EC1340-Fall 2023
Problem Set 5 solutions

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Matt Turner

1. The discounted present value of damage is:

$$\begin{aligned}
&\sum_{t=100}^{\infty} \delta^t 50 \\
&= \delta^{100} \sum_{t=0}^{\infty} \delta^t 50 \\
&= 50 \frac{\delta^{100}}{1-\delta}
\end{aligned}$$

But $\delta = 1/(1+r)$, so this equals

$$\begin{aligned}
&50 \frac{(\frac{1}{1+r})^{100}}{1-(\frac{1}{1+r})} \\
&= 50 \frac{1}{r} \left(\frac{1}{1+r}\right)^{99}
\end{aligned}$$

2. Prove that $\sum_{t=1}^{\infty} \left(\frac{1}{1+r}\right)^t = \frac{1}{r}$.

$$\begin{aligned}
\sum_{t=1}^{n} \left(\frac{1}{1+r}\right)^t &= \left(\frac{1}{1+r}\right) + ... + \left(\frac{1}{1+r}\right)^n \\
\left(\frac{1}{1+r}\right) \sum_{t=1}^{n} \left(\frac{1}{1+r}\right)^t &= \left(\frac{1}{1+r}\right)^2 + ... + \left(\frac{1}{1+r}\right)^{n+1}
\end{aligned}$$

Subtracting the second equation from the first gives

$$\left[1 - \left(\frac{1}{1+r}\right)\right] \sum_{t=1}^{n} \left(\frac{1}{1+r}\right)^t = \left(\frac{1}{1+r}\right) - \left(\frac{1}{1+r}\right)^{n+1}$$

Simplifying gives

$$\begin{aligned}
\sum_{t=1}^{n} \left(\frac{1}{1+r}\right)^t &= \left(\frac{1+r}{r}\right) \left[\left(\frac{1}{1+r}\right) - \left(\frac{1}{1+r}\right)^{n+1}\right] = \left[\frac{1}{r} - \frac{1}{r}\left(\frac{1}{1+r}\right)^n\right] \\
&= \left[\frac{1}{r} - \frac{1}{r}\left(\frac{1}{1+r}\right)^n\right]
\end{aligned}$$

Taking a limit gives us the result.

$$\sum_{t=1}^{\infty} \left(\frac{1}{1+r}\right)^t = \lim_{n\to\infty} \sum_{t=1}^{n} \left(\frac{1}{1+r}\right)^t = \frac{1}{r}$$

3. a. The present value of a one hundred dollar payoff 100 years from now as a function of the interest rate is

$PV\left(100{,}r\right) = 100\left(\frac{1}{1+r}\right)^{100}$

b. The value of our 100$ payoff 100 years from now for r = 10%, 5.5% and 1.3% interest is:

$$PV(100{,}.013) = 100\left(\frac{1}{1+.013}\right)^{100} \approx 27.4$$

$$PV(100{,}.055) = 100\left(\frac{1}{1+.055}\right)^{100} \approx 0.47$$

$$PV(100{,}.1) = 100\left(\frac{1}{1+.1}\right)^{100} \approx 0.007$$

4. Here is the problem with not discounting. The problem with discounting is that it seems to give "too little" weight to the future. Suppose instead that we give all future years exactly the same weight as the current year. Thus, given streams of payoffs $(c_t)_{t=1}^{\infty}$ we will rank them according to their sum. There are two problems with this method.

a. Let $c_t = \begin{cases} -1000 & if \quad t=0 \\ 0.00001 & if \quad t \geq 1 \end{cases}$. Let $c_t' = 0$ for all t. If I do not discount future benefits, which policy should I choose?

b. Consider two policies

$$c_t = \begin{cases} 1 & if \quad t \text{ odd and greater than 1} \\ 2 & if \quad t \text{ even} \\ 1.1 & if \quad t = 1 \end{cases}$$

and

$$c_t' = \begin{cases} 2 & if \quad t \text{ odd} \\ 1 & if \quad t \text{ even} \end{cases}$$

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If you use the undiscounted sum of payoffs can you rank these two policies?

A. If I do not discount then, since there are an infinite number of future generations, imposing an arbitrarily large cost on the current generation in exchange for an arbitrarily small benefit to all future generations always maximizes welfare. More formally, $\lim_{n\to\infty}\sum_{t=1}^{n} c_t = \infty$ while $\lim_{n\to\infty}\sum_{t=1}^{n} 0 = 0$.

B. There are two problems. One, the undiscounted present value of both policies is infinite. Therefore, if I do not discount, my welfare criteria does not allow me to chose between the two policies. Suppose, I try to get around this by choosing policy c_t' if $\lim_{n\to\infty}\left(\sum_{t=1}^{n} c_t^{'} - \sum_{t=1}^{n} c_t\right) > 0$. But this limit does not converge and is alternately greater and less than zero. Again, undiscounted present value leaves me unable to choose between the two policies.