`EC1410 Topic #8`

Systems of Cities

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Outline

1. Some stylized facts
2. Zipf's law
3. Systems of Cities

Stylized facts about cities

This discussion draws on Duranton and Puga (2000).

- What does the size distribution of cities look like? How does it change over time? Do cities change their position?
- What are patterns of sectoral specialization? Does this change over time?
- Can we explain these patterns as a consequence of spatial equilibrium?

To begin, we need to define 'specialized' and 'diversified' cities. Let

$$s_{ij} \sim \text{Share of industry } j \text{ employment in sector } i$$
$$ZI_i \equiv \max_j(s_{ij}) \sim \text{ specialization}$$

Tthe specialization of a city is the share of employment in its largest sector. Its also useful to think about relative specialization,

$$sj \sim \text{Share of industry } j \text{ national employment.}$$
$$RZI_i \equiv \max_j(s_{ij}/s_j) \sim \text{ specialization}$$

Providence is relatively specialized in the manufacture of submarines, even though it is a small share of overall employment.

Measuring diversity is a little trickier. What does it mean to say that one set of jobs is ‘less different than another’?

Consider, the two sets of jobs,

$$\{2 \text{ mechanics}, 1 \text{ brain surgeon}\}$$
$$\{1 \text{ Banker}, 1 \text{ Baker}, 1 \text{ Carpenter}\}$$

Which is more diverse?

The industry standard for answering this question is the ‘Herfindahl Index’,

$$DI_i = \left(\sum_j s_{ij}^2\right)^{-1}$$

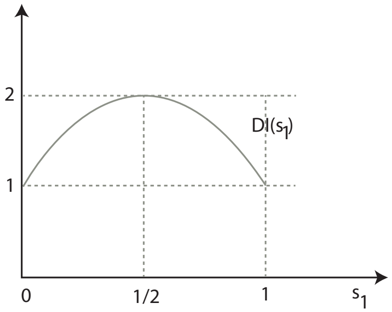
For s_{ij} the share of industry j in city i’s employment.

To see how this works, suppose $j = 1, 2$ and drop the i subscript.

Then we have $s_2 = 1 - s_2$ and

$$DI_i = (s_1^2 + s_2^2)^{-1} = (s_1^2 + (1 - s_1)^2)^{-1}$$
$$= (s_1^2 + (1 - 2s_1 - s_1^2))^{-1} = (2s_1^2 + 1 - 2s_1)^{-1}$$

If we plot this as s_1 ranges from 0 to 1, we get something like this,



So 'most diverse' means employment is uniformly distributed across all alternatives.

The corresponding relative diversity index is also sometimes useful,

$$RDI_i = \left(\sum_j (s_{ij} - s_j)^2 \right)^{-1}$$

- Most of the literature is based on indexes like those defined here.
- A city can be relatively specialized and relatively diverse!

With definitions of specialization and diversity in place, we can state our first two stylized facts.

FACT 1: *specialized and diversified cities coexist*.

Table 1. Most and least specialised and diversified US cities in 1992

	Specialisation		Diversity	
Rank	City (sector)	RZI	City	RDI
1	Richmond, VA (tobacco)	64.4	Cincinnati, OH	166.6
2	Macon, GA (tobacco)	55.0	Oakland, CA	161.2
3	Lewiston, ME (leather)	49.6	Atlanta, GA	159.4
4	Galveston, TX (petroleum)	49.1	Philadelphia, PA	151.4
5	Bangor, ME (leather)	45.6	Salt Lake City, UT	120.8
6	Owensboro, KY (tobacco)	44.4	Buffalo, NY	110.1
7	Corpus Christi, TX (petroleum)	37.6	Columbus, OH	108.3
8	Cheyenne, WY (petroleum)	33.4	Portland, OR	94.1
315	Buffalo, NY (rubber and plastics)	1.6	Lawton, OK	2.4
316	Cincinnati, OH (chemicals)	1.5	Richland, WA	2.4
317	Chicago, IL (metal products)	1.5	Steubenville, OH	2.4

Source: Black and Henderson data set.

Duranton and Puga (2000)

Fact 2: *Larger cities are more diversified than small cities.*

Figure 1. The size–diversity relation for US cities in 1992. *Source:* Black and Henderson (1998) data set.

Duranton and Puga (2000)

- All cities have a large share of employment is non-traded sectors, e.g., services, so all cities are pretty diversified.

- Cities $>$ 500k are more specialized in business services; Finance, Insurance, Real Estate (FIRE), less in manufacturing.
- Cities 50-500k are mmore specialized in ‘mature industries’ like ‘textiles’, less in ‘new industries’, like ‘instruments’.
- Big cities are relatively specialized in ‘new industries’.

All together, there is evidence for cities as nurseries of new industrial processes. New processes start in large diverse congested places and once the process is established, migrate to smaller, less congested and less diverse cities.

Fact 3: *The distribution of city sizes is stable over time (more soon), the ranking of cities by size is less stable, and the sectoral specialization is still less stable.*

TABLE 1—RANKINGS AND CHANGES FOR THE TEN LARGEST US METROPOLITAN AREAS, 1977 AND 1997

Rank in 1977 (and change in rank between 1977 and 1997)	Total population	Apparel	Transportation equipment	Instruments
New York	1 (0)	1 (+1)	3 (+20)	2 (+2)
Los Angeles	2 (0)	2 (−1)	2 (0)	5 (−3)
Chicago	3 (0)	11 (−3)	8 (+4)	4 (+5)
Washington	4 (0)	13 (+7)	29 (+8)	16 (−6)
Philadelphia	5 (+1)	3 (+3)	13 (−3)	7 (+4)
Boston	6 (+1)	4 (0)	17 (+24)	3 (−2)
Detroit	7 (+1)	10 (0)	1 (0)	20 (−1)
San Francisco	8 (−3)	15 (−12)	9 (0)	6 (−3)
Cleveland	9 (+5)	18 (+26)	6 (+15)	12 (+6)
Dallas	10 (−1)	5 (+4)	7 (−2)	13 (−5)
Total rank variation	(12)	(57)	(76)	(37)

Notes: The first number in each column is the 1977 ranking (among 272 metropolitan areas). It is followed (in parentheses) by the change in ranking between 1977 and 1997 (where an increase is a decline). The final row sums the absolute value of all the changes above. All calculations use the year 2000 definition of US metropolitan areas. Columns 3 to 5 give the ranking (and the 1977–1997 change) for total employment in SIC23 (apparel and other textile products), SIC37 (transportation equipment), and SIC38 (instruments and related products).

Source: US Census, County Business Patterns, and author's calculations.

Duranton (2007). When thinking about sectoral stability, it matters how aggregated are the sectors. Employment in 'furniture making' is less stable that employment in 'manufacturing'.

Fact 4: *City employment and population are related to specialization and diversity.*

Fact 5abc: *Plants/firms have short half-lives, about 5 years. Most innovations occur in big diversified cities. Most firm relocations are from big diversified cities to smaller less diversified cities.*

A subset of the literature on agglomeration investigates the relationship between diversity, specialization and productivity. Short answer: knowledge intensive activities tend to better in diverse cities, established processes do better in specialized cities (i.e., factory towns).

Zipf's Law/Rank-Size Rule, Gibrat's law

Zipf's law is an extraordinary feature of the size distribution not cities.
Formally, Zipf's law is that the size distribution of cities follows a Pareto distribution with exponent equal to 1.

Less obscurely, it is a rank size rule. If city size is N and the rank of the city in the set of cities under consideration (usually a country) is $r(N)$, then

$$\ln r(N) = \ln A - \zeta \ln N$$
$$\zeta = 1$$

Reorganizing and using $\zeta = 1$, we have

$$\begin{aligned} \ln r(N) &= \ln\left(A/N\right) \\ \implies r &= A/N \\ \implies N &= A/r \end{aligned}$$

Consider the first and second and third ranked cities, N_1, N_2, N_3. Then

$$\frac{N_2}{N_1} = \frac{A/2}{A/1} = 1/2$$
$$\frac{N_3}{N_1} = \frac{A/3}{A/1} = 1/3.$$

That is, the largest city is twice as large as the second, three times as large as the third, and so on.

This is an odd property, and not one that we would expect to be true. It is also really easy to verify. We can just do the regression

$$\ln r(N) = \ln A - \zeta \ln N + \varepsilon$$

and see what we estimate for ζ.

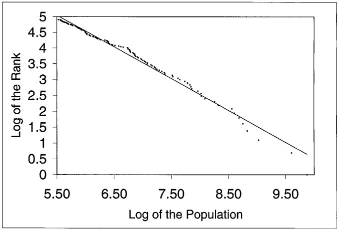
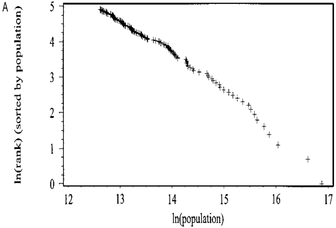


FIGURE I
Log Size versus Log Rank of the 135 largest U. S. Metropolitan Areas in 1991
Source: Statistical Abstract of the United States [1993].



$$\ln \text{Rank} = \underset{(.010)}{10.53 - 1.005} \ln \text{Size}, \tag{1}$$

- This looks pretty good for Zipf's law.
- These figures are showing the same data, 10 years apart, but the *x*-axes are a lot different. Can you explain this? Hint $ln(1000) \approx 6.9$.

This figure is remarkable, and it must be telling us something important about how cities grow. Since cities growing is pretty clearly central to the process of economic growth and development, this means that Zipf's law is telling us something important about this process.

Another way to say this, we can reject any model of the development process that does not give us a city size distribution that satisfies Zipf's law. Since Zipf's law is such a specific prediction, this let's us eliminate a lot of models.

This has led to two questions. First, does Zipf's law really hold. Second, why might we expect Zipf's law to hold.

Why should Zipf's law hold? I

In a remarkable paper, Gabaix (1999) shows that Zipf's law is 'like' the central limit theorem.

The Central Limit Theorem is one of the most important results in statistic. It says that if you take many draws from an arbitrary distribution and average them, and repeat this many time, then the distribution of the sums will be Normal, that is, a 'bell curve'.

Here is an animation demonstrating this, `https://www.youtube.com/watch?v=XAuMfxWg6eI`.

Gabaix's argument rests on two assumptions and a theorem.

The assumptions are that,

- Cities grow by a random multiplicative share each year. This share can be drawn from more-or-less any distribution.
- The size of cities has a strictly positive lower bound (with multiplicative growth, once a city hits zero, it stays there. This assumption stops that.)

To state the theorem, let N be an arbitrary size threshold, an N' be the size of a city drawn at random from the sample of cities, then

$$Pr(N' > N) = A/N^{\zeta}.$$

This is called a Pareto distribution. The theorem Gabaix appeals to is that such a Pareto distribution implies the rank size rule (in expectation).

Recalling the Central limit theorem, what should happen if the growth rate of cities is random in each year? Over time, the distribution of city sizes should approach a normal distribution (really log normal because the shocks are multiplicative).
What Gabaix shows is that with the lower bound on city sizes, the city size distribution converges to a Pareto distribution with parameter one, which is equivalent to the rank size rule.

This means that if city growth rates are unrelated to city size, a claim known as 'Gibrat's law', then Zipf's law is implied.

Does Zipf's law really hold? I

There has been a lot of research checking whether Zipf's law holds. This research has taken three basic approaches.

- More careful analysis of the 135 largest US cities shown earlier
- Expansion of the set of cities.
- Extension to other countries.

As Gabaix (1999) points out, even in the set of 135 cities, the data deviates slightly from a perfectly straight line. There is often an outlier, often a capital. Also, the middle size cities often decrease in size 'too fast'. This is visible in our figure as the plot being a little flatter than 1 as it approaches zero. Looking at the figure very closely, the plot of points looks slightly concave.
? investigate whether Zipf's law holds more broadly in the US.

The city size distribution depends in large part on how the boundaries of MSAs are drawn. Thus, the dramatic figures we have seen above may reflect the rules used for drawing MSA boundaries rather than important economic fundamentals.

To address this issue, they check whether the rank size rule applies to 85,287 6×6 mile squares drawn on a regular grid covering the continental US in 2000.

Zipf's law

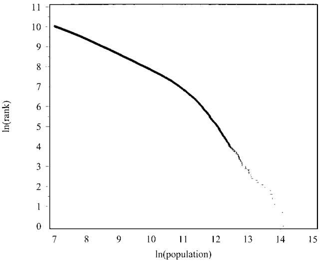


Fig. 3.5 Square-level Zipf plot for continental United States (all 23,974 squares with population at least 1,000)

Table 3.5 **Six-by-six-square-level Zipf regression results (squares with population 1,000 and above)**

		Piecewise linear				Linear	
Sample of squares	N	Kink	Slope1	Slope2	R^2	Slope	R^2
All squares with population ≥ 1,000	23,974	10.89	.747	1.937	.998	.833	.969
By Census division							
New England	1,027	9.96	.569	1.521	.996	.763	.930
Middle Atlantic	2,184	10.28	.669	1.249	.997	.759	.965
East North Central	4,313	10.92	.784	1.982	.999	.861	.975
West North Central	2,337	11.04	.886	2.607	.999	.941	.984
South Atlantic	4,977	10.72	.756	2.175	.995	.857	.959
East South Central	2,898	10.48	1.010	2.357	.997	1.072	.983
West South Central	3,078	11.17	.786	2.834	.997	.857	.969
Mountain	1,383	11.55	.723	3.662	.997	.791	.964
Pacific	1,777	11.21	.521	1.872	.992	.646	.922

'piecewise linear regression allows the slope of the regression line to change at log population 11.

This does not look as good for Zipf's law.

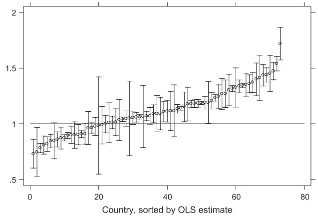
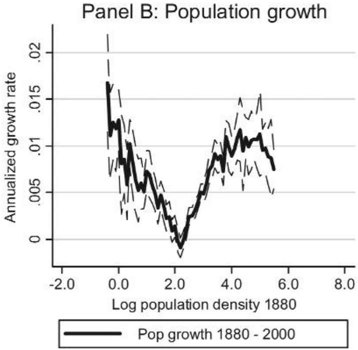


Fig. 1. Values of the OLS estimate of the Pareto exponent with the 95% confidence interval, for the full sample of 73 countries for the latest available period, sorted according to the Pareto exponent.

Soo (2005) estimates Zipf's law for a sample of 73 countries.

y-axis is Zipf coefficient and confidence interval. There is clearly some noise around a slope of one in the rank-size rule.

Since Zipf's law is implied by a model of random growth and scael invariant growth rates, it is interesting to check if city growth rates are invariant to city size.



Michaels et al. (2012) do exactly this using 120 years of US data.
This does not look like scale invariant growth rates.
Summing up,

- The Zipf's law figures are very dramatic, and surely tell us something about the process governing city growth.
- There has been considerable research on this, but (I think) not much progress.
- The literature is bogged down in trying to decide how close a straight line with slope one the data needs to be before we can decide it is one.
- Gabaix's result that Zipf's law is a consequence of a random growth process is a big step forward, but...
- Gibrat's law, is more problematic than Zipf's law empirically, and

- a 'random growth model' is not very satisfying as an explanation for how the world works.

Costs and Benefits of Cities I

To think about systems of cities, we need a simple description of how the costs and benefits of cities vary with their size.

For this purpose, 'benefits' are only increasing returns to scale in production, 'costs' are only commuting costs.

Use the same notation as for agglomeration effects; Production, Y; labor/population, N. Also following the earlier analysis,

$$Y_i = AN^{1+\sigma}$$

be total output, and

$$w_i = (AN^{\sigma})$$

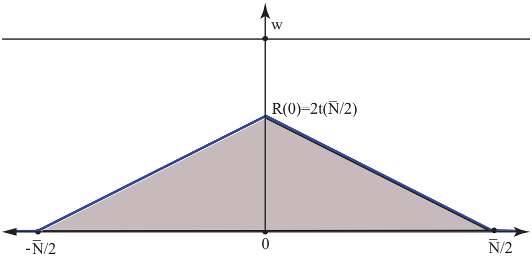
be the market wage. Recall, agents ignore their effect on aggregate output.

To describe commuting costs, we recall the monocentric city model and make the following assumptions:

- No housing, everyone consumes one unit of land.
- Commute costs are $2t$ per unit distance.
- Agricultural land rent is zero.

This means that the length of the city is equal to its population, $N = 2\overline{x}$.

We want to calculate total and average commuting costs.



Total commute cost is the shaded area.

Calculate by,

- Integrating:

$$
\begin{aligned}
TC(N) &= 2\int_0^{N/2} 2tx dx \\
&= 2[2t(\frac{1}{2}x^2]_0^{N/2}] \\
&= tN^2/2
\end{aligned}
$$

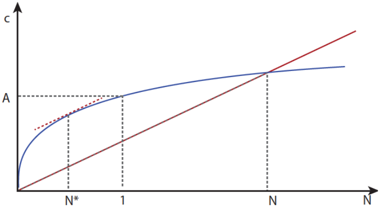
- Recalling that the area of a triangle is $1/2\times$ width $\times$ height. This gives $TC(N) = 2 \times \frac{1}{2} \times \frac{N}{2} \times 2t(\frac{N}{2}) = tN^2/2$.

To simplify a little bit, suppose that all residents get the average commute, and that land rents are distributed uniformly to all residents (rather than going to absentee landlords).

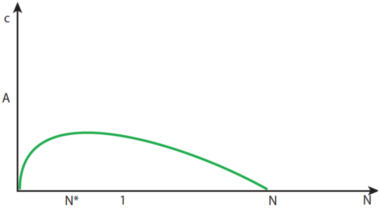
We can calculate consumption as a function of city size for a city resident as the difference between wages and commute costs.

$$\begin{aligned} c(N) &= w(N) - \frac{TC(N)}{N} \\ &= AN^{\sigma} - \frac{1}{L}\frac{tN^2}{2} \\ &= AL^{\sigma} - \frac{t}{2}L \end{aligned}$$

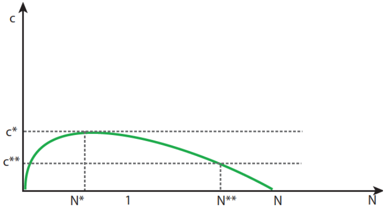
here we think σ is in the neighborhood of 0.05 and much smaller than 1.



Components of $c(L)$



$c(L)$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Duranton, G. (2007). Urban evolutions: The fast, the slow, and the still. *American Economic Review*, 97(1):197–221.

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