# Comments on: "Optimal Urban Transportation Policy: Evidence from Chicago"

Matthew A. Turner

Brown University

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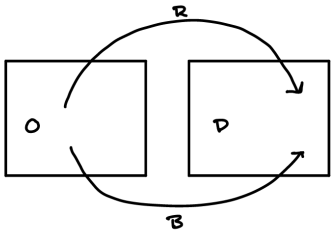
# Toy model I




Consider a toy special case,

- One origin-destination pair, $N$ travelers (exogenous), all choose mode $j$ from rail (R) or bus (B).
- Agent type $\theta$ gets payoff from mode $j$

$$u(\theta, j) = A_j - t_j - p_j + \varepsilon_j(\theta) \equiv v_j + \varepsilon_j(\theta),$$

For $A_j \sim$ mean trip value, $t_j \sim$ travel time, $p_j \sim$ fare, $\varepsilon_j \sim$ Frechet taste shock (not Gumbel) with dispersion $\eta$.

Each household chooses $R$ or $B$, to solve

$$j(\theta) = argmax_{j\in\{R,B\}}\left\{v_j + \varepsilon_j(\theta)\right\}$$
$$\Longrightarrow$$
$$q_j^D = N\frac{v_j^\eta}{v_R^\eta + v_B^\eta}, j \in \{R, B\}$$

This is demand for travel for each mode.

Comment: *People do not reschedule trips in response to $p_j$ or $t_j$. This would be really hard to fix, but it is an important margin of adjustment empirically.*

$v_j \equiv A_j - t_j - p_j$; $A_j \sim$ mean trip value; $t_j \sim$ travel time; $p_j \sim$ fare.

Supply is defined implicity by

$$\begin{bmatrix} t_B \\ t_R \end{bmatrix} = T\left(\begin{bmatrix} q_B^S \\ q_R^S \end{bmatrix}, \begin{bmatrix} k_B \\ k_R \end{bmatrix}\right)$$

Inverting,

$$\begin{bmatrix} q_B^S \\ q_R^S \end{bmatrix} = T^{-1}\left(\begin{bmatrix} t_B \\ t_R \end{bmatrix}, \begin{bmatrix} k_B \\ k_R \end{bmatrix}\right)$$

Comment: *With buses mostly empty, the supply relationship should be singular. Are you sure you can invert it?*

$q_j \sim$ trips; $t_j \sim$ travel time; $k_j \sim$ fleet size.

Equilibrium is demand $=$ supply,

$$\begin{bmatrix} q_B^D \\ q_R^D \end{bmatrix} = \begin{bmatrix} q_B^S \\ q_R^S \end{bmatrix}$$

$$\begin{bmatrix} N\frac{v_B^\eta}{v_R^\eta+v_B^\eta} \\ N\frac{v_R^\eta}{v_R^\eta+v_B^\eta} \end{bmatrix} = \quad T^{-1}\left(\begin{bmatrix} t_B \\ t_R \end{bmatrix}, \begin{bmatrix} k_B \\ k_R \end{bmatrix}\right)$$

$$\Longrightarrow \begin{bmatrix} q_B(p_R, p_B, k_R, k_B) \\ q_R(p_R, p_B, k_R, k_B) \end{bmatrix}$$

The planner solves a second best problem.

Planner chooses fleet sizes, $k_j$, and fares, $p_j$, to maximize welfare taking equilibrium response as given.

With Frechet shocks, average utility is

$$E(u(\theta,.)) = \Gamma\left(\frac{\eta-1}{\eta}\right)\left(v_B^\eta + v_R^\eta\right)^{1/\eta}$$

Planner maximizes average utility minus the 'social' cost of running the system minus the social cost of congestion, subject to equilibrium and budget constraints.

$$W(p,k) = \max_{p,k} \Gamma\left(\frac{\eta-1}{\eta}\right)\left(v_B^\eta + v_R^\eta\right)^{1/\eta} - E(\text{Cost})$$
$$\text{s.t. } p_R q_R + p_B q_B \geq E(\text{Cost})$$
$$\text{Equilibrium responses}$$

This seems just right.

Comment: *Where is the discussion of taste dispersion? You cannot evaluate consumer surplus without knowing it.*

Comment: *Note parallel to CES. $\eta$ looks like elasticity of substitution across types.*

Now suppose we have two origin destination pairs. People can't move, and number of trips is fixed. Let $N_1 = N_2$ to make things easy. Then

$$W(p,k) = \max_{p,k} \frac{1}{2}\Gamma\left(\frac{\eta-1}{\eta}\right)\left[\left(v_{B1}^{\eta} + v_{R1}^{\eta}\right)^{1/\eta} + \left(v_{B2}^{\eta} + v_{R2}^{\eta}\right)^{1/\eta}\right] - E(\text{Cost})$$

$$\text{s.t. } p_{R1}q_{R1} + p_{B1}q_{B1} + p_{R2}q_{R2} + p_{B2}q_{B2} \geq E(\text{Cost})$$
$$\text{Equilibrium responses}$$

Just like for one pair, but planner gets twice as many choices.

Comment: $\eta$ *acts like inequality aversion across types, within a location, but not across locations. This is because people don't choose locations. Is this how we want to pick transit policy?*

Is this problem well behaved?

Consider,

$$\frac{dE(u(\theta,.))}{dv_B} = \frac{d)}{dv_B}\Gamma\left(\frac{\eta-1}{\eta}\right)\left(v_B^\eta + v_R^\eta\right)^{1/\eta}$$
$$= \Gamma\left(\frac{\eta-1}{\eta}\right)\left(v_B^\eta + v_R^\eta\right)^{(1/\eta)-1} v_B^{\eta-1}$$

Comments

- This is singular as $v_b \to 0$ if $\eta < 1$ (but $\eta > 1$).
- This is singular if $v_B$ and $v_R \to 0$ for $\eta > 1$. I think the structure of the problem requires service of some sort, everywhere. This results is going to be by assumption.
- I think I can make examples where the second derivative of $E(u(\theta,.))$ is positive, but it's too much algebra to show.