EC1410-Spring 2025
Problem Set 6 solutions

(Updated 29 October 2024)

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1. In this problem, we will work through an example of the discrete choice model with heterogeneous agents. Consider a discrete linear city with three neighborhoods $i \in \{1,2,3\}$. Let x_i denote a neighborhood's distance from the CBD, with x_1 = 1, x_2 = 2, x_3 = 3. The cost to commute one unit distance is τ. The city is populated by households indexed by j. Each household chooses a neighborhood i, pays land rent R_i, and commutes to the center, at location 0, to earn wage w. A household's utility is $V_{ij} = A_i \cdot c_i z_{ij}$ where $A_i = i$ is the amenity value in location i, c_i is consumption and z_{ij} is the household and location specific valuation. All z_{ij} are drawn from a Frechet distribution, $F(z) = e^{-Tz^{-\epsilon}}$.

   (a) Let consumption be $c_i = w - R_i + i\tau$. Set up the household's problem.

   $$V_{ij} = A_i.z_{ij} \cdot c_i$$

   Using $c_i = w - R_i + i\tau$ and $A_i = i$,

   $$V_{ij} = i \cdot z_{ij} \cdot (w - R_i - i\tau)$$

   Therefore, the household's problem is to choose a discrete location i that maximizes its utility.

   $$\max \{V_{1j}, V_{2j}, V_{3j}\}$$

   (b) Using the big theorem from the lecture, solve for the share of household s_i in each location.

   $$s_i = \frac{[i \cdot (w - R_i - i\tau)]^{\epsilon}}{\sum_1^{k=3}[k \cdot (w - R_k - k\tau)]^{\epsilon}}$$

   (c) Let the share of households in each location $s_1 = s_2 = s_3 = \frac{1}{3}$, wage $w = 5$ and the price of agricultural land $\bar{R} = 1$. Assume that the land rent at $x = 3$ is equal to $\bar{R}$. Solve for R_1, R_2 and R_3 in terms of τ.

   Setting $s_1 = s_2 = s_3 = \frac{1}{3}$ we get the following three equations:

   $$\begin{aligned} s_1 = s_2 \implies w - R_1 - \tau &= 2(w - R_2 - 2\tau) \\ 2R_2 - R_1 &= w - 3\tau \end{aligned}$$

   $$\begin{aligned} s_2 = s_3 \implies 2(w - R_2 - 2\tau) &= 3(w - R_3 - 3\tau) \\ 2R_2 - 3R_3 &= 5\tau - w \end{aligned}$$

$$s_1 = s_3 \implies w - R_1 - \tau = 3(w - R_3 - 3\tau)$$
$$3R_3 - R_1 = 2w - 8\tau$$

Plugging in $w = 5$ and $R_3 = \bar{R} = 1$, we get:

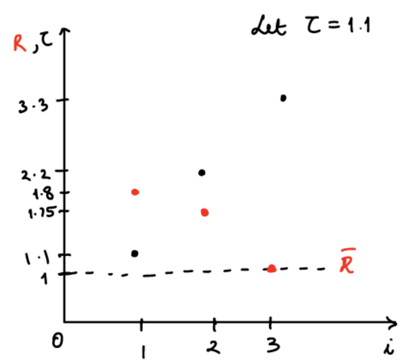
$$R_1 = 8\tau - 7$$
$$R_2 = 2.5\tau - 1$$
$$R_3 = 1$$

(d) Solve for consumption in terms of τ.

Substituting R_i from part c and w in $c_i = w - R_i + i\tau$

$$c_1 = 12 - 9\tau$$
$$c_2 = 6 - 4.5\tau$$
$$c_3 = 4 - 3\tau$$

(e) Plot land rent and commuting costs as a function of i. How does this compare to the monocentric city model with a continuum of locations?



In a monocentric city model with a continuum of locations, for each unit decrease in commute costs, land rent increases by exactly the same amount. This is not always the case in a discrete space model with heterogeneous agents. Panel A above shows that the land rent declines less than the increase in commuting costs, given $\tau > 1$.

(f) Do all households at location i have the same utility? What does this suggest about the usefulness of R to measure welfare?

All households in location i do not have the same utility since the utility form contains z_{ij}, which is a household and location specific valuation. This makes evaluating welfare much more complicated in these models. Research using these models often disregards land rent altogether in welfare calculations.