École Centrale Nantes

EMARO-CORO M1

Mechanical Design methods in Robotics (DESRO)

Project

Report of:

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Introduction

The aim of this project was to design a parallel manipulator applying the concepts previously analysed in class. To do so, we had to pass through the steps of the Design Process Layout after French: task definition (the formulation of the optimization problem based on the specifications of the chosen project), conceptual design (the synthesis of a certain number of options that could fulfil the requirements and the evaluation of their complexity) and embodiment (the realization of the chosen option in CAD and its animation).
As a group we chose Project 1, which consisted in the design of a three degrees of freedom planar parallel manipulator with the following specifications:

- The manipulator should have a cylindrical regular workspace of diameter equal to 100 mm, its height corresponding to the range of rotation of the moving-platform.
- The range of rotation of the moving platform should be higher than 60° throughout the regular workspace.
- The inverse condition numbers of its normalized forward and inverse Jacobian matrices should be larger than 0.1 throughout the regular workspace.
- The manipulator should be compact, i.e., the overall size of the manipulator should be as small as possible.
- For a force equal to 10 N applied on the moving-platform along the normal to the plane of motion in its home configuration, the point-displacement of the moving-platform should be lower than 0.1 mm.

Task definition: optimization problem

We defined the optimization problem as shown below:

OPTIMIZATION PROBLEM

- variables : R, r, L1, L2
- function to optimize: sum(R, r, L1, L2)
- constraints :

$$-(OP_x + OP_y^2) + 100^2 \le 0$$

with $OP = R\underline{h}_i + L_1\underline{u}_i + L_2\underline{v}_i + r\underline{k}_i \qquad i = 1,2,3$

$$\underline{h}_i = [\cos\theta_0 \quad \sin\theta_0]^T$$

$$\underline{u}_i = \left[\frac{R\cos\theta_0 + L_1\cos\theta_1}{|R\cos\theta_0 + L_1\cos\theta_1|} \quad \frac{R\sin\theta_0 + L_1\sin\theta_1}{|R\sin\theta_0 + L_1\sin\theta_1|} \right]^T$$

$$\underline{v}_i = \left[\frac{R\cos\theta_0 + L_1\cos\theta_1 + L_2\cos\theta_2}{|R\cos\theta_0 + L_1\cos\theta_1 + L_2\cos\theta_2|} \quad \frac{R\sin\theta_0 + L_1\sin\theta_1 + L_2\sin\theta_2}{|R\sin\theta_0 + L_1\sin\theta_1 + L_2\sin\theta_2|} \right]^T$$

$$\underline{k}_i = \left[\frac{R\cos\theta_0 + L_1\cos\theta_1 + L_2\cos\theta_2 + r\cos\theta_3}{|R\cos\theta_0 + L_1\cos\theta_1 + L_2\cos\theta_2 + r\cos\theta_3|} \quad \frac{R\sin\theta_0 + L_1\sin\theta_1 + L_2\sin\theta_2 + r\sin\theta_3}{|R\sin\theta_0 + L_1\sin\theta_1 + L_2\sin\theta_2 + r\sin\theta_3|} \right]^T$$

The angles are given by the IGM:

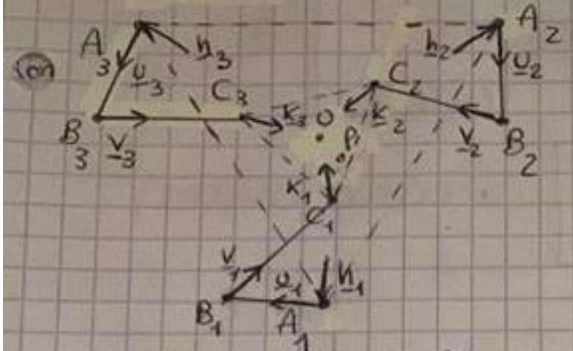
$$\begin{cases} L_1^2 = (x_{A_i} - x_{B_i})^2 + (y_{A_i} - y_{B_i})^2 \\ L_2^2 = (x_{C_i} - x_{B_i})^2 + (y_{C_i} - y_{B_i})^2 \end{cases}$$

- $\theta_{1i} = \arctan2\left(\frac{y_A}{x_A}\right)$ angle corresponding to the A_i variable

$\theta_{2i} = \arctan2\left(\frac{y_B - y_A}{x_B - x_A}\right)$ angle corresponding to the B_i variable

$\theta_{3i} = \arctan2\left(\frac{y_B - y_C}{x_B - x_C}\right)$ angle corresponding to the C_i variable

We considered as design variables to optimize both radii, of the base (R) and of the moving-platform (r) respectively, and the lengths of the joints' links $L_1$and L_2. To satisfy the first and the second specifications, we computed the IGM and from its results we computed the angles.
A third constraint is also needed to satisfy the third specification, checking that the values given by the first two constraints do not lead to a singularity configuration. Thus, we computed the DKM of the robot to get the direct and inverse Jacobian matrices, as shown below (we assumed to always have three legs, whichever configuration chosen):



O = centre of actuator

P = centre of base

To optimize:

- radius actuator: r
- radius base: R
- lenght of link AB: L
- lenght of link B_iC_i: K

$$\begin{cases} \overrightarrow{OP} = \overrightarrow{PA_1} + \overrightarrow{A_1B_1} + \overrightarrow{B_1C_1} + \overrightarrow{C_1O} \\ \overrightarrow{OP} = \overrightarrow{PA_2} + \overrightarrow{A_2B_2} + \overrightarrow{B_2C_2} + \overrightarrow{C_2O} \\ \overrightarrow{OP} = \overrightarrow{PA_3} + \overrightarrow{A_3B_3} + \overrightarrow{B_3C_3} + \overrightarrow{C_3O} \end{cases}$$

$$\begin{cases} \underline{P} = R\underline{h}_1 + L\underline{u}_1 + k\underline{v}_1 + r\underline{k}_1 \\ \underline{P} = R\underline{h}_2 + L\underline{u}_2 + k\underline{v}_2 + r\underline{k}_2 \\ \underline{P} = R\underline{h}_3 + L\underline{u}_3 + k\underline{v}_3 + r\underline{k}_3 \end{cases}$$

$$\begin{cases} \dot{\underline{P}} = \underline{0}_2 + L\dot{\theta}_1\underline{E}\,\underline{u}_1 + k(\dot{\beta}_1+\dot{\theta}_1)\underline{E}\,\underline{v}_1 + r(\dot{\theta}_1+\dot{\beta}_1+\dot{\gamma}_1)\underline{E}\,\underline{k}_1 \\ \dot{\underline{P}} = \underline{0}_2 + L\dot{\theta}_2\underline{E}\,\underline{u}_2 + k(\dot{\theta}_2+\dot{\beta}_2)\underline{E}\,\underline{v}_2 + r(\dot{\theta}_2+\dot{\beta}_2+\dot{\gamma}_2)\underline{E}\,\underline{k}_2 \\ \dot{\underline{P}} = \underline{0}_2 + L\dot{\theta}_3\underline{E}\,\underline{u}_3 + k(\dot{\theta}_3+\dot{\beta}_3)\underline{E}\,\underline{v}_3 + r(\dot{\theta}_3+\dot{\beta}_3+\dot{\gamma}_3)\underline{E}\,\underline{k}_3 \end{cases}$$

With $\dot{\theta}_i + \dot{\beta}_i + \dot{\gamma}_i = \phi_i$

$$\begin{cases} \underline{v}_1^T\dot{\underline{P}} = L\dot{\theta}_1\underline{v}_1^T\underline{E}\,\underline{u}_1 + \cancel{k(\dot{\beta}_1+\dot{\theta}_1)\underline{v}_1^T}\underline{E}\,\underline{v}_1 + r\dot{\phi}_1\underline{v}_1^T\underline{E}\,\underline{k}_1 \\ \underline{v}_2^T\dot{\underline{P}} = L\dot{\theta}_2\underline{v}_2^T\underline{E}\,\underline{u}_2 + \cancel{k(\dot{\beta}_2+\dot{\theta}_2)\underline{v}_2^T}\underline{E}\,\underline{v}_2 + r\dot{\phi}_2\underline{v}_2^T\underline{E}\,\underline{k}_2 \\ \underline{v}_3^T\dot{\underline{P}} = L\dot{\theta}_3\underline{v}_3^T\underline{E}\,\underline{u}_3 + \cancel{k(\dot{\beta}_3+\dot{\theta}_3)\underline{v}_3^T}\underline{E}\,\underline{v}_3 + r\dot{\phi}_3\underline{v}_3^T\underline{E}\,\underline{k}_3 \end{cases}$$

$$\begin{cases} \underline{v}_1^T\dot{\underline{P}} - r\dot{\phi}_1\underline{v}_1^T\underline{E}\,\underline{k}_1 = L\dot{\theta}_1\underline{v}_1^T\underline{E}\,\underline{u}_1 \\ \underline{v}_2^T\dot{\underline{P}} - r\dot{\phi}_2\underline{v}_2^T\underline{E}\,\underline{k}_2 = L\dot{\theta}_2\underline{v}_2^T\underline{E}\,\underline{u}_2 \\ \underline{v}_3^T\dot{\underline{P}} - r\dot{\phi}_3\underline{v}_3^T\underline{E}\,\underline{k}_3 = L\dot{\theta}_3\underline{v}_3^T\underline{E}\,\underline{u}_3 \end{cases}$$

$$\begin{bmatrix} -r v_1^T E k_1 & v_1^T \\ -r v_2^T E k_2 & v_2^T \\ -r v_3^T E k_3 & v_3^T \end{bmatrix} t = \begin{bmatrix} L v_1^T E u_1 & 0 & 0 \\ 0 & L v_2^T E u_2 & 0 \\ 0 & 0 & L v_3^T E u_3 \end{bmatrix} \dot{\theta}$$

with $t = \begin{bmatrix} \dot{\phi} \\ \dot{p} \end{bmatrix}$ and $\dot{\theta} = \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix}$ (actuated angles)

Singularities :

(1) SERIAL : $\det B = L^3 (v_1^T E u_1)(v_2^T E u_2)(v_3^T E u_3) = L^3 \prod_{i=1}^{3} v_i^T E u_i = 0$

iff $v_1^T // u_1$, $v_2^T // u_2$, $v_3^T // u_3$

(2) PARALLEL : $\det A = 0$ if $v_1^T // v_2^T // v_3^T$ or $v_1^T // k_1$, $v_2^T // k_2$, $v_3^T // k_3$

We then normalized the forward Jacobian matrix dividing it for the radius of the moving platform r and computed its inverse condition number by the means of the Matlab function cond(). This way, we put a constraint on the serial singularities and therefore imposed our optimization to stop whenever a singular configuration was found.

We implemented the optimization problem in Matlab, by the means of the fmincon() function. As required by the function, we created three different files: a main to call it, an objective function and a constraint one (see the files joint to the report). Note that to implement the problem, we also needed to put additional constraints to avoid the design variables from going below 0 and the difference between the two radii getting negative.

Our starting point was:

```
%startopt.m
OP=[10,1];
R=80;
r=90;
L1=120;
L2=100;
phi_0=pi*1/2;
Theta=IGM(OP,phi_0,R,r,L1,L2);
Param_opt=[R,r,L1,L2];
```

The optimization problem took 21 iterations:

Iter	F-count	f(x)	Feasibility	First-order optimality	Norm of step
0	16	3.900000e+02	1.000e+01	9.997e-01	
1	33	3.900025e+02	9.875e+00	9.997e-01	8.827e-02
2	49	3.893740e+02	0.000e+00	9.997e-01	7.053e+00
3	65	3.859851e+02	0.000e+00	7.629e-01	1.701e+00
4	81	3.664793e+02	0.000e+00	7.743e-01	1.295e+01
5	97	3.231929e+02	6.196e-02	7.996e-01	2.384e+01
6	113	3.116026e+02	5.827e-02	8.066e-01	5.949e+00
7	129	2.313774e+02	4.377e-02	8.560e-01	4.108e+01
8	145	2.309946e+02	3.838e-02	8.562e-01	2.100e-01
9	161	2.213656e+02	0.000e+00	8.621e-01	6.224e+00
10	180	2.078433e+02	0.000e+00	8.702e-01	8.506e+00
11	201	1.802644e+02	0.000e+00	8.972e-01	1.698e+01
12	217	1.523268e+02	6.314e-03	9.277e-01	1.731e+01
13	235	1.523013e+02	5.792e-03	9.277e-01	3.337e-02
14	251	1.524701e+02	0.000e+00	1.000e-01	9.406e-01
15	267	1.521047e+02	0.000e+00	5.495e-02	2.589e+00
16	283	1.517766e+02	0.000e+00	5.259e-02	5.953e+00
17	299	1.517920e+02	0.000e+00	4.909e-02	8.392e-01
18	315	1.517540e+02	0.000e+00	2.166e-02	5.302e-01
19	331	1.516879e+02	0.000e+00	4.015e-03	2.331e-01
20	347	1.516720e+02	0.000e+00	2.962e-04	5.251e-02
21	363	1.516720e+02	0.000e+00	4.004e-05	2.775e-04

```
Local minimum possible. Constraints satisfied.

fmincon stopped because the size of the current step is less than
the selected value of the step size tolerance and constraints are
satisfied to within the default value of the constraint tolerance.

<stopping criteria details>
```

The final results at the 21th iterations were the ones shown below:

```
R =

   50.0000


r =

   21.6716


L1 =

   40.0000


L2 =

   40.0004
```

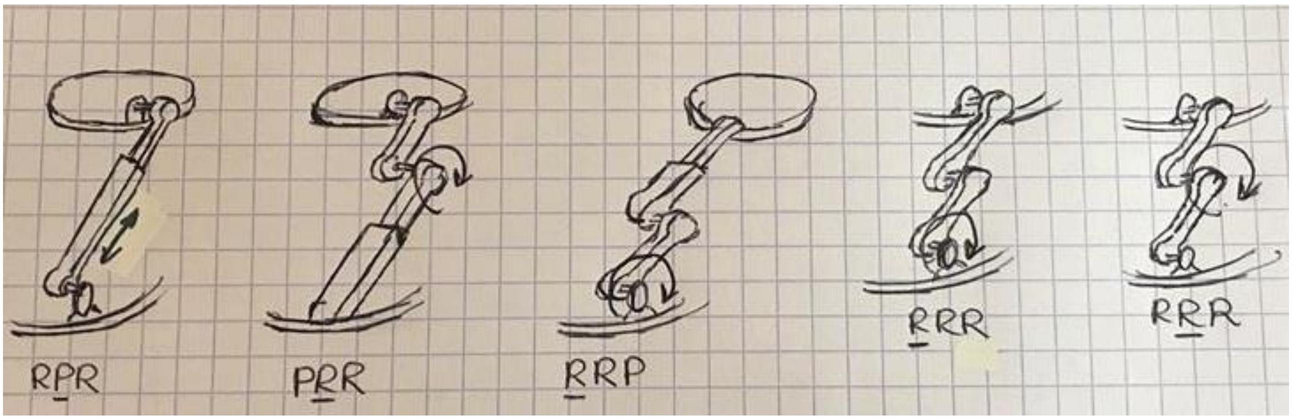
Conceptual design: synthesis of different options

After having obtained the optimization problem's results, we synthetized a list of different manipulators options that might fulfil the given requirements. We considered 5 different possibilities:

1. RRR manipulator, with three revolute joints of which the first one is actuated
2. RRR manipulator, with three revolute joints of which the second one is actuated
3. RPR manipulator, with two revolute joints and a prismatic one in the middle; we still opted for the actuation of the first revolute joint

4. PRR manipulator, with a prismatic joint attached to the lower base and two revolute ones, of which the first one is actuated
5. RRP manipulator, with two revolute joints of which the first one is actuated and a prismatic attached to the moving platform

We decided to use 3 legs in whichever case chosen (as previously stated). Find below a rough sketch of the 5 manipulators synthetized:



Conceptual design: evaluation of complexity and choice of the best option

We then proceeded with the complexity evaluation, to choose the best option between the four listed above. We considered the complexity as expressed by the formula below:

$$K = w_N K_N + w_L K_L + w_J K_J + w_B K_B$$

We assumed the coefficients w_N, w_L, w_B and w_B as equal for all the four manipulators proposed for lack of more information. Since the number of joints considering all the three legs is the same for all of them (N=9, $\mathrm{N}_m = 9$), we got the same joint-number complexity K_N value for all five:

$$K_N = 1 - \exp(-q_N N) = 0.899$$

with $q_N = -\frac{\ln(0.1)}{N_m} = 0.2558$.

As in the previous case, even for the loop complexity K_L the values are all equal. This is due to the fact that for all five architectures we are indeed using three legs, thus the number of kinematic loops is always 3 while the minimum number of loops is always 0. From these values we got L = 3, so we get $\mathrm{L}_m = 3$ and $q_L = -\frac{\ln(0.1)}{L_m} = 0.7675$, which gives a value of:

$$K_L = 1 - \exp(-q_L L) = 0.899$$

As for the joint-type complexity evaluation, we divided our architectures into two different groups, with two different K_J values:

- RRR and RRR, that present three joints of the same type

$$K_J = \frac{1}{n}\left(3K_{G|R}\right) = 0.5234$$

- RPR, PRR and RRP that present two revolute and one prismatic joint

$$K_J = \frac{1}{n}\left(2K_{G|R} + K_{G|P}\right) = 0.6823$$

with $K_{G|R} = 0.5234$ and $K_{G|P} = 1$.

As for the link diversity component K_B, we computed as shown below:

3-R$\underline{P}$R : RP B_2, PR B_2 $\}$ $B_2 = 2$ $\quad b_1 = \frac{B_2}{N_{max}} = \frac{2}{9}$

$B = 0{,}4822 \quad k_{B_1} = \frac{B}{B_{max}} = 0{,}2078$

3-P$\underline{R}$R : PR B_2, RR B_1 $\}$ $B_2 = B_1 = 1$ $\quad b_1 = \frac{1}{9} \quad B = 0{,}3522 \quad k_{B_2} = 0{,}1518$

3-$\underline{R}$RP : RR B_1, RP B_2 $\}$ $B_1 = B_2 = 1$ $\quad b_1 = \frac{1}{9} \quad B = 0{,}3522 \quad k_{B_3} = 0{,}1518$

3-$\underline{R}$RR and 3-R$\underline{R}$R : RR B_1, RR B_1 $\}$ $B_1 = 2$ $\quad b_1 = \frac{2}{9} \quad B = 0{,}4822$

$k_{B_4} = 0{,}2078$

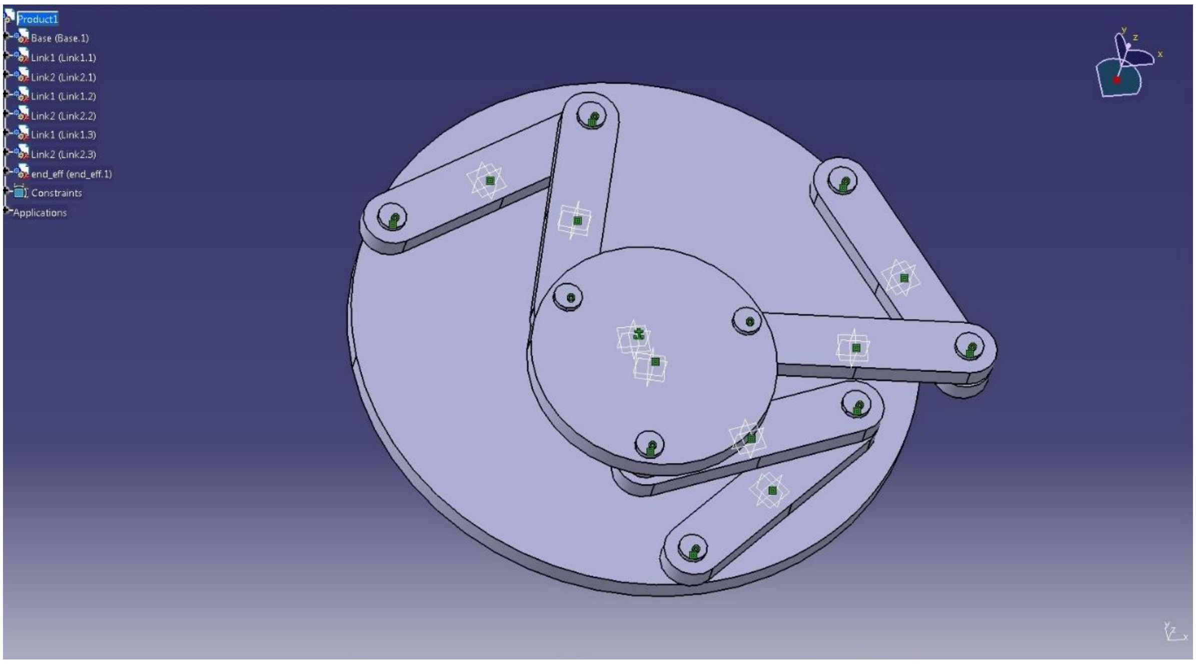
Thus, the five architectures present complexity values equal to:

$$K_{RPR} = 2.6881$$
$$K_{PRR} = K_{RRP} = 2.6321$$
$$K_{RRR} = K_{RRR} = 2.5292$$

As we can see, the architecture R$\underline{P}$R has the highest complexity value, while P$\underline{R}$R and $\underline{R}$RP have a slightly lower one. However, $\underline{R}$RR and R$\underline{R}$R have the lowest complexity value between all architectures synthetized. Consequently, we discarded the first three manipulators and considered only the remaining two. We then arbitrarily decided to choose $\underline{R}$RR and continued with the next and last part of our project: the realization of the model in CAD and its animation.

Embodiment: realization in CAD and animation

For this last step, we realized the CAD model by the means of the CATIA software previously used during the labs (see the files joint to the report). Note that during the realization process we tried to keep the parts as interchangeable as possible to satisfy the corollary axioms. We modelled only one part for the base, one for the moving platform, one for the connection between the first and the second joint of the leg and one for the connection between the remaining two. To find the optimal robot's proportions, we then applied the values found thanks to the optimization problem and obtained the architecture as it can be seen in 3RRRmanip.CATProduct, shown below:



Conclusions

During this project, we applied the concepts of design strategies previously analysed in class. We followed the Design Process Layout after French frame to design a three degrees of freedom planar parallel manipulator. Thus, we formulated the optimization problem to meet the specifications, finding the proper parameters' values to insert during the last step in the CAD model. During the evaluation of complexity phase, we selected the <u>R</u>RR architecture between the ones synthetized due to its lower value of complexity and then passed to the embodiment phase, implementing the parallel manipulator model in CATIA.