

EC1340 Topic #2

Trends in climate and CO_2

Matthew A. Turner
Brown University
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Last time, we...

- Introduced the BDICE model and started to fill in some of its parameters.
- Measured CO₂ emissions.
- Carbon cycle, and the remarkable fact that about 45% of emissions are absorbed by the biosphere.
- Introduced RCPs.

Summary of progress

Recall BDICE :

$$\max_{I, M} u(c_1, c_2) \quad (1)$$

$$\text{s.t. } W = c_1 + I + M \quad (2)$$

$$c_2 = (1 + r)I - \gamma(T_2 - T_1)I \quad (3)$$

$$E = (1 - \rho_4 \frac{M}{W})(\rho_5(c_1 + I)) \quad (4)$$

$$P_2 = \rho_0 E + P_1 \quad (5)$$

$$T_2 = \rho_1(P_2 - P_1) + T_1 \quad (6)$$

- Last time we talked about emissions and consumption, emissions, and the carbon cycle. We picked up W (world income) and M (mitigation) for free.
- Today we want climate and the relationship between CO_2 and climate, the last constraint.

Trends in Climate, CO₂ and how they are related

- 1 Review
- 2 Short Run Climate
- 3 Long run climate
- 4 Long run CO₂
- 5 CO₂ and climate
- 6 Global warming potential (GWP)
- 7 Conclusion

Endowment of climate - outline

We want to try to understand our endowment of climate. This will be T_1 , which we will define as the pre-industrial mean temperature.

Learning T_1 is a measurement problem.

- ① Measured temperature and the problem of measuring temperature.
- ② Other measured aspects of climate.
- ③ Icecores and oxygen isotope ratios, other climate proxies.

Restricting attention to temperature is a useful simplification for our model, but we really care about other aspects of climate, so we'll investigate this a little bit, too.

Measured Temperature

This is what modern weather stations look like:



(JFK airport in NY)



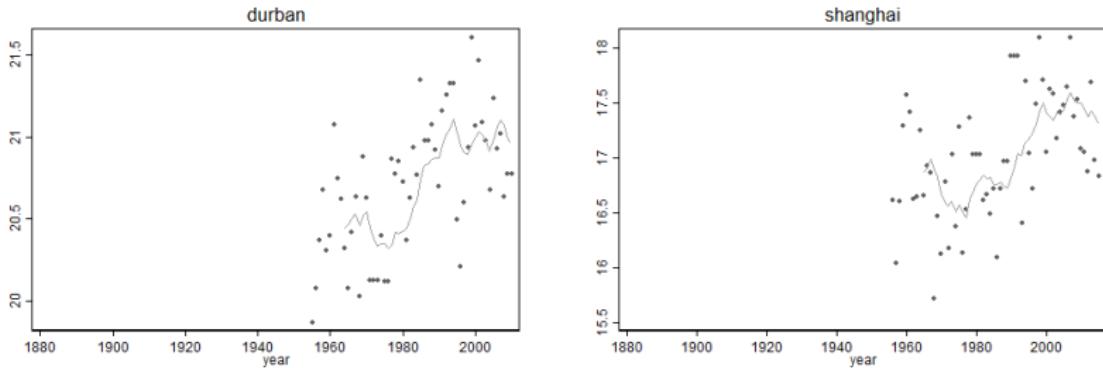
(Antarctic weather station)

<http://www4.ncdc.noaa.gov/cgi-win/wwcgi.dll?wwDi~StnPhoto~2001941B~a~000>

http://www.gdargaud.net/Antarctica/DC2005/20050406_10_WeatherStation.jpg

- We have measured temperature data going back about 100 years.
- There are also satellite, radiosonde and various ocean instrumental measurements, but for the most part they go back only 30-40 years.

Mean annual temperature, Durban and Shanghai



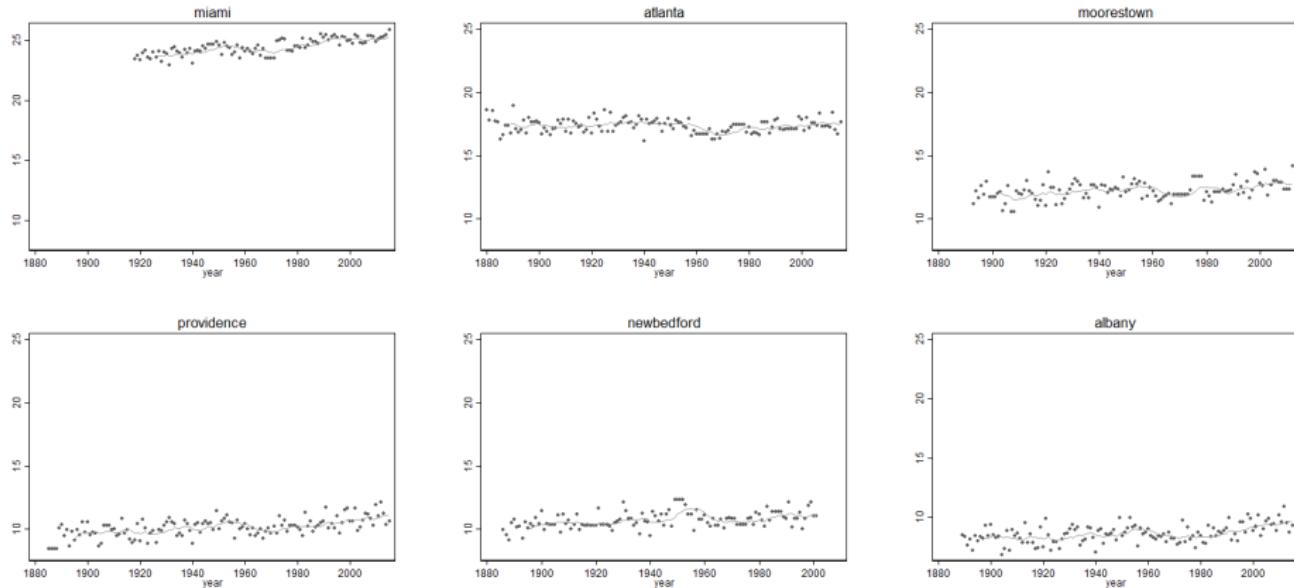
Black line is 10 year trailing average,

$$T_{10,t} = 0.1 \times (T_{t-9} + T_{t-8} + \dots + T_{t-0}).$$

Source:

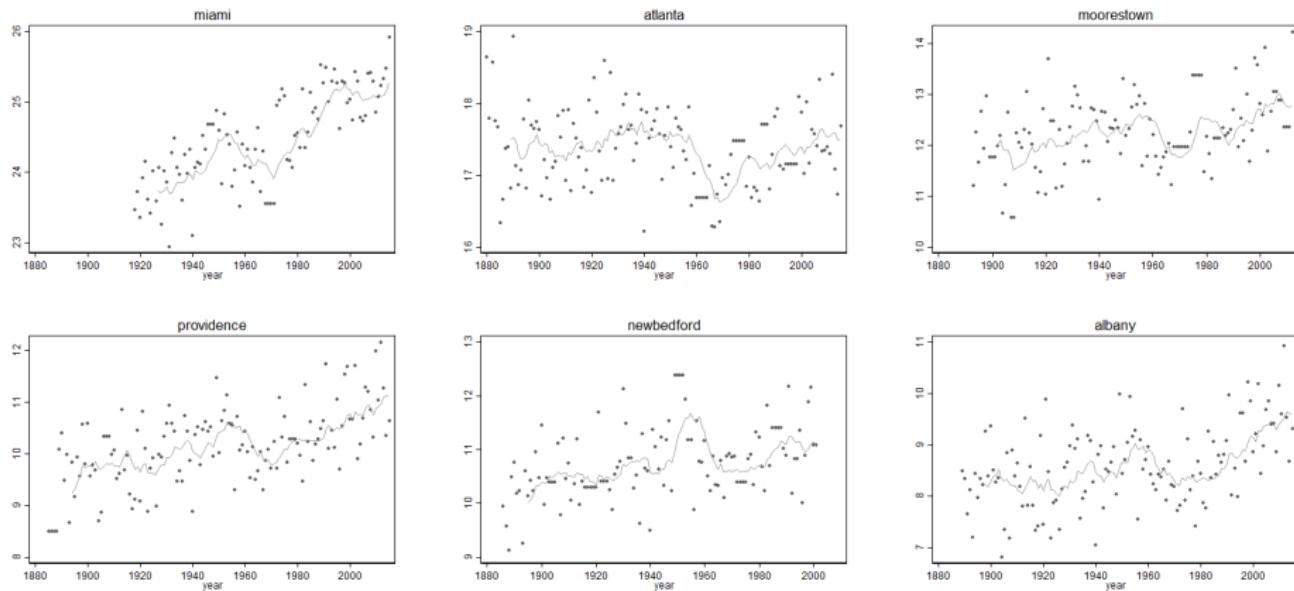
http://data.giss.nasa.gov/gistemp/station_data/

Mean annual temperature, US Atlantic coast I



Note: (1) Latitude affects temperature more than time. (2) More warming in the north.

Mean annual temperature, US Atlantic coast II



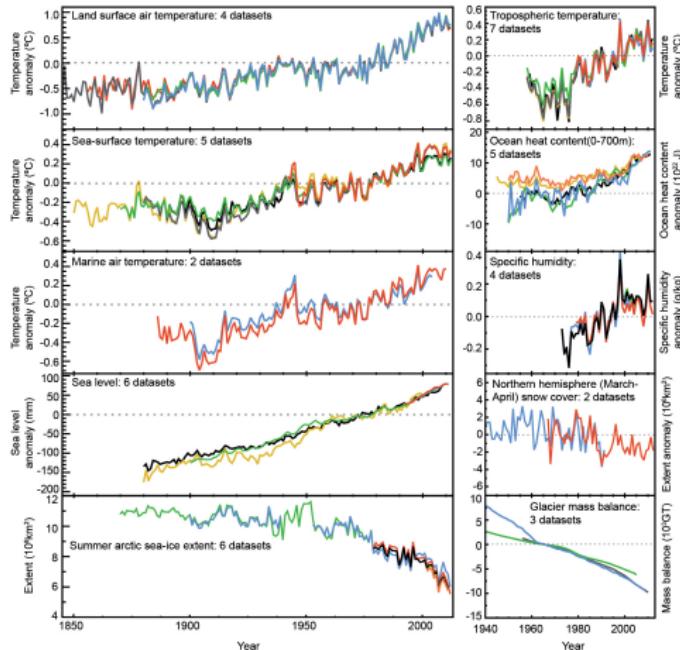
Same graphs with different axes. Change is about 1C for 100years.

Issues with measured temperature

- Measurement error within instrument.
- Different types of instruments.
- Sample of stations/instruments varies over time.
- What does ‘world average temperature’ mean? Average over what?(see next slide)
- Very little coverage of the ocean.
- Urban heat Islands (see IPCC 2013 TS 2.2 this effect is probably small).

Many of these issues can be resolved with statistics. Some cannot.

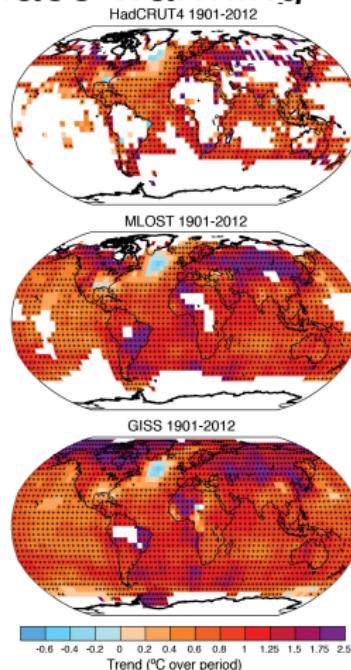
IPCC summary of lots of instrumental measurements



IPCC 2013 figure TS.01

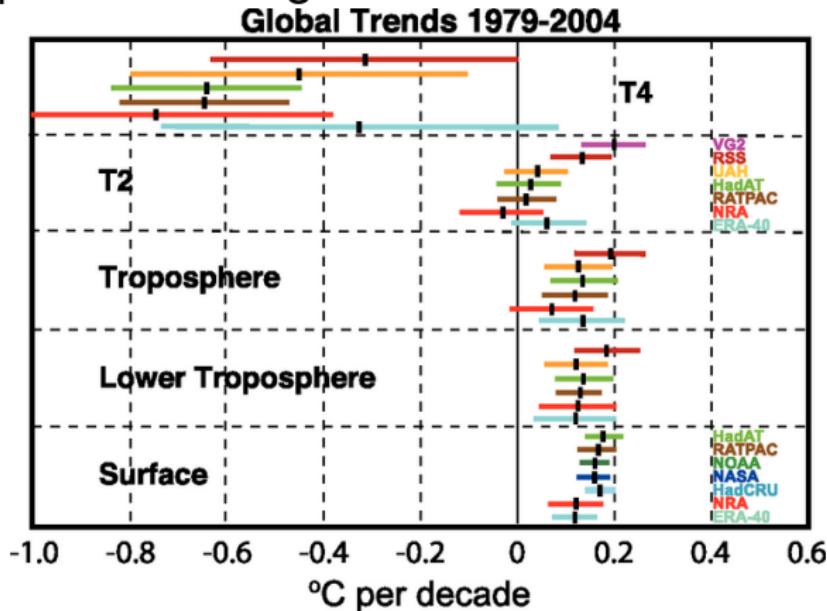
- T_1 is pre-industrial norm. Current is about 1°C warmer.
- Note: warming hiatus during about 2000-2010.

Distribution of surface warming



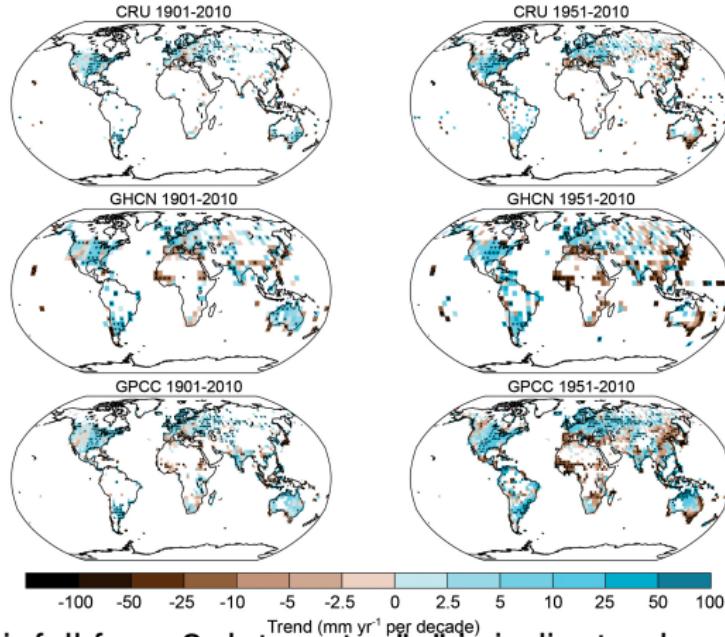
Distribution of change in surface temperature 1901-2012 as determined by linear trends for three data sets. White indicates missing data. IPCC 2013 Physical Science Basis figure TS.02

Stratospheric cooling



Stratosphere begins about 3 miles up. It has been cooling. (1) The world is complicated, some places on the surface cool too. (2) There is very little energy stored in the Stratosphere. IPCC 2007 figure 3-18

Changes in rainfall I.



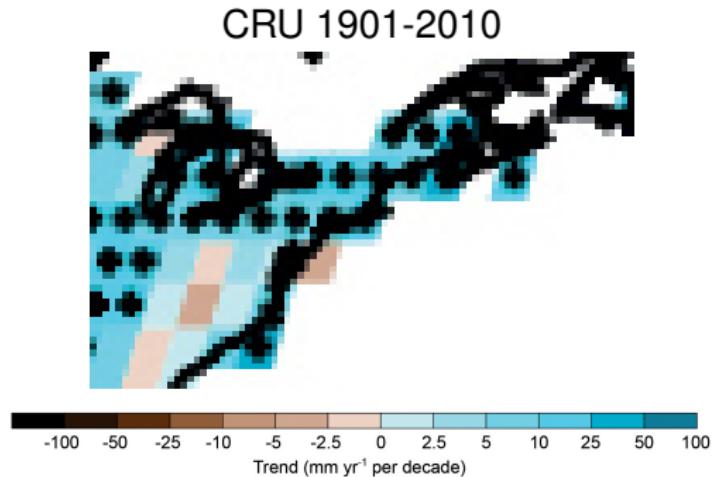
Trends in rainfall from 3 data sets. "+"'s indicate changes statistically different from zero. Mean annual rainfall in Providence is about 47 inches ~ 1175mm. In Salt Lake, annual average precipitation is 455. Are these changes big?

IPCC Physical Science Basis 2013 figure

TS.TFE.1.2

Changes in rainfall II.

Zoom in on Providence:



Providence had 2.5-5mm increase in annual precipitation each decade, for 11 decades $\sim 27.5\text{-}55\text{mm}$ increase on a base of about 1175mm. This is a 2-5% increase.

Sea level rise \approx 20cm since 1860

From 1961 on, measured sea level rise is just less than 2mm/year. This occurs because water expands as it gets warmer and ice melts. The IPCC attributes about 60% of sea level rise to warming.

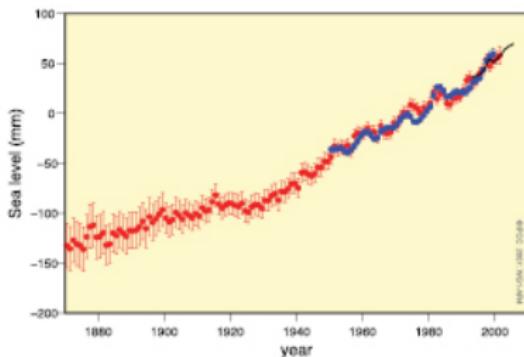


Figure TS.18. Annual averages of the global mean sea level based on reconstructed sea level fields since 1870 (red), tide gauge measurements since 1950 (blue) and satellite altimetry since 1992 (black). Units are in mm relative to the average for 1961 to 1990. Error bars are 90% confidence intervals. IPCC 2007, fig. TS.18
Ocean acidity is also increasing.

Long run climate proxies

Climate is something that happens on geological time scales. Some of the recent rise in global measured temperature may be part of normal climate fluctuations. To assess this, we need longer time series. Since no longer series of measured temperatures exist, we use proxies. There are many sorts of proxy data that can give us information about historical temperature.

- Boreholes.
- Tree rings.
- Glacial extent.
- Species range.

Generally, all point toward warming. Among these ice cores are of particular interest.

Icecores I



(Extracting an ice core)



(Extracting an ice core)



(sections of an icecore)

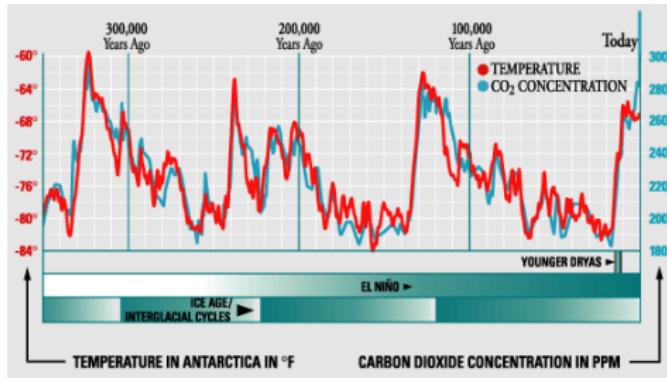
<http://kaira.sgo.fi/2011/05/ice-cores-part-1.html>

<http://kaira.sgo.fi/2011/05/ice-cores-part-1.html>

http://www.gdargaud.net/Antarctica/DC2005/20050406_10_

Icecores are records of the water deposited on the site over many years. Water consists of several isotopes. In particular, some water molecules contain oxygen atoms with 8 neutrons, while others contain 10. The ratio of these isotopes in polar snow varies systematically with temperature. Thus, icecores allow us to track temperature back into the very distant past, about 800,000 years in some cases.

Isotope ratio from Vostok Antarctica ice core



Red is temperature imputed from isotope ratio. This core goes back about 400,000 years, others go back 800,000 and show the same thing. It is warm now by the standards of the last several hundred thousand years.(NB: blue is CO₂ from the same core).

<http://www.koshland-science-museum.org/exhibitgc/historical02.jsp>

The current state of climate

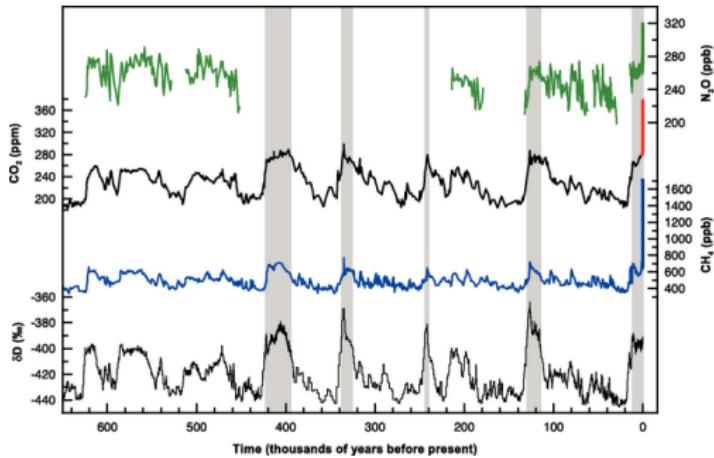
We've measured T_1 with instrumental measurements and proxies

- Measured temperature data, though subject to some problems, suggests dramatic warming.
- Other short run proxies, glacial extent and species range, confirm measured temperature data.
- Ice core data shows that the last 200 years are warmer than the preceding 650,000+. Other long run proxies (tree rings, boreholes) are broadly consistent, though they also have problems.
- Weak evidence of increased rainfall in North America.

Icecore CO₂ Measurements

Icecores also allow the construction of long series of CO₂ measurements. Small air bubbles are trapped in icecores. By checking the CO₂ concentration in this air, it is possible to create much longer series of atmospheric CO₂, though with somewhat lower frequency (we can't reconstruct daily CO₂ concentrations from icecores). Similarly for other trace gases.

Trends in CO₂ and other trace gases



Antarctic ice and recent atmospheric measurements of N₂O (top), CO₂ (second), CH₄ (third). Data cover 650,000 years and the shaded bands indicate current and previous interglacial warm periods. Bottom graph is isotope ratio

CO₂ is very high now.

IPCC 2007 Physical Science Basis, figure TS 1

The current state of atmospheric CO₂

We've measured CO₂ directly with instruments and with icecore bubbles,

- Mauna Loa measured CO₂ shows steady increase since 1959.
- Icecore record of CO₂ and other trace gases show that the levels of last 200 years are without precedent in the preceding 650,000+ years.

Next step is to look at how atmospheric CO₂ affects temperature.

CO₂ and climate – Outline

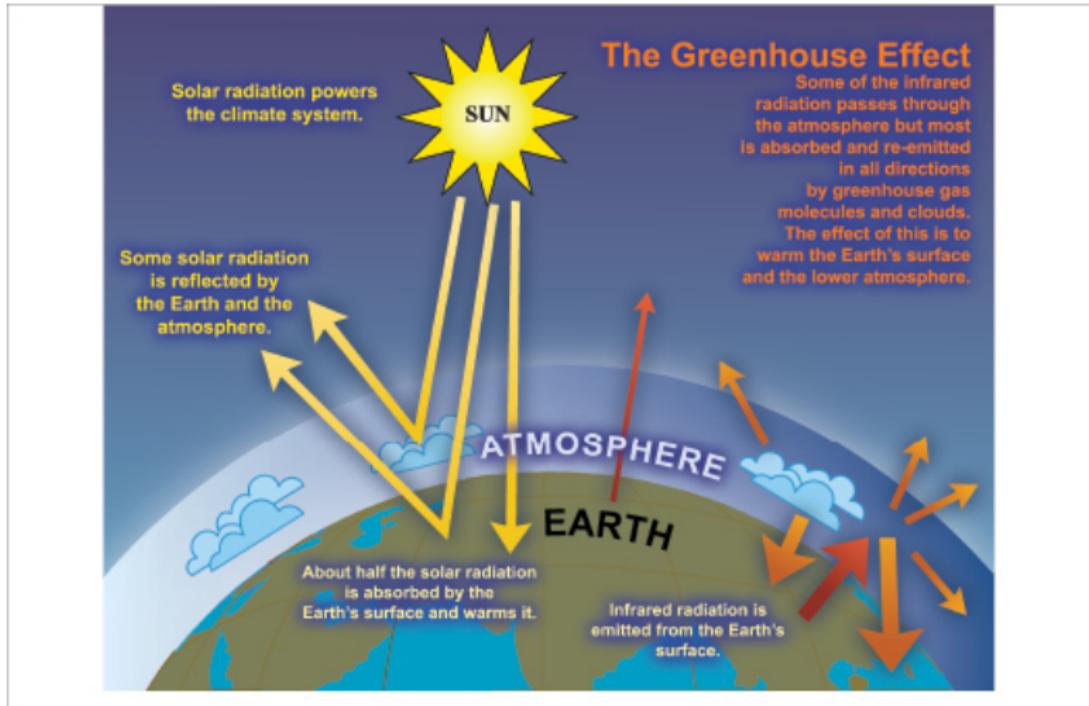
We now begin to investigate the relationship between atmospheric CO₂ and future climate (In BDICE , this relationship is determined by the parameter ρ_1). More specifically, we'll look at

- ① Basic physics of the greenhouse effect
- ② Correlation of carbon levels and temperature
- ③ Complications: Astronomical cycles, aerosols, heat transmission
- ④ Climate models
- ⑤ Climate projections
- ⑥ Sea level rise

Physics of the greenhouse effect

- ① Solar radiation arrives at Earth at a known and nearly constant rate.
- ② About 59% of this radiation is absorbed by Earth (or its atmosphere), the rest is reflected back to space.
- ③ All radiation of heat from the Earth occurs at the upper edge of the atmosphere.
- ④ All radiation from the surface occurs at the surface.
- ⑤ GHG molecules reflect heat radiating from the surface back to the surface and prevent it from radiating to space.

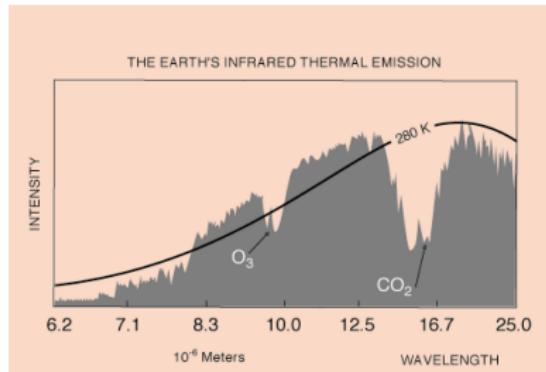
The physics of this process are elementary, are well understood and uncontroversial. See 'Principles of Atmospheric Science', J. E. Frederick, Jones and Bartlett, Sudbury MA, 2008.



http://www.ipcc.ch/publications_and_data/ar4/wg1/en/faq-1-3.html

Spectral evidence for CO₂ warming effect

CO₂ only stops the radiation of certain wavelengths of light from making out to space. These wavelengths can be measured in the laboratory. When satellites look back at Earth and measure the intensity of reflected spectra, there is a gap where CO₂ absorbs reflected light in the relevant wavelength. That is, the physicists are right. CO₂ traps heat.



Graph taken from online physics forum, but it looks just like figure 2.11 in 'Principles of Atmospheric Science' J. Frederick, 2008

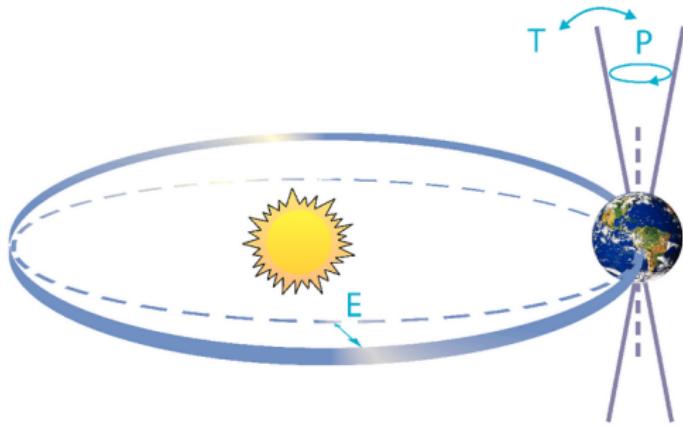
Copyright 2023, Matthew Turner

Complications

That more atmospheric CO₂ causes warming seems clear, other issues are less clear. Here are some of the problems:

- non-constant solar radiation
- Aerosols
- transmission of heat in the atmosphere
- transmission of heat to and from the Ocean

Cycles in solar radiation



Schematic of the Earth's orbital changes (Milankovitch cycles) that drive the ice age cycles. 'T' denotes changes in the tilt (or obliquity) of the Earth's axis, 'E' denotes changes in the eccentricity of the orbit (due to variations in the minor axis of the ellipse), and 'P' denotes precession, that is, changes in the direction of the axis tilt at a given point of the orbit.

http://www.ipcc.ch/publications_and_data/ar4/wg1/en/faq-6-1-figure-1.html

Aerosols

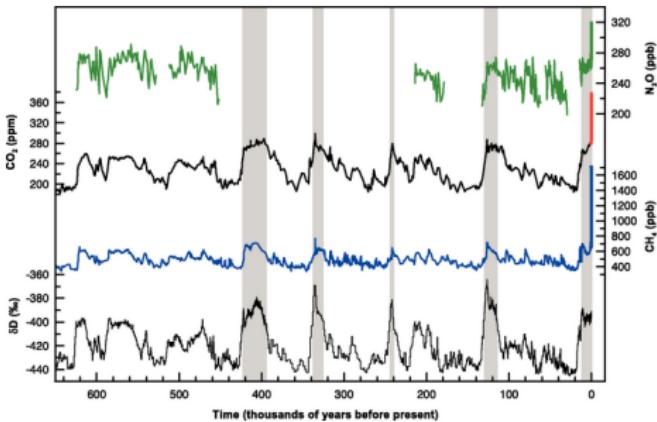
- Aerosols are small particles that float in the atmosphere, e.g., dust, diesel exhaust. They reflect light back out into space and thus promote global COOLING.
- Evidence in support of this theory is found in the cooler years that follow major volcanic eruptions.
- Aerosols are also increasing.

Heat transmission

- Radiative heat transmission appears to be a fairly straightforward problem for physicists. Heat transmission by convection is very hard.
- Heat transmission in the atmosphere COULD be quite different from what models suggest. Stay tuned for more on problems with climate models.
- Heat transmission to the ocean and within the ocean is not very well understood and is important. Water holds a lot more energy than air.

CO₂ and climate, empirical relationship I

One way to guess at the total effect of CO₂ and complicating factors is to look at the historical record. This is the plot of icecore isotope ratios and CO₂ that we have seen before.



Note that spikes in CO₂ and spikes in the isotope ratio line up, more-or-less. (IPCC 2007 Physical Science Basis, figure TS 1)

CO₂ and climate, empirical relationship II

In AR6, the IPCC assembles data on climate proxies that date back almost 60 million years. (IPCC 2022, Working Group I, figure TS 1)

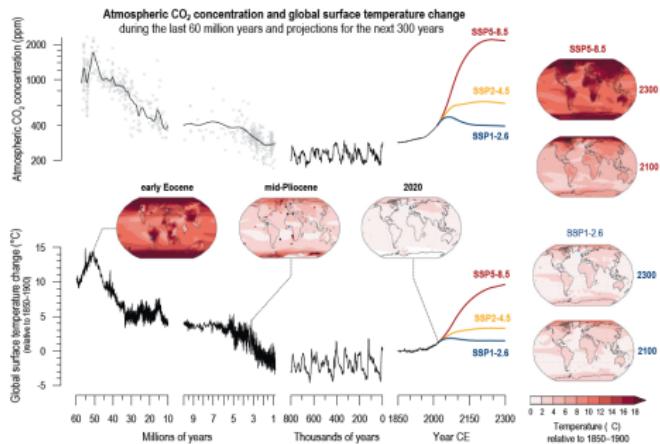
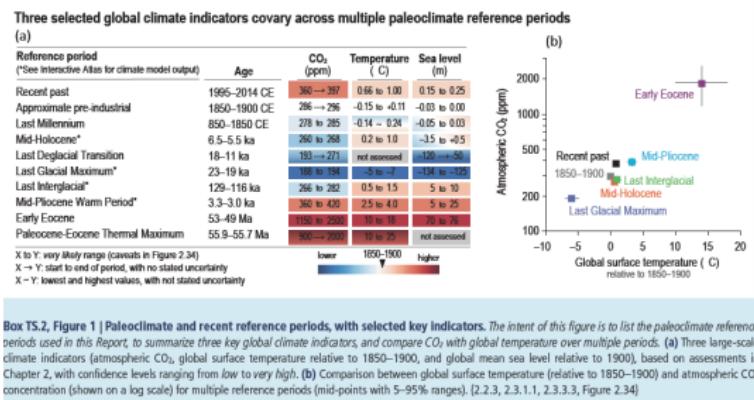


Figure TS.1 | Changes in atmospheric CO₂ and global surface temperature (relative to 1850–1900) from the deep past to the next 300 years. The intent of

(The right panel is model based projections of future climate). This longer horizon makes the relationship even clearer.

CO₂ and climate, empirical relationship III

Here is an even better way to present these data (IPCC 2022, Working Group I, figure TS2.1)



Box TS.2, Figure 1 | Paleoclimate and recent reference periods, with selected key indicators. The intent of this figure is to list the paleoclimate reference periods used in this Report, to summarize three key global climate indicators, and compare CO₂ with global temperature over multiple periods. (a) Three large-scale climate indicators (atmospheric CO₂, global surface temperature relative to 1850–1900, and global mean sea level relative to 1900), based on assessments in Chapter 2, with confidence levels ranging from low to very high. (b) Comparison between global surface temperature (relative to 1850–1900) and atmospheric CO₂ concentration (shown on a log scale) for multiple reference periods (mid-points with 5–95% ranges). [2.2.3, 2.3.1.1, 2.3.3.3, Figure 2.34]

This shows the relationship between CO₂ and temperature very clearly.

BUT, (1) the current warming cycle is clearly not like the ones before it, and (2) these are long run relationships.

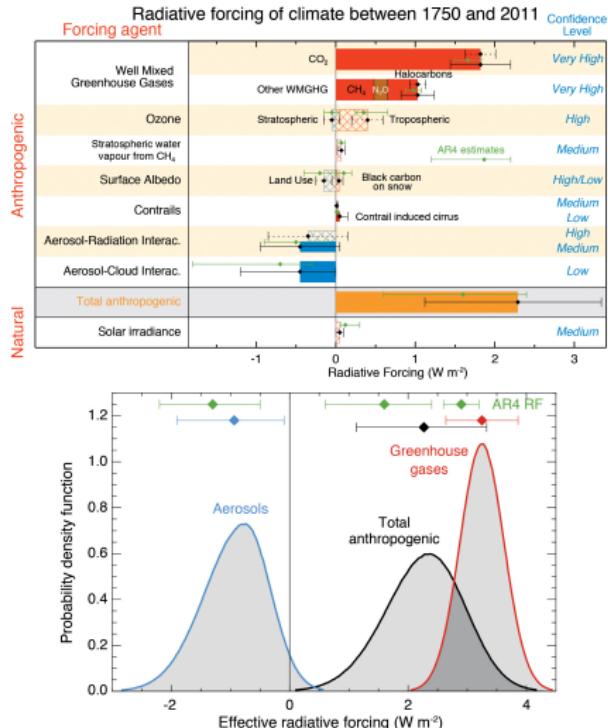
CO₂ and climate forcing I

The other way to guess at the total effect of CO₂ and complicating factors is to try to add them all up in a model. The effect of different forces on climate is measured in Watts/m². (To compare different heat sources, climate scientists measure ‘climate forcing’ in Watts/m².)

- baseline solar radiation \approx 240 Watts/m².
- CO₂ \approx +1.82 Watts/m², CH₄ \approx +0.48 Watts/m², N₂O \approx +0.17 Watts/m²
- changes in solar radiation \approx +0.12 Watts/m²
- aerosols \approx -1.0 Watts/m²
- reflectivity from buildings \approx -0.2 Watts/m²

Others: airplane contrails, Montreal Protocol gases, volcanic eruptions, changes to water vapor content of air, black carbon.

CO₂ and climate forcing II



IPCC Physical Science basis 2013, Fig TS.06

Models of CO₂ and climate I

If we want to think about what will happen to the climate, rather than just account for change in heat, then we need to worry about feedbacks. For example:

- As polar ice melts, reflectivity decreases.
- As climate warms, atmospheric water vapor increases (water vapor is a GHG), tundra melts and gives off CH₄, and methane hydrates give off CH₄.
- As temperature changes weather changes, and with it cloud cover. Clouds reflect sunlight and trap heat.
- How does the ocean absorb heat? Will this change as the climate warms? (This seems to be a very hard problem).

There are LOTS of others (I think).

Models of CO₂ and climate II

To learn the effect of forcings and feedback on climate we need to do something similar, that is, make a model of the climate in which all of the different forces can act and see what happens.

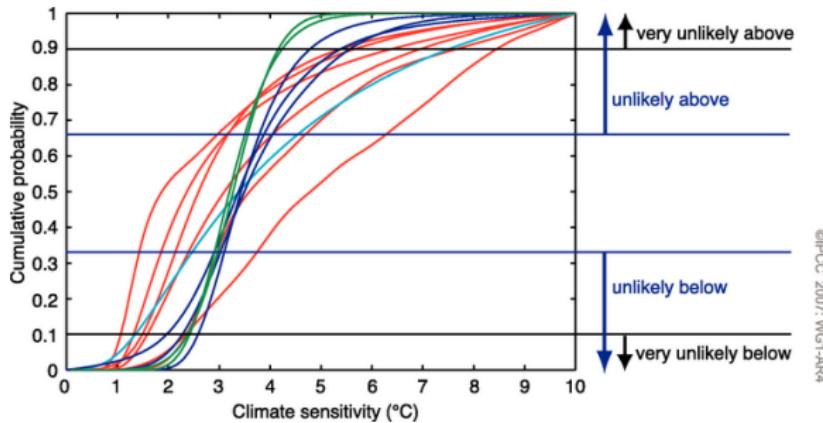
- This is going to be much harder than just adding up all of the forcings.
- There is going to be LOTS of uncertainty around model predictions.

Five minutes on climate models I

Suppose atmospheric CO₂ stabilizes at 550ppm. (Recall: 2022 level is 420, growth rate about 2ppm/year.) What is our best guess at ultimate equilibrium temperature based on:

- Extrapolation from 20th century measured temperature and CO₂ (red)
- AOGCM's Atmosphere Ocean General Circulation Models
- Extrapolation from long run proxy data (light blue).
- Climate models(green).

Five minutes on climate models II



Baseline is pre-industrial climate, about 0.7 degrees Celsius cooler than current. Light blue line (historical proxy data), if CO₂ stabilizes at 550 ppm the probability of an ultimate equilibrium climate 7 degrees Celsius warmer than pre-industrial norm is about 0.1.

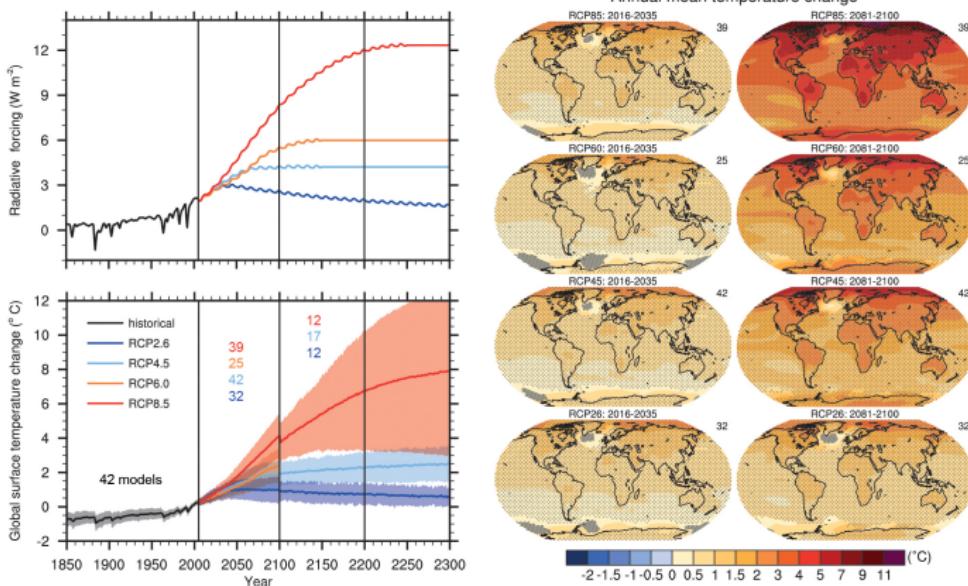
IPCC Physical Science basis 2007 Figure TS.25

Five minutes on climate models III

Issues:

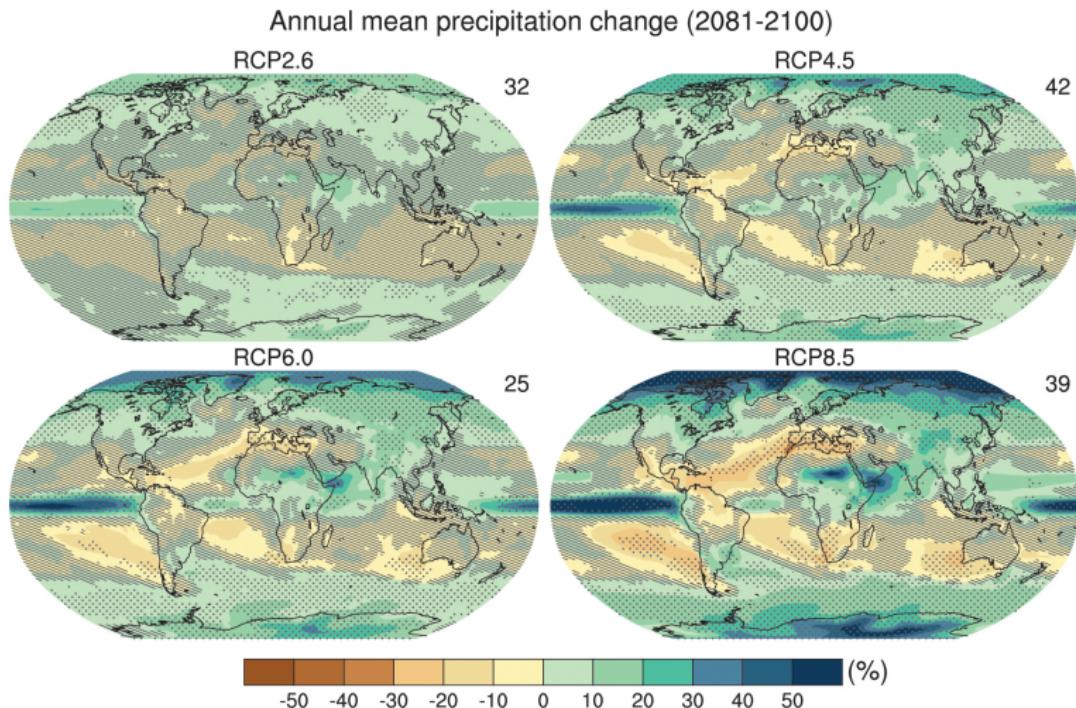
- How to choose between models? If each is equally likely then resulting uncertainty is much larger
- These types of models, by construction, underestimate uncertainty.
- Remember, these models are trying to do something really hard, they're probably not very accurate. It looks like they may predict too much warming, they failed to predict the warming hiatus around 2000.

Projected temperature from Fifth Assessment Report

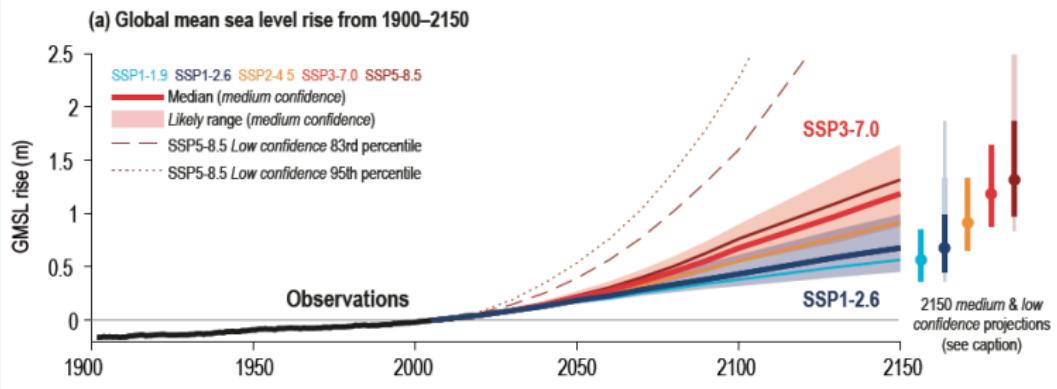


This does not move much in AR6 (more later).

Projected rainfall from Fifth Assessment Report



Projected sea level rise from Sixth Assessment Report



NB: (1) Really scary sea level rise is probably not in the cards over this time frame. (2) Figure reports ‘confidence intervals’ based on ‘structured expert judgement’. (IPPC 2022 AR6 WGI figure TS4.)

Stern Report steady state climate projections I

The Stern report also summarizes the output of a bunch of climate models, in what is probably a more useful way (for our purposes).

The following table relates atmospheric concentrations of CO₂ to the PROBABILITY that climate stabilizes at a particular level.

NB: We're ignoring the 2-300 year adjustment period, which is when most of us will live.

Stern Report steady state climate projections II

This table provides an indicative range of likelihoods of exceeding a certain temperature change (at equilibrium) for a given stabilisation level (measured in CO₂ equivalent). For example, for a stock of greenhouse gases stabilised at 550 ppm CO₂e, recent studies suggest a 63 - 99 % chance of exceeding a warming of 2°C relative to the pre-industrial.

The data shown is based on the analyses presented in Meinshausen (2006), which brings together climate sensitivity distributions from eleven recent studies (chapter 1). Here, the 'maximum' and 'minimum' columns give the maximum and minimum chance of exceeding a level of temperature increase across all eleven recent studies. The 'Hadley Centre' and 'IPCC TAR 2001' columns are based on Murphy *et al.* (2004) and Wigley and Raper (2001), respectively. These results lie close to the centre of the range of studies (Box 1.2). The 'IPCC TAR 2001' results reflect climate sensitivities of the seven coupled ocean-atmosphere climate models used in the IPCC TAR. The individual values should be treated as approximate.

The red shading indicates a 60 per cent chance of exceeding the temperature level; the amber shading a 40 per cent chance; yellow shading a 10 per cent chance; and the green shading a less than a 10 per cent chance.

Stabilisation Level (CO ₂ e)	Maximum	Hadley Centre Ensemble	IPCC TAR 2001 Ensemble	Minimum
Probability of exceeding 2°C (relative to pre-industrial levels)				
400	57%	33%	13%	8%
450	78%	78%	38%	26%
500	96%	96%	61%	48%
550	99%	99%	77%	63%
650	100%	100%	92%	82%
750	100%	100%	97%	90%
Probability of exceeding 3°C (relative to pre-industrial levels)				
400	34%	3%	1%	1%
450	50%	18%	6%	4%
500	61%	44%	18%	11%
550	69%	69%	32%	21%
650	94%	94%	57%	44%
750	99%	99%	74%	60%
Probability of exceeding 4°C (relative to pre-industrial levels)				
400	17%	1%	0%	0%
450	34%	3%	1%	0%
500	45%	11%	4%	2%
550	53%	24%	9%	6%
650	66%	58%	25%	16%
750	82%	82%	41%	29%
Probability of exceeding 5°C (relative to pre-industrial levels)				
400	3%	0%	0%	0%
450	21%	1%	0%	0%
500	32%	3%	1%	0%
550	41%	7%	2%	1%
650	53%	24%	9%	5%
750	62%	47%	19%	11%

Nordhaus' rule of thumb

Doubling CO₂ from preindustrial level (280ppm) to 560ppm give a temperature increase of about 3 degrees Celsius above pre-industrial level.^{e.g. Nordhaus p54.}

From our model of the climate change problem,

$$T_2 = \rho_1(P_2 - P_1) + T_1 \quad (7)$$

or

$$\rho_1 = \frac{T_2 - T_1}{P_2 - P_1} \quad (8)$$

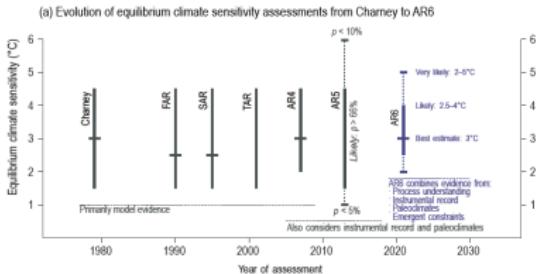
$$= \frac{3}{280 \text{ ppm CO}_2} \frac{\text{Degrees}}{\text{ppm CO}_2} \quad (9)$$

Nordhaus' rule of thumb

ρ is also called ‘climate sensitivity’.

Climate sensitivity is degrees of warming over 100 years that follows from doubling pre-industrial CO₂ concentration.

Interestingly, estimates of ρ have hardly moved over the last 30 years. Here are estimates from all IPCC reports:



(IPCC AR6 WG1 TS 16.)

Summary: CO₂ and climate

Nordhaus, Stern report and IPCC climate projections give us the relationship between atmospheric CO₂ and climate.

- Nordhaus' rule of thumb fits our framework neatly.
- Stern and IPCC are more complicated because time is not just two periods, climate is not just temperature, and climate will not change the same way in all places.
- Other aspects of our environment will change as a consequence of higher CO₂ levels and warming:
 - Sea level will rise
 - Ocean acidification
 - Ecological changes/ species extinctions
 - Changes to regional weather, e.g., monsoon patterns, heat spells, droughts, extreme weather.

Summary of progress

Recall BDICE :

$$\max_{I,M} u(c_1, c_2) \quad (10)$$

$$\text{s.t. } W = c_1 + I + M \quad (11)$$

$$c_2 = (1+r)I - \gamma(T_2 - T_1)I \quad (12)$$

$$E = (1 - \rho_4 \frac{M}{W})(\rho_5(c_1 + I)) \quad (13)$$

$$P_2 = \rho_0 E + P_1 \quad (14)$$

$$T_2 = \rho_1(P_2 - P_1) + T_1 \quad (15)$$

- We can now fill in the last, ‘climate model’ constraint.
- $\rho = 3/280$, P_1 is 280 ppm CO₂ (pre-industrial), P_2 is whatever we choose, T_1 is pre-industrial climate.

What is ‘Global Warming Potential’? I

- We would like to calculate the rate at which we are willing to trade CH₄ for CO₂. To do this, define α_t as the change in radiative forcing from one unit of concentration of the gas at t , i.e. $\frac{W}{m^2 \text{PPM}}$. Define GWP as warming potential relative to CO₂,

$$GWP(\text{CH}_4) = \frac{\sum_{t=0}^3 \alpha^{\text{CH}_4}(t) dt}{\sum_{t=0}^3 \alpha^{\text{CO}_2}(t) dt}$$

- Example, suppose $t = 1, 2, 3$ and for CO₂, $(\alpha^{\text{CO}_2}(1), \alpha^{\text{CO}_2}(2), \alpha^{\text{CO}_2}(3)) = (1, 1, 1)$ and for CH₄, $(\alpha^{\text{CH}_4}(1), \alpha^{\text{CH}_4}(2), \alpha^{\text{CH}_4}(3)) = (2, 0, 0)$.
- Then $GWP(\text{CH}_4) = \frac{2+0+0}{1+1+1} = \frac{2}{3}$

What is ‘Global Warming Potential’? II

Issues:

- T is completely arbitrary. GWP of CH_4 for $T = 20, 100, 500$ is about 63, 21, 9.
- We care about damage, not radiative forcing (see Schmalensee, The Energy Journal (1993)) . To see this consider,
 - If we anticipate that the world will end in 20 years, then neither CO_2 nor CH_4 is very harmful, but CH_4 causes a lot more warming over that time than CO_2
 - If we plan to spend the next 20 years as we do now, but then to permanently convert to an economy based on penguin farming, then CH_4 emissions now are not very harmful, but CO_2 is (because it is persistent).

What is ‘Global Warming Potential’? III

- We can't really assign relative values to CO_2 and CH_4 until we solve for the whole optimal path of emissions for both gases, so any definition of GWP is going to be unsatisfactory.
- This means that CO_2e is a pretty suspicious concept.

Conclusion I

- We posed the problem of climate change as a simple optimization problem, ‘the Baby Dynamic Integrated Climate Economy model’. To solve this problem we need to understand our endowment of climate and carbon, and how they are related. We’ve started on this today.
- Climate:
 - Measured and Icecore data show increases in temperature over past 650,000+years.
 - Climate is complicated. Measured and proxy data show other changes in climate; rainfall, sea level, ocean acidity, ice extent, etc.
- CO₂ :
- Atmospheric CO₂ concentrations are very high relative to their levels over the past 6-800k years.

Conclusion II

- measured CO_2 has increased monotonically to 421 ppm at Mauna Loa observatory (July 2022).
- ice core record suggests we are at or above atmospheric CO_2 concentrations observed over past 6-800k years.
- Climate and CO_2
 - Visible in icecore data
 - The physics relating atmospheric CO_2 to warming is elementary and uncontroversial. But there are many complications; aerosols, heat transmission, feedbacks.
 - Stern, IPCC and Nordhaus all make climate projections, we'll mostly use Nordhaus just because it is simple.