

CONTROL METHODS

Contents

Method#01 - Optimal control: Linear quadratic (LQ) trajectory-tracking problem	2
Method#02 - PID control: Tuning PID controller of the LTI, SISO system	4
Method#03 - H infinity control: LTI system	8
References	9

Method#01 - Optimal control: Linear quadratic (LQ) trajectory-tracking problem

Let's consider a linear time-variant 1st order dynamic system

$$\begin{aligned}\dot{x} &= ax + u + \tilde{f}, \\ y &= x + du.\end{aligned}\tag{1.01}$$

The **tracking error** for the dynamic system (1.01) in time t is

$$e(t) = y - z,\tag{1.02}$$

The **control criterion** is

$$J = \frac{1}{2} \cdot f \cdot ((x(T) - z(T))^2 + \frac{1}{2} \cdot \int_0^T (q \cdot e^2 + r \cdot u^2) dt)\tag{1.03}$$

The matrix differential **Riccati equation** and a solution of the solution of the linear vector differential equation are (see [1])

$$\begin{aligned}\dot{k} &= m \cdot k - 2 \cdot l \cdot k - s, \\ \dot{g} &= (k \cdot m - l) \cdot g + (k \cdot n - w) \cdot z + k \cdot \tilde{f}.\end{aligned}\tag{1.04}$$

The **boundary condition** for the equations (1.04) is

$$\begin{aligned}k(T) &= f, \\ g(T) &= f \cdot z(T).\end{aligned}$$

The matrices of equations (1.04) are

$$\begin{aligned}l &= a - q \cdot d / (1 + q \cdot d^2), \\ m &= 1 / (1 + q \cdot d^2), \\ n &= q \cdot d / (1 + q \cdot d^2), \\ s &= q - q^2 \cdot d^2 / (1 + q \cdot d^2), \\ w &= q - q^2 \cdot d^2 / (1 + q \cdot d^2).\end{aligned}$$

The **optimal control** of the dynamic system (1.01) with the quadratic criterion (1.03) is

$$u^* = \frac{(g + q \cdot d \cdot z - (k + q \cdot d) \cdot x)}{(r + q \cdot d^2)}.\tag{1.05}$$

Case study - Input data

$$\begin{aligned}T &= 8.5 \text{ (s)} \\ a &= -1, d = 1, \tilde{f} = 1, \\ f &= 0.5, q = 50, r = 0.5 \\ z(t) &= 0.25p^3 + 0.75p^2 - 1.5p - 2, p \in [-5, 3.5].\end{aligned}$$

Case study - Simulation results

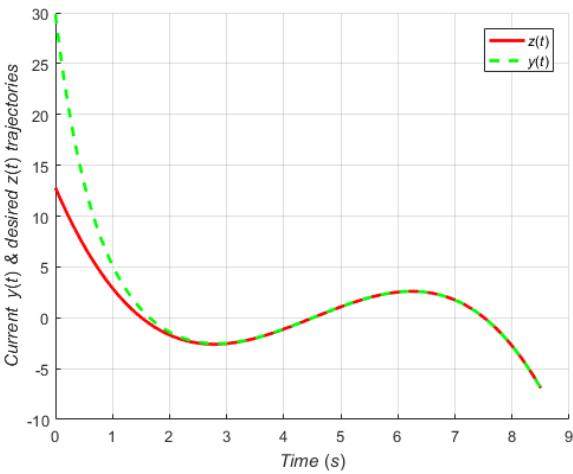


Figure 1.01 - Desired and current trajectories

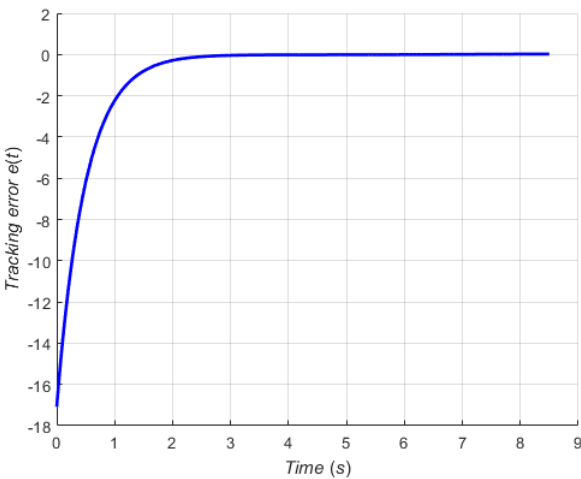


Figure 1.02 - Tracking error

Method#02 - PID control: Tuning PID controller of the LTI, SISO system

Let's consider the following UAV stabilization system

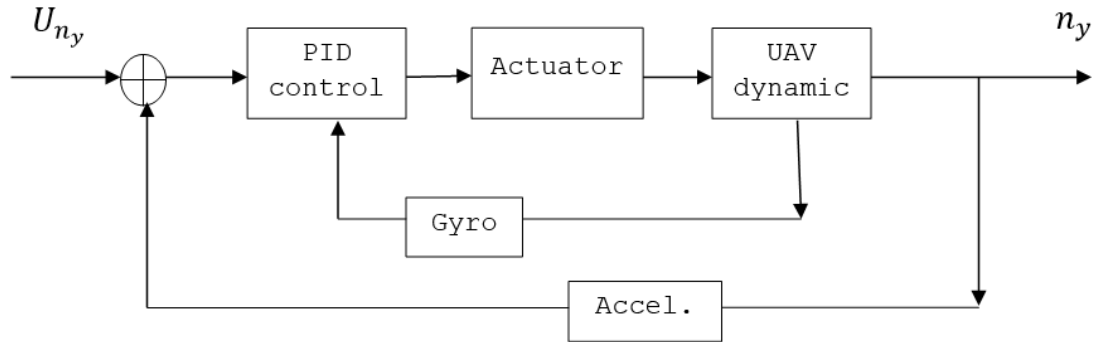


Fig.2.01 - Functional block-diagram of the UAV stabilization system

Assumptions

Measurement noise & errors of the Gyro and Accelerometer aren't taking into account in the model: $W_{gyro}(s)=1$, $W_{accel}(s)=1$.

...

PID controller

$$\delta(t) = K_p e(t) + K_D \dot{e}(t) + K_I \int_0^t e(t) dt, \quad (2.01)$$

Actuator

$$W_{act} = \frac{1}{T_{act}s + 1}, \quad (2.02)$$

where $T_{act} = \frac{1}{K_{act}}$ is actuator time constant, $K_{act} = 20$.

UAV dynamics

$$W_{\delta}^{\omega_z} = \frac{K(T_1 s + 1)}{T_2^2 s^2 + 2\xi T_2 s + 1}, \quad W_{\omega_z}^{\dot{\theta}} = \frac{1}{T_1 s + 1}, \quad W_{\dot{\theta}}^{n_y} = \frac{V}{g}, \quad (2.03)$$

Where

$$K = 1,$$

$$T_1 = 0.7 \text{ (s)}, \quad T_2 = 0.5 \text{ (s)},$$


$$\xi = 0.3.$$

1st step - Initial PID coefficients load into Workspace

```
Command Window
>> clear all, close all
>> uiopen('D:\! MATLAB\!GitHub\Control\!done\2_PID\C02_PID_tuning_SISO.slx',1)
>> Kp = 0.35; Kd = -0.65; Ki = 0.06;
fx >> |
```

2nd step - Main characteristics of step response

- Overshoot is calculated is
 $100\% \times [\max(\text{output value}) - \text{final value}] / \text{final value}$.
Recommended value is 15...20%
- Settling time is the time it takes for the output signal to enter the error band)
- Rise time - it depends on inertia characteristics of an UAV

 Block Parameters: Check Step Response Characteristics (tuning PID controller) ✕

Check Step Response Characteristics

Assert that the input signal satisfies bounds specified by step response characteristics.


Bounds Assertion

☒ Include step response bound in assertion

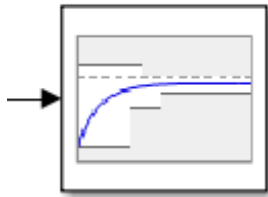
Step time (seconds):	<input type="text" value="0"/>		
Initial value:	<input type="text" value="0"/>	Final value:	<input type="text" value="20"/>
Rise time (seconds):	<input type="text" value="1"/>	% Rise:	<input type="text" value="90"/>
Settling time (seconds):	<input type="text" value="3"/>	% Settling:	<input type="text" value="1"/>
% Overshoot:	<input type="text" value="20"/>	% Undershoot:	<input type="text" value="1"/>

☒ Enable zero-crossing detection

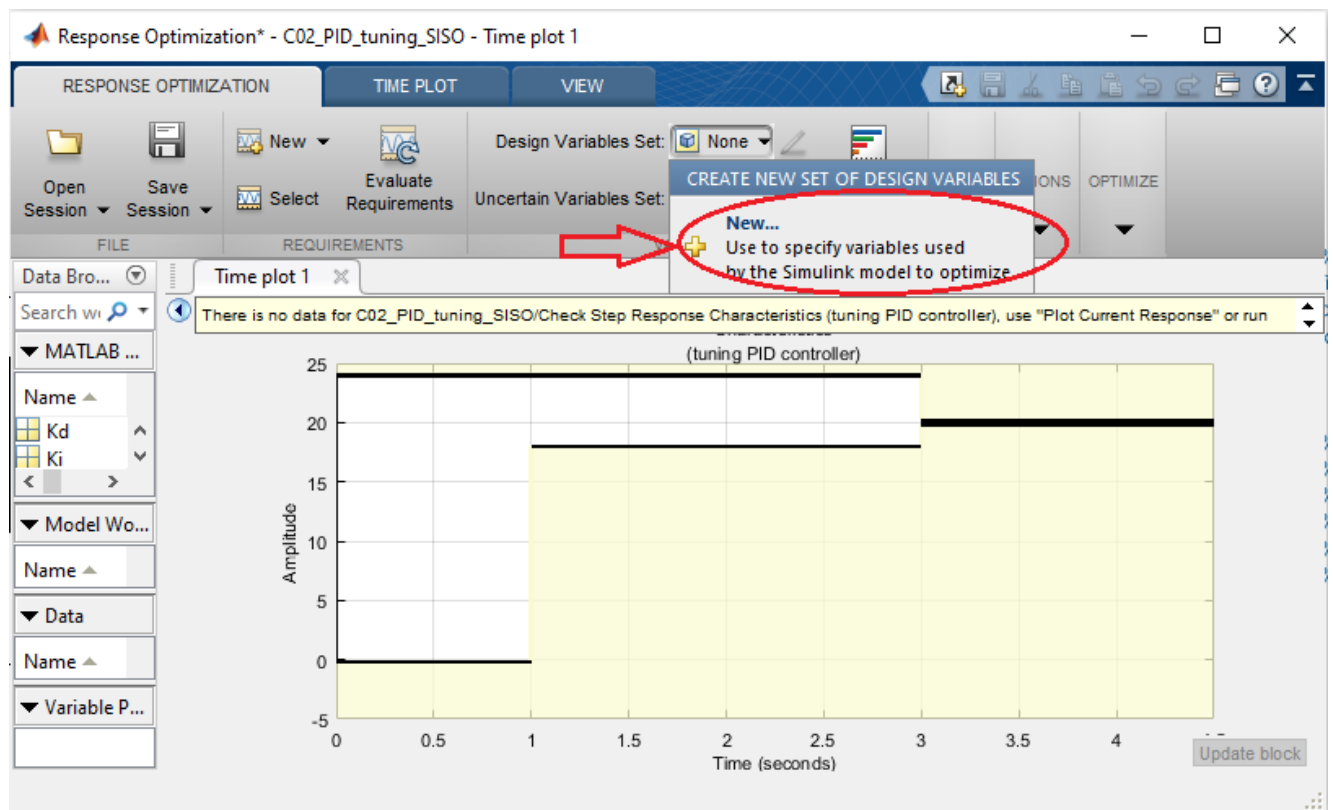
Show Plot ☐ Show plot on block open Response Optimization...

 OK Cancel Help Apply

3rd step - Response Optimization setting



Check Step Response
Characteristics
(tuning PID controller)



Create Design Variables Set

Create Design Variables set: DesignVars

Add model variables to optimize.

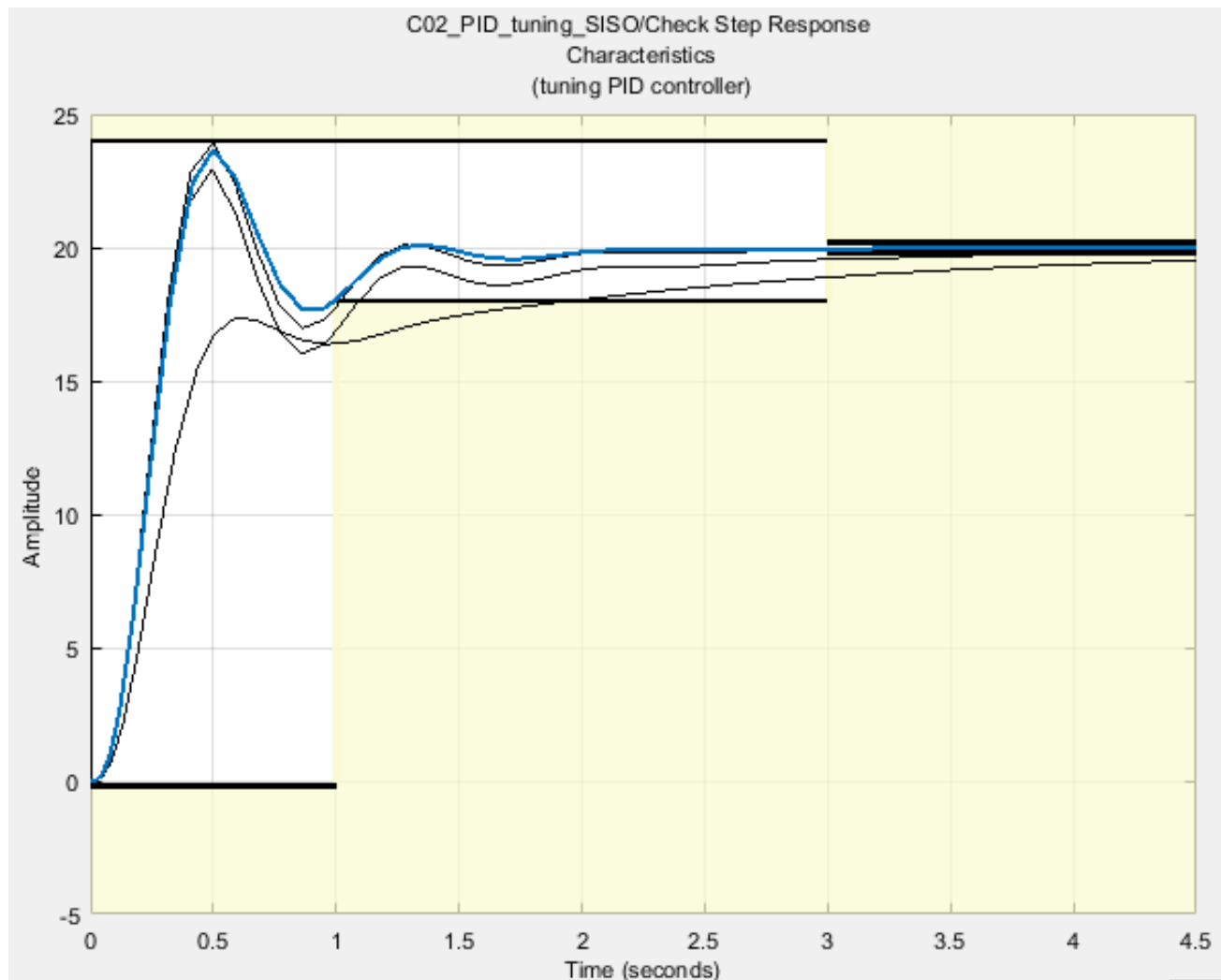
Update model variables

Variable	Current value	Used By
Kd	-0.65	C02_PID_tuning_SISO/PID controller/Diff
Ki	0.06	C02_PID_tuning_SISO/PID controller/Integ
Kp	0.35	C02_PID_tuning_SISO/PID controller/Prop

Specify expression indexing if necessary (e.g., a(3) or s.x)

OK Cancel Help

4th step - Optimize



5th step - Analysis of the optimization results

Optimization Progress Report

Iteration	F-count	Check Step Response Characteristics (tuning PID controller) (Upper) (<=0)
0	7	83.2226
1	14	16.4927
2	21	2.5773
3	28	0.3768
4	35	0.0122
5	42	-0.0025
6	49	-0.0025

Optimized variable values written to 'DesignVars' in the Design Optimization workspace
'C02_PID_tuning_SISO' updated with optimized values
Optimized requirement values written to 'ReqValues' in the Design Optimization workspace

Optimization solver output:

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in feasible directions to within the selected value of the optimality tolerance.

Save Iteration... Display Options... Optimize

Data Browser

Search workspace variables

MATLAB Workspace

Name	Value
Kd	-2.4934
Ki	0.4485
Kp	0.3115

Model Workspace (C02_PID_tuning_SISO)

Name	Value
sintez_sys_stab_op...	1x1 Sessio...

Data

Name	Value
BlockReq	1x1 BlockR...
DesignVars	3x1 Contin...
ReqValues	1x1 struct

Variable Preview

Method#03 - H infinity control: LTI system

...coming soon...

References

[1] V.Bobronnikov & M.Trifonov. Solving of the some special control problems of launch vehicle at the initial flight part using the AKOR method. In *AIP Conference Proceedings*. Vol.2318, No. 1, p. 110003. AIP Publishing LLC. 2021.