

# EC1410-Spring 2024

## Problem Set 6

(Updated 19 March 2024)

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When you write up your answers, your goals should be to (1) be correct, and (2) convince your reader that your answer is correct. It is always helpful if your work is legible and if all steps are presented, possibly with a line of explanation. Answers that do not achieve these goals will not be awarded full credit.

### Problems

1. In this problem, we will work through an example of the discrete choice model with heterogeneous agents. Consider a discrete linear city with three neighborhoods  $i \in \{1, 2, 3\}$ . Let  $x_i$  denote a neighborhood's distance from the CBD, with  $x_1 = 1$ ,  $x_2 = 2$ ,  $x_3 = 3$ . The cost to commute one unit distance is  $\tau$ . The city is populated by households indexed by  $j$ . Each household chooses a neighborhood  $i$ , pays land rent  $R_i$ , and commutes to the center, at location 0, to earn wage  $w$ . A household's utility is  $V_{ij} = A_i \cdot c_i z_{ij}$  where  $A_i = i$  is the amenity value in location  $i$ ,  $c_i$  is consumption and  $z_{ij}$  is the household and location specific valuation. All  $z_{ij}$  are drawn from a Frechet distribution,  $F(z) = e^{-Tz^{-\epsilon}}$ .
  - (a) Let consumption be  $c_i = w - R_i - i\tau$ . Set up the household's problem.
  - (b) Using the big theorem from the lecture, solve for the share of household  $s_i$  in each location.
  - (c) Let the share of households in each location  $s_1 = s_2 = s_3 = \frac{1}{3}$ , wage  $w = 5$  and the price of agricultural land  $\bar{R} = 1$ . Assume that the land rent at  $x = 3$  is equal to  $\bar{R}$ . Solve for  $R_1$ ,  $R_2$  and  $R_3$  in terms of  $\tau$ .
  - (d) Solve for consumption in terms of  $\tau$ .
  - (e) Plot land rent and commuting costs as a function of  $i$ . How does this compare to the monocentric city model with a continuum of locations?
  - (f) Do all households at location  $i$  have the same utility? What does this suggest about the usefulness of  $R$  to measure welfare?