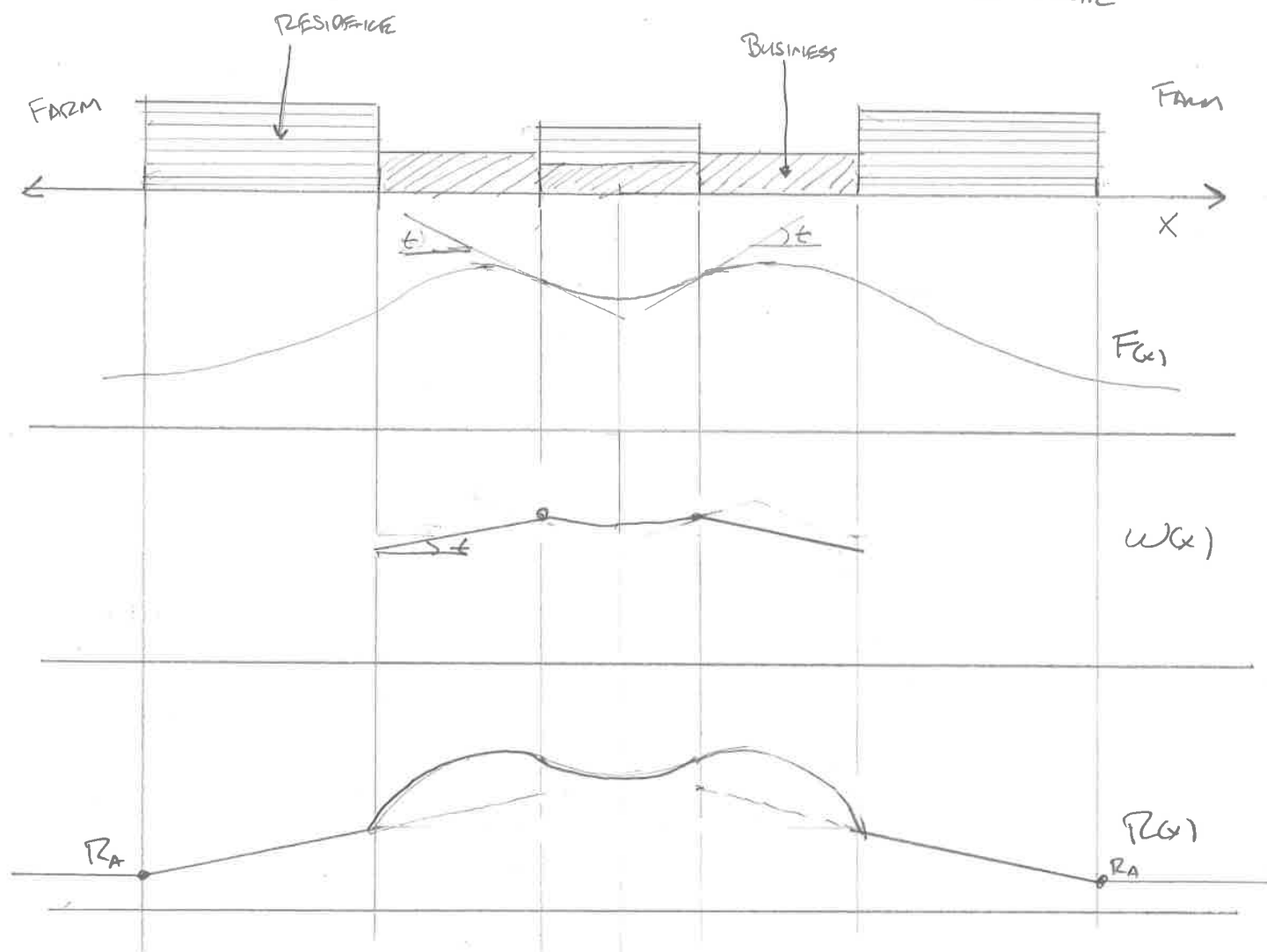


## FUSHA + OGAWA PARTIALLY MIXED EQUIL



1. CENTER IS MIXED  $\Rightarrow$  WORK WHERE YOU LIVE  
 $\Rightarrow$  SLOPE OF WAGE GRADIENT LESS THAN  $t$ .
2. PEOPLE IN ADJACENT BUSINESS DISTRICTS COMMUTE FROM OUTSIDE  $\Rightarrow$  WAGE GRADIENT HAS SLOPE  $t$   
 AND RENT GRADIENT ALWAYS ABOVE  $t$ .

①

- $s \in [s_1, s_2] \sim$  AMENITY  
 $X \sim$  NUMERAI'RE CONSUMPTION GOOD  
 $l^c \sim$  RESIDENTIAL LAND  
 $l^p \sim$  PRODUCTIVE LAND  
 $l \sim$  UTILITY LEVEL  
 $w, I \sim$  WAGE, NON-WAGE INCOME  
 $r \sim$  LAND RENT

$N \sim$  LABOUR = PEOPLE  
 $l^p \sim$  PRODUCTIVE LAND  
 $X = f(N, l^p; s)$   
 $\sim$  CRS.

$$\textcircled{1} L = l^c + l^p$$

FREE MOBILITY

- CONSUMERS SOLVE:  $\text{MAX } U(X, l^c; s)$   
 $\text{s.t. } X + r l^c = w + I$   
 $\Rightarrow \textcircled{2} V(w, r; s) = k$

- FIRMS HAVE UNIT COST FUNCTION

$$\textcircled{3} C(w, r; s) = 1$$

↑ FREE ENTRY +  $\pi = 0 + P_X = 1$ .

WITH CRS  $\Rightarrow \textcircled{4} C_r = \frac{l^p}{X}, \textcircled{5} C_w = \frac{N}{X}$

AN EQUILIBRIUM MUST SATISFY  $\textcircled{1} - \textcircled{5}$ .

TOTAL DIF  $\textcircled{2} + \textcircled{3} \Rightarrow V_w \frac{dw}{ds} + V_r \frac{dr}{ds} = +V_s \textcircled{6}$

$C_w \frac{dw}{ds} + C_r \frac{dr}{ds} = -C_s \textcircled{7}$

SOLVE  $\textcircled{6} + \textcircled{7}$  FOR  $\frac{dr}{ds} = \frac{-V_w C_s + V_s C_w}{V_w C_r - V_r C_w} \equiv \frac{V_w C_s - V_s C_w}{\Delta}$

SOLVE  $\textcircled{6} + \textcircled{7}$  FOR  $\frac{dw}{ds} = \frac{-V_s C_r + V_r C_s}{V_w C_r - V_r C_w} \equiv \frac{V_s C_r - V_r C_s}{\Delta}$

(2)

USING (4) + (5) WE HAVE

$$\Delta = -V_r C_w + V_w C_r = -V_r \frac{N}{x} + V_w \frac{l^p}{x}$$

$$= V_w \left[ -\frac{V_r}{V_w} \frac{N}{x} + \frac{l^p}{x} \right]$$

USING ROLL'S IDENTITY,

$$= V_w \left[ +l^c \frac{N}{x} + \frac{l^p}{x} \right]$$

$$= +V_w \left[ \frac{l^c N + l^p}{x} \right]$$

$$= \frac{+V_w L}{x}$$

FROM USING CRS OF C(.) IN (7)

$$\Rightarrow -C_s = \frac{N}{x} \frac{dw}{ds} + \frac{l^p}{x} \frac{dr}{ds}$$

$$\Rightarrow C_s = - \left[ \frac{N}{x} \frac{dw}{ds} + \frac{l^p}{x} \frac{dr}{ds} \right] \quad (8)$$

USING CRS OF C(.) IN (6)

$$\Rightarrow \frac{dw}{ds} + \frac{V_r}{V_w} \frac{dr}{ds} = \frac{V_s}{V_w} \quad (\text{USE ROLL'S IDENTITY})$$

$$\Rightarrow \frac{dw}{ds} = l^c \frac{dr}{ds} + \frac{V_s}{V_w} \quad \left[ \frac{V_r}{V_w} = -l^c \right]$$

$$\Rightarrow \frac{dw}{ds} = l^c \left[ \frac{-V_w C_s + V_s C_w}{\Delta} \right] + \frac{V_s}{V_w}$$

$$\Rightarrow \frac{dw}{ds} = \frac{+x l^c}{V_w L} \left[ -V_w C_s + V_s \frac{N}{x} \right] + \frac{V_s}{V_w}$$

→

(8)

$$\Rightarrow \frac{dw}{ds} = -\frac{X l^c}{L} C_s - \frac{N l^c}{L} \cdot \frac{U_s}{U_w} + \frac{U_s}{U_w}$$

$$P_s^* \equiv U_s/U_w \Rightarrow \frac{dw}{ds} = -\frac{X l^c}{L} C_s - \frac{N l^c}{L} P_s^* + P_s^*$$

$$\frac{dw}{ds} = -\frac{X l^c}{L} C_s + \frac{L - N l^c}{L} P_s^*$$

$$\frac{dw}{ds} = -\frac{X l^c}{L} C_s + \frac{l^p}{L} P_s^* \quad (9)$$

USING (8) TO SUBSTITUTE FOR  $C_s$  IN (9)

$$\frac{dw}{ds} = +\frac{X l^c}{L} \left[ C_w \frac{dw}{ds} + C_r \frac{dr}{ds} \right] + \frac{l^p}{L} P_s^*$$

USING CRS

$$= +\frac{X l^c}{L} \left[ \frac{N}{X} \frac{dw}{ds} + \frac{l^p}{X} \frac{dr}{ds} \right] + \frac{l^p}{L} P_s^*$$

$$\frac{l^p}{L} P_s^* = \frac{dw}{ds} - \frac{X l^c}{L} \left[ \frac{N}{X} \frac{dw}{ds} + \frac{l^p}{X} \frac{dr}{ds} \right]$$

$$\frac{l^p}{L} P_s^* = \frac{dw}{ds} - \frac{l^c N}{L} \frac{dw}{ds} - \frac{l^c l^p}{L} \frac{dr}{ds}$$

$$P_s^* = \left( \frac{L}{l^p} + \frac{l^c N}{l^p} \right) \frac{dw}{ds} - l^c \frac{dr}{ds}$$

$$\Rightarrow \boxed{P_s^* = \frac{dw}{ds} - l^c \frac{dr}{ds}} \quad \square$$