

# EC2410-Spring 2023

## Problem Set 1

(Updated 15 January 2023)

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When you write up your answers, your goals should be to (1) be correct, and (2) convince your reader that your answer is correct. It is always helpful if your work is legible and if all steps are presented, possibly with a line of explanation.

Answers which do not achieve these goals will not be awarded full credit.

### Problems

1. Consider a city with measure one of land. All city residents receive a wage  $w$  and consume land inelastically so that measure 1 of residents occupies measure 1 of land, i.e., fills the city up. Residents pay land rent  $R \geq 0$  and derive utility from consumption. The set of potential city residents is the set  $[0, \Theta]$ , with measure  $\Theta$  and is indexed by  $\theta$ . Agent  $\theta$ 's utility is,

$$u(\theta) = \begin{cases} w - R & \text{if } \theta \text{ in city} \\ \theta & \text{else} \end{cases}$$

That is, agents get utility from consuming  $w - R$  in the city, and an idiosyncratic reservation value outside the city. Consider two cases,  $\Theta \geq 1 > w$  and  $\Theta \geq w > 1$ . Land markets are perfectly competitive.

- (a) Characterize a free mobility equilibrium for this economy, and in particular, find land rent for all locations in the city.
  - (b) Calculate aggregate land rent and consumers' surplus in equilibrium.
  - (c) Is land rent as interesting a measure of welfare in this model as in the linear city model? Explain briefly.
2. Consider the linear city model developed in class. Recall the following notation:

$l = 1 \sim$  unit land consumption for all residents

$c \sim$  composite consumption

$x \sim$  distance to center

$R(x) \sim$  unit land rent

$t \sim$  unit cost of transportation

$w \sim$  fixed wage paid to all workers at city center

$\bar{u} \sim$  reservation utility level

$\bar{R} \sim$  agricultural land rent

Consumers solve

$$\begin{aligned} \max_x & u(c) \\ \text{s.t. } & w = c + R(x) + 2tx. \end{aligned}$$

In equilibrium all consumers are indifferent between all locations in the city and their outside option. Land rent is collected by absentee landlords and leaves the model.

- (a) Find the equilibrium extent of the city,  $\bar{x}$ .
  - (b) Consider an increase in the wage from  $w$  to  $w'$ .
    - i. Calculate the resulting change in aggregate land rent.
    - ii. Calculate the resulting change in aggregate wage income. Assume that migrants' wages are also  $w$  before they move.
    - iii. Draw a graph to illustrate both quantities.
    - iv. Is an infinitesimal wage increase 'completely capitalized' into land rent? Explain briefly.
3. This question asks you to find the rent gradient three different ways. Suppose that  $u(h, z) = h^\alpha z^{1-\alpha}$  where  $h$  is housing,  $z$  is consumption. Suppose agents choose location  $x$ , have income  $w$  and pay unit transportation cost  $t$ .
- (a) Find  $p(x)$  using the Marshallian method.
  - (b) Find  $p(x)$  using the Bid-rent approach.
  - (c) Find  $\frac{dp}{dx}$  using the expenditure function approach. For extra credit, verify that  $p(x)$  you found in the first two parts of this question satisfies this definition of  $\frac{dp}{dx}$ .
4. Consider the monocentric city model with housing developed in Breuckner(2004) and discussed in class. Find the sign of  $\frac{\partial r}{\partial t}$  and  $\frac{\partial D}{\partial t}$ . That is, the rate of change of the land rent gradient and the population density gradient as transportation costs change.