Method#01 - Optimal control: Linear quadratic (LQ) trajectory-tracking problem

Let's consider a linear time-variant 1st order dynamic system

$$\dot{x} = ax + u + \tilde{f}, \qquad (1.01)$$
$$y = x + du.$$

The **tracking error** for the dynamic system (1.01) in time t is

$$e(t) = y - z, \qquad (1.02)$$

The **control criterion** is

$$J = \frac{1}{2} \cdot f \cdot \left((x(T) - z(T))\right)^2 + \frac{1}{2} \cdot \int_0^T \left(q \cdot e^2 + r \cdot u^2\right) dt \qquad (1.03)$$

The matrix differential **Riccati equation** and a solution of the solution of the linear vector differential equation [1] are

$$\dot{k} = m \cdot k - 2 \cdot l \cdot k - s, \qquad (1.04)$$
$$\dot{g} = (k \cdot m - l) \cdot g + (k \cdot n - w) \cdot z + k \cdot \tilde{f}.$$

The **boundary condition** for the equations (1.04) is

$$k(T) = f,$$
$$g(T) = f \cdot z(T).$$

The matrices of equations (1.04) are

$$l = a - q \cdot d / (1 + q \cdot d^2),$$
$$m = 1 / (1 + q \cdot d^2),$$
$$n = q \cdot d / (1 + q \cdot d^2),$$
$$s = q - q^2 \cdot d^2 / (1 + q \cdot d^2),$$
$$w = q - q^2 \cdot d^2 / (1 + q \cdot d^2).$$

The **optimal control** of the dynamic system (1.01) with the quadratic criterion (1.03) is

$$u^* = \frac{(g + q \cdot d \cdot z - (k + q \cdot d) \cdot x)}{(r + q \cdot d^2)}. \qquad (1.05)$$

Case study - Input data

$T = 8.5$ (s)

$a = -1, d = 1, \tilde{f} = 1,$

$f = 0.5, q = 50, r = 0.5$

$z(t) = 0.25p^3 + 0.75p^2 - 1.5p - 2,\ p \in [-5, 3.5]$.

Case study - Simulation results

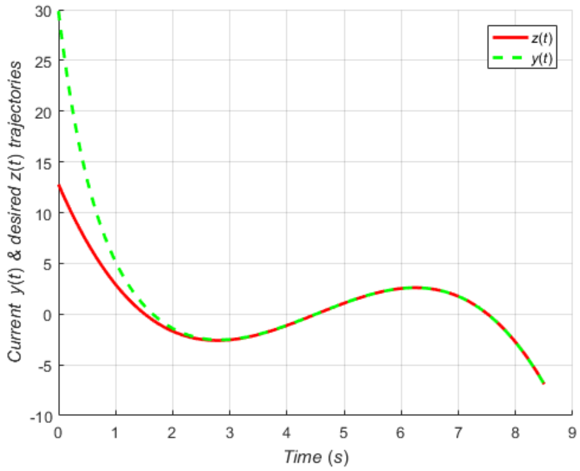

Figure 1.01 – Desired and current trajectories

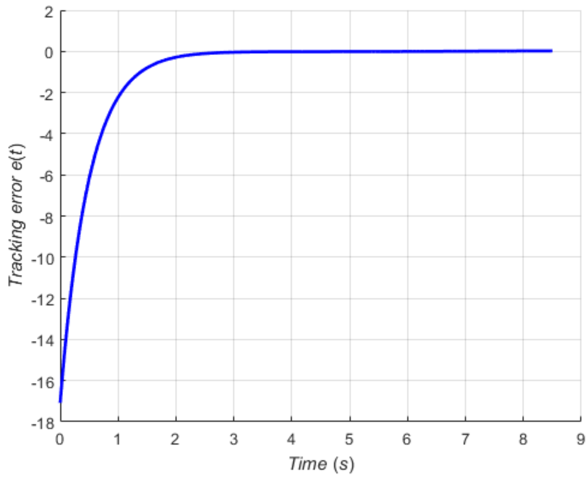

Figure 1.02 – Tracking error

References

[1] V.Bobronnikov & M.Trifonov. Solving of the some special control problems of launch vehicle at the initial flight part using the AKOR method. In *AIP Conference Proceedings*. Vol.2318, No. 1, p. 110003. AIP Publishing LLC. 2021.