EC2410-Spring 2019
Problem Set 4

(Updated 4 April 2019)

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When you write up your answers, your goals should be to (1) be correct, and (2) convince your reader that your answer is correct. It is always helpful if your work is legible and if all steps are presented, possibly with a line of explanation.

Answers which do not achieve these goals will not be awarded full credit.

Problems

1. This problem asks you to work through the details of equilibrium in an economy with monopolistic competition and differentiated products, the hardware used for several of the papers we'll read.

   Let T denote a tradition good. This is a composite good with perfectly elastic supply (e.g., a commodity available on an international market) that sells for price p^T. Let M denote the measure of a set of differentiated products indexed by i. Specifically, let $q(i)$ denote a quantity consumed of good of type i for $i \in [0,M]$, let $x(i)$ denote the quantity of this good produced and, finally, let $p(i)$ denote the price of variety i.

   Let

   $$U = \frac{Q^\mu T^{1-\mu}}{\mu^\mu (1-\mu)^{1-\mu}}$$

   bet the utility of a representative agent, where Q is a CES consumption aggregator

   $$Q = \left[\int_0^M q(i)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$$

   Production of the differentiated good is done by monopolistically competitive firms, each of which has the identical technology and produces exactly one variety (the possibility that firms could gain market power by controlling a segment of the product space is assumed away). The cost for firm i to produce $x(i)$ is

   $$C(x(i)) = A + Bx(i),$$

   so each firm has decreasing average costs.

   Suppose that there is free entry of firms so that firm profits are all zero in equilibrium, that consumers have exogenous wage w, and that there are measure N of consumers.

   (a) Find the equilibrium price and quantity of variety i.

   (b) Verify the constant mark-up rule, i.e., that profit maximizing firms charge a constant mark up over marginal cost in equilibrium.

   (c) Can you verify that utility goes up with N in this model?

2. Consider a world consisting of $N = 5$ people indexed by j, each of whom makes a binary choice from action set $A = \{U,R\}$, 'urban' or 'rural'. Agents choosing rural get a payoff $u_j = \bar{u} = 0$. Agents choosing urban get a payoff that depends on the number of other agents who also choose urban. Letting n denote the sum of agents choosing urban, this payoff is $u(n) = \min\{-4+3x,4-x\}$.

(a) In this game, a strategy for agent j, s_j is simply a choice of action. A strategy profile is a choice of strategy for all N agents. Characterize the set of pure strategy Nash equilibria of this game.

(b) In this game, a mixed strategy is a choice of $p_j \in [0,1]$, the probability of choosing urban. A mixed strategy profile is a choice of mixed strategy for all N agents. Give the condition that describes the set of mixed strategy equilibria. This should give you a 4th order polynomial in p. You don't have to solve it, but try to describe what the mixed equilibria might look like.

(c) What is the Pareto optimal outcome?

(d) Consider the problem above, but with two cities instead of one. In this case, the action set for an agent is $A = \{U_1,U_2,R\}$, where each city is just like the single city $u(n)$. Find all of the pure strategy Nash equilibrium configurations of this economy and also the Pareto optimal configuration.

(e) How would this problem change if, instead of two cities, we had three?