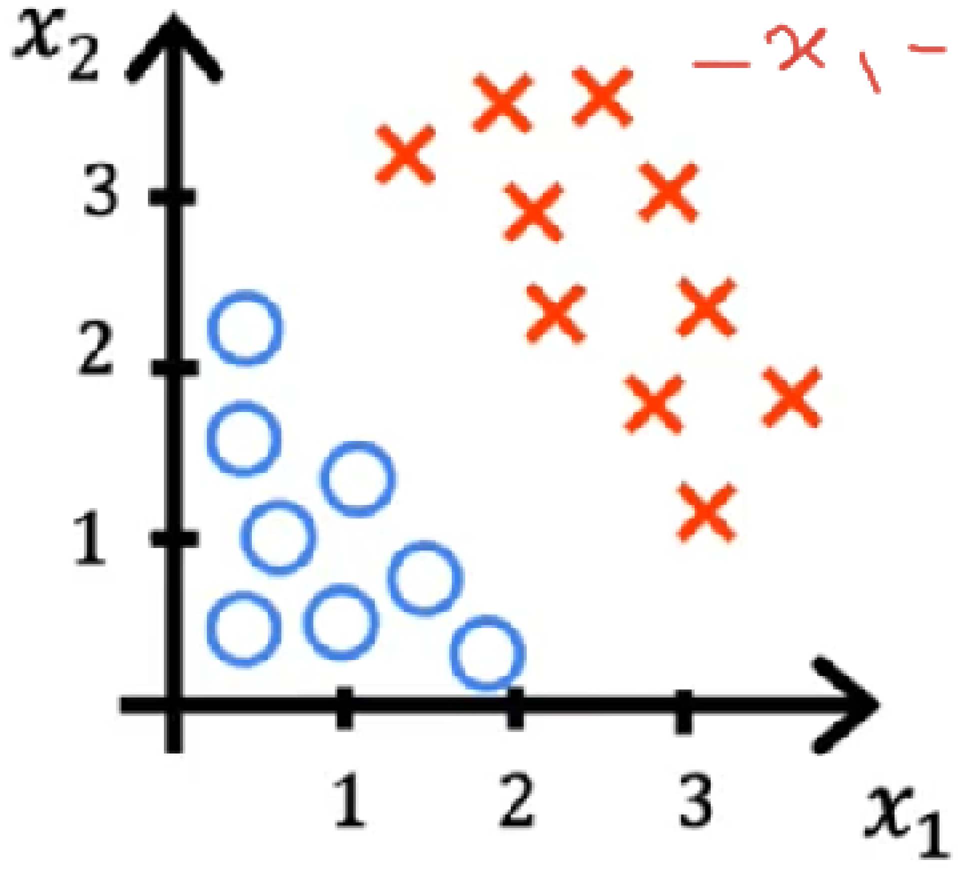
$$g(z) = \frac{1}{1+e^{-z}}$$

$$f_{\vec{w},b}(\vec{x}) = g(z) = g(\boxed{w_1 x_1 + w_2 x_2 + b})$$

Decision boundary $\quad z = \boxed{\vec{w} \cdot \vec{x} + b = 0}$



Decision boundary



$-x_1 - x_2 + 3 = 0$

$$\left.\begin{array}{l} w1 = 1 \\ w2 = 1 \\ b = -3 \end{array}\right.$$

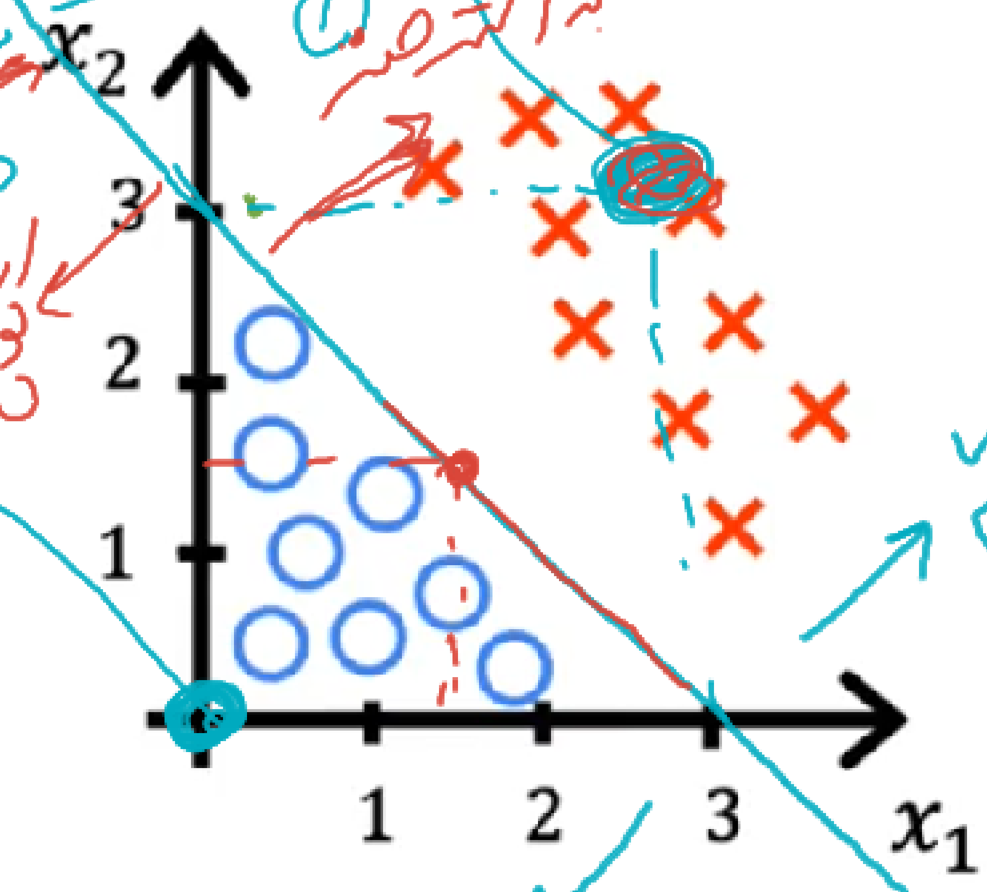
$$z = \boxed{x_1 + x_2 - 3} = 0$$

$$x_1 + x_2 = 3$$

$3 \geq 0 \rightarrow 1$

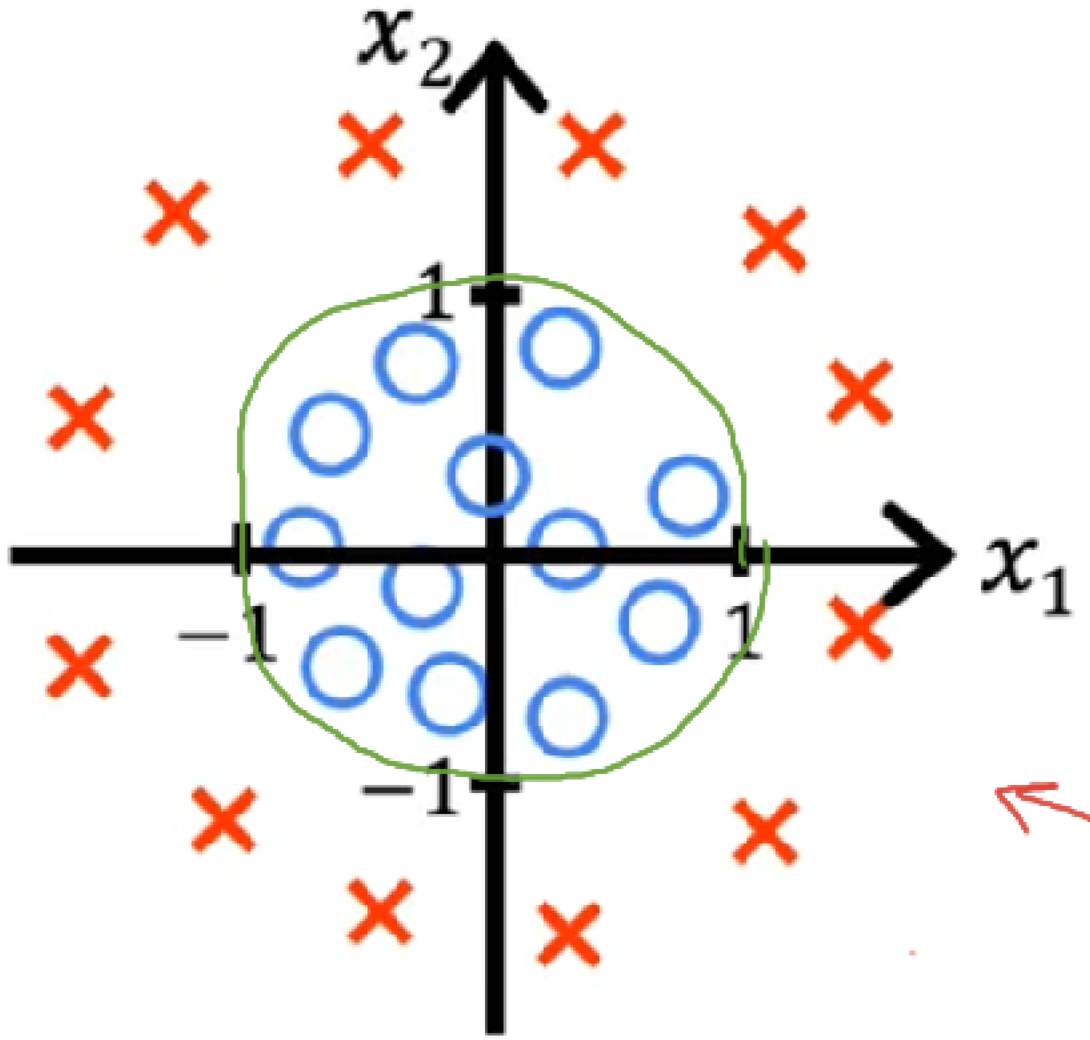
$w_1x_1 + w_2x_2 + b = 0$

$-3 < 0$



$$f_{\vec{w},b}(\vec{x}) = g(z) = g(\boxed{w_1x_1 + w_2x_2 + b})$$

Decision boundary $z = \boxed{\vec{w} \cdot \vec{x} + b = 0}$



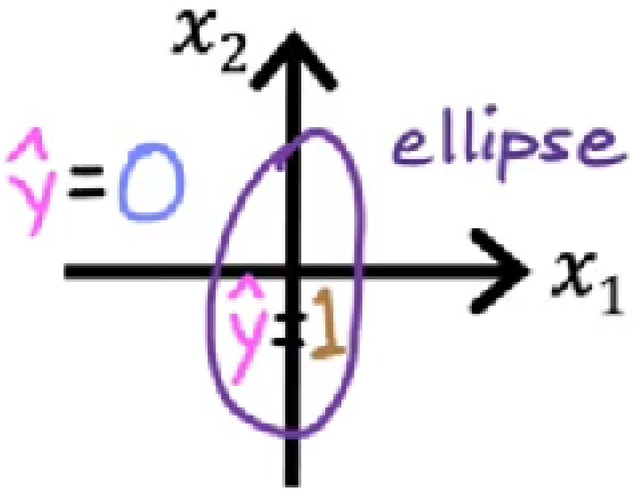
$$x_1^2 + x_2^2 = 1 \rightarrow x_1^2 + x_2^2 - 1 = 0$$

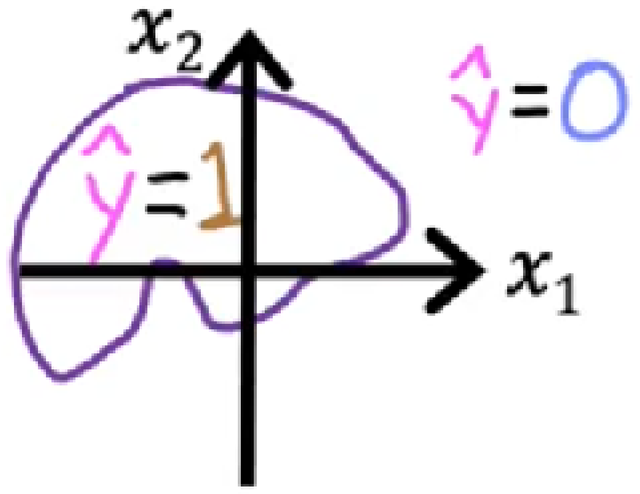
$$(0,0) \longrightarrow -1 < 0 \longrightarrow 0$$

$$(1,1) \longrightarrow 1 > 0 \longrightarrow 1$$

$$f_{\vec{w},b}(\vec{x}) = g(z) = g(\overbrace{w_1 x_1^2 + w_2 x_2^2 + b}^{z})$$

Non-linear decision boundaries





$$f_{\vec{w},b}(\vec{x}) = g(z) = g(w_1x_1 + w_2x_2 \\ + w_3x_1^2 + w_4x_1x_2 + w_5x_2^2 \\ + w_6x_1^3 + \cdots + b)$$