$$u(\theta) = \begin{cases} W - R & \text{IF } \theta \text{ IN CITY} \\ \theta & \text{ELSE} \end{cases}$$

$\theta \in [0, \Theta]$.

(a) IN EQUILIBRIUM, H.H. CHOOSE CITY IFF $W - R \geq \theta$.
IF $\Theta > 1 > W$, THEN EVEN IF $R = 0$ WE HAVE H.H.
WITH $\theta > W$ OUTSIDE THE CITY $\Rightarrow$ LESS THAN
MEASURE 1 OF LAND OCCUPIED IN CITY FOR ANY $R \geq 0 \Rightarrow$ MARGINAL
CITY LAND IS VACANT $\Rightarrow R = 0$.

IF $\Theta > W > 1$ THEN $R^* = W - 1$ IS EQUILIBRIUM
RENT. IN THIS CASE $\theta = W - R^* = 1$
SO THE CITY IS FULLY OCCUPIED AND NO ONE WANTS
TO MOVE OR CHANGE R.

(b) IF $\Theta > 1 > W$, THEN $R = 0$ AND

$\theta \in [0, W]$ IN CITY, $\theta \in (W, \Theta)$ OUT.
SINCE RENT IS ZERO, LAND RENT IS ZERO. BUT

$$CS = \int_0^W (W - \theta)\, d\theta = \frac{1}{2} W^2$$

IF $\Theta > W > 1$ THEN $R = W - 1$ AND 1 UNIT OF
HH OCCUPY CITY

$$CS = \int_0^1 W - R - \theta \, d\theta = \int_0^1 1 - \theta \, d\theta = \frac{1}{2}$$

$$\text{RENT} = (W - 1) \cdot 1 = W - 1$$

(c) WITH HETEROGENEOUS OUTSIDE OPTIONS, WE NEED TO
WORRY ABOUT CS AS WELL AS RENT.

-4-

(2) MAX $u(c)$

$w = c + R(x) + 2tx$

(a) LET $c^* = u^{-1}(\bar{u})$.

THEN WE HAVE $R(x) + 2tx = w - c^*$ (1) $\forall x \in [-\bar{x}, \bar{x}]$

IN PARTICULAR, $\bar{R} + 2t\bar{x} = w - c^*$

$$\Rightarrow \quad \bar{x} = \frac{w - c^* - \bar{R}}{2t}$$

(b)(i) FROM ABOVE, WHEN w INCREASES TO w', WE HAVE

$$\bar{x} = \frac{w - c^* - \bar{R}}{2t}$$

AND $\bar{x}' = \bar{x} + \dfrac{w' - w}{2t}$

From (1)

$$R(x) = \begin{cases} w - c^* - 2tx & x \in [-\bar{x}, \bar{x}] \\ \bar{R} & \text{ELSE} \end{cases}$$

$$R'(x) \begin{cases} w' - c^* - 2tx & x \in [-\bar{x}', \bar{x}'] \\ \bar{R} & \text{ELSE.} \end{cases}$$

THUS

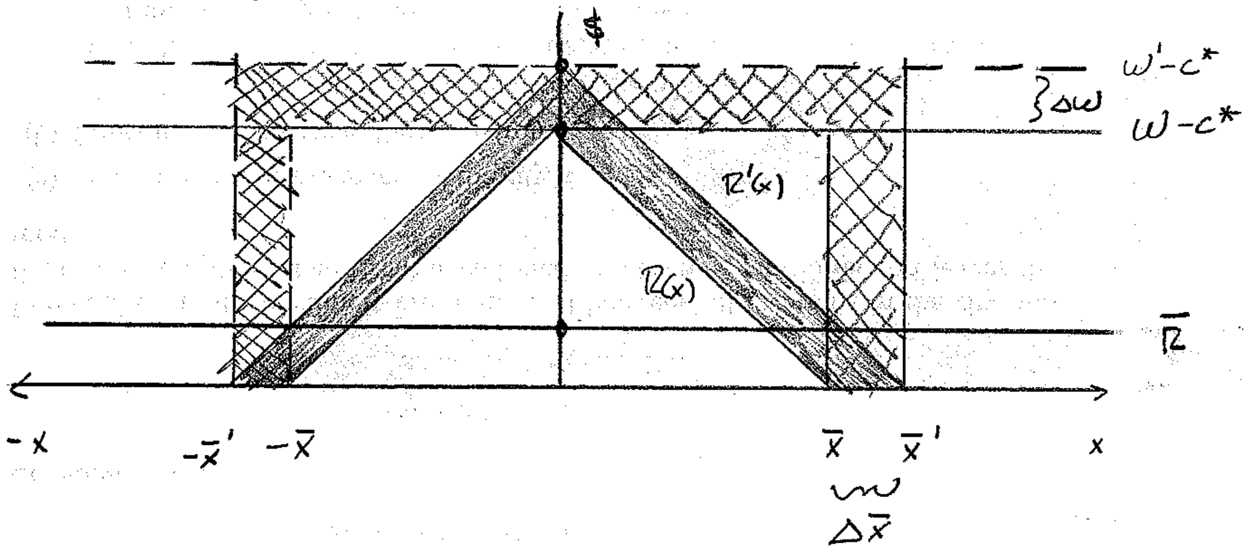
$$R'(x) - R(x) = \begin{cases} w' - w & x \in [-\bar{x}, \bar{x}] \\ w' - c^* - 2tx - \bar{R} & x \in \{[-\bar{x}', -\bar{x}] \cup [\bar{x}, \bar{x}']\} \\ 0 & \text{ELSE} \end{cases}$$

$\rightarrow$

-5-

$$\text{so} \quad \int_{-\infty}^{\infty} R' - R \, dx = 2\left[(w'-w)\bar{x} - \frac{1}{2}(w - c^* - 2t\bar{x}' - \bar{R})(\bar{x}' - \bar{x})\right]$$

(ii) IF NEW MIGRANTS WERE ALSO PAID w, THEN AGG. WAGE INCREASE IS $2\bar{x}(w'-w)$.



THE HATCHED AREA GIVES CHANGE IN AGGREGATE WAGE INCOME RESULTING FROM Δw.

THE SHADED AREA GIVES INCREASE IN AGGREGATE LAND RENT FROM Δw

THE INCREASE IN WAGES IS <u>LARGER</u> THAN THE INCREASE IN RENT BY ABOUT $2\Delta\bar{x}\, w'$.

HOWEVER, THE INCREASE IN WAGES <u>NET OF COMMUTING</u> IS COMPLETELY CAPITALIZED INTO RENT.

[a] MAX $h^\alpha z^{1-\alpha}$

S.T. $ph + z = \omega - \tau x$

$$\Rightarrow \quad z(p) = (1-\alpha)(\omega - \tau x)$$
$$h(p) = \frac{\alpha}{p}(\omega - \tau x)$$

WITH FREE MOBILITY

$$[h(p)]^\alpha [z(p)]^{1-\alpha} = \underline{u}$$

$$\Rightarrow \quad p(x) = \left[\frac{(\omega - \tau x)\alpha^\alpha (1-\alpha)^{1-\alpha}}{\underline{u}}\right]^{1/\alpha}$$

[b] SEE DURANTON + PUGA HANDBOOK p8

[c] $$\frac{d}{dx} e(p, \underline{u}) = \frac{\partial e}{\partial p} \cdot \frac{\partial p}{\partial x} = -\tau$$

SINCE $\frac{d}{dx}(\omega - \tau x) = -\tau$

BUT $\frac{\partial e}{\partial p} = h$ SO $\frac{\partial p}{\partial x} = \frac{-\tau}{h}$

(1) $\max V(c, q)$ [CONSUMER PROBLEM]

S.T. $y = c + pq + tx$

F.O.C $\quad pV_c = V_q \quad (1)$

FREE-MOBILITY $\quad V(y - pq - tx, q) = \bar{u} \quad (2)$

$(2) \Rightarrow \quad V_c\left[-pq_t - p_t q - x\right] + V_q q_t = 0 \quad (3)$

$(1) \rightarrow (3) \Rightarrow \quad p_t = -x/q < 0$ FOR $x > 0$. (4)

$\max_S \; l\left[ph(s) - r - iS\right]$ [HOUSING FIRM PROBLEM]

F.O.C $\Rightarrow \quad ph_s = i \quad (5)$

FREE-ENTRY $\Rightarrow \quad ph(s) - r - iS = 0 \quad (6)$

$(6) \Rightarrow \quad p_t h + ph' \cdot S_t - r_t - iS_t = 0$

USING (5) $\Rightarrow \quad r_t = h \cdot p_t$

$$\boxed{\therefore \text{ USING (4) WE HAVE } r_t = \frac{-xh}{q} < 0 \text{ FOR } x > 0}$$

$D \equiv hq^{-1}$ SO WE HAVE:

$$D_t = \frac{-h(s) q_t + h' S_t q}{q^2} \quad (7)$$

$h, h', q, q^2 > 0$ BY ASSUMPTION. THAT JUST LEAVES q_t, S_t.

BUT $q_t > 0$ B/C $p_t < 0$ AND q_t IS A COMPENSATED DEMAND.

$\longrightarrow$

TOTAL DIFFERENTIATING (5), WE HAVE

$$p_t h' + p h'' \cdot s_t = 0$$

$$\Rightarrow \quad s_t = \frac{-p_t h'}{p h''} = -\left[-\frac{x}{f} \frac{h'}{p h''} \right] < 0 \qquad (8)$$

USING (8) IN (7), TOGETHER WITH $f_t > 0$, WE HAVE $D_t < 0$

THAT IS, AS $t \uparrow$, AT ANY x, WE HAVE LAND PRICES AND DENSITY $\downarrow$.

INTUITIVELY, AS $t \uparrow$ COMMUTE COSTS GO UP AT EACH x. AS COMMUTE COST $\uparrow$, LAND PRICES $\downarrow$ TO PRESERVE CONSTANT u. BUT THIS MEANS (1) THE CAPITAL LAND RATIO SHOULD FALL (2) HOUSING PER PERSON $\uparrow$. TOGETHER THIS MEANS $D \downarrow$.

ANOTHER WAY TO THINK ABOUT THIS IS, AS $t \uparrow$ WE ARE RESCALING THE x AXIS, AND EACH x "LOOKS LIKE" A LOCATION THAT WAS MORE REMOTE WITH SMALLER t.