

EC1340-Fall 2023 Problem Set 1 solutions

(Updated 31 July 2023)

Matt Turner

1. (a)

$$\begin{aligned}
 T_2 - T_1 &= \rho_1(P_2 - P_1) \\
 &= \rho_1 \rho_0 E \\
 &= \rho_1 \rho_0 \left(\left(1 - \rho_4 \frac{M}{W}\right) (\rho_5 (c_1 + s)) \right) \\
 &= \rho_1 \rho_0 \left(1 - \rho_4 \frac{M}{W}\right) \rho_5 (W - M)
 \end{aligned}$$

(b)

$$\begin{aligned}
 W &= 77 \times 10^{15} \text{USD} = 77,000 \text{bUSD} \\
 \rho_0 &= 0.26 \frac{\text{ppm}}{\text{Gt C}} \\
 \rho_1 &= \frac{3 \text{ C}^\circ}{280 \text{ ppm}} \\
 \rho_5 &= 0.17 \frac{\text{kg C}}{\text{USD}} = \frac{0.17 \text{ t C}}{1000 \text{ USD}} = \frac{0.17 \text{ Gt C}}{1000 \text{ bUSD}}
 \end{aligned}$$

The last one is the tricky one. You need to remember to divide by 1000 to go from kg to tons. With $M = 0$, we have

$$\begin{aligned}
 T_2 - T_1 &= \rho_1 \rho_0 \rho_5 W \\
 &= \frac{3}{280} \times 0.26 \times \frac{0.17}{1000} \times 77,000 \\
 &= 0.0365 \text{C}^\circ
 \end{aligned}$$

- (c) If we double W and multiply ρ_5 by $1/3$, then, since the expression above just involves multiplication, we multiply the outcome by $2/3$ to get 0.242C° .
- (d) To start, look at p36 of Notes #1 which shows the various RCP's. The divergence between our two period framework and the IPCC's richer dynamic is stark and makes it difficult to compare with this problem. I think there are two ways to think about it: (1). Part 1b describes a 100 year problem with emissions only in the first year. In this case it matches most closely with RCP2.5, which forecasts very little warming. (2) We can think about extending the policy of part 1b to a 100 year horizon by simply multiplying W by 100. In this case, it is easy to see that we'll calculate 3.65C° of warming. This is a pretty good match, both in emissions and 100 year temperature change (see p 36 of Notes #1) to RCP 6.0.
2. (a) From the left axis of the figure, burning coal reserves creates about 400gt of C emissions. At 2.12 GtC per ppm of concentration, this works out to about

103ppm increase in carbon. From the right axis, burning all coal reserves gives enough carbon for about a 200ppm increase in concentration. If 0.55 stays in the atmosphere this gives us a 110ppm increase in C concentration.

- (b) Nordhaus' rule of thumb is that doubling atmospheric carbon from pre-industrial levels of 280ppm to 560ppm causes 3 degree Celsius of warming in 100 years. Thus, a 280ppm increase causes 3 degrees of warming. It follows that a 110ppm increase causes about a 1.2 degree increase after 100 years.
 - (c) From the figure, world carbon emissions from coal were about 3200Mt or 3.2 Gt in 2010 (Note that the figure in the book probably actually ends in 2009, I'm using the last year in the figure). If we continue to burn coal at this rate for 100 year we will have 320Gt of carbon emissions. To convert this to coal, note that this is $3.7 \times 320\text{Gt}$ or about 1180Gt of CO_2 . At two tons of CO_2 emissions per ton of coal, this requires burning 590Gt of coal.
 - (d) Of the 320Gt of carbon emissions, $0.55(320)=176\text{Gt}$ stays in the atmosphere (if the carbon cycle doesn't change). Dividing by 2.12, this is an 83ppm increase in atmospheric concentrations. Using Nordhaus' rule of thumb, this causes $(83/280) \times 3$ or about 0.9 degrees Celsius of extra warming.
3. Let's take current US population to be 300m and current world population to be 7b. This isn't exact, but it's nice round numbers.

We know from Deschenes and Greenstone that 4 degrees F of warming causes 6 extra deaths per day in the US. We're asked to find how many extra daily deaths a tank of gas today causes in 100 years.

First, scale up to world population. If the whole world is like the US then 4 degrees of warming causes $\frac{7b}{300m} \times 6 = 23 \times 6 = 138$ extra daily deaths world wide, dividing by 4 we have 34 extra daily deaths per degree. (In fact, the rest of the world is poorer and hotter than the US so you might want to adjust this up some.)

435 liters of gas gives us 1000kg of CO_2 or 270kg of C. Thus 50 liters gives us 50/435 of this amount or 31kg of C emissions.

Using Nordhaus' rule of thumb, 280 ppm of C concentration causes 3 degrees C, or about 5 degree F of warming in 100 years. So each ppm causes 5/280 degrees of warming.

Since about 0.55 of each ton of emissions is retained in the atmosphere, it takes about $2.12/0.55 = 3.85$ Gt of C emissions to create 1ppm of atmospheric concentration.

Altogether, our tank of gas causes $\frac{31kg}{3.85Gt} \times \frac{5}{280}$ degrees of warming. Each degree of warming causes 34 daily extra deaths. Thus, our tank of gas causes $\frac{31kg}{3.85Gt} \times \frac{5}{280} \times 34 = 4.8 \times 10^{-12}$.

This suggests that the future death toll from this tank of gas ought to be pretty low. warming in 100 years.

4. Gasoline is about 87% c by weight and weighs about 6.3 pounds per gallon. Multiplying, we have about 5.5 pounds c per gallon or $5.5/2.2 = 2.5\text{kg/c}$ per gallon. At 1000kg/ton, this is 0.0025 tc/gallon.

If the tax of c is τ_c /ton, the corresponding tax on gasoline is about $\tau_{g,c} = 0.025\tau_c$.

Since one unit of c equals about $44/12$ of CO_2 , at tax of τ on c is equivalent to a tax of $\tau_{\text{CO}_2} = \frac{12}{44}\tau_c$. So, $\tau_{g,\text{CO}_2} = 0.025\frac{44}{12}\tau_{\text{CO}_2}$.

So we have

τ_c	$\tau_{g,c}$	τ_{CO_2}	τ_{g,CO_2}
20	0.05	20	.185
50	0.125	50	.463
100	0.25	100	.925
200	0.50	200	1.85

The difference between Italian and US gas prices is about 3.00/gallon. Using the tax on $\text{CO}_2 = 100$ that we calculated above as a starting point, we would need a tax on CO_2 of about $3.00/0.92 = 330\$/\text{ton}$ to make up this difference.