EC1340-Fall 2025
Problem Set 8 solutions

(Updated 29 July 2025)

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1. (a) Fisher i chooses x_i to solve

$$\max \frac{x_i}{\sum_{j=1}^N x_j}\left(\sum_{j=1}^N x_j\right)^\alpha K^{1-\alpha} - wx_i$$

taking as given the behavior of the other fishers.
The optimal choice of x_i is determined by the first order condition

$$\begin{aligned}0 &= \frac{d\pi}{dx_i}\\ &= \left[\frac{1}{\sum_{j=1}^N x_j} - \frac{x_i}{(\sum_{j=1}^N x_j)^2}\right]\left(\sum_{j=1}^N x_j\right)^\alpha K^{1-\alpha} + \left[\frac{x_i}{(\sum_{j=1}^N x_j)}\right]\alpha\left(\sum_{j=1}^N x_j\right)^{\alpha-1} K^{1-\alpha} - w\end{aligned}$$

Now assume symmetry. That is, let $X = \sum_{j=1}^N x_j$ and $x_i = x_j = X/N$ for all i,j.

$$\Longrightarrow$$
$$0 = \left[\frac{1}{X} - \frac{X/N}{X^2}\right](X)^\alpha K^{1-\alpha} + \left[\frac{X/N}{X}\right]\alpha\,(X)^{\alpha-1} K^{1-\alpha} - w$$

As $N \to \infty$ all of the terms involving $X/N \to 0$ and we are left with,

$$w = \left[\frac{1}{X}\right](X)^\alpha K^{1-\alpha}$$

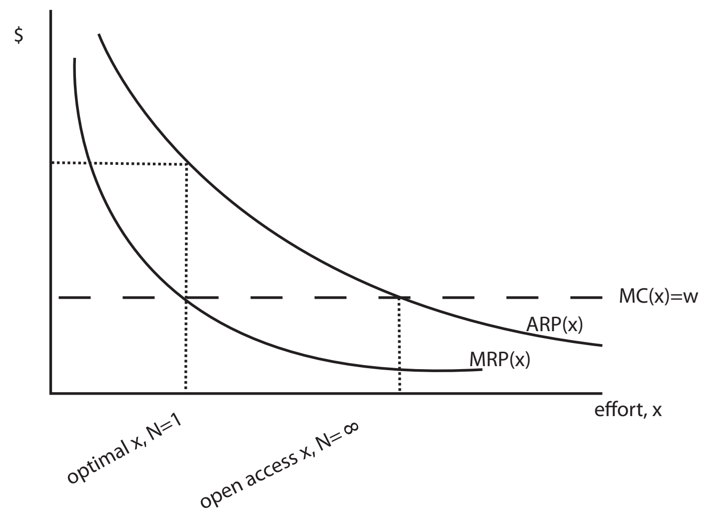
which is ARP(x)=MC(x), just as in Gordon.

(b) if $N = 1$ then $x = X$ and our first order condition is

$$\begin{aligned}w &= \left[\frac{1}{X} - \frac{1}{X}\right](X)^\alpha K^{1-\alpha} + \alpha\,(X)^{\alpha-1} K^{1-\alpha}\\ &= \alpha\,(X)^{\alpha-1} K^{1-\alpha}\end{aligned}$$

but this is just $MRP(x) = MC(x)$

(c) Your graph should look like this:



In the first part of this problem we have $ARP = w$ in the second part we have $MRP = w$.

2. (a) Our Fisher solves

$$\max_{h_i} \; h_i + \frac{1}{N}\left(K_1 - \sum_{j=1}^{N} h_j\right)^{1/2} \frac{1}{1+r}$$

Differentiating with respect to h_i we have our first order condition

$$\frac{d\pi}{dh_i} = 0 \tag{1}$$

$$\Longrightarrow 1 + \frac{1}{(1+r)N}\left[\frac{1}{2}\left(K_1 - \sum_{j=1}^{N} h_j\right)^{-1/2}(-1)\right] = 0 \tag{2}$$

$$\Longrightarrow \frac{1}{2(1+r)N}\left(K_1 - \sum_{j=1}^{N} h_j\right)^{-1/2} = 1 \tag{3}$$

$$\Longrightarrow \left(K_1 - \sum_{j=1}^{N} h_j\right)^{1/2} = \frac{1}{2N(1+r)} \tag{4}$$

$$\Longrightarrow K_1 - \sum_{j=1}^{N} h_j = \left(\frac{1}{2N(1+r)}\right)^2 \tag{5}$$

$$\Longrightarrow \sum_{j=1}^{N} h_j = K_1 - \left(\frac{1}{2N(1+r)}\right)^2 \tag{6}$$

If all fishers are identical then $h_1 = \frac{1}{N}\sum_{j=1}^{N} h_j$ and we have

$$h_i^* = \frac{1}{N}\left[K_1 - \left(\frac{1}{2N(1+r)}\right)^2\right]$$

(b) Note that from 6, as $N \to \infty$, $\sum h_j \to K_1$ so that aggregate profits are close to K_1.

If instead, a single fisher exploits the whole fishery we have, from 2

$$1 + \frac{1}{(1+r)}\left[\frac{1}{2}(K_1 - h_1)^{-1/2}(-1)\right] = 0$$
$$\Longrightarrow h_1^* = K_1 - \left(\frac{1}{2(1+r)}\right)^2$$

If we substitute this value of h_1 into the expression for profits we find that profits for the fishery when it is managed by a single profit maximizing owner are

$$\pi^* = \left(K_1 - \left(\frac{1}{2(1+r)}\right)^2\right) + \left(K_1 - \left(K_1 - \left(\frac{1}{2(1+r)}\right)^2\right)\right)^{1/2} \frac{1}{1+r}$$
$$= K_1 + \frac{1}{4}\left(\frac{1}{(1+r)}\right)^2$$

This is always strictly larger than K_1, so in a large fishery a single individual can always buy out all of the other individuals and have about $\frac{1}{4}\left(\frac{1}{(1+r)}\right)^2$ left over.

In fact, this doesn't happen in reality, and you might want to think about why this is.

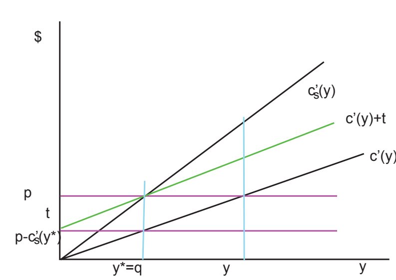
3. (a) The steel mill solves

$$\max pf(l_s) - wl_s$$
$$\Longrightarrow pf'(l_s) = w$$

(b) The fishery solves

$$\max qg(l_c,h) - wl_c$$
$$\Longrightarrow q\frac{\partial g(l_c,h)}{\partial l_c} = w$$

(c) Since the steel mill does not account for the damage it causes the fishery in its production decisions, we should expect the economy to produce too much steel and too few fish relative to the amounts that maximize total profits.



(d) The conglomerate solves

$$\max_{l_c, l_s} pf(l_s) + qg(l_c, h) - w(l_c + l_s)$$

$$\Longrightarrow$$

$$q\frac{\partial g(l_c, h)}{\partial l_c} = w$$

$$pf'(l_s) + q\frac{\partial g(l_c, h)}{\partial h}\frac{\partial h}{\partial s}f'(l_s) = w$$

The term involving q in the second first order condition describes the lost revenues in the fishery associated with the marginal unit of labor used to make steel.

(e) Under joint ownership, the conglomerate tries to maximize total profits to the fishery and the steel mill. In this case, the steel mill chooses steel output in a way that reflects its total cost.