EC1410-Spring 2026
Problem Set 5 solutions

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1. In this question we will examine the dissimilarity index of a fictional city.

   Assume there are three census tracts in a city, each with population 1. Assume that, initially, the black population of the city is zero. At the end of the period, the black population increases to $\frac{1}{7}$ of the total city population (i.e., the city is around 14% black), while the non-black population does not change. Answer the following questions about the population distribution at the end of the period.

   (a) What is the total city population and black population?

   The black population becomes $\frac{1}{7}$ of the total population. Letting the black population be x, we have:

$$
\begin{aligned}
x &= \frac{3+x}{7} \\
7x &= 3+x \\
6x &= 3 \\
x &= \frac{1}{2} \text{ is the black population} \\
3+x &= 3.5 \text{ is the total city population}
\end{aligned}
$$

   (b) Assume that the black population is equally divided among the three tracts. What is the dissimilarity index?

   In this case, the black and non-black populations of the three tracts are (1/6, 1), (1/6,1) and (1/6,1). The total black population of the city is 0.5 and the total non-black population of the city is 3. So the dissimilarity index is:

$$
\begin{aligned}
\text{dissimilarity index } &= \frac{1}{2}\sum_{i=1}^{3}\Big|\frac{B_i}{B}-\frac{W_i}{W}\Big| \\
&= 0.5\Big(\Big|\frac{1/6}{0.5}-\frac{1}{3}\Big| + \Big|\frac{1/6}{0.5}-\frac{1}{3}\Big| + \Big|\frac{1/6}{0.5}-\frac{1}{3}\Big|\Big) \\
&= 0.5\Big(3*\Big|\frac{1}{3}-\frac{1}{3}\Big|\Big) \\
&= 0.5(3*0) \\
&= 0
\end{aligned}
$$

   (c) Assume that the entire black population is in one census tract. What is the dissimilarity index in this case?

   In this case, the black and non-black populations of the three tracts are (0.5, 1), (0,1) and (0,1). The total black population of the city is 0.5 and the total non-black population of the city is 3. So the dissimilarity index is:

$$\begin{aligned}
\text{dissimilarity index } &= \frac{1}{2}\sum_{i=1}^{3}\left|\frac{B_i}{B}-\frac{W_i}{W}\right| \\
&= 0.5\Big(\left|\frac{0.5}{0.5}-\frac{1}{3}\right| + \left|\frac{0}{0.5}-\frac{1}{3}\right| + \left|\frac{0}{0.5}-\frac{1}{3}\right|\Big) \\
&= 0.5\Big(\left|1-\frac{1}{3}\right| + \left|-\frac{1}{3}\right| + \left|-\frac{1}{3}\right|\Big) \\
&= 0.5\Big(\left|\frac{2}{3}\right| + \left|0-\frac{1}{3}\right| + \left|-\frac{1}{3}\right|\Big) \\
&= 0.5\Big(\frac{2}{3}+\frac{1}{3}+\frac{1}{3}\Big) \\
&= 0.5\Big(\frac{4}{3}\Big) \\
&= \frac{2}{3}
\end{aligned}$$

(d) Given the above, how should we interpret the rapid increase in the time series of the dissimilarity index in Figure 1 of Cutler et al. (1999)?

Unless the black population is equally distributed across all of the tracts in a city, any inflow of black population will lead to a non-zero dissimilarity index. That is, much of the increase in the dissimilarity increase could be caused mainly by an increase in black population instead of an increase in segregation, per se.

(e) Why is it important that the empirical analysis in Cutler et al. (1999) focuses on the variation in the dissimilarity index across cities, instead of over time?

The dissimilarity index could change over time due to inflows and outflows of the black and non-black populations. What is more interesting is how, in cities which may have faced similar amounts of migration, black residents ended up with different living patterns.

2. How much lower is the black than non-black college graduation rate in 1990 in high segregation cities compared to low segregation cities?

The black college graduation rate in 1990 was 4.9% in high segregation cities and 4.4% in low segregation cities. The non-black college graduation rate in 1990 was 10.6% in low segregation cities and 14.7% in high segregation cities. Therefore, the black college graduation rate is $4.9 - 4.4 = 0.5$ percentage points higher in high segregation cities than low segregation cities, and the non-black graduation rate is $14.7 - 10.6 = 4.1$ percentage points higher in high segregation cities than low segregation cities. Overall, then, the black college graduation rate is $4.1 - 0.5 = 3.6$ percentage points lower than the non-black college graduation rate in high segregation cities compared to low segregation cities.

3. In this problem, we will examine bid-rent functions with two types of agents.

Assume there are two types of agents, the rich and the poor, whose only difference (for now) is their wage levels, with $w_r > w_p$. Assume also that everyone takes the bus, which has cost

$$(w_r * t^b + c^b)|x| \text{ for the rich type, where } t^b, c^b > 0$$
$$(w_p * t^b + c^b)|x| \text{ for the poor type, where } t^b, c^b > 0$$

(a) Set up the household problem from the monocentric city model for each type of agent.

$$\max_{c,x} u(c) \text{ such that } w_r = c + R(x)\bar{l} + (w_r * t^b + c^b)|x| \text{ for the rich type}$$
$$\max_{c,x} u(c) \text{ such that } w_p = c + R(x)\bar{l} + (w_p * t^b + c^b)|x| \text{ for the poor type}$$

(b) Assume $u(c^*) = \overline{u}$ for both types. For both types, substitute an expression for c^* in terms of $\overline{u}$ into the constraint from the previous part. Call these functions $R_r(x)$ and $R_p(x)$, respectively.
If $u(c^*) = \overline{u}$, then $c^* = u^{-1}(\overline{u})$. Substituting into the constraints above, we have, for the rich type:

$$\begin{aligned} w_r &= c + R_r(x)\bar{l} + (w_r * t^b + c^b)|x| \\ w_r &= u^{-1}(\overline{u}) + R_r(x)\bar{l} + (w_r * t^b + c^b)|x| \\ R_r(x) &= \frac{w_r - u^{-1}(\overline{u}) - (w_r * t^b + c^b)|x|}{\bar{l}} \end{aligned}$$

For the poor type, we have:

$$\begin{aligned} w_p &= c + R_p(x)\bar{l} + (w_p * t^b + c^b)|x| \\ w_p &= u^{-1}(\overline{u}) + R_p(x)\bar{l} + (w_p * t^b + c^b)|x| \\ R_p(x) &= \frac{w_p - u^{-1}(\overline{u}) - (w_p * t^b + c^b)|x|}{\bar{l}} \end{aligned}$$

(c) Evaluate $R_r(0)$ and $R_p(0)$. Which is larger?

$$\begin{aligned} R_r(x) &= \frac{w_r - u^{-1}(\overline{u}) - (w_r * t^b + c^b)|x|}{\bar{l}} \\ R_r(0) &= \frac{w_r - u^{-1}(\overline{u})}{\bar{l}} \\ R_p(0) &= \frac{w_p - u^{-1}(\overline{u})}{\bar{l}} \\ R_r(0) - R_p(0) &= \frac{w_r - w_p}{\bar{l}} \\ w_r &> w_p \\ R_r(0) &> R_p(0) \end{aligned}$$

(d) Evaluate $\frac{\partial R_r(x)}{\partial x}$ and $\frac{\partial R_p(x)}{\partial x}$. Which is steeper?

$$\begin{aligned}
R_r(x) &= \frac{w_r - u^{-1}(\overline{u}) - (w_r * t^b + c^b)|x|}{\bar{l}} \\
\frac{\partial R_r(x)}{\partial x} &= \frac{-(w_r * t^b + c^b)}{\bar{l}} \\
\frac{\partial R_p(x)}{\partial x} &= \frac{-(w_p * t^b + c^b)}{\bar{l}} \\
\frac{\partial R_r(x)}{\partial x} - \frac{\partial R_p(x)}{\partial x} &= \frac{(-w_r + w_p) * t^b}{\bar{l}} < 0
\end{aligned}$$

That is, the gradient of the bid-rent function with respect to distance is more negative for the rich, so the line is steeper for the rich.

(e) Plot $R_r(x)$ and $R_p(x)$ on one graph. Indicate the areas in which each type has the higher willingness to pay. Describe the resulting equilibrium briefly.

This graph should look like the one on page 53 of the lecture notes. The rich have a higher willingness to pay at $x = 0$, but their willingness to pay declines more steeply with distance than does the willingness to pay of the poor, so the rich bus riders live near the center and the poor bus riders live further out.