Method#01 – Zero-bearing guidance method

Initial state of the distance between a vehicle#1 and a vehicle#2

$$r_0 = \sqrt{(Y_{C0} - Y_0)^2 + (X_{C0} - X_0)^2}. \qquad (1.01)$$

Initial state of the sight angle

$$\varphi_0 = arctg\left(\frac{Y_{C0} - Y_0}{X_{C0} - X_0}\right). \qquad (1.02)$$

Kinematic equations of the center of mass motion

$$\begin{aligned} \frac{dr}{dt} &= V_C \cos(\varphi - \theta_C) - V\cos(\varphi - \theta), \\ r\frac{d\varphi}{dt} &= -V_C \sin(\varphi - \theta_C) + V\sin(\varphi - \theta), \end{aligned} \qquad (1.03)$$

where φ is a sight angle.

Dynamic equations of angular motion

$$\frac{d\theta}{dt} = \frac{P + C_Y^{\alpha} S\rho(H)V^2/2}{mV}\alpha. \qquad (1.04)$$

Equation for a calculation of ideal constraint

$$\varepsilon = \zeta^* - \zeta, \qquad (1.05)$$

where ζ is the bearing angle.

Condition for zero-bearing guidance method is $\zeta^* = 0$.

Constraint equation is

$$\alpha = \varphi - \theta\ . \qquad (1.06)$$

Equations for a calculation of vehicles coordinates

$$\begin{aligned} \frac{dX}{dt} &= \frac{dL}{dt} = V\cos\theta, \quad \frac{dX_C}{dt} = \frac{dL_C}{dt} = V_C\cos\theta_C, \\ \frac{dY}{dt} &= \frac{dH}{dt} = V\sin\theta, \quad \frac{dY_C}{dt} = \frac{dH_C}{dt} = V_C\sin\theta_C. \end{aligned} \qquad (1.07)$$

Equation for a calculation of a g-force

$$n_y = \frac{V}{g}\dot{\theta} + \cos\theta \qquad (1.08)$$

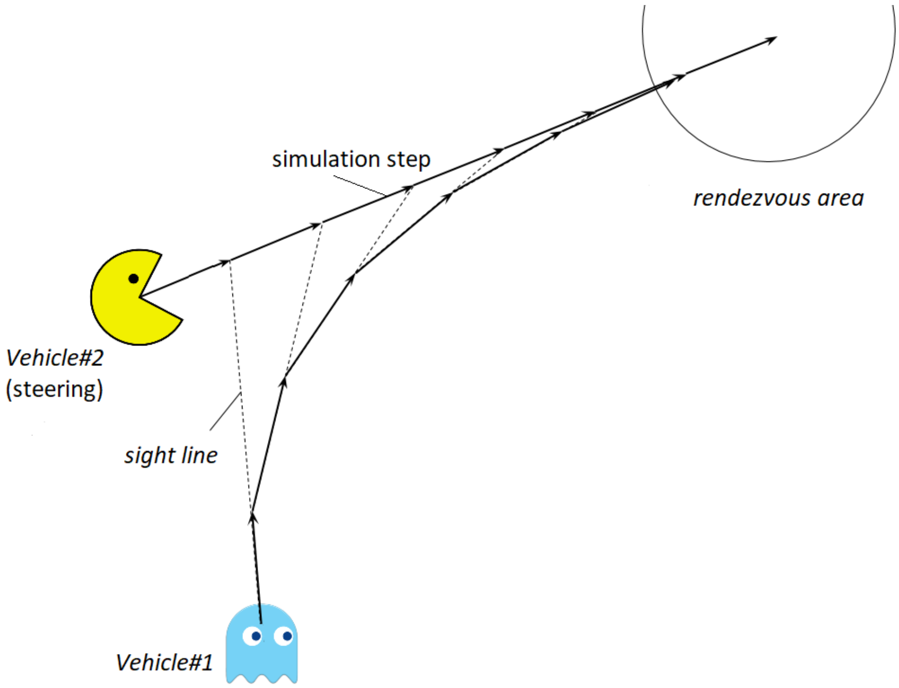


Figure 1.01 Visualization of the zero-bearing guidance method

Method#02 – Fixed lead angle guidance method

(constant bearing guidance method)

Initial state of the distance between a vehicle#1 and a vehicle#2

$$r_0 = \sqrt{(Y_{C0} - Y_0)^2 + (X_{C0} - X_0)^2}. \tag{2.01}$$

Initial state of the sight angle

$$\varphi_0 = arctg\left(\frac{Y_{C0} - Y_0}{X_{C0} - X_0}\right). \tag{2.02}$$

Kinematic equations of the center of mass motion

$$\begin{aligned} \frac{dr}{dt} &= V_C \cos(\varphi - \theta_C) - V \cos(\varphi - \theta), \\ r\frac{d\varphi}{dt} &= -V_C \sin(\varphi - \theta_C) + V \sin(\varphi - \theta). \end{aligned} \tag{2.03}$$

Equation for a calculation of ideal constraint

$$\varepsilon = \eta^* - \eta = 0 \tag{2.04}$$

Condition for fixed lead angle guidance method is $\eta^* = const \ (\varphi = const)$.

Constraint equation is

$$\theta = \varphi - \eta \tag{2.05}$$

It follows from the equations (2.03) and (2.05) assuming $\dot{\varphi}=0$, the lead angle is

$$\eta = \arcsin\left(\frac{V_C}{V}\sin(\varphi-\theta_C)\right) \tag{2.06}$$

For each lead angle η, two straight-line trajectories can be found by condition, oriented at angles φ_1 and φ_2

$$\begin{aligned}\varphi_1 &= \arcsin\left(\frac{V}{V_C}\sin\eta\right),\\ \varphi_2 &= \pi-\arcsin\left(\frac{V}{V_C}\sin\eta\right).\end{aligned} \tag{2.07}$$

Equation for a calculation of a g-force

$$n_y = \frac{V\dot{\theta}}{g} = \frac{V}{g}\left(\frac{V_c\sin\varphi - V\sin\eta}{r}\right) \tag{2.07}$$

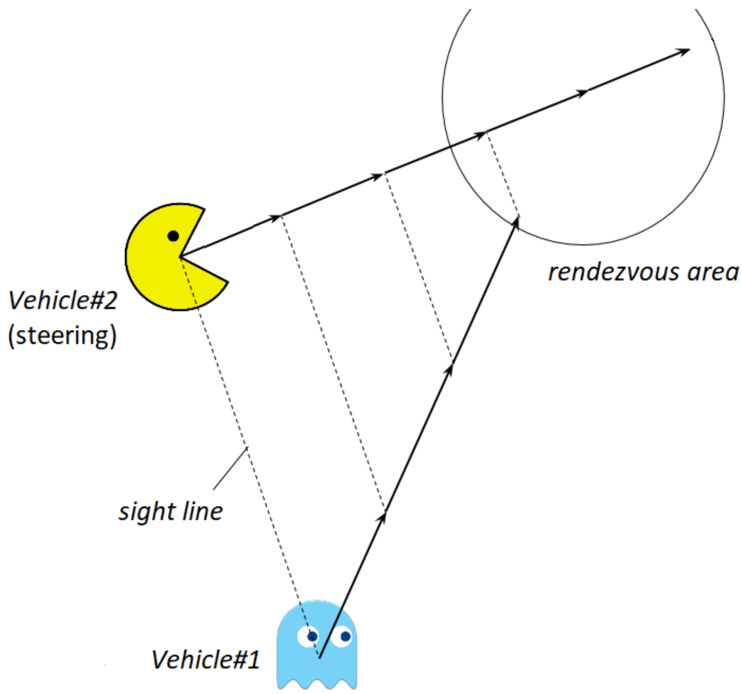


Figure 2.01 Visualization of the fixed lead angle guidance method

Case study - Input data

Vehicle#1	Vehicle#2 (steering)
$V = 630\ (m/s)$	$V_C = 300\ (m/s)$
$X_0 = 0\ (m)$, $Y_0 = 15000\ (m)$	$X_{C0} = 12000\ (m)$
$S = 0.3\ (m^2)$, $m = 350\ (kg)$	$Y_C = 9000\ (m)$
$P = 5250\ (N)$, $C_y^{\alpha} = 4$	$\theta_C = -115\ (grad)$

Case study - Simulation results

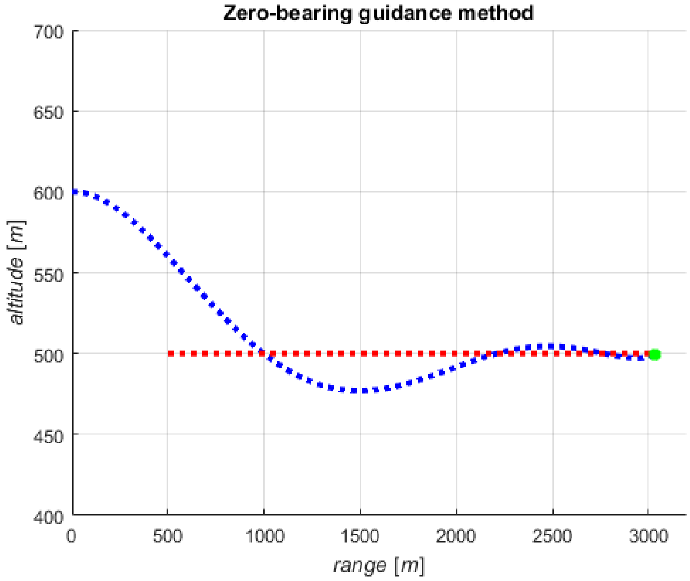


Figure 3.01 Docking trajectory (method#1)

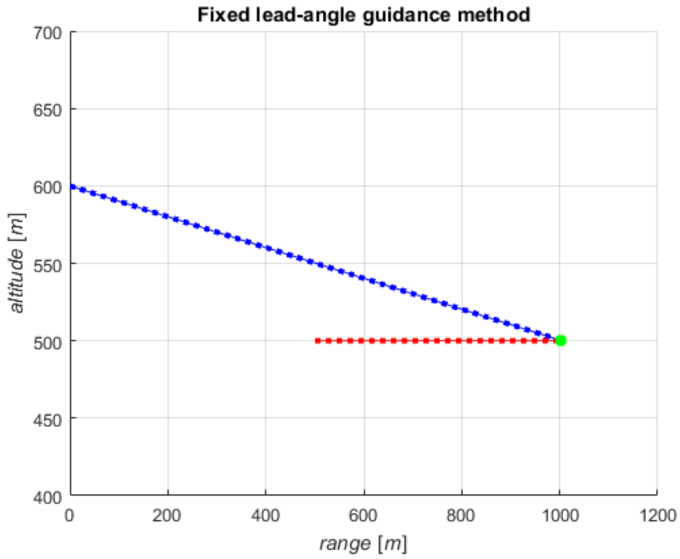
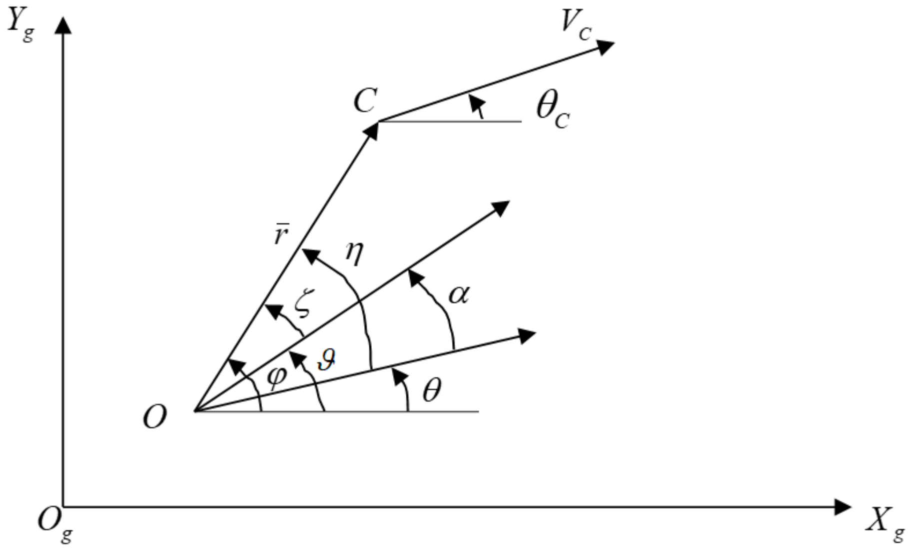


Figure 4.01 Docking trajectory (method#2)

Appendix A. Coordinate frame and angles



References

[1] https://www.red3d.com/cwr/steer/gdc99/