EC1410-Spring 2025
Problem Set 1

(Updated 29 October 2024)

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When you write up your answers, your goals should be to (1) be correct, and (2) convince your reader that your answer is correct. It is always helpful if your work is legible and if all steps are presented, possibly with a line of explanation. Answers that do not achieve these goals will not be awarded full credit.

Problems

1. In this problem, we will work through an example of the monocentric city model. Assume we have a linear, open city. Let w=3, $\bar{l} = 1$, $p_c = 1$, $\overline{R}$ = 0.5, $\overline{u}$ = 0, and $A = 1$. Let $u(c) = \ln(c - 1)$.

    (a) Set up the household's problem. Assume we are in a spatial equilibrium, so everyone is optimizing and no one wants to move. Call consumption in this equilibrium c^*. What is $u(Ac^*)$ equal to?

    (b) Find c^*.

    (c) Using the constraint from the household's problem, find an expression for $\overline{x}$ in terms of $w, c^*, \overline{R}, \bar{l}$ and t.

    (d) Use the assumption that there is one unit of land at each x to derive an expression for N^* in terms of $\overline{x}$ and $\bar{l}$.

    (e) Use the household's equilibrium budget constraint and the equilibrium extent of the city to solve for the equilibrium rent gradient, $R^*(x)$.

    (f) Take derivatives of your expressions for $\overline{x}, N^*$, and $R^*(x)$ with respect to t. How do the city extent, population, and equilibrium rent gradient change as transportation costs increase? Provide some intuition.

    (g) Assume that transportation costs increase from $t_0 = 1$ to $t_1 = 2$. What is the boundary of the city now? What is $R^*(0)$? Use these three points to draw a picture of how the rent gradient changes when t increases. Please label $R^*(0), \overline{R}$ and $\overline{x}$.

    (h) How would total land rent within the boundaries of the city change if we go from $t_0 = 1$ to $t_1 = 2$?

2. In this problem, we will analyze property taxes in the monocentric city model.

    (a) Assume we have an open, linear city with property tax rate τ_0. $R_0(x)$ is the land rent in this city. Set up the household's problem (you don't need to solve it).

    (b) Assume the tax rate increases from τ_0 to τ_1, where $1 + \tau_1 = (1.10)(1 + \tau_0)$. Set up the household's problem with this new tax rate.

    (c) Using what you know about c^* in an open city equilibrium, solve for $R_1(x)$ in terms of $R_0(x)$. How does the sum of rent and property taxes change?

(d) Suppose landlords are responsible for paying the property tax. What does this suggest about the relationship between what tenants pay and property taxes?

3. In this problem, we will examine rental gradients in practice. Using Zillow or some similar real estate website, pick a radial road out from the center of Providence (for example, along Angell Street from Kennedy Plaza) and plot the prices of at least 15 similar properties as distance to the center increases. What do you find? You can do this for another city if you would like.