An application: Explaining gentrification and blight with bid-rent functions

Lecture notes #3: EC2410
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In their article, 'Paradise lost and regained,' LeRoy and Sonstelie (1983) develop a generalization of the moncentric city model with housing which allows for two classes of households, rich and poor, and two modes of commuting, car and bus. This model is interesting for three reasons. First, it provides a nice illustration of how we can use bid rent functions to characterize a spatial equilibrium in more complicated environments. Second, the model address the problem of central city decline and resurgence. This is a problem that has been the subject of an older literature about the decline of US central cities, e.g., Bradford and Kelejian (1973), Boustan (2010), and more recently their resurgence and gentrification, e.g., Couture and Handbury (2017). Third, it provides a nice demonstration of the power of the basic logic of the monocentric city and spatial equilibrium. With care, we can rationalize complicated location choices, under weak assumptions, by thinking carefully about how people respond to transportation costs.

The model developed in LeRoy and Sonstelie (1983) allows households to choose between two commute modes, 'bus' and 'car', each of which involves a fixed cost, a variable time cost, and a variable monetary cost. In any equilibrium, condition on location choice, households choose the cheapest commute mode. Conditional on these choices, each location is occupied by the class of household willing to pay the most for land at the location.

The notation to describe this problem, is as follows;

$$\begin{aligned} h &\sim \text{housing} \\ x &\sim \text{composite consumption} \\ d &\sim \text{distance (location)} \\ w &\sim \text{wage} \\ u(h,x) &\sim \text{utility.} \end{aligned}$$

This is all standard. We will sometimes use subscripts $j \in \{p,r\}$, 'poor' and 'rich', to indicate attributes of the two different classes of households.

Commute costs are intuitive, if complicated and non-standard. There are two commute modes, car and bus, indexed by $i \in \{a,b\}$. Each involves a fixed monetary cost f^i and a variable monetary cost, c^i. In addition, each commute mode involves a variable time cost, t^i. This parameter measures the fraction of the work day foregone per unit distance of commuting. Thus we have the total cost of commuting distance d by car is $f^a + (wt^a + c^a)d$, while the total cost of commuting by bus is $f^b + (wt^b + c^b)d$. Consistent with our intuition about how buses and cars work, assume $c^a > c^b$, $f^a > f^b$, and $t^a < t^b$. That is, the fixed and variable monetary variable costs favor buses, but time costs favor cars. Thus, holding distance constant, we can always find a wage high enough that cars are preferred to buses. We expect households to always choose the cheapest available mode at each distance.

Given this commuting technology, we would like to know how we should expect rich and poor to arrange themselves in equilibrium. In particular, when should we expect to see the rich located centrally with the poor on the periphery, and when should we expect the opposite. With this in place, we would like to know whether the model is able to rationalize the big patterns that we saw in the course of the late 19th to mid-20th century US, that is, a shift from cities where the rich lived centrally and rich and poor used the same transportation technology to cities where the rich live in the suburbs and the poor live in the center. The big result of the model is that the model will make this prediction in response to the arrival of a new transportation technology, the car, and it will make it even more strongly if we assume that as incomes increase, people are progressively more inclined to trade housing for consumption.

LeRoy and Sonstelie (1983) proceeds informally in the context of the monocentric city model with housing. We will simplify to consider the linear city without housing, and with a slightly simpler transportation technology. This eases exposition, increases transparency, and allows a more formal statement of results.

To proceed, we consider the 'baby Leroy and Sonstelie model'. This model is based on the standard linear city model without housing. The notation that we need to describe this model is as follows,

$$\begin{aligned} \overline{h} &\sim \text{ fixed housing/land consumption} \\ x &\sim \text{ composite consumption} \\ d &\sim \text{ distance (location)} \\ w &\sim \text{ wage} \\ u(x) &\sim \text{ utility} \\ \overline{u} &\sim \text{ reservation utility.} \end{aligned}$$

As before, we will sometimes use r and p subscripts to index rich and poor specific quantities. To lighten notation, we suppose that the rent for land not in urban use is zero.

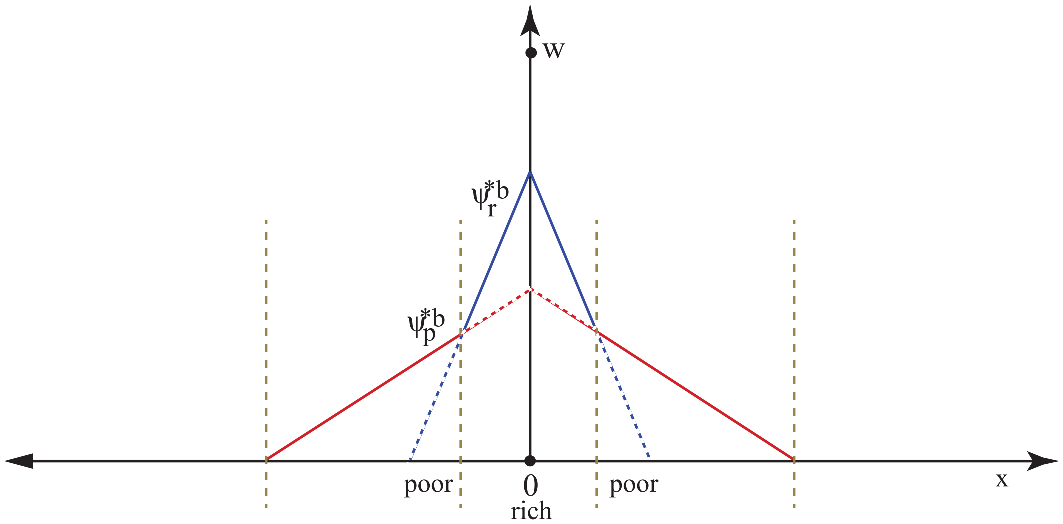
The cost structure of commuting is the same as in described above except that we set the fixed cost of taking the bus to zero, set the monetary cost of both modes to zero, and prohibit the poor from driving. Under these assumptions, the unit cost of commuting from d for the poor is $w_p t^b d$. The rich choose between bus and car at each distance. Given optimizing behavior, this means that their commute cost from each distance is $\min\{f^a + w^r t^a d, w^r t^b d\}$, where $f^a > 0$ and $t^a < t^b$.

For the purpose of characterizing equilibrium, we would like to think of there as being three types of households, poor, rich bus riders, and rich drivers. We proceed by calculating a bid-rent function for each. In equilibrium, each location is occupied by the type that values it most highly, and the equilibrium land rent is the upper envelope of the three bid-rent functions.

Having framed the problem in this way, the bid-rent function for the poor is

$$\Psi_p(w_p, t^b, d, \overline{u}) = \max_d \left\{ \frac{w_p - x - w_p t^b d}{\overline{h}} | u(x) \geq \overline{u} \right\} \tag{1}$$

Figure 1: Equilibrium with rich and poor households and only bus travel



if we define $x_p^* = u^{-1}(\overline{u})$, then this becomes,

$$\Psi_p^*(d) = \frac{w_p - x_p^* - w_p t^b d}{\overline{h}} \tag{2}$$

A similar logic lets us derive the bid rent function for rich bus riders,

$$\Psi_r^{*b}(d) = \frac{w_r - x_r^* - w_r t^b d}{\overline{h}}, \tag{3}$$

and rich drivers

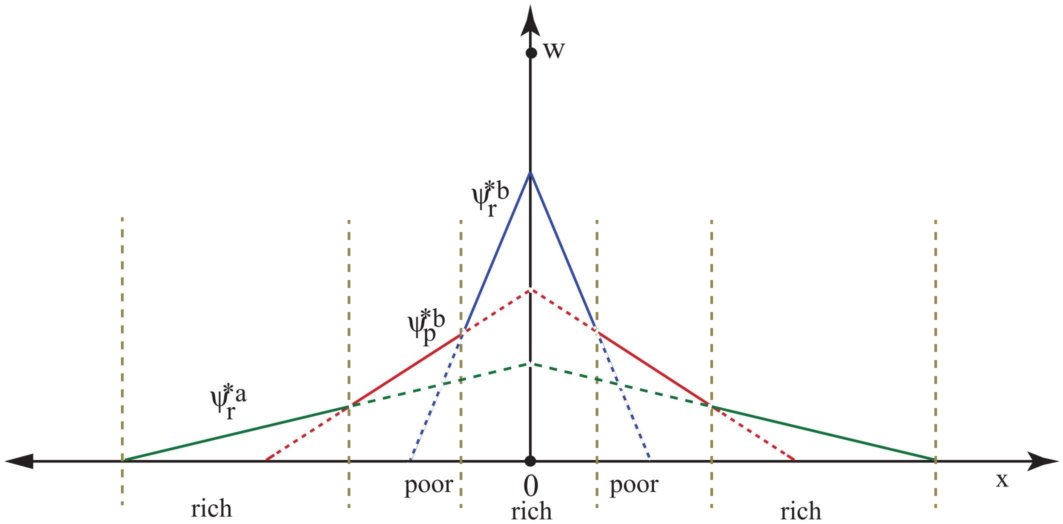
$$\Psi_r^{*a}(d) = \frac{w_r - f^a - x_r^* - w_r t^a d}{\overline{h}}, \tag{4}$$

When $w_r > w_p$ and $f^a > 0$, we must have that rich bus riders outbid other households for parcels sufficiently close to zero, as long as rich and poor consumer the same amount of housing. Beyond this, there is not too much that can be said in general.

To see the sorts equilibrium that can emerge, suppose that cars are so expensive that even the rich cannot afford them, that is, $f^a \to \infty$. In this case, we can concern ourselves with just two classes of households, rich and poor bus riders. Figure 1 illustrates bid rent functions. The steeper blue lines give the bid rent function for the rich bus riders, the flatter red lines the bid-rent function for the poor bus riders. Equilibrium and rent is the upper envelope of these two, and the resulting equilibrium has the rich in the center and the poor in the periphery. This is the configuration that LeRoy and Sonstelie (1983) have in mind as paradise, prosperous central cities populated by the rich, with the poor commuting from the periphery.

We can now consider what happens as the price of the car falls. In this case, we must consider three classes of agents, rich and poor bus riders and rich drivers. Figure 2

Figure 2: Equilibrium with rich and poor households and cars for the rich



illustrates a possible equilibrium. In this figure, bid rent functions for the two types of bus riding households are unchanged, but the fixed and variable costs of an automobile have fallen sufficiently that rich households can outbid poor bus riders for the far suburbs and some of the agricultural land beyond. The bid rent function for the rich drivers is given by the green lines in the figure. Equilibrium land rent is the upper envelope of the three bid-rent functions, and in the resulting equilibrium, the center is populated by rich bus riders, the next ring by poor bus riders, and furthest ring by rich drivers. This is 'paradise partially lost'. A partial abandonment of the core by the rich in response to the availability of the automobile.

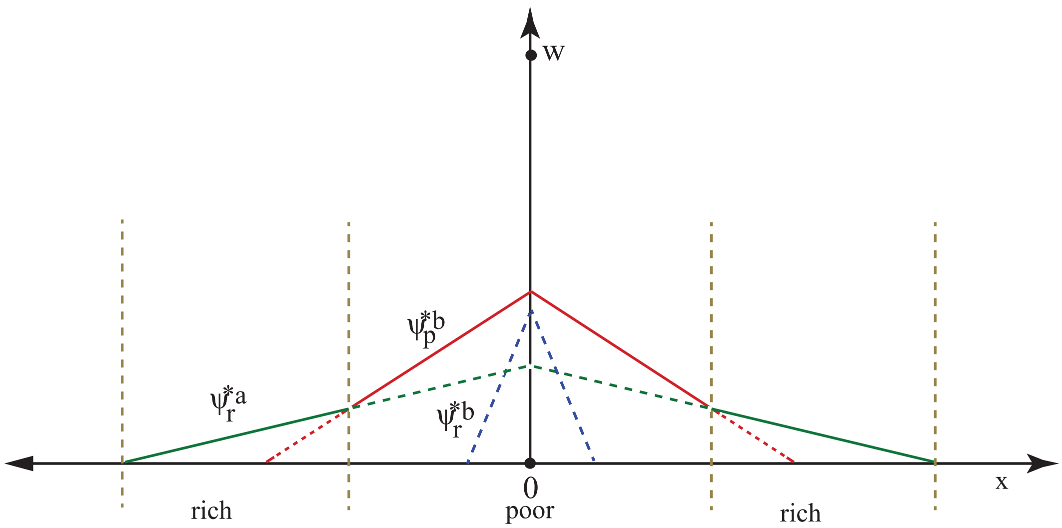
In order to get the rich to entirely abandon the center, we need to make an assumption about preferences. In particular, we need to suppose that as incomes rise, households shift their consumption towards housing. In the context of this model, this means that $\overline{h}_r > \overline{h}_p$. By inspection of the bid rent functions for the rich and poor, if the rich have a sufficiently large increase in the demand for housing, then this can lead to a situation where poor bus riders can outbid the rich for the most central locations.

Figure 3 gives an example. In this example, housing demand by the rich is so high that poor bus riders outbid the rich for the very center. Since the rich also have higher value of time, this means that poor bus riders outbid the rich for all locations in the city, rich bus riders disappear. However, if the car is cheap enough, then it may be that rich drivers will outbid poor bus riders for suburban locations and for locations beyond the edge of the formerly bus riding city. This is the 'paradise lost' scenario that Leroy and Sonstelie envision. The central city is given over entirely to bus riding poor, while the rich commute in from remote suburbs.

In fact, LeRoy and Sonstelie (1983) imagine a third equilibrium, in which the price of cars falls so low that they are available to the poor and rich drivers are able to outbid poor bus riders for the center. In this case, we could again get the rich in the center and the poor commuting from the periphery, 'paradise regained'.

A few comments about this model seem relevant. First, it is worth noting that Glaeser

Figure 3: Equilibrium with rich and poor households and cars and more money for the rich



et al. (2008) redoes much of the analysis in LeRoy and Sonstelie (1983), but with the addition of a third mode, walking. They establish similar comparative statics to LeRoy and Sonstelie (1983), but also find conditions under which rich walkers end up downtown. Both papers provide interesting examples of when reduction in transportation costs, whether the horse drawn trolley or the car, led to a dramatic changes in how rich and poor arranged themselves within cities.

Second, both Glaeser et al. (2008) and LeRoy and Sonstelie (1983) work in the context of the monocentric city model with housing. This makes it more difficult to explicitly state bid rent curves, but easier to specify the rate at which households' preferences over consumption and housing must change with income.

Third, it is worth noting that with an open city model, the types of comparative statics we have considered not only involve changes in the way rich and poor organize themselves within cities, they also involve changes in the total numbers of urban versus rural rich and poor. One of the more important economic phenomena of the 21st century US seems to have been a ruralization of poverty. The model we discussed may suggest a basis for understanding this phenomena. As incomes and transportation costs have shifted to allow the rich to occupy progressively larger fractions of urban land, the poor may have been outbid and left to choose their rural outside option.

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