EC1340-Fall 2021
Problem Set 6 solutions

(Updated 31 May 2021)

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1. (a) We want to solve

$$
\begin{aligned}
&\max_{s,c_1} \frac{c_1^{1-\eta}}{1-\eta} + \frac{1}{1+\rho}\frac{c_2^{1-\eta}}{1-\eta} \\
&\text{s.t. } W = c_1 + s \\
&\quad c_2 = (1+r)s \\
\implies &\max_{c_1} \frac{c_2^{1-\eta}}{1-\eta} + \frac{1}{1+\rho}\frac{((1+r)(W-c_1))^{1-\eta}}{1-\eta}
\end{aligned}
$$

Differentiating we get the first order condition:

$$
\begin{aligned}
&c_1^{-\eta} + \frac{1}{1+\rho}((1+r)(W-c_1))^{-\eta}(-(1+r)) = 0 \\
\implies &\left(\frac{c_1}{(1+r)(W-c_1)}\right)^{-\eta} = \frac{1+r}{1+\rho} \\
\implies &c_1^* = \frac{w(1+r)^{\frac{\eta-1}{\eta}}(1+\rho)^{\frac{1}{\eta}}}{1+(1+r)^{\frac{\eta-1}{\eta}}(1+\rho)^{\frac{1}{\eta}}}
\end{aligned}
$$

From the constraints, we have that $c_2 = (1+r)(W-c_1)$. Substituting our solution for c_2^* we have

$$
\begin{aligned}
c_2^* &= (1+r)\left(W - \frac{w(1+r)^{\frac{\eta-1}{\eta}}(1+\rho)^{\frac{1}{\eta}}}{1+(1+r)^{\frac{\eta-1}{\eta}}(1+\rho)^{\frac{1}{\eta}}}\right) \\
&= \frac{w(1+r)}{1+(1+r)^{\frac{\eta-1}{\eta}}(1+\rho)^{\frac{1}{\eta}}}
\end{aligned}
$$

(b)

$$
\begin{aligned}
\frac{c_1^*}{c_2^*} &= \frac{w(1+r)^{\frac{\eta-1}{\eta}}(1+\rho)^{\frac{1}{\eta}}}{w(1+r)} \\
&= \left(\frac{1+\rho}{1+r}\right)^{1/\eta}
\end{aligned}
$$

with $\rho < r$, we have $\left(\frac{1+\rho}{1+r}\right) < 1$. For $\eta = 1$ this means that $\frac{c_1^*}{c_2^*} = \left(\frac{1+\rho}{1+r}\right)$ and consumption in the two period ought to be about the same (if r and ρ are small). As η approaches zero, the exponent gets large and the expression goes to zero. This means that consumption in the first period approaches zero and consumption becomes a unequal as possible. Thus changes in η cause consumption in the two period to be more or less different.

(c) As ρ increases, the expression above increases. This means that first period consumption increases at the expense of second period consumption. This is consistent with less patience.

(d) As r goes up, the expression above decreases. This means that second period consumption is increasing. As r goes it becomes 'cheaper' in terms of first period consumption to provide second period consumption. The consumer substitutes toward the cheaper good.

2. (a) The cost of a reduction in emissions of proportion α of the whole is $\Lambda_0 Y_0 = \frac{2}{3}\alpha^3 Y_0$.

(b) We need to accomplish αE_0 of emissions in country A. To do this, we need

$$\begin{aligned}\alpha E_0 &= \alpha^A E_A \\ &= \alpha^A E_0/2\end{aligned}$$

so that $\alpha^A = 2\alpha$.

The cost of this reduction to country A is,

$$\begin{aligned}\Lambda_A Y_A &= \frac{2}{3}(\alpha^A)^3 Y_A \\ &= \frac{2}{3}(2\alpha)^3 \frac{1}{2} \\ &= \frac{8}{3}\alpha^3\end{aligned}$$

3. We want to find x to solve,

$$\begin{aligned}x\Lambda_A Y_A &= \Lambda_0 Y_0 \\ x\frac{8}{3}\alpha^3 &= \frac{2}{3}\alpha^3 Y_0 \\ \implies x &= 4\end{aligned}$$

That is, it costs 4 times as much to accomplish our reduction in country A alone.