EC1410-Spring 2024
Problem Set 4 solutions

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Matt Turner

1. This problem will use the information presented in the slides to estimate urban and rural incomes in 1820 and 2000.

   (a) First, use the figures based on Bolt and Van Zanden (2014) to determine real per capita GDP in 1820 and 2000, in constant 2011 dollars.

   Based on the figure entitled "US GDP from 1800 to 2016", real per capita GDP in 2011 dollars was around \$2,000 in 1820 and \$43,000 in 2000.

   (b) Then, use the figures in Boustan et al. (2013) to find the share of the population in urban areas, and the urban wage premium, in both 1820 and 2000.

   Based on Figure 1 in Boustan et al. (2013), the share of the population in urban areas was around 8% in 1820 and 79% in 2000.

   Based on Figure 3 in Boustan et al. (2013), the urban wage premium was around 21% in 1820 and 39% in 2000.

   (c) Combine the information you have collected above to estimate urban and rural incomes in both 1820 and 2000 (hint: GDP per capita is a weighted average of wages in rural and urban areas).

   In 1820, real GDP = rural population share * rural wage + urban population share * urban wage

$$\begin{aligned} \$2{,}000 &= 0.92x + 0.08(1.21x) \\ x &= \$1{,}967 \text{ is the average rural income} \\ 1.21x &= \$2{,}380 \text{ is the average urban income} \end{aligned}$$

   Similarly for 2000:

$$\begin{aligned} \$43{,}000 &= 0.21x + 0.79(1.39x) \\ x &= \$32{,}872 \text{ is the average rural income} \\ 1.39x &= \$45{,}692 \text{ is the average urban income} \end{aligned}$$

2. In this problem, we will combine assumptions about urban and rural amenities with the income figures you produced above to estimate urban population growth between 1820 and 2000.

   Denote urban and rural amenities by A_U and A_R, respectively. Let $c_R = w_R$. Let $u(Ac) = \ln(Ac - 1)$. Finally, let A_R/A_U be proportional to the ratio of rural to urban death rates (you can assume that the ratio in 1820 is the same as it is in 1870, and that in 2000 this ratio is 1).

   (a) In a spatial equilibrium, $u(A_R c_R) = u(A_U c_U) = \overline{u}$. Assuming we are in a spatial equilibrium, write an expression for c_U in terms of rural wages and the ratio of urban to rural amenities.

$$
\begin{aligned}
u(A_R c_R) &= u(A_U c_U) \\
\ln(A_R c_R - 1) &= \ln(A_U c_U - 1) \\
A_R c_R - 1 &= A_U c_U - 1 \\
A_R c_R &= A_U c_U \\
A_R w_R &= A_U c_U \\
c_U &= \frac{A_R}{A_U} w_R
\end{aligned}
$$

(b) Write down the household's problem for the household living at $\overline{x}$ (you do not need to solve it).

$$\max_{c_U} \ln(A_U c_U - 1) \text{ subject to } w_U = c_U + 2t\overline{x} + \overline{R}\overline{l}$$

(c) Assuming $\overline{R} = 0$, solve the constraint in the above problem to get an expression for $\overline{x}$.

$$
\begin{aligned}
w_U &= c_U + 2t\overline{x} + \overline{R}\overline{l} \\
w_U &= c_U + 2t\overline{x} \\
\overline{x} &= \frac{w_U - c_U}{2t}
\end{aligned}
$$

(d) Recall that the city extends from $-\overline{x}$ to $\overline{x}$, and that each household consumes an exogenous amount of land $\overline{l}$. This means that the city population is given by $N^* = \frac{2\overline{x}}{\overline{l}}$.

Write an expression for N^* based on your expression for $\overline{x}$.

$$
\begin{aligned}
N^* &= \frac{2\overline{x}}{\overline{l}} \\
\overline{x} &= \frac{w_U - c_U}{2t} \\
N^* &= \frac{2}{\overline{l}} \frac{w_U - c_U}{2t} = \frac{w_U - c_U}{t\overline{l}}
\end{aligned}
$$

(e) Assume that t in 2000 is 1/2 of what it was in 1820, and that $\overline{l}$ is constant over time. Use the information about wages from the previous problem, as well what we know about the ratio of rural to urban amenities, to write expressions for N^* in 1820 and 2000.

For 1820, recall $\frac{A_R}{A_U}$ is proportional to the ratio of rural to urban

crude death rates in 1870, which was $(1.38)^{-1}$. This follow from the fact that the urban to rural crude death rate is 1.38 in 1870. Thus, we have:

$$\begin{aligned}
N^*_{1820} &= \frac{w_{U,1820} - c_{U,1820}}{t_{1820}\bar{l}} \\
c_{U,1820} &= \frac{A_{R,1820}}{A_{U1820}} w_{R,1820} \\
&= (1.38)^{-1} w_{R,1820} \\
&= (1.38)^{-1} * 1967 \\
N^*_{1820} &= \frac{2380 - (1.38)^{-1} * 1967}{t_{1820}\bar{l}}
\end{aligned}$$

For 2000, $\frac{A_R}{A_U}$=1:

$$\begin{aligned}
N^*_{2000} &= \frac{w_{U,2000} - c_{U,2000}}{t_{2000}\bar{l}} \\
c_{U,2000} &= \frac{A_{R,2000}}{A_{U2000}} w_{R,2000} \\
&= 1 * w_{R,2000} \\
&= 32872 \\
N^*_{2000} &= \frac{45692 - 32872}{t_{2000}\bar{l}} \\
&= \frac{45692 - 32872}{0.5 * t_{1820}\bar{l}}
\end{aligned}$$

(f) How much does your model imply the urban population grew between 1820 and 2000? That is, what is N^*_{2000}/N^*_{1820}?

$$\begin{aligned}
N^*_{2000}/N^*_{1820} &= \frac{45692 - 32872}{0.5 * t_{1820}\bar{l}} \div \frac{2380 - (1.38)^{-1} * 1967}{t_{1820}\bar{l}} \\
&= \frac{2 * (45692 - 32872)}{2380 - (1.38)^{-1} * 1967} \\
&= 26.86
\end{aligned}$$

(g) How does the urban population growth you computed above compare with the actual growth in the urban population over that time period? (For your reference, according to the Census, the US population in 1820 was 9,638,453 and in 2000 was 281,421,906). Why do you think there is a discrepancy?

Eyeballing figure 1 of Boustan et al. (2013), the urban shares in 1820 was about 10% and in 2000, about 80%. Together with the data given in the problem, that gives urban populations in the two years of about 900,000 and 224,000,000. It follows that the urban US population grew by a factor of $\frac{224{,}000{,}000}{900{,}000} \approx 270$.

Our model predicts a very large increase in the urban population, though it misses the exact number by a lot too. To me, this seems pretty good for such a simple model. If we wanted to try to get the model to fit more precisely, we would want to start by using more accurate data, in particular for transportation costs. We would also want to worry about the fact that we have a model of a single city that we are using to predict the share of people in all cities. Thus, we are ignoring the fact that urban populatin can increase because there are more cities, as well as because existing cities get bigger.