

# EC1340-Fall 2023

## Problem Set 8

(Updated 31 July 2023)

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When you write up your answers, your goals should be to (1) be correct, and (2) convince your reader that your answer is correct. It is always helpful if your work is legible and if all steps are presented, possibly with a line of explanation.

In the case of empirical exercises, your goal should be to provide enough information to allow a reader to replicate your answer. This requires a description of data and data sources as well as a description of your analysis of the data.

Answers which do not achieve these goals will not be awarded full credit.

### Problems

1. Consider a fishery with  $N$  fishers. Let

$K$  = stock of fish

$x_i$  = effort by fisher  $i$ .

$$\left( \sum_{j=1}^N x_j \right)^\alpha K^{1-\alpha} = \text{harvest}, \alpha < 1.$$

$w$  = price of effort.

This exercise asks you to characterize market and rent maximizing exploitation behavior.

- (a) Find the first order conditions for  $x_i^1$  under open access. To do this assume symmetry, so that  $X = \sum_{j=1}^N x_j$  and  $x_j = X/N$ . Make this substitution in the first order conditions and take the limit as  $N \rightarrow \infty$ .
  - (b) Let  $N = 1$  and find the first order conditions.
  - (c) Explain the differences between the two sets of FOC's. Draw a graph like those that Gordon draws to illustrate both sets of FOC's.
2. Consider a fishery consisting of  $N$  people, each of whom lives for two days, and a pond. At the beginning of day 2, the pond contains  $K_2 = (K_1 - H)^{0.5}$  pounds of fish, where  $H$  is the total period 1 harvest and  $K_t$  is the stock of fish in period  $t$ . Let  $h_i \in [0,1]$  be the harvest by fisher  $i$  in period 1. Harvesting fish is costless. In days 2, all fishers harvest equal shares of the second period stock. If the fishers discount day 2 at the market rate of interest,  $r$ , and the price of fish is one, then a profit maximizing fisher chooses  $h_i$  to solve

$$\max_{h_i} h_i + \frac{1}{N} \left( K_1 - \sum_{j=1}^N h_j \right)^{0.5} \frac{1}{1+r}$$

- (a) Solve for the fisher's choice of  $h_i$ .

- (b) Does it make sense for a single individual to buy the whole fishery?
3. Suppose a steel company produces steel,  $s$  from labor,  $l_s$ , according to a production function  $s = f(l_s)$ . Also suppose that for every  $s$  units of steel produced, the steel company dumps  $h(s)$  tons of fish killing pollutants into the nearby river. A nearby fishery produces fish,  $c$ , according to how much labor,  $l_c$  it uses, and the level of pollution. That is  $c = g(l_c, h)$ . Let the prices of steel, fish, and labor be;  $p$ ,  $q$ , and  $w$ . Assume  $h' > 0$ ,  $g_h < 0$ ,  $f' > 0$ ,  $f'' < 0$  and  $g_{hh} < 0$ .
- Find the first order condition for the steel mill's profit maximization problem.
  - Find the first order conditions for the fishery's profit maximization problem.
  - Why do you expect that the output levels satisfying (3a) and (3b) will not be socially optimal? Draw a graph to go with your answer.
  - Suppose that a single conglomerate runs the fishery and the steel mill. Write down the conglomerate's profit maximization problem, and find the first order conditions for  $l_s$  and  $l_c$ . (You will need to do partial differentiation to generate these first order conditions.)
  - On the basis of your answers to (3a), (3b), and (3d), argue that joint ownership is socially preferred to separate ownership.