EC1410-Spring 2025
Problem Set 9 solutions

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Matt Turner

1. The standard deviation of the sprawl index in Eid et al. (2008) is 0.281. Use this to evaluate the relationship between a one standard deviation change in sprawl and BMI (using the results from both the cross-sectional regression and the first-differences regression).

   The cross-sectional relationship is given by:

   $$BMI_{it} = \beta x_{it} + 0.46 * \text{Sprawl}$$

   This implies that if sprawl increases by 0.281, then BMI will increase by 0.46*0.281=0.12926.

   The first-differences relationship is given by:

   $$\Delta_t BMI_t = \beta \Delta_t x_t - 0.04 * \Delta_t \text{Sprawl}$$

   This implies that if sprawl increases by 0.281, then BMI will decrease by 0.04*0.281=0.01124.

2. Use Allcott et al. (2019) to determine, on average, by how much does a $20,000 increase in household income change the grams of sugar added per 1,000 calories of food on your grocery store shelf? Explain.

   Starting from the average amount of added sugar per 1,000 calories that a household with an income of $10,000 experiences, increasing household income by $20,000 decreases the average amount of added sugar per 1,000 calories by four grams. The same is true if you start with a household income of $20,000 and increase household income by $20,000. You can continue computing this with different starting incomes. Along the range of household incomes represented in the graph, the median decrease in added sugar per 1,000 calories for a $20,000 increase in household income is 2 grams. The mean decrease is around 2 grams as well.

3. (a) Using the right two panels of Figure 1 of Bayer et al. (2007), estimate the willingness to pay per point for a neighborhood with better test scores.

   Moving across the boundary, test scores increased by about 75 points, and house prices increased by about $25,000. This gives a willingness to pay per point of test scores of

   $$\frac{\$25{,}000}{75} = \$333.33$$

   (b) Using the same logic, estimate the value of a 1 percentage point higher college share, black share, and $1,000 in neighborhood income.

   Moving across the boundary, college share increased by about 5 percentage points, black share decreased by about 2 percentage points, and neighborhood income increased by about $2,000. Since house prices increased by about

\$25,000 moving across the boundary, our estimates of willingness to pay for a 1 percentage point higher college share, black share, and \$1,000 in neighborhood income are, respectively:

$$\frac{\$25{,}000}{5} = \$5{,}000, \quad \frac{\$25{,}000}{-2} = -\$12{,}500, \quad \frac{\$25{,}000}{2} = \$12{,}500$$

(c) Using these four estimates, what is the implied price increase when you cross the boundary into a higher quality school district?

The implied price increase when crossing the boundary is the sum across factors of the willingness to pay per point of change in each factor times the amount that each factor changed. Using the willingness to pay calculated above,

$$\$333.33 * 75 + \$5{,}000 * 5 - \$12{,}500 * (-2) + \$12{,}500 * 2 = \$100{,}000$$

(d) Compare this to the actual price increase. Your estimate should have been much bigger than what is observed. Why did this happen?

We attributed the entire observed \$25,000 increase in house prices to four different factors. So when we then add up the implied willingness to pay for each of those factors, we get four times the observed increase in house prices.

4. In this problem, we will work through an example of a hedonic model. Consider a housing market where houses are differentiated only by quality z. Let $p(z)$ be the market price for a house of quality z.

(a) Let the firms cost function be $c(z,\theta) = \frac{z^{1/\beta}}{\theta}$. Set up the firms profit maximization problem. Derive the first order condition.

The firms profit function can be written as:

$$\max_z \ \pi(z, p(z); \theta) = p(z) - \frac{z^{1/\beta}}{\theta}$$

$$FOC \implies \frac{dp}{dz} = \beta \frac{z^{\frac{1}{\beta}-1}}{\theta}$$

$$\theta(z) = \frac{z^{\frac{1}{\beta}-1}}{\frac{dp}{dz}\beta}$$

(b) Each household consumes unit housing to attain a utility $u(z, p(z); \gamma) = \gamma z^\alpha + m - p(z)$. Set up the households profit maximization problem. Derive the first order condition.

$$\max_z \ u(z, p(z); \gamma) = \gamma z^\alpha + m - p(z)$$

$$FOC \implies \frac{dp}{dz} = \gamma \alpha z^{\alpha-1}$$

$$\gamma(z) = \frac{\frac{dp}{dz}}{\alpha z^{\alpha-1}}$$

(c) Let $p(z) = z^2$, $\alpha = 2$ and $\beta = \frac{1}{3}$. Assuming perfectly assortative matching, solve for the optimal housing quality that clears the market. What is the number of firms making houses with the optimal housing quality?

Perfectly assortative matching implies that:

$$\begin{aligned}
\gamma(z) &= \theta(z) \\
\frac{\frac{dp}{dz}}{\alpha z^{\alpha-1}} &= \frac{z^{\frac{1}{\beta}-1}}{\frac{dp}{dz}\beta} \\
\text{Setting } \frac{dp}{dz} &= 2z : \\
\\
\frac{\alpha}{\beta} z^{\alpha+\frac{1}{\beta}-2} &= 4z^2 \\
z &= \frac{2}{3} \\
\theta(z) &= 1
\end{aligned}$$