EC1340 Topic #4

Climate damage (part I)

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Outline

1. Introduction
2. Ohlmstead and Rhode (2011)
3. Technical aside: partial differentiation
4. Technical aside: OLS
5. Mendehsohn, Nordhaus and Shaw (1995)
6. Conclusion: a climate damage function?

Progress so far...

Recalling our statement of the global warming problem,

$$\max_{I,M} u(c_1, c_2) \tag{1}$$

$$\text{s.t. } W = c_1 + I + M \tag{2}$$

$$c_2 = (1+r)I - \gamma(T_2 - T_1)I \tag{3}$$

$$E = (1 - \rho_4 \frac{M}{W})(\rho_5(c_1 + I)) \tag{4}$$

$$P_2 = \rho_0 E + P_1 \tag{5}$$

$$T_2 = \rho_1(P_2 - P_1) + T_1 \tag{6}$$

We have discussed the last two constraints, the climate model and the carbon cycle. We've also discussed emissions and the relationship between consumption and emissions.

Our next target is γ.

Costs of climate change

If we are going to find a big impact of climate change on productivity, it should be in a sector which is sensitive to the climate. Let's start by looking at agriculture. Presumably the effects are bigger here than they are in 'jewelry', 'manufacturing' or 'business services', which appear to be less sensitive to the climate. Thus, looking at the effect of climate on agriculture ought to give us an upper bound on the effect of climate on overall productivity.

Will climate change be an agricultural disaster? I

O & R track expansion wheat cultivation across North America during the 19th and 20th century. We see rapid expansion of wheat cultivation into areas and climates where wheat could not previously be grown.

Figure 2: Klippart's "Potential Wheat-Producing Area" in the North America in 1858.

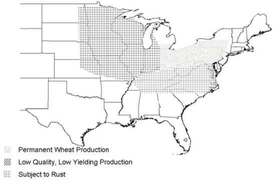
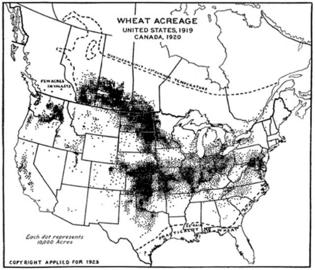
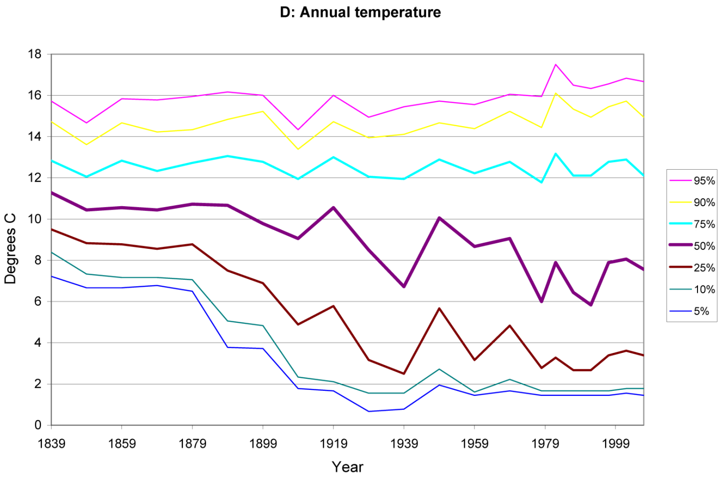


Figure 3: North American Acreage in Wheat in 1919-20.





Olmstead and Rhode (2011), Figure 4

In particular, wheat moves into colder, drier regions.
This is good for two reasons

- Agriculture can adapt quickly
- As the planet warms and get wetter, we'll be reverting to conditions more like those in the parts of North America that were first successfully settled/farmed. We know we can live in these conditions

What we can't learn from this paper is whether this adaptation will be easy, or if it will be hard, like the community of Selkirk described in the paper.
To answer this, we'll need to actually try to measure how agricultural productivity changes with climate.

Partial differentiation I

Given a univariate function $f : R \to R$, or $f(x) \in R$, we have

$$\frac{df}{dx} = \lim_{\epsilon \to 0} \frac{f(x + \epsilon) - f(x)}{\epsilon}$$

This is the 'instantaneous slope' of f at x.
Partial differentiation is the generalization of this idea to surfaces.
Consider a function $F : R^2 \to R$, or $F(x_1, x_2) \in R$. This function describes a surface, a height for each point in the plane. How do we think about the slope of such a surface? What we want is a tangent plane rather than a tangent line.

Partial differentiation II

With partial differentiation, we think about the slope of such a plane along one axis. Thus, given $F(x_1, x_2)$, we define

$$\frac{\partial F}{\partial x_1} = \lim_{\epsilon \to 0} \frac{F(x_1 + \epsilon, x_2) - F(x_1, x_2)}{\epsilon}$$

This is exactly analogous to the univariate derivative, if we imagine that we are finding the slope of a ‘slice’ of the surface parallel to the x_2 axis.
Mechanically, treat the ‘other variables’ as constant and use all the rules you know from univariate differentiation.

Partial differentiation III

Example:

$$
\begin{aligned}
F(x, y) &= 2x + 3y^2 + 2xy \\
\implies \frac{\partial F}{\partial x} &= 2 + 2y \\
\implies \frac{\partial F}{\partial y} &= 6y + 2x
\end{aligned}
$$

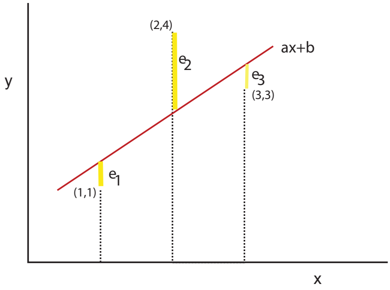
This should be in your calculus book.

OLS I

We'll need to use regression analysis to figure out the effect of climate on agricultural productivity.

- Regression analysis is a collection of mathematical techniques for drawing a line through the 'center' of a bunch of points.
- The first step is to define, mathematically, what 'center' means'.
- There are many ways to do this. One of the simplest is 'Ordinary Least Squares' or OLS.

OLS II



We want *a* and *b* so that line goes through the ‘center’ of these points.
Define the residuals as $e = y - (ax + b)$ so that
$e_1 = -(a + b - 1)$, $e_2 = -(2a + b - 4)$, $e_3 = -(3a + b - 3)$.

OLS III

Choose *a* and *b* to minimize the sum of squared residuals,

$$\begin{aligned} &min_{a,b} e_1^2 + e_2^2 + e_3^2 \\ =&min_{a,b} (-(a+b-1))^2 + (-(2a+b-4))^2 + (-(3a+b-3))^2 \end{aligned}$$

First order conditions are $\frac{\partial F}{\partial a} = 0$ and $\frac{\partial F}{\partial b} = 0$.

OLS IV

Evaluating $\frac{\partial F}{\partial a} = 0$,

$$
\begin{aligned}
& 2(a+b-1) + 2(2a+b-4)2 + 2(3a+b-3)3 = 0 \\
\implies & b = 3 - \frac{14}{6}a
\end{aligned}
$$

Evaluating $\frac{\partial F}{\partial b} = 0$,

$$
\begin{aligned}
& 2(a+b-1) + 2(2a+b-4) + 2(3a+b-3) = 0 \\
\implies & b = \frac{8}{3} - 2a
\end{aligned}
$$

Equating the two expressions for b we get $a = 1$. Using this in either of the expressions for b we get $b = 2/3$.

OLS V

Thus we have $y = \frac{2}{3} + x + e$, or $\hat{y} = \frac{2}{3} + x$.

(Aside) Actually, we'll think of e as a random variable, what we'll actually have is $E\hat{a}$, $E\hat{b}$, $E\hat{e}$. This means that the parameters themselves may be estimated with error.

OLS VI

We can rewrite these first order conditions in a really useful way,

$$
\begin{aligned}
\frac{\partial F}{\partial a} &= 0 \\
2(a+b-1)1 + 2(2a+b-4)2 + 2(3a+b-3)3 &= 0 \\
(a+b-1)1 + (2a+b-4)2 + (3a+b-3)3 &= 0 \\
e_1 x_1 + e_2 x_2 + e_3 x_3 &= 0 \\
\frac{1}{3} e_1 x_1 + \frac{1}{3} e_2 x_2 + \frac{1}{3} e_3 x_3 &= 0 \\
COV(ex) &= 0
\end{aligned}
$$

OLS VII

$$\begin{aligned} \frac{\partial F}{\partial b} &= 0 \\ 2(a+b-1)+2(2a+b-4)+2(3a+b-3) &= 0 \\ 2e_1+2e_2+2e_3 &= 0 \\ \frac{1}{3}e_1+\frac{1}{3}e_2+\frac{1}{3}e_3 &= 0 \\ E(e) &= 0 \end{aligned}$$

- That is, $E(e) = 0$ and $COV(ex)$ are equivalent to the two first order conditions for OLS.
- This is general.
- It means that if we think either of these conditions fails, then OLS will not let us draw the 'correct' line through our data.

OLS VIII

- But what if we estimate the wrong model?
- Suppose the true process is

$$y = b + ax + cz + e$$

and we estimate

$$y = \hat{b} + \hat{a}x + \hat{e}$$

Really this means that $\hat{e} = e + cz$.

- If $cov(x, \hat{e}) = cov(x, e + cz) = 0$ then this is OK and we will still get $\hat{a} = a, \hat{b} = b$.
- But if $cov(x, \hat{e}) \neq 0$ we don't.

OLS IX

Consider the following simple example of school funding and achievement.
Suppose

- y is mean test scores at a school
- x is spending per student at each school
- z is the unobserved average ability of students at each school.
- $x = c_1 z$ is the funding rule. Usually, more money goes to disadvantaged/low ability students so that $c_1 < 0$.

OLS X

Suppose that the true data generating process is

$$y = b + ax + cz + e$$

where $a, c > 0$ so test scores increase with funding and ability. Rearranging the funding rule we have $z = x/c_1$. If we substitute this into the data generating process we get

$$\begin{aligned} y &= b + ax + \frac{c}{c_1}x + e \\ &= b + (a + \frac{c}{c_1})x + e \end{aligned}$$

Since we don't observe z, we can only estimate

$$y = \hat{b} + \hat{a}x + \hat{e},$$

where $\hat{e} = \frac{c}{c_1}x + e$.

OLS XI

- It is not too hard to show that

$$COV(x, \hat{e}) = COV(x, \frac{c}{c_1}x + e) \neq 0.$$

  It follows that OLS cannot give us the right answer.
- Instead, we will get $\hat{a} = (a + \frac{c}{c_1})$.
- Since $c > 0$ and $c_1 < 0$ (often), this is going to mean that $E\hat{a} < a$.

OLS XII

- In fact, these kinds of regressions often find a negative effect of school funding on test scores.
- In this problem we say that x is an ‘endogenous variable’ in this estimating equation and that our estimate of the effect of spending on attainment is biased.
- We will need to worry about this sort of thing when we try to figure out the effect of climate on agricultural productivity.

Climate and agricultural productivity I

Based on Mendelsohn et al., AER 1995.
First we need some notation to describe agriculture:

- $q \sim$ output, e.g., wheat
- $p \sim$ output price
- $R \sim$ unit land price
- $K \sim$ other inputs, e.g. machinery, labor, fertilizer
- $w \sim$ price of other inputs
- $T \sim$ climate; $q = F(K, T) \sim$ output as a function of inputs for one unit of land, $\frac{\partial F}{\partial K} > 0, \frac{\partial F}{\partial T} = ?$

Climate and agricultural productivity II

- Suppose we collect data which describes wheat yields per acre, input use and climate. Then we can estimate, the production relationship as a linear function of inputs and climate

$$
\begin{aligned}
q &= F(K, T) + \epsilon \\
&= A_0 + A_1 K + A_2 T + \epsilon,
\end{aligned}
$$

  where ϵ is unobserved determinants of output, e.g., irrigation, farmer skill.
- A_2 will reflect the sensitivity of yields to climate, which is the number we want.
- (To make things easy on ourselves, we're requiring that F be linear.)

Climate and agricultural productivity III

- There are (at least) two problems with this approach.

Problem I – Endogeneity

Say that q also depends on farmer skill. Let I denote the extent of (unobserved) skill and suppose that skill depends in a deterministic, linear way on climate. That is, that $I = B_0 T$. In this case, if the true model of agricultural production is

$$q = A_0 + A_1 K + A_2 T + A_3 I + \mu$$

and we have

$$\begin{aligned} &= A_0 + A_1 K + A_2 T + A_3 B_0 T + \mu \\ &= A_0 + A_1 K + (A_2 + A_3 B_0) T + \mu \end{aligned}$$

Thus, if we estimate

$$q = \hat{A}_0 + \hat{A}_1 K + \hat{A}_2 T + \epsilon$$

we'll end up with $\hat{A}_2 = A_2 + A_3 B_0$. That is, we'll confound the effect of climate with the fact that more(less) skilful farmers outbid less(more) skilful farmers for better climates.

- This is another example of the endogenous variable problem we discussed in the context of school funding. It means that we don't have $cov(T, \epsilon) \neq 0$ in our estimating equation.
- This endogeneity problem arises almost any time you try to estimate a production function, so this is something you really should not do.

Problem II – Constraining crop substitution

When we estimate

$$q = A_0 + A_1 K + A_2 T + \epsilon$$

we are implicitly constraining farmers not to change crops (or to change to pasture or urban) as climate changes. (In fact, these sorts of studies are often done with data from experimental farms). This means that we are constraining the sorts of adjustments a farmer can make to climate.

We estimate the effect of climate on q yields correctly (subject to endogeneity bias) but overestimate the effect of climate on the value of agricultural productivity.

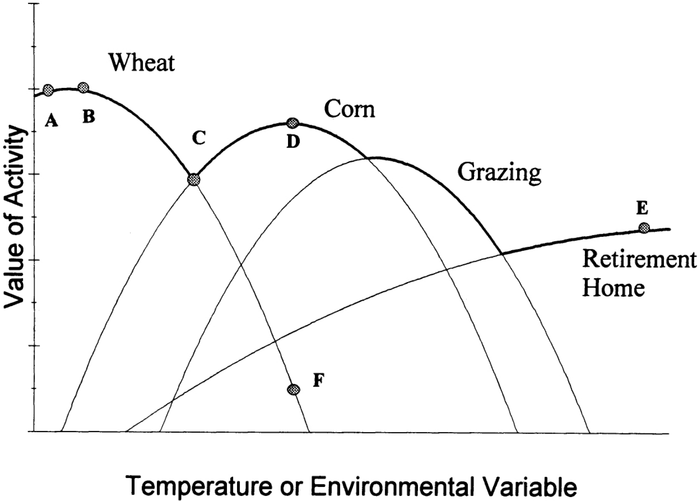


FIGURE 1. BIAS IN PRODUCTION-FUNCTION STUDIES

from Mendelsohn et al, AER 1995

Using land rent to infer the value of climate

To get around the problem of constraining crop choice, we will look at the effect of climate on agricultural land values. To understand why this works, we need more theory.
Farmers use the same technology and face the same prices as before and solve,

$$\max pF(K, T) - wK - R$$
$$\Longrightarrow p\frac{\partial F}{\partial K} = w$$

Solving this gives us the profit maximizing choice of K for given prices (really, a factor demand equation), $K^*(p, w, T)$.

Farmers enter at each location until profits are zero (this is perfect competition), so we have

$$pF(K^*, T) - wK^* - R = 0$$
$$\Longrightarrow R = pF(K^*, T) - wK^*$$

Differentiating with respect to T,

$$\Longrightarrow \frac{\partial R}{\partial T} = p\frac{\partial F(K^*, T)}{\partial T} + \left[p\frac{\partial F(K^*, T)}{\partial K^*}\frac{\partial K^*}{\partial T} - w\frac{\partial K^*}{\partial T}\right]$$
$$\Longrightarrow \frac{\partial R}{\partial T} = p\frac{\partial F(K^*, T)}{\partial T} + \left[p\frac{\partial F(K^*, T)}{\partial K^*} - w\right]\frac{\partial K^*}{\partial T}$$

and using the first order condition

$$\Longrightarrow \frac{\partial R}{\partial T} = p\frac{\partial F(K^*, T)}{\partial T}.$$

That is, since climate is not priced (you can't buy it separately) the value of climate is reflected in land rent. Say that 'the value of climate is capitalized into land rent'.

How do we estimate the land rent equation?

There is no time series variation in climate, so use cross-section.
Idea: compare similar farms in Kansas and North Dakota and attribute difference in land prices to climate.
Land price data is at the county level, climate data is at the weather station level.
To get climate at the center of each county:

- Get 1951-81 monthly average of 5511 weather stations for Jan, Apr, Jul, Oct, temp and rainfall.
- Predict weather at each station as a function of weather at neighboring stations, latitude, longitude, elevation, distance to stations.

Resulting ‘climate surface’, gives climate at every point, in particular, all county centers. (The details are complicated, the idea is easy).

Not all counties are equally agricultural. We learn more from a county A with 100% in agriculture with 50$ of revenue than we do from county B that is 1% in agriculture and generates 2$ of revenue.

To fix this (loose), draw two samples

- Area weighted: 100 replications of county A, 1 of B
- Revenue weighted: 50 replications of county A, 2 of B

In this way, we give more weight to places with more agriculture

Should we do this? Maybe unsuitability for agriculture is informative too?

Which weights should we prefer? Revenue weights give more weight to what happens in high value, non-grain crops. Area weighted gives more weight to grain.

Last problem: We observe land prices, not land rental price. Since the model is about rental prices, we need to convert sale prices to rental prices.
This takes a bit of theory.
Say a farmer can borrow M dollars at interest rate $r \Longrightarrow$ pay back $(1+r)M$ in one year. Suppose the price of land is A per unit.

- If $R > rA$ then farmer borrows A, buys land, farms, sells land for A, pays back $(1+r)A$ and is better off than if he had rented the land. Thus, if $R > rA$, there is no demand for rental land and rental prices fall.
- Conversely, if $R < rA$ then land owners sell their land and rent, and rental prices rise.

In equilibrium, we should have $R = rA$. That is, to get land rental rate, multiply by the interest rate, Mendelsohn et al use r=5%. As long as interest rates are the same everywhere (we'll just look at US here, so this is fine) and the asset price of land doesn't change for other reasons, e.g., close to a city (more problematic), this lets us go back and forth between land price and rental price.

With all these data in place, we can estimate the effect of climate on land rent. That is,

$$R = A_0 + A_1 T + A_2 T^2 + A_3 x + \epsilon$$

where A_1 and A_2 are the parameters we care about, x is soil characteristics, and ϵ is unobserved determinants of land rent. R is measured in 1978 and 1982.

Endogeneity of irrigation?

Suppose that irrigation is a function of climate, that is we irrigate the dry places, but not the wet ones. Then we have, e.g., $I = BT$, $B > 0$ and our true agricultural land rent depends on irrigation too,

$$\begin{aligned} R &= A_0 + A_1 T + A_2 T^2 + A_3 x + A_4 I + \epsilon \\ &= A_0 + A_1 T + A_2 T^2 + A_3 x + A_4 BT + \epsilon \\ &= A_0 + (A_1 + A_4 B) T + A_2 T^2 + A_3 x + \epsilon \end{aligned}$$

Thus, if we estimate

$$R = \hat{A}_0 + \hat{A}_1 T + \hat{A}_2 T^2 + \hat{A}_3 x + \epsilon$$

we will actually get $\hat{A}_1 = (A_1 + A_4 B)$. Since $B > 0$, $\hat{A}_1 > A_1$, so we overestimate the sensitivity of rent on unirrigated land to climate. Alternately, $\hat{A}_1$ gives us the total effect of climate on productivity after we allow time for people to build irrigation (This means we understate damages by the cost of building new irrigation).

Mendehsohn, Nordhaus and Shaw (1995)

TABLE 3—REGRESSION MODELS EXPLAINING FARM VALUES

	Cropland weights			Crop-revenue weights	
Independent variables	1982 (i)	1982 (ii)	1978 (iii)	1982 (iv)	1978 (v)
Constant	1,490	1,329	1,173	1,451	1,307
	(71.20)	(60.18)	(57.95)	(46.36)	(52.82)
January temperature	−57.0	−88.6	−103	−160	−138
	(6.22)	(9.94)	(12.55)	(12.97)	(13.83)
January temperature squared	−0.33	−1.34	−2.11	−2.68	−3.00
	(1.43)	(6.39)	(11.03)	(9.86)	(14.11)
April temperature	−137	−18.0	23.6	13.6	31.8
	(10.81)	(1.56)	(2.23)	(1.00)	(2.92)
April temperature squared	−7.32	−4.90	−4.31	−6.69	−6.63
	(9.42)	(7.43)	(7.11)	(9.44)	(11.59)
July temperature	−167	−155	−177	−87.7	−132
	(13.10)	(14.50)	(18.07)	(6.80)	(12.55)
July temperature squared	−3.81	−2.95	−3.87	−0.30	−1.27
	(5.08)	(4.68)	(6.69)	(0.53)	(2.82)
October temperature	351.9	192	175	217	198
	(19.37)	(11.08)	(11.01)	(8.89)	(9.94)
October temperature squared	6.91	6.62	7.65	12.4	12.4
	(6.38)	(7.09)	(8.93)	(12.50)	(15.92)
January rain	75.1	85.0	56.5	280	172
	(3.28)	(3.88)	(2.81)	(9.59)	(7.31)
January rain squared	−5.66	2.73	2.20	−10.8	−4.09
	(1.86)	(0.95)	(0.82)	(3.64)	(1.72)
April rain	110	104	128	82.8	113
	(4.03)	(4.44)	(5.91)	(2.34)	(4.05)
April rain squared	−10.8	−16.5	−10.8	−62.1	−30.6
	(1.17)	(1.96)	(1.41)	(5.52)	(3.35)
July rain	−25.6	−34.5	−11.3	−116	−5.28
	(1.87)	(2.63)	(0.94)	(6.06)	(0.34)
July rain squared	19.5	52.0	37.8	57.0	34.8
	(3.42)	(9.43)	(7.54)	(8.20)	(6.08)
October rain	−2.30	−50.3	−91.6	−124	−135
	(0.09)	(2.25)	(4.45)	(3.80)	(5.15)
October rain squared	−39.9	2.28	0.25	171	106
	(2.65)	(0.17)	(0.02)	(14.17)	(11.25)
Income per capita		71.0	65.3	48.5	47.1
		(15.25)	(15.30)	(6.36)	(7.39)
Density		1.30	1.05	1.53	1.17
		(18.51)	(16.03)	(18.14)	(17.66)
Density squared		-1.72×10^{-4}	-9.33×10^{-5}	-2.04×10^{-4}	-9.38×10^{-5}
		(5.31)	(3.22)	(7.47)	(4.57)
Latitude		−90.5	−94.4	−105	−85.8
		(6.12)	(6.95)	(5.43)	(5.33)
Altitude		−0.167	−0.161	−0.163	−0.149
		(6.09)	(6.41)	(4.72)	(5.20)
Salinity		−684	−416	−582	−153
		(3.34)	(2.20)	(2.59)	(0.81)
Flood-prone		−163	−309	−663	−740
		(3.34)	(6.98)	(8.59)	(11.99)
Wetland		−58.2	−57.5	762	230
		(0.47)	(0.51)	(4.41)	(1.72)
Soil erosion		−1,258	−1,513	−2,690	−2,944
		(6.20)	(8.14)	(8.21)	(11.23)
Slope length		17.3	13.7	54.0	30.9
		(2.91)	(2.49)	(6.24)	(4.54)
Sand		−139	−35.9	−288	−213
		(2.72)	(0.77)	(4.16)	(3.95)
Clay		86.2	67.3	−7.90	−18.0
		(4.08)	(3.47)	(0.22)	(0.63)
Moisture capacity		0.377	0.510	0.206	0.450
		(9.69)	(14.21)	(3.82)	(10.07)
Permeability		−0.002	−0.005	−0.013	−0.017
		(1.06)	(2.53)	(5.58)	(8.61)
Adjusted R^2:	0.671	0.782	0.784	0.836	0.835
Number of observations:	2,938	2,938	2,941	2,941	2,941

Notes: The dependent variable is the value of land and buildings per acre. All regressions are weighted. Values in parenthesis are t statistics.

from Mendelsohn et al, AER 1995

Results:

- Warmer temperatures are bad in Jan, Apr, Jul, don't matter in Oct.
- Each unit of warming is worse than the one before.
- Jan, Apr rain is good, Jul, Oct rain is bad.

no surprises, really.
Also look at farm REVENUE as a function of climate. This is less interesting. It's harder to interpret because costs could change with climate, too. How would land values change if everyplace got

$5°\mathrm{F} = 2.8°\mathrm{C}$ warmer and 8% wetter?

TABLE 5—PREDICTED IMPACT OF GLOBAL WARMING ON FARMLAND VALUES AND FARM RENTS

		Change in farmland values (billions of dollars, 1982 prices)		Change in farmland rents (percentage of 1982 farm marketings)	
Year	Weight	Impact	Truncated impact	Impact	Truncated impact
1982	Cropland	−\$125.2	−\$118.8	−4.4	−4.2
1978	Cropland	−\$162.8	−\$141.4	−5.7	−4.9
1982	Crop revenue	\$34.5	\$34.8	1.2	1.2
1978	Crop revenue	−\$14.0	\$21.0	−0.5	0.7

Notes: The global-warming scenario is a uniform 5°F increase with a uniform 8-percent precipitation increase. The "impact" column shows the estimated loss; the "truncated impact" columns show the impact when the loss in farmland value in each county is limited to the original value of the land. The last two columns are annualized impacts, as explained in the text, as a percentage of 1982 farm marketings.

Mendehsohn, Nordhaus and Shaw (1995)

With a lot of difference in the effect from one place to another...

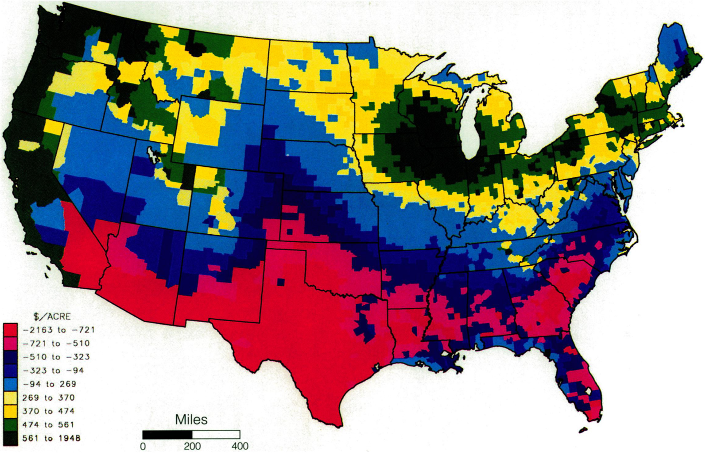
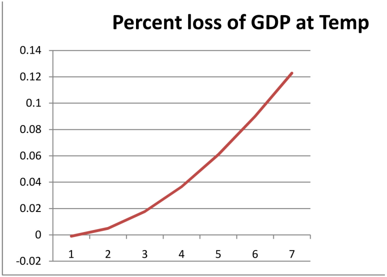


FIGURE 2. INFLUENCE OF CURRENT CLIMATE ON FARM VALUES: CROPLAND WEIGHTS
Note: Farm value is measured as the difference in dollars per acre from the sample average, 1982 prices.

Where does this leave us?

Using the Mendelsohn et al. study, and others like it, Nordhaus proposes a damage function like this,



My calculations, based on Nordhaus fig 3-3 and, appendix, and Nordhaus' website

Here,

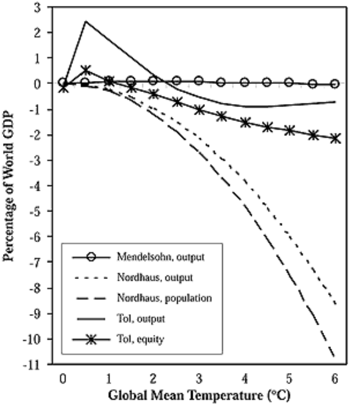
$$\%\Delta\text{GDP} = -\left(1 - \frac{1}{1 - 0.0045\Delta T + 0.0035(\Delta T)^2}\right)$$

So a 3 degree Celsius increase translates into about 2.8% decrease in income.

This is in line with other estimates (probably partly because others read Nordhaus)

- IPCC 2007 ‘Impacts, Adaptation, and Vulnerabilities’ gives 1-5% gdp loss from 4 degrees Celsius of warming.
- Stern says 5.3-13.8% loss from warming in 2200(not 2100) from 3.9 degrees Celsius of warming (p 176) (though it is hard to be sure exactly what the underlying assumptions are).

- Stern fig 6.2 summarizes other versions Nordhaus' cost curve (really, IPCC 2001 fig 19.4):



Monetary impacts as a function of level of climate change (measured as percentage of global GDP).

I think there are two important points to make about these functions:

- They are all talking about levels of GDP, not growth.
- They all find pretty small effects for 3 degrees of warming. Large effects come at much higher levels, that are not expected (probably) for 200 years.
- Note that these damage curves give us the next piece of our puzzle, the relationship between warming and future consumption.
- Note that they are also, all, quite speculative.