EC1340 Topic #10

Tragedy of the commons and taxes

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Outline

Today we examine the incentives that people face with regard to carbon emissions. This will lead to an understanding of why there is ‘too much’ CO_2 , and also provide a foundation for thinking about how to regulate CO_2 emissions.
Specifically, today we’ll talk about the ‘Tragedy of the Commons’ in five different ways

- Words – Hardin
- Pictures — Gordon
- Nash Equilibrium – Cheung
- Dynamic Nash Equilibrium – no reading
- Divergence of private and social cost

Tragedy of the Commons - Hardin 1968

Story: There are many shepherds exploiting a small pasture. Each spring, each shepherd gets a free goat and must decide whether to put it on the communal pasture to graze or to chase it out into the desert.

- If shepherds maximize their own income, then they put their goat on the pasture as long as the goat provides any positive value at the end of the season.
- This results in a crowded pasture and skinny goats, each of which is worth almost zero.
- If, instead, a central authority restricts the number of goats, then at the end of the season the community could have fewer, but fatter more valuable goats whose total value was larger

This is the ‘Tragedy of the commons’

Issues:

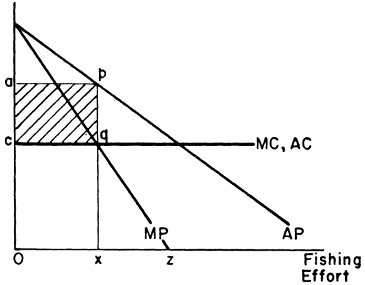
- There is considerable empirical support for this model. Elinor Ostrom won the Nobel Prize in 2009 for work which documented when the Tragedy of the Commons does and does not occur.
- Hardin was worried about the population of people, not goats!
- Hardin's solution is 'mutual coercion, mutually agreed upon'. This is an alternative (centralized) institution for resource allocation rather than the decentralized institution of the example.

We'd eventually like to describe such alternative institutions more precisely.

Tragedy of the Commons - Gordon 1954

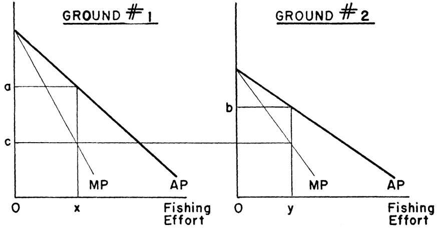
- E is fishing effort, w is the price for one unit of effort.
- $q = F(E)$ is harvest from effort E.
- $MP(E) = p\frac{dF}{dE}$ (really MRP, marginal revenue product)
- $AP(E) = p\frac{F}{E}$ (really ARP, average revenue product)
- to solve max $\pi(E) = pF(E) - wE$, choose E so that $MP(E) = MC(E) = w$. This is the key: effort gets average rather than marginal return.

The picture for this is (from Gordon 1954):



Using the figure's notation, total rent/profits for the fishery are $x(p-q)$. This is the aggregate benefit to fishermen relative to everyone selling their effort for E in the residual labor market. (NB: Should be MRP and ARP)

Suppose that the fishers have the choice of two adjacent, but different fishing grounds. Then the optimum looks like this (from Gordon 1954):



In this case, the $AP(E)$ is *a* in #1 and *b* in #2. This can't happen if small fishermen are choosing the best place to work.
In equilibrium, should have, $AP(E) = w$ in all fisheries.

Issues:

- This is exactly the same intuition as Hardin had.
- Note that we can think of *MP* as the ‘social value of effort’ and *AP* as private value.
- this situation is often called an ‘open access’ resource, as opposed to ‘common property’, which now means something under collective management.
- Sometimes say ‘all rents are dissipated’ under open access.
- Few fisheries are managed this way anymore.
- How is this like CO_2 ? Read ‘emissions’ for ‘effort’ and set $w = 0$ and you’ll be pretty close.

Tragedy of the Commons - Cheung 1970

Cheung provides a bit more formal development of Gordon's argument.
Notation:

- x_i is effort for $i = 1, .., N$ fishers.
- K size of fish stock
- $H = F(\sum_{i=1}^{N} x_i, K)$ harvest of fish
- $h_i = \frac{x_i}{\sum_{i=1}^{N} x_i} F$ so that i's share of harvest is share of effort

With price of fish $p = 1$, each fisher solves

$$\max \frac{x_i}{\sum_{i=1}^{N} x_i} F(\sum_{i=1}^{N} x_i, K) - wx_i$$

To solve, set x_i derivative equal to zero and solve for optimal effort, taking other fishers' efforts as given.

To evaluate this derivative, first notice that (for example)

$$\begin{aligned}\frac{\partial}{\partial x_1}\sum_{i=1}^{N} x_i &= \frac{\partial}{\partial x_1}\left(x_1 + x_2 + \ldots + x_N\right)\\ &= 1\end{aligned}$$

and recall that

$$\frac{d}{dx}\frac{u(x)}{v(x)} = \frac{vdu - udv}{v^2}$$

so that with $u = x_i$ and $v = \sum_{i=1}^{N} x_i$, we have

$$\frac{\partial}{\partial x_i}\frac{x_i}{\sum_{i=1}^{N} x_i} = \frac{\sum x_i - x_i}{(\sum x_i)^2}$$

Also recall the product rule

$$\frac{d}{dx}u(x)v(x) = uv' + u'v$$

and the chain rule

$$\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$$

Using these rules and facts we have the FOC for our problem

$$\frac{\partial \pi}{\partial x_i} = 0$$

$$\frac{\partial}{\partial x_i}\left(\frac{x_i}{\sum_{i=1}^{N} x_i} F(\sum_{i=1}^{N} x_i, K) - w x_i\right) = 0$$

$$\frac{\partial}{\partial x_i}\left(\frac{x_i}{\sum_{i=1}^{N} x_i}\right) F(\sum_{i=1}^{N} x_i, K) + \frac{x_i}{\sum_{i=1}^{N} x_i}\frac{\partial}{\partial x_i} F(\sum_{i=1}^{N} x_i, K) - w = 0$$

$$\frac{\sum x_i - x_i}{(\sum x_i)^2} F(\sum_{i=1}^{N} x_i, K) + \frac{x_i}{\sum_{i=1}^{N} x_i}\frac{\partial F}{\partial x_i}\frac{\partial}{\partial x_i}\left(\sum_{i=1}^{N} x_i\right) - w = 0$$

$$\frac{\sum x_i - x_i}{(\sum x_i)^2} F(\sum_{i=1}^{N} x_i, K) + \frac{x_i}{\sum_{i=1}^{N} x_i}\frac{\partial F}{\partial x_i} - w = 0$$

To get any further, we want to assume that all fishers are identical and let

$$\sum x_i = X$$
$$x_i = X/N$$

Note that it matters whether we assume symmetry before or after we differentiate – it describes quite different things.
Imposing symmetry on our FOC we have

$$\begin{aligned}
&\frac{\sum x_i - x_i}{(\sum x_i)^2} F(\sum_{i=1}^{N} x_i, K) + \frac{x_i}{\sum_{i=1}^{N} x_i} \frac{\partial F}{\partial x_i} - w = 0 \\
\implies &\left(\frac{1}{X} - \frac{X/N}{X^2}\right) F(X, K) + \frac{X/N}{X} \frac{\partial F}{\partial x_i} - w = 0 \\
\implies &\left(\frac{1}{X} - \frac{1}{NX}\right) F(X, K) + \frac{1}{N} \frac{\partial F}{\partial x_i} - w = 0
\end{aligned}$$

as N gets large, this becomes $\frac{F}{X} - w = 0$ or $AP(X) = w$, exactly what Gordon gets!

Issues:

- Stories (1) Fishing draws down stock so fish are scarcer and harder to catch (No – no time) (2) Boats get in each other's way (Yes). Hence, sometimes this is called a congestion externality or problem. It's also decreasing returns to a fixed factor.
- Gordon's result is a limit result. If fishermen are infinitely small, get Hardin. If not, then not all rents are dissipated (quite). Which is relevant for CO_2 ?

Tragedy of the Commons - Dynamic example

- The Hardin/Gordon/Cheung story is static.
- Since we are interested in global warming, we’re really interested in time, and in particular on peoples’ incentives to save an open access resource, for us the climate.
- This example extends the Tragedy of the Commons to an environment with a resource that can be saved or exploited now.

- N fishers live for two periods and support themselves exclusively by fishing from a single pond. Fishers maximizes the total value of their harvest over two periods and the price of fish is one.
- $x_i \in [0, 1]$ is harvest in period one for $i = 1, .., N$ fishers.
- $K = N$ size of fish stock
- Fish stock in period two is $(1 + R) \left(N - \sum_{i=1}^{N} x_i\right)$. That is, saved fish grow at rate $1 + R$.
- All fishers split tomorrow's stock equally tomorrow.
- Fishers value harvest today and tomorrow equally (no impatience). Harvesting is free in both periods.

Each fisher's maximization problem is:

$$\max_{x_i} x_i + \frac{1}{N}(1+R)\left(N - \sum_{j\neq i} x_j - x_i\right)$$

$$\Longrightarrow \max_{x_i} x_i - \frac{(1+R)}{N}x_i + \frac{(1+R)\left(N - \sum_{j\neq i} x_j\right)}{N}$$

$$\Longrightarrow \max_{x_i} \left(1 - \frac{(1+R)}{N}\right)x_i + \frac{(1+R)\left(N - \sum_{j\neq i} x_j\right)}{N}$$

$$\Longrightarrow \text{fisher } i \text{ chooses} \begin{cases} x_i = 1 & \textit{if} \quad \left(1 - \frac{(1+R)}{N}\right) > 0 \\ x_i = 0 & \textit{if} \quad \left(1 - \frac{(1+R)}{N}\right) < 0 \\ \text{anything} & \textit{if} \quad \left(1 - \frac{(1+R)}{N}\right) = 0 \end{cases}$$

- Since all fishers face the same decision, this means that there is zero savings if $\left(1 - \frac{(1+R)}{N}\right) > 0$.
- Note that as N gets large, this will always be true.
- This problem is a bit odd to make the math easy, but the intuition is general.

If we wanted to maximize aggregate profits in this example, we would solve, letting $x = \sum x_j$,

$$\begin{aligned}
&\max_{x_i} x + (1+R)(N-x) \\
\implies &\max_{x_i} x_i - (1+R)x + (1+R)N \\
\implies &\max_{x_i} (1-(1+R))x + (1+R)N \\
\implies &\max_{x_i} -Rx + (1+R)N
\end{aligned}$$

So, to maximize aggregate profits, always save everything $(x = 0)$ as long as $R > 0$.

- If $R > 0$ but $\left(1 - \frac{(1+R)}{N}\right) > 0$, the decentralized fishery won't save enough.
- Like the tragedy of the commons, decentralized behavior causes all to be poorer.
- This is similar to the logic of the tragedy of the commons. In the tragedy of the commons, people disregard the fact that their actions harm others today.
- In this example, people disregard the fact that their actions are beneficial to others tomorrow.

What is the global warming versions of this? We undertake too little mitigation today b/c most of the benefit goes to others.

Two period Tragedy of the Commons

What happens if we put the static Tragedy of the Commons intuition together with the intuition from the dynamic example?
Notation:

- Single agent maximizes profits over two periods, $\pi = \pi_1 + \pi_2$
- K_t, X_t are effort and stock in period $t = 1, 2$.
- $H_t = G(X_t, K_t)$ harvest at t
- $K_2 = F(K_1 - G(X_1, K_1))$ 'growth equation of stock'

We again let the price of fish $p = 1$ to make things easier and get the agents maximization problem

$$\max_{X_1, X_2} (G(X_1, K_1) - wX_1) + (G(X_2, K_2) - wX_2)$$
$$\text{s.t. } K_2 = F(K_1 - G(X_1, K_1))$$

a constrained maximization problem in two variables
Substituting the constraint into the objective, we have

$$\max_{X_1, X_2} (G(X_1, K_1) - wX_1) + (G(X_2, F(K_1 - G(X_1, K_1)) - wX_2)$$

an unconstrained maximization problem in two variables

To solve this problem we must satisfy two first order conditions, one for effort in each period.
In period two,

$$
\begin{aligned}
& \frac{\partial \pi}{\partial X_2} = 0 \\
\Longrightarrow & \frac{\partial}{\partial X_2}\left[(G(X_1, K_1) - wX_1) + (G(X_2, K_2) - wX_2)\right] = 0 \\
\Longrightarrow & (\frac{\partial G}{\partial X_2} - w) = 0
\end{aligned}
$$

Thus, we choose second period effort as in Gordon's static problem, so that marginal revenue product equals marginal cost.

In period one,

$$
\begin{aligned}
&\frac{\partial \pi}{\partial X_1} = 0 \\
\Longrightarrow &\frac{\partial}{\partial X_1}\left[(G(X_1, K_1) - wX_1) + (G(X_2, K_2) - wX_2)\right] = 0 \\
\Longrightarrow &(\frac{\partial G}{\partial X_1} - w) + \frac{\partial G}{\partial K_2} F'(-\frac{\partial G}{\partial X_1}) = 0
\end{aligned}
$$

The first period optimum is different than Gordon. In the first period. We want to exploit a little less today because each unit of harvest today drives up the cost of tomorrow's harvest by reducing tomorrow's stock.

We've just examined what happens in the two period fishery when there is a single agent, e.g., a social planner, who maximizes total value of the fishery.

What happens if there are many small fishers? Letting $X_t = \sum x_{it}$ and x_{it} be i's effort at t, each fisher wants to choose their effort in each period to solve,

$$\max_{x_{i1}, x_{i2}} \frac{x_{i1}}{X_1}(G(X_1, K_1) - wx_{i1}) + \frac{x_{i2}}{X_2}(G(X_2, K_2) - wx_{i2})$$
$$\text{s.t. } K_2 = F(K_1 - G(X_1, K_1))$$

We just solved this problem for $N = 1$. What happens as N becomes large?

- Like dynamic example, you only get $\frac{1}{N}$th of the return to your savings, so as N gets big, there is no incentive to save. This means that the FOCs for first and second period will be the same, i.e., the extra bit of first period FOC drops out.
- Without savings, we're in Gordon's world, and in each period, we end up with effort chosen so that ARP=MC.

Thus, the two FOCs for this problem are

$$w = \frac{G(X_1, K_1)}{X_1}$$
$$w = \frac{G(X_2, K_2)}{X_2}$$

but working it out is a bit of a mess (but is a good exercise).

Externalities etc.

- The incentive problem illustrated by the Tragedy of the Commons is central to understanding the problem of global warming – it lets us understand why there seems to be ‘too much’ CO_2 .
- There are, other ways to describe this incentive problem, some of which are discussed in the reading. These are, Market Failure, Externality, and Public Good Problem.
- These are really, just different names for the same incentive problem.

- *Market Failure*

  If there were a markets where each fisher paid each of the others for the harm that his fishing caused them, so that each fisher would make $N - 1$ such payments, and these payments were set so that there sum was the difference between *ARP* and *MRP* at the optimal effort, then the tragedy of the commons would not occur. Each fisher would, in practice, face the *MRP* rather than the *ARP* and would choose effort so that $MRP = MC$.

  Similarly, if there were markets where each fisher each fisher compensated the others for the benefit they obtain from each others' contribution to communal savings.

Alternately, if there were markets where you paid the other 7 billion people in the world for the harm your CO_2 emissions caused (and they paid you)...

- *Externalities*

  An externality is the result of an involuntary, non-market exchange. For example: (1) someone makes you worse off by getting in your way while you are fishing and doesn't compensate you for this harm, (2) Sneezes on you (3) puts CO_2 in the atmosphere.

  In economic environments with externalities people do not face the right incentives and we typically see problematic outcomes.

  Note that 'externality' is not a very well defined idea. Does robbery count? There was a 40 year academic argument about the definition, which (thankfully) has just about stopped.

- *Public goods*

  Pure public good:

  - c_i private consumption for consumer $i = 1, ..N$.
  - S public good such as; national defense, sewage treatment.
  - utility for each i; $u_i(c_i, S)$; everyone is forced to consume the same amount of the public good.
  - The private MRS between c_i and S is $\frac{\frac{\partial u_i}{\partial S}}{\frac{\partial u_i}{\partial c_i}}$. The social value of this same unit is $\sum_{i=1}^{N} \frac{\frac{\partial u_i}{\partial S}}{\frac{\partial u_i}{\partial c_i}}$, which will generally be much larger.

  The atmospheric concentration of CO_2 is a pretty good approximation to this sort of pure public good. Everyone ‘consumes’ the same CO_2 concentration, and to get the full value of climate change, we should sum over 7 billion.

...alternately, we have congestible public goods:

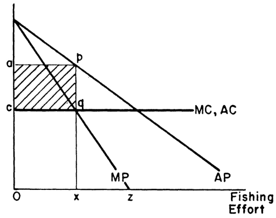
- c_i private consumption for consumer $i = 1, ..N$.
- s_i congestible public good for i
- utility for i is $u(c_i, s_i, \sum_{j \neq i} s_j)$

Most ‘public goods’ are not actually ‘pure public goods’, they are congestible, e.g., roads, parks, air quality. CO_2 is probably a pure public good, capacity of atmosphere to assimilate emissions is, maybe, congestible public good ($N =$6.7 billion).

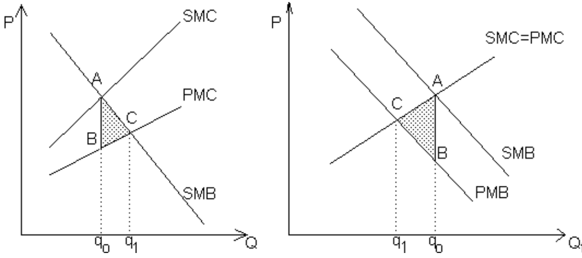
Note that for our fishery problem we had $\pi_i = \pi_i(H_i, \text{ Stock } - \sum_{j \neq i} H_j)$. That is, Tragedy of the Commons is a congestible public goods problem.

- *Divergence of Private and Social Cost*
  Another way of thinking about this incentive problem is as a divergence between private and social cost/benefits.

  For example, in Gordon's main picture, *ARP* is the private benefit to a unit of effort and *MRP* is the Social benefit, the private benefit less the cost of congestion caused by each marginal unit. Since the private benefit is larger than the social benefit, in equilibrium, we get too much effort.



This terminology finds more general application. For example; Social versus private cost of smoke, Social versus private benefit of emissions, social versus private benefit of constant climate, etc.



figures from

`http://www.siilats.com/docs/welfare/26extern.htm`

This is, I think, the easiest language to use to talk about this class of incentive problems.

Conclusion

- We observe that there seems to be too much atmospheric CO_2 .
- We have developed models that explain why this might occur. There is a fundamental incentive problem surrounding CO_2 emissions: the emitter experiences only the tiniest cost from their emissions – my drive to the store affects climate only imperceptibly–but the social cost, 7 billion times my cost is not (necessarily) trivial.
- There are lots of words/models for thinking about this problem, and we've run through most of them today.
- This hardware provides a foundation for thinking about regulation to reduce emissions to their optimal level.
- Note that the intuition we've developed here is central to environmental economics and applies much more broadly than just to climate change.