EC1410 Topic #1

The Monocentric City Model

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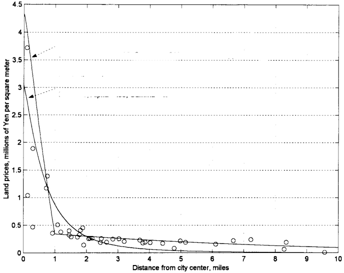
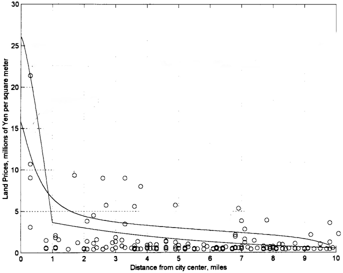
Outline

Real Life Land Rent Gradients I

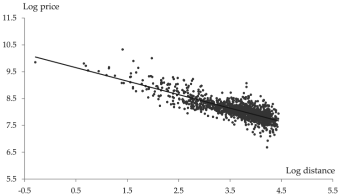


1991 land prices in Hiratsuka, Japan

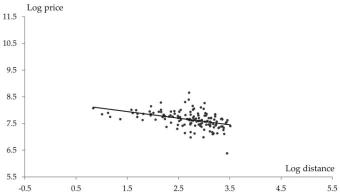


1991 land prices in Yokohama, Japan

Figures from Lucas et al. (2001) showing how land rent declines (very fast) with radial distance from the center of two Japanese cities.



2012 land prices in Paris, France



1991 land prices in Dijon, France

These figures are from Combes et al. (2019). They show the decline of the natural logarithm of rent with the logarithm of radial distance to the center of two French cities.

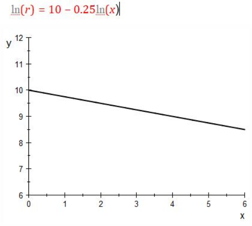
- Japan and France don't look the same, but they are. The Japanese figures plot R against x. The French figures plot $\ln R$ against $\ln x$. If you unpack this, they all show the same exponential decline in rents with distance from the center.
- To see this, you need to remember the rules for logarithms.

$$
\begin{aligned}
\ln R &= A + B \ln x \\
\implies \ln R &= \ln e^A + \ln x^B \\
\implies \ln R &= \ln e^A x^B \\
\implies exp[\ln R] &= exp[\ln e^A x^B] \\
R &= e^A x^B
\end{aligned}
$$

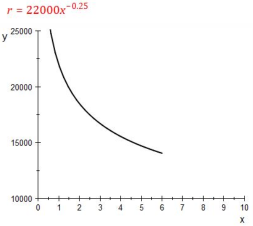
Eye-balling the figure for Paris, we get the intercept is about 10 and the slope about $-\frac{1}{4}$.

$$\begin{aligned} \ln R &= 10 - \frac{1}{4}\ln x \\ \Longrightarrow R &= e^{10}x^{-\frac{1}{4}} \\ \Longrightarrow R &\approx 22,000x^{-\frac{1}{4}} \end{aligned}$$

and so we have,



$\Longleftrightarrow$



So, land rent really does behave the same way in France as it does in Japan. This is pretty neat. It did not have to be true.

- Cities almost everywhere show this sort of log-linear decline in rent with distance to the center.
- Aside #1. Economists often find the world is well described by 'log linear' relationships like this, so it's worth learning how to go back and forth.
- Aside #2. Log-linear relationships have another advantage. The coefficient on $\ln x$ is an elasticity. It tells us the percentage change in rent that results from a one percent change in distance. (Elasticities are really handy because you don't need to keep track of the units that you use to measure x and R, or whatever variables you are interested in.)

Monocentric City Model I

The monocentric city model is one of the most successful economic models I know. It is one of the two or three most heavily commonly used model for urban economists. We start with a simple version, and build up.

It assumes: Spatial equilibrium, Commuting costs, and agglomeration economies. It predicts downward sloping land rent, density and building height gradients and several other comparative statics that are consistent with observation.

Here is how it works:

- City is on a featureless plane, we'll start with a city on a line.
- Location is x. One unit of land at each x (This is a little fishy. Really, 'there is a uniform density of land', which is a little better.)
- Central Business District (CBD) at $x = 0$.
- $|x|$ is distance to center, $x > 0$ is right, $x < 0$ is left.
- City is populated by identical workers/households.
- All workers commute to the CBD to provide one unit of labor and earn wage w.
- Each household consumes $\overline{\ell}$ units of land and c, a composite consumption good.
- Rent per unit of land at x is $R(x)$ in urban use. Reservation rent (e.g. in agriculture) is $\overline{R}$.

- $p_c = 1$, so c is the 'numeraire good'.
- cost to commute unit distance is t.
- N is population of the city.
- $\overline{u}$ is reservation utility, or outside option for all households. They can always move away and get this payoff.
- All land rent, urban and agricultural, is collected by 'absentee landlords' and leaves the model (more on this later).

Each household chooses their location, commutes to work and divides w between commuting, c and rent. This means that a household's problem is

$$\max_{c,x} u(c)$$
$$\text{s.t. } w = c + R(x)\overline{\ell} + 2t|x|$$

Spatial Equilibrium I

- To finish the model, we need to describe equilibrium.
- In much of micro-economics, the notion of equilibrium is something like 'markets clear' or 'no excess demand'.
- We want 'spatial equilibrium'. This will have lots of flavors, but boils down to 'everyone optimizes and no one wants to move'.
- For our purpose, we want 'all households solve the household problem and no one wants to move'.
- This is one of the main ideas of the course. We'll see that it has many important and sometimes surprising implications.
- Example: Can an land rent gradient be a spatial equilibrium if it is discontinuous?

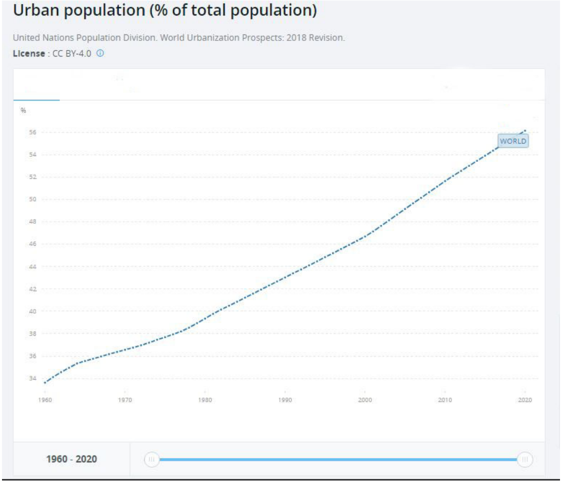
- For the monocentric city model, spatial equilibrium comes in two main flavors, ‘open city’ and ‘closed city’.
    - ‘Open City’. Here all agents are indifferent between all locations in the city and their outside option. In this model, population adjusts until all agents are indifferent between locations in the city or the outside option.
    - ‘Closed City’. Here all agents are indifferent between all locations in the city, but they are not allowed to move away. The population of the city is fixed, and the constant utility level adjusts.

  The ‘open city’ equilibrium is usually more realistic (I think)and usually leads to easier math problems.
- Open and Closed city equilibria are special cases where migration is free or infinitely expensive. Reality is going to lie in between. We’ll talk about this case later.

- Spatial equilibrium is sometimes called ‘free mobility’. That is, people move for tiny gains.

  Note that we don’t need everyone to move freely, just enough people to set prices. This is easier to justify. Once you have decided to move, you can choose one location or another for (almost) free, so ‘free mobility’ seems pretty reasonable for price setting agents.

- Here is 60 years of history of the world urban share. As a starting point, ‘open cities’ look pretty good.



Solving the Monocentric City model I

Assume an open city. Then it must be that

$$u(c^*) = \overline{u} \tag{1}$$

at all occupied locations, when the households optimize. Otherwise, someone wants to move.

- We can solve this expression for $c^* = u^{-1}(\overline{u})$.
- For example, if $u(c) = c^{1/2}$ then we have $(c^*)^{1/2} = \overline{u}$ and $c^* = (\overline{u})^2$.
- From the household's problem, consumption must be the same everywhere in a spatial equilibrium. Therefore,

$$w - c^* = R(x)\overline{\ell} + 2tx. \tag{2}$$

That is, with wages and consumption fixed for all households, commuting costs and land rent must vary in such a way that they always sum to a constant.

- Let $\overline{x}$ denote the most remote occupied location. At this location, we must have

$$w - c^* = \overline{R\ell} + 2t\overline{x}. \tag{3}$$

That is, at the edge of the city, the cost to commute is such that a household can just pay the reservation cost for land and commuting costs, and still buy the reservation consumption bundle.

- Reorganizing, we have

$$\overline{x} = \frac{w - c^* - \overline{R\ell}}{2t}. \tag{4}$$

- Since the city extends from $-\overline{x}$ to $\overline{x}$ and each household consumes an exogenously fixed amount of land

$$N^* = \frac{2\overline{x}}{\overline{\ell}}. \tag{5}$$

Note how we use the assumption that there is one unit of land at each x.

- Using the equilibrium budget constraint (2) and the equilibrium extent of the city, we can solve for the equilibrium rent gradient,

$$R^*(x) = \begin{cases} \frac{w - c^* - 2t|x|}{\overline{\ell}} & \text{if} \quad |x| \leq \overline{x} \\ \overline{R} & \text{if} \quad |x| > \overline{x} \end{cases} \tag{6}$$

Monocentric City Model Example I

Suppose

$$u(c) = ln(c)$$
$$\overline{R} = 0$$
$$\overline{u} = 0$$
$$\overline{\ell} = 1$$

Then the household's problem is

$$\max_{c,x} ln(c)$$
$$\text{s.t. } w = c + R(x) + 2t|x|$$

Spatial equilibrium plus the open city assumption means that

$$ln(c^*) = 0 \Longrightarrow c^* = 1 \text{ all } x.$$

Using the budget constraint we have

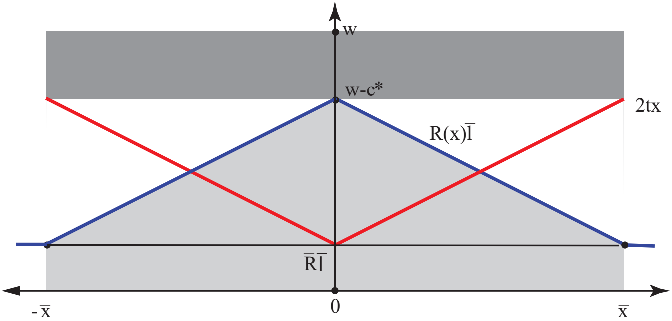
$$w - 1 = R(x) + 2t|x|$$

which means that

$$R(x) = \begin{cases} w - 1 - 2tx & \text{if} \quad 0 < x < (w-1)/2t \\ w - 1 + 2tx & \text{if} \quad 0 > x > -(w-1)/2t \\ 0 & \text{if} \quad |x| > (w-1)/2t \end{cases}$$

and that the edges of the city are at $\bar{x} = \pm(w-1)/2t$. Since $\bar{\ell} = 1$ this means that $N^* = w - 1/t$

Monocentric City in two pictures I



- Red is commute costs, $2t|x|$.
- Blue is land rent.
- In equilibrium, land rent and commute costs have to sum to a constant, $R(x)\overline{\ell} = w - c^{*}$. This will be important later.
- Say that ‘land rent capitalizes the cost/value of commuting’.

Note: View of the city of Providence as seen from the dome of the new State House. Drawn by M. D. Mason, published in the Providence Sunday journal, Nov. 15, 1896.

This is a model of a ‘mill town’. Everyone travels to the center to work.

How we are doing so far?

- We started with the fact of downward sloping land rent gradients.
- We have a model that assumes: transportation is costly, everyone wants to work in the center, people arrange themselves so that no one wants to move, i.e., spatial equilibrium.
- With just these assumptions, we get a downward sloping rent gradient.
- Does the model make other predictions we can check? Comparative statics for
  - wages, coming up.
  - transportation costs, coming up.
  - population density, need an extension.
  - building height, need an extension.
  - amenities, need an extension.

- Two final comments:
  - The shape of the rent gradient is wrong, linear not loglinear. Adding a description of housing (as oppposed to just land) will help with this.
  - Why are people in the center? This is a central assumption. Implicitly, there is a Mill or big factory in the center where people are more productive than if they work elsewhere. More on this when we talk about ‘agglomeration economies’.

A Simple Closed City I

Suppose we know population but not reservation utility. Then we can figure out the size of the city just by knowing household land consumption,

$$\overline{N} = 2\overline{x}/\overline{\ell} \Longrightarrow \overline{x} = \frac{1}{2}\overline{N}\,\overline{\ell}$$

Once we know this, we can figure out consumption by requiring that rent at the edge of the city equal agricultural rent,

$$\begin{aligned} R^*(\overline{x}) &= \frac{w - c^* - 2t\overline{x}}{\overline{\ell}} = \overline{R} \\ \Longrightarrow c^* &= (w - \overline{R}\overline{\ell}) - 2t\overline{x} \\ &= w - (\overline{R} + tN)\overline{\ell} \end{aligned}$$

A Simple Closed City II

and this means that rent at the center is $\overline{R} + tN$ per unit land, and so $c^* = w - (\overline{R} + tN)\overline{\ell}$ for the person at the center.

With spatial equilibrium, everyone gets the same consumption

Thus, $u^* = u(c^*) = u(w - (\overline{R} + tN)\overline{\ell})$.

With open city equilibrium, the reservation utility is exogenous, and the population of the city adjusts until the marginal person pays just the agricultural rent. With closed city equilibrium, population is fixed and utility adjusts so the person at zero pays just enough that the person at the edge doesn't want to oubid him for the central spot.

A Circular City I

Suppose we relax the (silly) assumption that the city is on a line, and think about a symmetric city on a plane, keeping everything else the same.

- This doesn't change the household's problem at all, so we still have

$$\max_{c,x} u(c) \tag{7}$$
$$\text{s.t. } w = c + R(x)\bar{\ell} + 2t|x|$$

- and so, with an open city,

$$u(c^*) = \bar{u} \tag{8}$$

- Consumption must be the same everywhere in a spatial equilibrium,

$$w - c^* = R(x)\bar{\ell} + 2tx.$$

- Let $\bar{x}$ denote the most remote occupied location. At this location, we must have

$$w - c^* = \overline{R}\bar{\ell} + 2t\bar{x}.$$

Reorganizing, we have

$$\bar{x} = \frac{w - c^* - \overline{R}\bar{\ell}}{2t}.$$

This is the same as for the linear city, but it is now the edge of a circular city.

- The area of our circular city is $\pi\overline{x}^2$, so population is

$$N^* = \frac{\pi\overline{x}^2}{\overline{\ell}}.$$

Here, there is $2\pi x$ of land at each x, instead of 1 as in the linear city.

- Land rent doesn't change

$$R^*(x) = \begin{cases} \frac{w-c^*-2t|x|}{\overline{\ell}} & \text{if} \quad |x| \leq \overline{x} \\ \overline{R} & \text{if} \quad |x| > \overline{x} \end{cases}$$

- but total land rent is a little messier,

$$
\begin{aligned}
&= \int_0^{\overline{x}} 2\pi x R(x) dx \\
&= \int_0^{\overline{x}} 2\pi x \left[\frac{(w - c^*) - 2tx}{\overline{\ell}} \right] dx \\
&= \frac{2\pi}{\overline{\ell}} \int_0^{\overline{x}} (w - c^*)x - 2tx^2 dx \\
&= \frac{2\pi}{\overline{\ell}} \left[(w - c^*)x^2/2 - 2tx^3/3 \right]_0^{\overline{x}} \\
&= \frac{2\pi}{\overline{\ell}} \left[(w - c^*)\overline{x}^2/2 - 2t\overline{x}^3/3 \right]
\end{aligned}
$$

- So, really, the circular city isn't much different from the linear city, except that you have to keep track of more math... so we'll stick with the linear city whenever we can.

Partial differentiation I

Given a univariate function $f : R \to R$, or $f(x) \in R$, we have

$$\frac{df}{dx} = \lim_{\epsilon \to 0} \frac{f(x+\epsilon) - f(x)}{\epsilon}$$

This is the 'instantaneous slope' of f at x.
Partial differentiation is the generalization of this idea to surfaces. Consider a function $F : R^2 \to R$, or $F(x_1, x_2) \in R$. This function describes a surface, a height for each point in the plane. How do we think about the slope of such a surface? What we want is a tangent plane rather than a tangent line.

Partial differentiation II

With partial differentiation, we think about the slope of such a plane along one axis. Thus, given $F(x_1, x_2)$, we define

$$\frac{\partial F}{\partial x_1} = \lim_{\epsilon \to 0} \frac{F(x_1 + \epsilon, x_2) - F(x_1, x_2)}{\epsilon}$$

This is exactly analogous to the univariate derivative, if we imagine that we are finding the slope of a ‘slice’ of the surface parallel to the x_1 axis.
Mechanically, treat the ‘other variables’ as constant and use all the rules you know from univariate differentiation.

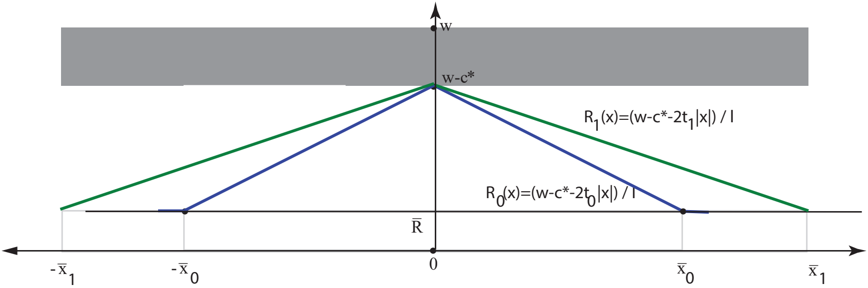
Partial differentiation III

Example:

$$
\begin{aligned}
F(x, y) &= 2x + 3y^2 + 2xy \\
\implies \frac{\partial F}{\partial x} &= 2 + 2y \\
\implies \frac{\partial F}{\partial y} &= 6y + 2x
\end{aligned}
$$

This should be in your calculus book.

Comparative statics, t, open city



Here, we have $t_1 < t_0$.

As transportation cost falls,

- Utility and consumption stays the same.
- Rent gradient flattens, intercept unchanged.
- City gets longer $\Longrightarrow$ population increases (land per person fixed).
- Larger share of people live outside any fixed radius. (Odd thing to note, but useful later).
- Aggregate land rent increases.

Notice that all of the benefit of a reduction in transportation costs either gets used up with more commuting, or is collected by absentee landlords. Utility of residents is unchanged.
We can do this analytically using partial derivatives.

$$\begin{aligned}\frac{\partial R(x)}{\partial t} &= \frac{\partial}{\partial t}\frac{w - c^* - 2t|x|}{\bar{\ell}} \\ &= \frac{-2|x|}{\bar{\ell}}\end{aligned}$$

So as t increases, rent falls at each x, and conversely (we're ignoring the corner where $R = \overline{R}$.)

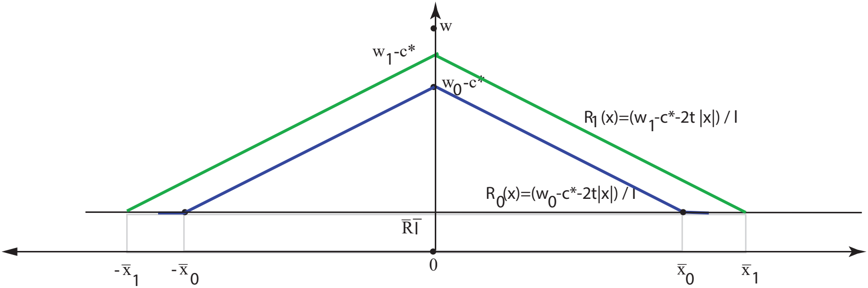
$$\begin{aligned}\frac{\partial \overline{x}}{\partial t} &= \frac{\partial}{\partial t}\frac{w - c^* - \overline{R}\overline{\ell}}{2t} \\ &= -\frac{w - c^* - \overline{R}\overline{\ell}}{2t^2} < 0.\end{aligned}$$

So as t increases, the length of the city falls, and conversely.
Since $N = 2\overline{x}/\bar{\ell}$, we can use the chain rule to get

$$\begin{aligned}\frac{\partial N}{\partial t} &= \frac{\partial}{\partial t} 2\overline{x}/\bar{\ell} \\ &= \frac{2}{\bar{\ell}} \frac{\partial \overline{x}}{\partial t}\end{aligned}$$

and so the population of the city changes in response to t just like its length.

Comparative statics, w, open city I



Here, $w_1 > w_0$.

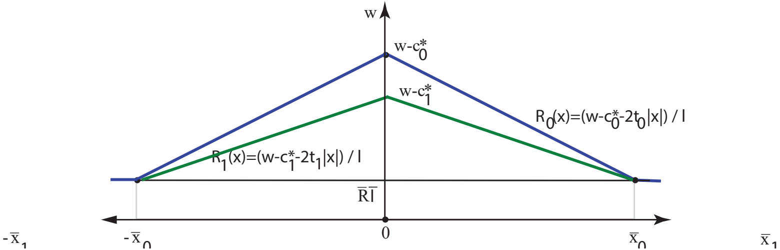
As the productivity of labor (wages) rises in the center,

- Utility and consumption stays the same.
- Slope of rent gradient is unchanged, intercept increases by $\Delta w = w_1 - w_0$.
- City gets longer $\implies$ population increases (land per person fixed).
- Aggregate land rent increases by almost the exact amount as the total wage bill.

Notice that almost all of the benefit of an increase in wages either gets used up with more commuting, or is collected by absentee landlords. Utility of residents is unchanged.

What does this suggest about the incidence of an increase in local minimum wages?

Comparative statics, t, closed city I



Here, we have $t_1 < t_0$.

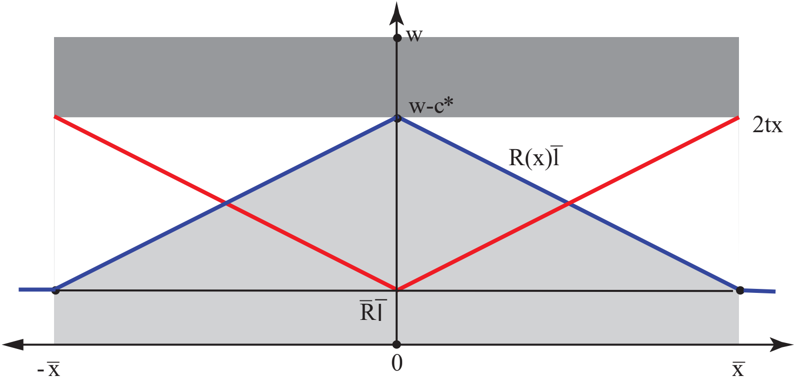
As transportation cost falls,

- Slope of rent gradient is decrease (gradient flattens), intercept stays constant.
- Utility and consumption increase.
- City size stays the same. This follows from closed city assumption and fixed land consumption.
- Aggregate land rent decreases.
- With a closed city, all of the benefit of the improved transportation costs are captured by residents. Landlords are worse off.

It is interesting to compare incidence of e.g. wage changes in open and closed city to tax incidence with elastic and inelastic demand.

- With open city, supply of people is perfectly elastic. All changes fall on landlords, good or bad.
- With closed city, supply of people is perfectly inelastic, so changes fall on households.
- This highlights the importance for policy evaluation of understanding how responsive is migration to local economic conditions.

Land rent and Welfare I



Land rent and Welfare II

- In an open city equilibrium, each household gets $u(c^*) = \overline{u}$.
- This means aggregate land rent is a measure of welfare. It is the collective willingness to pay for this city.
- Similarly, changes in land rent indicate changes in welfare. This is really useful and is widely used, e.g., to value school quality or other place based attributes.
- Land rent is easy to observe (at least compared to utility).
- This result breaks down in a closed city model or with heterogenous agents.

Is a monocentric city optimal? I

We've described how the monocentric city is organized in equilibrium. This is what happens when everyone pursues their own narrow self interest.

What would happen if a rent maximizing developer organized the city, subject to free mobility for the households? If an 'optimal' city is one that maximizes aggregate land rent (why is this defensible?), is an optimal city different from an equilibrium city.

Spoiler – Recall the First Welfare Theorem: in a market economy, if an equilibrium exists, it is Pareto optimal. Something similar will occur here.

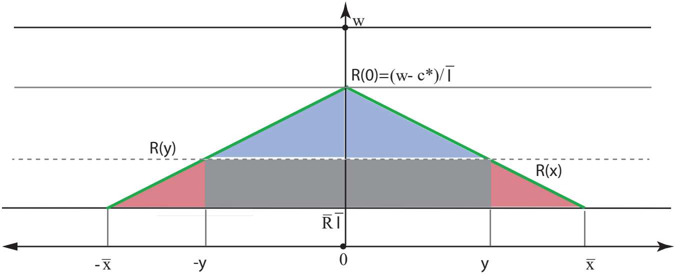
Is a monocentric city optimal? II

In an optimal city, we still allow free mobility, so, as in the equilibrium city, we must have

$$
\begin{aligned}
c^* &= u^{-1}(\overline{u}) \\
w - c^* &= R^{**}(x)\overline{\ell} + 2t|x| \\
\implies R^{**}(x) &= \frac{w - c^* - 2t|x|}{\overline{\ell}}
\end{aligned}
$$

Given this, our planner wants to choose the extent of the city, y to maximize total land rent.

Is a monocentric city optimal? III



- Suppose $y < \bar{x}$. Then aggregate land rent is less than at $\bar{x}$
- Suppose $y > \bar{x}$. Then the marginal increase in urban land rent does not offset foregone agricultural land rent, and total land rent declines.

Is a monocentric city optimal? IV

It follows that the decentralized city is ‘optimal’ in the sense that it maximizes land rent. This is the expected result, it is slightly weaker than the first welfare theorem because surplus (rent) maximization is implied by Pareto Optimality, but not conversely.

Amenities I

Suppose our city has an amenity A that affects the utility of residents. This could be something like good/bad weather, crime, pollution, parks. How does this affect equilibrium?

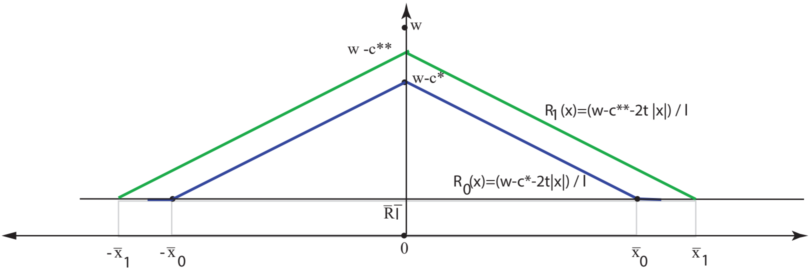
To illustrate ideas as simply as possible, suppose a household's utility is $u(Ac)$, almost just as before. So, $A > 1$ is something good, $A < 1$ is something bad. How does this change the open city equilibrium?

With an open city, we have

$$u(Ac^{**}) = \overline{u}$$
$$\implies c^{**} = \frac{1}{A} u^{-1}(\overline{u})$$

Amenities II

If $A = 1$ we get back the basic case we've already done and $c^{**} = c^{*}$. If $A > 1$, then $c^{**} < c^{*}$. If a city has an amenity that contributes to utility then households can attain their reservation utility level at lower levels of consumption. That is, people accept less consumption to get better weather. Nothing else about the model changes. How does this affect the equilibrium city?



Amenities III

As $A \uparrow$ and amenities get better,

- equilibrium consumption falls.
- Rent gradient intercept increases, rent goes up everywhere.
- the city grows in extent and population.

Sunny cities should be bigger and more expensive than snowy ones. Another way of saying this, land rents capitalize amenities. This is a widely used insight for valuing place specific attributes/policies.

Property taxes I

Suppose land in our city is subject to a property tax rate τ. How does this change the equilibrium?

Let $R_\tau(x)$ be the land rent in the taxed city. Then the household problem is

$$\begin{aligned} &\max_{c,x} u(c) \\ &\text{s.t. } w = c + (1+\tau)R_\tau(x)\bar{\ell} + 2t|x| \end{aligned} \tag{9}$$

With an open city, we have

$$\begin{aligned} u(c^*) &= \overline{u} \\ \implies c^* &= u^{-1}(\overline{u}) \end{aligned}$$

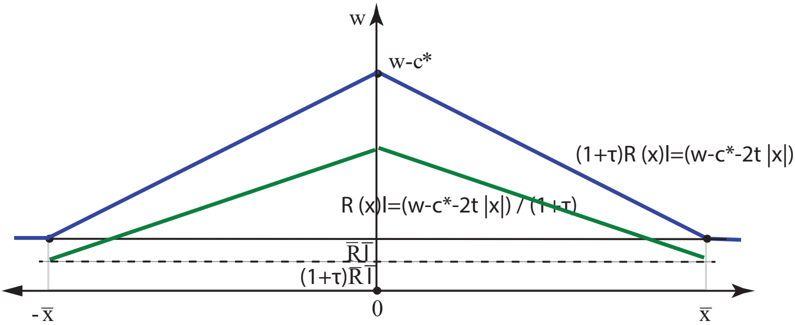
Property taxes II

So that

$$
\begin{aligned}
w - c^* &= (1+\tau)R_\tau(x)\bar{\ell} + 2t|x| \\
\implies (1+\tau)R_\tau(x) &= (w - c^* - 2t|x|)/\bar{\ell} \\
&= R(x)
\end{aligned}
$$

That is, the sum of rent and taxes with a property tax is exactly equal to the rent without a property tax. This means that nothing about the city changes, except that some of the money that would have been collected by absentee landlords is collected by the government.

Property taxes III



Property taxes IV

Thus, property taxes are a non-distortionary tax. There is no deadweight loss, because no one changes behavior to reduce their tax burden. Such taxes are pretty rare. This result is known as the 'Henry George Theorem'.

A few comments:

- With heterogenous agents, it starts to break down.
- It's important to also tax agricultural land. Otherwise there is an effect on the extensive margin. Some people move away from the edges of the city because untaxed farmers outbid them for land.

Land Rent and Capitalization I

How are rent and asset price related?

- Let ρ be the real interest rate. One dollar today turns into $1 + \rho$ in a year. P is the purchase price of a property and R the rental price for one year.
- If $\rho P < R$ then renters should buy their properties and pocket the difference. If $\rho P > R$ then owners should sell and become renters. Only when $\rho P = R$ is there no opportunity for intertemporal arbitrage.
- So, we should have $\rho P = R$. That is rent equals one year of interest on the asset price of the property.

There is another interesting way to get this result. Suppose the rent on a property is R every year, forever. The sales price the be the value today of this stream of payments.

- R in one year is worth $R_1 = \frac{1}{1+\rho} R$ today.
- R in two years is worth $R_2 = \frac{1}{(1+\rho)^2} R$ today, and so on.
- R every year forever, starting in one year is worth

$$\begin{aligned} P &= \frac{1}{(1+\rho)} R + \frac{1}{(1+\rho)^2} R + \frac{1}{(1+\rho)^3} R + ... \\ &= \sum_{t=1}^{\infty} \frac{1}{(1+\rho)^t} R \end{aligned}$$

- This turns out to be easy to evaluate. To do this, let $\delta = \frac{1}{(1+\rho)}$ to ease notation (δ is called the 'discount factor').

$$P = \sum_{t=1}^{\infty} \delta^t R$$

So, we also have

$$\delta P = \delta \sum_{t=1}^{\infty} \delta^t R$$

- subtracting the second from the first,

$$
\begin{aligned}
P - \delta P &= \sum_{t=1}^{\infty} \delta^t R - \delta \sum_{t=1}^{\infty} \delta^t R \\
\implies (1-\delta)P &= \delta R + \delta^2 R + \ldots \\
&\quad - \delta^2 R - \delta^3 R - \ldots \\
&= \delta R
\end{aligned}
$$

- Rearranging, $P = \frac{1}{1-\delta} R = \frac{1}{\rho} R$.
- That is, asset price of a property is the discount present value of the stream of rental payments. Alternatively, the rental price is the annual cost of capital, and the two ways of figuring out how rent and sale price are related are equivalent (also pretty neat).
- What does this logic mean will be the implication of a change in property taxes for the sale price of a house?

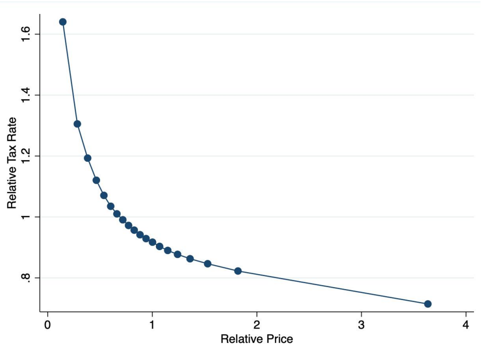
Aside: Fair assessment of property taxes

Consider the problem of unfair property tax assessments currently under debate in Chicago,

https://apps.chicagotribune.com/news/watchdog/cook-county-property-tax-divide/assessments.html.

At issue is the possibility that either assessors deliberately discriminate against the residents of less expensive houses, and assign them higher mill rates, or that poor people live in houses that are 'unobservably bad', and so honest assessors over value them, and they end up paying too much tax.

The following figure, from Berry (2021) illustrates the extent of the problem



Notes: Relative price is property sale price divided by jurisdiction average price in year of sale. Relative tax rate is property's tax rate divided by jurisdiction average tax rate in the year of sale. A jurisdiction is defined as the same county, city, and school district. The tax rate is the tax due in the year of sale divided by the sale price. Binned scatter plot shows average relative tax rate and average relative price by 20 quantiles of relative sale price. Based on 26 million residential sales contained in Corelogic tax and deed database.

Given what we have done so far, there are two problems with this discussion

- We know that asset price is function of rental price, and rental price is a function of property taxes, so it is not clear how much information in the price is independent of the tax burden. To see this, let V denote unobserved 'full rental price' of the house converted to an annual rental rate, R be the transaction price converted to a an annual rental rate, and T annual tax assessment. Then with 100% capitalization, we have

$$V = T + P$$

Now suppose that tax assessments are ad valorem, $T = \tau P$, then using $\ln(1 + x) \approx x$ for x small,

$$\tau = \ln V - \ln P.$$

So 100% capitalization implies a slope of -1 if you regress the tax rate on log of the sales price. It is not clear if the figure is different from this. It definitely looks log linear. In this case, the downward slope of a plot of tax rate on sales prices is purely a function of capitalization.

- Suppose assessors are being unfair. What are the welfare implications of this? ‘Over-taxed’ houses transact for less money, and their owners write larger checks to the city and smaller checks to the bank each year.
- The problem is that you pay property taxes on improvements to your house, too. If you add a room, you pay property tax on the value of this addition forever. If you are subject to a higher property tax, home improvements cost more. This means that a high property tax disincentivizes home improvement and maintenance, or said another way, incentivizes blight.

Aside: The Border Discontinuity Design I

- An interesting implication of the monocentric city model is that land rent can never be discontinuous. To see this, imagine the rent gradient is discontinuous. In this case, the household at the high side of the discontinuity can move to the low side, experience almost zero change in commute costs, and a discrete drop in rent. This contradicts the idea that this was an equilibrium to start with. A household can move and make themselves better off.
- Actually, we can have a discontinuous rent gradient if amenities vary discontinuously. In this case, spatial equilibrium requires that rent vary discontinuously in order to equalize utility across locations.

Aside: The Border Discontinuity Design II

- This intuition motivates the ‘border discontinuity design’ for empirical work. This research design provides a basis for valuing location specific amenities that vary discretely as we move across the landscape.
- Black (1999) uses this idea to examine the value of school quality by looking at how housing prices vary when we cross a school district boundary where school quality varies.

Black (1999) I

Black (1999) considers the relationship between school quality and real estate prices between 1993 and 1995 for three counties in Massachusetts. Here is what school attendance zone boundaries look like.

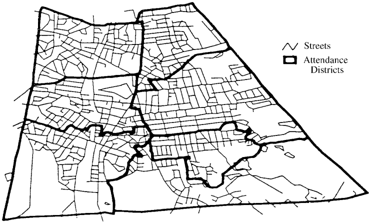


FIGURE I
Example of Data Collection for One City: Melrose
Streets, and Attendance District Boundaries

Black (1999) II

- To this map, she adds data describing elementary school average test scores (school quality) and real estate transaction data.
- We know that school quality varies discretely at an attendance zone boundary. How much is this worth?
- In a spatial equilibrium, this value must be reflected in a discontinuous change in land prices at the border.

Black (1999) III

- To measure this gap (if it is present), she restricts attention to transactions within a few hundred yards of a school attendance zone boundary, and estimates the following regression,

$$\begin{aligned}\text{ln}(\text{House price}_i) = A_0 + A_1 \text{test score}_i + \\ A_2 \text{border indicators} + \text{controls}_i + \varepsilon_i\end{aligned}$$

  The parameter A_1 tells us the size (in log points) of the change in house prices at the border.
- If real estate markets are in 'spatial equilibrium' this should tell us the value of improving test scores.

Distance from boundary:	(1) All houses[d]	(2) 0.35 mile from boundary (616 yards)	(3) 0.20 mile from boundary (350 yards)	(4) 0.15 mile from boundary (260 yards)	(5) 0.15 mile from boundary (260 yards)
Elementary school test score[c]	.035 (.004)	.016 (.007)	.013 (.0065)	.015 (.007)	.031 (.006)
Bedrooms	.033 (.004)	.038 (.005)	.037 (.006)	.033 (.007)	.035 (.007)
Bathrooms	.147 (.014)	.143 (.018)	.135 (.024)	.167 (.027)	.193 (.028)
Bathrooms squared	−.013 (.003)	−.017 (.004)	−.015 (.005)	−.024 (.006)	−.025 (.007)
Lot size (1000s)	.003 (.0003)	.005 (.0005)	.005 (.0005)	.005 (.0007)	.004 (.0006)
Internal square footage (1000s)	.207 (.007)	.193 (.01)	.191 (.01)	.195 (.02)	.191 (.012)
Age of building	−.002 (.0003)	−.002 (.0002)	−.003 (.0005)	−.003 (.0006)	−.002 (.0004)
Age squared	.000003 (.000001)	.000003 (.0000006)	.00001 (.000002)	.000009 (.000003)	.000005 (.000002)
Boundary fixed effects	NO	YES	YES	YES	NO
Census variables	Yes	No	No	No	Yes
N	22,679	10,657	6,824	4,594	4,589
Number of boundaries	N/A	175	174	172	N/A
Adjusted R^2	0.6417	0.6745	0.6719	0.6784	.6564

The top row gives the estimates of A_1 that we want. It ranges between 0.013 and 0.031, so a 1 point increase in test score gives about a 1-3% increase in housing prices.

Conclusion/Summary I

We've now developed the most basic version of the monocentric city model pretty thoroughly.

This model assumes: spatial equilibrium, costly commuting, central employment.

The model makes the following predictions.

- $R(x)$ decreasing in x. We've seen this is correct. More to come.
- As commuting costs, t, decrease,
  - utlity and consumption, $\overline{u}$, c^* constant(by assumption).
  - Rent gradient gets flatter, intercept stays the same.
  - City gets longer, $\overline{x} \uparrow$.
  - Population increases, $N \uparrow$.

Conclusion/Summary II

  - A larger share of population lives outside any given distance from the center.
  - Aggregate rent goes up (and this measures welfare).
- As wages, w, increase,
  - utlity and consumption, $\overline{u}$, c^* constant(by assumption).
  - Slope of rent gradient unchanged, intercept increases by Δw.
  - City gets longer, $\overline{x} \uparrow$.
  - Population increases, $N \uparrow$.
  - Aggregate rent goes up (and this measures welfare).
- As agricultural rent changes, what happens? Not done. No empirical results on this, so there's not really anything we can check.
- As amenities, A increase,
  - utlity constant, but consumption c^{**} falls.

Conclusion/Summary III

- Slope of rent gradient unchanged, intercept increases.
- City gets longer, $\overline{x} \uparrow$.
- Population increases, $N \uparrow$.
- Aggregate rent goes up (and this measures welfare).

- Changes in property taxes do not change anything except how much rent is collected by absentee landlords. This is called the Henry George Theorem.
- Spatial equilibrium requires that rent gradients be continuous, unless something that people value about the location changes discontinuously. This intuition gives rise to the widely used border discontinuity research design for evaluating location specific attributes.

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