EC2410-Spring 2016
Problem Set 2

(Updated 13 February 2018)

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1. Consider a city with measure one of land. All city residents receive a wage w and consume land inelastically so that measure on of residents occupies measure 1 on land, i.e., fills the city up. Residents pay land rent $R \geq 0$ and derive utility from consumption. The set of potential city residents is the set $[0,\Theta]$, with measure Θ and is indexed by θ. Agent θ's utility is,

$$u(\theta) = \begin{cases} w - R & \text{if } \theta \quad \text{in city} \\ \theta & \text{else} \end{cases}$$

   That is, agents get utility from consuming $w - R$ in the city, and an idiosyncratic reservation value outside the city. Consider two cases, $\Theta \geq 1 > w$ and $\Theta \geq w > 1$.

   (a) Characterize a free mobility equilibrium for this economy, and in particular, find land rent for all locations in the city.

   (b) Calculate aggregate land rent and consumers' surplus in equilibrium.

   (c) Is land rent as interesting a measure of welfare in this model as in the linear city model? Explain briefly.

2. Consider a 'partially mixed' land use distribution in a linear city. Specifically, suppose that the central region of the city is mixed, i.e., occupied by firms an households. This central mixed region is surrounded by two symmetric business districts, occupied solely by firms. Finally, these business districts are surrounded by purely residential regions. Suppose that the details of this economy are as described in the Fujita and Ogawa we discussed in class. Can you construct land rent, agglomeration and wage gradients such that this spatial configuration is an equilibrium? Draw a graph to illustrate and explain briefly.

3. This question asks you to find the rent gradient three different ways. Suppose that $u(h,z) = h^{\alpha} z^{1-\alpha}$ where h is housing, z is consumption. Suppose agents choose location x, have income w and pay unit transportation cost t.

   (a) Find $p(x)$ using the Marshallian method.

   (b) Find $p(x)$ using the Bid-rent approach.

   (c) Find $\frac{dp}{dx}$ using the expenditure function approach. For extra credit, verify that $p(x)$ you found in the first two parts of this question satisfies this definition of $\frac{dp}{dx}$.

4. (Extra credit) Derive the equation $p_s^* = l^c \frac{dr}{ds} + \frac{dw}{ds}$ from equation (5) of Roback (1982).