2020 OPhO Invitational Contest
Answers Submission

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Solution 1:

(a) let x distance from right side of till mass not goes on the tablecloth. the tablecloth moves with constant speed so $t = \frac{D-x}{v}$ The acceleration of body on the table cloth is $\mu_k g$ so it's gonna gain velocity u that $u = \frac{\mu_k g(D-x)}{v}$. And distance mass went is $\frac{D}{2} - x = \frac{\mu_k g(D-x)^2}{2v^2}$. and it must stop till the side so $u^2 = 2\mu_k g x$. combining equations, substituting one unknown from another yields

$$v \geq 3\sqrt{\frac{gD\mu_k}{8}}$$

(b) The time will be $t = \frac{7-x}{v}$. distance mass went $0.5 - x = \frac{\mu_k g(7-x)^2}{2v^2}$, velocity of mass $u = \mu_k g\frac{7-x}{v}$. And the energy $u^2 = 2\mu_k g x$. from here we find $x = 0.25$ and

$$v \geq \sqrt{91.125\mu_k g} \approx 16.37$$

(c)

(d)

Solution 2:

(a) Let P is power output of the sun. The power reaches to sail is $P\frac{A}{4\pi r^2}$ where r is distance between sail and sun. Let $\mathrm{d}N$ number of photons that collide with sail with time $\mathrm{d}t$. We can write following formula $P\frac{A}{4\pi r^2}\mathrm{d}t = \mathrm{d}NE$, where E is energy of a photon. The energy of a photon is $E = pc$, p is momentum. So we can find total impulse that photons give to sail. The total momentum change will be $2\mathrm{d}Np$ since sail is refractive. The total momentum will be $2P\frac{A}{4\pi r^2 c}\mathrm{d}t$. So $F = \frac{\mathrm{d}p}{\mathrm{d}t}$, force exerted on sail will be $F = 2P\frac{A}{4\pi r^2 c}$. And if we set this equal to gravitational force we will get maximum area density:

$$2P\frac{A}{4\pi \cancel{r^2} c} = \frac{GMm}{\cancel{r^2}}$$

and area density is

$$\sigma = \frac{P}{2\pi GM}$$

(b) The maximum area density is different this time because there's absorbtion instead of reflection. The force balance will be

$$\frac{P\pi R^2}{4\pi r^2 c} = \frac{GM\sigma_{max}4\pi R^2}{r^2}$$

and our shell's is $\sigma = \frac{\sigma_{max}}{2} = \frac{P}{32\pi GMc}$. Coming to actual situation if we write work energy theorem

$$\int_0^\infty \left(\frac{P\pi R^2}{4\pi r^2 c} - \frac{GMm}{r^2}\right)\mathrm{d}r = \frac{mv^2}{2}$$

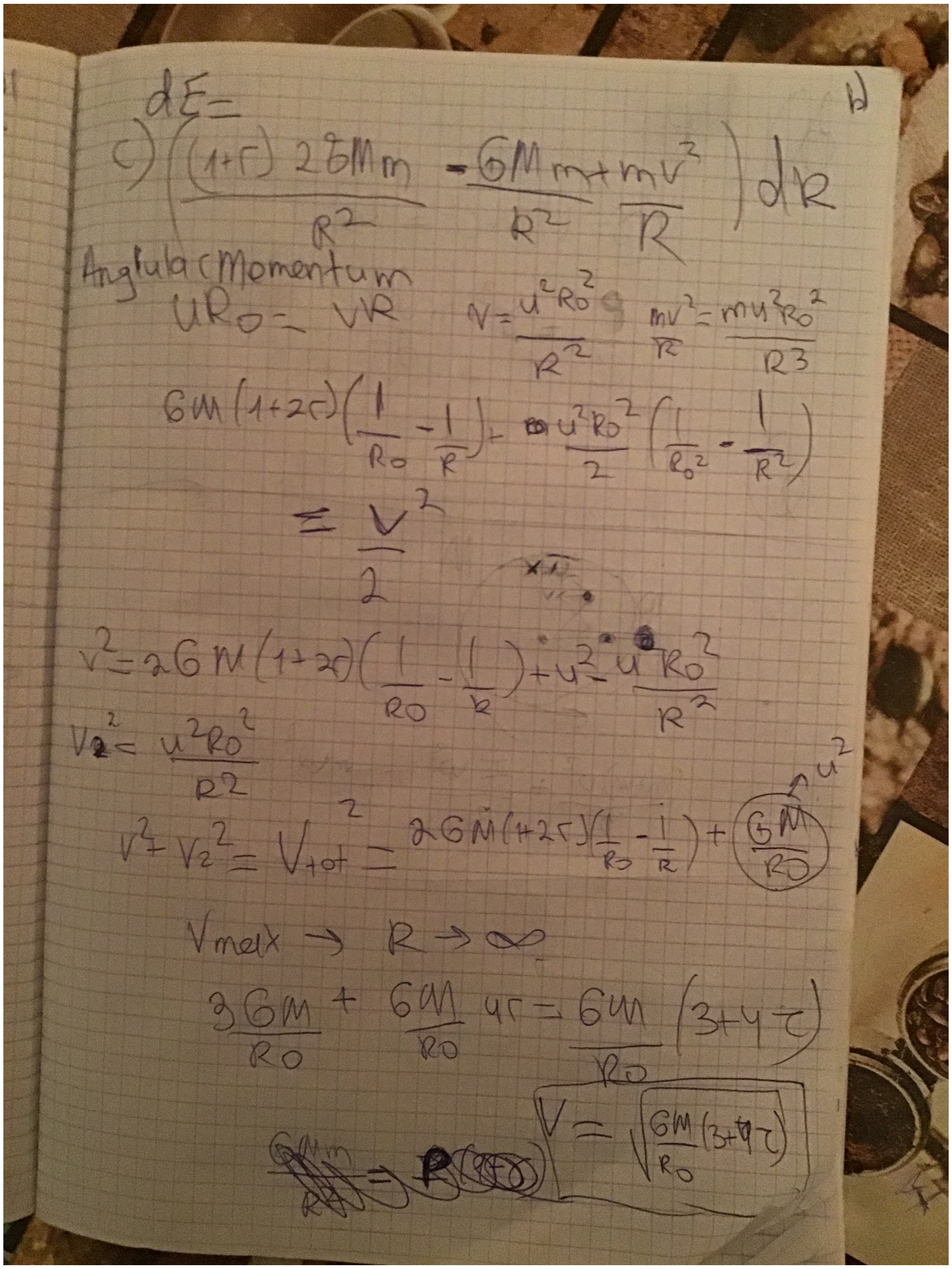
and substituting what we got earlier we get

$$\int_0^\infty \frac{GMm}{r^2}\mathrm{d}r = \frac{1}{2}mv^2$$

evaluating integral gives us $\frac{GMm}{r}$ and our final speed is

$$v = \sqrt{\frac{2GM}{r}} \approx 42124.18m/s$$

(c) here is a solution with picture(we couldn't compile it within the time)



$dE =$

c) $\left(\frac{(1+r)\,2GMm}{R^2} - \frac{GMm}{R^2} + \frac{mv^2}{R} \right) dR$

Angular Momentum

$uR_0 = vR \qquad v = \frac{u^2R_0^2}{R^2} \qquad \frac{mv^2}{R} = \frac{mu^2R_0^2}{R^3}$

$GM(1+2r)\left(\frac{1}{R_0} - \frac{1}{R}\right) = \frac{u^2R_0^2}{2}\left(\frac{1}{R_0^2} - \frac{1}{R^2}\right)$

$= \frac{v^2}{2}$

$v^2 = 2GM(1+2r)\left(\frac{1}{R_0} - \frac{1}{R}\right) + u^2 - \frac{u^2R_0^2}{R^2}$

$V_2^2 = \frac{u^2R_0^2}{R^2}$

$v^2 + V_2^2 = V_{tot}^2 = 2GM(1+2r)\left(\frac{1}{R_0} - \frac{1}{R}\right) + \frac{GM}{R_0}$ u^2

$V_{max} \to R \to \infty$

$\frac{3GM}{R_0} + \frac{GM}{R_0}4r = \frac{GM}{R_0}(3+4r)$

$\boxed{V = \sqrt{\frac{GM}{R_0}(3+4r)}}$

(d) We assume that the laser is so powerful, its power does not change due distance from laser. So we can write work energy theorem for the sail and then differentiate it.

$$\gamma mc^2 - \frac{P}{c}(1+r)x = const$$

differentiating it we get $\gamma^3 m\dfrac{\mathrm{d}v}{\mathrm{d}t} = \dfrac{P}{c}(1+r)$. So seperating variables we get :

$$\int_0^{0.2c} \frac{m}{\left(1-\frac{v^2}{c^2}\right)^{3/2}}\mathrm{d}v = \int_0^t \frac{P}{c}(1+r)\mathrm{d}t$$

we can integrate it by substituting $\dfrac{v}{c} = \sin\theta$. We get

$$mc\frac{\frac{v}{c}}{\sqrt{1-\frac{v^2}{c^2}}}\Bigg|_0^{0.2c} = \frac{P}{c}(1+r)t$$

So power of the laser is $P = \dfrac{mc^2 0.2}{\sqrt{1-0.2^2}(1+r)t}$

(e)

Solution 3:

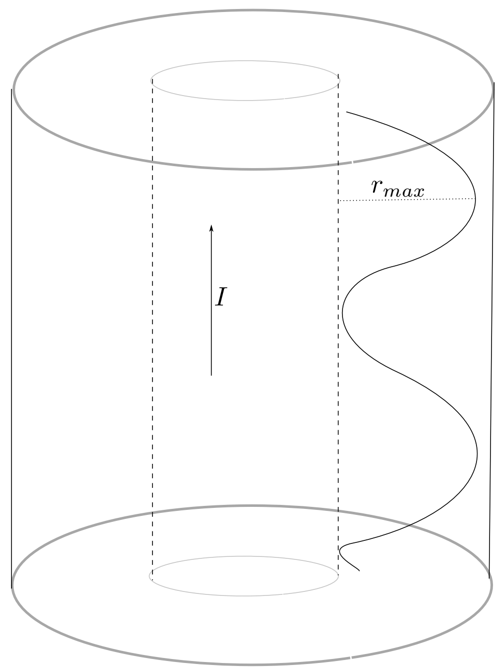
(a) Since energy doesn't stuck in anywhere we can write conservation of energy for a cylinder height h and radius r :

$$-\kappa 2\pi r\cancel{h}\frac{\mathrm{d}T}{\mathrm{d}r} = \left(I\frac{\pi r^2}{\pi R^2}\right)^2 \rho\frac{\cancel{h}}{\pi r^2}$$

so we can seperate and integrate. we get $T = -\dfrac{I^2 r^2 \rho}{4\pi^2 R^4 \kappa} + \mathcal{C}$. We can find constant from $T(r=R) = T_0$. So $\mathcal{C} = T_0 + \dfrac{I^2\rho}{4\pi^2 R^2 \kappa}$. Finally

$$T(r) = T_0 + \frac{I^2\rho}{4\pi^2 R^2\kappa}\left(1-\frac{r^2}{R^2}\right)$$

(b) The magnetic force acts perpendicular to velocity. and electric fieald is always acts along radial. So first accelerates and at the same time it rotates with increasing radius. and after v_r being 0 electric force try to slow it down and this proccess happens over and over again.



(c) Since there're two cylinders with potential of $-V$ and 0 we can assume that system is a cylinderic condenstor. So we can find E field inside of condensator.

$$\int_a^b \frac{\lambda}{2\pi\varepsilon_0 r}\mathrm{d}r = V$$

we can find the formula for E with taking cylinderic gaussian surface. And substituting λ to equation of E we get that $E = \dfrac{V}{\ln\left(\frac{b}{a}\right)r}$.

Let v_r and v_z will be radial and along wire components of velocity respectively.The magnetic fieald of infinite wire is $B = \dfrac{\mu_0 I}{2\pi r}$. Using right hand rule we see that magnetic force due radial velocity acts on parallel velocity. We can write force equation for v_z:

$$m\frac{\mathrm{d}v_z}{\mathrm{d}t} = e\frac{\mu_0 I}{2\pi r}v_r$$

multipliying both sides by $\mathrm{d}t$ and using $v_r\mathrm{d}t = \mathrm{d}r$ and integrating we get

$$mv_z = e\frac{\mu_0 I}{2\pi}\ln\frac{r_{max}}{a}$$

We can write work-energy equation for electron. since magnetic force doesn't change the velocity we can write

$$\int_a^{r_{max}} e\frac{V}{\ln\left(\frac{b}{a}\right)r}\mathrm{d}r = \frac{mv_z^2}{2}$$

$$2e\frac{V}{\ln\left(\frac{b}{a}\right)}\ln\frac{r_{max}}{a} = mv_z^2$$

At maximum distance there will not be radial speed. So if we square our force equation and substitute mv_z^2 we get

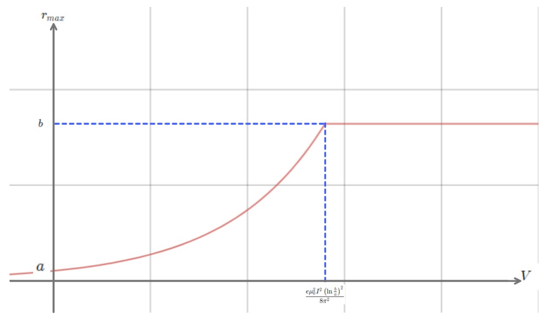
$$2\cancel{e}\frac{V}{\ln\left(\frac{b}{a}\right)}\cancel{\ln\frac{r_{max}}{a}} = \frac{e^{\cancel{2}}\mu_0^2 I^2}{4\pi^2 m}\left(\ln\left(\frac{r_{max}}{a}\right)\right)^{\cancel{2}}$$

And r_{max} is

$$r_{max} = a\exp\left(\frac{8\pi^2 mV}{\ln\frac{b}{a}e\mu_0^2 I^2}\right)$$

(d)

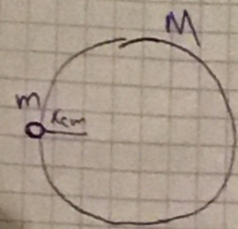
(e) We didn't succeed at previous part so we are gonna show graph of non-relavitistic case



Solution 4:

Problem 4

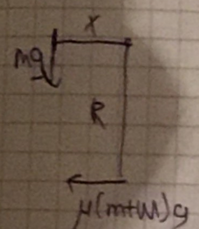
a)



From conservation of momentum,

$$V_{cm} = \frac{m u_0}{m+M}$$

$$x_{cm} = \frac{m\,0 + M \cdot R}{m+M} = \frac{MR}{m+M}$$



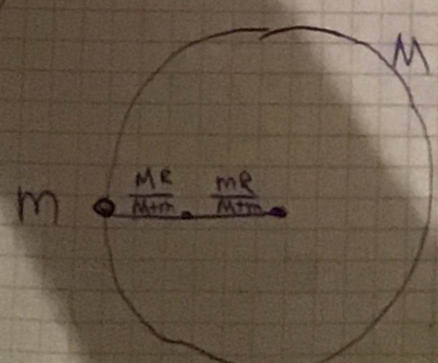
$$\tau = mgx - \mu(m+M)gR$$

If $mgx > \mu(m+M)gR$ (accordingly, $\mu < \frac{mM}{(m+M)^2}$), it will rotate in ↻ direction.

If $\mu > \frac{mM}{(m+M)^2}$, it will rotate in ↺ direction

If $\mu = \frac{mM}{(m+M)^2}$ it will not rotate

b)



Torque about centre of mass

$$\tau = \mu(m+M)gR - \frac{mg \cdot MR}{m+M} = I\beta$$

$\beta \to$ angular acceleration

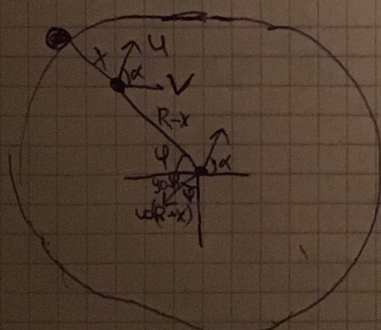
From Hugens-Stainer theorem;

$$I = \underbrace{m\cdot x^2}_{\text{Moment of Inertia of Bullet}} + \underbrace{M\cdot\left(R^2-(R-x)^2\right)}_{\text{Moment of Inertia of Cylinder}} =$$

$$= \frac{mM^2R^2}{(M+m)^2} + MR^2 - \frac{M\cdot m^2R^3}{(M+m)^2}$$

Hence, $$\beta = \frac{\mu(m+M)g - \frac{mg\cdot M}{m+M}}{\left(M + \frac{mM^2}{(M+m)^2} - \frac{Mm^2}{(M+m)^2}\right)R}$$

c)



$$u\cos\alpha = v$$
$$u\sin\alpha = \omega(R-x)\cos\varphi$$

Due to no vertical velocity of center

$$\Rightarrow u^2 = v^2 + \frac{\omega^2\cdot m^2R^3}{(M+m)^2}\cos^2\varphi$$

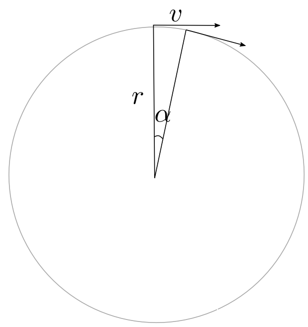
$$E_{K_S} = \frac{1}{2}(m+M)u^2 + \frac{I\omega^2}{2} =$$

$$E_{K_b} = \frac{1}{2}mu_0^2$$

$$\Delta E = E_{K_S} - E_{K_b} = \frac{1}{2}(m+M)v^2 + \frac{1}{2}\frac{m^2\omega^2R^2\cos^2\varphi}{M+m} + \frac{1}{2}MR^2\omega^2 + \frac{1}{2}\frac{mM^2R^2\omega^2}{(m+M)^2}$$
$$- \frac{1}{2}\frac{Mm^2R^2\omega^2}{(m+M)^2} - \frac{1}{2}mu_0^2$$

Solution 5:

(a) Assume that body is moving with speed v on circle radius r.



Since we are lookig at small interval of time the α is small angle. We can approximate the distance body went $x = vt$. At the same time due to small interval $x = r\alpha$ here there's no derivative it's just definiton of radian. The acceleration of body is directed downward at that interval of time. So we can approximate that $v_y = at$ where $a = \frac{v^2}{r}$. And at the same time $v_y^2 = v^2 - v_x^2$ where $v_x = v\cos\alpha$. So substituting all we get that $\sin^2\alpha = \alpha^2$. Here we can use the trigoometric identity $\sin^2 x = \frac{1-\cos 2x}{2}$. And we get

$$\cos 2\alpha = 1 - 2\alpha^2 \quad \text{Substitute } 2\alpha = x$$
$$\cos x = 1 - \frac{x^2}{2}$$

(b)

(c)

Solution 6:

(a) We can find acceleration due gravity with Gaussian law for gravity

$$\oint g\mathrm{d}S = 4\pi G M_{enclosed}$$

Since R is very big we can assume earth is infinite plate,so at the center g is perpendicular to earth. We can choose a cylinder gaussian surface:

$$g \cdot 2A = 4\pi G m$$
$$g = 2\pi G\sigma$$

Let θ_1 would be equlibrium position. We can write that $\tau = m\ell(\omega^2\ell\sin\theta_1\cos\theta_1 - g\sin\theta_1) = 0$. And fundamental law between angular acceleration and torque $I\ddot{\theta} = \tau$, where $I = m\ell^2$. So if we change angle by θ torque will be $\tau = m\ell(\omega^2\ell\cos 2\theta_1 - g\cos\theta_1)\theta$ (due to taylor series) And

$$\ell\ddot{\theta} + \theta(g\cos\theta_1 - \omega^2\ell\cos 2\theta_1) = 0$$

So we see that

$$\Omega_1^2(0) = \frac{2\pi G\sigma\cos\theta_1 - \omega^2\ell\cos 2\theta_1}{\ell}$$

(b) For equlibrium position we need to solve equation for τ:

$$m\ell^2\sin\theta\Big(\omega^2\cos\theta - \frac{g}{\ell}\Big) = 0$$

Let $\omega_0 = \sqrt{\frac{g}{\ell}}$ There's two solutions to these equation:

$$\begin{cases} \sin\theta = 0, \text{or} \quad \theta = 0, \theta = \pi \\ \cos\theta = \frac{\omega_0^2}{\omega^2} \end{cases}$$

For the stability we should have $\frac{\mathrm{d}\tau}{\mathrm{d}\theta} < 0$ otherwise there will not be oscillations.

$$\frac{\mathrm{d}\tau}{\mathrm{d}\theta} = m\ell^2(\omega^2(\cos^2\theta - \sin^2\theta) - \omega_0^2\cos\theta)$$

If $\theta = 0$

$$\frac{\mathrm{d}\tau}{\mathrm{d}\theta} = m\ell^2(\omega^2 - \omega_0^2)$$

in this case we see that $\omega^2 < \frac{g}{l}$ for stability.

If $\theta = \pi$

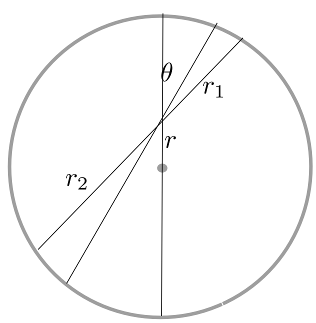
$$\frac{\mathrm{d}\tau}{\mathrm{d}\theta} = m\ell^2(\omega^2 + \omega_0^2)$$

we see that in this case the position is not stable so there will not be oscillations.

If $\theta = \arccos\Big(\frac{\omega_0^2}{\omega^2}\Big)$ and plugging value applying condition we get that $\omega^2 > \frac{g}{\ell}$

So maximum angular velocity is $\omega^2 = \frac{2\pi G\sigma}{\ell}$

(c) For this part there will be non-zero component of g parallel to earth.



$$\mathrm{d}\vec{g} = \frac{G\sigma r\mathrm{d}r\mathrm{d}\theta}{r^2}\vec{e_r}$$

$$\int \mathrm{d}g = \int G\sigma\ln\Big(\frac{r_2}{r_1}\Big)\cos\theta\mathrm{d}\theta$$

Using law of cosines

$$R^2 = r^2 + r_1^2 + 2rr_1\cos\theta$$
$$r_1 = \sqrt{R^2 - r^2\sin^2\theta} - r\cos\theta$$
$$R^2 = r^2 + r_2^2 - 2rr_2\cos\theta$$
$$r_2 = \sqrt{R^2 - r^2\sin^2\theta} + r\cos\theta$$

So our equation become

$$2G\sigma\int_0^{\frac{\pi}{2}} \ln\left(\frac{\sqrt{R^2 - r^2\sin^2\theta} + r\cos\theta}{\sqrt{R^2 - r^2\sin^2\theta} - r\cos\theta}\right)\cos\theta \mathrm{d}\theta$$

So we have a monstrous integral. We can approxiamte it with provided $r \ll R$. We ignore the terms with r^2 and our integral becomes

$$g_{\|} = \int_0^{\frac{\pi}{2}} \frac{4G\sigma r}{R}\cos^2\theta \mathrm{d}\theta$$

And this integral is $\mathbb{TRIVIAL}$ than other ones in OPHO. Integrate with trig identity and we get that

$$\vec{g_{\|}} = \frac{\pi G\sigma\vec{r}}{R}$$

We can write equation for equlibrium point

$$2\pi G\sigma\left(\sin\theta - \frac{r}{2R}\cos\theta\right) - \omega^2(r - \ell\sin\theta)\cos\theta = 0$$

solving this with assuming $r \ll R$ we get that $\tan\theta = \frac{\left(\frac{2\pi G\sigma}{2R} + \omega^2\right)r}{2\pi G\sigma}$. We can write torque equation:

$$\tau = -(mg_{\perp}\ell\sin\theta - g_{\|}\ell\cos\theta - \omega^2(r - \ell\sin\theta)\ell\cos\theta)$$
$$\tau = -m\ell(2\pi G\sigma(\sin\theta - \frac{r\cos\theta}{2R}) - \omega^2(r^2 - \ell\sin\theta)\cos\theta)$$
$$\frac{\mathrm{d}\tau}{\mathrm{d}\theta} = -m\ell(2\pi G\sigma(\cos\theta + \frac{r\sin\theta}{2R}) + \omega^2(r\sin\theta + \ell\cos 2\theta))$$
$$\ell\ddot{\theta} + \theta(2\pi G\sigma\left(\cos\theta_1 + \frac{r\sin\theta_1}{2R}\right) + \omega^2(r\sin\theta_1 + \ell\cos 2\theta)) = 0$$

And frequency is

$$\Omega^2(r) = \frac{2\pi G\sigma\left(\cos\theta_1 + \frac{r\sin\theta_1}{2R}\right) + \omega^2(r\sin\theta_1 + \ell\cos 2\theta)}{\ell}$$

(d)

Solution 7:

(a)

(b)

(c)

(d)

Solution 8:

(a) We can find power output of sunn by using

$$\int_0^\infty \frac{8\pi h\nu^3}{c^3}\frac{\mathrm{d}\nu}{e^{\frac{h\nu}{kT_\odot}}-1}$$

substitute $x = \dfrac{h\nu}{kT_\odot}$ we get

$$\frac{8\pi k^4 T_\odot^4}{c^3h^3}\int_0^\infty \frac{x^3}{e^x-1}\mathrm{d}x$$

For solving this integral we need to do some advanced stuff. Take a look at Gamma function

$$\Gamma(x) = \int_0^\infty t^{x-1}e^{-t}\mathrm{d}t$$

let's substitude $t = nu$, where $n \in \mathbb{N}$ we simplify a little bit divide equation by n^x and get that

$$\Gamma(x)\frac{1}{n^x} = \int_0^\infty u^{x-1}(e^{-u})^n\mathrm{d}u$$

and we can take sum of both sides $n = 1 \to \infty$ and note that on the right hand side we have a geometric sequence. And solving int we get

$$\Gamma(x)\sum_{n=1}^{\infty}\frac{1}{n^x} = \int_0^\infty \frac{u^{x-1}}{e^u-1}\mathrm{d}u$$

for our case we need to take $x = 4$. And get our integral changing domain

$$(4-1)!\sum_{1}^{\infty}\frac{1}{n^4} = \int_0^\infty \frac{x^3}{e^x-1}\mathrm{d}x$$

Now a bigger problem. What the heck is sum of fourth powers??? okay we need more advanced stuff. For this case take a look at Fourier transformation

$$f(x) = a_0 + \sum_{n=1}^{\infty}\left(a_n\cos(nx) + b_n\sin(nx)\right)$$

And the parseval's theorem(comes from Fourier) says that

$$\frac{1}{\pi}\int_{-\pi}^{\pi}(f(x))^2\mathrm{d}x = 2(a_0)^2 + \sum_{n=1}^{\infty}(a_n^2+b_n^2)$$

Where

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi}f(x)\mathrm{d}x$$
$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi}f(x)\cos(nx)\mathrm{d}x$$
$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi}f(x)\sin(nx)\mathrm{d}x$$

Choose $f(x) = x^2$ solving for coefficents with integration by parts and using odd/even fucntion yields

$$\begin{cases} a_0 = \dfrac{\pi^2}{3} \\ a_n = \dfrac{4(-1)^n}{n^2} \\ b_n = 0 \end{cases}$$

What we got from here we can substitute them to parseval's theorem and see what we get

$$\frac{\pi^4}{45} = 2\sum_{n=1}^{\infty}\frac{1}{n^4}$$

and we finally did it!! $\sum_{n=1}^{\infty}\frac{1}{n^4} = \frac{\pi^4}{90}$ So $\int_0^{\infty}\frac{x^3}{e^x-1}\mathrm{d}x = 6\frac{\pi^4}{90} = \frac{\pi^4}{15}$. And our integral is $\frac{8\pi^5k^4T_{\odot}^4}{15c^3h^3}$ So the power output of the star will be $\frac{32\pi^6R_{\odot}^2k^4T_{\odot}^4}{15c^2h^3}$ Energy flux on the planet will be power output times $\frac{1}{4\pi r_{ES}^2}$ and solving for temperature

$$T_{\odot} = \sqrt[4]{\frac{15c^2h^3J_0r_{ES}^2}{\pi^5R_{\odot}^2k^4}}$$

(b) The energy that produced by a fusion reaction is $E = (4m_p - m_{He})c^2$. Let $\mathrm{d}N$ number of reactions happens in $\mathrm{d}t$ time. We can assume power output of sun is due nuclear fusion. So

$$P(R_{\odot})\mathrm{d}t = \mathrm{d}N(4m_p - m_{He})c^2$$

and $\frac{\mathrm{d}N}{\mathrm{d}t} = \frac{P(R_{\odot})}{(4m_p - m_{He})c^2}$ and number of protons per second is $4\frac{\mathrm{d}N}{\mathrm{d}t}$

The number of protons that can react :$\frac{2\eta M_{\odot}}{m_p}$ so

$$t = \frac{(4m_p - m_{He})c^2\eta M_{\odot}}{2P(R_{\odot})m_p}$$

(c) We can find pressure due gravity. Take a small piece with area A, thicness $\mathrm{d}r$ at distance r from center. The force acting on piece is $\frac{3GM^2Sr\mathrm{d}r}{4\pi R^6}$ So we can write force balance on this piece

$$p(r+\mathrm{d}r)S - p(r)S = \frac{3GM^2Sr\mathrm{d}r}{4\pi R^6}$$

using the definition of derivative and integrating, using initial value $p(R) = 0$ we get

$$p(r) = \frac{3GM^2}{8\pi R^6}(R_{\odot}^2 - r^2)$$

(d)

(e)

(f) The force is because momentum change. The momentum change is $p_0(1+\gamma)$ p_0 is inital momentum. and there's relation between momentum and enrgy $E = pc$ so

$$F = \frac{\pi\sigma T_{\odot}^4R_{\odot}^2r_E^2(1+\gamma)}{cr_{SE}^2}$$

(g) Since exoplanet is on thermal equlibrium we an write

$$\frac{\pi\sigma T_{\odot}^4R_{\odot}^2\cancel{r_E^2}(1-\gamma)}{r_{SE}^2} = \sigma T_E^4 4\pi\cancel{r_E^2}$$

$$T_E = T_{\odot}\sqrt[4]{\frac{R_{\odot}^2(1-\gamma)}{4r_{SE}^2}}$$

(h) Why same question that voided in open round? we don't even try it because last time we did 3 different ways 3 wrong answers

Solution 9:

(a) Let v_x will be velocity of particles. And u velocity of piston. Assuming collisions are elastically and m/llM we can write that $v'_x = v_x - 2u$ The energy change of particle will be $\frac{1}{2}m(v_x - 2u)^2 - \frac{1}{2}mv_x^2 = -2muv_x$ number of the particles collides with the piston is $\mathrm{d}N = \frac{1}{2}nSv_x\mathrm{d}t$

$$\mathrm{d}E_{total} = \mathrm{d}N\Delta E = nmuv_x^2\mathrm{d}t = Mu\mathrm{d}u$$
$$M\frac{\mathrm{d}u}{\mathrm{d}t} = nmv_x^2 \tag{1}$$

Assuming all the velocity components are the same we can find v_x:

$$\frac{m(v_x^2 + v_y^2 + v_z^2)}{2} = \frac{3mv_x^2}{2} = \frac{3kT}{2}$$

With eq 1 we also have $M\ddot{x} = PS$ So we can write change in energy

$$\mathrm{d}U = -nSmv_x^2u\mathrm{d}t = -nmv_x^2\mathrm{d}V$$

also internal energy is $U = \frac{3nmv_x^2V}{2}$. so substituting we get $\mathrm{d}U = -\frac{2}{3}\frac{\mathrm{d}V}{V}$ integrating and using $U = C_VT$ we get $TV^{2/3} = const$ or $PV^{5/3} = const$
using that fact we can derive formula for pressure at distance x piston moved:

$$P = \frac{P_0V_0^{5/3}}{V^{5/3}} = \frac{NRT_0L_0^{2/3}}{N_AA(L_0 + x)^{5/3}}$$

And pressure is acting on piston so

$$M\ddot{x} = fracNRT_0L_0^{2/3}N_AA(L_0 + x)^{5/3}$$

(b)

(c)

(d)

(e)

Solution 10:

(a)

(b)

(c)

(d)

(e)

(f) Here We will use cylindrical coordinate system for the sake of simplicity

$$B_z = \frac{\mu_0 m}{4\pi(r^2 + z^2)^{\frac{3}{2}}}\left(\frac{3z^2}{r^2 + z^2} - 1\right)$$

$$B_r = \frac{3\mu_0 mrz}{4\pi(r^2+z^2)^{\frac{5}{2}}}$$

let's calculate flux through ring

$$\phi = \int_0^R B_z 2\pi r dr = \frac{\mu_0}{2}\int_0^R \frac{mr}{(r^2+z^2)^{\frac{3}{2}}}\left(\frac{3z^2}{r^2+z^2}-1\right)dr = \frac{\mu_0 mR^2}{2(R^2+z^2)^{\frac{5}{2}}}$$

fro superconductor ring net flux ought to be zero ,so there will be current compensate this flux ,we will write it in positive form ,I doesn't matter at all

$$LI = \frac{\mu_0 mR^2}{2(R^2+z^2)^{\frac{5}{2}}}$$

so here $I = \dfrac{\mu_0 mR^2}{2L(R^2+z^2)^{\frac{5}{2}}}$ Writing energy conservation

$$\frac{Mv_0^2}{2} = \frac{Mv^2}{2} + \frac{LI^2}{2}$$

$$\frac{Mv_0^2}{2} = \frac{Mv^2}{2} + \frac{(\mu_0 mR^2)^2}{8L(R^2+z^2)^5}$$

Result:

$$v = \sqrt{v_0^2 - \frac{(\mu_0 mR^2)^2}{4ML(R^2+z^2)^5}}$$

(g) as we know that $\varepsilon = \dfrac{\mathrm{d}\phi}{\mathrm{d}t} = \dfrac{3vzmR^2}{2(R^2+z^2)^{\frac{5}{2}}}$

$$I = \frac{\varepsilon}{\frac{2\pi R}{\sigma dz}}$$

force acting on this ring : $F = 2\pi RIB_z$ so

$$F_{total} = \int_{-\infty}^{\infty} 2\pi\sigma RB_r^2 v dz = \frac{9\mu_0^2 m^2 R^3\sigma}{8\pi}\int_{-\infty}^{\infty}\frac{z^2}{(z^2+R^2)^5}dz$$

so dividing it by v and calculating the integral will give us the damping parameter

$$k = \frac{9\mu_0^2 m^2\sigma}{1024R^4}$$

(h) when body reaches its terminal velocity ,then there is no net force acting on it $kv = Mg$ and from this

$$v = \frac{Mg}{k}$$