EC1410-Spring 2026
Problem Set 9 solutions

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1. Suppose that the shares of three industries in city i are

$$S_{i1} = \frac{1}{10},\ S_{i2} = \frac{1}{10},\ S_{i3} = \frac{8}{10}$$

(a) Evaluate the Herfindahl index for this city.

$$\begin{aligned}
\text{Herfindahl index for city } i &= \Big(\sum_j s_{ij}^2\Big)^{-1} \\
&= \left(\Big(\frac{1}{10}\Big)^2 + \Big(\frac{1}{10}\Big)^2 + \Big(\frac{8}{10}\Big)^2\right)^{-1} \\
&= (0.01 + 0.01 + 0.64)^{-1} \\
&= (0.66)^{-1} \\
&= \frac{100}{66} \approx 1.5
\end{aligned}$$

(b) Suppose the share of each industry in national employment is $\frac{1}{3}$. Evaluate the relative specialization for city i, RZI_i.

$$\begin{aligned}
RZI_i &= \max_j(s_{ij}/s_j) \\
&= \max(\frac{1/10}{1/3}, \frac{1/10}{1/3}, \frac{8/10}{1/3}) \\
&= \max(\frac{3}{10}, \frac{3}{10}, \frac{24}{10}) \\
&= \frac{24}{10} = 2.4
\end{aligned}$$

2. In this problem we will repeat the derivation of $c_E(N)$ for a circular city. Recall that household consumption is the difference between wages and average commute costs, or

$$c_E(N) = w(N) - \frac{TC(N)}{N} = AN^\sigma - \frac{TC(N)}{N}$$

(a) First, assuming that the city extends to $\overline{x}$ and that $\bar{l} = 1$, what is the population of the city?

The population of the city is the total amount of land divided by the land consumed per person. There is now $2\pi x$ of land at each x, so to compute the total amount of land, and population, we have:

$$\begin{aligned}
\text{Total land} &= \int_0^{\overline{x}} 2\pi x dx \\
&= \pi x^2 \Big]_0^{\overline{x}} \\
&= \pi \overline{x}^2 \\
N &= \frac{\text{Total land}}{\bar{l}} \\
&= \frac{\pi \overline{x}^2}{1} \\
&= \pi \overline{x}^2
\end{aligned}$$

(b) Recall that for an individual located at x, commuting costs are $2t|x|$. What is the total commuting cost of this city?

There is now $2\pi x$ of land at each x, and commuting costs are $2t|x|$ for every individual located at x, so total commuting costs are:

$$\begin{aligned}
\text{Total commuting cost} &= \int_0^{\overline{x}} 2\pi x(2tx)\ dx \\
&= \int_0^{\overline{x}} 4x^2 \pi t\ dx \\
&= 4\pi t \frac{x^3}{3}\Big]_0^{\overline{x}} \\
&= \frac{4}{3}\pi t \overline{x}^3
\end{aligned}$$

(c) What is the average commuting cost?

$$\begin{aligned}
\text{Average commuting cost} &= \frac{\text{Total commuting cost}}{N} \\
&= \frac{\frac{4}{3}\pi t \overline{x}^3}{\pi \overline{x}^2} \\
&= \frac{4}{3} t \overline{x} \\
&= \frac{4}{3} t \sqrt{\frac{N}{\pi}}
\end{aligned}$$

(d) Plug the average commuting cost into the formula for $c_E(N)$. Plot $c_E(N)$.

$$\begin{aligned}
c_E(N) = w(N) - \frac{TC(N)}{N} &= AN^\sigma - \frac{TC(N)}{N} \\
&= AN^\sigma - t\frac{4}{3\sqrt{\pi}}\sqrt{N}
\end{aligned}$$

The plot of $c_E(N)$ should demonstrate that the second component of $c_E(N)$ is now proportional to the square root of N, instead of linear in N (as was the case with the linear city).

3. Consider the relationship between city size and consumption as given below. Note that the utility for being the only person in a city is zero, which is the same as the rural utility.

   (a) Suppose we must assign 6 people to cities. What possible equilibrium configurations can be maintained? Are there any unstable equilibria?

   In a city of size one, urban consumption equals rural consumption. Everyone being in cities of size one is not a stable configuration - if one more person enters the city, the consumption of both residents increases. That is, people would want to leave their solitary city and move somewhere else.

   Everyone in cities of size three (let's call them city A and city B) is a stable configuration. If someone left city A for city B, they would be worse off in city B (utility of a city of size four is lower than for a city of size three). If someone left from either city to form their own city, their utility would not change.

   Everyone in cities of size two is a stable configuration. If you left your city and either created a new city or created a city of size three, your utility in that city would be lower than it is in your city of size two.

   Two cities of size one and two cities of size two is not a stable configuration. The two people in cities of size one would want to move to each other's city.

   One city of each size is not a stable configuration. The person in the city of size one would join the city of size two to create a city of size three that would give them a higher payoff.

   (b) What would a real estate developer do? Why?

   A real estate developer wants to maximize consumption, so that they can extract the highest possible rent payments. To do this, they would create three cities of size two, since per-capita consumption is highest for cities of size two. Note that we assume here that real estate developers can create cities of a given size (ie, they can both induce people to enter and exclude people from entering).