EC1410-Spring 2024
Problem Set 8

(Updated 12 December 2023)

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When you write up your answers, your goals should be to (1) be correct, and (2) convince your reader that your answer is correct. It is always helpful if your work is legible and if all steps are presented, possibly with a line of explanation. Answers that do not achieve these goals will not be awarded full credit.

Problems

1. Let $Y = f(x) = x^\alpha$ describe the production process, where f is the production technology.

    (a) Verify that f is increasing returns to scale if $\alpha > 1$.

    (b) Verify that f is constant returns to scale if $\alpha = 1$.

    (c) Verify that f is decreasing returns to scale if $\alpha < 1$.

2. Suppose $f(n_i) = n_i^\alpha$ is decreasing returns to scale in individual labor n_i, and output is given by

$$y_i(n_i) = AN^\sigma f(n_i), \text{ for } N = \sum_i n_i$$

    Verify that

$$\frac{\partial y_i}{\partial n_i} \approx AN^\sigma f'(n_i)$$

    as n_i gets small.

3. Consider an economy with two firms (call them Firm 1 and Firm 2) choosing between three locations, A, B and C.

    (a) Create a table with all of the possible combinations of firm/location choice.

    (b) Assuming the firms are choosing location randomly, create a new table with the share of outcomes where one firm is in A and the other is in B, where both firms are in A, etc., for each location pair you listed above.

    (c) Define pairwise distance, d_{ij}, to be 1 if the firms are in different locations, and 0 if the firms are in the same location. Add a column for pairwise distance to the table from the previous step.

    (d) Assuming that the firms choose location at random, plot three histograms about pairwise distances: First, if the firms are not in the same location, what are the relative frequencies of d_{ij} being 0 versus 1? Second, if the firms are in the same location, what are the relative frequencies of d_{ij} being 0 versus 1? Third, compute a weighted sum of these two histograms, weighing each by the relative frequency with which it occurs in your table, to create a "mean" histogram of pairwise distance.

(e) Assume that you have data on 100 industries, each of which has two firms with three possible choices of location. You observe that in 50 industries, the two firms are in the same location, and in the other 50 industries, the firms are in different locations. Do you think the observed location choices are consistent with firms randomly choosing locations? Explain briefly.

4. Suppose you observe only the part of the Basic Pharmaceuticals graphs (Figure 2 from Duranton and Overman, 2005) for pairwise distances between 88 and 92km. Would you conclude that pharmaceuticals are more agglomerated than would occur by chance? Explain briefly.

5. Zipf's law (which we will encounter again later) tells us that the nth largest city in a country is $\frac{1}{n}$ times as large as the largest city.

   (a) How many times would the 32$^{\text{nd}}$ largest city need to double to be the same size as the largest city?

   (b) Assume our current best estimate of agglomeration economies is $\sigma = 0.04$. How much more productive would we expect a unit of labor to be in the largest city than in the 32$^{\text{nd}}$ largest city?