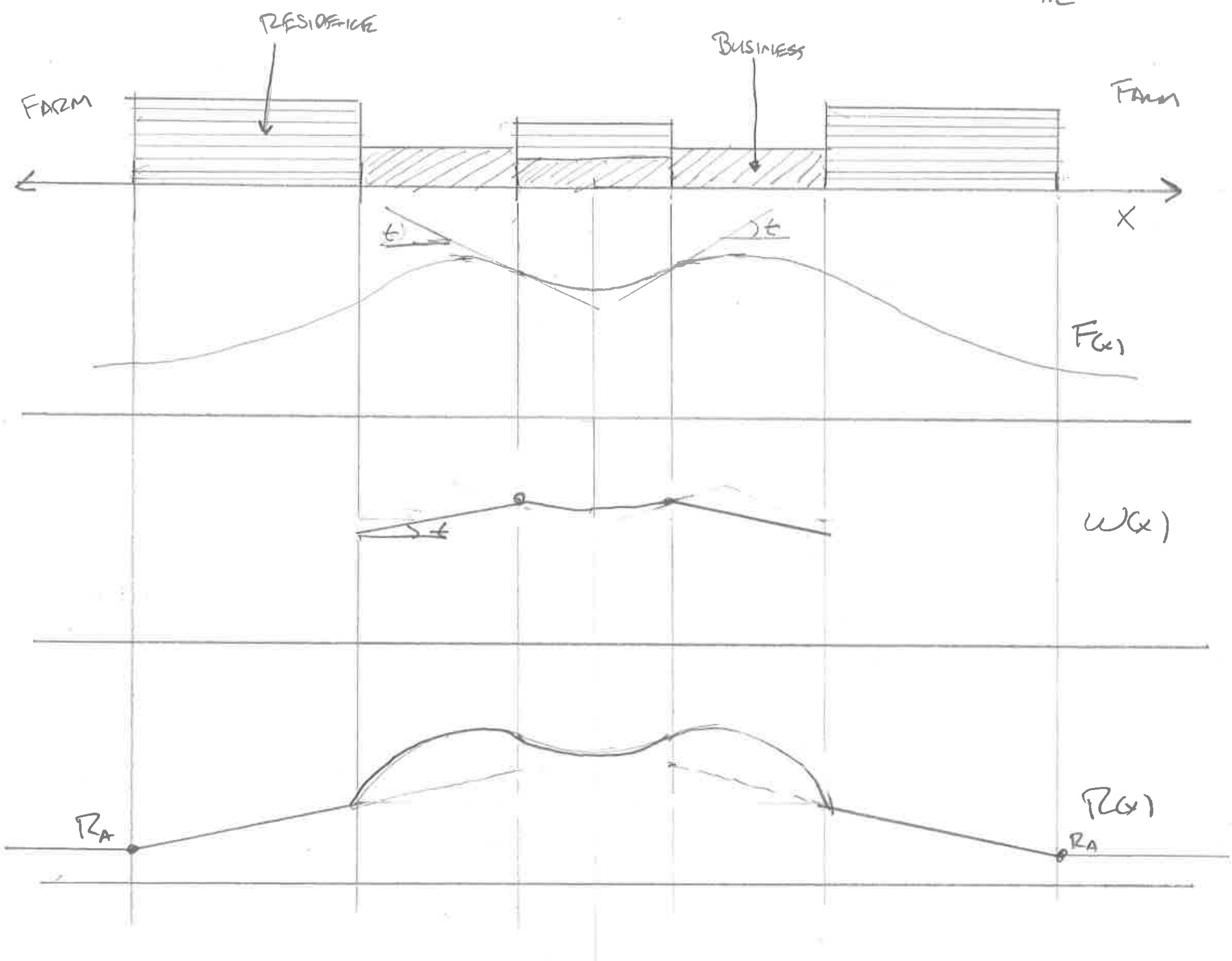
FUJITA + OGAWA PARTIALLY MIXED EQUIL



1. CENTER IS MIXED ⇒ WORK WHERE YOU LIVE
⇒ SLOPE OF WAGE GRADIENT LESS THAN t.

2. PEOPLE IN ADJACENT BUSINESS DISTRICTS COMMUTE FROM OUTSIDE ⇒ WAGE GRADIENT HAS SLOPE t AND RENT GRADIENT ALWAYS ABOVE tx.

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- $s \in [s_1, s_2]$ ~ AMENITY
  x ~ NUMERAIRE CONSUMPTION GOOD
  ℓ^c ~ RESIDENTIAL LAND
  k ~ UTILITY LEVEL
  w, I ~ WAGE, NON-WAGE INCOME
  r ~ LAND RENT

  N ~ LABOR = PEOPLE
  ℓ^p ~ PRODUCTION LAND
  $X = f(N, \ell^p; s)$
  ~ CRS.
  (1) $L = \ell^c + \ell^p$

- CONSUMERS SOLVE: $\max u(x, \ell^c; s)$ s.t. $x + r\ell^c = w + I$ $\Rightarrow$ (2) $V(w, r; s) = k$
  FREE-MOBILITY

- FIRMS ~~HAVE~~ UNIT COST FUNCTION

$$(3)\quad c(w, r; s) = 1$$

FREE ENTRY + $\pi = 0$ + $P_x \equiv 1$.

WITH CRS $\Rightarrow$ (4) $c_r = \frac{\ell^p}{X}$, (5) $c_w = N/X$

AN EQUILIBRIUM MUST SATISFY (1)–(5).

TOTAL DIF (2) + (3) $\Rightarrow$

$$V_w \frac{dw}{ds} + V_r \frac{dr}{ds} = +V_s \quad (6)$$

$$c_w \frac{dw}{ds} + c_r \frac{dr}{ds} = -c_s \quad (7)$$

SOLVE (6)+(7) FOR

$$\frac{dr}{ds} = \frac{-V_w c_s + V_s c_w}{V_w c_r - V_r c_w} \equiv \frac{V_w c_s - V_s c_w}{\Delta}$$

SOLVE (6)+(7) FOR

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(2)

USING (4) + (5) WE HAVE

$$\Delta = -V_r C_w + V_w C_r = -V_r \frac{N}{X} + V_w \frac{l^p}{X}$$

$$= V_w \left[-\frac{V_r}{V_w} \frac{N}{X} + \frac{l^p}{X} \right]$$

USING ROY'S IDENTITY,

$$= V_w \left[+l^c \frac{N}{X} + \frac{l^p}{X} \right]$$

$$= +V_w \left[\frac{l^c N + l^p}{X} \right]$$

$$= \frac{+V_w L}{X}$$

FROM USING CRS OF $C(\cdot)$ IN (7)

$$\Rightarrow \quad -C_s = \frac{N}{X} \frac{dw}{ds} + \frac{l^p}{X} \frac{dr}{ds}$$

$$\Rightarrow \quad C_s = -\left[\frac{N}{X} \frac{dw}{ds} + \frac{l^p}{X} \frac{dr}{ds} \right] \quad (8)$$

USING CRS OF $C(\cdot)$ IN (6)

$$\Rightarrow \quad \frac{dw}{ds} + \frac{V_r}{V_w} \frac{dr}{ds} = \frac{V_s}{V_w}$$

(USE ROY'S IDENTITY) $\left[\frac{V_r}{V_w} = -l^c \right]$

$$\Rightarrow \quad \frac{dw}{ds} = l^c \frac{dr}{ds} + \frac{V_s}{V_w}$$

$$\Rightarrow \quad \frac{dw}{ds} = l^c \left[\frac{-V_w C_s + V_s C_w}{\Delta} \right] + \frac{V_s}{V_w}$$

$$\Rightarrow \quad \frac{dw}{ds} = \frac{+X l^c}{V_w L} \left[-V_w C_s + V_s \frac{N}{X} \right] + \frac{V_s}{V_w}$$

→

$$\Rightarrow \quad \frac{dw}{ds} = \frac{-Xl^c}{L}\cdot c_s - \frac{Nl^c}{L}\cdot\frac{V_s}{V_w} + \frac{V_s}{V_w} \qquad (8)$$

$$P_s^* \equiv V_s/V_w \quad \Rightarrow \quad \frac{dw}{ds} = -\frac{Xl^c}{L}c_s - \frac{Nl^c}{L}P_s^* + P_s^*$$

$$\frac{dw}{ds} = -\frac{Xl^c}{L}c_s + \frac{L - Nl^c}{L}P_s^*$$

$$\frac{dw}{ds} = -\frac{Xl^c}{L}c_s + \frac{l^P}{L}P_s^* \qquad (9)$$

USING (8) TO SUBSTITUTE FOR c_s IN (9)

$$\frac{dw}{ds} = +\frac{Xl^c}{L}\left[c_w\frac{dw}{ds} + c_r\frac{dr}{ds}\right] + \frac{l^P}{L}P_s^*$$

USING CRS

$$= +\frac{Xl^c}{L}\left[\frac{N}{X}\frac{dw}{ds} + \frac{l^P}{X}\frac{dr}{ds}\right] + \frac{l^P}{L}P_s^*$$

$$\frac{l^P}{L}P_s^* = \frac{dw}{ds} - \frac{Xl^c}{L}\left[\frac{N}{X}\frac{dw}{ds} + \frac{l^P}{X}\frac{dr}{ds}\right]$$

$$\frac{l^P}{L}P_s^* = \frac{dw}{ds} - \frac{l^c N}{L}\frac{dw}{ds} - \frac{l^c l^P}{L}\frac{dr}{ds}$$

$$P_s^* = \left(\frac{L}{l^P} + \frac{l^c N}{l^P}\right)\frac{dw}{ds} - l^c\frac{dr}{ds}$$

$$\Rightarrow \quad \boxed{P_s^* = \frac{dw}{ds} - l^c\frac{dr}{ds}}$$