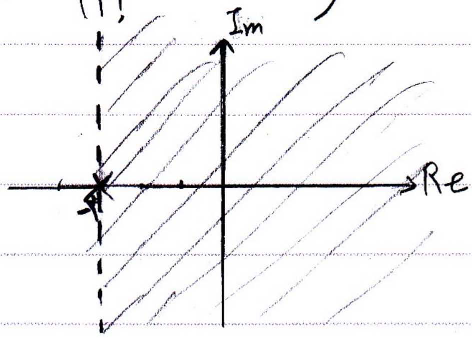
Date: .... **Subject:** سری ۶ تمارین سیگنال و سیستم‌ها

س۱:

a) $x(t) = 3t^2 e^{-3t} u(t)$

طبق جدول: $\frac{t^{n-1}}{(n-1)!} e^{-at} u(t) \xleftrightarrow{\mathcal{L}} \frac{1}{(s+a)^n} \quad Re\{s\} > -a$

$$3\times 2!\times\left(\frac{t^2}{2!} e^{-3t} u(t)\right) \xleftrightarrow{\mathcal{L}} 3\times 2! \times \frac{1}{(s+3)^3} = \frac{6}{(s+3)^3} \quad Re\{s\} > -3$$



صفر: ندارد

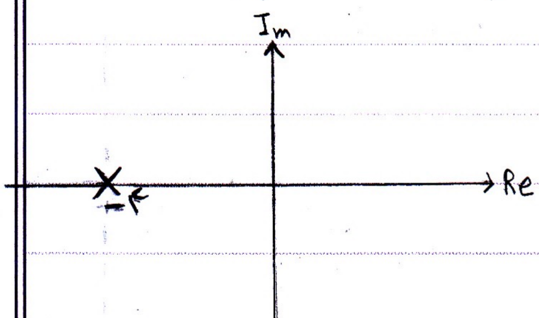
قطب: ۳- با ۳ بار تکرار

b) $x(t) = |t| e^{-4t} = t e^{-4t} u(t) - t e^{-4t} u(-t)$

$$\mathcal{L}\{t e^{-4t} u(t)\} = \frac{1}{(s+4)^2} \quad Re\{s\} > -4$$

$$\mathcal{L}\{-t e^{-4t} u(-t)\} = \frac{1}{(s+4)^2} \quad Re\{s\} < -4$$

$ROC = \emptyset \Rightarrow$ تبدیل لاپلاس موجود نیست



صفر: ندارد

قطب: ۴- با ۲ بار تکرار

c) $x(t) = (t-3) e^{-2t} u(t-3)$

$e^{-2(t-3)} \times e^{-6} (t-3) u(t-3)$

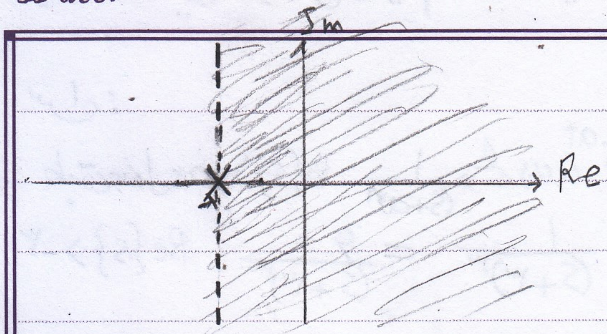
$$\mathcal{L}\{t e^{-2t} u(t)\} = \frac{1}{(s+2)^2} \quad Re\{s\} > -2$$

انتقال زمانی

$$\mathcal{L}\{(t-3) e^{-2(t-3)} u(t-3)\} = e^{-3s} \frac{1}{(s+2)^2} \quad Re\{s\} > -2$$

$$\Rightarrow X(s) = \frac{e^{-6} \times e^{-3s}}{(s+2)^2} = \frac{e^{-3(s+2)}}{(s+2)^2} \quad Re\{s\} > -2$$

Date: Subject:

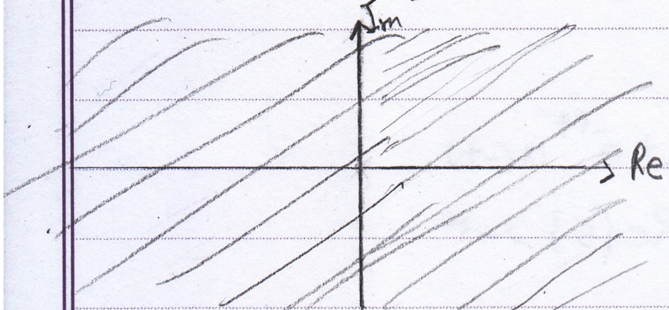


صفر: $+\infty$

قطب: -2 ، ۲ بار تکرار

d) $x(t) = \begin{cases} 1 & 0 \le t \le 1 \\ 0 & \text{o.w} \end{cases} = u(t) - u(t-1)$

$$\mathcal{L}\{x(t)\} = \frac{1}{s} - e^{-s}\frac{1}{s} = \frac{1}{s}(1-e^{-s})$$



صفر: ندارد

قطب: ندارد $\lim_{s\to 0} \frac{1-e^{-s}}{s} \overset{H}{=} \lim_{s\to 0} \frac{e^{-s}}{1} = 1$

س۲:

a) $X(s) = \frac{s}{s^2+9} \quad Re\{s\} > 0$

$$x(t) = \cos(3t)\, u(t)$$

b) $X(s) = \frac{s+2}{s^2+7s+12} \quad -4 < Re\{s\} < -3$

$$\frac{s+2}{s^2+7s+12} = \frac{s+2}{(s+4)(s+3)} = \frac{A}{s+4} + \frac{B}{s+3} \Rightarrow As+3A+Bs+4B = s+2$$

$A+B=1$, $3A+4B=2$

$A=+2$, $B=-1$

$$= \frac{2}{s+4} - \frac{1}{s+3}$$

$$\mathcal{L}^{-1}\left\{\frac{2}{s+4},\ Re\{s\} > -4\right\} = 2e^{-4t}u(t)$$

$$\mathcal{L}^{-1}\left\{\frac{-1}{s+3},\ Re\{s\} < -3\right\} = +e^{-3t}u(-t)$$

$$\Rightarrow x(t) = 2e^{-4t}u(t) + e^{-3t}u(-t)$$

Date: Subject:

c) $X(s) = \dfrac{s-1}{(s+2)(s+3)(s^2+s+1)}$

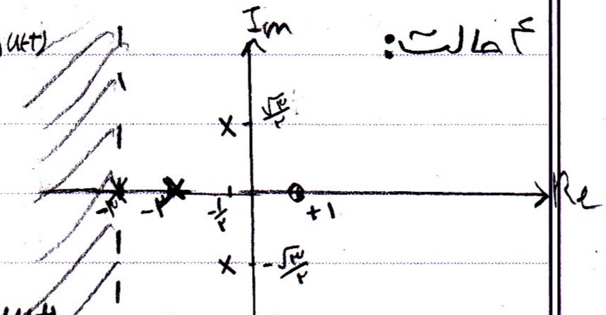
$s^2+s+1=0 \Rightarrow s = \dfrac{-1 \pm \sqrt{-3}}{2} = \dfrac{-1 \pm \sqrt{3}\,j}{2}$

$X(s) = \dfrac{3s+1}{7(s^2+s+1)} - \dfrac{1}{s+2} + \dfrac{4}{7(s+3)} =$

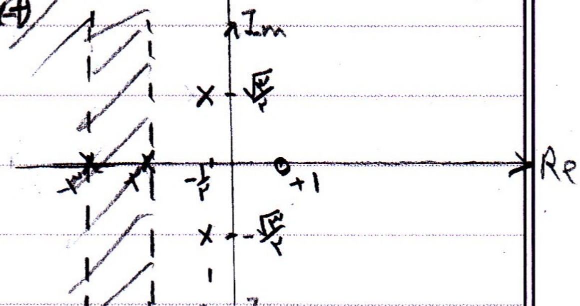
$= \dfrac{3}{7}\dfrac{s}{(s+\frac{1}{2})^2+(\frac{\sqrt{3}}{2})^2} + \dfrac{1}{7\left((s+\frac{1}{2})^2+(\frac{\sqrt{3}}{2})^2\right)} - \dfrac{1}{s+2} + \dfrac{4}{7(s+3)}$

۴ حالت:

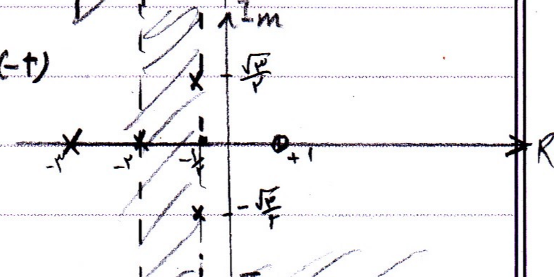
① $x(t) = -\frac{3}{7} e^{-\frac{t}{2}} \cos(\frac{\sqrt{3}}{2}t)\, u(-t) + \frac{e^{-\frac{t}{2}}}{7\sqrt{3}} \sin(\frac{\sqrt{3}}{2}t)\, u(-t)$
$+ e^{-2t} u(-t) - \frac{4}{7} e^{-3t} u(-t)$



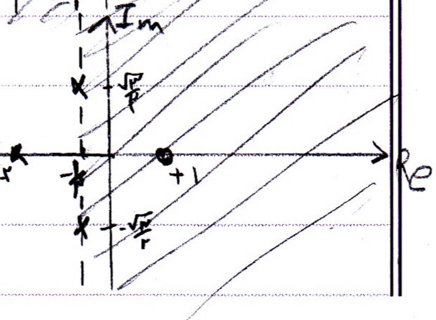
② $x(t) = -\frac{3}{7} e^{-\frac{t}{2}} \cos(\frac{\sqrt{3}}{2}t)\, u(-t) + \frac{e^{-\frac{t}{2}}}{7\sqrt{3}} \sin(\frac{\sqrt{3}}{2}t)\, u(-t)$
$+ e^{-2t} u(-t) + \frac{4}{7} e^{-3t} u(-t)$



③ $x(t) = -\frac{3}{7} e^{-\frac{t}{2}} \cos(\frac{\sqrt{3}}{2}t)\, u(-t) + \frac{e^{-\frac{t}{2}}}{7\sqrt{3}} \sin(\frac{\sqrt{3}}{2}t)\, u(-t)$
$- e^{-2t} u(t) + \frac{4}{7} e^{-3t} u(-t)$



④ $x(t) = \frac{3}{7} e^{-\frac{t}{2}} \cos(\frac{\sqrt{3}}{2}t)\, u(t) - \frac{e^{-\frac{t}{2}}}{7\sqrt{3}} \sin(\frac{\sqrt{3}}{2}t)\, u(t)$
$- e^{-2t} u(t) + \frac{4}{7} e^{-3t} u(t)$



سوال ۳:

$$\mathcal{L}\{x(t)\} = \mathcal{L}\{y(t)\}$$

$$x(t) = e^{-۵t} u(t-1) = e^{-۵(t-1)} \times e^{-۵} \times u(t-1)$$

$$\mathcal{L}\{x(t)\} = e^{-۵} \mathcal{L}\{e^{-۵(t-1)} u(t-1)\} = e^{-۵} \times \frac{1}{s+۵} \times e^{-s} = \frac{e^{-(s+۵)}}{s+۵} \quad Re\{s\} > -۵$$

$$\mathcal{L}\{y(t)\} = \mathcal{L}\{A e^{-۵t} u(-t-t_0)\} = \frac{e^{-(s+۵)}}{s+۵} \quad Re\{s\} < -۵$$

$$\Longrightarrow A = -1 \;,\; t_0 = -1$$

سوال ۴: الف)

$$x(t) = e^{-۴t} u(t) - t e^{-۴t} u(t)$$

$$X(s) = \frac{1}{s+۴} - \frac{1}{(s+۴)^۲} = \frac{s+۳}{(s+۴)^۲} \quad Re\{s\} > -۴$$

$$y(t) = t e^{-۴t} u(t)$$

$$Y(s) = \frac{1}{(s+۴)^۲} \quad Re\{s\} > -۴$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{\frac{1}{(s+۴)^۲}}{\frac{s+۳}{(s+۴)^۲}} = \frac{1}{(s+۳)} \quad Re\{s\} > -۳$$

$$\Longrightarrow h(t) = e^{-۳t} u(t)$$

ب)

$$x_1(t) = e^{-۲t} u(t) \qquad X(s) = \frac{1}{s+۲} \quad Re\{s\} > -۲$$

$$Y(s) = X(s) H(s) = \frac{1}{s+۲} \times \frac{1}{s+۳} = \frac{1}{s+۲} - \frac{1}{s+۳} \quad Re\{s\} > -۲$$

$$y(t) = e^{-۲t} u(t) - e^{-۳t} u(t)$$

ج)

$$x_۲(t) = e^{۲t} \xrightarrow{\text{لاپلاس و فوریه ندارد}}$$

$$y(t) = x_۲(t) * h(t) = \int_{-\infty}^{+\infty} h(\tau) x_۲(t-\tau) d\tau = \int_{-\infty}^{\infty} e^{-۳\tau} u(\tau) e^{۲(t-\tau)} d\tau$$

$$= \int_0^{\infty} e^{-۵\tau} \times e^{۲t} d\tau = e^{۲t} \times -\frac{1}{۵} e^{-۵\tau} \Big|_0^{\infty} = \frac{e^{۲t}}{۵}$$

د)

$$H^{-1}(s) = \frac{1}{\frac{1}{s+۳}} = s+۳ \qquad ROC = \text{all } s\text{-plane}$$

س۵ :

$$\frac{dx(t)}{dt} = -2y(t) + \delta(t) \xrightarrow{\mathcal{L}} sX(s) = -2Y(s) + 1$$

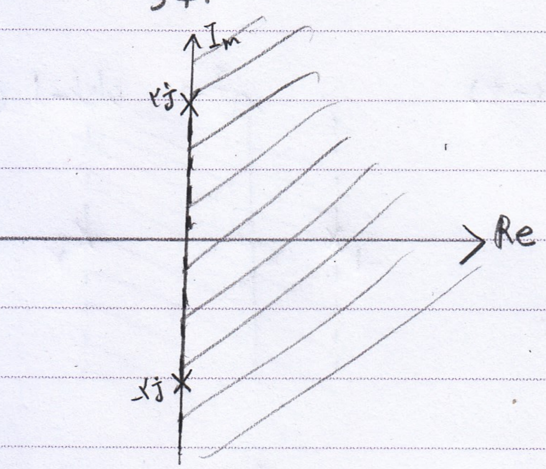
$$\frac{d}{dt}y(t) = 2x(t) \xrightarrow{\mathcal{L}} sY(s) = 2X(s)$$

$$\Rightarrow X(s) = \frac{sY(s)}{2} \Rightarrow \frac{s^2Y(s)}{2} = -2Y(s) + 1 \Rightarrow \frac{s^2+4}{2} = \frac{1}{Y(s)}$$

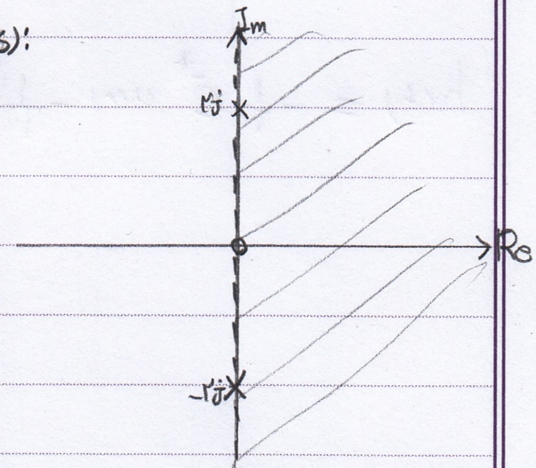
$$\Rightarrow Y(s) = \frac{2}{s^2+4} \quad Re\{s\} > 0$$

$$\Rightarrow X(s) = \frac{s}{s^2+4} \quad Re\{s\} > 0$$

Y(s):



X(s):



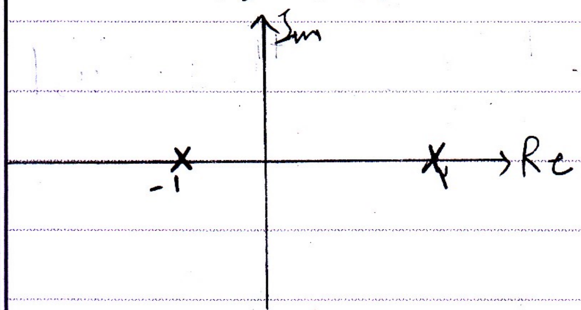
Date: .... Subject: ....

سؤال ۶: الف)

$$\frac{d^2}{dt^2}y(t)-\frac{d}{dt}y(t)-2y(t)=x(t)$$

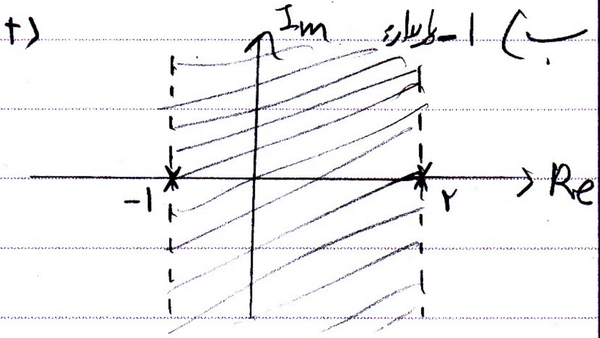
$$s^2Y(s)-sY(s)-2Y(s)=X(s)$$

$$H(s)=\frac{1}{s^2-s-2}=\frac{1}{(s-2)(s+1)}=\frac{1}{3(s-2)}-\frac{1}{3(s+1)}$$



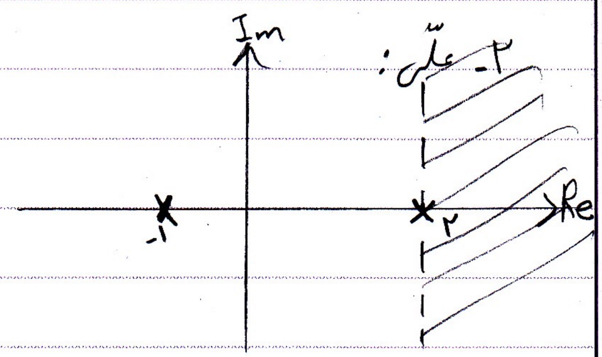
ب) ۱- پایدار:

$$h(t)=-\frac{1}{3}e^{-t}u(t)-\frac{1}{3}e^{2t}u(-t)$$



۲- علّی:

$$h(t)=-\frac{1}{3}e^{-t}u(t)+\frac{1}{3}e^{2t}u(t)$$



۳- نه علی و نه پایدار:

$$h(t)=\frac{1}{3}e^{-t}u(-t)-\frac{1}{3}e^{2t}u(-t)$$