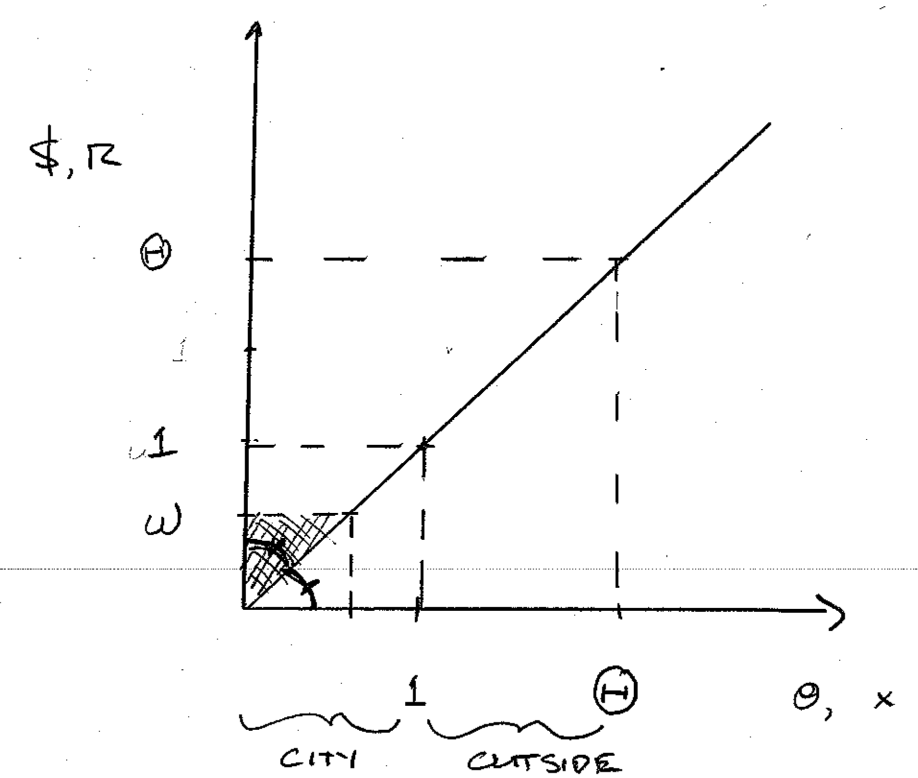
(1) FIRST NOTE THAT AN AGENT WITH A LOW θ WILL ALWAYS OUT-BID AN AGENT WITH A HIGH θ FOR A SPOT IN THE CITY.

THEN, EQUILIBRIUM LOOKS LIKE THIS:



SUPPOSE $\Theta \geq 1 > \omega$ AS DRAWN. THEN IF $R = 0$ AND

$\theta \leq \omega$ LIVE IN THE CITY AND $\theta > \omega$ LIVE OUTSIDE, THEN NO ONE WANTS TO MOVE, AND NO LANDLORD CAN INCREASE THEIR RENT. THIS IS A FREE-MOBILITY EQUILIBRIUM WITH THE CITY PARTLY OCCUPIED. AGGREGATE LAND RENT IS ZERO AND CONSUMERS' SURPLUS IS THE HATCHED REGION,

$$\int_0^{\omega} (\omega - \theta)\, d\theta$$

IF $\Theta \geq \omega > 1$ THEN THE CITY IS FULLY OCCUPIED BY $\{\theta \mid 0 \leq \theta \leq 1\}$. THE MARGINAL RESIDENT IS $\theta = 1$. RENT ADJUSTS

SO THAT SHE IS INDIFFERENT BETWEEN THE CITY AND THE OUTSIDE OPTION, SO $R = W-1$. IN THIS CASE, NO ONE WANTS TO MOVE AND LANDLORDS CAN'T RAISE THEIR RENT.

AGGREGATE LAND RENT IS $\int_0^1 (W-1)dx = W-1$ AND

AGGREGATE CONSUMERS' SURPLUS IS $\int_0^1 ((W-1)-\theta)d\theta$.

IN AN ENVIRONMENT WHERE AGENTS HAVE HETEROGENEOUS OUTSIDE OPTIONS <u>OR</u> WHERE THEIR TASTE FOR A LOCATION, WE NEED TO WORRY ABOUT SURPLUS FOR INFRA-MARGINAL AGENTS WHEN WE CALCULATE WELFARE.

IN THE LINEAR CITY MODEL, AGENTS ARE HOMOGENEOUS AND THIS ISSUE DOES NOT ARISE.

2 FOR LINEAR CITY W/O SUBWAY, BID RENT SOLVES

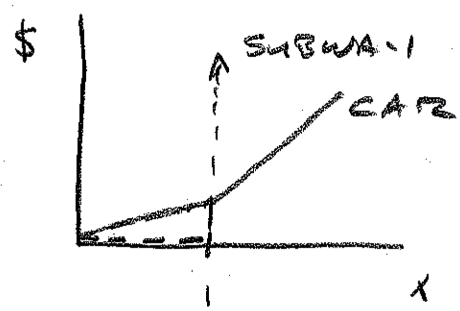
$$\max_{r,x} \; w - tx - r$$

$$(w - tx - r)^{\beta} \geq \bar{u}$$

$$\Rightarrow \quad w - tx - r = \bar{u}^{1/\beta}$$

$$\Rightarrow \quad R_0(x) = w - tx - \bar{u}^{1/\beta}$$

FOR LINEAR CITY WITH SUBWAYS, COMMUTE COSTS LOOK LIKE THIS



THE BID RENT FOR SUBWAY COMMUTERS IS

$$R^S(x) = \begin{cases} w - \bar{u}^{1/\beta} & \\ 0 & x > 1 \end{cases}$$

AND FOR DRIVERS

$$R^d(x) = \begin{cases} w - \bar{u}^{1/\beta} - \frac{t}{2}x & x \leq 1 \\ w - \bar{u}^{1/\beta} - \frac{t}{2} - t(x-1) & x > 1 \end{cases}$$

THE UPPER ENVELOPE, $\max\{R^S, R^d\}$, IS

$$R_1(x) = \begin{cases} w - \bar{u}^{1/\beta} & x \leq 1 \\ w - \bar{u}^{1/\beta} - t/2 - t(x-1) & x > 1 \end{cases}$$

NOTE THAT R_1 IS DISCONTINUOUS AT 1.

EQUILIBRIUM CITY SIZE W/O SUBWAYS SATISFIES

$$R_0(\bar{x}_0) = 0 \Rightarrow 0 = w - t\bar{x}_0 - \bar{u}^{1/\beta}$$

$$\Rightarrow \bar{x}_0 = \frac{w - \bar{u}^{1/\beta}}{t}$$

EQUILIBRIUM CITY SIZE WITH SUBWAYS IS DETERMINED BY MARGINAL DRIVER

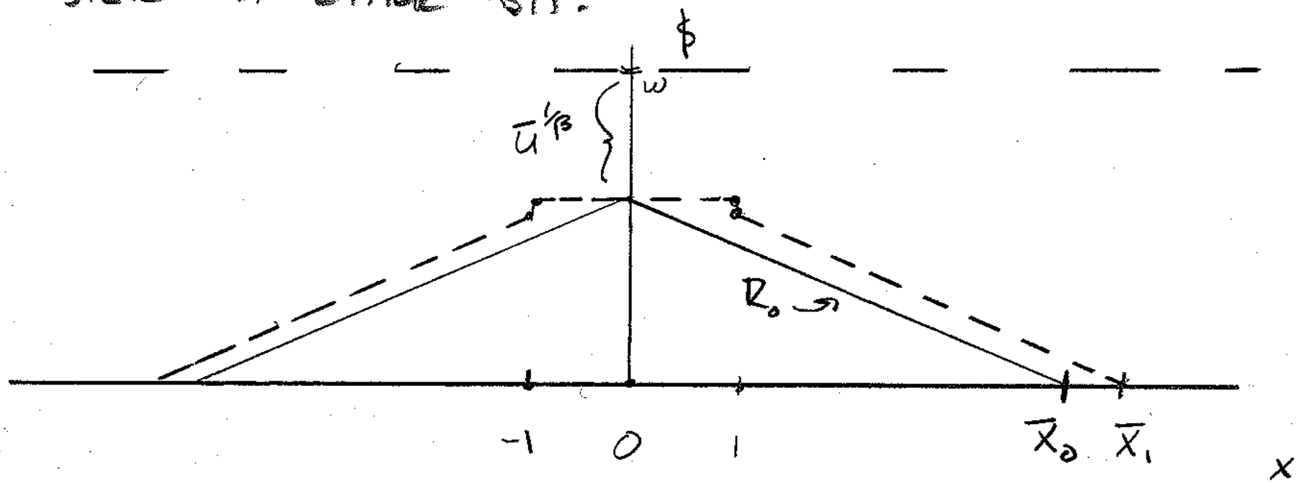
$$R_1(\bar{x}_1) = 0 \Rightarrow w - \bar{u}^{1/\beta} - t/2 - t(\bar{x}_1 - 1) = 0$$

$$\Rightarrow \frac{w - \bar{u}^{1/\beta} - t/2}{t} + 1 = \bar{x}_1$$

$$\Rightarrow \frac{w - \bar{u}^{1/\beta}}{t} + \frac{1}{2} = \bar{x}_1 = \bar{x}_0 + 1/2$$

SO $\bar{X}_1 = \bar{x}_0 + 1/2$, SO SUBWAYS INCREASE CITY SIZE A LITTLE BIT.

(2)



(3) TO MAKE THIS EASY, ASSUME ONE WAY COMMUTING.

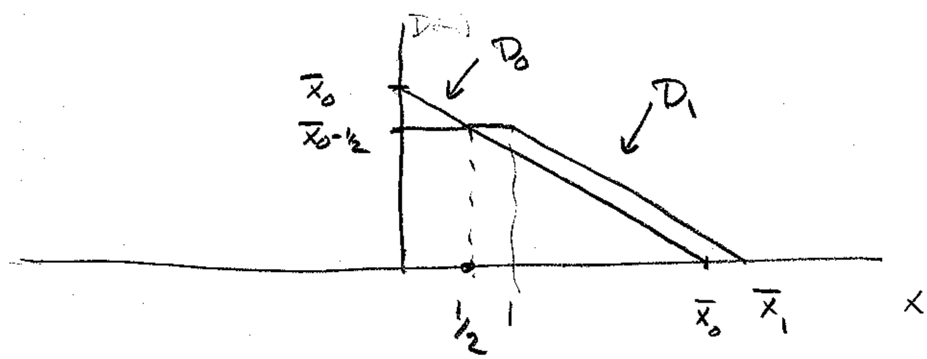
WHEN EVERYONE DRIVES, THE NUMBER OF CARS PAST x IS

$$D_0(x) = \begin{cases} \bar{x}_0 - x & x \in [0, \bar{x}_0] \\ -[\bar{x}_0 - x] & x \in [0, -\bar{x}_0] \\ 0 & \text{ELSE} \end{cases}$$

WITH SUBWAYS, NONE OF THE COMMUTERS IN $x \in [-1, 1]$ DRIVE, SO WE HAVE

$$D_1(x) = \begin{cases} \bar{x}_1 - 1 & x \in [-1, 1] \\ (\bar{x}_1 - 1) - x & x \geq 1 \\ -(\bar{x}_1 + 1 - x) & x < -1 \\ 0 & \text{ELSE} \end{cases}$$

OR



WHERE $D_0(x) = D_1(x)$ WHEN $\bar{x}_0 - x = \bar{x}_1 - 1$

$$\Rightarrow \bar{x}_0 - x = \bar{x}_0 + \tfrac{1}{2} - 1$$

$$\Rightarrow x = \tfrac{1}{2}$$

WITHOUT SUBWAY, TOTAL DRIVING IN CENTRAL CITY

i.e. SUBWAY CATCHMENT IS

$$2\int_0^1 D_0(x)\,dx = 2\int_0^1 \bar{x}_0 - x\,dx = 2\left[\bar{x}_0 x - \tfrac{1}{2}x^2\right]_0^1$$
$$= 2\left[\bar{x}_0 - \tfrac{1}{2}\right]$$

TOTAL DRIVING IN THE WHOLE CITY IS

$$2\int_0^{\bar{x}_0} D_0(x)\,dx = 2\int_0^{\bar{x}_0} = \bar{x}_0^2$$

WITH SUBWAY, TOTAL DRIVING IN THE CENTRAL CITY IS,

$$2\int_0^1 D_1(x)\,dx = 2\left[\bar{x}_0 - 1/2\right]$$

IN THE WHOLE CITY

$$2\left[\bar{x}_0 - \tfrac{1}{2}\right] + \int_1^{\bar{x}_0 + 1/2} (\bar{x}_0 - 1/2 - x)\,dx = \text{(BY INSPECTION)}$$
$$= 2\left[\tfrac{1}{2}\bar{x}_1^2 - \tfrac{1}{2}\right] = \left[(\bar{x}_0 + \tfrac{1}{2})^2 - 1\right]$$

SO AS LONG AS $\bar{x}_0$ IS BIG ENOUGH, TOTAL DRIVING INCREASES WITH SUBWAY. BUT DRIVING IN THE CENTRAL PART DECREASES. AS LONG AS POLLUTION IS LOCAL, THIS RATIONALIZES DURANTON + TURNER AER 2011 AND CHEN AND WHALLEY AEJ 2012.

[a] MAX $h^{\alpha} z^{1-\alpha}$

S.T. $ph + z = \omega - \tau x$

$$\Rightarrow \quad z(p) = (1-\alpha)(\omega - \tau x)$$

$$h(p) = \frac{\alpha}{p}(\omega - \tau x)$$

WITH FREE MOBILITY

$$[h(p)]^{\alpha}[z(p)]^{1-\alpha} = \underline{u}$$

$$\Rightarrow \quad p(x) = \left[\frac{(\omega - \tau x)\alpha^{\alpha}(1-\alpha)^{1-\alpha}}{\underline{u}}\right]^{1/\alpha}$$

[b] SEE DURANTON + PUGA HANDBOOK p8

[c] $\frac{d}{dx} e(p, \underline{u}) = \frac{\partial e}{\partial p} \cdot \frac{\partial p}{\partial x} = -\tau$

SINCE $\frac{d}{dx}(\omega - \tau x) = -\tau$

BUT $\frac{\partial e}{\partial p} = h$ SO $\frac{\partial p}{\partial x} = \frac{-\tau}{h}$