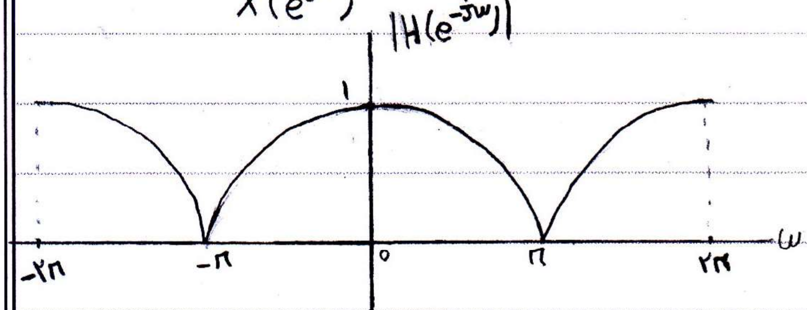
س۱: الف) در $y_1[n]$ چون دارد مقدار متوسط دو سیگنال را می گیرد، در مقابل تغییر مقاومت می کند. فرکانس های کم (حول مضرب زوج π) ایجاد می کند پس یک فیلتر پایین گذر است. در $y_2[n]$ تفاضل دو سیگنال را می گیرد (در واقع دلتا می گیرد) و در مقابل عدم تغییر مقاومت می کند فرکانس های بالا (حول مضرب فرد π) ایجاد می کند پس یک فیلتر بالا گذر است.

ب)

$$y_1[n] = \frac{x[n] + x[n-1]}{2}$$

$$Y_1(e^{j\omega}) = f\left\{\frac{x[n] + x[n-1]}{2}\right\} = \frac{1}{2}X(e^{j\omega})\left(1 + e^{-j\omega}\right)$$

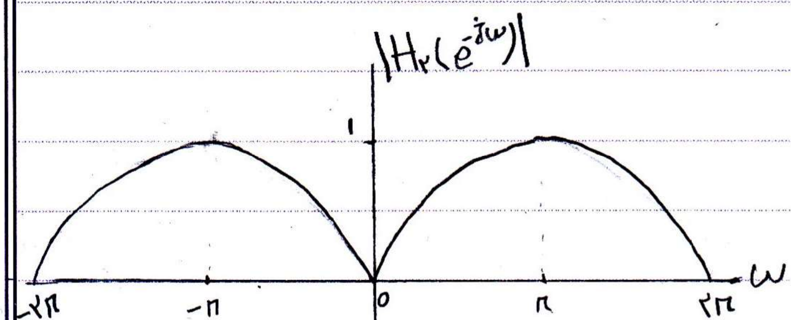
$$H_1(e^{j\omega}) = \frac{Y_1(e^{j\omega})}{X(e^{j\omega})} = \frac{1 + e^{-j\omega}}{2}$$



$$y_2[n] = \frac{x[n] - x[n-1]}{2}$$

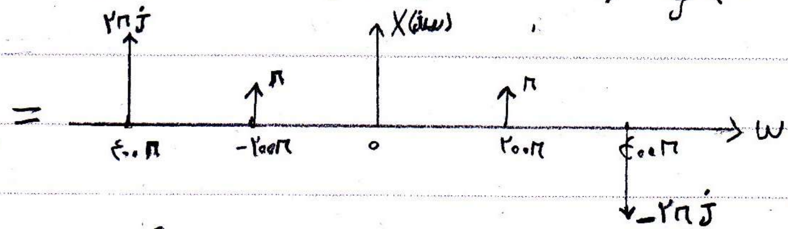
$$Y_2(e^{j\omega}) = f\left\{\frac{x[n] - x[n-1]}{2}\right\} = \frac{1}{2}X(e^{j\omega})\left(1 - e^{-j\omega}\right)$$

$$H_2(e^{j\omega}) = \frac{Y_2(e^{j\omega})}{X(e^{j\omega})} = \frac{1 - e^{-j\omega}}{2}$$



$\cancel{x(t) = \cos(200\pi t) + 2\sin(400\pi t)}$

س ۲: $X(j\omega) = \pi\left(\delta(\omega - 200\pi) + \delta(\omega + 200\pi)\right) + \frac{2\pi}{j}\left(\delta(\omega - 400) - \delta(\omega + 400\pi)\right)$

$=$ 

$g(t) = x(t)\sin(400\pi t)$

$w(t) = g(t)\sin(400\pi t) = x(t)(\sin(400t))^2 = x(t)\left(\frac{1-\cos(800t)}{2}\right)$

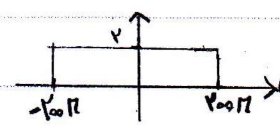
$W(j\omega) = X(j\omega) * \left(2\pi\delta(\omega) - \frac{\pi}{2}\left(\delta(\omega - 800\pi) + \delta(\omega + 800\pi)\right)\right) \times \frac{1}{2\pi}$

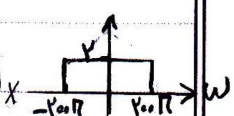
$= \frac{X(j\omega)}{4} * \left(2 - \delta(\omega - 800\pi) + \delta(\omega + 800\pi)\right)$

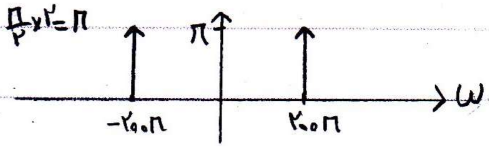
$= \frac{X(j\omega)}{2} - \frac{X(j\omega - 800\pi)}{4} + \frac{X(j\omega + 800\pi)}{4}$

خروجی:

خارج فیلتر

$W(j\omega) \times$ 

$= \left(\frac{X(j\omega)}{2} - \cancel{\frac{X(j\omega - 800\pi)}{4}} + \cancel{\frac{X(j\omega + 800\pi)}{4}}\right) \times$ 

$=$ 

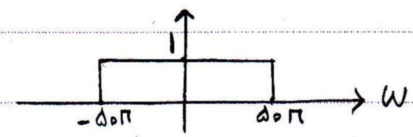
$= \pi\delta(\omega + 200\pi) + \pi\delta(\omega - 200\pi)$

$w(t) = \mathcal{F}^{-1}\{X(j\omega)\} = \cos(200\pi t)$

س۳ : الف)

$$x(t) = \left(\frac{\sin(50\pi t)}{\pi t}\right)$$

$$X(w) = f\{x(t)\} = f\left\{\frac{\sin(50\pi t)}{\pi t}\right\} = rect\left(\frac{50w}{2}\right)$$

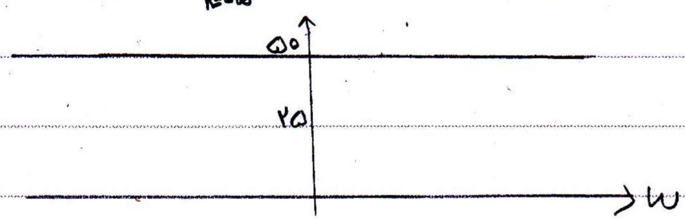
$=$ 

$$w_s = 50\pi \quad , \quad T = \frac{2\pi}{50\pi} = \frac{1}{25}$$

$$X_p(jw) = \frac{1}{T}\sum_{k=-\infty}^{+\infty} X(j(w - kw_s))$$

هم‌پوشانی‌ها

$$X_p(jw) = 25\sum_{k=-\infty}^{+\infty} X(j(w - k50\pi)) = 25 \times 2 = 50$$

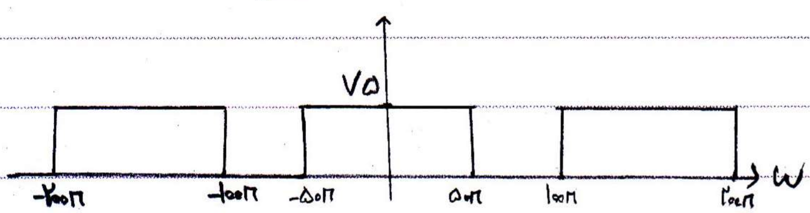


$2w_M > w_s$

قابل بازیابی نیست.

$$w_s = 150\pi \quad , \quad T = \frac{2\pi}{150\pi} = \frac{1}{75}$$

$$X_p(jw) = 75\sum_{k=-\infty}^{\infty} X(j(w - k150\pi))$$



$2w_M < w_s$

قابل بازیابی است.

با فیلتر پایین گذر با فرکانس قطع $50\pi < w_c < 100\pi$ و بهره $1/75$.

ب) حداقل فرکانس نمونه برداری 100π است.

$$2w_M < w_s \Rightarrow 2\times 50\pi = 100\pi < w_s$$

Date: .... Subject: ....

س۴:

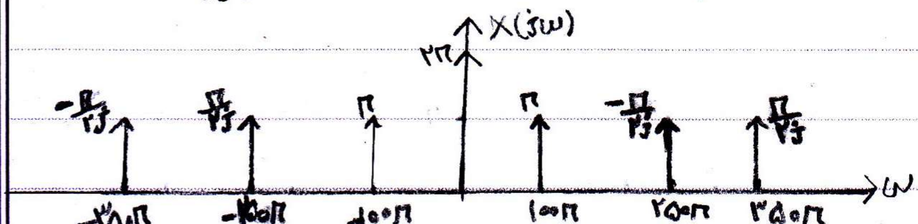
a) $x(t)=e^{-۵t}\,u(t)$

$X(j\omega)=\dfrac{1}{۵+j\omega}=0 \Rightarrow \omega$ ندارد $\Leftarrow$ نرخ نایکوشت ندارد

b) $x(t)=1+\cos(۱۰۰\pi t)+\overbrace{\cos(۳۰۰\pi t)\sin(۵۰\pi t)}^{\frac{1}{۲}(\sin(۳۵۰\pi t)-\sin(۲۵۰\pi))}$

$X(j\omega)=۲\pi\delta(\omega)+\pi(\delta(\omega-۱۰۰\pi)+\delta(\omega+100))+\frac{\pi}{۲j}(\delta(\omega-۳۵۰\omega)-\delta(\omega+۳۵۰\omega))$

$-\frac{\pi}{۲j}(\delta(\omega-۲۵۰\pi)-\delta(\omega+۲۵۰\pi))=0$



$X(j\omega)=0$

$|\omega|>۳۵۰\pi=\omega_M$

نرخ نایکوشت $۲\omega_M=۷۰۰\pi$

c) $x(t)=u(t)-u(t-۴)=rect\left(\frac{t-۲}{۴}\right)$

$X(j\omega)=e^{-۲j\omega}\,\mathcal{F}\{rect(\frac{t}{۴})\}=e^{-۲j\omega}\times ۲\frac{\sin(۲\omega)}{\omega}$

$=(\cos(۲\omega)-j\sin(۲\omega))\,۲\frac{\sin(۲\omega)}{\omega}$

$=\frac{۲\cos(۲\omega)\sin(۲\omega)}{\omega}-۲\frac{\sin^۲(۲\omega)}{\omega}=\frac{\sin(۴\omega)-۲\sin^۲(۲\omega)}{\omega}$

نرخ نایکوشت ندارد.

سری ۵: تناوب اصلی $N_0 = \frac{2\pi}{\frac{3\pi}{7}} \times 3 = 14$

فرکانس اصلی $\omega_0 = \frac{2\pi}{14} = \frac{\pi}{7}$

$x[n] = 2 + \cos(\frac{3\pi}{7}(n-1))$

$= 2 + \frac{e^{j(\frac{3\pi}{7}(n-1))} + e^{-j(\frac{3\pi}{7}(n-1))}}{2}$

$\frac{\frac{3\pi}{7}}{\frac{\pi}{7}} = 3 \Rightarrow a_3 = \frac{e^{-j\frac{3\pi}{7}}}{2}$

$\frac{-\frac{3\pi}{7}}{\frac{\pi}{7}} = -3 \Rightarrow a_{-3} = \frac{e^{j\frac{3\pi}{7}}}{2}$

$\frac{0}{\frac{\pi}{7}} = 0 \Rightarrow a_0 = 2$

$(K \neq 3, -3, 0 \Rightarrow a_K = 0)$

$\Rightarrow a_1 = 0$

سری ۶:

a) $x[n] = 1 + \cos(n\frac{\pi}{2}) + \sin(n\pi) = 1 + \frac{e^{j(\frac{n\pi}{2})} + e^{-j(\frac{n\pi}{2})}}{2} + \frac{e^{jn\pi} - e^{-jn\pi}}{2j}$

دوره تناوب جملات: $N_1 = \frac{2\pi}{\frac{\pi}{2}} = 4$, $N_2 = \frac{2\pi}{\pi} = 2 \xrightarrow{\text{ک.م.م}} N_0 = 4$ تناوب اصلی

$\omega_0 = \frac{2\pi}{4} = \frac{\pi}{2}$ فرکانس اصلی

$\frac{\frac{\pi}{2}}{\frac{\pi}{2}} = 1 \Rightarrow a_1 = \frac{1}{2}$ $\qquad \frac{-\frac{\pi}{2}}{\frac{\pi}{2}} = -1 \Rightarrow a_{-1} = \frac{1}{2}$

$\frac{\pi}{\frac{\pi}{2}} = 2 \Rightarrow a_2 = \frac{1}{2j} = -\frac{j}{2}$ $\qquad \frac{-\pi}{\frac{\pi}{2}} = -2 \Rightarrow a_{-2} = -\frac{1}{2j} = \frac{j}{2}$

$\frac{0}{\frac{\pi}{2}} = 0 \Rightarrow a_0 = 1$

b) $N_0 = 8$

$a_K = \frac{1}{N_0} \sum_{n=\langle N \rangle} x[n] e^{-jK\omega_0 n} = \frac{1}{8} \sum_{n=0}^{7} x[n] e^{-jK(\frac{2\pi}{8})n}$

$= \frac{1}{8}\left(4 + 3e^{-jK(\frac{\pi}{4})} + 2e^{-jK(\frac{\pi}{2})} + e^{-jK(\frac{3\pi}{4})} + 0 + e^{-jK(\frac{5\pi}{4})} + 2e^{-jK(\frac{3\pi}{2})} + 3e^{-jK(\frac{7\pi}{4})} \right)$

سری فوریه: $x[n] = \frac{1}{8} \sum_{K=0}^{7} a_K e^{jK(\frac{2\pi}{8})n} = \frac{1}{8} \sum_{K=0}^{7} a_K e^{jK(\frac{\pi}{4})n}$

سوال ۷

$N=۳ \Rightarrow \omega = \frac{۲\pi}{۳}$

$\frac{\frac{۲\pi}{۳}}{\frac{۲\pi}{۳}} = ۱ \Rightarrow a_1 = \frac{۱}{۳}\sum_{n=<۳>} x[n]\, e^{-j\frac{۲\pi}{۳}\times ۱\times n} = \frac{۱}{۳}\sum_{n=۰}^{۲} x[n]\, e^{-j\frac{۲\pi}{۳}n}$

$= \frac{۱}{۳}\left(۰ + e^{-j\frac{۲\pi}{۳}} + ۲e^{-j\frac{۴\pi}{۳}}\right) = \frac{۱}{۳}\left(e^{-j\frac{۲\pi}{۳}} + ۲e^{-j\frac{۴\pi}{۳}}\right)$

$x[n] \xleftrightarrow{FS} a_k$

$y[n] = x[n]\times H(e^{j\omega}) \xleftrightarrow{FS} b_k = a_k \times H(e^{j\omega})$

$\Rightarrow b_1 = a_1 \times H(e^{+j\frac{۲\pi}{۳}}) = \frac{۱}{۳}\left(e^{-j\frac{۲\pi}{۳}} + ۲e^{-j\frac{۴\pi}{۳}}\right)\times H(e^{j\frac{۲\pi}{۳}})$

$H(e^{j\omega}) = -\frac{\omega}{\pi} + ۱ \Rightarrow H(e^{j\frac{۲\pi}{۳}}) = -\frac{\frac{۲\pi}{۳}}{\pi} + ۱ = \frac{۱}{۳}$

$\Rightarrow b_1 = \frac{۱}{۹}\left(e^{-j\frac{۲\pi}{۳}} + ۲e^{-j\frac{۴\pi}{۳}}\right)$

Date: .... Subject: ....

a) $x[n] = (n+1)a^n u[n] \qquad |a|<1$ س۸:

راه اول: طبق جدول کتاب

$$X(e^{j\omega}) = \frac{1}{(1-ae^{-j\omega})^2}$$

راه دوم:

$$x[n] = na^n u[n] + a^n u[n]$$

$$X(e^{j\omega}) = \mathcal{F}\{na^n u[n]\} + \mathcal{F}\{a^n u[n]\}$$

$$\mathcal{F}\{a^n u[n]\} = \frac{1}{1-ae^{-j\omega}}$$

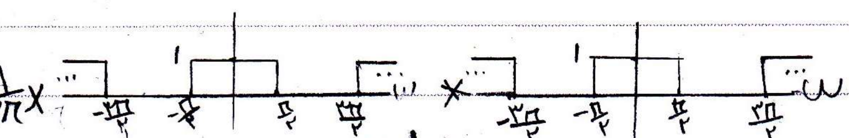
(مشتق فرکانسی)

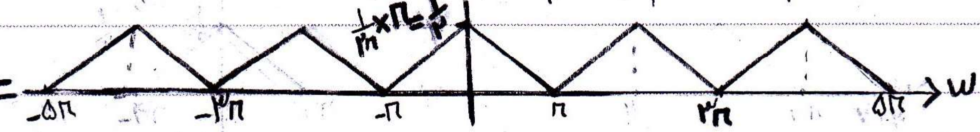
$$\mathcal{F}\{na^n u[n]\} = j\frac{d}{d\omega}\mathcal{F}\{a^n u[n]\} = j\left(\frac{-(+jae^{-j\omega})}{(1-ae^{-j\omega})^2}\right)$$

$$= \frac{ae^{-j\omega}}{(1-ae^{-j\omega})^2} \Rightarrow X(e^{j\omega}) = \frac{ae^{-j\omega}+1-ae^{-j\omega}}{(1-ae^{-j\omega})^2} = \frac{1}{(1-ae^{-j\omega})^2}$$

b) $x[n] = \dfrac{\sin^2\left(\frac{n\pi}{2}\right)}{(n\pi)^2}$

$$X(e^{j\omega}) = \frac{1}{2\pi}\mathcal{F}\left\{\frac{\sin\left(\frac{n\pi}{2}\right)}{n\pi}\right\} * \mathcal{F}\left\{\frac{\sin\left(\frac{n\pi}{2}\right)}{n\pi}\right\}$$

$= \frac{1}{2\pi} \times$ 

$=$ 

Date: .... Subject: ....

س ۹: پارسوال: $E_\infty = \sum_{n=-\infty}^{\infty} |x[n]|^2 = \frac{1}{2\pi}\int_{2\pi} |X(\omega)|^2 d\omega$ ①

$X(\omega)$ زوج و حقیقی است در نتیجه $x[n]$ هم زوج و حقیقی است. $\Leftarrow x[-n]=x[n]$

$$\Rightarrow E_\infty = \sum_{n=-\infty}^{-1} |x[n]|^2 + |x[0]|^2 + \sum_{n=1}^{\infty} |x[n]|^2 = |x[0]|^2 + 2\sum_{n=1}^{\infty} |x[n]|^2 \quad ②$$

$$E_\infty = \frac{1}{2\pi}\int_{-\pi}^{\pi} |X(\omega)|^2 = \frac{1}{2\pi}\left(\int_{-\pi}^{0} \left|-\frac{2}{\pi}\omega\right|^2 d\omega + \int_{0}^{\pi} \left|\frac{2\omega}{\pi}\right|^2 d\omega\right)$$

$$= \frac{4}{\pi^3}\left(\int_0^{\pi} \omega^2 d\omega\right) = \frac{4}{\pi^3}\,\frac{\omega^3}{3}\Big|_0^{\pi} = \frac{4}{\pi^3}\times\frac{\pi^3}{3} = \frac{4}{3}$$

$$x[0] = \frac{1}{2\pi}\int_{2\pi} X(\omega)e^{j\omega\times 0} = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\omega)\,d\omega = \frac{1}{2\pi}\times 2\pi = 1$$

$$①,②:\ \frac{4}{3} = |1|^2 + 2\sum_{n=1}^{\infty} |x[n]|^2 \Rightarrow \sum_{n=1}^{\infty} |x[n]|^2 = \frac{1}{6}$$

س ۱۰: الف)

$$y[n] + \frac{1}{2}y[n-1] = x[n]$$

$$Y(e^{j\omega}) + \frac{1}{2}e^{-j\omega}Y(e^{j\omega}) = X(e^{j\omega})$$

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{1}{1+\frac{1}{2}e^{-j\omega}}$$

ب)

a) $x[n] = (0.5)^n u[n]$

$$X(e^{j\omega}) = \frac{1}{1-0.5e^{-j\omega}} \Rightarrow Y(e^{j\omega}) = X(e^{j\omega})\times H(e^{j\omega}) = \frac{1}{(1-0.5e^{-j\omega})(1+\frac{1}{2}e^{-j\omega})}$$

$$\Rightarrow Y(e^{j\omega}) = \frac{1}{2(1+0.5e^{-j\omega})} + \frac{1}{2(1-0.5e^{-j\omega})}$$

$$\Rightarrow y[n] = \frac{1}{2}(-0.5)^n u[n] + \frac{1}{2}\times(0.5)^n u[n] = \frac{1}{2}u[n]\left(\left(-\frac{1}{2}\right)^n + \left(\frac{1}{2}\right)^n\right)$$

b) $x[n] = (-0.5)^n u[n]$

$$X(e^{j\omega}) = \frac{1}{1+0.5e^{-j\omega}} \Rightarrow Y(e^{j\omega}) = X(e^{j\omega})\times H(e^{j\omega}) = \frac{1}{(1+\frac{1}{2}e^{-j\omega})^2}$$

$$\Rightarrow y[n] = (n+1)(-0.5)^n u[n] = (n+1)\left(-\frac{1}{2}\right)^n u[n]$$

$$C)\ X(e^{j\omega}) = 1 + ۲e^{-۳j\omega}$$

$$Y(e^{j\omega}) = X(e^{j\omega}) \times H(e^{j\omega}) = \frac{1 + ۲e^{-۳j\omega}}{1+\frac{1}{۲}e^{-j\omega}} = \frac{1}{1+\frac{1}{۲}e^{-j\omega}} + \frac{۲e^{-۳j\omega}}{1+\frac{1}{۲}e^{-j\omega}}$$

$$\Rightarrow Y(e^{j\omega}) = \frac{1}{1+\frac{1}{۲}e^{-j\omega}} + \frac{۴e^{-۳j\omega}}{۲+e^{-j\omega}}$$

$$y(t) = (-۰/۵)^n u[n] + ۲(-۰/۵)^{(n-۳)} u[n-۳] = (-\tfrac{1}{۲})^n u[n] + ۲(-\tfrac{1}{۲})^{(n-۳)} u[n-۳]$$