

①

CONSUMER SAVES

$$(a) \quad \text{MAX} \left[ \int_0^M f(i)^{\frac{\sigma-1}{\sigma}} di \right]^{\frac{\mu}{\sigma-1}} T^{1-\mu}$$

$$\text{s.t.} \quad \int_0^M p(i) f(i) di + p^T T = w \quad (1)$$

FIRST ORDER CONDITION IS  $\forall i = \lambda p \quad j \in [0, M], T, \lambda \geq 0$ SET  $p^T = 1$ , W.O.L.G.

$$\left[ \int_0^M f(i)^{\frac{\sigma-1}{\sigma}} di \right]^{\frac{\mu}{\sigma-1}} T^{-\mu} (1-\mu) = \lambda \quad (2)$$

$$\mu \frac{\sigma}{\sigma-1} \left[ \int_0^M f(i)^{\frac{\sigma-1}{\sigma}} di \right]^{\frac{\mu}{\sigma-1} - 1} \frac{\sigma-1}{\sigma} f(j)^{-\frac{1}{\sigma}} T^{-\mu} = \lambda p(j), \quad j \in [0, M] \quad (3)$$

(2)  $\rightarrow$  (3)

$$\Rightarrow \mu \frac{\sigma}{\sigma-1} \left[ \int_0^M f(i)^{\frac{\sigma-1}{\sigma}} di \right]^{-1} \frac{\sigma-1}{\sigma} f(j)^{-\frac{1}{\sigma}} T = (1-\mu) p(j), \quad j \in [0, M]$$

$$\Rightarrow \mu \left[ \int_0^M f(i)^{\frac{\sigma-1}{\sigma}} di \right]^{-1} f(j)^{-\frac{1}{\sigma}} T = (1-\mu) p(j) \quad (4), \quad j \in [0, M]$$

$$(4) \Rightarrow \frac{p(k)}{p(j)} = \left[ \frac{f(k)}{f(j)} \right]^{-\frac{1}{\sigma}} \quad j, k \in [0, M]$$

$$\Rightarrow \left( \frac{p(j)}{p(k)} \right)^{\sigma} = \frac{f(k)}{f(j)} \Rightarrow f(k) = f(j) \left( \frac{p(j)}{p(k)} \right)^{\sigma} \quad (5) \\ j, k \in [0, M]$$

(2)

using (5) in (4)

$$\mu \left[ \int_0^M \left[ f(j) \left( \frac{p(j)}{p(i)} \right)^{\sigma} \right]^{\frac{\sigma-1}{\sigma}} di \right]^{-1} f(j)^{-\frac{1}{\sigma}} = (1-\mu) p(j)$$

$$\Rightarrow f(j)^{-\frac{1}{\sigma}} f(j)^{-\frac{(\sigma-1)}{\sigma}} \left[ \int_0^M \left[ \frac{p(j)}{p(i)} \right]^{\sigma-1} di \right]^{-1} = \frac{1-\mu}{\mu T} p(j)$$

$$\begin{aligned} f(j)^{-1} &= \left[ \int_0^M \left[ \frac{p(j)}{p(i)} \right]^{\sigma-1} di \right]^{-1} \frac{1-\mu}{\mu T} p(j) \\ &= p(j)^{\sigma-1} p(j)^{-\sigma} \left[ \int_0^M p(i)^{1-\sigma} di \right]^{-1} \frac{1-\mu}{\mu T} \\ &= p(j)^{-\sigma} \int_0^M p(i)^{1-\sigma} di \frac{1-\mu}{\mu T} \end{aligned}$$

$$\Rightarrow f(j) = \frac{p(j)^{-\sigma}}{\int_0^M p(i)^{1-\sigma} di} \frac{\mu T}{1-\mu} \quad (6)$$

(6)  $\rightarrow$  (2)

$$\int_0^M \frac{p(j)^{1-\sigma}}{\left[ \int_0^M p(i)^{1-\sigma} di \right]} \frac{\mu T}{1-\mu} dj + T = W$$

$$\Rightarrow \frac{\mu T}{1-\mu} + T = W$$

$$\Rightarrow \boxed{T^* = (1-\mu)W} \quad (7)$$

(3)

$$(7) \rightarrow (6) \Rightarrow \boxed{f^*(y) = \frac{p(y)^{-\sigma}}{\int_0^M p(i)^{-\sigma} di} \text{ MW}} \quad (8) \quad y \in [0, M]$$

$$\text{DEFINE } P = \left[ \int_0^M p(i)^{-(\sigma-1)} di \right]^{-\frac{1}{\sigma-1}} \quad \sim \text{"C.F.S. PRICE INDEX"}$$

$$\Rightarrow P^{-(\sigma-1)} = \left[ \int_0^M p(i)^{1-\sigma} di \right] \quad (9)$$

$$(9) \rightarrow (8) \Rightarrow f^*(y) = \frac{p(y)^{-\sigma}}{P^{-(\sigma-1)}} \text{ MW}$$

$$\Rightarrow \boxed{f^*(y) = P^{\sigma-1} p(y)^{-\sigma} \text{ MW}} \quad (10) \quad \left( \text{C.F. FUJITA + THISSER eqn 9.41} \right)$$

STRICTLY, IF  $K \subset [0, M]$  MEASURE ZERO AND  $f: K \rightarrow \mathbb{R}$  THEN  $f^* + f$  IS ALSO SOLVES THE CONSUMERS PROBLEM.

(4)

PROFITS FOR FIRM  $j$  ARE

$$\begin{aligned}\pi &= P(j) f(j) - A - B f(j) \\ &= (P(j) - B) f(j) - A\end{aligned}$$

From (10)

$$\begin{aligned}\Rightarrow \pi &= (P(j) - B) \frac{M W}{P^{1-\sigma}} P(j)^{-\sigma} - A && (\text{RECALL, FIRMS ARE MONOPOLISTS}) \\ &= \left[ P(j)^{1-\sigma} - B P(j)^{-\sigma} \right] \frac{M W}{P^{1-\sigma}} - A\end{aligned}$$

$$\frac{d\pi}{dP(j)} = \frac{M W}{P^{1-\sigma}} \left[ (1-\sigma) P(j)^{-\sigma} + B \sigma P(j)^{-\sigma-1} \right] = 0$$

$$\Rightarrow (1-\sigma) + B \sigma P(j)^{-1} = 0$$

$$(1-\sigma) = -B \sigma P(j)^{-1}$$

$$\Rightarrow P^*(j) = \frac{-B \sigma}{1-\sigma} \quad \left( \begin{array}{l} \text{NEED} \\ \sigma > 1 \end{array} \right) \quad (11)$$

USING (11) IN (8)

$$f^*(j) = \frac{\left[ \frac{-B \sigma}{1-\sigma} \right]^{-\sigma}}{M \left[ \frac{-B \sigma}{1-\sigma} \right]^{1-\sigma}} = \frac{1-\sigma}{-M B \sigma} = \frac{\sigma-1}{M B \sigma}, \quad j \in [0, M] \quad (12)$$

(5)

WITH FREE ENTRY OF FIRMS  $\pi = 0$ 

$$\Rightarrow P_f - A - B_f = 0$$

USING (11) + (12)

$$\Rightarrow \left( \frac{-B_g}{1-\sigma} \right) \left( \frac{\sigma-1}{MB_g} \right) - A - B \left( \frac{\sigma-1}{MB_g} \right) = 0$$

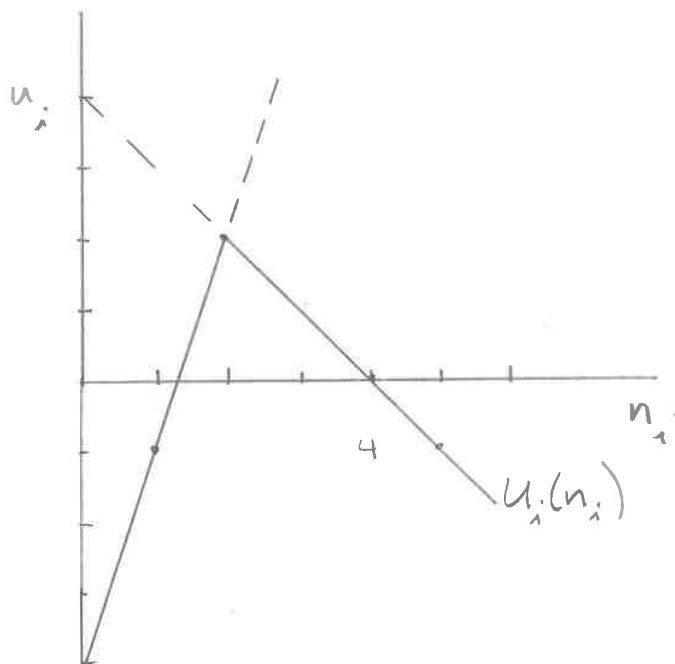
$$\Rightarrow \frac{\sigma}{M_g} - A - \frac{\sigma-1}{M_g} = 0$$

$$\Rightarrow \frac{1}{M_g} - A = 0, \quad (13)$$

HERE, UNLIKE F.T ON KRUGMAN, WAGE IS EXOGENOUS, SO WE ONLY GET ZERO ENTRY CONDITION TO HOLD IF PARAMETERS HAPPEN TO SATISFY (13).

(b) SEE REQ. (11) ABOVE - THIS IS CONSTANT MARK-UP BEYOND MARGINAL COST (= B)

(c) YOU CANNOT VERIFY THIS STATEMENT WITHOUT ENDOGENIZING THE WAGE.



① Suppose  $N=5$ . THERE ARE SIX CANDIDATE EQUILIBRIA,  
 (a) 0, 1, 2, 3, 4, 5 PEOPLE CHOOSE URBAN.

IF 0 PEOPLE CHOOSE URBAN, THEN EVERYONE GETS  $\bar{u}=0$   
 AND UNILATERAL DEVIATIONS ALTHO  $u(1) = -1$ , SO  $n_i=0$  IS  
 NASH.

IF  $n=4$ , THEN  $u_i(4) = \bar{u}=0$ . AN URBAN DWELLER  
 WHO DEVIATES TO NON-URBAN GETS 0. THIS IS NOT A  
 BENEFICIAL DEVIATION. A RURAL DWELLER WHO DEVIATES TO  
 URBAN GETS  $u_i(5) = -1$ , ALSO NOT A RATIONAL DEVIATION.

$\therefore$  ANY STRATEGIES S.T.  $n_i \in \{0, 4\}$  IS A NASH  
 EQUIL.

IF  $n_i \in \{1, 5\}$  THEN  $u = -1 < \bar{u}$  SO DEVIATIONS  
 TO NON-URBAN ARE RATIONAL.

IF  $n_i = 2$  THEN  $u_i = 2$  AND A DEVIATION TO  
 URBAN

(2)

GIVES PAI-OFF  $U(3) = 1 > \bar{U}$ , SO  $n_i = 2$  IS NOT NASH.

IF  $n_i = 3$ , THEN DEVIATIONS TO URBAN GIVE  $U(4) = 0 = \bar{U}$   
AND DEVIATIONS FROM URBAN GIVE  $\bar{U} = 0 < 1 = U(3)$ .  
 $\Rightarrow$  STRATEGIES S.T.  $n_i = 3$  ARE NASH.

THIS, STRATEGIES ARE NASH  $\Leftrightarrow n_i \in \{0, 3, 4\}$ .

(b) FOR MIXED STRATEGIES, EACH AGENT CHOOSES  $p_i \in [0, 1]$   
WE CAN USE THIS TO WRITE DOWN THE PROBABILITIES OF EACH CONFIGURATION.

$n_i$	PROB	$U(n_i)$
0	$(1-p)^5$	-4
1	$p(1-p)^4$	-1
2	$p^2(1-p)^3$	2
3	$p^3(1-p)^2$	1
4	$p^4(1-p)$	0
5	$p^5$	-1

FOR THE PURPOSE OF CALCULATING MIXED EQUIL, WE NEED TO KNOW THE EFFECT ON PAI-OFFS OF CHANGING  $p$  FOR ONE AGENT, HOLDING OTHERS FIXED AT  $p'$ .

(3)

IF ALL AGENTS BUT 1 CHOOSE  $P$ , AND 1 CHOOSES  $P'$ , THEN WHAT WE CARE ABOUT IS THE DISTRIBUTION OF EVERYONE ELSE'S MOVES. LET  $n'$  DENOTE # OF PLAYERS,  $\neq 1$ , CHOOSING UNBAIL. THEN

$n'$	$PR(n')$
0	$(1-p)^4$
1	$p(1-p)^3$
2	$p^2(1-p)^2$
3	$p^3(1-p)$
4	$p^4$

THEN, IF 1 CHOOSES  $P'$ ,

$$E(\pi_i) = (1-p') \cdot 0 + \quad (1 \text{ is, pure})$$

$$p' \left[ -1(1-p)^4 + 2(1-p)^3 p + 1(1-p)^2 p^2 \right. \\ \left. + 0 \cdot (1-p)p^3 - 1p^4 \right] \quad (*)$$

WE NEED TO FIND  $p'$  SUCH THAT

$$\frac{dE(\pi_i)}{dp'} = 0 \quad \text{AND} \quad p' = p$$

WHERE THE SECOND CONDITION IS B/C WE'VE RESTRICTED ATTENTION TO SYMMETRIC EQUIL.

BY INSPECTION OF (\*) WE NEED THE EXPRESSION IN BRACKETS TO BE ZERO, SO WE WILL NEED TO CHOOSE  $p$  SO THAT IT IS A ZERO OF THIS 4<sup>TH</sup> ORDER POLYNOMIAL.  $\Rightarrow$  EXPECT AS MANY AS 4 ROOTS.



(4)

(c) THE PARETO OPTIMAL OUTCOMES ARE  $n \in \{2, 3\}$   
 FOR EITHER URBAN POPULATION, THERE IS NO WAY  
 TO MAKE ONE PERSON BETTER W/O MAKING  
 ANOTHER WORSE.

(d) WE NEED  $n_1 + n_2 \leq 5$ .

(1)  $(n_1, n_2) \in \{(0, 0), (0, 4), (4, 0)\}$  ARE  
 ALL NASH. IN THESE EQUIL, EVERYONE GETS  $\bar{u}$ .

(2)  $(n_1, n_2) \in \{(2, 3), (3, 2)\}$

HERE THE URBAN PAY-OFF IS POSITIVE, SO  
 NO ONE MOVES TO RURAL. NO ONE WANTS  
 TO MOVE FROM THE SMALL TO THE BIG CITY,  
 THIS IS STRICTLY WORSE. MOVING FROM SMALL CITY  
 TO BIG DOES NOT CHANGE PAY-OFF FOR MARGINAL  
 AGENT.

(3) NO OUTCOME WITH  $n_1 \in \{1, 5\}$  IS NASH  
 B/C ONE AGENT CAN DEViate TO RURAL FOR  
 $0 > -1$ .

(4) NO OUTCOME  $n_1 = 2$  IS AN EQUILIBRIUM  
 UNLESS  $n_1 + n_2 = 5$ . OTHERWISE RURAL  
 AGENTS WILL DEViate

(5)  $(n_1, n_2) \in \{(0, 3), (3, 0)\}$  ARE ALSO EQUIL.  
 URBAN RESIDENTS DON'T WANT TO MOVE, AND  
 RURAL AGENTS WHO MOVE TO THE CITY ARE  
 INDIFFERENT.

→

(5)

Summing up, we have

		$n_1$					
		0	1	2	3	4	5
$n_2$	0	*	X	X	*	*	X
	1	X	X	X	X	X	X
	2	X	X	X	*	X	X
	3	*	X	*	X	X	X
	4	*	X	X	X	X	X
	5	X	X	X	X	X	X

~~||||~~  $\Rightarrow n_1 + n_2 > 5$

\*  $\Rightarrow$  NASH

X  $\Rightarrow$  NOT NASH

    $\Rightarrow$  T.O.

Therefore, NASH EQUIL IS CONSISTENT WITH TWO CITIES OF ABOUT OPTIMAL SIZE AND AN EMPTY COUNTRYSIDE, OR WITH ONE CITY LARGER THAN OPTIMAL WITH 1 OR 2 STILL RURAL.

- (c) ADDING ANOTHER CITY WOULD NOT QUALITATIVELY CHANGE THE EQUILIBRIUM OUTCOMES, IT WOULD JUST CREATE INDETERMINACY ABOUT WHICH 2 CITIES ARE OCCUPIED. THIS SEEMS SPECIAL TO THE CASE  $N=5$ .