

EC1340 Topic #9

## **Optimal warming policy**

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Fall 2021

*(Updated June 16, 2021)*

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# The problem of stabilizing CO<sub>2</sub>

Consider the problem we started with:

- World emissions of C in 2013 were about 13 Gt. Stabilizing atmospheric concentrations (not temp) requires cutting this about in half to 6.5Gt.
- There are about 7.4 bn people in the world as of 2015. Stabilization requires reducing emissions to  $6.5\text{Gt}/7.5\text{bn} < 1\text{t C emissions per person}$ .
- Current per capita emissions/incomes are about: US/CA/AU, 4.6t/55000\$; China is 1.8/7600\$; India is 0.5/1500\$.
- The US needs between a 50% and 80% reduction to hope to reach this target.

Stern/Gore/Hansen/Nordhaus disagree about the rate at which we should approach this goal, but not about the goal.

## Optimal mitigation policy

To tackle this problem, we wrote it as the BDICE model

$$\begin{aligned} \max_{I, M} \quad & \frac{c_1^{1-\alpha}}{1-\alpha} + \frac{1}{1+\rho} \frac{c_2^{1-\alpha}}{1-\alpha} \\ \text{s.t.} \quad & W = c_1 + I + M \\ & c_2 = (1+r)I - \gamma(T_2 - T_1)I \\ & E = (1 - \rho_4 \frac{M}{W})(\rho_5(c_1 + I)) \\ & P_2 = \rho_0 E + P_1 \\ & T_2 = \rho_1(P_2 - P_1) + T_1 \end{aligned}$$

The BDICE model organizes the main ideas, but leaves out some important things:

- Timing. When should we invest in education/factories and when in mitigation? To fix, use many time periods instead of two.
- Population growth. To allow this, Nordhaus uses exogenous population growth to match predictions. Endogenous would be better.
- Technical progress. Exogenous versus endogenous?  $\text{CO}_2$  reducing versus not? Both exogenous in Nordhaus.
- Non-linearities in climate response to  $\text{CO}_2$  and in carbon cycle. Nordhaus uses simple models calibrated to reproduce complicated models.
- Multiple countries. Not in our books, but treated in later work (RICE 2010)

## DICE model (partial)

Here are variable definitions for the DICE model

$Q(t)$  = total output (gdp) at  $t$

$L(t)$  = population at  $t$

$C(t)$  = aggregate consumption at  $t$

$c(t) = C(t)/L(t)$  per capita consumption

$I(t)$  = total savings at  $t$

$K(t)$  = capital at  $t$

$E(t)$  = emissions at  $t$

$A(t)$  = level of technology at  $t$

$\Lambda(t)$  = cost of mitigation as % of  $Q(t)$

$1 - \Omega(t)$  = loss of output from climate at  $t$

Here are main equations for the DICE model

$$W = \sum_{t=0}^{24} L(t) \frac{c(t)^{1-\alpha}}{1-\alpha} \left( \frac{1}{1+\rho} \right)^t \quad (1)$$

$$Q(t) = \Omega(t) [1 - \Lambda(t)] A(t) K(t)^\gamma L(t)^{1-\gamma} \quad (2)$$

$$Q(t) = C(t) + I(t) \quad (3)$$

$$K(t) = I(t) + (1 - \delta_K) K(t-1) \quad (4)$$

$$E(t) = \sigma(t) [1 - \mu(t)] A(t) K(t)^\gamma L(t)^{1-\gamma} \quad (5)$$

$$\Lambda(t) = \pi(t) \theta_1(t) \mu(t)^{\theta_2} \quad (6)$$

plus a description of the way climate, carbon, population and technology evolve.

What is all of this stuff!

- Equation 1 – utility function

$$\begin{aligned} W &= \sum_{t=0}^{24} L(t) \frac{c(t)^{1-\alpha}}{1-\alpha} \left( \frac{1}{1+\rho} \right)^t \\ &= L(0) \frac{c(0)^{1-\alpha}}{1-\alpha} + \\ &\quad L(1) \frac{c(1)^{1-\alpha}}{1-\alpha} + \left( \frac{1}{1+\rho} \right)^1 \\ &\quad L(2) \frac{c(2)^{1-\alpha}}{1-\alpha} + \left( \frac{1}{1+\rho} \right)^2 + \dots \end{aligned}$$

This is a generalization of the CRRA utility function to many periods AND weights each period by population.

- Equation 2 – production

$$Q(t) = \Omega(t) [1 - \Lambda(t)] A(t) K(t)^\gamma L(t)^{1-\gamma}$$

$Q(t)$  = total output (gdp) at  $t$

$L(t)$  = population at  $t$

$K(t)$  = capital at  $t$

$A(t)$  = level of technology at  $t$

$\Lambda(t)$  = cost of mitigation as % of  $Q(t)$

$1 - \Omega(t)$  = loss of output from climate at  $t$



- Equation 3 – Budget constraint at  $t$

$$Q(t) = C(t) + I(t)$$

$C(t)$  = aggregate consumption at  $t$

$I(t)$  = total savings at  $t$

- Equation 4 – Evolution of capital

$$K(t) = I(t) + (1 - \delta_K)K(t-1)$$

$I(t)$  = total savings at  $t$

$K(t)$  = capital at  $t$

$\delta_K$  is ‘depreciation rate’.

- Equation 5 – Emissions

$$E(t) = \sigma(t) [1 - \mu(t)] A(t) K(t)^\gamma L(t)^{1-\gamma}$$

$L(t)$  = population at  $t$

$K(t)$  = capital at  $t$

$E(t)$  = emissions at  $t$

$A(t)$  = level of technology at  $t$

$\mu(t)$  = share of mitigation at (policy variable)  $t$

$\sigma(t)$  = Gt Carbon per unit output at  $t$

The RHS of this basically eq 2 (output)  $\times \sigma(t)$ .

- Equation 6 – mitigation cost function

$$\Lambda(t) = \pi(t)\theta_1(t)\mu(t)^{\theta_2}$$

$\theta_1(t), \theta_2(t)$  = mitigation cost parameters at  $t$

$\pi(t)$  = participation cost markup at  $t$

$\Lambda(t)$  = cost of mitigation as share of  $Q(t)$

The ‘low cost backstop’ policy involves modifying  $\Lambda$  so each ton of mitigation is 5\$ (Nordhaus, p77). This is Nordhaus’ stylized description of geo-engineering.

- All together again,

$$W = \sum_{t=0}^{24} L(t) \frac{c(t)^{1-\alpha}}{1-\alpha} \left( \frac{1}{1+\rho} \right)^t$$

$$Q(t) = \Omega(t) [1 - \Lambda(t)] A(t) K(t)^\gamma L(t)^{1-\gamma}$$

$$Q(t) = C(t) + I(t)$$

$$K(t) = I(t) + (1 - \delta_K) K(t-1)$$

$$E(t) = \sigma(t) [1 - \mu(t)] A(t) K(t)^\gamma L(t)^{1-\gamma}$$

$$\Lambda(t) = \pi(t) \theta_1(t) \mu(t)^{\theta_2}$$

This is a partial description of the DICE model. Things I've skipped:

- How climate is affected by  $\text{CO}_2$  .
- How emissions affect atmospheric  $\text{CO}_2$  .
- How the stock of  $\text{CO}_2$  evolves in the atmosphere. It's all there, and looks like what we've talked about in class.
- The rate of return to capital,  $r$ , doesn't occur explicitly in this model. Instead we have, a social rate of time preference  $\rho$ , 'inequality aversion', here it's  $\alpha$ . Consumption growth is hidden. It's the rate of change of  $c_t$ . The rate of return to capital is  $\alpha g + \rho = r$ .

Using the DICE model, for any mitigation path we can find

- gdp at  $t$
- emissions at  $t$
- carbon concentration at  $t$
- climate at  $t$
- carbon price at  $t$

What is carbon price?  $\frac{dW}{dE}$  at  $t$ . We can also look at the mitigation

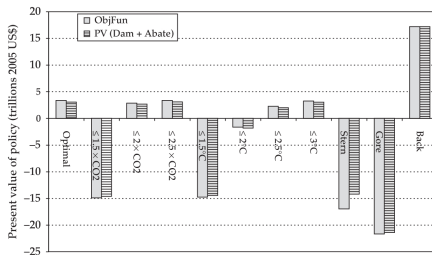
paths,  $\mu(t)$ , that accomplish different goals. For example:

- ①  $\mu(t) = 0$
- ②  $\mu(t)$  maximizes  $W$  – the optimal policy
- ③  $\mu(t)$  maximizes  $W$  and  $\text{CO}_2 \leq \text{cap}$
- ④  $\mu(t)$  maximizes  $W$  and temperature  $\leq \text{cap}$
- ⑤  $\mu(t)$  approximates Kyoto – a fraction of countries restrict emissions to 1990 levels
- ⑥ Stern review – strict cap on  $\text{CO}_2$
- ⑦ Gore plan – strict cap on  $\text{CO}_2$
- ⑧ low cost backstop

The ‘value’ of a policy 2-8 is the difference between  $W$  under that policy and policy (1).



## Value of different policies relative to 'do nothing for 250 years':



**Figure 5-1.** Present value of alternative policies. The difference in the present value of a policy relative to the baseline under two measures. The first bar is the value of the objective function in 2005 dollars (ObjFun), and the second is the present value of the sum of abatement and damages in the same units [PV (Dam + Abate)]. The policies are shown in Table 4-1. The baseline is omitted because it has zero present-value difference.

Note: World annual income was about 50T in 2005. This graph says global warming is a small problem! Why? Discounting and small damages.

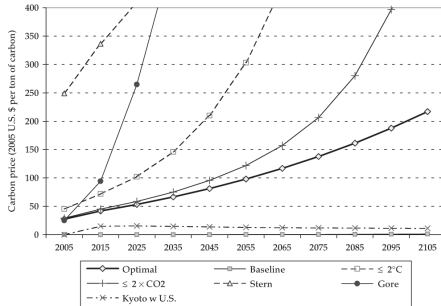
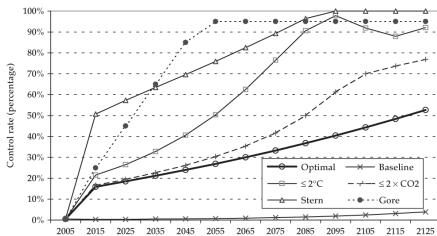
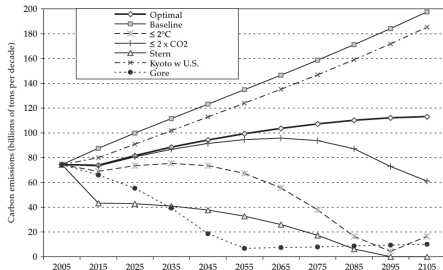


Figure 5-4. Carbon prices under different policies. The globally averaged carbon price under different policies over the next century. Note the upward tilt of the strategies. These prices are per ton of carbon; for prices per ton of  $\text{CO}_2$ , divide by 3.67.

Recall carbon price is change in  $W$  resulting from 1t C in that year.

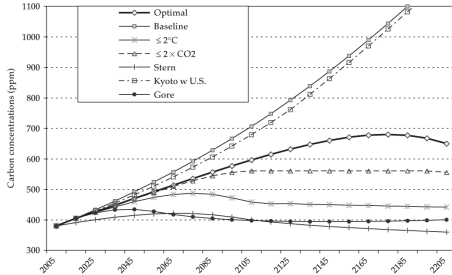


**Figure 5-5.** Emissions-control rates under different policies. The global emissions-control rate for CO<sub>2</sub> under different policies over the next century. Note the upward-tilted ramp of the strategies.



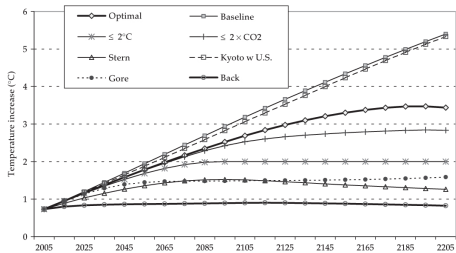
**Figure 5-6.** Global emissions of industrial CO<sub>2</sub> per decade under different policies. The global emissions of industrial CO<sub>2</sub> under different policies over the next century. The figure for 2005 is the actual value.

Note that emissions continue to increase on Nordhaus' optimal plan!

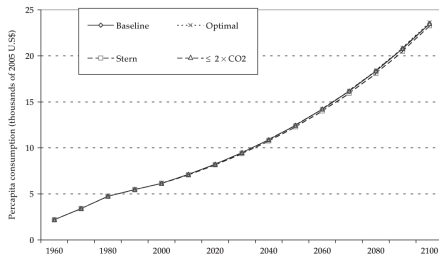


**Figure 5-7.** Atmospheric CO<sub>2</sub> concentrations under different policies. The atmospheric concentrations of CO<sub>2</sub> under different policies over the next century. The figure for 2005 is the actual value.

Concentrations start to fall under optimal plan, even with rising emissions. Why? (Carbon cycle, and 200 years vs 100 years on x-axis!)



**Figure 5-8.** Projected global mean temperature change under different policies. Increases are relative to the 1900 average.



**Figure 5-9.** Per capita consumption, major runs. The trend of per capita consumption is strongly rising in the DICE-2007 model projections. Also, the levels of consumption are virtually indistinguishable among the different policies.

This figure also shows that global warming is a ‘small problem’.  
The really important thing in this model is economic growth.

## Conclusion

Using Nordhaus' model we find that the optimal policy calls for a modest initial price of  $\text{CO}_2$  which rises over time.

This occurs because, at least for the next 50 years, it looks like we'll get rich much faster if we invest in capital than in mitigation. After 50 years, we'll be able to afford much more rapid mitigation, and it won't hurt as much because we're starting from a much higher income level.



## Issues with DICE

- DICE uses the following assumptions
  - $r=5.5\%$
  - Damages, more-or-less, from developed world agriculture. No fires, hurricanes or plagues, etc.
  - No growth effects
  - No uncertainty

That is, at every opportunity, DICE uses 'most conservative' defensible assumptions. That this still leads us to conclude that we need a carbon tax is compelling. There is lots of room to debate about whether the Carbon tax should be 40\$ per ton, or 400\$. That it should be positive seems settled.

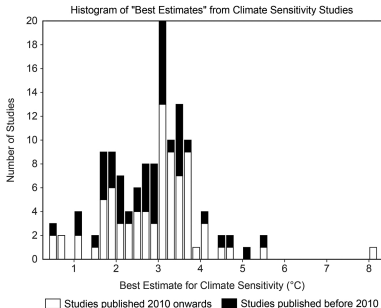
- We've spent the last several weeks talking about the quality of the underlying data. There is lots of room for improvement
- What about uncertainty, or 'stewardship' or sustainability? What about differences between countries?

## Pindyck Critique

Pindyck makes two critiques of the economic analysis of climate change. They are mainly directed at the literature based on IAMs, which builds on Nordhaus and is a lot more complex.

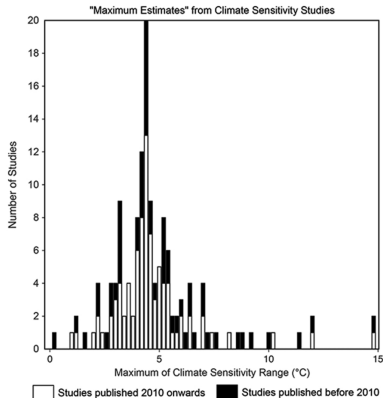
- That uncertainty is so important that precise estimates of the social cost of carbon are misleading. We should not expect more precision than we would expect for the choice of defense spending.
- That uncertainty is a fundamental part of the problem and means that we should be thinking about policy as insurance against catastrophes rather than moving the mean.

# Distribution of Climate Sensitivity



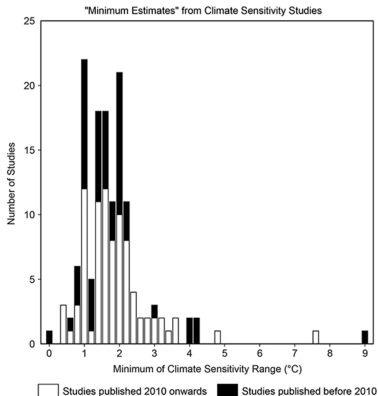
- Histogram of climate sensitivities drawn from 131 studies.
- Black is pre2010. White is post. Note that post-2010 density is flatter, so uncertainty is increasing. Note range.
- Note that time frame for climate sensitivities is not specified.
- Implicitly, this figure imagines that each study is a draw from the true distribution of climate sensitivities. Does this seem right?

# Distribution of Climate Sensitivity:Max



- Histogram of maximum climate sensitivities drawn from 131 studies.
- Black is pre2010. White is post.

# Distribution of Climate Sensitivity:Min



- Histogram of minimum climate sensitivities drawn from 131 studies.
- Black is pre2010. White is post.

# Insurance I

- Climate uncertainty together with a convex damage function creates a role for 'insurance'.
- Nordhaus damage function, share of GDP lost at  $\Delta T$ , is

$$L(\Delta T) = 1 - \frac{1}{1 - 0.0045\Delta T + 0.0035\Delta T^2}$$

Pindyck uses something a little simpler

$$L_P(\Delta T) = 1 - \frac{1}{1 + 0.01\Delta T^2}$$

## Insurance II

- So we have

$$L_P(0) = 0$$

$$L_P(2) = 0.04$$

$$L_P(4) = 0.14$$

- Consider two lotteries,
  - 2°warming for certain, so  $L_P = 0.04$
  - Equal chance of 0 or 4°of warming, so
$$E(L_P) = \frac{1}{2}L_P(0) + \frac{1}{2}L_P(4) = 0.07$$

so we would pay to avoid uncertainty.

- This is the sense in which climate change policy is ‘insurance’.

## Insurance III

- This issue is compounded by the fact that the utility function is also concave.

If you don't like Nordhaus' conclusion, 'climate change is a bad recession', Pindyck is giving you an out. There is too much uncertainty for this sort of precision, and the uncertainty is itself costly and something we want to avoid.