

EC2410-Spring 2017

Problem Set 3

(Updated 25 January 2017)

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When you write up your answers, your goals should be to (1) be correct, and (2) convince your reader that your answer is correct. It is always helpful if your work is legible and if all steps are presented, possibly with a line of explanation.

Answers which do not achieve these goals will not be awarded full credit.

Problems

1. This problem asks you to work through a little of the math in Ciccone and Hall, AER 1996. Let n denote the number of workers in a county, a the area of the county, and q the output of the county. There are two counties, aggregate population is N and share β of population is in county 1.

The output of a county is described by the production possibility frontier, $0 = n^\alpha \left(\frac{q}{a}\right)^{\frac{\lambda-1}{\lambda}} - \frac{q}{a}$, where $\alpha > 0$ and $\lambda > 1$. The area of the two counties is the same.

- (a) Write aggregate output as a function of β and determine the conditions under which $\beta^* = 1$.
 - (b) Evaluate the elasticity of aggregate output to an increase in aggregate population, keeping β constant.
 - (c) Verify that estimating $\ln Q = A + B \ln N$ will give us an estimate of the elasticity you just calculated.
2. This problem asks you to work through the details of equilibrium in an economy with monopolistic competition and differentiated products, the hardware used in several of the papers we've read.

Let T denote a tradition good. This is a composite good with perfectly elastic supply (e.g., a commodity available on an international market) that sells for price p^T . Let M denote the measure of a set of differentiated products indexed by i . Specifically, let $q(i)$ denote a quantity consumed of good of type i for $i \in [0, M]$, let $x(i)$ denote the quantity of this good produced and, finally, let $p(i)$ denote the price of variety i .

Let

$$U = \frac{Q^\mu T^{1-\mu}}{\mu^\mu (1-\mu)^{1-\mu}}$$

be the utility of a representative agent, where Q is a CES consumption aggregator

$$Q = \left[\int_0^M q(i)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$$

Production of the differentiated good is done by monopolistically competitive firms, each of which has the identical technology and produces exactly one variety (the

possibility that firms could gain market power by controlling a segment of the product space is assumed away). The cost for firm i to produce $x(i)$ is

$$C(x(i)) = A + Bx(i),$$

so each firm has decreasing average costs.

Suppose that there is free entry of firms so that firm profits are all zero in equilibrium, that consumers have exogenous wage w , and that there are measure N of consumers.

- (a) Find the equilibrium price and quantity of variety i .
- (b) Verify the constant mark-up rule, i.e., that profit maximizing firms charge a constant mark up over marginal cost in equilibrium.
- (c) Can you verify that utility goes up with N in this model?