CONTROL METHODS

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Method#01 - Optimal control: Linear quadratic (LQ) trajectory-tracking problem

Let's consider a linear time-variant 1st order dynamic system

$$\dot{x} = ax + u + \tilde{f},$$
$$y = x + du. \qquad (1.01)$$

The **tracking error** for the dynamic system (1.01) in time t is

$$e(t) = y - z\,, \qquad (1.02)$$

The **control criterion** is

$$J = \frac{1}{2}\cdot f\cdot\left((x(T)-z(T)\right)^2 + \frac{1}{2}\cdot\int_0^T\left(q\cdot e^2 + r\cdot u^2\right)dt \qquad (1.03)$$

The matrix differential **Riccati equation** and a solution of the solution of the linear vector differential equation are (see [1])

$$\dot{k} = m\cdot k - 2\cdot l\cdot k - s,$$
$$\dot{g} = (k\cdot m - l)\cdot g + (k\cdot n - w)\cdot z + k\cdot\tilde{f}. \qquad (1.04)$$

The **boundary condition** for the equations (1.04) is

$$k(T) = f,$$
$$g(T) = f\cdot z(T).$$

The matrices of equations (1.04) are

$$l = a - q\cdot d/(1+q\cdot d^2),$$
$$m = 1/(1+q\cdot d^2),$$
$$n = q\cdot d/(1+q\cdot d^2),$$
$$s = q - q^2\cdot d^2/(1+q\cdot d^2),$$
$$w = q - q^2\cdot d^2/(1+q\cdot d^2).$$

The **optimal control** of the dynamic system (1.01) with the quadratic criterion (1.03) is

$$u^* = \frac{(g + q\cdot d\cdot z - (k + q\cdot d)\cdot x)}{(r + q\cdot d^2)}\,. \qquad (1.05)$$

Case study - Input data

$T = 8.5$ (s)

$a = -1, d = 1, \tilde{f} = 1,$

$f = 0.5, q = 50, r = 0.5$

$z(t) = 0.25p^3 + 0.75p^2 - 1.5p - 2,\ p \in [-5, 3.5]$.

Case study - Simulation results

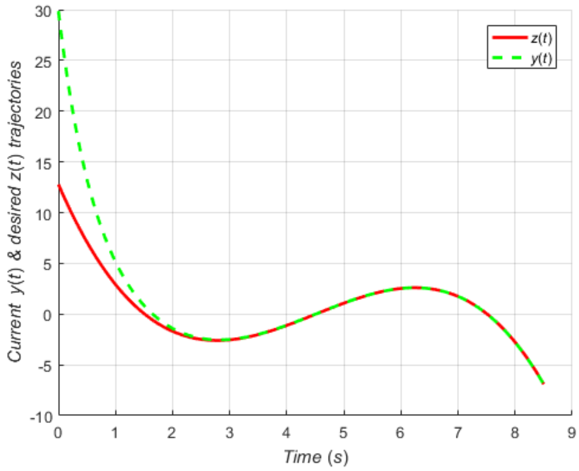


Figure 1.01 – Desired and current trajectories

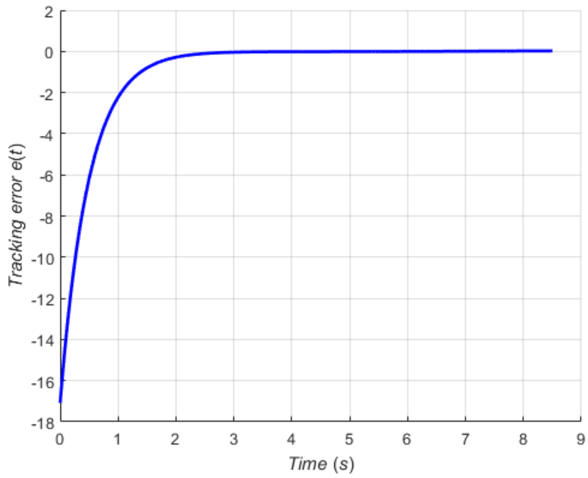


Figure 1.02 – Tracking error

Method#02 - PID control: Tuning PID controller of the LTI, SISO system

Let's consider the following UAV stabilization system

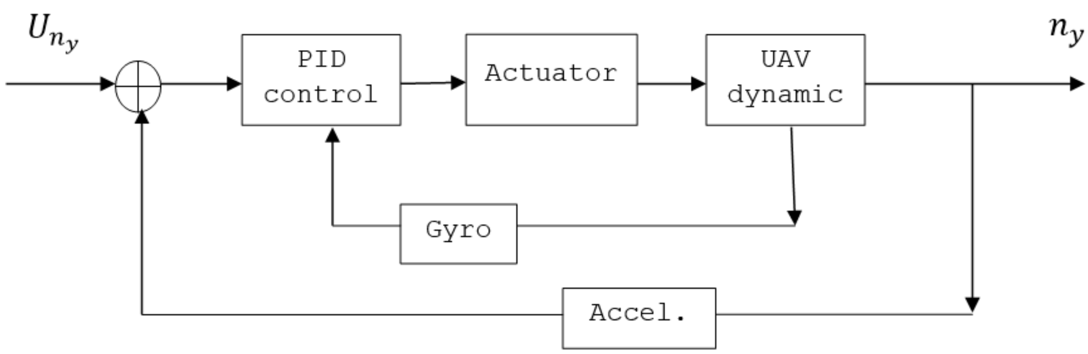


Fig.2.01 – Functional block-diagram of the UAV stabilization system

Assumptions

Measurement noise & errors of the Gyro and Accelerometer aren't taking into account in the model: $W_{gyro}(s)=1,\ W_{accel}(s)=1$.

…

PID controller

$$\delta(t) = K_P e(t) + K_D \omega_z(t) + K_I \int_0^T e(t)dt\ , \qquad (2.01)$$

Actuator

$$W_{act} = \frac{1}{T_{act}s+1}, \qquad (2.02)$$

where $T_{act} = \frac{1}{K_{act}}$ is actuator time constant, $K_{act} = 20$.

UAV dynamics

$$W_{\delta}^{\omega_z} = \frac{K(T_1 s+1)}{T_2^2 s^2 + 2\xi T_2 + 1},\ W_{\omega_z}^{\dot{\theta}} = \frac{1}{T_1 s+1},\ W_{\dot{\theta}}^{n_y} = \frac{V}{g}, \qquad (2.03)$$

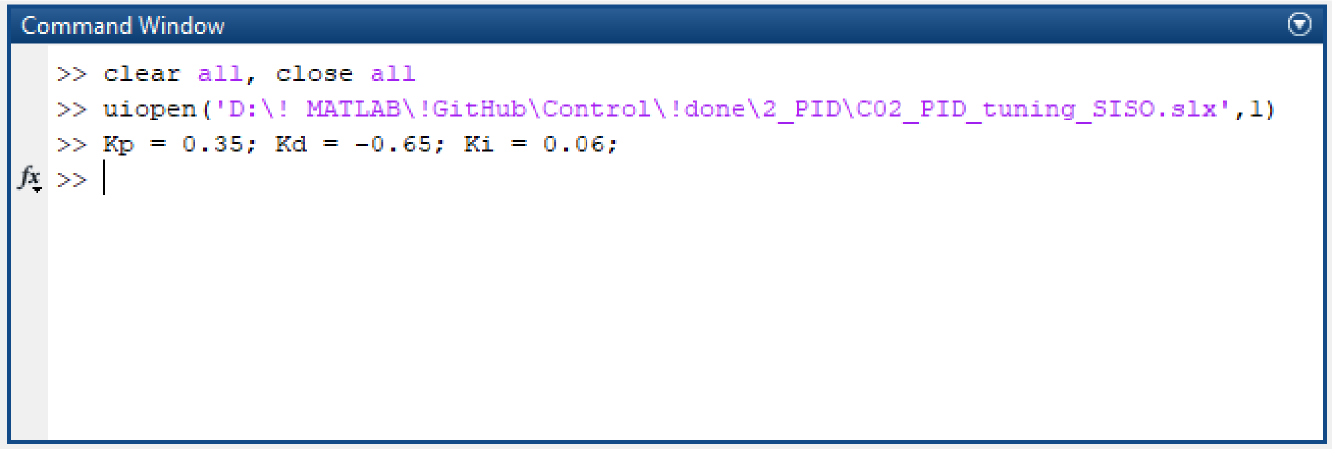
Where

$K=1,$

$T_1 =\ 0.7\ (s),\ T_2 = 0.5\ (s),$

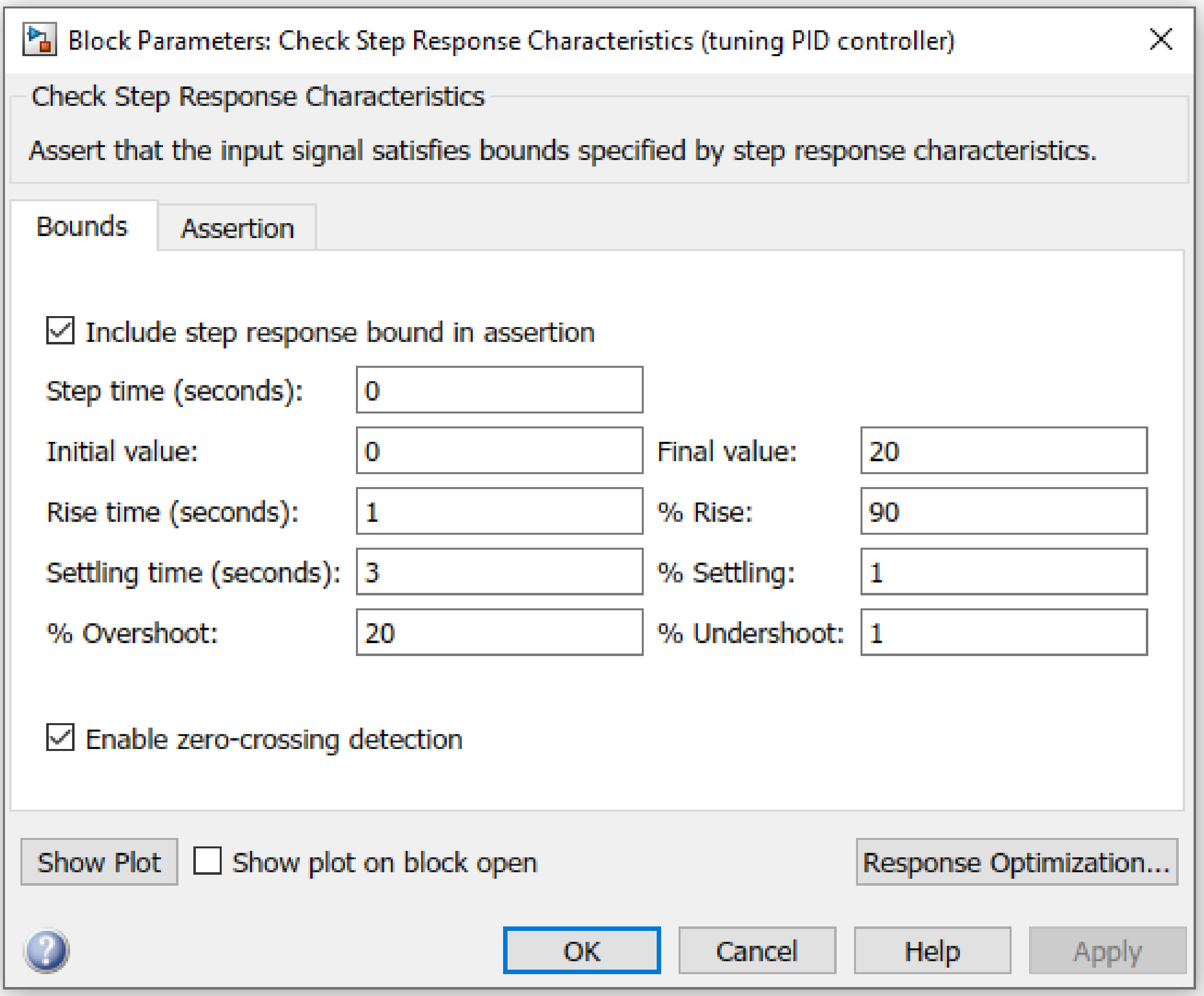
$\xi = 0.3.$

1st step - Initial PID coefficients load into Workspace

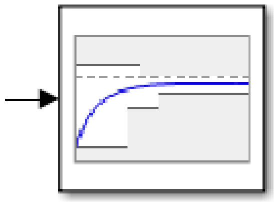


2nd step – Main characteristics of step response

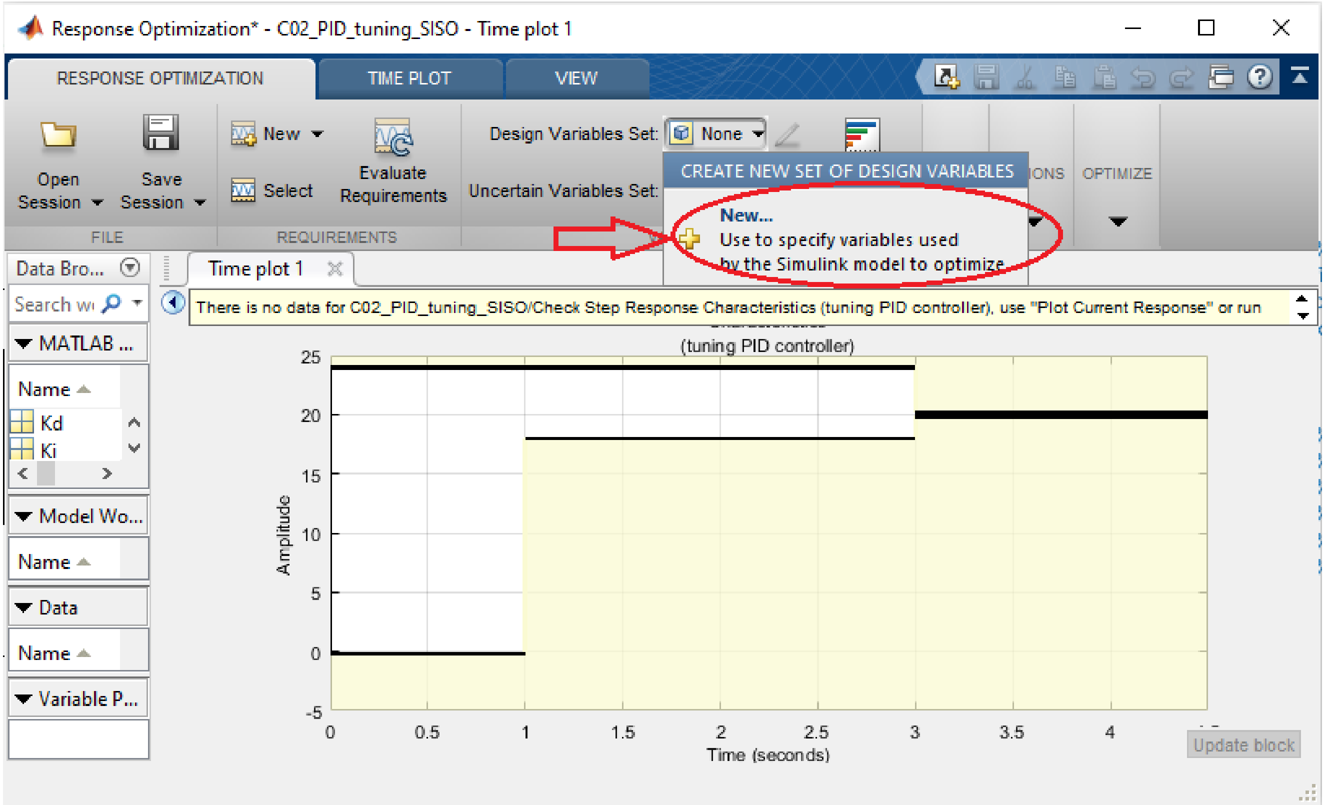
- Overshoot is calculated is 100%*[max(output value) – final value]/final value. Recommended value is 15…20%
- Settling time is the time it takes for the output signal to enter the error band)
- Rise time – it depends on inertia characteristics of an UAV

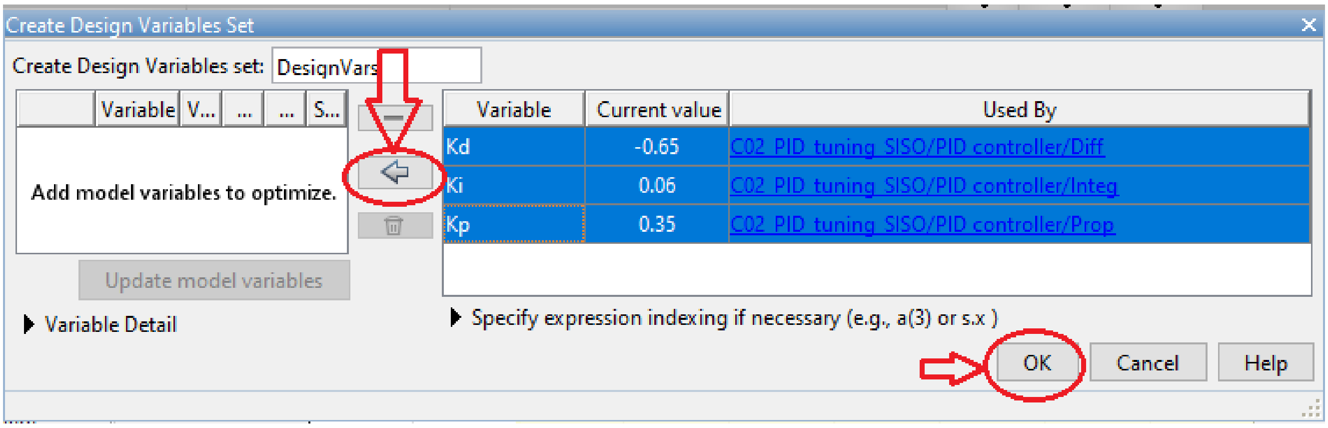


3td step – Response Optimization setting

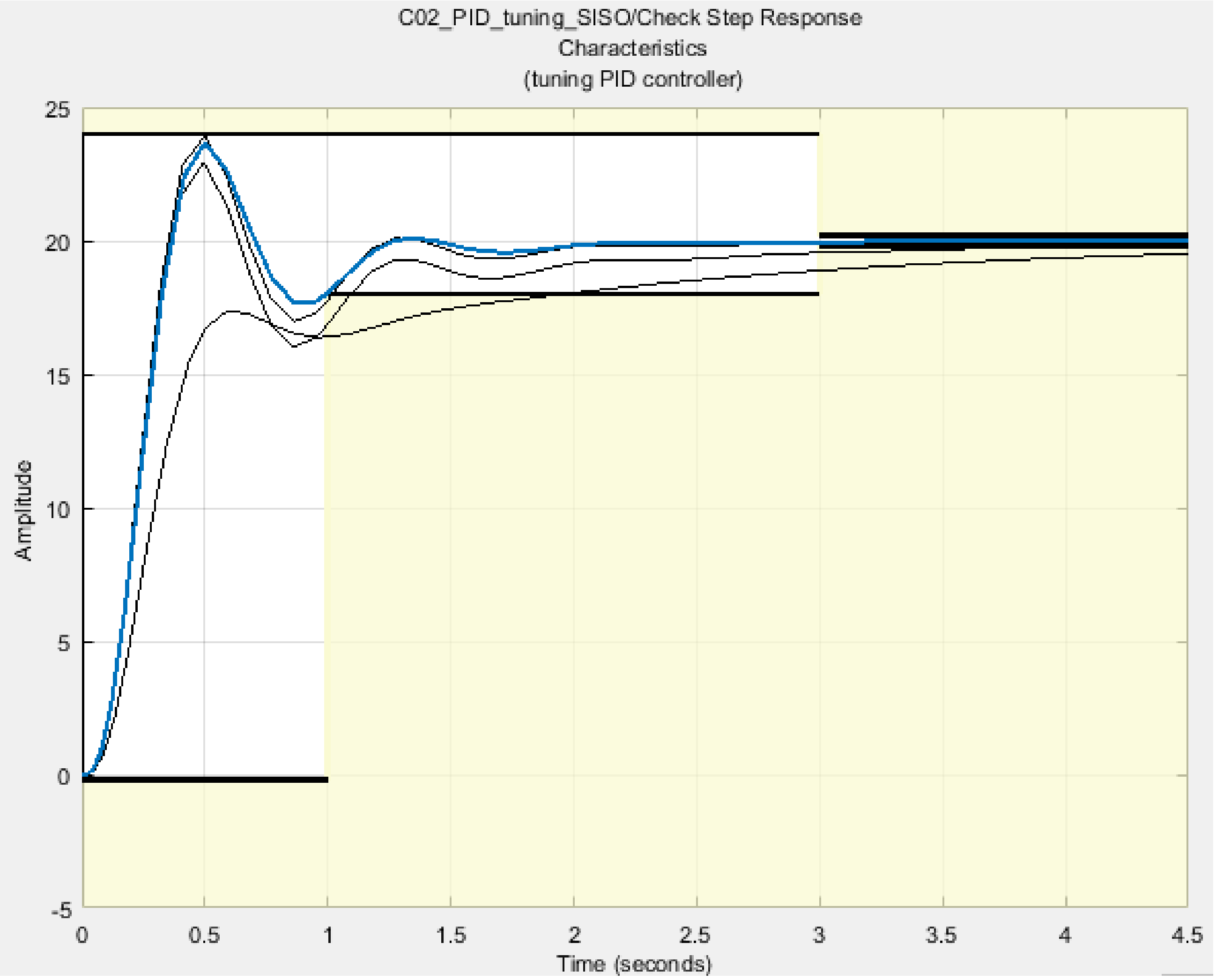


Check Step Response Characteristics (tuning PID controller)

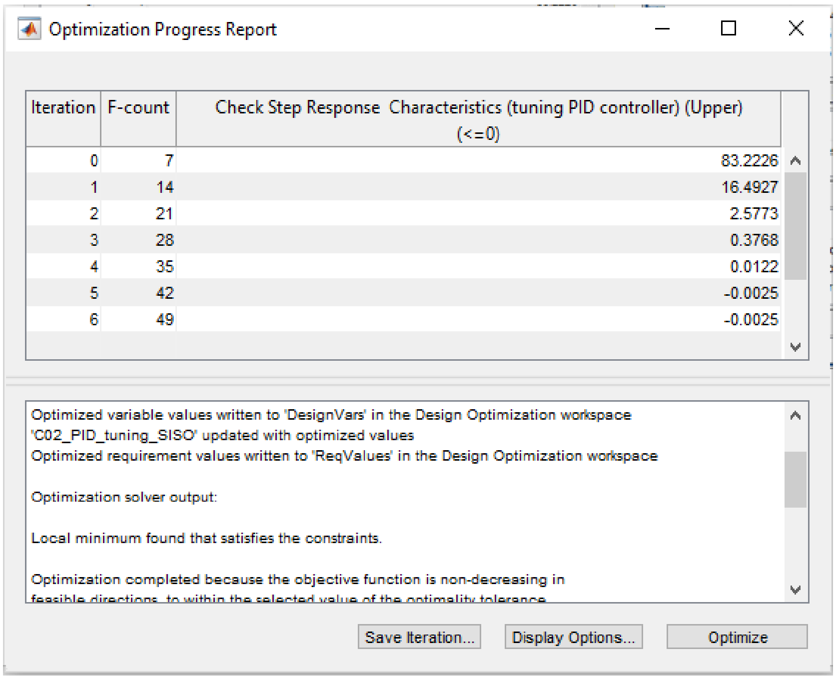


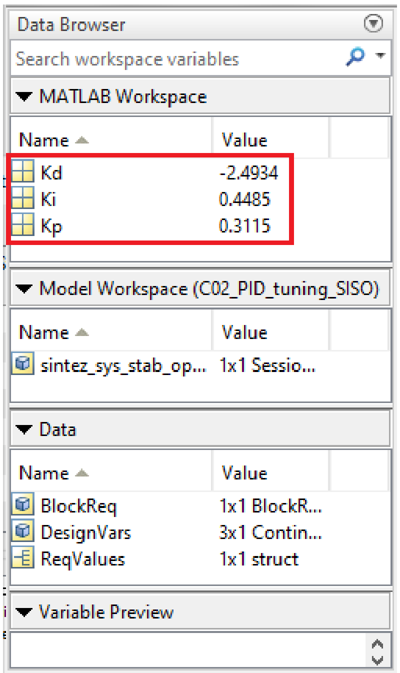


4th step – Optimize



5th step – Analysis of the optimization results





Method#03 – H infinity control: LTI system

…coming soon…

References

[1] V.Bobronnikov & M.Trifonov. Solving of the some special control problems of launch vehicle at the initial flight part using the AKOR method. In *AIP Conference Proceedings*. Vol.2318, No. 1, p. 110003. AIP Publishing LLC. 2021.