

# EC1410-Spring 2023

## Problem Set 4

(Updated 11 January 2023)

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When you write up your answers, your goals should be to (1) be correct, and (2) convince your reader that your answer is correct. It is always helpful if your work is legible and if all steps are presented, possibly with a line of explanation. Answers that do not achieve these goals will not be awarded full credit.

### Problems

1. This problem will use the information presented in the slides to estimate urban and rural incomes in 1820 and 2000.
  - (a) First, use the figures based on Bolt and Van Zanden (2014) to determine real per capita GDP in 1820 and 2000, in constant 2011 dollars.
  - (b) Then, use the figures in Boustan et al. (2013) to find the share of the population in urban areas, and the urban wage premium, in both 1820 and 2000.
  - (c) Combine the information you have collected above to estimate urban and rural incomes in both 1820 and 2000 (hint: GDP per capita is a weighted average of wages in rural and urban areas).

2. In this problem, we will combine assumptions about urban and rural amenities with the income figures you produced above to estimate urban population growth between 1820 and 2000.

Denote urban and rural amenities by  $A_U$  and  $A_R$ , respectively. Let  $c_R = w_R$ . Let  $u(Ac) = \ln(Ac - 1)$ . Finally, let  $A_R/A_U$  be proportional to the ratio of rural to urban death rates (you can assume that the ratio in 1820 is the same as it is in 1870, and that in 2000 this ratio is 1).

- (a) In a spatial equilibrium,  $u(A_R c_R) = u(A_U c_U)$ . Assuming we are in a spatial equilibrium, write an expression for  $c_U$  in terms of rural wages and the ratio of urban to rural amenities.
- (b) Write down the household's problem for the household living at  $\bar{x}$  (you do not need to solve it).
- (c) Assuming  $\bar{R} = 0$ , solve the constraint in the above problem to get an expression for  $\bar{x}$ .
- (d) Recall that the city extends from  $-\bar{x}$  to  $\bar{x}$ , and that each household consumes an exogenous amount of land  $\bar{l}$ . This means that the city population is given by  $N^* = \frac{2\bar{x}}{\bar{l}}$ .

Write an expression for  $N^*$  based on your expression for  $\bar{x}$ .

- (e) Assume that  $t$  in 2000 is  $1/2$  of what it was in 1820, and that  $\bar{l}$  is constant over time. Use the information about wages from the previous problem, as well what we know about the ratio of rural to urban amenities, to write expressions for  $N^*$  in 1820 and 2000.

- (f) How much does your model imply the urban population grew between 1820 and 2000? That is, what is  $N_{2000}^*/N_{1820}^*$ ?
- (g) How does the urban population growth you computed above compare with the actual growth in the urban population over that time period? (For your reference, the US population in 1820 was 9,638,453 and in 2000 was 281,421,906 according to the Census). Why do you think there is a discrepancy?