Date: .... Subject: سری ۳ تمارین سیگنال ها و سیستم ها

س ۱:

$$x(t) = 2j\sin\left(\tfrac{3}{2}t\right) + \cos\left(t - \tfrac{\pi}{3}\right) + 2$$

$$= e^{\frac{3}{2}jt} - e^{-\frac{3}{2}jt} + \tfrac{1}{2}\left(e^{j(t-\frac{\pi}{3})} + e^{-j(t-\frac{\pi}{3})}\right) + 2$$

$$= e^{\frac{3}{2}jt} - e^{-\frac{3}{2}jt} + \tfrac{1}{2}e^{-\frac{\pi}{3}j}\left(e^{jt}\right) + \tfrac{1}{2}e^{\frac{\pi}{3}j}\left(e^{-jt}\right) + 2$$

دوره تناوب جملات: $T_1 = \frac{2\pi}{\frac{3}{2}} = \frac{4\pi}{3}$ و $T_2 = 2\pi \xrightarrow{\text{ک.م.م}} T_0 = \frac{4\pi}{1}$ دوره تناوب اصلی سیگنال

فرکانس اصلی $\omega_0 = \frac{2\pi}{4\pi} = \frac{1}{2}$

$\frac{\frac{3}{2}}{\frac{1}{2}} = 3 \Rightarrow a_3 = 1$ $\qquad$ $\frac{1}{\frac{1}{2}} = 2 \Rightarrow a_2 = \frac{1}{2}e^{-\frac{\pi}{3}j}$

$\frac{-\frac{3}{2}}{\frac{1}{2}} = -3 \Rightarrow a_{-3} = -1$ $\qquad$ $\frac{-1}{\frac{1}{2}} = -2 \Rightarrow a_{-2} = \frac{1}{2}e^{\frac{\pi}{3}j}$

$a_0 = 2$ $\qquad$ ($K = 1$ یا -1 یا $|K| > 3 \Rightarrow a_K = 0$)

س ۲:

$$a_K = \frac{1}{T_0}\int_{T_0} x(t)e^{-jK\omega_0 t}\,dt$$

$$T_0 = 4 \Rightarrow \omega_0 = \frac{2\pi}{4} = \frac{\pi}{2}$$

$$a_K = \frac{1}{4}\int_{-1}^{3}\delta(t-2)e^{-jK(\frac{\pi}{2})t}\,dt = \frac{1}{4}e^{-jK\frac{\pi}{2}\times 2} = \frac{1}{4}e^{-jK\pi} = \frac{1}{4}(-1)^K$$

Date: Subject:

سؤال ۳:

a) $x(t-t_0)+x(t+t_0)$

$\quad\quad b_{1K} \quad\quad b_{2K}$

$b_{1K}=a_K e^{-jK\omega_0 t}=a_K e^{-jK(\frac{2\pi}{T})t_0}$ ویژگی time shifting (انتقال در زمان):

$b_{2K}=a_K e^{jK\omega_0 t}=a_K e^{jK(\frac{2\pi}{T})t_0}$

$b_K=b_{1K}+b_{2K}=a_K(e^{-jK(\frac{2\pi}{T})t_0}+e^{jK(\frac{2\pi}{T})t_0})$ ویژگی linearity (خطی بودن):

$x(t)\xleftrightarrow{FS} a_K$

$x(t-t_0)+x(t+t_0)\xleftrightarrow{FS} a_K(e^{-jK(\frac{2\pi}{T})t_0}+e^{jK(\frac{2\pi}{T})t_0})$ ضرایب سری فوریه:

$T_1=T \;,\; T_2=T \xrightarrow{\text{ک.م.م}} T_0=T$ دوره تناوب:

b) $Even\{x(t)\}=\frac{1}{2}(x(t)+x(-t))$

$\quad\quad b_{1K} \quad\quad b_{2K}$

$b_{1K}=a_K$

$b_{2K}=a_{-K}$ ویژگی time-reversal (معکوس پذیری در زمان):

$b_K=\frac{1}{2}(b_{1K}+b_{2K})=\frac{1}{2}(a_K+a_{-K})$ ویژگی linearity (خطی بودن):

$x(t)\xleftrightarrow{FS} a_K$

$Even\{x(t)\}\xleftrightarrow{FS}\frac{1}{2}(a_K+a_{-K})$ ضرایب سری فوریه:

$T_1=T \;,\; T_2=T \xrightarrow{\text{ک.م.م}} T_0=T$ دوره تناوب:

c) $\frac{d^2x(t)}{dt^2}$

$x(t)\xleftrightarrow{FS} a_K$

$\frac{dx}{dt}\xleftrightarrow{FS} jK\omega_0 a_K = jK(\frac{2\pi}{T})a_K$ ویژگی Diffrentiation (مشتق):

$\frac{d^2x}{dt^2}\xleftrightarrow{FS} jK(\frac{2\pi}{T})\times jK(\frac{2\pi}{T})a_K=-K^2(\frac{2\pi}{T})^2 a_K$ ضرایب سری فوریه:

دوره تناوب: $T_0=T$

d) $x(3t-1)$

$x(t-1)\xleftrightarrow{FS} a_K(e^{-jK(\frac{2\pi}{T})t_0})=a_K(e^{jK(\frac{2\pi}{T})})$ ویژگی time-shifting (انتقال در زمان):

$x(3t-1)\xleftrightarrow{FS} a_K(e^{+jK(\frac{2\pi}{T})})$ ویژگی time-scaling (مقیاس در زمان):

ادامه صفحه بعد

ضرایب سری فوریه:

$$x(3t-1) \xleftrightarrow{FS} a_k\left(e^{jk(\frac{2\pi}{T})}\right)$$

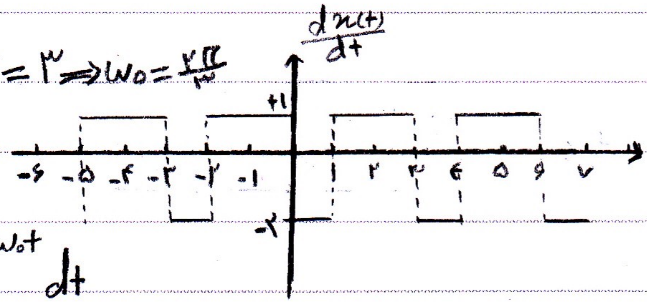
دوره تناوب:

$$T_0 = \frac{T}{3}$$

حل:

$$x(t) \xleftrightarrow{FS} a_k \qquad T=3 \Rightarrow \omega_0 = \frac{2\pi}{3}$$

$$\frac{dx(t)}{dt} \xleftrightarrow{FS} b_k = jk\omega_0 a_k$$



$$b_k = \frac{1}{3}\int_{-2}^{1} \frac{dx(t)}{dt} e^{-jk\omega_0 t}\,dt$$

$$= \frac{1}{3}\left(\int_{-2}^{0} 1 \times e^{-jk\omega_0 t}\,dt + \int_{0}^{1} -2 \times e^{-jk\omega_0 t}\,dt\right)$$

$$= \frac{1}{3}\left(\frac{1}{-jk\omega_0} e^{-jk\omega_0 t}\Big|_{-2}^{0} + \frac{2}{jk\omega_0} e^{-jk\omega_0 t}\Big|_{0}^{1}\right)$$

$$= \frac{1}{3}\left(\frac{1}{-jk\omega_0}\left(1 - e^{2jk\omega_0}\right) + \frac{2}{jk\omega_0}\left(e^{-jk\omega_0} - 1\right)\right)$$

$$= \frac{1}{3}\left(\frac{-3}{jk\omega_0} + \frac{e^{2jk\omega_0}}{jk\omega_0} + \frac{2e^{-jk\omega_0}}{jk\omega_0}\right)$$

$$a_k = \frac{b_k}{jk\omega_0} \Rightarrow a_k = \frac{1}{3}\left(\frac{-3}{-k^2\omega_0^2} + \frac{e^{2jk\omega_0}}{-k^2\omega_0^2} + \frac{2e^{-jk\omega_0}}{-k^2\omega_0^2}\right)$$

$$= \frac{1}{3k^2\omega_0^2}\left(3 - e^{2jk\omega_0} - 2e^{-jk\omega_0}\right)$$

$$= \frac{1}{3k^2 \times \frac{4\pi^2}{9}}\left(3 - e^{-2jk(\frac{2\pi}{3})} - 2e^{-jk(\frac{2\pi}{3})}\right)$$

$$= \frac{3}{4k^2\pi^2}\left(3 - e^{-\frac{4}{3}jk\pi} - 2e^{-\frac{2}{3}jk\pi}\right) \quad ; (k \neq 0)$$

$$k=0 \Rightarrow a_0 = \frac{1}{3}\int x(t)\,dt = \frac{1}{3}\left(\frac{3\times 2}{2}\right) = 1$$

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س۵:

$$x(t) = 3e^{j3t} + \cos(2t) + e^{j4t}$$
$$= 3e^{j3t} + \frac{1}{2}\left(e^{j2t} + e^{-j2t}\right) + e^{j4t}$$

دوره تناوب ها: $\frac{2\pi}{3}$, $\frac{2\pi}{2}$, $\frac{2\pi}{4}$ $\xrightarrow{\text{ک.م.م}}$ $T_0 = 2\pi$ دوره تناوب اصلی سیگنال

$\omega_0 = \frac{2\pi}{2\pi} = 1$ فرکانس اصلی سیگنال

$\frac{3}{1} = 3 \Rightarrow a_3 = 3$ $\qquad$ $\frac{2}{1} = 2 \Rightarrow a_2 = \frac{1}{2}$

$\frac{-2}{1} = -2 \Rightarrow a_{-2} = \frac{1}{2}$ $\qquad$ $\frac{4}{1} = 4 \Rightarrow a_4 = 1$ $\qquad$ ($K>4$ یا $K<2$ و $K \neq -2$ $\Rightarrow a_K = 0$)

رابطه پارسوال (Parseval):

$$\underset{\text{توان متوسط}}{P} = \frac{1}{T}\int_T |x(t)|^2 dt = \sum_{K=-\infty}^{\infty} |a_K|^2$$

$$P = |a_{-2}|^2 + |a_2|^2 + |a_3|^2 + |a_4|^2 = \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 + 3^2 + 1^2 = \frac{1}{4} + \frac{1}{4} + 9 + 1$$
$$= 10 + \frac{1}{2} = \frac{21}{2} = 10.5$$

س۶:

$$b_K = (-1)^K a_K + (-1)^K a_{-K} = e^{j\pi K} a_K + e^{j\pi K} a_{-K}$$

$$T_{0x} = 4 \Rightarrow \omega_{0x} = \frac{2\pi}{4} = \frac{\pi}{2} \Rightarrow b_K = e^{j\frac{\pi}{2}2K}(a_K + a_{-K}) = e^{-j(\frac{\pi}{2})K(-2)}(a_K + a_{-K})$$

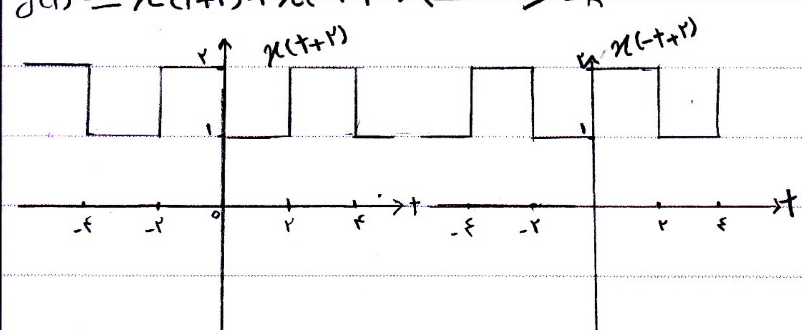
ویژگی خطی بودن: $b_K = e^{-j(\frac{\pi}{2})K(-2)} a_K + e^{-j(\frac{\pi}{2})K(-2)} a_{-K} = b_{1K} + b_{2K}$

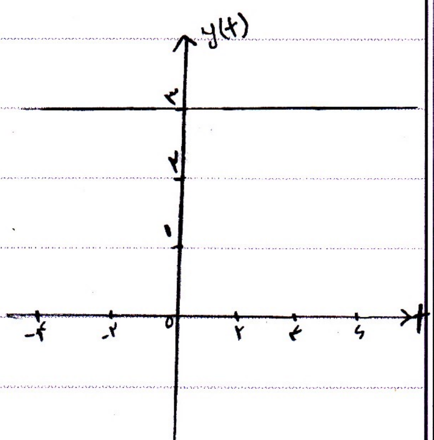
ویژگی انتقال در زمان: $b_{1K} = e^{-j(\frac{\pi}{2})K(-2)} a_K \xleftrightarrow{FS} x(t+2)$

ویژگی انتقال در زمان و معکوس زمان: $b_{2K} = e^{-j(\frac{\pi}{2})K(-2)} a_{-K} \longleftrightarrow x(-t+2)$

$x(t) \xleftrightarrow{FS} a_K$

$y(t) = x(t+2) + x(-t+2) \xleftrightarrow{FS} b_K$





Kian King

س ۷:

$T_0 = 6 \Rightarrow \omega_0 = \frac{2\pi}{3}$

$a_0 , a_{|k|>2} = 0 \qquad a_1 , a_{-1} , a_2 , a_{-2} = ?$

خاصیت انتقال در زمان و خطی بودن:

$x(t) \xleftrightarrow{FS} a_k$

$-x(t-3) \xleftrightarrow{FS} -e^{-j\omega_0 k t_0} a_k = -e^{-j(\frac{2\pi}{3})k(3)} a_k = -e^{-2\pi j k} a_k = -(-1)^k a_k \overset{\text{فرض}}{=} a_k$

$k=1 \Rightarrow a_1 = a_1 \;,\; k=-1 \Rightarrow a_{-1} = a_{-1} \;,\; k=2 \Rightarrow -a_2 = a_2 \Rightarrow a_2 = 0$

$k=-2 \Rightarrow -a_{-2} = a_{-2} \Rightarrow a_{-2} = 0$

$\Rightarrow x(t) = a_1 e^{j(1)(\frac{2\pi}{3})t} + a_{-1} e^{j(-1)(\frac{2\pi}{3})t} = a_1 e^{\frac{2\pi}{3}jt} + a_2 e^{-\frac{2\pi}{3}jt}$

حقیقی $x(t)$:

$a_k = a^*_{-k}$

$k=-1 \Rightarrow a_{-1} = a_1^* \xlongequal{a_1 \text{ حقیقی}} a_1 \Rightarrow a_{-1} = a_1$

$\Rightarrow x(t) = a_1 (e^{\frac{2\pi}{3}jt} + e^{-\frac{2\pi}{3}jt})$

فرض:

$\int_{-3}^{3} |x(t)|\, dt = 12\pi$ ①

$|x(t)| = |a_1 (e^{\frac{2\pi}{3}jt} + e^{-\frac{2\pi}{3}jt})| = |a_1 (\cos(\frac{2\pi}{3}t) + j\sin(\frac{2\pi}{3}t) + \cos(-\frac{2\pi}{3}t) + j\sin(-\frac{2\pi}{3}t))|$

$= |a_1 (2\cos(\frac{2\pi}{3}t))| \xlongequal{a_1 \text{ مثبت}} 2a_1 |\cos(\frac{2\pi}{3}t)|$

$\Rightarrow \int_{-3}^{3} |x(t)|\, dt = \int_{-3}^{3} 2a_1 |\cos(\frac{2\pi}{3}t)|\, dt = 2a_1 \int_{-3}^{3} |\cos(\frac{\pi}{3}t)|\, dt$

$= 2a_1 \left(\int_{-3}^{-\frac{3}{2}} -\cos(\frac{\pi}{3}t)\, dt + \int_{-\frac{3}{2}}^{\frac{3}{2}} \cos(\frac{\pi}{3}t)\, dt + \int_{\frac{3}{2}}^{3} -\cos(\frac{\pi}{3}t)\, dt \right)$

$= 2a_1 \left(-\frac{3}{\pi}\sin(\frac{\pi}{3}t) \Big|_{-3}^{-\frac{3}{2}} + \frac{3}{\pi}\sin(\frac{\pi}{3}t) \Big|_{-\frac{3}{2}}^{\frac{3}{2}} + \frac{-3}{\pi}\sin(\frac{\pi}{3}t) \Big|_{\frac{3}{2}}^{3} \right)$

$= 2a_1 \left(-\frac{3}{\pi}(-1-0) + \frac{3}{\pi}(1-(-1)) - \frac{3}{\pi}(0-1) \right)$

$= 2a_1 (\frac{3}{\pi} + \frac{6}{\pi} + \frac{3}{\pi}) = 2a_1 (\frac{12}{\pi}) = \frac{24 a_1}{\pi} \overset{①}{=} 12\pi \Rightarrow a_1 = \frac{\pi^2}{2}$

$\Rightarrow a_1 = a_{-1} = \frac{\pi^2}{2}$

$\Rightarrow x(t) = \frac{\pi^2}{2} (e^{\frac{2\pi}{3}jt} + e^{-\frac{2\pi}{3}jt}) = \frac{\pi^2}{2}(2\cos(\frac{2\pi}{3}t)) = \pi^2 \cos(\frac{2\pi}{3}t)$

حل:

$$a_K = \frac{1}{1}\int_{-\frac{1}{2}}^{\frac{1}{2}} \delta(t)\, e^{-jK\left(\frac{2\pi}{1}\right)t}\, dt = e^{0} = 1$$

$$H(j\omega_0 K) = \int_{-\infty}^{\infty} h(\tau)\, e^{-jK\omega_0\tau}\, d\tau = \int_{-\infty}^{\infty} e^{-4|\tau|}\, e^{-jK\omega_0\tau}\, d\tau$$

$$= \int_{-\infty}^{0} e^{\tau(4-jK\omega_0)}\, d\tau + \int_{0}^{\infty} e^{-\tau(4+jK\omega_0)}\, d\tau$$

$$= \frac{1}{4-jK\omega_0}\, e^{\tau(4-jK\omega_0)}\Big|_{-\infty}^{0} + \frac{1}{4+jK\omega_0}\, e^{-\tau(4+jK\omega_0)}\Big|_{0}^{\infty} = \frac{1}{4-jK\omega_0} - \frac{1}{4+jK\omega_0}$$

$$\overset{\omega_0=2\pi}{=} \frac{+4+jK\omega_0-4+jK\omega_0}{16+K^2(2\pi)^2} = \frac{2jK(2\pi)}{16+4\pi^2K^2} = \frac{jK\pi}{4+K^2\pi^2}$$

$$x(t) \xleftrightarrow{FS} a_K = 1$$

$$y(t) = x(t)\times H(jK\omega_0) \xleftrightarrow{FS} a_K \times H(jK\omega_0) = 1\times\frac{jK\pi}{4+K^2\pi^2} = \frac{jK\pi}{4+K^2\pi^2}$$

ضرایب