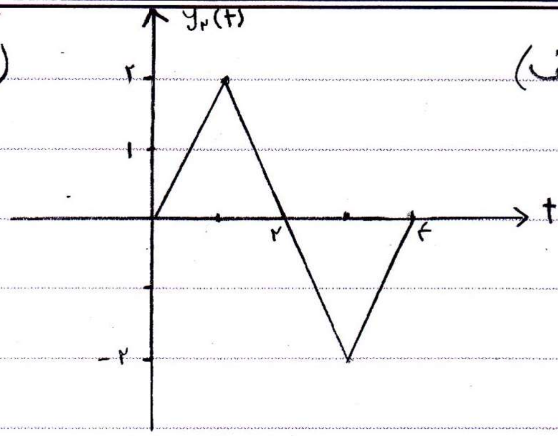
Date: .... Subject: ....

سوال ۱: الف)

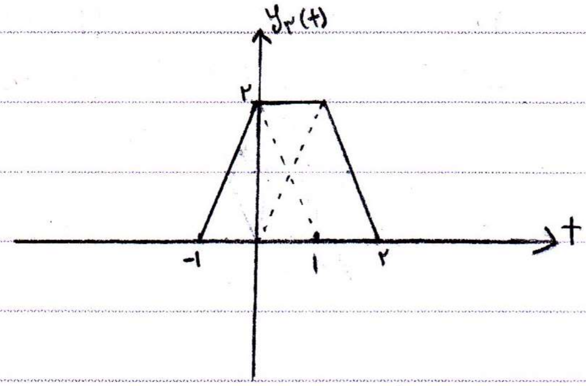
$$x_2(t) = x_1(t) - x_1(t-2)$$

$$\xrightarrow{LTI} y_2(t) = y_1(t) - y_1(t-2)$$



$$x_3(t) = x_1(t) + x_1(t+1)$$

$$\xrightarrow{LTI} y_3(t) = y_1(t) + y_1(t+1)$$



ب)

$$x(t) = u(t-1) - u(t-2)$$

$$\xrightarrow{LTI} y(t) = e^{-(t-1)} u(t-1) + u(-1-(t-1)) - e^{-(t-2)} u(t-2) - u(-1-(t-2))$$

$$= e^{-(t-1)} u(t-1) + u(-t) - e^{-(t-2)} u(t-2) - u(1-t)$$

سوال ۲:

a) $x(t) = u(t) - u(t-2)$, $h(t) = e^{-2t} u(t)$

$$x(t) * h(t) = \int_{-\infty}^{+\infty} x(\tau) h(t-\tau) d\tau = \int_{-\infty}^{+\infty} (u(\tau) - u(\tau-2)) e^{-2(t-\tau)} u(t-\tau) d\tau$$

$$= \int_{-\infty}^{\infty} u(\tau) e^{-2(t-\tau)} u(t-\tau) d\tau - \int_{-\infty}^{\infty} u(\tau-2) e^{-2(t-\tau)} u(t-\tau) d\tau$$

$t<0$: $x(t) * h(t) = \int_{-\infty}^{\infty} 0 \, d\tau - \int_{-\infty}^{\infty} 0 \, d\tau = 0$

$0 \le t < 2$: $x(t) * h(t) = \int_{-\infty}^{\infty} e^{-2(t-\tau)} u(\tau) u(t-\tau) d\tau - \int_{-\infty}^{\infty} 0 \, d\tau = \int_0^t e^{-2(t-\tau)} d\tau = \frac{e^{-2t}}{2} e^{2\tau}\Big|_0^t = \frac{1}{2}(1 - e^{-2t})$

$2 \le t$: $x(t) * h(t) = \int_{-\infty}^{\infty} e^{-2(t-\tau)} u(\tau) u(t-\tau) d\tau - \int_{-\infty}^{\infty} e^{-2(t-\tau)} u(\tau-2) u(t-\tau) d\tau = \int_0^t e^{-2(t-\tau)} d\tau$

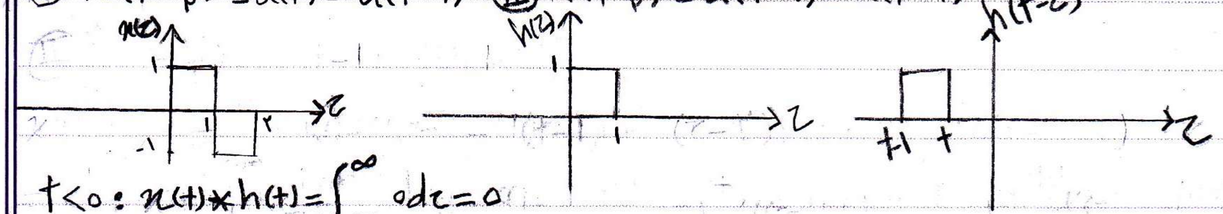
$$- \int_2^t e^{-2(t-\tau)} d\tau = \frac{1}{2} e^{-2t} \left(e^{2\tau}\Big|_0^t - e^{2\tau}\Big|_2^t \right) = \frac{1}{2} e^{-2t} (e^{2t} - 1 - e^{2t} + e^4) = \frac{1}{2} e^{-2t} (e^4 - 1)$$

$$x(t) * h(t) = \begin{cases} 0 & t<0 \\ \frac{1}{2}(1 - e^{-2t}) & 0 \le t < 2 \\ \frac{1}{2} e^{-2t} (e^4 - 1) & 2 \le t \end{cases}$$

b) $x(t)=\Pi(t-\frac{1}{2})-\Pi(t-\frac{3}{2})$, $h(t)=u(t)-u(t-1)$

$$x(t)*h(t)=\int_{-\infty}^{\infty}x(\tau)h(t-\tau)d\tau=\int_{-\infty}^{\infty}\left(\Pi(\tau-\tfrac{1}{2})-\Pi(\tau-\tfrac{3}{2})\right)\left(u(t-\tau)-u(t-(\tau-1))\right)d\tau$$

I $\Pi(t-\frac{1}{2})=u(t)-u(t-1)$ II $\Pi(t-\frac{3}{2})=u(t-1)-u(t-2)$



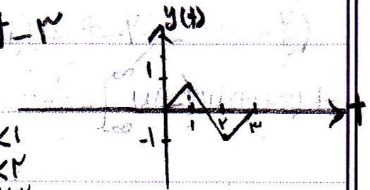
$t<0:\ x(t)*h(t)=\int_{-\infty}^{\infty}0\,d\tau=0$

$0\leqslant t<1:\ x(t)*h(t)=\int_{0}^{t}1\,d\tau=\tau\Big|_0^t=t-0=t$

$1\leqslant t<2:\ x(t)*h(t)=\int_{t-1}^{1}1\,d\tau-\int_{1}^{t}1\,d\tau=\tau\Big|_{t-1}^{1}-\tau\Big|_{1}^{t}=(1-(t-1))-(t-1)=-2t+3$

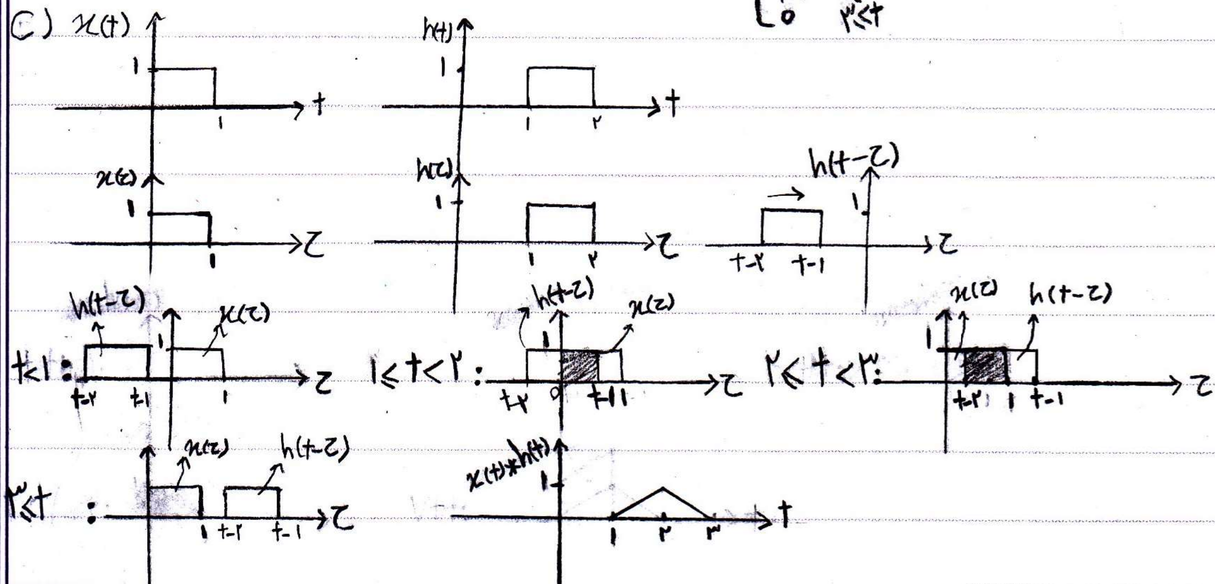
$2\leqslant t<3:\ x(t)*h(t)=\int_{t-1}^{2}-1\,d\tau=-\tau\Big|_{t-1}^{2}=-(2-(t-1))=+t-3$

$3\leqslant t:\ x(t)*h(t)=\int_{-\infty}^{\infty}0\,d\tau=0$



$$\Rightarrow x(t)*h(t)=\begin{cases}0 & t<0\\ t & 0\leqslant t<1\\ -2t+3 & 1\leqslant t<2\\ t-3 & 2\leqslant t<3\\ 0 & 3\leqslant t\end{cases}$$

c)



$t<1:\ \int_{-\infty}^{\infty}x(\tau)h(t-\tau)d\tau=\int_{-\infty}^{\infty}0\,d\tau=0$

$1\leqslant t<2:\ \int_{-\infty}^{\infty}x(\tau)h(t-\tau)d\tau=\int_{0}^{t-1}1\,d\tau=\tau\Big|_0^{t-1}=(t-1)-(0)=t-1$

$2\leqslant t<3:\ \int_{-\infty}^{+\infty}x(\tau)h(t-\tau)d\tau=\int_{t-2}^{1}1\,d\tau=\tau\Big|_{t-2}^{1}=1-(t-2)=-t+3$

$3\leqslant t:\ \int_{-\infty}^{+\infty}x(\tau)h(t-\tau)d\tau=\int_{-\infty}^{+\infty}0\,d\tau=0$

$$\Rightarrow x(t)*h(t)=\begin{cases}0 & t<1\\ t-1 & 1\leqslant t<2\\ -t+3 & 2\leqslant t<3\\ 0 & 3\leqslant t\end{cases}$$

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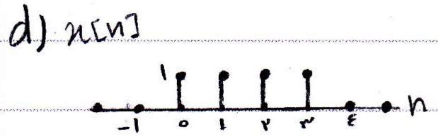
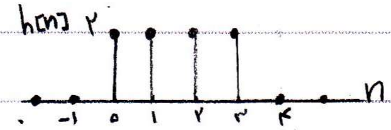
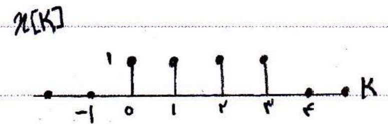
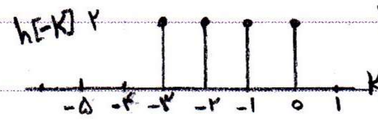
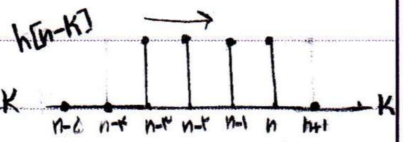
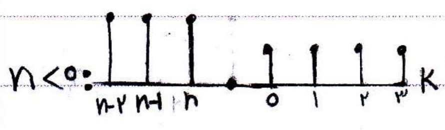
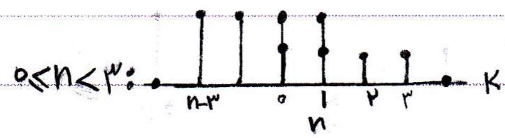
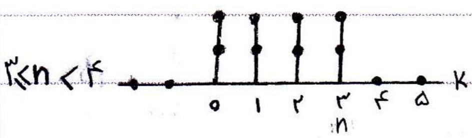
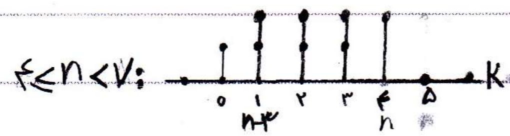
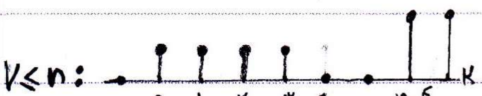
d)

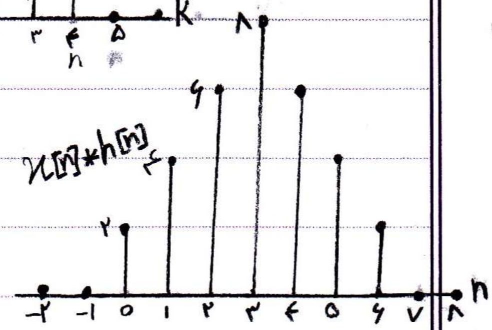







$n<0: \sum_{K=-\infty}^{+\infty} x[K]h[n-K] = \sum_{K=-\infty}^{\infty} 0 = 0$

$0 \leq n < 3: \sum_{K=-\infty}^{+\infty} x[K]h[n-K] = \sum_{K=0}^{n} 2 = 2(n+1) = 2n+2$

$3 \leq n < 4: \sum_{K=-\infty}^{+\infty} x[K]h[n-K] = \sum_{K=0}^{3} 2 = 2(3+1) = 8$

$4 \leq n < 7: \sum_{K=-\infty}^{\infty} x[K]h[n-K] = \sum_{K=n-3}^{3} 2 = 2(3-(n-3)+1) = -2n+14$

$7 \leq n : \sum_{K=-\infty}^{\infty} x[K]h[n-K] = \sum_{K=-\infty}^{\infty} 0 = 0 \Rightarrow x[n]*h[n] = \begin{cases} 0 & n<0 \\ 2n+2 & 0 \leq n < 3 \\ -2n+14 & 4 \leq n < 7 \\ 0 & 7 \leq n \end{cases}$

سوال ۳:

$$h(t) = \Big[\big((h_1(t) * h_2(t)) + (h_2(t) * h_2(t)) - (h_2(t) * h_1(t))\big) * h_1(t) + h_1^{-1}(t)\Big] * h_2^{-1}(t) = \left[h_1 * h_2 * h_1 + h_2 * h_2 * h_1 - h_2 * h_1 * h_1 + h_1^{-1}\right] * h_2^{-1}$$

(distributive)

$$= h_1 * h_2 * h_1 * h_2^{-1} + h_2 * h_2 * h_1 * h_2^{-1} - h_2 * h_1 * h_1 * h_2^{-1} + h_1^{-1} * h_2^{-1} \quad \text{(distributive)}$$

$$= h_1 * h_1 * (h_2 * h_2^{-1}) + h_2 * h_1 * (h_2 * h_2^{-1}) - h_1 * h_1 * (h_2 * h_2^{-1}) + h_1^{-1} * h_2^{-1} \quad \text{(Commutative)}$$

$$= h_1 * h_1 + h_2 * h_1 - h_1 * h_1 + h_1^{-1} * h_2^{-1} \quad (x(t) * h(t) * h^{-1}(t) = x(t))$$

$$= (h_2 * h_1) + (h_1^{-1} * h_2^{-1}) \quad (h_1 * h_1 - h_1 * h_1 = 0)$$

پاسخ نهایی: $h(t) = h_2(t) * h_1(t) + h_1^{-1}(t) * h_2^{-1}(t)$

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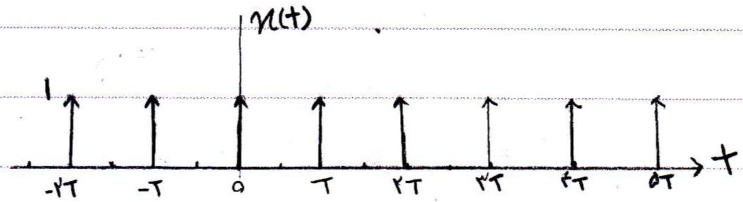
سؤال ۴: الف)

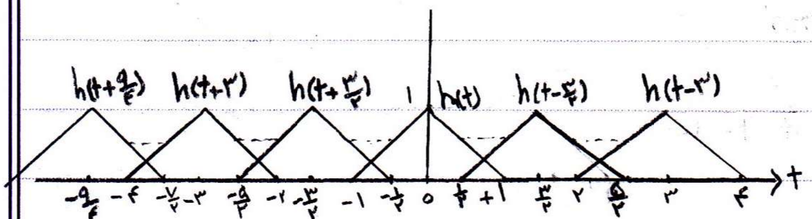
$x(t)=\sum_{K=-\infty}^{\infty}\delta(t-KT)$



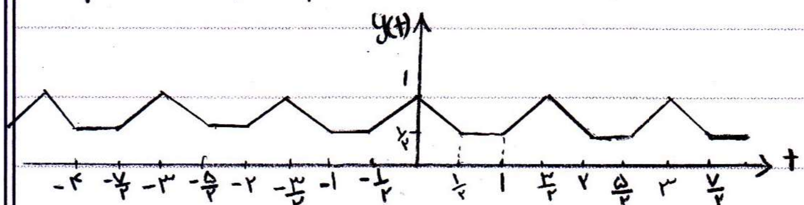
ب) $y(t)=x(t)*h(t)=\left(\sum_{K=-\infty}^{\infty}\delta(t-\frac{3}{2}K)\right)*h(t)=\dots+\delta(t+\frac{3}{2})*h(t)$

$+\delta(t)*h(t)+\delta(t-\frac{3}{2})*h(t)+\delta(t-3)*h(t)+\dots$

$\overset{①}{=}\dots+h(t+\frac{3}{2})+h(t)+h(t-\frac{3}{2})+h(t-3)=\sum_{K=-\infty}^{\infty}h(t-\frac{3}{2}K)$



$①: \delta(t-t_0)*h(t)=h(t-t_0)$



سؤال ۵:

a) $x[n]*(h[n]\cdot g[n])=(x[n]*h[n])\cdot g[n]$ غلط.

$g[n]=\delta[n]\xRightarrow{\text{سمت چپ}} x[n]*(h[n]\cdot g[n])=x[n]*(h[n]\cdot\delta[n])=x[n]*h[0]$

$\xRightarrow{\text{سمت راست}}(x[n]*h[n])\cdot g[n]=(x[n]*h[n])\cdot\delta[n]=(x[n]*h[n])\delta[n]$

بازای هر n برقرار نیست.

b) $y(t)=x(t)*h(t)\Rightarrow y(2t)=2x(2t)*h(2t)$ درست.

$y(t)=\int_{-\infty}^{\infty}x(\tau)h(t-\tau)d\tau$

$\Rightarrow y(2t)=\int_{-\infty}^{+\infty}x(\tau)h(2t-\tau)d\tau \xrightarrow[d\tau=2d\tau']{\tau=2\tau'}\int_{-\infty}^{\infty}2x(2\tau')h(2t-2\tau')d\tau'$

$=2\int_{-\infty}^{\infty}x(2\tau')h(2t-2\tau')d\tau'=2x(2t)*h(2t)$

c) $x(t), h(t)$ are odd $\Rightarrow y(t)=x(t)*h(t)$ is an even signal درست

$$y(-t)=x(-t)*h(-t)=\int_{-\infty}^{+\infty} x(-\tau)h(-t+\tau)d\tau$$

$x(-t)=-x(t)$
$h(-t)=-h(t)$ $\Longrightarrow y(-t)=\int_{-\infty}^{\infty}-x(\tau)\times h(-t+\tau)d\tau=\int_{-\infty}^{\infty}x(\tau)h(t-\tau)d\tau$

$$=x(t)*h(t)=y(t)$$

$\Longrightarrow y(t)=y(-t) \Longrightarrow y(t)$ is even ✓

س۲:

a) $h(t)=te^{-t}u(t)$

$t<0 \Rightarrow u(t)=0 \Rightarrow te^{-t}u(t)=0 \Rightarrow h(t)=0 \Rightarrow$ علّی است. (Causal)

$$\int_{-\infty}^{\infty}|h(\tau)|d\tau=\int_{-\infty}^{\infty}|\tau e^{-\tau}u(\tau)|d\tau=\int_{0}^{\infty}|\tau e^{-\tau}|d\tau=\int_{0}^{\infty}\tau e^{-\tau}d\tau$$

$$=\tau\times -e^{-\tau}\Big|_0^{\infty}-\int_0^{\infty}-e^{-\tau}d\tau=-\tau e^{-\tau}\Big|_0^{\infty}-e^{-\tau}\Big|_0^{\infty}=-e^{-\tau}(\tau+1)\Big|_0^{\infty}=0\times(\infty+1)-(-1(0+1))$$

$= 1<\infty \Longrightarrow$ پایدار است (Stable)

$t\neq 0 \Rightarrow t<0 \Rightarrow h(t)=0$
$\Rightarrow t>0 \Rightarrow h(t)=te^{-t}\neq 0 \Longrightarrow$ حافظه‌دار است (with memory)

b) $h[n]=(0.8)^n u[n+2]$

$n<0 \Rightarrow -2\leq n<0 \Rightarrow u[n+2]=1 \Rightarrow h[n]=(0.8)^n\neq 0$
$\Rightarrow n<-2 \Rightarrow u[n+2]=0 \Rightarrow h[n]=0$ } علّی نیست. (non causal)

$$\sum_{k=-\infty}^{\infty}|h[k]|=\sum_{k=-2}^{\infty}|0.8^n|=\frac{(0.8)^{-2}}{1-0.8}=\frac{\frac{1}{0.64}}{0.2}=\frac{1}{0.128}<\infty$$ پایدار است. (Stable)

$n\neq 0 \Rightarrow n>0 \Rightarrow h[n]=(0.8)^n u[n+2]\neq 0 \Rightarrow$ حافظه‌دار است

(With memory)

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c) $h(t) = e^{-6t}u(t+2)$

$t<0 \Rightarrow -2\leq t<0 \Rightarrow u(t+2)=1 \Rightarrow h(t)=e^{-6t}\neq 0 \Rightarrow$ علّی نیست. (non causal)

$\Rightarrow t<-2 \Rightarrow u(t+2)=0 \Rightarrow h(t)=0$

$$\int_{-\infty}^{\infty}|h(\tau)|d\tau = \int_{-\infty}^{\infty}|e^{-6\tau}u(\tau+2)|d\tau = \int_{-2}^{\infty}e^{-6\tau}d\tau = -\frac{1}{6}e^{-6\tau}\Big|_{-2}^{\infty}$$

$= -\frac{1}{6}(0-e^{12}) = +\frac{e^{12}}{6} < \infty \Rightarrow$ پایدار است. (Stable)

$t\neq 0 \Rightarrow -2\leq t<0 \Rightarrow u(t+2)=1 \Rightarrow h(t)=e^{-6t}\neq 0 \Rightarrow$ حافظه دار است. (with memory)

d) $h[n] = 5^n u[3-n]$

$n<0 \Rightarrow u[3-n]=1 \Rightarrow h[n]=5^n\neq 0 \Rightarrow$ علّی نیست. (non-causal)

$$\sum_{k=-\infty}^{\infty}|5^k u[3-k]| = \sum_{k=-\infty}^{3}5^k = \frac{5^{-\infty}-5^4}{1-5} = \frac{625}{4} < \infty \Rightarrow$$ پایدار است. (Stable)

$n\neq 0 \Rightarrow n<0 \Rightarrow u[3-n]=1 \Rightarrow h[n]=5^n\neq 0 \Rightarrow$ حافظه دار است. (with memory)

سوال ۷: $y[n]=h[n] \Leftarrow x[n]=\delta[n] \Leftarrow$ پاسخ ضربه

$y[n]+2y[n-1]=x[n]$

$y[n]+2y[n-1]=\delta[n]$

معادله مشخصه: $r+2=0 \Rightarrow r=-2 \xRightarrow{\text{همگن}} y[n]_{(h)} = C(-2)^n$

جواب فرضی معادله: سیستم علّی $\Leftarrow$ سکون ابتدایی $\Rightarrow y[n]=C(-2)^n u[n]$

صدق در معادله $\Rightarrow C(-2)^n u[n] + 2C(-2)^{n-1}u[n-1] = \delta[n]$

$\Rightarrow C(-2)^n u[n] + 2\times(-2)^{-1}\times C(-2)^n u[n-1] = \delta[n]$

$\Rightarrow C(-2)^n u[n] - C(-2)^n u[n-1] = C(-2)^n(u[n]-u[n-1]) = \delta[n]$

$\xRightarrow{n=0} C(-2)^0 = 1 \Rightarrow C=1 \Rightarrow y[n]=(-2)^n u[n] = h[n]$:پاسخ ضربه