

$$u(\theta) = \begin{cases} W-R & \text{if } \theta \text{ in city} \\ 0 & \text{else} \end{cases}$$

$$\theta \in [0, H].$$

- (a) IN EQUILIBRIUM, H.H. CHOOSE CITY IFF  $W-R \geq 0$ .  
 IF  $H > 1 > W$ , THEN EVEN IF  $R=0$  WE HAVE H.H.  
 WITH  $\theta > W$  OUTSIDE THE CITY  $\Rightarrow$  LESS THAN  
 MEASURE 1 OF LAND OCCUPIED IN CITY FOR ANY  $R \geq 0 \Rightarrow$  MARGINAL  
 CITY LAND IS UNOCCUPIED  $\Rightarrow R=0$ .

IF  $H > W > 1$  THEN  $R^* = W-1$  IS EQUILIBRIUM  
 RENT. IN THIS CASE  $\theta = W-R^* = 1$

SO THE CITY IS FULLY OCCUPIED AND NO ONE WANTS  
 TO MAKE AN CHANGE  $R$ .

- (b) IF  $H > 1 > W$ , THEN  $R=0$  AND

$$\theta \in [0, W] \text{ IN CITY, } \theta \in (W, H) \text{ OUT.}$$

SINCE RENT IS ZERO, LAND RENT IS ZERO. THEN

$$CS = \int_0^W (W-\theta) d\theta = \frac{1}{2} W^2$$

IF  $H > W > 1$  THEN  $R = W-1$  AND 1 UNIT OF  
 H.H. OCCUPY CITY

$$CS = \int_0^1 W-R-\theta d\theta = \int_0^1 1-\theta d\theta = \frac{1}{2}$$

$$RENT = (W-1) \cdot 1 = W-1$$

- (c) WITH HETEROGENEOUS OUTSIDE OPTIONS, WE NEED TO  
 WORRY ABOUT CS AS WELL AS RENT.

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(2) MAX  $u(c)$ 

$$W = c + R(x) + 2tx$$

(a) LET  $c^* = u^{-1}(\bar{u})$ .THEN WE HAVE  $R(x) + 2tx = W - c^*$  ①  $\forall x \in [-\bar{x}, \bar{x}]$ IN PARTICULAR,  $\bar{R} + 2t\bar{x} = W - c^*$ 

$$\Rightarrow \bar{x} = \frac{W - c^* - \bar{R}}{2t}$$

(b) (i) FROM ABOVE, WHEN  $W$  INCREASES TO  $W'$ , WE HAVE

$$\bar{x} = \frac{W - c^* - \bar{R}}{2t}$$

$$\text{AND } \bar{x}' = \bar{x} + \frac{W' - W}{2t}$$

FROM ①

$$R(x) = \begin{cases} W - c^* - 2tx & x \in [-\bar{x}, \bar{x}] \\ \bar{R} & \text{ELSE} \end{cases}$$

$$R'(x) = \begin{cases} W' - c^* - 2tx & x \in [-\bar{x}', \bar{x}'] \\ \bar{R} & \text{ELSE} \end{cases}$$

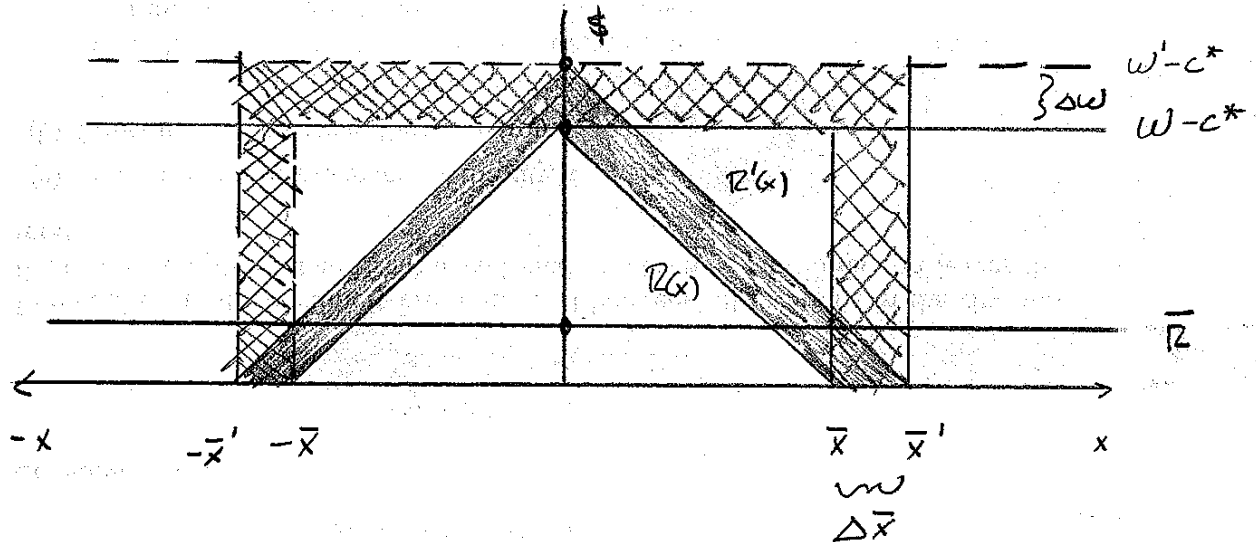
THUS

$$R'(x) - R(x) = \begin{cases} W' - W & x \in [-\bar{x}, \bar{x}] \\ W' - c^* - 2tx - \bar{R} & x \in [-\bar{x}', -\bar{x}] \cup [\bar{x}, \bar{x}'] \\ 0 & \text{ELSE} \end{cases}$$

→

$$\text{SO } \int_{-\infty}^{\infty} R' - R dx = 2 \left[ (w' - w) \bar{x} - \frac{1}{2} (w - c^* - 2\bar{x}' - \bar{r}) (\bar{x}' - \bar{x}) \right]$$

- (ii) IF NEW MIGRANTS WERE ALSO PAID  $w$ , THEN AGG. WAGE INCREASE IS  $2\bar{x}(w' - w)$ .



THE HATCHED AREA GIVES CHANGE IN AGGREGATE WAGE INCOME RESULTING FROM  $\Delta w$ .

THE SHADED AREA GIVES INCREASE IN AGGREGATE LAND RENT FROM  $\Delta w$

THE INCREASE IN WAGES IS LARGER THAN THE INCREASE IN RENT BY ABOUT  $2\Delta\bar{x} w'$ .

HOWEVER, THE INCREASE IN WAGES NET OF COMMUTING IS COMPLETELY CAPITALIZED INTO RENT.

[a]  $\text{MAX } h^\alpha z^{1-\alpha}$

S.T.  $ph + z = w - \tau x$

$\Rightarrow z(p) = (1-\alpha)(w - \tau x)$

$h(p) = \frac{\alpha}{p} (w - \tau x)$

WITH FREE MOBILITY

$[h(p)]^\alpha [z(p)]^{1-\alpha} = \underline{u}$

$\Rightarrow p(x) = \left[ \frac{(w - \tau x)^\alpha (1-\alpha)^{1-\alpha}}{\underline{u}} \right]^{1/\alpha}$

[b] SER DURAMITTI + PUGA HANDBOOK P8

[c]  $\frac{d}{dx} e(p, y) = \frac{\partial e}{\partial p} \cdot \frac{\partial p}{\partial x} = -\tau$

SINCE  $\frac{d}{dx} (w - \tau x) = -\tau$

BUT  $\frac{\partial e}{\partial p} = h$  SO  $\frac{\partial p}{\partial x} = \frac{-\tau}{h}$

①  $\text{MAX } V(c, f)$  [CONSUMER PROBLEM]

S.T.  $y = c + p_f f + t x$

F.O.C  $P V_c = V_f$  ①

FREE-MOBILITY  $V(y - p_f - t x, f) = U$  ②

②  $\Rightarrow V_c [-p_f - t] + V_f f_t = 0$  ③

②  $\rightarrow$  ③  $\Rightarrow P_t = -\frac{x}{f} < 0$  FOR  $x > 0$ . ④

$\text{MAX}_S$   $1 [p_h(s) - r - i s]$  [HEADING FIRM PROBLEM]

F.O.C  $\Rightarrow p_h s = i$  ⑤

FREE-ENTRY  $\Rightarrow p_h(s) - r - i s = 0$  ⑥

⑥  $\Rightarrow P_t h + p_h' s_t - r_t - i s_t = 0$

USING ⑤  $\Rightarrow r_t = h \cdot P_t$

$\therefore$  USING ④ WE HAVE  $r_t = -\frac{x h}{f} < 0$  FOR  $x > 0$

$D \equiv h_f^{-1}$  SO WE HAVE:

$D_t = \frac{-h(s) f_t + h' s_t f}{f^2}$  ⑦

$h, h', f, f^2 > 0$  BY ASSUMPTION. THAT JUST LEAVES  $f_t, s_t$ .

BUT  $f_t > 0$  B/C  $P_t < 0$  AND  $f_t$  IS A COMPLEMENTED DEMAND.

$\rightarrow$

TOTAL DIFFERENTIATING (5), WE HAVE

$$P_t h' + p h'' \cdot S_t = 0$$

$$\Rightarrow S_t = \frac{-P_t h'}{p h''} = - \left[ \frac{-x}{f} \frac{h'}{p h''} \right] < 0 \quad (8)$$

USING (8) IN (7), TOGETHER WITH  $f_t > 0$ , WE HAVE  $D_t < 0$

THAT IS, AS  $t \uparrow$ , AT ANY  $x$ , WE HAVE LAND PRICES AND DENSITY  $\downarrow$ .

INTUITIVELY, AS  $t \uparrow$  COMMUTE COSTS GO UP AT EACH  $x$ . AS COMMUTE COST  $\uparrow$ , LAND PRICES  $\downarrow$  TO PRESERVE CONSTANT  $U$ . BUT THIS MEANS (1) THE CAPITAL LAND RATIO SHOULD FALL (2) HOUSING PER PERSON  $\uparrow$ . TOGETHER THIS MEANS  $D \downarrow$ .

ANOTHER WAY TO THINK ABOUT THIS IS, AS  $t \uparrow$  WE ARE RESCALING THE  $x$  AXIS, AND EACH  $x$  "LOOKS LIKE" A LOCATION THAT WAS MORE REMOTE WITH SMALLER  $t$ .