FRIEDRICH-ALEXANDER-UNIVERSITÄT ERLANGEN-NÜRNBERG
SCHOOL OF ENGINEERING

Deep Reinforcement Learning

A. Maier, V. Christlein, K. Breininger, S. Vesal, F. Meister, C. Liu, S. Gündel, S. Jaganathan, N. Maul, M. Vornehm, L. Reeb, F. Thamm, M. Hoffmann, C. Bergler, F. Denzinger, W. Fu, B. Geissler, Z. Yang
Pattern Recognition Lab, Friedrich-Alexander-Universität Erlangen-Nürnberg
June 13, 2020

Outline

Sequential Decision Making

Reinforcement Learning

- Markov Decision Processes
- Policy Iteration
- Other Solution Methods

Deep Reinforcement Learning

- Deep Q Learning
- AlphaGo
- AlphaGo Zero

Sequential Decision Making

Sequential decision making: Multi-armed bandit problem

Action Formalize choosing a machine as **action** a at time t from a set A

Reward Action a_t has a **different**[1] **unknown pdf** $p(r|a)$ generating **reward** r_t

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Policy Formalize choosing an action a as pdf $\pi(a)$ which we call a **policy**

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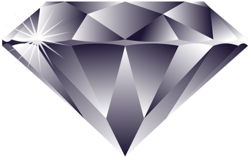
Evaluative Feedback



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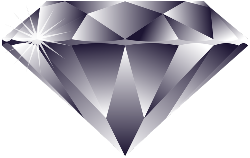

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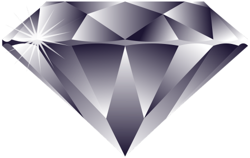

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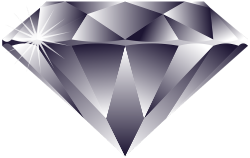

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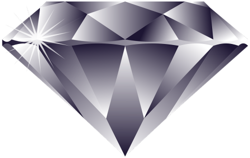

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- We call $Q_t(\mathbf{a})$ the action-value function, which changes with every new information

Incremental update of $Q_t(\mathbf{a})$

$$
\begin{aligned}
Q_{t+1}(\mathbf{a}) &= \frac{1}{t}\sum_{i=1}^{t}\mathbf{r}_i \\
&= \frac{1}{t}\left(\mathbf{r}_t + \sum_{i=1}^{t-1}\mathbf{r}_i\right) \\
&= \frac{1}{t}\left(\mathbf{r}_t + (t-1)\frac{1}{t-1}\sum_{i=1}^{t-1}\mathbf{r}_i\right) \\
&= \frac{1}{t}\left(\mathbf{r}_t + (t-1)Q_t(\mathbf{a})\right) \\
&= \frac{1}{t}\left(\mathbf{r}_t + t\,Q_t(\mathbf{a}) - Q_t(\mathbf{a})\right) \\
&= Q_t(\mathbf{a}) + \frac{1}{t}\left(\mathbf{r}_t - Q_t(\mathbf{a})\right)
\end{aligned}
$$

Exploitation

- Reward is maximized by a policy $\pi(a)$ choosing $\max_a Q_t(a)$
- We exploit a known good action
- This is a **deterministic**[1] policy called **greedy action selection**

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→ Sometimes explore by selecting other moves which could potentially be better

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Epsilon Greedy

$$\pi(a) = \begin{cases} 1 - \epsilon & \text{if } a = \max_a Q_t(a) \\ \epsilon/(n-1) & \text{else} \end{cases}$$

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Softmax

$$\pi(a) = \frac{e^{Q_t(a)/\tau_t}}{\sum_{n=1}^{|A|} e^{Q_t(a_n)/\tau_t}}$$

- τ_t is called **temperature** and used to decrease exploration over time

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- learned that **exploration** of different actions is necessary
- assumed rewards **didn't depend** on a **state** of the world
- and our action at time t **doesn't influence** the **rewards** from a at $t+1$

NEXT TIME

ON DEEP LEARNING

Deep Reinforcement Learning - Part 2

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Reinforcement Learning

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- Rewards now additionally **depend on** the **state** s_t:

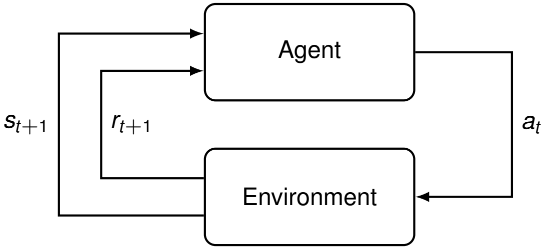
$$p(r_t | s_t, a_t)$$

- However this setting is known as **contextual bandit**
- In the full **reinforcement learning problem**, actions influence the state:

$$p(s_{t+1} | s_t, a_t)$$

Markov Decision Processes

Markov Decision Process



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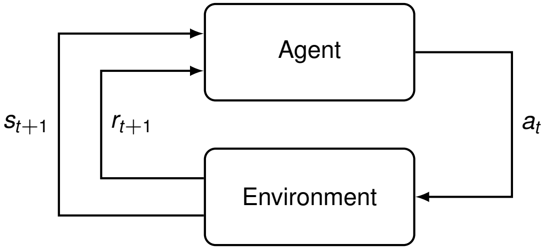
Markov Decision Process



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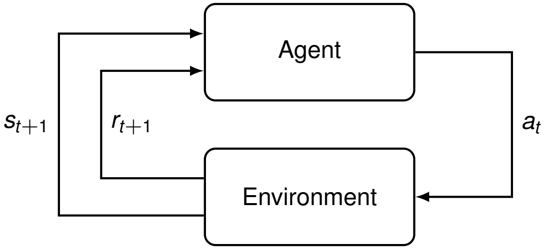

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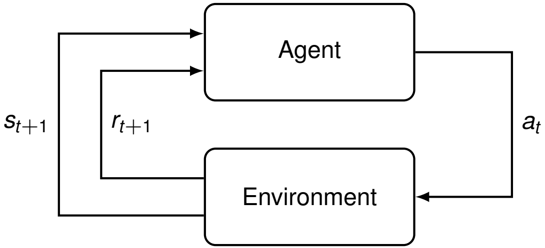

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Policy	Agents choose actions a_t by a policy $\pi(a\|s)$

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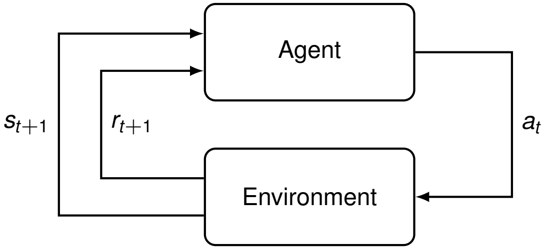

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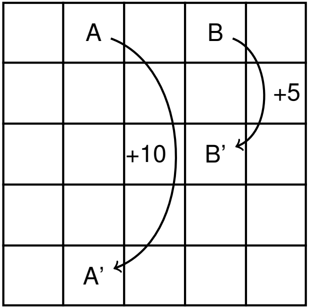
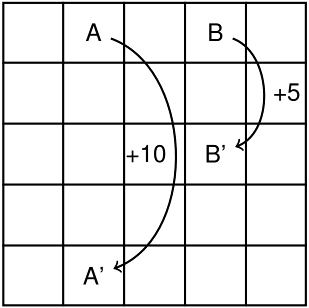
Policy Agents choose actions a_t by a policy $\pi(a|s)$

If all those sets are **finite** we call this a **finite MDP**



- ➔ Here s is the field we are currently on.
- The agent can **move** in all four directions
- Any action which would leave the grid has $p(s_{t+1}|a_t, s_t)$ equal to a δ distribution on $s_{t+1} = s_t$ and a similarly deterministic $r_t = -1$



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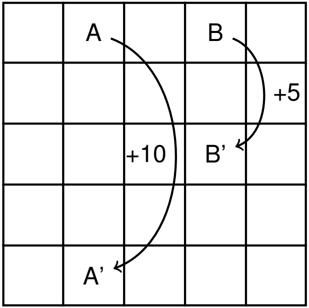

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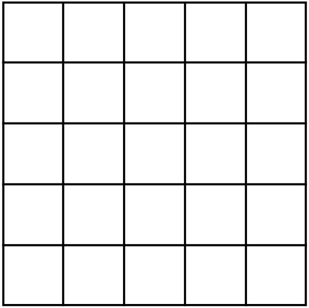
- Every state we reach other than tile A' and B' deterministically causes $r_t = 0$
- On A or B **any** action will take us to A' or B' respectively

Example policy



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- We can extend the **uniform random policy** to be independent from s_t

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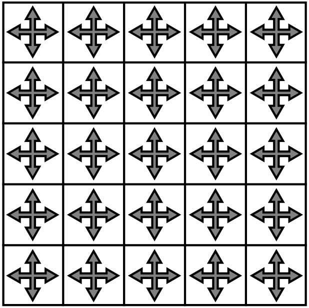

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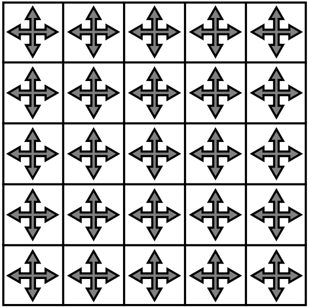

- Policies now depend on s_t
- We can extend the **uniform random policy** to be independent from s_t
- However there's no reason to believe that this policy is any good
- How can we **estimate good policies**?

What is a good policy?

→ We have to be precise about good

- Preliminary we have to state **two kinds** of tasks
    1. **Episodic** tasks which have an **end**
    2. **Continuing** tasks which are **infinitely** long
- **Unify them** using a **terminal state** in **episodic tasks** which only transition to themselves with deterministic $r_t = 0$
- Goal is to **maximize the future return**

$$\max_{\pi(s_t, a_t)} g_t = \sum_{k=t+1}^{T} \gamma^{k-t-1} r_k$$

- γ is a **discount** reducing influence of rewards **far** in the future
- $\gamma \in (0, 1]$ meaning that $\gamma = 1$ is allowed as long as $T \neq \infty$

NEXT TIME

ON DEEP LEARNING

Deep Reinforcement Learning - Part 3

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Policy Iteration

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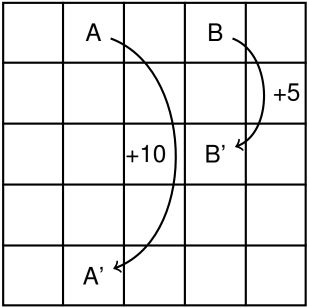
- We introduce the **state-value function** $V_\pi(s)$

$$V_\pi(s) = \mathbb{E}_\pi\left[g_t | s_t\right] = \mathbb{E}_\pi\left[\sum_{k=t+1}^{T} \gamma^{k-t-1} r_k | s_t\right]$$

State-value Function Example



The definition of the gridworld

- Recall our grid example

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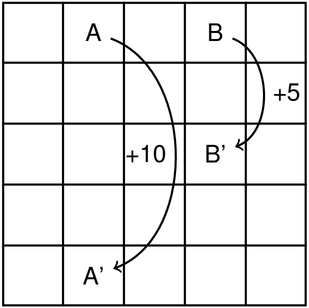

The definition of the gridworld

3.3	8.8	4.4	5.3	1.5
1.5	3.0	2.3	1.9	0.5
0.1	0.7	0.7	0.4	-0.4
-1.0	-0.4	-0.4	-0.6	-1.2
-1.9	-1.3	-1.2	-1.4	-2.0

$V_{\pi}(s)$ for the uniform random policy

- Recall our grid example
- Some **edge tiles** are negative since the policy **can't control the move**

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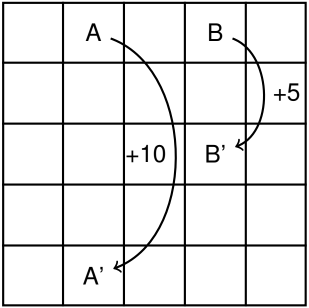

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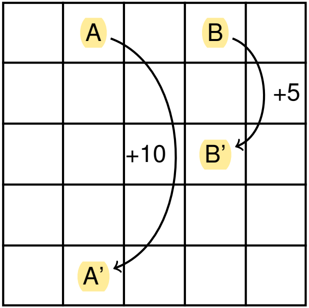

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$V_\pi(s)$ for the uniform random policy

- Recall our grid example
- Some **edge tiles** are negative since the policy **can't control the move**
- What if we use the **greedy action selection** policy on this $V_\pi(s)$?
- We get a **better policy**!

Action-value function

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Action-value function

- Before we used the action-value function $Q(a)$
- Now we introduced $V_\pi(s)$ filling a similar role
- We can also introduce the **action-value function** $Q_\pi(s,a)$
- Basically this accounts for the **transition probabilities**

$$Q_\pi(s,a) = \mathbb{E}_\pi\left[g_t | s_t, a_t\right] = \mathbb{E}_\pi\left[\sum_{k=t+1}^{T} \gamma^{k-t-1} r_k | s_t, a_t\right]$$

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[1] in a finite MDP

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- $Q^*(s,a)$ can also be defined and is related to $V^*(s_t)$ by:

$$Q^*(s,a) = \mathbb{E}\left[r_{t+1} + \gamma V^*(s_{t+1})\right] \tag{2}$$

[1] in a finite MDP

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$V_\pi(s)$ for the uniform random policy

22.0	24.4	22.0	19.4	17.5
19.8	22.0	19.8	17.8	16.0
17.8	19.8	17.8	16.0	14.4
16.0	17.8	16.0	14.4	13.0
14.4	16.0	14.4	13.0	11.7

V^*

- Observe that V^* is strictly positive since it's deterministic

Optimal Policies

- Policies can now be ordered: $\pi \geq \pi'$ if and only if $V_\pi(s) \geq V_{\pi'}(s), \forall s \in S$

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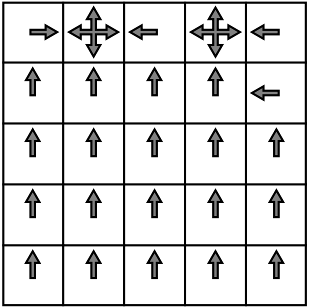
- Given either V^* or Q^* an optimal policy is directly obtained by **greedy action selection**

Greedy Action Selection on $V^*(s)$ or $Q^*(s, a)$



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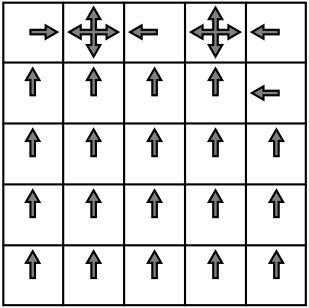

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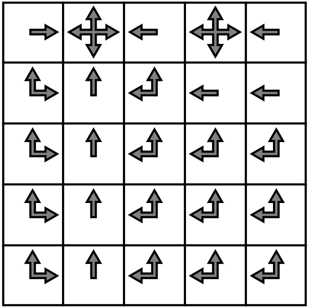

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A Tool to Compute Optimal Value-functions

- We **still need** to compute $V^*(s)$ and $Q^*(s, a)$

A Tool to Compute Optimal Value-functions

- We **still need** to compute $V^*(s)$ and $Q^*(s, a)$
- For this the **Bellman equations** can be utilized
- They are **consistency conditions** for the value functions

Bellman equation for $V_\pi(s)$

$$V_\pi(s) = \sum_a \pi(a|s) \sum_{s_{t+1},r} p(s_{t+1}, r|s, a) \left[r + \gamma V_\pi(s_{t+1})\right]$$

Policy Evaluation

- The Bellman equations form a **system of linear equations** which can be solved for **small** problems
- Better: **Iteratively solve**, by turning the Bellman equations into **update rules**:

$$V_{k+1}(s) = \sum_{a} \pi(a|s) \sum_{s_{t+1},r} p(s_{t+1}, r|s, a) \left[r + \gamma V_k(s_{t+1})\right]$$

For all $s \in S$

Policy Improvement

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- But is this **guaranteed** to work?

Policy Improvement Theorem

- We consider changing a single action a_t in state s_t but following π

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- So iteratively updating $V_\pi(s)$ and using **greedy action selection** is guaranteed to work here
- We terminate if the policy no longer changes
- Last remark: If we don't loop over all $s \in S$ for policy evaluation, but update the policy directly this algorithm is called **Value iteration**

NEXT TIME

ON DEEP LEARNING

Deep Reinforcement Learning - Part 4

A. Maier, V. Christlein, K. Breininger, S. Vesal, F. Meister, C. Liu, S. Gündel, S. Jaganathan, N. Maul, M. Vornehm, L. Reeb, F. Thamm, M. Hoffmann, C. Bergler, F. Denzinger, W. Fu, B. Geissler, Z. Yang
Pattern Recognition Lab, Friedrich-Alexander-Universität Erlangen-Nürnberg
June 13, 2020

Other Solution Methods

Limitations

- Both policy iteration and value iteration **require** using the **updated policies** during learning to obtain better approximations to $V^*(s)$

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- For this reason we call them **on-policy** algorithms
- Additionally we assumed the **state-transition** pdf and **reward** pdf are known
- Can we relax this?
- Yes. The methods differ mostly **how they perform policy evaluation**

Monte Carlo Techniques

Properties

- Only for **episodic** tasks
- Off-policy - learns $V^*(s)$ by following any **arbitrary** $\pi(s, a)$
- Does **not need** information about dynamics of the environment

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Scheme

- **Generate** an **episode** by using some policy
- Loop **backwards** over the episode accumulating the expected future reward $g_t = g_{t+1} + r_{t+1}$
- If a state was **not yet** visited append g_t to a list $returns(s_t)$
- Update $V_{s_t} = \frac{1}{N} \sum_{n=1}^{N} returns_n(s_t)$

Temporal Difference Learning

Properties

- On-policy
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Scheme

- Loop and follow $\pi(s_t, a_t)$
- Use *a* from $\pi(s_t, a_t)$, observe r_t, s_{t+1}
- Update: $V_{t+1}(s) = V_t(s) + \alpha \left[r_t + \gamma V_t(s_{t+1}) - V_t(s_t) \right]$

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- **Converges to the optimal solution**
- A variant of this estimates $Q_{(s,a)}$ and is known as SARSA

Q Learning

Properties

- Off-policy
- Temporal difference type of method
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Q Learning

Properties

- Off-policy
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Scheme

- Loop and follow $\pi(s_t, a_t)$ derived from $Q_t(s, a)$ e.g. ϵ-greedy
- Use a from $\pi(s_t, a_t)$, observe r_t, s_{t+1}
- Update: $Q_{t+1}(s, a) = Q_t(s_t, a_t) + \alpha \left[r_t + \gamma \max_a Q_t(s_{t+1}, a_t) - Q_t(s_t, a_t) \right]$

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- What about just parametrizing $\pi(s_t, a_t, \mathbf{w})$ by weights $\mathbf{w}$ and use some loss-function L?

➔ Known as **policy gradient** and this instance is called REINFORCE

- Generate an episode using $\pi(s_t, a_t, \mathbf{w})$
- Go forwards in the episode: $t = 0, \dots, T - 1$
- $\mathbf{w} = \mathbf{w} + \eta\gamma^t g_t \nabla_{\mathbf{w}} \ln\left(\pi(a_t|s_t, \mathbf{w})\right)$

NEXT TIME

ON DEEP LEARNING

Deep Reinforcement Learning - Part 5

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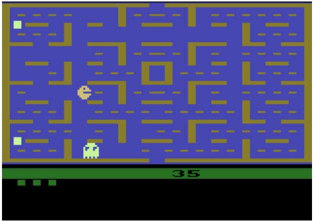
Deep Reinforcement Learning

Deep Q Learning

Atari Games: Human-level control through deep reinforcement learning [4]

- Volodymyr Mnih et al. (Google DeepMind) 2013/2015
- Idea: Let a neural network play Atari games!
- Input: Current and three subsequent video frames from game
- Processed by network trained with reinforcement learning
- Goal: learn best controller movements



Atari Pac-Man

Source: Human-level control through deep reinforcement learning [4]

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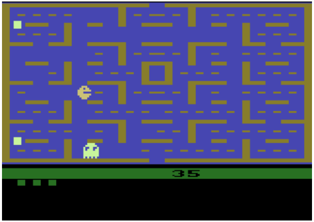
- Convolutional layers for frame processing, fully-connected for final decision making



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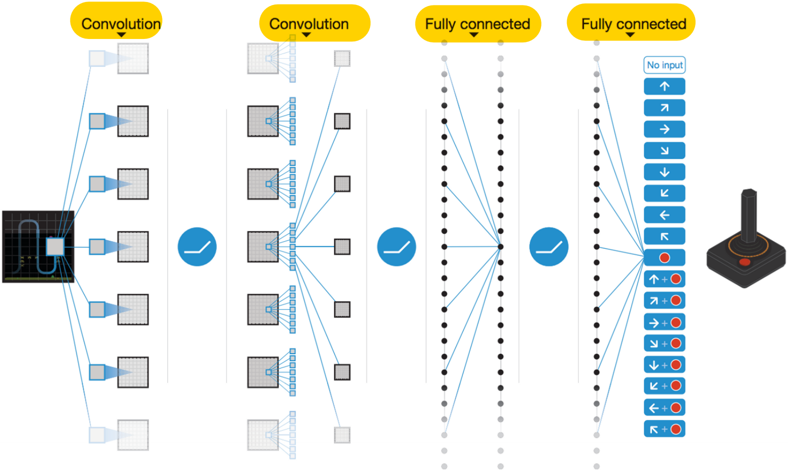
Learning Atari Games



Source: Human-level control through deep reinforcement learning [4]

Learning Atari Games

- **Deep Q-network**: Deep network that applies Q-learning
- State s_t of the game: current + 3 previous frames (image stack)
- 18 outputs associated with an action

→ Each output estimates optimal action value for "its" action given the input

- Instead of label & cost function, update to maximize reward
- Reward: +1/-1 when game score increased/decreased, 0 otherwise
- ϵ-greedy policy with ϵ decreasing to a low value during training
- Semi-gradient form of Q-learning to update network weights **w**
- Uses mini-batches to accumulate weight updates

Target Network

- Weight update:

$$\mathbf{w}_{t+1} = \mathbf{w}_t + \alpha \left[r_{t+1} + \gamma \max_a \hat{q}(s_{t+1}, a, \mathbf{w}_t) - \hat{q}(s_t, a_t, \mathbf{w}_t) \right] \cdot \nabla \mathbf{w}_t \hat{q}(s_t, a_t, \mathbf{w}_t)$$

- Problem: The target $\gamma \max_a \hat{q}(s_{t+1}, a, \mathbf{w}_t)$ is a function of $\mathbf{w}_t$.

➔ Target changes simultaneously with the weights we want to learn!

➔ Training can oscillate or diverge

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➔ Target changes simultaneously with the weights we want to learn!

➔ Training can oscillate or diverge

- Idea: Use a second **target network**:
- After each C steps, copy weights of action-value network to a duplicate network and keep them fixed
- Use output $\bar{q}$ of “target network” as a target to stabilize:

$$\gamma \max_a \bar{q}(s_{t+1}, a, \mathbf{w}_t)$$

Experience Replay

Goal: Reduce correlation between updates

- After performing action a_t for image stack s_t (state) and receiving reward r_t, add $\left(s_t, a_t, r_t, s_{t+1}\right)$ to **replay memory**

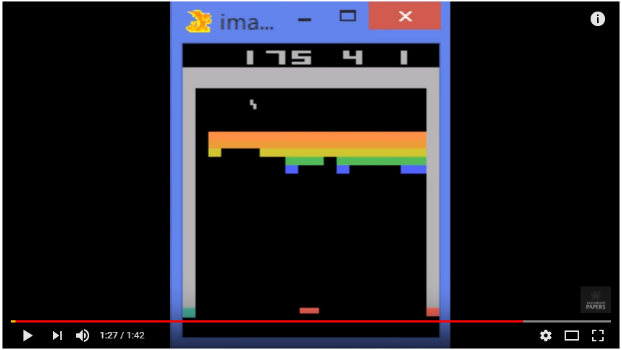
➔ Memory accumulates experiences

- To update the network, draw random samples from memory, instead of taking the most recent ones

➔ Removes dependence on current weights

➔ Increases stability

Atari Breakout Example



Video on learning Atari Breakout. Click here

AlphaGo

Mastering the game of Go with deep neural networks and tree search [1]

- Go is an ancient Chinese boardgame: Black plays against white for control over the board
- Simple rules but extremely high number of possible moves and situations
- Performance on par with professional human players thought years away

Traditional Go board

Source: https://commons.wikimedia.org/wiki/File:FloorGoban.jpg

Challenges in Go

- Go is a “perfect information” game: No hidden information and no chance
- Theoretically, we can construct a full game tree and traverse it with Minimax to find the best moves

Challenges in Go

- Go is a “perfect information” game: No hidden information and no chance
- Theoretically, we can construct a full game tree and traverse it with Minimax to find the best moves
- Problem: High number of legal moves ($\approx$ 250 – chess $\approx$ 35)
- Games involve many moves ($\approx$ 150)

➔ Exhaustive search is infeasible!

Challenges in Go (cont.)

- Search tree can be **pruned** if we have an accurate evaluation function
- For chess (DeepBlue) already extremely complex and based on massive human input
- For Go: "No simple yet reasonable evaluation function will ever be found for Go." (Müller 2002) [5]

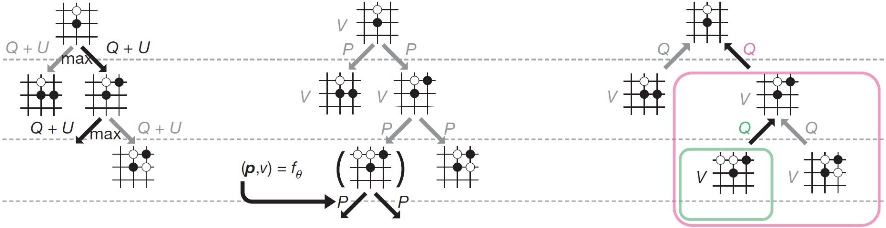
Challenges in Go (cont.)

- Search tree can be **pruned** if we have an accurate evaluation function
- For chess (DeepBlue) already extremely complex and based on massive human input
- For Go: “No simple yet reasonable evaluation function will ever be found for Go.” (Müller 2002) [5]
- Still: **AlphaGo beat Lee Sedol and Ke Jie, two of the world’s strongest players in 2016 and 2017!**

Mastering the game of Go with deep neural networks and tree search [1]

- AlphaGo was developed by Silver et al. (also Google DeepMind)
- Combination of multiple methods:
  - Deep neural networks
  - Monte Carlo tree search (MCTS)
  - Supervised learning **and**
  - Reinforcement learning
- First improvement compared to a full tree search: Monte Carlo Tree Search (MCTS)
- Networks to support efficient search through tree

Monte Carlo Tree Search



- Idea: Run many Monte Carlo simulations of episodes (=entire Go games) to select action (=where to place a stone)
- Starting from a root node representing the current state, MCTS iteratively extends the search tree

Source: Mastering the game of go without human knowledge [2]

Monte Carlo Tree Search (cont.)

Algorithm:

- **Selection**: Starting at root, traverse with tree policy to a leaf node
- **Expansion**: (Optional) add one or more child nodes to the current leaf
- **Simulation**: From the current or the child node, simulate episode with actions according to rollout policy
- **Backup**: Propagate the received reward back through the tree

- Repeat for a certain amount of time, then stop
- Then, choose action from root node according to accumulated statistics
- Start again with new root node

Monte Carlo Tree Search (cont.)

- Tree policy guides in how far successful paths are frequented more often.
- Typical exploration/exploitation trade-off.
- Problem: Estimation via MCTS not accurate enough for Go.

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- Ideas in AlphaGo:
  - Control tree expansion by using a neural network to find promising actions.
  - Improve value estimation by a neural network.
- More efficient extension & evaluation of search tree → better at Go!

Deep Neural Networks for Go

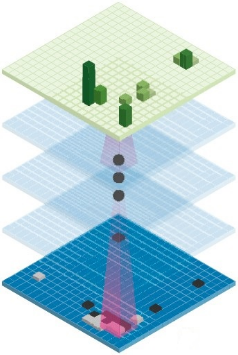
Utilization of three different networks:

- **Policy network**: Suggests the next move in leaf nodes for extension
- **Value network**: Given the current board position, get chances of winning
- **Rollout policy network**: Guide rollout action selection
- All networks are deep convolutional networks
- Input: Current board position and additional precomputed features

Source: Mastering the game of Go with deep neural networks and tree search [1]

Policy Network

- 13 conv-layers, one output for each point on the Go board.
- Huge database of expert human moves (30 mio) available.
- Start with **supervised** learning: Train network to predict the next move in **human expert plays**
- Further train network with reinforcement learning by playing against **older versions of itself**. Reward when winning the game
- Older versions avoid correlation and instability
- Training time: 3 weeks on 50 GPUs + 1 day for RL



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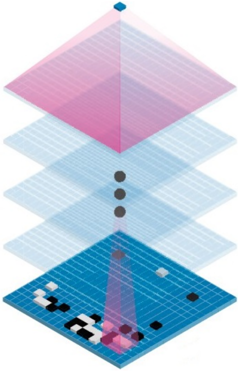
Value network

- Same architecture as policy network but just one output node
- Goal: Estimate how likely the current state leads to a win
- Training utilized self-play games of reinforcement learned policy

→ Trained using Monte-Carlo policy evaluation for 30 mio positions from these games

- Training time: 1 week on 50 GPUs



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Rollout policy network

- AlphaGo could use policy network to select moves during roll-out
- Problem: Inference comparatively high: 5 ms
- Solution: Train simpler, linear network on subset of data that provides actions **fast**
- Speedup of $\approx$ 1000 compared to policy network ➔ more simulations possible

AlphaGo Zero

AlphaGo Zero: Do we even need humans for training?

- After minor improvements, Silver et al. proposed AlphaGo Zero:

→ **Solely** trained with reinforcement learning & playing against itself!

- Simpler MCTS, no rollout policy
- Include MCTS in self-play games
- Multi-task training: Policy and value network share initial layers
- Further extension in Dec. ’17: AlphaZero [3] – able to also play chess and shogi

NEXT TIME

ON DEEP LEARNING

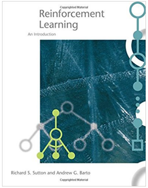
Next Time

- Algorithms to learn if we **don’t even observe rewards**
- How to benefit from **adversaries**
- Extensions to perform **image processing** tasks

Comprehensive Questions

- What is a policy?
- What are value functions?
- Explain the exploitation vs exploration dilemma.
- Describe typical solutions to the dilemma.
- What is the difference of a multi armed bandit problem to the full reinforcement learning problem?
- Describe a Markov decision process.
- Is an optimal policy necessarily unique?
- What do the Bellman equations represent?
- Describe policy iteration.
- Why does policy iteration work?
- How can you beat your friends in every Atari game?
- How can one master the game of Go?

Further Reading



Reinforcement Learning

Richard Sutton

- Link - the one real reference for Reinforcement learning in its 2018 draft, including Deep Q learning and Alpha Go details

References

References I

[1] David Silver, Aja Huang, Chris J Maddison, et al. "Mastering the game of Go with deep neural networks and tree search". In: Nature 529.7587 (2016), pp. 484–489.

[2] David Silver, Julian Schrittwieser, Karen Simonyan, et al. "Mastering the game of go without human knowledge". In: Nature 550.7676 (2017), p. 354.

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[4] Volodymyr Mnih, Koray Kavukcuoglu, David Silver, et al. "Human-level control through deep reinforcement learning". In: Nature 518.7540 (2015), pp. 529–533.

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