

Method#01 - Zero-bearing guidance method

Initial state of the distance between a vehicle#1 and a vehicle#2

$$r_0 = \sqrt{(Y_{c0} - Y_0)^2 + (X_{c0} - X_0)^2}. \quad (1.01)$$

Initial state of the sight angle

$$\varphi_0 = \arctg\left(\frac{Y_{c0} - Y_0}{X_{c0} - X_0}\right). \quad (1.02)$$

Kinematic equations of the center of mass motion

$$\begin{aligned} \frac{dr}{dt} &= V_c \cos(\varphi - \theta_c) - V \cos(\varphi - \theta), \\ r \frac{d\varphi}{dt} &= -V_c \sin(\varphi - \theta_c) + V \sin(\varphi - \theta), \end{aligned} \quad (1.03)$$

where φ is a sight angle.

Dynamic equations of angular motion

$$\frac{d\theta}{dt} = \frac{P + C_Y^\alpha S \rho(H) V^2 / 2}{mV} \alpha. \quad (1.04)$$

Equation for a calculation of ideal constraint

$$\varepsilon = \zeta^* - \zeta, \quad (1.05)$$

where ζ is the lead angle.

Condition for zero-bearing guidance method is $\zeta^* = 0$.

Constraint equation is

$$\alpha = \varphi - \theta. \quad (1.06)$$

Equations for a calculation of vehicles coordinates

$$\begin{aligned} \frac{dX}{dt} = \frac{dL}{dt} &= V \cos \theta, & \frac{dX_c}{dt} = \frac{dL_c}{dt} &= V_c \cos \theta_c, \\ \frac{dY}{dt} = \frac{dH}{dt} &= V \sin \theta, & \frac{dY_c}{dt} = \frac{dH_c}{dt} &= V_c \sin \theta_c. \end{aligned} \quad (1.07)$$

Equation for a calculation of a g-force

$$n_y = \frac{V}{g} \dot{\theta} + \cos \theta \quad (1.08)$$

Method#02 - Fixed lead angle guidance method

Initial state of the distance between a vehicle#1 and a vehicle#2

$$r_0 = \sqrt{(Y_{c0} - Y_0)^2 + (X_{c0} - X_0)^2}. \quad (2.01)$$

Initial state of the sight angle

$$\varphi_0 = \arctg\left(\frac{Y_{c0} - Y_0}{X_{c0} - X_0}\right). \quad (2.02)$$

Kinematic equations of the center of mass motion

$$\begin{aligned} \frac{dr}{dt} &= V_c \cos(\varphi - \theta_c) - V \cos(\varphi - \theta), \\ r \frac{d\varphi}{dt} &= -V_c \sin(\varphi - \theta_c) + V \sin(\varphi - \theta). \end{aligned} \quad (2.03)$$

Equation for a calculation of ideal constraint

$$\varepsilon = \eta^* - \eta = 0 \quad (2.04)$$

Condition for fixed lead angle guidance method is $\eta^* = \text{const}$ ($\varphi = \text{const}$).

Constraint equation is

$$\theta = \varphi - \eta \quad (2.05)$$

It follows from the equations (2.03) and (2.05) assuming $\dot{\varphi} = 0$, the lead angle is

$$\eta = \arcsin\left(\frac{V_c}{V} \sin(\varphi - \theta_c)\right) \quad (2.06)$$

For each lead angle η , two straight-line trajectories can be found by condition, oriented at angles φ_1 and φ_2

$$\begin{aligned} \varphi_1 &= \arcsin\left(\frac{V}{V_c} \sin \eta\right), \\ \varphi_2 &= \pi - \arcsin\left(\frac{V}{V_c} \sin \eta\right). \end{aligned} \quad (2.07)$$

Equation for a calculation of a g-force

$$n_y = \frac{V \dot{\theta}}{g} = \frac{V}{g} \left(\frac{V_c \sin \varphi - V \sin \eta}{r} \right) \quad (2.07)$$

Case study - Input data

Vehicle#1	Vehicle#2
$V = 630 \text{ (m / s)}$	$V_C = 300 \text{ (m / s)}$
$X_0 = 0 \text{ (m)} , \quad Y_0 = 15000 \text{ (m)}$	$X_{C0} = 12000 \text{ (m)}$
$S = 0.3 \text{ (m}^2\text{)} , \quad m = 350 \text{ (kg)}$	$Y_C = 9000 \text{ (m)}$
$P = 5250 \text{ (N)} , \quad C_y^\alpha = 4$	$\theta_C = -115 \text{ (grad)}$

Case study - Simulation results

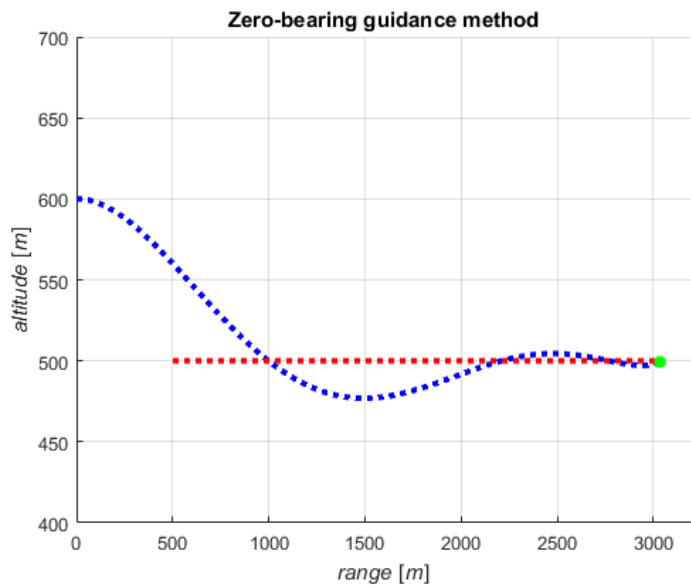


Figure 1.01 Docking trajectory (method#1)

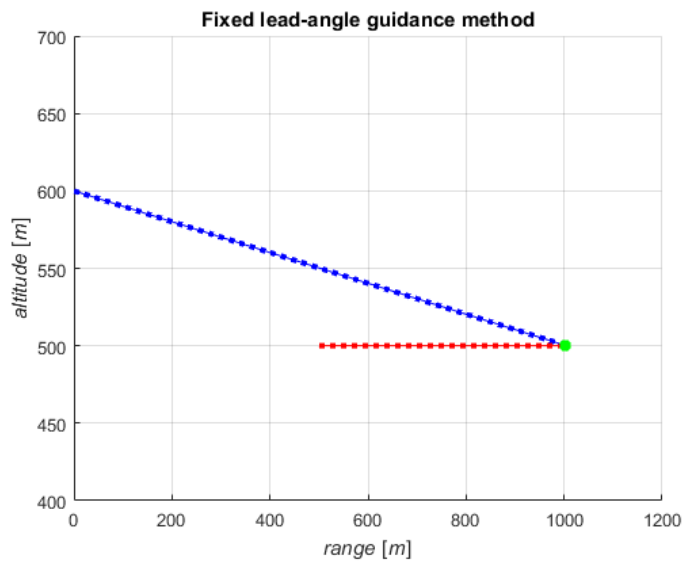
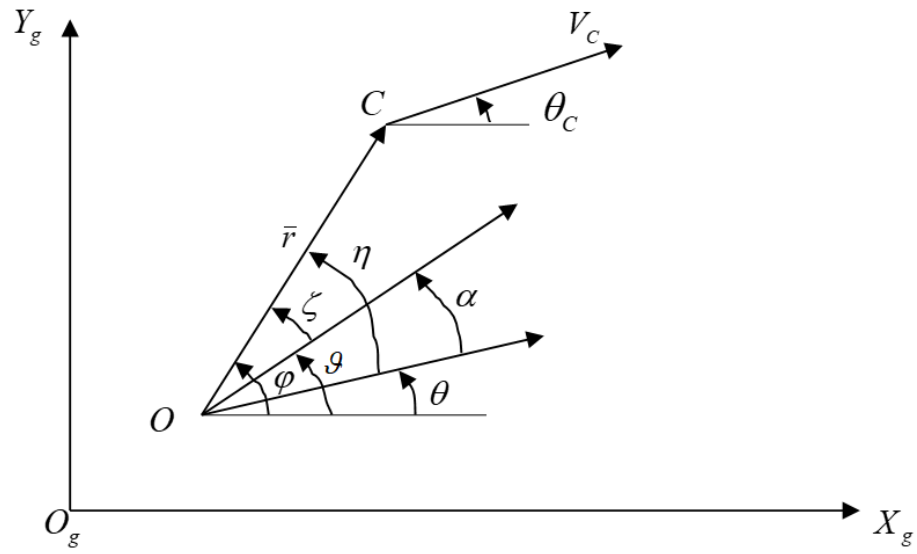


Figure 2.01 Docking trajectory (method#2)

Appendix A. Coordinate frame and angles



References

- [1] <https://www.red3d.com/cwr/steer/gdc99/>