(1)

CONSUMER SOLVES

(a) $$\text{MAX} \left[ \int_0^M q(i)^{\frac{\sigma-1}{\sigma}} di \right]^{\mu \frac{\sigma}{\sigma-1}} T^{1-\mu}$$

$$\text{S.T.} \quad \int_0^M p(i) q(i) di + p^T T = \omega \qquad (1)$$

FIRST ORDER CONDITION IS $\nabla u = \lambda p \qquad j \in [0, M], T, \ \lambda \geq 0$

SET $p^T = 1$, W.O.L.G.

$$\left[ \int_0^M q(i)^{\frac{\sigma-1}{\sigma}} di \right]^{\mu \frac{\sigma}{\sigma-1}} T^{-\mu} (1-\mu) = \lambda \qquad (2)$$

$$\mu \frac{\sigma}{\sigma-1} \left[ \int_0^M q(i)^{\frac{\sigma-1}{\sigma}} di \right]^{\frac{\mu\sigma}{\sigma-1} - 1} \frac{\sigma-1}{\sigma} q(j)^{-\frac{1}{\sigma}} T^{1-\mu} = \lambda p(j), \quad j \in [0, M] \qquad (3)$$

(2) → (3)

$$\Rightarrow \quad \mu \frac{\sigma}{\sigma-1} \left[ \int_0^M q(i)^{\frac{\sigma-1}{\sigma}} di \right]^{-1} \frac{\sigma-1}{\sigma} q(j)^{-\frac{1}{\sigma}} T = (1-\mu) p(j), \quad j \in [0, M]$$

$$\Rightarrow \quad \mu \left[ \int_0^M q(i)^{\frac{\sigma-1}{\sigma}} di \right]^{-1} q(j)^{-\frac{1}{\sigma}} T = (1-\mu) p(j) \qquad (4) \quad , j \in [0, M]$$

$$(4) \Rightarrow \quad \frac{p(k)}{p(j)} = \left[ \frac{q(k)}{q(j)} \right]^{-\frac{1}{\sigma}} \quad ; \ j, k \in [0, M]$$

$$\Rightarrow \quad \left( \frac{p(j)}{p(k)} \right)^{\sigma} = \frac{q(k)}{q(j)} \Rightarrow q(k) = q(j) \left( \frac{p(j)}{p(k)} \right)^{\sigma} \qquad (5)$$

$$j, k \in [0, M]$$

2

USING 5 IN 4

$$\mu\left[\int_0^M\left[q(j)\left(\frac{p(j)}{p(i)}\right)^{\sigma}\right]^{\frac{\sigma-1}{\sigma}}di\right]^{-1} q(j)^{-\frac{1}{\sigma}}T = (1-\mu)\,p(j)$$

$$\Rightarrow \quad q(j)^{-\frac{1}{\sigma}}\,q(j)^{-\frac{(\sigma-1)}{\sigma}}\left[\int_0^M\left[\frac{p(j)}{p(i)}\right]^{\sigma-1}di\right]^{-1} = \frac{1-\mu}{\mu T}\,p(j)$$

$$q(j)^{-1} = \left[\int_0^M\left[\frac{p(j)}{p(i)}\right]^{\sigma-1}di\right]\frac{1-\mu}{\mu T}\,p(j)$$

$$= p(j)^{\sigma-1}p(j)\left[\int_0^M p(i)^{1-\sigma}di\right]\frac{1-\mu}{\mu T}$$

$$= p(j)^{\sigma}\int_0^M p(i)^{1-\sigma}di\;\frac{1-\mu}{\mu T}$$

$$\Rightarrow \quad q(j) = \frac{p(j)^{-\sigma}}{\int_0^M p(i)^{1-\sigma}di}\;\frac{\mu T}{1-\mu} \qquad (6)$$

6 → 2

$$\int_0^M \frac{p(j)^{1-\sigma}}{\left[\int_0^M p(i)^{1-\sigma}di\right]}\frac{\mu T}{1-\mu}dj + T = w$$

$$\Rightarrow \quad \frac{\mu T}{1-\mu} + T = w$$

$$\Rightarrow \boxed{T^* = (1-\mu)w} \qquad (7)$$

3

(7) → (6) ⇒

$$\boxed{q^*(j) = \frac{p(j)^{-\sigma}}{\int_0^M p(i)^{1-\sigma}\,di}\, MW} \quad (8) \quad j \in [0, M]$$

DEFINE $\mathbb{P} = \left[ \int_0^M p(i)^{-(\sigma-1)}\, di \right]^{-\frac{1}{\sigma-1}}$ ~ "C.E.S. PRICE INDEX"

$$\Rightarrow \quad \mathbb{P}^{-(\sigma-1)} = \left[ \int_0^M p(i)^{1-\sigma}\, di \right] \quad (9)$$

(9) → (8)

$$\Rightarrow \quad q^*(j) = \frac{p(j)^{-\sigma}}{\mathbb{P}^{-(\sigma-1)}}\, MW$$

$$\Rightarrow \quad \boxed{q^*(j) = \mathbb{P}^{\sigma-1} p(j)^{-\sigma} MW} \quad (10)$$

(c.f. FUJITA + THISSE eqn 9.4)

STRICTLY, IF $K \subset [0, M]$ MEASURE ZERO AND $f: K \to \mathbb{R}$ THEN $q^* + f$ IS ALSO SOLVES THE CONSUMERS PROBLEM.

→

(4)

PROFITS FOR FIRM $j$ ARE

$$\pi = p(j)\, q(j) - A - B\, q(j)$$

$$= \big(p(j) - B\big) q(j) - A$$

FROM (10)

$$\Rightarrow \pi = \big(p(j) - B\big) \frac{\mu w}{\mathbb{P}^{1-\sigma}}\, p(j)^{-\sigma} - A$$

(RECALL, FIRMS ARE MONOPOLISTS)

$$= \left[p(j)^{1-\sigma} - B\, p(j)^{-\sigma}\right] \frac{\mu w}{\mathbb{P}^{1-\sigma}} - A$$

$$\frac{d\pi}{dp(j)} = \frac{\mu w}{\mathbb{P}^{1-\sigma}} \left[(1-\sigma)\, p(j)^{-\sigma} + B\sigma\, p(j)^{-\sigma-1}\right] = 0$$

$$\Rightarrow \quad (1-\sigma) + B\sigma\, p(j)^{-1} = 0$$

$$(1-\sigma) = -B\sigma\, p(j)^{-1}$$

$$\Rightarrow p^*(j) = \frac{-B\sigma}{1-\sigma} \quad \left(\begin{matrix}\text{NEED}\\ \sigma > 1\end{matrix}\right) \quad (11)$$

USING (11) IN (8)

$$q^*(j) = \frac{\left[\frac{-B\sigma}{1-\sigma}\right]^{-\sigma}}{M\left[\frac{-B\sigma}{1-\sigma}\right]^{1-\sigma}} = \frac{1-\sigma}{-MB\sigma} = \frac{\sigma-1}{MB\sigma}, \quad j \in [0, M]$$

(12)

(5)

WITH FREE ENTRY OF FIRMS $\Pi = 0$

$$\Rightarrow \quad Pq - A - Bq = 0$$

USING (11) + (12)

$$\Rightarrow \left(\frac{-B\sigma}{1-\sigma}\right)\left(\frac{\sigma-1}{MB\sigma}\right) - A - B\left(\frac{\sigma-1}{MB\sigma}\right) = 0$$

$$\Rightarrow \quad \frac{\sigma}{M\sigma} - A - \frac{\sigma-1}{M\sigma} = 0$$

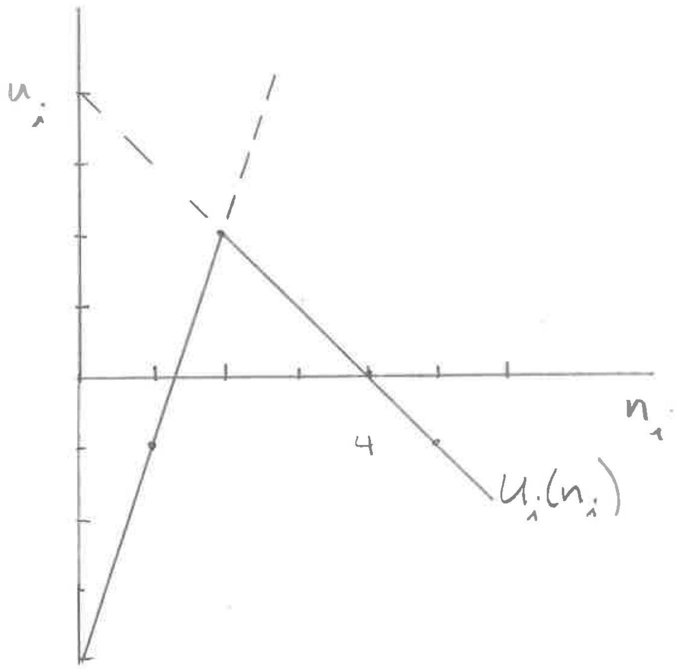
$$\Rightarrow \quad \frac{1}{M\sigma} - A = 0 \quad , \quad (13)$$

HERE, UNLIKE F-T OR KRUGMAN, WAGE IS EXOGENOUS, SO WE ONLY GET ZERO ENTRY CONDITION TO HOLD IF PARAMETERS HAPPEN TO SATISFY (13).

(b) SEE EQN (11) ABOVE – THIS IS CONSTANT MARK-UP BEYOND MARGINAL COST $(= B)$

(c) YOU CANNOT VERIFY THIS STATEMENT WITHOUT ENDOGENIZING THE WAGE.

(1)




(1) (a) SUPPOSE $N=5$. THERE ARE SIX CANDIDATE EQUILIBRIA, 0, 1, 2, 3, 4, 5 PEOPLE CHOOSE URBAN.

IF 0 PEOPLE CHOOSE URBAN, THEN EVERYONE GETS $\bar{u}=0$ AND UNILATERAL DEVIATIONS ALLOW $u(1)=-1$, SO $n_i=0$ IS NASH.

IF $n=4$, THEN $u_i(4)=\bar{u}=0$. AN URBAN DWELLER WHO DEVIATES TO NON-URBAN GETS 0. THIS IS NOT A BENEFICIAL DEVIATION. A RURAL DWELLER WHO DEVIATES TO URBAN GETS $u_i(5)=-1$, ALSO NOT A RATIONAL DEVIATION.

$\therefore$ ANY STRATEGIES S.T. $n_i \in \{0,4\}$ IS A NASH EQUIL.

IF $n \in \{1,5\}$ THEN $u=-1<\bar{u}$ SO DEVIATIONS TO NON-URBAN ARE RATIONAL.

IF $n=2$ THEN $u_i=2$ AND A DEVIATION TO URBAN

(2)

GIVES PAYOFF $u_i(3) = 1 > \bar{u}$, SO $n_i = 2$ IS NOT NASH.

IF $n_i = 3$, THEN DEVIATIONS TO URBAN GIVE $u_i(4) = 0 = \bar{u}$ AND DEVIATIONS FROM URBAN GIVE $\bar{u} = 0 < 1 = u(3)$.

$\Rightarrow$ STRATEGIES S.T. $n = 3$ ARE NASH.

THUS, STRATEGIES ARE NASH $\iff n \in \{0, 3, 4\}$.

(b) FOR MIXED STRATEGIES, EACH AGENT CHOOSES $p_j \in [0,1]$. WE CAN USE THIS TO WRITE DOWN THE PROBABILITY OF EACH CONFIGURATION.

| $n_i$ | PROB | $u(n_i)$ |
|---|---|---|
| 0 | $(1-p)^5$ | $-4$ |
| 1 | $p(1-p)^4$ | $-1$ |
| 2 | $p^2(1-p)^3$ | 2 |
| 3 | $p^3(1-p)^2$ | 1 |
| 4 | $p^4(1-p)$ | 0 |
| 5 | $p^5$ | $-1$ |

FOR THE PURPOSE OF CALCULATING MIXED EQUIL, WE NEED TO KNOW THE EFFECT ON PAYOFFS OF CHANGING $p$ FOR ONE AGENT, HOLDING OTHERS FIXED AT $p'$.

(3)

IF ALL AGENTS BUT 1 CHOOSE P, AND 1 CHOOSES P', THEN WHAT WE CARE ABOUT IS THE DISTRIBUTION OF EVERYONE ELSE'S MOVES. LET $n'$ DENOTE # OF PLAYERS, ≠1, CHOOSING URBAN. THEN

| $n'$ | $Pr(n')$ |
|---|---|
| 0 | $(1-p)^4$ |
| 1 | $p(1-p)^3$ |
| 2 | $p^2(1-p)^2$ |
| 3 | $p^3(1-p)$ |
| 4 | $p^4$ |

THEN, IF 1 CHOOSES $p'$,

$$E(\pi_1) = (1-p')\cdot 0 + \quad (1 \text{ is rural})$$

$$p'\left[-1(1-p)^4 + 2(1-p)^3 p + 1(1-p)^2 p^2 + 0\cdot(1-p)p^3 - 1p^4\right] \quad (*)$$

WE NEED TO FIND $p'$ SUCH THAT

$$\frac{dE(\pi_1)}{dp'} = 0 \quad \underline{\underline{\text{AND}}} \quad p' = p$$

WHERE THE SECOND CONDITION IS B/C WE'VE RESTRICTED ATTENTION TO SYMMETRIC EQUIL.

BY INSPECTION OF (*) WE NEED THE EXPRESSION IN BRACKETS TO BE ZERO, SO WE WILL NEED TO CHOOSE P SO THAT IT IS A ZERO OF THIS 4TH ORDER POLYNOMIAL.
$\Rightarrow$ EXPECT AS MANY AS 4 ROOTS.

(4)

(c) THE PARETO OPTIMAL OUTCOMES ARE $n \in \{2,3\}$ FOR EITHER URBAN POPULATION, THERE IS NO WAY TO MAKE ONE PERSON BETTER W/O MAKING ANOTHER WORSE.

(d) WE NEED $n_1 + n_2 \leq 5$.

(1) $(n_1, n_2) \in \{(0,0), (0,4), (4,0)\}$ ARE ALL NASH. IN THESE EQUIL, EVERYONE GETS $\bar{u}$.

(2) $(n_1, n_2) \in \{(2,3), (3,2)\}$

HERE THE URBAN PAYOFF IS POSITIVE, SO NO ONE MOVES TO RURAL. NO ONE WANTS TO MOVE FROM THE SMALL TO THE BIG CITY, THIS IS STRICTLY WORSE. MOVING FROM SMALL CITY TO BIG DOES NOT CHANGE PAYOFF FOR MARGINAL AGENT.

(3) NO OUTCOME WITH $n_i \in \{1,5\}$ IS NASH B/C ONE AGENT CAN DEVIATE TO RURAL FOR $0 > -1$.

(4) NO OUTCOME $n_i = 2$ IS AN EQUILIBRIUM UNLESS $n_i + n_j = 5$. OTHERWISE RURAL AGENTS WILL DEVIATE

(5) $(n_1, n_2) \in \{(0,3), (3,0)\}$ ARE ALSO EQUIL. URBAN RESIDENTS DON'T WANT TO MOVE, AND RURAL AGENTS WHO MOVE TO THE CITY ARE INDIFFERENT.

→

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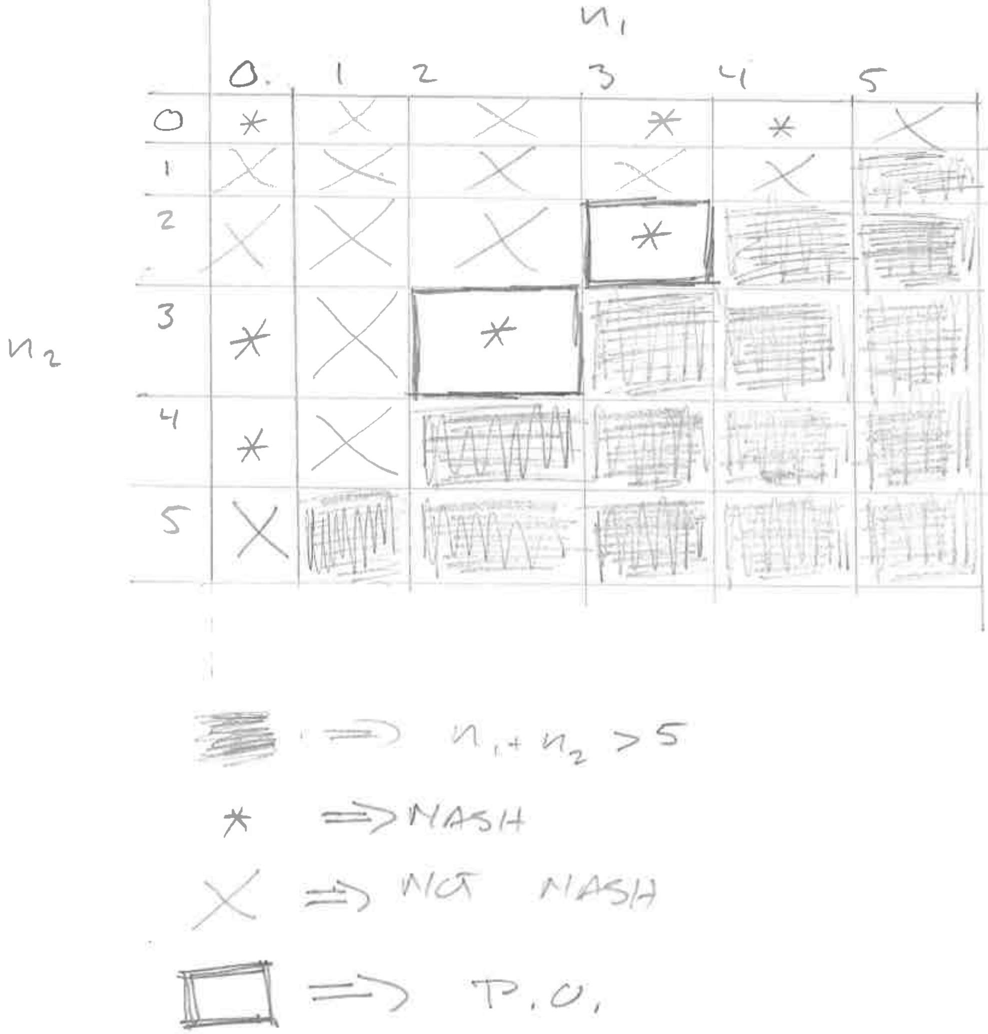
SUMMING UP, WE HAVE




THEREFORE, NASH EQUIL IS CONSISTENT WITH TWO CITIES OF ABOUT OPTIMAL SIZE AND AN EMPTY COUNTRYSIDE, OR WITH ONE CITY LARGER THAN OPTIMAL WITH 1 OR 2 STILL RURAL.

(e) ADDING ANOTHER CITY WOULD NOT QUALITATIVELY CHANGE THE EQUILIBRIUM OUTCOMES, IT WOULD JUST CREATE INDETERMINACY ABOUT WHICH 2 CITIES ARE OCCUPIED. THIS SEEMS SPECIAL TO THE CASE $N=5$.