

EC1340-Fall 2023

Problem Set 6 solutions

(Updated 31 July 2023)

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1. (a) We want to solve

$$\begin{aligned} & \max_{s, c_1} \frac{c_1^{1-\eta}}{1-\eta} + \frac{1}{1+\rho} \frac{c_2^{1-\eta}}{1-\eta} \\ & \text{s.t. } W = c_1 + s \\ & \quad c_2 = (1+r)s \\ \Rightarrow & \max_{c_1} \frac{c_1^{1-\eta}}{1-\eta} + \frac{1}{1+\rho} \frac{((1+r)(W-c_1))^{1-\eta}}{1-\eta} \end{aligned}$$

Differentiating we get the first order condition:

$$\begin{aligned} & c_1^{-\eta} + \frac{1}{1+\rho} ((1+r)(W-c_1))^{-\eta} (-(1+r)) = 0 \\ \Rightarrow & \left(\frac{c_1}{(1+r)(W-c_1)} \right)^{-\eta} = \frac{1+r}{1+\rho} \\ \Rightarrow & c_1^* = \frac{w(1+r)^{\frac{\eta-1}{\eta}} (1+\rho)^{\frac{1}{\eta}}}{1 + (1+r)^{\frac{\eta-1}{\eta}} (1+\rho)^{\frac{1}{\eta}}} \end{aligned}$$

From the constraints, we have that $c_2 = (1+r)(W-c_1)$. Substituting our solution for c_1^* we have

$$\begin{aligned} c_2^* &= (1+r) \left(W - \frac{w(1+r)^{\frac{\eta-1}{\eta}} (1+\rho)^{\frac{1}{\eta}}}{1 + (1+r)^{\frac{\eta-1}{\eta}} (1+\rho)^{\frac{1}{\eta}}} \right) \\ &= \frac{w(1+r)}{1 + (1+r)^{\frac{\eta-1}{\eta}} (1+\rho)^{\frac{1}{\eta}}} \end{aligned}$$

- (b)

$$\begin{aligned} \frac{c_1^*}{c_2^*} &= \frac{w(1+r)^{\frac{\eta-1}{\eta}} (1+\rho)^{\frac{1}{\eta}}}{w(1+r)} \\ &= \left(\frac{1+\rho}{1+r} \right)^{1/\eta} \end{aligned}$$

with $\rho < r$, we have $\left(\frac{1+\rho}{1+r} \right) < 1$. For $\eta = 1$ this means that $\frac{c_1^*}{c_2^*} = \left(\frac{1+\rho}{1+r} \right)$ and consumption in the two period ought to be about the same (if r and ρ are small). As η approaches zero, the exponent gets large and the expression goes to zero. This means that consumption in the first period approaches zero and consumption becomes as unequal as possible. Thus changes in η cause consumption in the two period to be more or less different.

- (c) As ρ increases, the expression above increases. This means that first period consumption increases at the expense of second period consumption. This is consistent with less patience.
- (d) As r goes up, the expression above decreases. This means that second period consumption is increasing. As r goes it becomes 'cheaper' in terms of first period consumption to provide second period consumption. The consumer substitutes toward the cheaper good.
2. (a) The cost of a reduction in emissions of proportion α of the whole is $\Lambda_0 Y_0 = \frac{2}{3}\alpha^3 Y_0$.
- (b) We need to accomplish αE_0 of emissions in country A. To do this, we need

$$\begin{aligned}\alpha E_0 &= \alpha^A E_A \\ &= \alpha^A E_0 / 2\end{aligned}$$

so that $\alpha^A = 2\alpha$.

The cost of this reduction to country A is,

$$\begin{aligned}\Lambda_A Y_A &= \frac{2}{3}(\alpha^A)^3 Y_A \\ &= \frac{2}{3}(2\alpha)^3 \frac{1}{2} \\ &= \frac{8}{3}\alpha^3\end{aligned}$$

3. We want to find x to solve,

$$\begin{aligned}x \Lambda_A Y_A &= \Lambda_0 Y_0 \\ x \frac{8}{3}\alpha^3 &= \frac{2}{3}\alpha^3 Y_0 \\ \implies x &= 4\end{aligned}$$

That is, it costs 4 times as much to accomplish our reduction in country A alone.