EC1340-Fall 2023
Problem Set 9

(Updated 31 July 2023)

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When you write up your answers, your goals should be to (1) be correct, and (2) convince your reader that your answer is correct. It is always helpful if your work is legible and if all steps are presented, possibly with a line of explanation.

In the case of empirical exercises, your goal should be to provide enough information to allow a reader to replicate your answer. This requires a description of data and data sources as well as a description of your analysis of the data.

Answers which do not achieve these goals will not be awarded full credit.

Problems

1. Suppose two firms produce steel and have cost functions $c_1(y_1) = 3y_1^2$, and $c_2(y_2) = y_2^2$. The market price of steel is $p = 5$. Steel is jointly produced with smoke, and a planner has determined that social welfare is maximized when only one unit of steel is produced.

   (a) In the absence of regulation, how much steel is produced?

   (b) What tax on steel will cause the two firms to reduce production to the point that industry production is 1 unit – the social optimum?

   (c) Suppose that the planner chooses to regulates production by issuing each firm non-tradable quota to produce half a unit of steel. What are the total costs to produce the one unit of steel? What are firms' profits?

   (d) Do the firms prefer taxes or quotas? Does the planner prefer taxes or quotas? Explain.

   (e) Now suppose that firms are able to trade their initial half units of quota at a market price p_Q Show that the price at which the quota market clears is the same as the tax that you found in part 1c.

   (f) What is the most that the government can charge firms for their initial allocation of tradable quota, and still expect firms to prefer tradable quota to non-tradable quota?

2. This problem asks you to identify the optimal type of regulatory instrument—price or quantity—in an environment where the planner is uncertain about the firm's costs.

   Let

$$\begin{aligned} B(y) &= 2y - \frac{1}{2}y^2 \\ C(y) &= \eta y + \frac{1}{2}y^2, \end{aligned}$$

   where η is a random variable that affects the firm's costs. Define η as follows:

$$\eta = \begin{cases} 1 & p = \frac{1}{2} \\ 0 & p = \frac{1}{2} \end{cases}.$$

(a) Find y^*, the magnitude of the optimal quantity regulation that solves $\max_y E\left(B(y) - C(y)\right)$.

(b) Evaluate expected social welfare at the value of y^* that you just found. (You have just found the optimal quota and the associated level of expected welfare.)

(c) Find the firm's response function $\widehat{y}(p)$. That is, find $\widehat{y}(p)$ to solve $\max_y py - C(y)$.

(d) Find $\widehat{p}$ to solve $\max_p E\left(B\left(\widehat{y}\left(p\right)\right) - C\left(\widehat{y}\left(p\right)\right)\right)$.

(e) Evaluate $\widehat{y}(p)$ at the value of $\widehat{p}$ that you just found. Note that this value will be random. (You have just found the optimal price regulation and the firm's response to this regulation.)

(f) Evaluate expected social welfare given the firm's response to the optimal price regulation. That is, evaluate $B\left(\widehat{y}\left(\widehat{p}\right)\right) - C\left(\widehat{y}\left(\widehat{p}\right)\right)$.

(g) Draw a graph that illustrates: Marginal cost function (in each state of the world), the marginal benefit function, $\widehat{p}$,$\widehat{y}\left(\widehat{p}\right)$,$y^*$. Also illustrate the deadweight loss from each instrument in each state of the world.

3. Let $C(x,\phi)$ denote a firm's cost of abating pollution to level x, where $c(\overline{x},\phi) = 0$, $c(0,\phi) > 0$, and $c_x < 0$. ϕ is a variable that affect the firm's costs. This parameter is known to the firm, but is uncertain for the planner. Let $D(x)$ denote the social damage caused by pollution level x. Finally, let $P(x; s,l,p) = sx + p\max[x - l,0]$ be the penalty function that the planner uses to regulate pollution. Thus, the planner's problem is

$$\min_x E[D(x) + C(x,\phi)]$$

and the firm's problem is

$$\min_x P(x) + C(x,\phi).$$

Let $C(x,\phi) = 10 - \phi x - \frac{1}{2}x^2$ and ϕ be one or zero with equal probability. Let $D(x) = \frac{11}{10}x + \frac{1}{4}x^2$.

(a) Solve graphically for the firm's response to $P(x; s,l,p)$.

(b) Find the planner's optimum value of x for each value of ϕ.

(c) Find the welfare maximizing choice(s) of s,l,p.