EC1410-Spring 2022
Problem Set 3

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When you write up your answers, your goals should be to (1) be correct, and (2) convince your reader that your answer is correct. It is always helpful if your work is legible and if all steps are presented, possibly with a line of explanation. Answers that do not achieve these goals will not be awarded full credit.

Problems

1. This problem will examine the change in population density and its gradient over time based on Clark's 1951 study of population density in major metropolitan areas in Australia, the US and Europe.

   (a) Using Table 1 from Clark (1951), calculate the population density of London at the CBD (x=0), and 3 miles from the CBD, in 1801, 1841, and 1939.

   (b) How does the ratio of population density at the CBD to the population density three miles from the CBD change between 1801 and 1939?

2. This problem will examine the relationship between population density and transportation costs.

   In 1, you saw that the population density gradient flattened out (there is relatively more population at $x = 3$ compared to $x = 0$ over time). This problem examines if decreased transportation costs could explain this flattening of the density gradient.

   Assume we have the setup of the monocentric city model with housing, as in the lecture. Assume as well that housing production is perfectly competitive. Let $\bar{u} = 3$.

   (a) Let the household's problem be given by:

   $$\max_{c,h,x} c^{1/2}h^{1/2} \text{ subject to } w = c + ph + 2tx$$

   Let $\widetilde{w} = w - 2tx$. Use the first-order condition of the household's problem with respect to h to find h^* in terms of p and $\widetilde{w}$.

   (b) Use the fact that utility is $\bar{u} = 3$ everywhere to solve for p^* in terms of $\widetilde{w}$.

   (c) Substitute your expressions for p^* and $\widetilde{w}$ into your expression for h^* to write h^* in terms of w, t, and x.

   (d) Let the developer's problem be given by

   $$\max_{S} pS^{2/3} - iS - R$$

   where S is the capital to land ratio, and p, i and R are the costs of housing, capital, and land, respectively. For the remainder of the problem, let $i = \frac{1}{33}$. Use the first-order condition of this problem with respect to S to solve for S^* in terms of p.

(e) Substitute in your expression for p^* from earlier to obtain an expression for population density, $\frac{S^*}{h^*}$, in terms of w, t and x.

(f) Solve for the population density at $x = 1$ and $x = 2$ for $t = 1$ and $t = 0.5$. How does population density outside of the city center change when transportation costs fall?

(g) In order to obtain a more general result about the population density gradient and transportation costs, take the derivative of your expression for population density with respect to t.

(h) Evaluate the derivative from the previous part at $x = 0$. Does your expression for population density at $x = 0$ depend on t?

(i) Based on your results to the previous two parts, could this model of falling transportation costs explain the decreasing population density at the center and flattening of the population density gradient that you examined in the previous problem?