# EC1340 – Fall 2018

Midterm

10:30-11:50am, October 23, 2018

Matt Turner

*You will have 80 minutes to complete this exam. No notes or books are allowed but you may use a calculator. Cell phones and any device with a wireless connection must be off. Anyone still working on their exam after time is called will be subject to an automatic 10 point penalty.*

*When you write up your answers, your goal should be to (1) be correct, and (2) convince your reader that your answer is correct. Answers which do not achieve these goals will not be awarded full credit. To accomplish the second objective, it is helpful if your work is legible and if all steps are presented, possibly with a line of explanation. Total points =100/Share of total grade =25% .*

***This exam has TWO pages.***

1. (30 points in total) The most recent IPCC report, "Global Warming of 1.5C" makes the case that climate change will begin to have important economic impacts starting in about 30 years. This problem asks you to think about the mitigation costs we should be willing to accept now in order to avoid climate damage in 30 years.

   Let $t = 0,1,2,...$ index years. Suppose that one ton of $CO_2$ emissions today causes 0\$ of damage for $t < 30$ and 100\$ of damage for $t \geq 30$. Let $M$ denote the amount spent on mitigation at $t = 0$. If the interest rate is $r$ how much will a planner who maximizes the discount present value of consumption be willing to spend on abatement to reduce future damage to zero.

2. (30 points in total ) Dell, Jones and Olken derive the following relationship between economic growth rates and climate,

   $$g_{it} = g_i + (\beta + \gamma)T_{it} - \beta T_{it-1},$$

   and estimate

   $$g_{it} = B_0 + (-0.9)T_{it} + (-0.4)T_{it-1} + \epsilon_{it}$$

   for a sample of poor countries (this is from Table 3, Column 2).

   Consider a series of annual temperature shocks, $(T_{i0},T_{i1},T_{i2},T_{i3},T_{i4},T_{i5}) = (0,0,1/2,1/2,1,1)$ and a country for which $g_{i0} = 0$, $Y_{i0} = 1$ and $L_{it} = 1$ for all $t$.

   (a) (25) Using estimates above, evaluate and plot the path of $1 + g_{it}$ and $Y_{it}$ for $t = 0,...,5$ for $i$ a poor country.

   (b) (5) On the basis of your work above, what do the results above suggest is the effect of this particular path of climate change.

   Hint: Don't forget that $g_{it}$ is a percentage.

3. (22 points in total) Consider the 'savings problem' that we discussed in class. That is,

$$\begin{aligned} \max_{s,c_1} \; & \frac{c_1^{1-\eta}}{1-\eta} + \frac{1}{1+\rho}\frac{c_2^{1-\eta}}{1-\eta} \\ \text{s.t. } & W = c_1 + s \\ & c_2 = (1+r)s \end{aligned}$$

where $c_t$ is consumption in period $t$, $s$ is savings, $W$ is initial income, $\rho > 0$ is the pure rate of time preference, $0 < \eta < 1$ is inequality or risk aversion, and $r > \rho$ is the rate of return to capital. Define $g = c_2/c_1 - 1$ to be the rate of consumption growth.

   (a) (15) Derive derive the equation, $\eta g + \rho = r$, that we talked about in class. *Hints: (1) solve the constraints so that you can write $c_2$ as a function of $c_1$, (2) organize your first order condition so that you recognize the expression for $c_2$ from part (1) and substitute, (3) solve for the ratio of $c_2/c_1$ and substitute with the expression for $g$ (4) take logarithms and recall that $ln(1+x) \sim x$ for $x$ small.*

   (b) (5) Restate the equation $\eta g + \rho = r$ in words.

   (c) (2) In a few sentences, explain why this relationship is important to an analysis of the problem of climate change?

4. (18 points in total) We discussed each of the following three papers in class

   (a) *Adapting to Climatic Challenges: A Progress Report on Studies of the Historical Evolution of Wheat Production*, unpublished working paper by Olmstead and Rhode, 2011.

   (b) *Civil conflicts are associated with the global climate*, published in Nature by Hsiang, Meng and Cane, 2011.

   (c) *Global climate change, war and population decline in recent human history*, published in the Proceedings of the National Academy of Science (PNAS), by Zhang et al., 2007

   For each paper provide a one or two sentence description of; (1) the data on which the paper is based (2 points per paper),(2) the conclusion that the paper reaches (2 points per paper), and (3) why this conclusion is relevant to the economics of climate change.(2 points per paper).

   A good answer need not be longer than three or four sentences.

# EC1340-Fall 2018

Midterm solutions
October 23, 2018
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1. The discounted present value of damage is:

$$\begin{aligned}
&\sum_{t=30}^{\infty} \delta^t 100 \\
&= \delta^{30} \sum_{t=0}^{\infty} \delta^t 100 \\
&= 100 \frac{\delta^{30}}{1-\delta}
\end{aligned}$$

But $\delta = 1/(1+r)$, so this equals

$$\begin{aligned}
&= 100 \frac{(\frac{1}{1+r})^{30}}{1-(\frac{1}{1+r})} \\
&= 100 \frac{1}{r} \left( \frac{1}{1+r} \right)^{29}
\end{aligned}$$

It follows that a planner who maximizes the discounted present value of income should be willing to spend anything up to this amount in climate change mitigation in order to avoid this sequence of damages.

2. (a) We have that $g_{it} = g_i - 0.9T_{it} - 0.4T_{it-1}$ for a poor country. With $g_i = 0$, this becomes $g_{it} = -0.9T_{it} - 0.4T_{it-1}$.

GDP develops according to $\frac{Y_{it+1}}{L_{it+1}} = (1+g_{it})\frac{Y_{it}}{L_{it}}$ With $L_{it} = 1$, this becomes $Y_{it+1} = (1+g_{it})Y_{it}$.

In fact, this isn't quite right. Because all of the estimations in the paper treat $g$ as a percentage, in order to get the path of GDP correct, we need to divide by 100. So, the development of GDP should really be $Y_{it+1} = (1+\frac{g_{it}}{100})Y_{it}$.

All together, we have,

| T | poor g | 1+g | Y |
|---|---|---|---|
| . | 0 | 1 | 1 |
| 0 | 0 | 1 | 1 |
| $\frac{1}{2}$ | $\frac{-1}{100}[0.9\frac{1}{2}] = -0.0045$ | 0.9955 | 0.9955 |
| $\frac{1}{2}$ | $\frac{-1}{100}[0.9\frac{1}{2} + 0.4\frac{1}{2}] = -0.0065$ | 0.9935 | 0.990025 |
| 1 | $\frac{-1}{100}[0.9 + 0.4\frac{1}{2}] = -0.0110$ | 0.9890 | 0.9791 |
| 1 | $\frac{-1}{100}[0.9 + 0.4] = -0.0130$ | 0.9870 | 0.9664 |

This answers (a). (b) We see that the growth rate declines with temperature along this path, and that the rate of decline is increasing in the change in temperature.

(c) From the table, growth is -1.3% for a poor country, and this growth rate is constant after the second period of a permanent temperature change of 1 degree.

3. (a)

$$\begin{aligned}\max_{s,c_1} \frac{c_1^{1-\eta}}{1-\eta} + \frac{1}{1+\rho}\frac{c_2^{1-\eta}}{1-\eta} \\ \text{s.t. } W = c_1 + s \\ c_2 = (1+r)s\end{aligned}$$

Now, reorganize the two constraints to get

$$\begin{aligned}\max_{s,c_1} \frac{c_1^{1-\eta}}{1-\eta} + \frac{1}{1+\rho}\frac{c_2^{1-\eta}}{1-\eta} \\ \text{s.t. } c_2 = (1+r)(W-c_1)\end{aligned}$$

Next, substitute our constraint into the utility function,

$$\max_{c_1} \frac{c_1^{1-\eta}}{1-\eta} + \frac{1}{1+\rho}\frac{((1+r)(W-c_1))^{1-\eta}}{1-\eta}$$

The first order condition for this (complicated) unconstrained maximization problem in one variable is

$$\begin{aligned}0 = (1-\eta)\left(\frac{c_1^{-\eta}}{1-\eta}\right) + \\ \frac{1}{1+\rho}(1-\eta)\frac{((1+r)(W-c_1))^{-\eta}(-(1+r))}{1-\eta} \\ 0 = c_1^{-\eta} + \frac{-(1+r)}{1+\rho}((1+r)(W-c_1))^{-\eta}\end{aligned}$$

Using the constraint, $c_2 = (1+r)(W-c_1)$, we have

$$\begin{aligned}c_1^{-\eta} + \frac{-(1+r)}{1+\rho}c_2^{-\eta} = 0 \\ \left(\frac{c_1}{c_2}\right)^{-\eta} = \frac{(1+r)}{1+\rho} \\ \left(\frac{c_2}{c_1}\right)^{\eta}(1+\rho) = (1+r)\end{aligned}$$

If we let $g$ be the 'rate of consumption growth', $g = \frac{c_2}{c_1} - 1$, then this is

$$(1+g)^{\eta}(1+\rho) = 1+r$$

Recall that, for $x$ small $\ln(1+x) \approx x$. Taking logs and using this approximation, we have

$$\begin{aligned} \ln((1+g)^{\eta}(1+\rho)) &= \ln(1+r) \\ \eta \ln(1+g) + \ln(1+\rho)) &= \ln(1+r) \\ \eta g + \rho &= r \end{aligned}$$

(b) The real return to capital is equal to the subjective rate of time preference plus the product of inequality aversion and the rate of consumption growth. That is, the real return to capital depends on peoples' impatience, and on the extent to which consumption growth can lead to unequal consumption over the life cycle.

(c) The discount rate governs the way that discounted present value calculations work, and discounted present value is the economist's best/favorite tool for ranking complicated and long lasting consumption paths. Ranking such paths is at the heart of climate change policy, so anything that sheds light on the choice of discount rate is important to climate policy.

4. (a) *Adapting to Climatic Challenges: A Progress Report on Studies of the Historical Evolution of Wheat Production*, unpublished working paper by Olmstead and Rhode, 2011.

This paper examines the spread of wheat production across the US and Canada from about 1840 until about 2000. It finds that the area over which people were able to cultivate wheat expanded quickly, and in particular it expanded in to regions previously thought to have climates where wheat could not be grown. This paper provides evidence about the rate at which people can adapt to new climates.

(b) *Civil conflicts are associated with the global climate*, published in Nature by Hsiang, Meng and Cane, 2011.

This paper looks at data describing El Nino induced climate variation and a measure of civil violence (conflict between government and non-government forces that results in fatalities). It finds that civil violence increases with El Nino induced hot seasons. This is of interest to the study of climate change economics because it provides and example of how climate change might be costly, apart from its immediate impact on the productivity with which we make stuff.

(c) *Global climate change, war and population decline in recent human history*, published in the Proceedings of the National Academy of Science (PNAS), by Zhang et al., 2007.

This paper assembles data describing fatalities in wars, al over the world, from about 1400 on, with data describing the climate over the same period. This period contains 'the little ice-age', a period that was anomalously cold. It shows a pattern between cold temperatures and conflict. This paer suggests that climate stress, here 'too cold' rather than 'too hot' can impose important costs on society that will be hard to observe in a study of productivity or agricultural yields.