EC2410-Spring 2016
Problem Set 2

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1. Consider a city with measure one of land. All city residents receive a wage w and consume land inelastically so that measure on of residents occupies measure 1 on land, i.e., fills the city up. Residents pay land rent $R \geq 0$ and derive utility from consumption. The set of potential city residents is the set $[0,\Theta]$, with measure Θ and is indexed by θ. Agent θ's utility is,

$$u(\theta) = \begin{cases} w - R & \text{if } \theta \quad \text{in city} \\ \theta & \text{else} \end{cases}$$

   That is, agents get utility from consuming $w - R$ in the city, and an idiosyncratic reservation value outside the city. Consider two cases, $\Theta \geq 1 > w$ and $\Theta \geq w > 1$.

   (a) Characterize a free mobility equilibrium for this economy, and in particular, find land rent for all locations in the city.

   (b) Calculate aggregate land rent and consumers' surplus in equilibrium.

   (c) Is land rent as interesting a measure of welfare in this model as in the linear city model? Explain briefly.

2. There is pretty good empirical support for the following two claims. First, that subway systems reduce central city air pollution (Chen and Whalley, AEJ 2012). Second, that public transit provision in a city has very little impact on the amount of travel by car in the metropolitan region. That is, people take driving trips until the roads are full whether there is transit or not (Duranton and Turner, AER 2011). Since reductions in central city pollution reflect reductions in driving, it is not obvious how these two facts can both be true.

   The following model investigates this problem. In particular, it asks whether subways can reduce central city driving, and hence pollution, while simultaneously increasing (or holding fixed) metropolitan area driving.

   We want to compare total driving in a city with subways to total driving in a city without. To do this, consider a linear city as follows:

$$\begin{aligned} w &= \text{wage for all workers at CBD} \\ R(x) &= \text{land rent at location } x \\ \overline{u} &= \text{reservation utility level} \\ c &= \text{composite consumption} \\ \tau(x) &= \text{commute cost to CBD from } x \\ u(c) &= c^{\beta}, \beta \in (0,1). \end{aligned}$$

   In equilibrium, all agents choose location to maximize utility and all agents are indifferent between their location choice and the outside option. Agents consume one unit of land inelastically and the value of land in agriculture is zero.

   Suppose that without subways, $\tau(x) = tx$ and that with subways, $\tau(x) = 0$ for $x < 1$ and that for $x > 1$, $\tau(x) = \frac{t}{2} + (x-1)t$. That is, agents living in the catchment area of the subway commute for free. For agents living further away, the subway reduces the cost of driving within the subway catchment area, but does not affect it further out. Implicitly, the cost of mode changes is large.

(a) Find the rent gradient analytically for both types of city.

(b) Graph rent gradients, commute costs and equilibrium city sizes.

(c) Calculate the number of cars that pass each point x each day for both cities.

(d) Explain how this model rationalizes the two stylized facts given at the beginning of this problem.

3. This question asks you to find the rent gradient three different ways. Suppose that $u(h,z) = h^{\alpha} z^{1-\alpha}$ where h is housing, z is consumption. Suppose agents choose location x, have income w and pay unit transportation cost t.

   (a) Find $p(x)$ using the Marshallian method.

   (b) Find $p(x)$ using the Bid-rent approach.

   (c) Find $\frac{dp}{dx}$ using the expenditure function approach. For extra credit, verify that $p(x)$ you found in the first two parts of this question satisfies this definition of $\frac{dp}{dx}$.