THE COSTS OF ANTI-SPRAWL POLICIES

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Abstract

This paper employs analytical and numerical general equilibrium models to examine the overall efficiency costs, the distributional costs to different agents along the city and the urban spatial structure resulting from a range of anti-sprawl policies, including development taxes, urban growth boundaries, property taxes and gasoline taxes.

We find that from an overall efficiency perspective, development taxes and urban growth boundaries are equivalent instruments and the most cost-effective anti-sprawl policies. For plausible parameter values, our results indicate that the overall costs of achieving anti-sprawl targets under the gasoline tax or the property tax are substantially higher, in the order of two to three times more, than the development tax (or urban growth boundary).

We also find that if the choice of anti-sprawl instruments is influenced by distributional considerations, then it is no longer clear that the development tax or the urban growth boundary will be the preferred instruments. In fact, our results suggest that the preferred instrument is closely related to the location of land and the mechanism of revenue-recycling.

Finally, if the goal is to achieve density at the city core, our results suggest the dominance of the gasoline tax over any other instrument.

1. Introduction

The issue of urban sprawl and its potential adverse consequences has become one of the most popular and controversial policy issues in the United States. Critics of Sprawl argue that an uncontrolled pattern of land development has produced excessive losses of agricultural land, losses of open space, increases in commuting time and levels of air pollution. As a consequence, state and local governments have adopted a variety of policies – the so-called 'smart-growth' initiatives – that aim to restrict the spatial expansion of cities, and therefore control the externalities associated with sprawl.

Despite this growing public concern, little is known about the efficiency, distributional and urban spatial structure impacts of these alternative anti-sprawl policies. To guide policymakers in the choice of alternative instruments, this paper raised three questions: First, what are the overall efficiency costs of alternative policies to control urban sprawl? Second, how are these costs distributed across space and agents? And, third, what are the impacts of anti-sprawl policies on urban spatial structure?

Two classes of models have been used in the literature to examine the impacts of growth controls: the so-called 'amenity-based' and 'supply-restriction based' models. Both models aim to offer explanations of the regular empirical finding that growth controls raise housing prices and discuss the potential winners and losers of growth controls. The 'amenity-based' models (e.g. Frankena and Scheffman (1981), Brueckner (1990) and Engle *et. al.* (1992)) are open-city models and predict that the increase in the housing prices is a result of the capitalization effect of the amenity. In this setting, landowners as a class experience a welfare gain while consumers – due to the open city assumption – are unaffected. The main result from this class of models is that, the introduction of growth controls will typically increase welfare, at least up to a point, reflecting the increase in the 'quality of life' of the community. In contrast, the supply-restriction based model of growth controls (Brueckner, (1995)) asserts that the control is related to the increase in land prices as a simple consequence of a restriction in the supply of developable land (having no connection with amenity effects). This model assumes a closed city and the growth control harm consumers while enriching landowners. As a consequence, growth controls are only adopted if landowners have political power.

Several issues motivate our work. First, one of our major concerns is that, while the existing models in the literature have helped us to understand the motivation for growth controls in cities, they certainly provide little (if any) guidance for the choice of alternative anti-sprawl policies. Therefore, there is a need to develop a framework that allows for systematic comparisons of different instruments and decomposes the various channels of efficiency exploited by each instrument. Second, while it is appropriate to suggest that landowners are the winners of growth controls from an overall efficiency perspective, it may not be totally appropriate to think of landowners as a 'group' which is equally affected by the control. Think of a control imposed at the boundary of a city and the consequent increase in amenity; in this case one would expect that the *location* of the plot of the land would determine whether a specific landowner will (or not) experience a welfare increase. The key issue that we are raising here is that the distribution of the costs and benefits of growth controls in space can be critical for the choice of instruments, yet it has been completely neglected in the literature. We consider important to explore this issue because one would expect that if different instruments will have different 'spatial' incidences, depending on their locations landowners may prefer one instrument over another and in turn, lobby the planning agency for the implementation of certain policies. This insight may helps us to explain an empirically regularity that suggests that quantity controls (e.g. Urban Growth Boundaries) are widely more implemented than price controls (e.g. development taxes). Finally, while most of the literature has looked at the motivation for growth controls based on efficiency criteria, we believe that, in practice planning agencies may also have pure planning goals, such as increasing the density of the urban core. It is therefore critical that we develop a method of ranking instruments not only on efficiency and distributional grounds but also that takes into consideration planning goals.

The present paper builds on prior work by considering a wide range of alternative instruments and examining both incremental and large amounts of land saved. Using a consistent analytical and numerical general equilibrium framework, we examine development taxes, urban growth boundaries, property taxes and gasoline taxes. Our focus is how the efficiency costs, distributional costs and urban spatial structure impacts of these different instruments vary with different targets of saved land. Our analytical model considers the behavior of a representative household who decides where to locate and how much to consume of housing and a composite good. We also model the behavior of developers who maximize profits and allocate their land to either residential or agricultural use. We model the effects of each policy instruments within the framework of the 'amenity-based' model for the following reasons. First, most anti-sprawl policies are introduced at the city level. In this case, one should think of the community that is implementing the control as a small open economy where households are freely mobile within the metropolitan area. Second, the 'amenity-based' model is an open-city model, which means that population is endogenously determined. This is a feature we consider crucial for our problem, in the sense that it only makes sense to us to talk about anti-sprawl policies in a context with 'growing' population. We should however note that, as a consequence of the 'amenity-based' model households welfare will not be impacted by any anti-sprawl policy and all welfare effects will be measured in terms of changes in the value of land to landowners. In our model instruments exploit two main sources of efficiency denoted as *city size a*nd *height effects.*

Our results suggest that from an efficiency perspective, development taxes and urban growth boundaries are equivalent instruments. Furthermore, the gasoline and property taxes are roughly 2 to 3 more times costly. This is because, unlike the urban growth boundary and the development tax which exploit the city size effect and the heigh effect (due to the capitalization of the amenity), the gasoline tax and the property tax alter the heights of buildings more than optimal. For example, in the case of a property tax, the overall density of the city actually reduces, a reflection of the fact that the property tax penalizes capital in addition to land; while in the case of a gasoline tax, the density of the city core will increase more than under any other instrument, a reflection of the fact that the gasoline tax transfers income towards the city center.

One of the most novel features of our work is to show that there is a substantial variation in 'spatial incidence' of the different instruments. We consider the costs of land to landowners at different points in the city and we find that, from a policy instrument choice perspective, one should think of cities divided into 6 areas: the urban core, the first suburb, the second suburb, the urban fringe and rural areas. Our analyses sheds light to the empirical regularity that suggests that urban growth boundaries are more popular than development taxes and suggests that, this is the case simple because the share of developers who prefer urban growth boundaries over development taxes is higher. An interesting aspect of the 'spatial incidence' of policies is that, at the urban core landowners typically prefer a gasoline tax if revenues from this tax are distributed lump sum to households. In this case, the gasoline tax transfers income towards the city center and, in turn, housing bids increase at the center. In contrast, at the rural areas landowners typically prefer a property tax if revenues from this tax are distributed lump sum to all landowners. This is because, the property tax effectively subsidizes land in agriculture and this subsidy is greater than with any other instrument.

Finally, our paper also suggests that if the goal is densification of the urban core, then the gasoline tax dominates any other instrument. In contrast, the property tax is clearly not a desired instrument, in that densities will actually reduce.

The rest of the paper is organized as follows: Section 2 presents an analytical model that reveals the different efficiency impacts of the policy instruments. Section 3 presents the simulation model and section 4 provides the simulation results. Finally, section 5 offers conclusions.

2. The Analytical Model

This section discusses an analytical model used to compare the gross costs, the distributional costs to different landowners throughout the city, the housing density and the overall population levels resulting from different anti-sprawl policy instruments. We examine four policies – a development tax, an urban growth boundary, a property tax and a gasoline tax. Subsections A-E lay out the model assumptions and discuss the general equilibrium impacts of the different policy instruments.

A. Model Assumptions

Consider a metropolitan area divided into J cities. We assume that the landscape throughout the metropolitan area is homogenous and therefore we abstract from spatial heterogeneity resulting from natural geographical variation in amenities or level of local public goods provided in each community. We think of each of these cities within the metropolitan area as a small open economy with a central business district (*CBD*).

We develop a static model in which a representative household enjoys utility from housing (H), a composite consumption good (Z) and open space (O). The household utility function is given by:

$$U = u\left(H, Z\right) + \phi(O) \tag{2.1}$$

$u(.)$ is utility from non-environmental goods and is quasi-concave. $\phi(.)$ is utility from open space. The separability restriction in (2.1) implies the demands for H and Z do not vary directly with changes in O. The way we have modeled open space deserves some comments. Our notion of open space mimics the idea of a greenbelt and thus it is open space located at the urban fringe. Also, we assume that all households in the city value in the same way the existence of the greenbelt regardless of the relative distance of this open space from their place of residence. Hence, the separability in the utility function seems a reasonable assumption.

The household budget constraint is:

$$Z + p_x H = Y - t_x \tag{2.2}$$

where p_x is the rental price of H at location x from the CBD and t_x is the transportation cost from location x to the CBD. For simplicity, we have set the price of the composite good equal to unity. Households choose x, Z and H to maximize utility (2.1) subject to the budget constraint (2.2), taking the level of open space as given. From the resulting first-order conditions and (2.2) we obtain the uncompensated demand functions conditional on location x:

$$Z(p_x, Y - t_x, x) \qquad H(p_x, Y - t_x, x) \tag{2.3}$$

Substituting these equations into (2.1) gives the indirect utility function:

$$V = v(p_x, Y - t_x, O, x) \tag{2.4}$$

A representative household chooses x that maximizes (2.4) and p_x and t_x adjust so that:

$$V(p_x, Y - t_x, O, x) = \bar{V} \tag{2.5}$$

Equation (2.5) states households must achieve the same level of utility no matter their place of residence and thus implies an open city with no moving costs. Under this assumption, overall household utility will not change in response to any policy, in the sense that, households can always move within the metropolitan area and achieve the same level of utility they had prior to any policy intervention. From a welfare perspective this class of agents should not enter in the welfare costs calculations of any policy. Another implication of the open city assumption is that population in the city is endogenously determined. This is an aspect we consider crucial to capture the features of our problem, since it only make sense to us to consider anti-sprawl policies within models with endogenous population.

Equation (2.5) implicitly defines the housing bid rent function as:

$$p_x = p(t_x, Y, O, \bar{V}, x) \tag{2.6}$$

Land is assumed to be owned by absentee landowners[1]. Let L_x denote the amount of land at distance x from the *CBD*. Landowners maximize the returns to land by deciding its use. We consider two possible uses of land: residential use and agriculture use. For simplicity, we assume an exogenous rate of return to land in agriculture of r_a. When land is allocated to residential use, the landowner combines land (L_x) with capital (K_x) to produce housing in order to maximize the rent from residential land. The housing production function is given by:

$$H_x = H(L_x, K_x) \tag{2.7}$$

and exhibits constant returns to scale. The return to land in residential use is given by:

$$r_u = \underset{S_x}{Max}\ p_x h_x(S_x) - p_K S_x \tag{2.8}$$

where $S_x = \dfrac{K_x}{L_x}$ denotes improvements per acre of land, a proxy for housing density or building height, and p_K the price of capital. From the resulting first-order condition we obtain the optimal density level:

[1] At a first glance, this assumption will appear to be extremely restrictive. It is obviously the case that residents own some of the land in cities. However, note that under the open-city model households utility will not change in response to policies therefore it is simpler to think of the owners of land – the single class of agents who will be impacted by policies – as absentee landowners.

$$S_x = S(x,t,Y,O,\bar{V}) \tag{2.9}$$

At each location x, landowners choose the use that maximizes the value of his plot of land:

$$\underset{r_x}{Max}\{r_a, r_u\} \tag{2.10}$$

Therefore, if $r_a < r_u$ a plot of land will be converted into residential use.

Finally, the two closing conditions of the model state residential land rent must equal the exogenous agricultural rent at the boundary of the city $(\bar{x})$:

$$r_u(t,Y,O,\bar{V},\bar{x}) = r_a \tag{2.11}$$

From (2.11) we get the city boundary:

$$\bar{x} = \bar{x}(t,Y,O,r_a,\bar{V}) \tag{2.12}$$

The second closing condition of the model requires equilibrium in the housing market at all locations. This means at a particular location x, the total housing demand should equal total housing supply:

$$N_x H_x = h_x(S_x) \tag{2.13}$$

where N_x denotes the total population at location x.

Given the assumption of an open city, total population in the city will be endogenously determined and equal to:

$$N = \int_0^{\bar{x}} 2\pi x \frac{h_x(S_x)}{H_x} dx \tag{2.14}$$

Note that 2π is the constant number of radians of land available for housing at each distance and, therefore, $2\pi x \frac{h_x(S_x)}{H_x} dx$ denotes the total population at the ring of available land located at distance x from the *CBD*.

With this framework, we can now analyze the gross efficiency costs, the distributional costs to different landowners, the densities and the overall population change of various anti-sprawl policies. To calculate the gross efficiency costs of a policy, we note that prior to a policy intervention, the total value of land in the city is given by the sum of total value of land in residential use with the total value of land in agriculture use:

$$R = \int_{0}^{\bar{x}} r_u(t, x, Y, O, \bar{V}) 2\pi x dx + \int_{\bar{x}}^{\bar{m}} r_a 2\pi x dx \tag{2.15}$$

where $\bar{m}$ denotes the geographical boundary between one city and another.

The gross efficiency costs of a policy are calculated as changes in the value of land resulting from a policy intervention. Let b denote the pre-policy situation and ai the after-policy situation. Then society's costs, $SC(i)$, under instrument i are calculated as:

$$SC(i) = \left(\int_{0}^{(\bar{x})^b} R^b(x) 2\pi x dx - \int_{0}^{(\bar{x})^{ai}} R^{ai}(x) 2\pi x dx \right) \tag{2.16}$$

To analyze the distributional impacts of the policies we divide landowners into three main groups. First, we consider the set of landowners who allocate their land to residential use prior and post policy. Second, we examine the set of landowners who alter their behavior in response to the policy by not converting land. And finally, we consider the set of landowners who allocate their land to agriculture before and after the policy.

To measure the impacts of the policies on Urban Spatial Structure, we calculate the impacts of the policies on density and overall population. In subsections B-E we provide and interpret key equations that decompose the effects of each policy. Complete derivations are provided in Appendix A.

B. Development Tax

Consider a revenue-neutral tax of t_D per unit of residential land, with tax revenues being redistributed lump sum to all landowners (i.e. landowners who allocate their land to residential and agriculture uses). The results presented below are sensitive to different mechanisms of revenue recycling. We have assumed the case of a uniform redistribution of revenues from the development tax to all landowners, since this mechanism seemed the most natural to us. In practice, policymakers could earmark the revenues from this tax to specific spatially delineated projects. For simplicity of the exposition here, we have abstracted from those considerations.

When deciding the best use, each developer will now compare:

$$\underset{r_x}{Max}\{r_a + g; r_u - t_D + g\} \tag{2.16B}$$

where g denotes the government transfer to each developer and t_D the amount of tax paid. Landowners will choose S_x, such that:

$$r_u^{t_D} = \underset{S_x}{Max}\, p_x h(S_x) - p_K S_x - t_D + g \tag{2.17B}$$

The first order condition for the optimal density level S_x is given by:

$$p_x \frac{\partial h_x(S_x)}{\partial S_x} = p_K \tag{2.18B}$$

Note that the development tax doesn't affect the optimal choice of S_x directly. However, it positively affects density levels indirectly through a capitalization effect on housing prices of the open space induced by the policy.[2]

The government budget constraint under a development tax is given by:

$$\int_0^{\bar{m}} g 2\pi x dx = \int_0^{\bar{x}} t_D 2\pi x dx \tag{2.19B}$$

and therefore, the transfer to each developer can be written as:

$$g = t_D \frac{\pi \bar{x}^2}{\pi \bar{m}^2} \tag{2.20B}$$

Note that $\pi \bar{x}^2$ is the area of land allocated to residential use while $\pi \bar{m}^2$ denotes the total area of the city. The development tax reduces the return to land in residential use uniformly throughout the city in the amount $-t_D + g$ and subsidizes the use of land for agriculture in the amount of g. To the extent revenues from the development tax are being transferred to owners of agricultural land, it has to be the case that $t_D > g$, and hence, the tax indeed penalizes residential use[3].

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[2] Differentiating (2.18B) for t_D we get $\frac{\partial S_x}{\partial t_D} = -\frac{\frac{\partial p_x}{\partial t_D}\frac{\partial h_x(S_x)}{\partial S_x}}{p_x \frac{\partial^2 h_x(S_x)}{\partial S_x^2}} > 0$.

[3] We note this tax is non-spatial because all owners of land allocated to residential use will pay the exact same amount. In practice policymakers could consider spatial-explicit development taxes if they want to

B1. Gross Cost of the Development Tax

We now consider an incremental, revenue neutral increase in t_D. The gross efficiency costs of this policy can be expressed as (see Appendix A):

$$\frac{dR}{dt_D} = \underbrace{(r_u(\bar{x}) - r_a)2\bar{x}\pi\frac{d\bar{x}}{dt_D}}_{dR^S} + \underbrace{\int_0^{\bar{x}_{tD}} \left[-2\pi\bar{x}\frac{d\bar{x}}{dt_D}\frac{\partial p_x}{\partial O}h_x(S_x) \right] 2\pi x dx}_{dR^C} \tag{2.21B}$$

The term labeled dR^S represents the cost from the *size effect*. This is the cost associated with the reduction in the total amount of land developed and is given by the reduction in the return of land due to the change in land use induced by the policy, multiplied by the amount of land saved. The term labeled dR^C represents the benefit from the *capitalization effect*. This is the gain associated with households responding to the higher level of open space by increasing their bids for housing. The *capitalization effect* equals the increase in open space multiplied by the willingness to pay for open space. The total variation in the value of land in the city is therefore given by the costs from the *size effect*[4] net of the gains from the *capitalization effect*. Our simulations discussed below suggest a dominance of the *size effect*.

B2. Distributional Costs of the Development Tax

Next we examine the distribution of costs of an incremental increase in t_D across the different landowners. The gross costs of this policy to the set of landowners who continue to allocate land to residential use is given by:

$$\frac{dR}{dt_D} = \underbrace{\int_0^{\bar{x}_{tD}} \left[-2\pi\bar{x}\frac{d\bar{x}}{dt_D}\frac{\partial p_x}{\partial O}h_x(S_x) \right] 2\pi x dx}_{dR^C} + \underbrace{\int_0^{\bar{x}_{tD}} \left[-1 + \frac{dg}{dt_D} \right] 2\pi x dx}_{dR^{NT}} \tag{2.22B}$$

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promote development in targeted areas in the city. Alternatively, they should use a non-uniform mechanism for revenue recycling.

[4]Note that the *size effect* is negative since $\frac{\partial\bar{x}}{\partial t_D} = \frac{1}{\frac{\partial r_u}{\partial\bar{x}} - 2\pi\bar{x}\frac{\partial r_u}{\partial\partial O}} < 0$

In contrast, the gross costs to the set of landowners whose behavior is altered in response to the policy is:

$$\frac{dR}{dt_D} = \underbrace{(r_u(\bar{x}) - r_a - \frac{dg}{dt_D})2\pi \bar{x} \frac{d\bar{x}}{dt_D}}_{dR^S} \tag{2.23B}$$

Finally, the gross costs to the group of landowners who continue to allocate land to agriculture is:

$$\frac{dR}{dt_D} = \underbrace{\int_{\bar{x}_{t_D}}^{\bar{m}} \frac{dg}{dt_D} 2\pi x dx}_{dR^{TR}} \tag{2.24B}$$

Equations (2.22B)-(2.24B) highlight several interesting effects regarding the distributional impacts of a marginal increase in the development tax. First, all benefits from the *capitalization effect* (labeled as dR^C in equation (2.22B)) are captured by the landowners who continue to allocate land to residential use after the policy is in place. To this group of agents, the overall costs of the policy consist of the *capitalization effect* and the *net tax effect* (labeled as dR^{NT} in equation (2.22B)). It is therefore not possible to determine the direction of the total effect to this group of agents. In the particular case where the *capitalization effect* would dominate the *net tax effect*, these agents would experience a welfare gain under the development tax. Second, we note all costs from the *size effect* (labeled as dR^S) fall on the group of landowners who switch uses in response to the policy. They are the main losers from this policy even though they are partially compensated through the revenue-recycling mechanism. Finally, we note landowners at the urban fringe are the ones who benefit the most from this policy intervention. Effectively they are subsidized under this policy and the value of their land increases by the amount dR^{TR} in equation (2.24B), a reflection of the revenue-recycling mechanism chosen.

B3. Impacts of the Development Tax on Urban Spatial Structure

The impact of an incremental increase in t_D on total population is given by:

$$\frac{dN}{dt_D} = \underbrace{2\pi \bar{x} \frac{h_{\bar{x}}(S_{\bar{x}})}{H_{\bar{x}}} \frac{d\bar{x}}{dt_D}}_{dN^S} + \underbrace{\int_0^{\bar{x}_{t_D}} 2\pi x \frac{\partial \left(\frac{h_x(S_x)}{H_x} \right)}{\partial t_D} dx}_{dN^C} \qquad (2.25B)$$

Two opposite effects determine the overall population under the development tax. The term labeled dN^S represents the reduction in population due to the *size effect*. The term labeled dN^C represents the increase in population due to the *capitalization effect*. This effect translates into an increase in densities throughout the city. For the particular case where the *size effect* dominates the *capitalization effect*, the city will experience a reduction in total population.

C. Urban Growth Boundary

Now consider the impact of an urban growth boundary. In this model, where landowners are homogenous, we can think of this policy as one where the government chooses the overall acceptable level of city size, $\bar{x}_{UGB}$. The key difference between this policy and the development tax is it does not raise revenues for the government and consequently does not subsidize agricultural land.

The closing condition of the model, which implicitly defines the city boundary, now becomes:

$$r_u(t, \bar{x}_{UGB}, Y, O, \bar{V}) = r_a \qquad (2.26C)$$

C1. Gross Costs of the Urban Growth Boundary

The general equilibrium gross efficiency cost of an incremental increase in the UGB level can be decomposed as follows (see appendix A):

$$\frac{dR}{d\bar{x}_{UGB}} = \underbrace{(r_u(\bar{x}) - r_a) 2\pi \bar{x} \frac{d\bar{x}}{d\bar{x}_{UGB}}}_{dR^S} + \underbrace{\int_0^{\bar{x}_{UGB}} \left[-2\pi \bar{x} \frac{d\bar{x}}{d\bar{x}_{UGB}} \frac{\partial p_x}{\partial O} h_x(S_x) \right] 2\pi x dx}_{dR^C} \qquad (2.27C)$$

A comparison of (2.26C) with (2.20B) reveals the urban growth boundary is equal to the development tax, whenever both instruments are set to achieve the same amount of land preservation. Thus, from an efficiency gross costs perspective, the two instruments are equivalent.

C2. Distributional Impacts of the Urban Growth Boundaries

The main difference between the urban growth boundary and the development tax is the distributional impacts of the policy on the different landowners.

Lets consider first the impact of the urban growth boundary on landowners who allocate land to residential use both prior and post policy. The change in the return to land to this group of agents is given by:

$$\frac{dR}{d\bar{x}_{UGB}} = \underbrace{\int_0^{\bar{x}_{UGB}} \left[-2\pi \bar{x} \frac{d\bar{x}}{d\bar{x}_{UGB}} \frac{\partial p_x}{\partial O} h_x(S_x) \right] 2\pi x dx}_{dR^C} \tag{2.28C}$$

Equation (2.28C) distinguishes from (2.22B) in the absence of the *net tax effect* term, dR^{NT}. For this class of landowners, the urban growth boundary is clearly preferable to the development tax. In fact, (2.28C) is strictly positive while the sign of (2.22B) is indeterminate since the *net tax effect* may dominate the *capitalization effect*. Under an urban growth boundary policy, landowners who keep their land in residential use are the ones who benefit the most from the policy.

Next, consider the impact of an incremental change in the UGB on landowners who alter their behavior by changing land uses. The change in the return to land for this group of agents is given by:

$$\frac{dR}{d\bar{x}_{UGB}} = \underbrace{(r_u(\bar{x}) - r_a) 2\pi \bar{x} \frac{d\bar{x}}{d\bar{x}_{UGB}}}_{dR^S} \tag{2.29C}$$

A comparison of (2.29C) and (2.23B) suggests, for the same amount of land saved, this group of landowners is worse off under the urban growth boundary compared to the development tax. Under the development tax their loss in profits was given by $r_u(\bar{x}) - r_a - \frac{dg}{dt_D}$, with $\frac{dg}{dt_D} > 0$ and under the urban growth boundary their loss in profits is $r_u(\bar{x}) - r_a$.

Finally, consider the impact of the urban growth boundary on landowners who allocate their land to agriculture pre and post policy. Because the UGB does not generate revenues, this policy instrument will not affect this group of agents. The impact of the policy is thus:

$$\frac{dR}{d\bar{x}_{UGB}} = 0 \tag{2.30C}$$

Comparing (2.24B) with (2.30C) reveals that this group of landowners would be better off under the development tax due to the revenue-recycling effect embedded in the latter policy instrument.

C3. Impacts of the Urban Growth Boundary on Urban Spatial Structure

The impact of a marginal increase of an urban growth boundary on total population is given by:

$$\frac{dN}{d\bar{x}_{UGB}} = \underbrace{2\pi \bar{x} \frac{h_{\bar{x}}(S_{\bar{x}})}{h_{\bar{x}}^{d}} \frac{d\bar{x}}{d\bar{x}_{UGB}}}_{dN^{S}} + \underbrace{\int_{0}^{\bar{x}_{UGB}} 2\pi x \frac{\partial\left(\frac{h_x(S_x)}{h_x^d}\right)}{\partial \bar{x}_{UGB}} dx}_{dN^{C}} \tag{2.31C}$$

and is equivalent to the impact of a development tax, given by (2.25B). This allows us to conclude that both instruments would lead to the same densification of the city and population change.

We conclude the discussion of this instrument by noting from an efficiency gross costs as well as from an urban planning densification and migration control perspective, the urban growth boundary and the development tax are two equivalent instruments. However, if political or distributional considerations are taken into account in the choice of the instruments, one instrument may dominate the other. It is likely landowners who allocate their land to residential use will influence the planning agency to implement an urban growth boundary while landowners who allocate their land to agricultural use will pressure the agency to implement a development tax.

D. Property Tax

Next consider the impacts of a revenue-neutral tax t_H per unit of housing produced, with revenues returned lump sum to all landowners.

The government budget constraint under a property tax is given by:

$$\int_0^{\bar{m}} g 2\pi x dx = \int_0^{\bar{x}_{t_H}} t_H h^s(S_x) 2\pi x dx \tag{2.32D}$$

When deciding how to allocate the land, landowners will now choose the land use that maximizes their land value r_x:

$$\underset{r_x}{Max}\{r_a + g, r_u^{t_H}\} \tag{2.33D}$$

where,

$$r_u^{t_H} = \underset{S_x}{Max}(p_x - t_H)h_x(S_x) - p_K S_x + g \tag{2.34D}$$

and g denotes the transfer.

In contrast with the development tax, a property tax has a direct influence on the optimal density level. Lets assume g does not influence landowners' density level decisions[5]. The first order condition for S_x under a property tax is given by:

$$(p_x - t_H)\frac{\partial h_x(S_x)}{\partial S_x} = p_K \tag{2.35D}$$

Comparing (2.35D) with (2.18B) one infers density levels are inferior under a property tax relative to a development tax or an UGB.

D1. Gross Costs of the Property Tax

The general equilibrium gross efficiency cost of an incremental increase in the property tax t_H is given by:

$$\frac{dR}{dt_H} = \underbrace{\left[r_u(\bar{x}) - r_a\right]2\pi \bar{x}\frac{d\bar{x}}{dt_H}}_{dR^S} + \underbrace{\int_0^{\bar{x}_{t_H}}\left[-2\pi \bar{x}\frac{d\bar{x}}{dt_H}\frac{\partial p_x}{\partial O}h_x(S_x)\right]2\pi x dx}_{dR^C} + \underbrace{\int_0^{\bar{x}_{t_H}} t_H \frac{\partial h_x(S_x)}{\partial S_x}\frac{\partial S_x}{\partial t_H}2\pi x dx}_{dR^H}$$

(2.36D)

[5] Landowners may not know exactly how much of the tax will they get back and what is the density level chosen by the others landowners. So when deciding the optimal density level they only take into account the tax paid for each unit of housing produced even though g will change the value of residential land.

Comparing (2.36D) with (2.21B) or (2.27C), the gross cost of the property tax is higher than under the development tax or the urban growth boundary due to the additional *adverse height effect* denoted by dR^H in equation (2.36D) and a weaker *capitalization effect,* captured by the term dR^C. The *adverse height effect* reflects the property tax penalizing capital (in addition to land) and, as a consequence, the optimal height decisions are affected as shown by (2.35D). Also the *capitalization effect* under a property tax will not be as strong as the *capitalization effect* under the development tax or the UGB because density levels are inferior under a property tax relative to a development tax or an UGB. This means housing production, $h_x(S_x)$, under a property tax is inferior to that under a development tax or an UGB for all $x \in \left[0, \bar{x}_{t_H}\right]$. Note however the *size effect* is the same as the previous two policy instruments.

D2. Distributional Costs of the Property Tax

Let us now consider the distributional costs to landowners along the city resulting from a marginal increase in the property tax. First, consider the impact of the policy to landowners who allocate land to residential use prior and post policy. The costs of an increase in the property tax to these set of landowners is given by:

$$\frac{dR}{dt_H} = \underbrace{\int_o^{\bar{x}_{t_H}} \left[-2\pi\bar{x}\frac{\partial\bar{x}}{\partial t_H}\frac{\partial p_x}{\partial O}h_x(S)\right] 2\pi x dx}_{dR^C} + \underbrace{\int_0^{\bar{x}_{t_H}} \left[(p_x\frac{\partial h_x}{\partial S_x} - p_k)\frac{\partial S_x}{\partial t_H}\right] 2\pi x dx}_{dR^H} + \underbrace{\int_0^{\bar{x}_{t_H}} \left(\frac{\partial g}{\partial t_H} - h_x(S_x) - t_H\frac{\partial h_x(S_x)}{\partial S_x}\frac{\partial S_x}{\partial t_H}\right) 2\pi x dx}_{dR^{NT}}$$

(2.37D)

It is clearly the case that this set of landowners would be worse off under this policy than under the UGB. However, it is not possible to infer a priori if the property tax is preferable (or not) to the development tax. A comparison between (2.37D) and (2.22B) reveals that if dR^{NT} (from equation 2.22B) is smaller than $dR^H - dR^{NT}$ (from equation 2.37D), the development tax will be preferable.

Next consider the effect of the property tax to the class of landowners who altered their behavior in response to the policy. The gross cost of the policy to this set of agents is given by:

$$\frac{dR}{dt_H} = \underbrace{\left[r_u(\bar{x}) - r_a + \frac{\partial g}{\partial t_H} \right] 2\pi \bar{x} \frac{d\bar{x}}{dt_H}}_{dR^S} \tag{2.38D}$$

Similar to the development tax and the UGB, under the property tax this set of landowners will bear the full cost from the *size effect*. However, they will prefer a property tax to an urban growth boundary or even a development tax because under the property tax the amount of transfer received may be greater. It is possible $\frac{\partial g}{\partial t_H} > \frac{\partial g}{\partial t_D}$.

Finally, like the development tax, landowners who allocate land to agriculture will be subsidized under the property tax. Again, to the extent the property tax might generate more revenues than the development tax, this class of agents might prefer the property tax relative to any other instrument. Their welfare change is given by:

$$\frac{dR}{dt_H} = \underbrace{\int_{\bar{x}}^{\bar{m}} \frac{\partial g}{\partial t_H} 2\pi x dx}_{dR^T} \tag{2.39D}$$

D3. Impacts of the property tax on Urban Spatial Structure

The impact of an incremental increase in t_H on total population is given by:

$$\frac{dN}{dt_H} = \underbrace{2\pi \bar{x} \frac{h_{\bar{x}}(S_{\bar{x}})}{H_{\bar{x}}} \frac{d\bar{x}}{dt_H}}_{dN^S} + \underbrace{\int_0^{\bar{x}_{t_D}} 2\pi x \frac{\partial \left(\frac{h_x(S_x)}{H_x} \right)}{\partial t_H} dx}_{dN^{C+H}} \tag{2.40D}$$

As before two opposite effects determine the overall population under the property tax. On the one hand we have a reduction in population due to the *size effect*, captured by the term dN^S, and on the other hand we have a change in population due to changes in density levels throughout the city, captured by the term dN^{C+H}.

In contrast with the UGB or the development tax, the sign of this latter effect cannot be inferred analytically since it depends on the magnitudes of the *capitalization effect* and the *adverse height effect*[6]. If the *adverse height effect* is stronger than the *capitalization effect,* then a property tax will lead to lower levels of improvements per acre, as discussed in formula (2.35D). If the housing demand remains constant, then shorter buildings implies a decline in population density, with fewer households living on each acre of land. However the property tax also has an effect on the housing demand through higher housing prices. In response, housing demand also decreases. But a smaller demand for housing implies an increase in population density, which offsets the density decline caused by the lower level of improvements. So the net effect of the property tax on density levels throughout the city is uncertain. However, because of the *adverse height effect,* under the property tax any decrease in city's population will always be higher than under the UGB or the development tax.

E. Gasoline Tax

Finally, we consider the impact of an incremental increase in the gasoline tax, t_G. In our model, a gasoline tax is a tax levied per unit of miles traveled. We assume the revenues from the gasoline tax are returned lump sum to households. Under this policy, the new household budget constraint can be written as:

$$Y - t_x - t_{Gx} + g = Z + p_x H \qquad (2.41\text{E})$$

and the government budget constraint is given by:

$$\int_0^{\bar{x}_{t_G}} N(x) g 2\pi x dx = \int_0^{\bar{x}_{t_G}} N(x) t_{Gx} 2\pi x dx \qquad (2.42\text{E})$$

To better understand the impacts of the gasoline tax in the model, consider two distinct households. First, consider a household located at the city center. Because the gasoline tax increases with distance from the CBD, it is likely that the amount of transfer received from the government will more than compensate the amount of tax paid. In contrast, for a

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[6] Differentiating (2.35D) for t_H we get $\frac{\partial S_x}{\partial t_H} = -\frac{\left(\frac{\partial p_x}{\partial t_H} - 1\right)\frac{\partial h_x(S_x)}{\partial S_x}}{(p_x - t_H)\frac{\partial^2 h_x(S_x)}{\partial S_x^2}} \gtrless 0$.

household located in the *suburbs*[7], it is likely that the amount of tax paid will outweigh the transfer. Therefore, the gasoline tax redistributes income towards households located in the city core. As a consequence, households will alter their bids for housing. We call this effect the *disposable income effect.*

E1. The Gross Costs of the gasoline tax

The gross efficiency costs of a marginal increment in the gasoline tax t_G are given by:

$$\frac{dR}{dt_G} = \underbrace{\left[r_u(\bar{x}) - r_a \right] 2\pi \bar{x} \frac{d\bar{x}}{dt_G}}_{dR^S} + \underbrace{\int_0^{\bar{x}_{t_G}} \left[-2\pi \bar{x} \frac{d\bar{x}}{dt_G} \frac{\partial p_x}{\partial O} h_x(S_x) \right] 2\pi x dx}_{dR^C} + \underbrace{\int_0^{\bar{x}_{t_G}} \left[\frac{\partial p_x}{\partial Y^d} \frac{\partial Y^d}{\partial t_G} h_x(S_x) \right] 2\pi x dx}_{dR^Y}$$

(2.43E)

The key difference between this policy and the development tax or the urban growth boundary is given by the term dR^Y, which reflects the *disposable income effect*. This effect can be an efficiency gain (or loss), depending on whether the overall change in disposable income translates into higher (or lower) bids for housing[8]. A priori from an efficiency perspective, it is not possible to determine whether this instrument would be preferred to any other instrument.

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[7] By suburb we mean land inside the urban boundary but outside the "urban core," not separate autonomous cities.

[8] The impact of the gas tax on housing prices is obtained as: $\frac{\partial p_x}{\partial t_G} = -2\pi \bar{x} \frac{\partial p_x}{\partial O} \frac{\partial \bar{x}}{\partial t_G} + \frac{\partial p_x}{\partial Y^d} t_2(-x + x^j + t_G \frac{\partial x^j}{\partial t_G})$, where $t_2(x^j + t_G \frac{\partial x^j}{\partial t_G}) = \frac{\partial g}{\partial t_G}$, $t_G \in [0,1]$, $\frac{\partial x^j}{\partial t_G} < 0$, x^j is the location where households pay a gasoline tax bill exactly equal to the transfer g they get back from the government. The total impact of the gas tax on disposable income is given by: $\frac{dY^d}{dt_G} = t_2(-x + x^j + t_G \frac{\partial x^j}{\partial t_G})$. Note that $x^j + t_G \frac{\partial x^j}{\partial t_G}$ is the new location where the transfer received by a household now exactly equals the gas tax paid and its sign depends on the location in the city:

$$\frac{dY^d}{dt_G} = \begin{cases} > 0 & if \quad x < x^j + t_G \frac{\partial x^j}{\partial t_G} \\ = 0 & if \quad x = x^j + t_G \frac{\partial x^j}{\partial t_G} \\ < 0 & if \quad x > x^j + t_G \frac{\partial x^j}{\partial t_G} \end{cases}$$

E2. Distributional Costs of the gasoline tax

First consider the impact of an increase in the gasoline tax for the set of landowners who continue to allocate their land to residential use. The cost of the policy to these agents is given by:

$$\frac{dR}{dt_G} = \underbrace{\int_0^{\bar{x}t_G} \left[-2\pi \bar{x} \frac{d\bar{x}}{dt_G} \frac{\partial p_x}{\partial O} h_x(S_x) \right] 2\pi x dx}_{dR^C} + \underbrace{\int_0^{\bar{x}t_G} \left[\frac{\partial p_x}{\partial Y^D} \frac{dY^D}{dt_G} + \frac{\partial p_x}{\partial O} \frac{\partial O}{\partial t_G} \right] h_x(S_x) 2\pi x dx}_{dR^Y}$$

(2.44E)

This formula suggests, like the previous policies, this set of landowners will fully capture the welfare gain from the *capitalization effect*. In addition, this set of agents can have an additional source of welfare gain if their plot of land is located in an area where households are effectively subsidized under the gasoline (in other words, if $\frac{\partial Y^D}{\partial t_G}$ >0). In this case, households would bid higher prices for housing and therefore the value of land will increase more.

In sum, a key insight from our model is that, depending on the location of their plots of land, landowners could be made better off under the policy. Therefore, there is certainly a sub-group of landowners who allocate land for residential use, for which the gasoline tax is the preferred instrument.

For agents who alter behavior in response to the gasoline tax and for landowners who continue to allocate land to agriculture, the gasoline tax is equivalent to the urban growth boundary in that it does not compensate landowners for any losses. Therefore the welfare effects for the group of landowners who switch uses and for those who allocate land to agriculture are given respectively by:

$$\frac{dR}{dt_G} = \underbrace{\left[r_u(\bar{x}) - r_a \right] 2\pi \bar{x} \frac{d\bar{x}}{dt_G}}_{dR^S} \tag{2.45E}$$

$$\frac{dR}{dt_G} = 0 \tag{2.46E}$$

E3. Impacts of the gasoline tax on Urban Spatial Structure

The impact of an incremental increase in t_G on total population is given by:

$$\frac{dN}{dt_G} = \underbrace{2\pi \bar{x} \frac{h_{\bar{x}}(S_{\bar{x}})}{H_{\bar{x}}} \frac{d\bar{x}}{dt_G}}_{dN^S} + \underbrace{\int_0^{\bar{x}_{t_D}} 2\pi x \frac{\partial\left(\frac{h_x(S_x)}{H_x}\right)}{\partial t_G} dx}_{dN^{C+Y}} \tag{2.47E}$$

It is not possible to infer the sign of $\frac{\partial S_x}{\partial t_G}$ a priori because it depends on the location in the city, therefore one cannot say whether total city population will increase or decrease when a gasoline tax is imposed. If the increase in densities at the city core does not compensate the decrease in density levels in the *suburbs* plus the decrease in population due to the *size effect* (captured by the term dN^S), then we will have a reduction in total population. However it is not possible to determine analytically whether this decrease is bigger than under a UGB or a development tax. But due to a *disposable income effect* that is positive for certain locations, total population is still higher under a gasoline tax than under a property tax. Also, one would expect the following behavior for the density levels under a gasoline tax.

$$S_x^{t_G} \begin{cases} > S_x^{t_D} = S_x^{UGB} > S_x^{t_H} & for \quad x \in \left[0, x^t\right] \\ < S_x^{t_D} = S_x^{UGB} & for \quad x \in \left]x^t, \bar{x}_{t_G}\right] \\ \geq S_x^{t_H} & for \quad x \in \left]x^t, \bar{x}_{t_G}\right] \end{cases} \tag{2.48E}$$

So from (2.48E), a gasoline tax promotes the greatest densification of the city core compared to the alternative policy instruments. Graphically, this means the density function under a gasoline tax will rotate clockwise at location $x = x^t$ relative to the density function in the benchmark. For a development tax or a UGB, we have an upward parallel shift of the density function due to the *capitalization effect*, and for a property tax we have a downward nonparallel shift because the net burden of the property tax on landowners decreases with distance and the *adverse height effect* dominates the *capitalization effect*.

3. Numerical Model

In this section we numerically examine the costs of the alternative policies described in section 2 to protect land from development and their distribution across landowners within a city. Subsection A specifies the functional forms used in the simulation model and subsection B describes the parameter values used to calibrate the model. A complete description of the model is provided in appendix B.

A. Model Structure

Household Behavior

Consistent with the analytical model, households derive utility from a housing good, $H(x)$, a composite good, $Z(x)$, and from open space O. Households´ preferences are represented by a Cobb Douglas utility function:

$$U(H,Z,O) = H(x)^{\alpha} Z(x)^{(1-\alpha)}(1+\delta\sqrt{O}) \qquad (3.49A)$$

where α denotes the percentage of income net of transportation costs spent on housing and δ is a positive constant.

At each distance from the CBD, households' budget constraint is given by:

$$p(x)H(x) + Z(x) + tx = Y \qquad (3.50A)$$

where x is the radial distance from the CBD, $p(x)$ is the market housing price at "location x", Y is the annual income and t is the annual commuting cost per round trip mile, which includes both money and time components. Let t_1 represent the unitary commuting time cost and t_2 the unitary commuting money cost so that $t = t_1 + t_2$.

From the location no-arbitrage condition, we get the households' housing bid price function at distance x defined as[9]:

$$p(x) = \left\{ \left[\frac{\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}}{\overline{V}} \right] (Y - tx)(1+\delta\sqrt{O}) \right\}^{\frac{1}{\alpha}} \qquad (3.51A)$$

where $\overline{V}$ is the exogenous utility level.

[9] Note that given the assumption of homogeneous landscape we get symmetric circular equilibriums where housing prices and also residential land rents are the same for all locations with the same distance from the CBD.

Landowners Behavior

We use a constant-elasticity-of-substitution (CES) form to describe the housing production function:

$$H^s(x) = \psi\left[\beta K(x)^{\frac{\sigma-1}{\sigma}} + (1-\beta)L(x)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}},\ \sigma \neq 1 \tag{3.52A}$$

where $H^s(x)$ is housing supply at distance x, $K(x)$ is structural capital, $L(x)$ is land, β is the share of capital in the production function, σ is the elasticity of substitution between capital and land and ψ is a positive constant.

In the absence of government intervention, the density level that maximizes residential use at distance x is given by:

$$S(x) = \left[\frac{1-\beta}{\left[\frac{p_K}{\psi p(x)\beta}\right]^{\sigma-1} - \beta}\right]^{\frac{\sigma}{\sigma-1}} \tag{3.53A}$$

And urban rent at distance x is defined as:

$$r_u(x) = (1-\beta)^{\frac{\sigma}{\sigma-1}}\left[(\psi p(x))^{1-\sigma} - \beta^{\sigma} p_K{}^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \tag{3.54A}$$

Finally, prior to any government intervention, the total amount of land that will be devoted to residential use is computed as:

$$D^b \equiv \left\{(x,\theta) : x \leq \frac{Y}{t} - \frac{1}{t}\frac{\overline{V}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}(1+\delta\sqrt{O})}\left[\frac{\left[\beta^{\sigma}(p_K)^{1-\sigma} + (1-\beta)^{\sigma}(r_a)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}}{\psi}\right]^{\alpha}\right\} \tag{3.55A}$$

where D^b represents the area of a circle defined in polar coordinates centered at the origin.

B. Parameter values

We now discuss the parameter values used in our benchmark simulations[10].

The Bureau of Labor Statistics-CE (2001) as well as the U.S. Department of Commerce-Bureau of Economic Analysis (2001) reported for the year 2000 that on average, American households spent 33% of their disposable income on housing and around 17% on transportation, 92% of which was on automobile transportation. It is also reported in 2000, American households had an average annual disposable income of $38045 and average total monetary cost of owing and operating an automobile of 0.51 cents per mile[11]. Given these statistics, we have chosen a value of $\alpha = 0.4$, a disposable income $Y = \$40000$ and a commuting cost parameter per mile/year $t = \$600$ [12]. Also the unitary price and income elasticities implied by our Cobb Douglas utility function[13] are well within the range of values found in empirical studies (Polinsky, 1977, Polinsky and Ellwood, 1979; Rosen, 1979; Harmon, 1988).

The agriculture rent value was chosen based on the Farm Real Estate Value[14] Indicator reported by the United States Department of Agriculture for the year 2000 and set equal to $1000 per acre per year, corresponding to a rent per square mile of $640000.

Based on empirical studies which estimate the elasticity of substitution between land and capital in the structure production function (review in McDonald (1981)), the current wisdom is that σ lies between 0.6 and 0.8. On the basis of this review we set σ equal to 0.8 and a value of 0.95 for the share of capital, β, in the housing production function.

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[10] The simulation model was solved using Mathematica. Details of the computer programs are available from the authors upon request.

[11] Average total cost per mile include both variable costs (gas, oil, maintenance and tires) and fixed costs (insurance, license, registration, taxes, depreciation and finance charges) and all figures in the report reflect the average cost of operating a vehicle 15000 miles per year in stop and go conditions.

[12] The commuting cost parameter value includes both money and time costs components as follows: an out-of-pocket cost of $0.54 per mile and 250 round trips per year leads to a value of $270 per year for the money cost of commuting per mile (t_2). A yearly income of $40000 implies an hourly wage of $20, which yields a time cost per mile of $0.66 assuming a traffic speed of 30 miles per hour and commuting time valued at the full wage. Time cost (t_1) is then $330 per mile per year, yielding total money plus time cost of $600.

[13] In the Cobb Douglas case all own total expenditure elasticities are one, all own price elasticities are minus one and all cross price elasticities are zero.

[14] Farm Real Estate Value is defined as the value at which all land and buildings used for agriculture production could be sold under current market conditions, if allowed to remain on the market for a reasonable amount of time. It is an indicator of the financial condition of the farm sector and its value is influenced by net returns from agricultural production, capital investment in farm structures, interest rates, government commodity programs and nonfarm demands for farmland.

The unitary price of capital structures varies widely depending on the type of housing built (apartments versus single houses), type and quality of materials used and topographic conditions. We have used the construction cost data from the R.S. Means Company[15] to infer the cost of capital to be used in our simulation model[16]. Given the numbers reported we have used \$100 as a reasonable guess for the unitary cost of capital.

The exogenous radius of total available land, $\overline{m}$, was set equal to 20 miles and finally the parameters δ and ψ as well as the exogenous utility level, $\overline{V}$, were set arbitrarily to generate realistic cities. Their values are 0.0003,1 and 2200, respectively.

Table 1, in Appendix C, summarizes our benchmark metropolitan area. Prior to government intervention to control excessive urban growth, the city has a radius of 11.65 miles and approximately one and a half million people living within its boundaries[17].

4. Numerical Results

This section presents results from the numerical model. Subsections A and B compare the marginal and total cost, respectively, of the different anti-sprawl policies. Subsection C considers the distribution of total costs to landowners located at different distances from the CBD. Subsection D compares the density levels obtained by each policy instrument for a target percentage of saved land. Finally, subsection E offers further sensitivity analysis.

It should be noted our emphasis is on qualitative, rather than quantitative, differences across policies. The quantitative differences can vary depending on housing production parameters, level of exogenous agricultural rent and utility-function parameters, which determine the relative contributions of the size, capitalization, (adverse) height, net-tax and disposable income effects. It is also important to recognize our analysis abstracts from different levels of heterogeneity, including household heterogeneity in the valuation of open space, heterogeneity in the costs of development and heterogeneity of the

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[15] This firm computes construction costs per square foot of living area for single-family homes in a wide variety of American and Canadian cities, based on surveying construction companies.

[16] The Means data on construction costs include material costs, labor costs and equipment costs. Given that no land costs are included, the construction cost data reported by the R.S. Means Company are for the physical structure itself. In general, the marginal construction cost of an apartment is the price of building up. As a basic number, the Means data suggests that construction costs for a building in a typical high rise of from 8 to 24 stories was nearly \$110 per square foot in New York City in 1999.

[17] For an assumed 3 people per household.

landscape. Such heterogeneity can affect the relative costs of policies by imposing additional informational burdens. These impacts may vary depending on the policy considered, and thus heterogeneity can alter the relative cost-effectiveness of different policies. Finally, we should also acknowledge we abstract from strategic interaction amongst cities in the metropolitan area. Our model assumes only one city implements the anti-sprawl policy and the others cities are passive. However, it is likely neighboring cities will respond, which could potentially raise the costs of anti-sprawl policies.

A. *Marginal Costs to Landowners*

We first examine the marginal costs to landowners of alternative anti-sprawl policies. In figure 1, on the horizontal axis we measure percent of land saved (up to 30%) and on the vertical axis marginal costs. The main goals of this figure are to decompose the different sources of efficiency discussed in section 2 and discuss their contribution to the overall cost of the different policies.

$MC^{t_D\&UGB}$ represents both the marginal costs of the development tax and the UGB since, as already shown in the analytical model, these instruments are equivalent from the gross costs perspective. This curve reflects the two opposing efficiency channels exploited by these two instruments: the *capitalization* and the *size effects*. $MC^{t_D\&UGB}$ has a negative intercept due to the capitalization of open space, and is upward sloping, reflecting the increasing marginal cost of saving land. The crucial point from this curve is that while the *capitalization effect* is significant for low levels of land saved, the contribution of the *size effect* to the costs of the policy quickly increases. For example, doubling the percentage of land saved more than doubles the cost. This is not surprising, since as more land gets saved, we move towards the city center where housing prices are higher and thus residential land is more valuable.

MC^{t_H} shows the marginal costs of saving land under the property tax. The vertical distance between this curve and the $MC^{t_D\&UGB}$ curve reflects the (marginal) *adverse height effect*. This effect is a welfare loss because the property tax penalizes simultaneously land and capital, thus producing a sub-optimal level of density throughout the city as seen in (2.35D). As we can see from the figure, for 10% of land saved, the contribution of the *adverse height effect* to the total cost of the policy is approximately three-quarters. However, as the percentage of land saved increases the contribution of this effect reduces. At 20% of land saved it is only three-fifths.

Finally, MC^{t_G} shows the marginal costs under the gasoline tax. This curve equals the $MC^{t_D\&UGB}$ plus the (marginal) cost from the *disposable income effect*. Under our central parameter estimates, this figure suggests that the welfare cost from the *disposable income effect* is always lower than that from the *adverse height effect*, which means from a cost-effectiveness perspective the gasoline tax is preferable to the property tax. Furthermore, the *size effect* predominates for all policies.

B. Total Costs of the alternative policies

Figure 2 shows the total costs of land saved under the various policies. Locus $TC^{t_D\&UGB}$ represents both the total cost curves for a development tax and for an UGB because from an overall efficiency perspective, these two policies are equivalent, even though the development tax generates revenues. However this equivalence can be altered if the heterogeneity of the landscape is included in the model or if the revenue-recycling rule is different. An interesting feature from fig.2 concerning the development tax and the UGB is that up to a certain amount of land saved (around 13%), these growth control policies do not impose a cost to society. Instead they benefit society by increasing the overall value of land.

In contrast, the total costs of preserving land can be substantially higher if the policymaker adopts a property or a gasoline tax. The property tax is more costly than the urban growth boundary or the development tax for all amounts of land saved. Note however the total costs of the property tax tend to decline relative to the UGB and development tax, a reflection of the relative lesser importance of the *adverse height effect* to the total costs of the policy as more land is saved. At 25% land saved, the property tax is roughly three and a half times more expensive than the UGB or development tax. However, up to around 3% land saved, the property tax is actually beneficial to society.

Finally, the gasoline tax is also more costly than the development tax or UGB because of the negative *disposable income effect*, but it is less costly than the property tax. At 25% land saved, the gas tax is roughly double the cost of the UGB or development tax, however up to 6% land saved, the gasoline tax also benefits society.

In summary, figures 1 and 2 not only confirm the main prediction of the analytical model, which suggested an equivalence between the UGB and the development tax but also shed light on some interesting features. First up to a certain percentage of land saved growth controls can impose benefits and not costs to the overall society because of a strong *capitalization effect*. However given the *adverse height effect* and a negative *disposable income effect* for the property tax and the gas tax respectively, only for very small percentages of land saved will these two policies benefit society as a whole. Furthermore, these adverse effects make these two latter instruments always less cost effective than an UGB or a development tax for the same amount of land saved. Finally, under our central estimates and assumptions regarding revenue recycling, the figures suggest the gasoline tax dominates the property tax from a cost effectiveness perspective.

C. Distribution of Costs to Landowners along the city

We now consider the distribution of costs to different landowners across the city. Figure 3 compares the total costs to different landowners of saving 10% of land.

$Cost^{UGB}$ shows the costs under the urban growth boundary. Consistent with the analytical model, the urban growth boundary effectively benefits landowners inside the city's urban area. In fact, landowners at the city center capture the entire welfare gain from the *capitalization effect*. Under our central estimates this source of welfare gain is significant and therefore $Cost^{UGB}$ crosses the horizontal axis below zero. $Cost^{UGB}$ also shows that this policy is neutral for landowners who were allocating land to agricultural use prior to the policy intervention. Finally, all the burden of the policy falls in the group of landowners who will be restricted from allocating land to residential use. These are the set of landowners who own land located between the city boundary prior to the policy intervention (located at 11.858 miles in the figure) and the new boundary (located at 11.205 miles from the CBD). They bear the full cost of the *size effect*. Note that this cost equals the difference between the residential rent (under no policy) and the agricultural rent, and therefore this cost is decreasing as we move further away from the CBD.

In contrast, the distribution of costs under the development tax, denoted by $Cost^{t_D}$, is very different than under the UGB. While the urban growth boundary mainly benefits landowners at the city center, the development tax penalizes these landowners and subsidizes landowners who do not develop. Therefore the costs under the development tax are positive and increasing up to the new city boundary, as the constant costs from the *net tax effect* outweigh the benefits from the *capitalization effect*. The development tax does not directly alter the landowners' density decisions and therefore is a fixed payment for all landowners who allocated land to residential use (i.e. between the CBD and the new boundary).

The set of landowners who own land located between the old and the new city boundary are better off with the development tax than under the urban growth boundary. This is because part of the revenues generated by this tax is returned to them as shown in (2.23B). Finally landowners who keep their land under agricultural use are also substantially better off under the development tax (relative to the UGB).

Note however, while a development tax for 10% of land saved decreases the value of residential land, it also increases the value of undeveloped land. From fig. 2 one infers the increase in land values from the revenue recycling policy more than compensates the decrease in land values from the dominance of the *net tax effect* over the *capitalization effect*. So a development tax saving 10% of land benefits society as a whole.

Next consider the effects of a property tax. The curve $Cost^{t_H}$ shows the distribution of costs under the property tax. Unlike the development tax, the property tax falls both on land and capital infrastructures. As a consequence, the property tax alters the optimal landowners' decision on density. The closer to the central business district, the higher the burden from the property tax because density levels decrease with distance. Another interesting point is that compared with the development tax, for the same amount of land saved, the property tax generates more revenue.

For the set of landowners whose land is located between the old and the new boundary, as well as for those who allocate land to agricultural use, the property tax is always the preferred instrument. For the majority of landowners who keep land in residential use this instrument implies a bigger cost when compared to the costs imposed by the development tax or the UGB.

Finally consider the impacts of a gasoline tax, represented in figure 3 by $Cost^{t_G}$. This curve contrasts the most with the property tax because it subsidizes landowners at the city center. The gasoline tax is a tax on miles driven with revenues returned lump sum to all households. As a consequence, the further away from the city a household lives, the higher their tax will be for the same amount of return. Hence, the curve increases at an increasing rate as we move away from the CDB. For landowners located after the new boundary, this policy is equivalent to the Urban Growth Boundary. This is because the revenues were distributed to households instead to landowners.

The two key insights from figure 3 are that both space and the way revenues from different policies are recycled determines the relative winners and losers from the different anti-sprawl policies. In particular, the figure suggests we can divide the city into areas: First, the *urban core* located between 0 to 6 miles from the CBD. For the landowners located in this region, the gasoline tax is always the preferred instrument. In fact, under this policy landowners will have a welfare gain. For this class of agents, the property tax is the less desired instrument. Second, the *suburbs* located between 6 and 11 miles from the CBD. For this group of landowners, the urban growth boundary will dominate the development tax, the gasoline tax and the property tax. The property tax continues to be the less desired instrument until 8.5 miles, at which point the gasoline tax becomes the least desirable instrument. Finally at the *urban fringe*, the property tax is always the preferred instrument and the gasoline and UGB are the worst (and equivalent).

D. Impact of Anti-Sprawl Policies on Urban Spatial Structure

We evaluate the impacts of different anti-sprawl policies on urban spatial structure by looking at the housing densities and overall population in the city resulting from the policy intervention. In the case of density, we are interested in examining the density impacts of policies at different distances from the CBD. We are particularly interested in the impact of the policies on the densification and re-habilitation of the city core, as this is often a major goal of smart-growth programs. In the case of population, we are interested in quantifying the overall population change in the city that has implemented the policy and, in turn, the magnitude of the spillover effect into neighboring communities.

First consider the impacts on density. In figure 4, we plot for each distance from the CBD the resulting densities from the different policies for a 10% savings in land. The figure highlights three important points. First, the level of density resulting from a property tax

– labeled in the figure as $Dens^{t_H}$ - is uniformly less relative to the density under the development tax or the UGB (denoted by $Dens^{t_D \& UGB}$). This suggests that the direct *adverse height effect* from the property tax dominates the *capitalization effect* and thus $\frac{\partial S_x}{\partial t_H} < 0$ for all $x \in \left[0, \bar{x}_{t_H}\right]$.

Second, the density curve under the gasoline tax – labeled as $Dens^{t_G}$ - is obtained by a clock-wise rotation around the point of the UGB density curve (approximately at 6 miles from the CBD) where the gas tax bill equals the lump sum transfer received by the households. So between 0 and 6 miles from the CBD, the density under the gasoline tax is higher relative to that under the urban growth boundary or development tax, but for distances greater than 6 miles the density is actually lower. This is because the *disposable income effect* will be positive and thus will reinforce the *capitalization effect* at the city core. After 6 miles, that is, in the *suburbs*, the *disposable income effect* is negative and dominates the *capitalization effect.*

Third, the density under the gasoline tax is always higher than under the property tax and is more so at the city center. However as predicted in the analytical model (see (2.48E)), because densities decrease with distance while the gas tax bill increases with distance, as one moves away from the city core the *adverse height effect* of the property tax decreases while the negative *disposable income effect* increase. At 11.65 miles, the city boundary for 10% saved land, the density levels under a gas tax and a property tax are exactly the same[18].

The figure also sheds some light on policy debates surrounding the re-vitalization and densification of city core. Policymakers often argue the densification of the city core could potentially result in a reduction of total vehicle miles driven (VMT) and resulting externalities (congestion and air pollution) because it will be easier to provide better and more frequent public transportation services. If indeed that is the goal, Figure 4 suggests the gasoline tax would be the most effective way to densify the city core. Again, we caution the reader that this result is dependant on the assumptions underlying the revenue recycling policy.

[18] For the analytical proof of this result see Appendix A.

Finally from the above discussion and because population is a function of housing density, if population spillovers are a concern, for 10% saved land the UGB and development tax produce the least spillover with a population 96.5% of the benchmark. The gasoline and property tax displace more people, with populations of 95.2% and 88% of the benchmark respectively.

E. Sensitivity Analysis

We end this section by exploring the sensitivity of the above results to some additional parameters variations. We focus on the cases where a change in the parameters would likely affect the magnitude of the efficiency channels embedded under the different policies and we include the benchmark as the reference case. In particular we change the price of capital, the elasticity of substitution between capital and land and the share of capital in the production function in order to mainly explore the sensitivity of the *adverse height effect*. We also vary the utility function parameters for the share of housing as well as for the capitalization weight to analyze the sensitivity of the *capitalization effect*. Finally we change total available land for development to mainly explore the magnitudes of the *disposable income effect,* the *capitalization effect* and *size effect*. Table 2 in Appendix C, summarizes the results of the sensitivity analysis.

(i) Changes in the share of capital (β), elasticity of substitution between capital and land (σ) and cost of capital (p_k): In the second row in table 2, we vary the share of capital between 0.93 and 0.97, in the third row we vary σ between 0.7 and 0.9 and on the forth row we vary the price of capital between \$80 and \$120. Increasing the share of capital or increasing the elasticity of substitution between capital and land increase both the optimal amount of density chosen by landowners and the size of the city. This on the one hand increases the relative magnitude of the *adverse height effect* and on the other hand also increases the magnitude of the *capitalization effect*. By increasing total population, it allows landowners to capture more open space rents through housing prices and by increasing optimal density levels it imposes landowners a bigger property tax burden. These two effects combined lead to lower total costs for the UGB, development tax and gasoline tax, but higher total costs for the property tax. Furthermore, in terms of

the distribution of costs among landowners we see lower costs for the UGB at the city center and city edge, lower costs at the city center and higher costs at the city edge for the development tax and gasoline tax, higher costs at the city center and lower costs at the city edge for the property tax, and finally greater subsidies to agriculture.

Decreasing the price of capital has nearly the exact same effects as increasing the share of capital or increasing the elasticity of substitution between capital and land, but instead of higher costs for the development tax at the city edge, we obtain lower costs. This occurs because the burden of the development tax imposed on developers is still the same, but rents from residential land use have increased.

(ii) Changes in the share of housing expenditures (α) and in the open space capitalization weight (δ): In the sixth row of table 2 we vary α between 0.37 and 0.43 and on the seventh row we vary δ between 0 and 0.005. Increasing the share of housing expenditures decreases the size of the city as well as density levels throughout the city. As a consequence, we see a reduction in total population for all the policy instruments. These effects also affect the relative magnitudes of the *capitalization effect*, the *disposable income effect* and the *adverse height effect*. Although open space has increased, the reduction in total population implies a decrease in the magnitude of the *capitalization effect*. Also the decrease in city size implies a reduction in the magnitude of the negative *disposable income effect* and the reduction in density levels through the city leads to a lighter property tax burden. As a result, costs under the UGB and development tax increase, while costs under the property tax and the gas tax decrease. However, the cost effectiveness ranking of the policies remains unchanged.

For landowners at the city center, an increase in the share of housing expenditure increases the cost of the UGB and the gasoline tax but lowers the cost of a development tax and a property tax, while landowners at the city edge see an increase in the cost of the UGB but a decrease in the other three polices. Also, due to the small city size and small revenues collected, the subsidy to rural landowners is decreased to virtually nil.

Increasing the capitalization parameter *(δ)* leads to a larger city and total population and higher housing densities. It also leads to a decrease in total costs making all policies beneficial to society as a whole at the 10% level, and decreases the cost of all policies at all points in the city.

(iii) Change in total available land for development ($\overline{m}$): In the fifth row of table 2 we vary $\overline{m}$ between 15 and 25 miles. Increasing the available land leads to an increase in the number of landowners who receive redistributed rents and an increase in the amount of open space. It also increases the size of the city, total population and the total costs of all policies as well as the costs of all policies at all points in the city. While it decreases the subsidy to agriculture it also leaves density levels essentially unchanged. The interesting point that stands out from the sensitivity analysis of this parameter is that the costs to society of the alternative policies are sensitive to the revenue recycling policy. A decrease in the $\overline{m}$ mimics a decrease in the number of landowners outside the urban area being subsidized, therefore it decreases the magnitude of the net tax effects as well as the magnitudes of the *adverse height effect* and the negative *disposable income effect*. At the same time it increases not only the relative magnitude of the *capitalization effect* but also its absolute value because we have an increase in the amount of undeveloped land. For 10 % of land saved, a UGB, a development tax and a gasoline tax are growth policies that benefit society as a whole because they impose a strong *capitalization effect* and thus big increases in the city overall land value. While the property tax is still a growth policy that penalizes society as a whole, its costs are smaller compared to the benchmark case.

As a final note, for the range of parameter values chosen, though the size of the cities and heights of the cities varied dramatically, the cost efficiency rankings, the distribution of costs, and the relative densities in the city remained the same, even if the resulting values were very different than the benchmark. Also a very interesting insight from the sensitivity analysis is that growth control policies can indeed be beneficial to society because of the enormous *capitalization effect* provided.

5. Conclusions

This paper has employed analytical and numerical general equilibrium models to compare the overall efficiency costs, the distributional costs and the urban spatial structure resulting from a range of anti-sprawl policy instruments. We find that from an efficiency perspective, development taxes and urban growth boundaries are equivalent instruments. In contrast, the costs of the gasoline or the property tax can be substantially higher.

We also find that, if distributional considerations matter, our model supports an empirical regularity that suggests that urban growth boundaries are a popular instrument to control sprawl. This is because, if landowners would have to choice amongst potential instruments, the urban growth boundary would be the instrument that will have greater political support.

Finally, while not cost-effective from the perspective of saving land, if the goal is to increase density, the gasoline tax is the instrument that achieves greater densities at the city core.

Some limitations in the present study deserve attention. First, our analysis does not incorporate heterogeneity of the landscape. Heterogeneity augments the information burdens faced by regulators, and consequently implies that urban growth boundaries will tend to be less efficient than suggested here, because a blind line that delimits growth may not separate two plots of land with very distinct characteristics.

Second, this study concentrates solely on the costs to the city that implements the policy and implicitly assumes the behavior of neighboring cities to be passive. Future work that integrates strategic responses into cost models of anti-sprawl policies may provide additional useful and highly practical policy insights.

APPENDIX A: THE ANALYTICAL MODEL

We now present a complete derivation of the key equations of the analytical model described and discussed in section 2 of the paper.

Deriving Equation (2.21B)

To calculate the gross efficiency costs of an incremental increase in the development tax, we differentiate the city total aggregated land value equation $\int_0^{\bar{x}(t_D)} r_u(p_x(O,\overline{V},Y^d), p_k, g-t_D)2\pi x dx + \int_{\bar{x}(t_D)}^{\bar{m}} (r_a+g)2\pi x dx$ with respect to t_D:

$$\int (\frac{\partial r_u}{\partial O}\frac{\partial O}{\partial t_D} + \frac{\partial r_u}{\partial t_D} + \frac{\partial r_u}{\partial g}\frac{\partial g}{\partial t_D})2\pi x dx + (r_u(\bar{x}) - t_D + g)2\pi \bar{x}\frac{\partial \bar{x}}{\partial t_D} + \int \frac{\partial g}{\partial t_D}2\pi x dx + (r_a+g)2\pi \bar{x}\frac{\partial \bar{x}}{\partial t_D} \quad \text{(A.1B)}$$

Differentiating the residential rent (2.17B) with respect to t_D, we get:

$$\frac{\partial r_u}{\partial t_D} = -1 \quad \text{(A.2B)}$$

Noticing that total open space in the city can be written as $O = \pi\left[\bar{m}^2 - \bar{x}^2\right]$, then:

$$\frac{\partial O}{\partial t_D} = -2\pi\bar{x}\frac{\partial \bar{x}}{\partial t_D} \quad \text{(A.3B)}$$

Differentiating the government budget constraint (2.20B) with respect to the development tax we have that:

$$t_D 2\pi\bar{x}\frac{\partial \bar{x}}{\partial t_D} = \frac{\partial g}{\partial t_D}\pi(\bar{m})^2 - \pi(\bar{x})^2 \quad \text{(A.4B)}$$

Finally, substituting (A.2B), (A.3B) and (A.4B) into (A.1B) yields equation (2.21B).

Deriving Equation (2.25B)

Total population under a development tax is given by:

$$N = \int_0^{\bar{x}} 2\pi x \frac{h_x(S_x)}{H_x} dx \quad \text{(A.5B)}$$

where density levels, S_x, are computed through the first order condition (2.18B). Equation (2.25D) is easily obtained by differentiating (A.5B) with respect to t_D.

In order to show that $\dfrac{\partial\left(\dfrac{h_x(S_x)}{H_x}\right)}{\partial t_D} > 0, \forall x \in \left[o, \bar{x}\right]$ we differentiate $r_u(t, Y, O, \bar{V}, \bar{x}) - t_D = r_a$ with respect to the city boundary $\bar{x}$ and then using (A.3B) and the fact that residential rents decrease with distance from the CBD, that is, $\dfrac{\partial r_u}{\partial x} = \dfrac{\partial p_x}{\partial x} h_x(S_x) < 0$ we get the impact of a development tax in the city boundary as:

$$\frac{\partial \bar{x}}{\partial t_D} = \frac{1}{\dfrac{\partial r_u}{\partial \bar{x}} - 2\pi \bar{x} \dfrac{\partial r_u}{\partial \partial O}} < 0 \tag{A.6B}$$

So the size effect represented by the term $\underbrace{2\pi \bar{x} \dfrac{h_{\bar{x}}(S_{\bar{x}})}{H_{\bar{x}}} \dfrac{d\bar{x}}{dt_D}}_{dN^S}$ in equation (2.25B) is negative. In order to find the impact of a development tax on population at distance x from the CBD, we differentiate $\dfrac{h_x(S_x)}{H_x}$ with respect to t_D, which yields:

$$\frac{\partial\left(\dfrac{h_x(S_x)}{H_x}\right)}{\partial t_D} = \frac{\dfrac{\partial h_x(S_x)}{\partial S_x}\dfrac{\partial S_x}{\partial t_D} H_x - \dfrac{\partial H_x}{\partial t_D} h_x(S_x)}{(H_x)^2}$$

(A.7B) Improvements per acre, S_x, are indirectly positively affected by the development tax through a capitalization effect. Differentiating the FOC for the optimal density level, given by (2.17B) and making use of (A.3B), (A.6B) and because the intensive housing production function exhibits diminishing returns to improvements, we have that the impact of the development tax on improvements is positive:

$$\frac{\partial S_x}{\partial t_D} = 2\pi \bar{x} \frac{\dfrac{\partial p_x}{\partial O}\dfrac{\partial h_x(S_x)}{\partial S_x}}{p_x \dfrac{\partial^2 h(S_x)}{\partial S_x^2}} \frac{\partial \bar{x}}{\partial t_D} > 0 \tag{A.8B}$$

To evaluate the sign of the dwelling size effect, represented by $\frac{\partial H_x}{\partial t_D} h_x(S_x)$ in equation (A.7B), we differentiate the optimal amount of housing demand with respect to t_D:

$$\frac{\partial H_x}{\partial t_D} = -2\pi \bar{x} \frac{\partial H_x}{\partial p_x} \frac{\partial p_x}{\partial O} \frac{\partial \bar{x}}{\partial t_D} < 0 \tag{A.9B}$$

Plugging (A.8B) and (A.9B) into (A.7B) we have that under a development tax densities increase all over the urban area, that is,

$$\frac{\partial \left(\frac{h_x(S_x)}{H_x} \right)}{\partial t_D} > 0, \forall x \in \left[o, \bar{x} \right] \tag{A.10B}$$

Thus under a development tax the improvements effect runs in the same direction as the dwelling size effect and thus the final impact of t_D in total population depends on the magnitude of the size effect $\left(\underbrace{2\pi \bar{x} \frac{h_{\bar{x}}(S_{\bar{x}})}{H_{\bar{x}}} \frac{d\bar{x}}{dt_D} < 0}_{dN^S} \right)$ compared to the magnitude of the impact of the tax on density levels through out the city $\left(\underbrace{\int_0^{\bar{x}_{t_D}} 2\pi x \frac{\partial \left(\frac{h_x(S_x)}{H_x} \right)}{\partial t_D} dx > 0}_{dN^C} \right)$.

Deriving Equation (2.27C)

Following the analogous steps as above for equation (2.21B), we differentiate $\int_0^{\bar{x}_{UGB}} r_u(p_x(O,\bar{V},\overline{Y}^d), p_k) 2\pi x dx + \int_{\bar{x}_{UGB}}^{\bar{m}} r_a 2\pi x dx$ with respect to $\bar{x}_{UGB}$ to compute the gross efficiency costs of an incremental increase in the urban growth boundary, which yields the following expression:

$$\int \frac{\partial r_u}{\partial O} \frac{\partial O}{\partial \bar{x}_{UGB}} 2\pi x dx + (r_u(\bar{x}) - r_a) 2\pi \bar{x} \frac{\partial \bar{x}}{\partial \bar{x}_{UGB}} \tag{A.11C}$$

Note that under a UGB no revenues are generated and given policies were designed to achieve the same percentage of saved land, then we have that $\frac{\partial \bar{x}}{\partial \bar{x}_{UGB}} = \frac{\partial \bar{x}}{\partial t_D}$. Plugging (A.3B) into (A.11C) yields (2.27C).

Deriving equation (2.36D)

Under a property tax, the government budget constraint is given by:

$$\int_0^{\bar{m}} g 2\pi x dx = \int_0^{\bar{x}_{t_H}} t_H h^s(S_x) 2\pi x dx \tag{A.12D}$$

Differentiating (A.12D) with respect to the property tax, t_H, we get that:

$$\pi \bar{m}^2 \frac{\partial g}{\partial t_H} - \int_0^{\bar{x}} h_x(S_x) 2\pi x dx = \int_0^{\bar{x}} t_H \frac{\partial h_x(S_x)}{\partial t_H} 2\pi x dx + t_H h_x(S_{\bar{x}}) 2\pi \bar{x} \frac{\partial \bar{x}}{\partial t_H} \tag{A.13D}$$

Following the analogous procedure used for equation (2.21B) we differentiate $\int_0^{\bar{x}_{t_H}} (r_u(p_x(O,\bar{V},Y^d), p_k, t_H) + g) 2\pi x dx + \int_{\bar{x}_{t_H}}^{\bar{m}} (r_a + g) 2\pi x dx$ with respect to t_H and then using (A.13D) gives us equation (2.36D), the gross efficiency costs of an incremental increase in the property tax, t_H.

Although $\frac{\partial p_x}{\partial O} \frac{\partial O}{\partial t_H}$ is the same for all policy instruments, the magnitude of the capitalization effect is not the same. For the case of the property tax there is a direct effect of t_H on the optimal density level as we can see from the FOC (2.35D). Given that $\frac{\partial^2 h_x(S_x)}{\partial S_x^2} < 0$ then the optimal S_x that satisfies (2.17B) is bigger than the one that satisfies (2.35D). Consequently and given $\frac{\partial \bar{x}}{\partial \bar{x}_{UGB}} = \frac{\partial \bar{x}}{\partial t_D} = \frac{\partial \bar{x}}{\partial t_H}$, a weaker capitalization effect rises under a property tax compared to the one under an UGB or a development tax:

$$\int_0^{\bar{x}_{t_H}} \left[-2\pi \bar{x} \frac{\partial \bar{x}}{\partial t_H} \frac{\partial p_x}{\partial O} h_x(S_x^{t_H}) \right] 2\pi x dx < \int_0^{\bar{x}_{t_D}} \left[-2\pi \bar{x} \frac{\partial \bar{x}}{\partial t_D} \frac{\partial p_x}{\partial O} h_x(S_x^{t_D}) \right] 2\pi x dx \qquad \text{(A.14D)}$$

Deriving equation (2.40.D)

To compute the impact of the property tax on total population we follow the same procedure as for equation (2.25B). So differentiating (A.5) with respect to t_H yields equation (2.40D):

$$\frac{dN}{dt_H} = \underbrace{2\pi \bar{x} \frac{h_{\bar{x}}(S_{\bar{x}})}{H_{\bar{x}}} \frac{d\bar{x}}{dt_H}}_{dN^S} + \underbrace{\int_0^{\bar{x}_{t_D}} 2\pi x \frac{\partial \left(\frac{h_x(S_x)}{H_x} \right)}{\partial t_H} dx}_{dN^{C+H}} \qquad \text{(A.15D)}$$

The first term in the right side of (A.15D) represents the size effect and the second term on the right side of (A.15D) represents the effect on densities within the urban area due to the property tax policy.

Deriving the sign of (2.40D)

To prove the size effect that emerges with the use of t_H is negative we first differentiate $r_u(t_H, O, t, Y, \bar{x}, \bar{V}) = r_a$:

$$\frac{\partial r_u}{\partial t_H} + \frac{\partial r_u}{\partial O} \frac{\partial O}{\partial t_H} + \frac{\partial r_u}{\partial \bar{x}} \frac{\partial \bar{x}}{\partial t_H} = 0 \qquad \text{(A.16D)}$$

Then we compute the impact of the property tax on residential rents through the direct adverse effect, the capitalization effect and through the size effect. Differentiating (2.34D) with respect to t_H gives:

$$\frac{\partial r_u}{\partial t_H} + \frac{\partial r_u}{\partial O} \frac{\partial O}{\partial t_H} = \left[\frac{\partial p_x}{\partial O} \frac{\partial O}{\partial t_H} - 1 \right] h_x(S_x) \qquad \text{(A.17D)}$$

Substituting (A.17 D) and $\frac{\partial O}{\partial t_H} = -2\pi \bar{x} \frac{\partial \bar{x}}{\partial t_H}$ as well as $\left. \frac{\partial r_u}{\partial x} \right|_{x=\bar{x}} = \left. \frac{\partial p_x}{\partial x} \right|_{x=\bar{x}} h_{\bar{x}}(S_{\bar{x}})$ into (A.16 D) and solving for $\frac{\partial \bar{x}}{\partial t_H}$ we get the impact of t_H on the city boundary is given by:

$$\frac{\partial \bar{x}}{\partial t_H} = \frac{h_x(S_x)}{\left.\frac{\partial p_x}{\partial x}\right|_{x=\bar{x}} h_{\bar{x}}(S_{\bar{x}}) - 4\pi \bar{x} \frac{\partial p_x}{\partial O} h_x(S_x)} < 0 \qquad \text{(A.18D)}$$

In order to analyze the impact of the property tax on improvements at distance x from the CBD, we differentiate $\frac{\partial h_x(S_x)}{\partial S_x}$ with respect to the property tax and we get as before the impact of the tax depends on the net effect between the improvements effect and the dwelling size effect:

$$\frac{\partial\left(\frac{\partial h_x(S_x)}{\partial S_x}\right)}{\partial t_H} = \frac{\frac{\partial h_x(S_x)}{\partial S_x}\frac{\partial S_x}{\partial t_H} H_x - h_x(S_x)\frac{\partial H_x}{\partial p_x}\frac{\partial p_x}{\partial O}\frac{\partial O}{\partial t_H}}{(H_x)^2} \qquad \text{(A.19D)}$$

Differentiating the optimal density level FOC, (2.35D), and making use of (A.3B), (A.6B) and that the intensive housing production function exhibits diminishing returns to improvements, we see the impact of t_H on improvements is unclear:

$$\frac{\partial S_x}{\partial t_H} = \frac{\left[-\frac{\partial p_x}{\partial O} 2\pi \bar{x} \frac{\partial \bar{x}}{\partial t_H} - 1\right]\frac{\partial h_x(S_x)}{\partial S_x}}{-[p_x - t_H]\frac{\partial^2 h(S_x)}{\partial S_x^2}} \gtrless 0 \qquad \text{(A.20D)}$$

From (A.20D) one can see the effect of the property tax on density levels within the city depends on whether the capitalization effect net of the adverse effect on densities is positive or negative. If it is negative, that is, $-\frac{\partial p_x}{\partial O} 2\pi \bar{x} \frac{\partial \bar{x}}{\partial t_H} < 1$ and the improvement effect dominates the dwelling size effect, $\frac{\partial h_x(S_x)}{\partial S_x}\frac{\partial S_x}{\partial t_H} h_x^d > h_x(S_x)\frac{\partial h_x^d}{\partial p_x}\frac{\partial p_x}{\partial O}\frac{\partial O}{\partial t_H}$, then the property tax leads to a reduction of densities within the urban area, which exacerbates the decrease in population due to the size effect.

Deriving equation (2.43E)

A gasoline tax increases household's commuting costs. However given the revenue recycling policy, this instrument can subsidize households in certain areas of the city by increasing their disposable income. Let x^j be a location where households pay a gasoline tax bill exactly equal to the transfer g they get back from the government. Then, we have for a household at location x^j:

$$g = t_2 t_G x^j \tag{A.21E}$$

But for a household located at any $x \in \left[0, \overline{x}_{t_G}\right] / x^j$ we have:

$$g \neq t_2 t_G x \Leftrightarrow t_2 t_G (x^j - x) \neq 0 \tag{A.22E}$$

The disposable income under a gasoline tax for a household located at distance x from the CBD can then be written as:

$$Y_x^d = Y - tx - t_2 x + g \tag{A.23E}$$

Inserting (A.22E) into (A.23E) we have:

$$Y_x^d = Y - tx + t_2 t_G (x_j - x) \tag{A.24E}$$

And from (A.23E) one can see for households located at:

(i) $x \in \left[0, x^j\right[$, there was an increase in their disposable income (A.25E)

(ii) $x = x^j$, there was no change in their disposable income (A.26E)

(iii) $x \in \left]x^j, \overline{x}_{t_G}\right]$, there was a decrease in their disposable income (A.27E)

Differentiating the household's bid for housing

$$p_x^a = p(Y^d(Y, t, x, t_G, g), \overline{V}, O) \tag{A.28E}$$

and noting the disposable income under a gas tax is given by (A.24E), the impact of the gas tax on housing prices is obtained as:

$$\frac{\partial p_x}{\partial t_G} = -2\pi \overline{x} \frac{\partial p_x}{\partial O} \frac{\partial \overline{x}}{\partial t_G} + \frac{\partial p_x}{\partial Y^d} t_2 (-x + x^j + t_G \frac{\partial x^j}{\partial t_G}) \tag{A.29E}$$

where $t_2 (x^j + t_G \frac{\partial x^j}{\partial t_G}) = \frac{\partial g}{\partial t_G}$, $t_G \in [0,1]$, $\frac{\partial x^j}{\partial t_G} < 0$ and the total impact of the gas tax on disposable income is given by:

$$\frac{dY^d}{dt_G} = t_2 (-x + x^j + t_G \frac{\partial x^j}{\partial t_G}) \tag{A.30E}$$

Note that $x^j + t_G \frac{\partial x^j}{\partial t_G}$ is the new location where the transfer received by a household now exactly equals the gas tax paid. The sign of (A.30E) depends on the location in the city:

$$\frac{dY^d}{dt_G} = \begin{cases} > 0 & if \quad x < x^j + t_G \frac{\partial x^j}{\partial t_G} \\ = 0 & if \quad x = x^j + t_G \frac{\partial x^j}{\partial t_G} \\ < 0 & if \quad x > x^j + t_G \frac{\partial x^j}{\partial t_G} \end{cases} \tag{A.31E}$$

Differentiating $r_u(\bar{x}, t_G) = r_a$ and solving for $\frac{\partial \bar{x}}{\partial t_G}$ while noting that $\frac{\partial r_u}{\partial t_G} = \left\{ \frac{\partial p_x}{\partial Y^d} \frac{dy^d}{dt_G} - 2\pi \bar{x} \frac{\partial p_x}{\partial O} \frac{\partial \bar{x}}{\partial t_G} \right\} h_x(S_x)$ we get the impact of the gasoline tax in the radius of the residential area as:

$$\frac{\partial \bar{x}}{\partial t_G} = \frac{-\frac{\partial p_x}{\partial Y_d} \left.\frac{dY^d}{dt_G}\right|_{x=\bar{x}} h_x(S_{\bar{x}})}{\frac{\partial r_u}{\partial \bar{x}} - 2\pi \bar{x} \frac{\partial p_{\bar{x}}}{\partial O} h_x(S_{\bar{x}})} < 0 \tag{A.32E}$$

From (A.29E) we have that $\left.\frac{dY^d}{dt_G}\right|_{x=\bar{x}} < 0$.

Finally, totally differentiating the government budget (2.42E), we get:

$$\int_0^{\bar{x}} \frac{\partial N_x}{\partial t_G}(x^j - x)t_2 t_G 2\pi x dx + \int_0^{\bar{x}} N_x \left(\frac{\partial g}{\partial t_G} - x t_2 \right) 2\pi x dx + N_{\bar{x}}(x^j - x)t_2 t_G 2\pi \bar{x} \frac{\partial \bar{x}}{\partial t_G} \tag{A.33E}$$

Following the analogous steps as before to compute the efficiency costs, we differentiate $\int_0^{\bar{x}(t_G)} r_u(p_x^a(O, \overline{V}, Y - tx - t_2 t_G x + g), p_k) 2\pi x dx + \int_{\bar{x}(t_G)}^{\bar{m}} r_a 2\pi x dx$ with respect to t_G and noting urban rents under a gasoline tax can also be written as

$r_u = p(Y^d(Y,t,x,t_G,g),\overline{V},O)h(S_x) - p_k S_x$ and inserting (A.32E) and (A.33E) we get equation (2.44E).

Deriving equation (2.47E)

(2.47E) was derived in the same fashion as (2.25B), (2.31C) and (2.40D).

Deriving the sign of (2.47E)

Because disposable income affects housing bids, $\frac{\partial p_x}{\partial Y^d} > 0$, and given the optimal density level depends positively on housing prices, then the gas tax also affects density levels through a disposable income effect.

Differentiating the FOC for the optimal density level under a gas tax and solving for $\frac{\partial S_x}{\partial t_G}$ we get:

$$\frac{\partial S_x}{\partial t_G} = -\frac{\left\{\frac{\partial p_x}{\partial Y^d}\frac{dY^d}{dt_G} - \frac{\partial p_x}{\partial O}2\pi\overline{x}\frac{\partial \overline{x}}{\partial t_G}\right\}\frac{\partial h_x(S_x)}{\partial S_x}}{p_x^a\frac{\partial^2 h_x(S_x)}{\partial S^2}} \tag{A.34E}$$

Given (A.31E) then the sign of (A.34E) is indeterminate since it depends on the location in the city.

Let $x^t = x^j + t_G\frac{\partial x^j}{\partial t_G}$,that is, x^t is the location where the transfer received equals the gas tax bill paid, then we have:

$$\frac{\partial S_x}{\partial t_G}\begin{cases} > 0 & for \quad x \in \left[0, x^t\right] \\ < 0 & for \quad x \in \left]x^t, \overline{x}_{t_G}\right] \end{cases} \tag{A.35E}$$

Note that for the case where $x = x^t$ then (A.34E) reduces to (A.8D), that is, to the impact we would have under an UGB or a development tax. This is because at that particular location there is no disposable income effect (see (A31.E)).

However at $x = x^t$:

$$\frac{\partial S_x}{\partial t_G} = \frac{\partial S_x}{\partial \bar{x}_{UGB}} = \frac{\partial S_x}{\partial t_D} > \frac{\partial S_x}{\partial t_H} \tag{A.36E}$$

But for $x \in \left] x^t, \bar{x}_{t_G} \right]$ we have the negative disposable income effect dominates the positive capitalization effect:

$$\frac{\partial S_x}{\partial \bar{x}_{UGB}} = \frac{\partial S_x}{\partial t_D} > \frac{\partial S_x}{\partial t_G} > \frac{\partial S_x}{\partial t_H} \tag{A.37E}$$

So there is a location where the negative disposable income effect converges to the adverse effect of the property tax such that:

$$\frac{\partial S_x}{\partial \bar{x}_{UGB}} = \frac{\partial S_x}{\partial t_D} > \frac{\partial S_x}{\partial t_G} = \frac{\partial S_x}{\partial t_H} \tag{A.38E}$$

Because densities decrease with distance, $\frac{\partial S_x}{\partial x} < 0$ while the tax bill increase with distance from the CBD, $\frac{\partial tx}{\partial x} > 0$, as one moves away from the city core the adverse effect of the property tax decreases while the negative disposable income effect becomes stronger. So there is an x where density levels under these two instruments will coincide. Comparing (A.20D) and (A.34E), this x must be such that $p_x - t_H = p_x^a$ and $\frac{\partial p_x}{\partial Y^d}\frac{\partial Y^d}{\partial t_G} = -1$. From the condition of equality at the city boundary between urban and agriculture rents, the location where the level of density under a housing tax equals the level of density under a gas tax is at the city boundary: $x = \bar{x}_{t_G} = \bar{x}_{t_H}$. Under a property tax we have at $x = \bar{x}_{t_H}$:

$$r_u(p_x - t_H, p_k) + g = r_a + g \Leftrightarrow r_u(p_x - t_H, p_k) = r_a \tag{A.39E}$$

Under a gasoline tax we have at $x = \bar{x}_{t_G}$:

$$r_u(p_x^a, p_k) = r_a \tag{A.40E}$$

From (A.39E) and (A.40E) it follows that at $x = \bar{x}_{t_H} = \bar{x}_{t_G}$:

$$r_u(p_x - t_H, p_k) = r_a = r_u(p_x^a, p_k) \Rightarrow p_x - t_H = p_x^a \tag{A.41E}$$

So one concludes density levels under a gasoline tax, $S_x^{t_G}$, are:

$$S_x^{t_G}\begin{cases} > S_x^{t_D} = S_x^{UGB} > S_x^{t_H} & for \quad x \in \left[0, x^t\right] \\ < S_x^{t_D} = S_x^{UGB} & for \quad x \in \left]x^t, \bar{x}_{t_G}\right] \\ \geq S_x^{t_H} & for \quad x \in \left]x^t, \bar{x}_{t_G}\right] \end{cases} \tag{A.42E}$$

and the gasoline tax promotes densification of the city core compared to the alternative policy instruments. Graphically, this means the density function under a gasoline tax will rotate clockwise at location $x = x^t$ relative to the density function in the benchmark. For a development tax or a UGB, we have a upward parallel shift of the density function due to the capitalization effect and for a property tax we have a downward, nonparallel shift because the net burden of the property tax on landowners decreases with distance and the adverse effect dominates the capitalization effect.

Therefore the sign of (2.47E) is also indeterminate.

APPENDIX B: THE NUMERICAL MODEL

We now present a complete description of the model introduced in section 3. We start by enumerating all the parameters and endogenous variables in households, firms and government behaviors and then we derive all the equations presented in the description of the numerical model in section 3. Except where otherwise noted, i ranges over t_D, t_G, t_H and $\overline{x}_{UGB}$, which represent a tax per unit of residential land, a gasoline tax per mile traveled, a tax per unit of housing and an urban growth boundary policy respectively.

I. Parameters

Landscape

θ	Angular direction in the Polar Plane
x	Radial distance "from location x" to the CBD in the Polar Plane
(θ,x)	Polar coordinate system to represent locations
$\overline{m}$	Exogenous circle radius of total available land
Origin	Location of the CBD in the Polar Plane

Household Behavior Parameters

α	Percentage of income net of transportation costs that is spent on housing
t	Total annual commuting cost per round trip mile
t_1	Total annual commuting time costs per round trip mile
t_2	Total annual commuting money costs per round trip mile
Y	Annual income
$\overline{V}$	Exogenous utility level
δ	Positive constant parameter in the utility function

Landowners Behavior Parameters

β	Share of capital in the housing production function
σ	Elasticity of substitution between capital and land in the housing production function
p_K	Exogenous rental cost of structural capital
r_a	Exogenous agricultural land rent per acre
ψ	Exogenous positive parameter value in the housing production function

Government Policy Parameters

O	Undeveloped land target surrounding the urban area

G Government spending (lump sum transfers to landowners or households)

II. Endogenous Variables

$H(x)$ Household demand for residential space in "location x"
$Z(x)$ Household demand for the composite good
$p(x)$ Bid price function for housing at distance x / market housing price at "location x"
$S(x)$ Density level at distance x / height of building at distance x
$r_u(x)$ Rent per unit of land in residential use at "location x"
$K(x)$ Structural capital demand for housing production at distance x
$L(x)$ Land for housing production at distance x
$(\bar{x})^b$ Radius of the city prior to any government intervention
D^b Total land area in residential use before the introduction of policy i
$N^b(x)$ Total population at distance x before government intervention
$r_u^b(x)$ Rent per unit of land in residential use at "location x" before policy i
t_D Tax per unit of residential land
t_H Tax per unit of housing
t_G Gasoline Tax
$\bar{x}_{UGB}$ Radius of the city imposed by an urban growth boundary
D^{ai} Total land area in residential use after the introduction of policy
$N^{ai}(x)$ Total population at distance x after the introduction of policy i
$(\bar{x})^{ai}$ Radius of the city after the introduction of policy i
$r_u^{ai}(x)$ Rent per unit of land in residential use at "location x" after policy i
$SC(i)$ Society's costs under policy i

III. Equations

Household problem: Housing and Composite Good Demand Functions and Housing bid Price Function

Households´ preferences are represented by:

$$U(H,Z,O) = H(x)^{\alpha} Z(x)^{(1-\alpha)} (1+\delta\sqrt{O}) \tag{B1}$$

At each at distance x, households maximize their utility function $(B1)$ subject to the budget constraint:

$$p(x)H(x) + Z(x) + tx = Y \tag{B2}$$

This maximization yields the following optimal housing and composite good demand functions prior to any policy:

$$H(x) = \frac{\alpha(Y - tx)}{p(x)} \tag{B3}$$

$$Z(x) = (1-\alpha)(Y - tx) \tag{B4}$$

Plugging equations $(B3)$ and $(B4)$ into the utility function $(B1)$ we get the indirect utility function:

$$V(x)=\frac{\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}(Y-tx)(1+\delta\sqrt{O})}{p(x)^{\alpha}} \tag{B5}$$

Given the locational no-arbitrage condition, in equilibrium all households must achieve the same level of utility. Setting the indirect utility function $(B5)$ equal to $\overline{V}$ and solving for $p(x)$ we get the following households′ bid price function for housing at distance x:

$$p(x)=\left[\frac{\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}}{\overline{V}}\right]^{\frac{1}{\alpha}}(Y-tx)^{\frac{1}{\alpha}}(1+\delta\sqrt{O})^{\frac{1}{\alpha}} \tag{B6}$$

Equation $(B6)$ along with equations $(B3)$ and $(B4)$ express the household's behavior at each distance x from the CBD.

Landowners Problem: density level, urban rent and amount of developed area

Housing is produced according to:

$$H^{s}(x)=\psi\left[\beta K(x)^{\frac{\sigma-1}{\sigma}}+(1-\beta)L(x)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}},\ \sigma\neq 1 \tag{B7}$$

Given that $H^{s}(x)$ is homogeneous of degree one and thus exhibits constant returns to scale we can express housing supply as a function of the capital-to-land ratio, which is a measure of the density level at a particular distance. Let $S(x)=\frac{K(x)}{L(x)}$. Then $(B7)$ can be expressed as:

$$h(S(x))=\psi\left[\beta S(x)^{\frac{\sigma-1}{\sigma}}+(1-\beta)\right]^{\frac{\sigma}{\sigma-1}} \tag{B8}$$

Landowners choose $S(x)$ in order to maximize land rent from residential use, taking P_K and $(B6)$ as given.

Residential rent per unit of land at distance x is given by:

$$r_u(x)=p(x)h(S(x))-P_K S(x) \tag{B9}$$

Inserting $(B6)$ and $(B8)$ into $(B9)$ and after manipulating the expression we get the residential rent per unit of land at distance x before any policy as:

$$r_u^{b}(x)=\left[\left[\frac{\alpha^{\alpha}\left(1-\alpha\right)^{(1-\alpha)}}{\overline{V}}\right]^{\frac{1}{\alpha}}(Y-tx)^{\frac{1}{\alpha}}(1+\delta\sqrt{O})^{\frac{1}{\alpha}}\right]\psi\left[\beta S(x)^{\frac{\sigma-1}{\sigma}}+(1-\beta)\right]^{\frac{\sigma}{\sigma-1}}-p_K S(x) \tag{B10}$$

The optimal density level that results from the maximization of $(B10)$ is given by:

$$S(x)=\left[\frac{1-\beta}{\left[\frac{p_K}{\psi p(x)\beta}\right]^{\sigma-1}-\beta}\right]^{\frac{\sigma}{\sigma-1}} \tag{B11}$$

Substituting $(B11)$ into $(B10)$ we get the maximum rent for land in residential use at distance x as:

$$r_u(x)=(1-\beta)^{\frac{\sigma}{\sigma-1}}\left[(\psi p(x))^{1-\sigma}-\beta^{\sigma}p_K{}^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \tag{B12}$$

Prior to any government intervention, total amount of land that will be devoted to residential use prior is given by:

$$D^b\equiv\{(x,\theta):r_u(x,\theta)\geq r_a\} \qquad \text{B13)}$$

that is,

$$D^b\equiv\left\{(x,\theta):x\leq\frac{Y}{t}-\frac{1}{t}\frac{\overline{V}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}(1+\delta\sqrt{O})}\left[\frac{\left[\beta^{\sigma}(p_K)^{1-\sigma}+(1-\beta)^{\sigma}(r_a)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}}{\psi}\right]^{\alpha}\right\} \tag{B14}$$

D^b represents the area of a circle, defined in polar coordinates, centered at the origin. The city boundary is defined by the radius of the circle computed in $(B14)$:

$$\left(\overline{x}\right)^b=\frac{Y}{t}-\frac{1}{t}\frac{\overline{V}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}(1+\delta\sqrt{O})}\left[\frac{\left[\beta^{\sigma}(p_K)^{1-\sigma}+(1-\beta)^{\sigma}(r_a)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}}{\psi}\right]^{\alpha} \tag{B15}$$

For simplicity total available land is circular with a radius equal to $\overline{m}$ miles. So, total undeveloped land can be computed as:

$$O=\pi\left[\overline{m}^2-\left(\left(\overline{x}\right)^b\right)^2\right] \tag{B16}$$

The number of households living in this city is determined as the sum for all urban area of the ratio between housing supply (with $(B11)$ introduced in $(B8)$) at distance x and individual housing demand, $(B3)$:

$$N^T=\int_0^{\left(\overline{x}\right)^b}\frac{p(x)h(S(x))}{\alpha(Y-tx)}2\pi x\,dx \tag{B17}$$

The benchmark city is fully characterized by equations (B14), (B15), (B16) and (B17).

Under a development tax, an UGB and a property tax there are no constraints imposed on households' behavior.

Development Tax

Under a development tax, rent per unit of land in residential use is given by:

$$r_u(x)=p(x)h(S(x))-P_K S(x)-t_D+g \tag{B18}$$

and its maximum value is computed as before and is given by:

$$R_u^{at_D}(x) = (1-\beta)^{\frac{\sigma}{\sigma-1}}\left[(\psi p(x))^{1-\sigma} - \beta^{\sigma}(p_K)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} - t_D + g \tag{B19}$$

Rent per unit of land in agriculture use is given by:

$$r_a + g \tag{B20}$$

where the lump sum transfer is computed as:

$$g = t_L\left(\frac{\left(\bar{x}\right)^{at_D}}{\bar{m}}\right)^2 \tag{B21}$$

and the new city boundary under the residential land tax policy:

$$\left(\bar{x}\right)^{at_D} = \frac{Y}{t} - \frac{1}{t}\frac{\bar{V}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}(1+\delta\sqrt{O})}\left[\frac{\left[\beta^{\sigma}(p_K)^{1-\sigma} + (1-\beta)^{\sigma}(r_a + t_D)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}}{\psi}\right]^{\alpha} \tag{B22}$$

From (B22) we get the amount of t_D that achieves the amount of open space defined by:

$$O = \pi\left[\bar{m}^2 - \left(\left(\bar{x}\right)^{at_g}\right)^2\right] \tag{B23}$$

$$O = a\%\pi\left(\left(\bar{x}\right)^{b}\right)^2 \tag{B24}$$

where equation $(B23)$ represents total amount of open space obtained with the imposition of the policy instrument and $(B24)$ correspond to government's target for undeveloped land, with $a\%$ the percentage of land that government wishes to save. In our simulation a ranges from 0 to 30%. To provide meaningful comparisons between the policy instrument all instruments are set to achieve the same percentage of land and the rule for the open space target used in all policy instruments is given by (B24).

Urban Growth Boundary

An UGB does not affect landowners' decision concerning densities levels (B11). It also does not affect the return landowners get from residential land given by (B12). It just imposes a restriction on the amount of land that can be developed. Thus the new city boundary under an UGB to achieve an amount of open space O given by (B24) is computed as:

$$O = \pi\left[\bar{m}^2 - \left(\bar{x}_{UGB}\right)^2\right] \tag{B25}$$

Property Tax

Under a property tax, rent per unit of land in residential use is given by:

$$r_u(x) = (p(x) - t_H)h(S(x)) - P_K S(x) + g \tag{B26}$$

and its maximum value is given by:

$$R_u^{at_H}(x) = (1-\beta)^{\frac{\sigma}{\sigma-1}}\left[(\psi(P(x) - t_H))^{1-\sigma} - \beta^{\sigma}(P_K)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} + g \tag{B27}$$

Rent per unit of land in agriculture use is given by:

$$r_a + g \tag{B28}$$

where the lump sum transfer is computed as:

$$g = \frac{1}{\pi \overline{m}^2} \int_0^{(\overline{x})^{atH}} t_H h(S^{at_H}(x)) 2\pi x dx \tag{B29}$$

and the optimal density level that maximizes (B27) is given by:

$$S^{at_H}(x) = \left[\frac{1-\beta}{\left[\frac{p_K}{\psi(p(x)-t_H)\beta} \right]^{\sigma-1} - \beta} \right]^{\frac{\sigma}{\sigma-1}} \tag{B30}$$

The city boundary under the property land tax policy is:

$$(\overline{x})^{at_H} = \frac{Y}{t} - \frac{1}{t} \frac{\overline{V}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}(1+\delta\sqrt{O})} \left[\frac{\left[\beta^{\sigma}(p_K)^{1-\sigma} + (1-\beta)^{\sigma}(r_a)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}}{\psi} + t_H \right]^{\alpha} \tag{B31}$$

From (B31) we get the amount of t_H that would allows to achieve the amount of open space defined by (B24).

Gasoline Tax

A Gasoline tax affects directly households' behavior by increasing commuting costs.
Under a gasoline tax, the unitary operating costs of transportation increase in the amount $t_g t_2$. The total annual commuting cost per round trip mile under this policy is then given by:

$$t^a = t_1 + t_2(1+t_g) \tag{B32}$$

and the budget constraint a household face under this policy instrument is:

$$p^a(x)H(x) + Z(x) + t^a x = Y + g \tag{B33}$$

where g is the lump sum transfer a household at distance x receives.

$$g = \frac{\int_0^{(\overline{x})^{atG}} N(x) t_G t_2 x 2\pi x dx}{N^{Ta}} \tag{B34}$$

where N^{Ta} is total population after the introduction of the gasoline tax.
Under the gasoline tax policy the housing bid function is represented by the following function:

$$p^{at_G}(x) = \left[\frac{\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}}{\overline{V}} \right]^{\frac{1}{\alpha}} (Y + g - t^a x)^{\frac{1}{\alpha}} (1+\delta\sqrt{O})^{\frac{1}{\alpha}} \tag{B35}$$

The problem solved by landowners is the same as described before but now the density level that maximizes residential land at distance x is given by:

$$S^{at_G}(x) = \left[\frac{1-\beta}{\left[\frac{p_K}{\psi p^{at_G}(x)\beta} \right]^{\sigma-1} - \beta} \right]^{\frac{\sigma}{\sigma-1}} \tag{B36}$$

Total population is computed as:

$$N^{Tat_G} = \int_0^{(\bar{x})^{at_G}} \frac{p^{at_G}(x) h(S^{at_G}(x))}{\alpha\left(Y + g - t^a x\right)} 2\pi x dx \tag{B37}$$

The rent per unit of residential land at "location x" when a gasoline tax is imposed is given by:

$$R_u^{at_G}(x) = (1-\beta)^{\frac{\sigma}{\sigma-1}} \left[(\psi p^{at_G}(x))^{1-\sigma} - \beta^{\sigma}(p_K)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \tag{B38}$$

The gasoline tax, t_G, that must be charged to attain the specific total amount of undeveloped land given by (B24) is calculated implicitly through the new city boundary equation:

$$\left(\bar{x}\right)^{at_g} = \frac{Y+g}{t^a} - \frac{1}{t^a} \frac{\bar{V}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}(1+\delta\sqrt{O})} \left[\frac{\left[\beta^{\sigma}(p_K)^{1-\sigma} + (1-\beta)^{\sigma}(r_a)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}}{\psi} \right]^{\alpha} \tag{B39}$$

Costs to society of the alternative policies

Let b denote the pre-policy situation and ai the after-policy i situation. Then society's costs, $SC(i)$, under instrument i are calculated as the change in aggregated land value due to the policy used:

$$SC(i) = \left(\int_0^{(\bar{x})^b} r_u^b(x) 2\pi x dx - \int_0^{(\bar{x})^{ai}} r_u^{ai}(x) 2\pi x dx \right) + \pi\left(r_a^{ai}\left(\bar{x}\right)^{2ai} - r_a^b\left(\bar{x}\right)^{2b} \right) \tag{B40}$$

The first component of the right hand side of equation (14) represents the change in total residential land value from the introduction of policy i. $r_u^b(x)$ and $r_u^{ai}(x)$ are landowners residential rent values at "location x" before and after the introduction of the policy instrument i, respectively. The second component represents the change in total agriculture land value.

APPENDIX C

Table 1-Parameters used in the simulation model

Parameter	Parameter Value
Annual disposable income (Y)	$40000
Annual money cost of commuting per mile (t_2)	$270
Annual time cost of commuting per mile (t_1)	$330
Percentage of income net of transportation cost spent on housing (α)	0.4
Exogenous utility level ($\overline{V}$)	2200
Elasticity of substitution between land and capital (σ)	0.8
Share of capital in housing production function (β)	0.95
Unitary cost of capital (P_K)	$100
Exogenous city radius ($\overline{m}$)	20
Exogenous amenity utility parameter (δ)	0.003
Exogenous housing production parameter (ψ)	1
Agriculture rent (r_a)	$1000

Table 2: Sensitivity Analysis Results

Benchmark	Policy for 10% saved land	City edge pre-policy	Total Cost	Cost at 0	Cost at Xbar	Subsidy to agriculture	Density at City Center	Density at Xbar	Total Households
	UGB	11.8587	-535928	-49	-5	0	680	79	446222
	Development	11.8587	-535928	112	156	75	680	79	446222
	Property	11.8587	$5.800*10^6$	1259	66	164	636	67	407527
	Gasoline	11.8587	$2.150*10^6$	-2363	231	0	748	67	440748
Share of capital in the production function									
0.93	UGB	9.0589	475543	-9	-2	0	224	55	122389
	Development	9.0589	475543	96	102	24	224	55	122389
	Property	9.0589	$1.397*10^6$	360	90	37	212	50	115269
	Gasoline	9.0589	921580	-727	127	0	243	50	121642
0.97	UGB	14.9388	$-3.174*10^7$	-544	-17	0	4071	144	$2.723*10^6$
	Development	14.9388	$-3.174*10^7$	-293	234	252	4071	144	$2.723*10^6$
	Property	14.9388	$3.333*10^7$	8348	-655	1141	3742	102	$2.391*10^6$
	Gasoline	14.9388	$-8.758*10^6$	-13534	487	0	4486	102	$2.666*10^6$
Elasticity of substitution of capital for land									
0.7	UGB	8.76284	434376	-4	-1	0	110	42	76272
	Development	8.76284	434376	79	82	17	110	42	76272
	Property	8.76284	810030	183	76	23	106	39	73310
	Gasoline	8.76284	644095	-414	99	0	117	39	75927
0.9	UGB	14.0515	$-6.219*10^7$	-1759	-16	0	17085	164	$5.461*10^6$
	Development	14.0515	$-6.219*10^7$	-1469	273	231	17085	164	$5.461*10^6$
	Property	14.0515	$9.478*10^7$	32848	-1527	2032	14948	112	$4.578*10^6$
	Gasoline	14.0515	$-2.263*10^7$	-37618	505	0	19148	112	$5.367*10^6$
Price of Capital									
80	UGB	15.8349	$-1.394*10^7$	-202	-12	0	1464	101	$1.189*10^6$
	Development	15.8349	$-1.394*10^7$	-53	137	193	1464	101	$1.189*10^6$
	Property	15.8349	$1.044*10^7$	2457	-216	545	1365	80	$1.073*10^6$
	Gasoline	15.8349	$-3.505*10^6$	-5163	330	0	1614	80	$1.169*10^6$
120	UGB	8.32668	466086	-12	-2	0	328	65	153457
	Development	8.32668	466086	118	128	24	328	65	153457
	Property	8.32668	$1.755*10^6$	575	111	42	309	57	142645
	Gasoline	8.32668	$1.028*10^6$	-1044	153	0	358	57	152279

Table 2, continued

	Policy for 10% saved land	City edge pre-policy	Total Cost	Cost at 0	Cost at Xbar	Subsidy to agriculture	Density at City Center	Density at Xbar	Total Households
Total available land for development									
	UGB	11.6652	$-2.671*10^6$	-77	-8	0	660	79	422573
15	Development	11.6652	$-2.671*10^6$	30	98	127	660	79	422573
	Property	11.6652	$3.338*10^6$	1072	-50	276	616	67	385822
	Gasoline	11.6652	-119909	-2323	226	0	726	67	417426
	UGB	12.0262	369940	-38	-4	0	700	79	467953
25	Development	12.0262	369940	151	185	50	700	79	467953
	Property	12.0262	$7.078*10^6$	1374	124	111	655	67	427252
	Gasoline	12.0262	$3.211*10^6$	-2428	235	0	770	67	462140
Share of housing in demand function									
	UGB	19.9342	$-3.835*10^8$	-3096	-78	0	3198	98	$4.547*10^6$
0.37	Development	19.9342	$-3.835*10^8$	-3030	-13	552	3198	98	$4.547*10^6$
	Property	19.9342	$-2.502*10^8$	5649	-1911	2450	2970	67	$4.031*10^6$
	Gasoline	19.9342	$-3.233*10^8$	-20633	539	0	3548	67	$4.435*10^6$
	UGB	2.26373	15174.5	0	0	0	106	68	5093.1
0.43	Development	2.26373	15174.5	32	32	0.4	106	68	5093.1
	Property	2.26373	18786.7	47	32	0.4	104	67	4987.9
	Gasoline	2.26373	15174.5	-91	33	0	111	67	5089.7
Capitalization weight									
	UGB	11.3894	$2.754*10^6$	0	0	0	626	78	387967
0	Development	11.3894	$2.754*10^6$	155	155	64	626	78	387967
	Property	11.3894	$7.756*10^6$	1150	83	136	587	67	355715
	Gasoline	11.3894	$4.882*10^6$	-2032	219	0	687	67	383555
	UGB	12.3011	$-4.943*10^8$	-4536	-242	0	1579	99	$1.677*10^6$
0.005	Development	12.3011	$-4.943*10^8$	-4280	13	86	1579	99	$1.677*10^6$
	Property	12.3011	$-4.075*10^8$	1153	-753	196	1412	67	$1.418*10^6$
	Gasoline	12.3011	$-4.575*10^8$	-15156	405	0	1830	67	$1.626*10^6$

Figure 1: Marginal Cost to Landowners

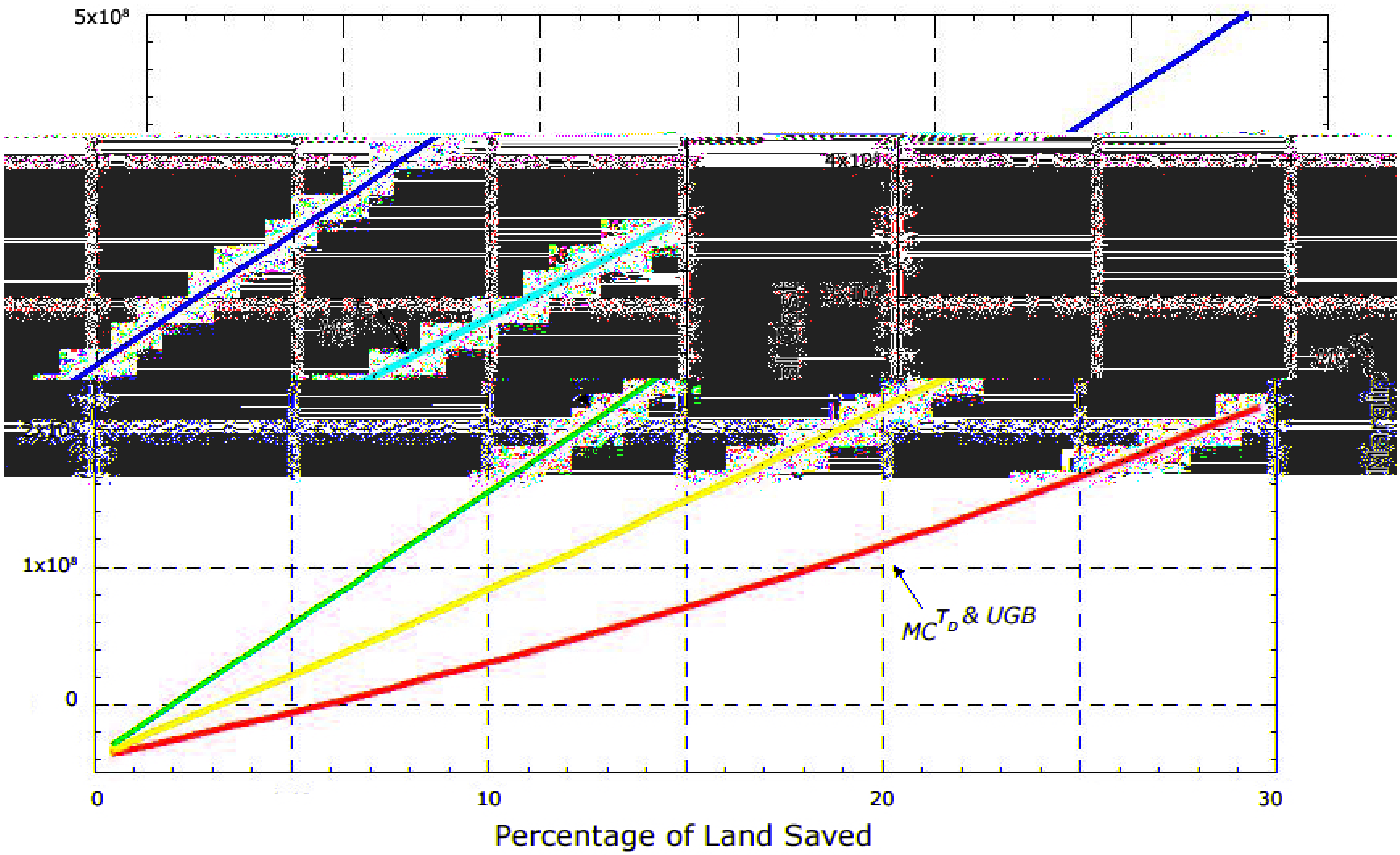
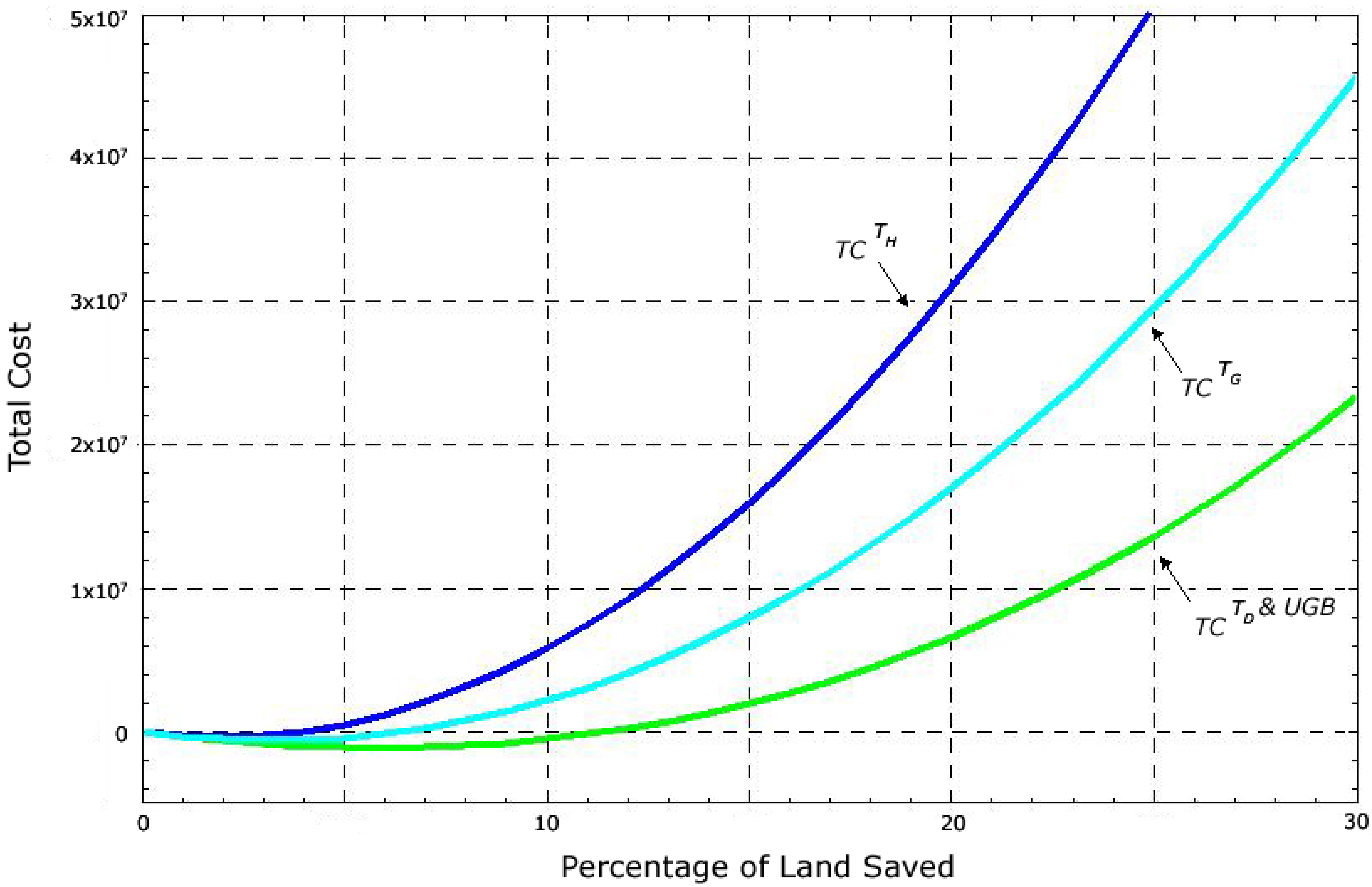


Figure 2: Total Cost to Landowners



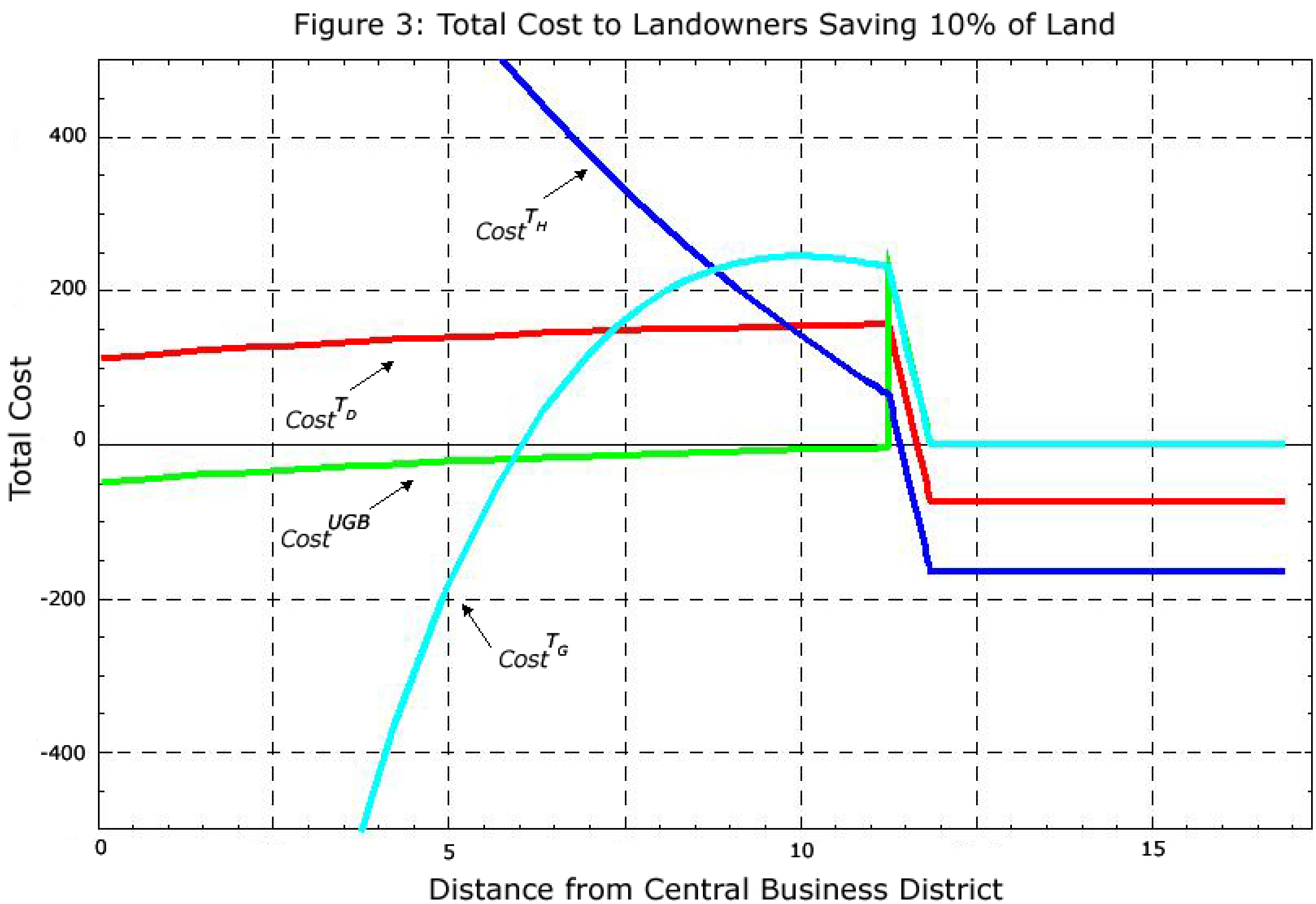


Figure 3: Total Cost to Landowners Saving 10% of Land
Total Cost
400
200
0
-200
-400
0
5
10
15
Distance from Central Business District
$Cost^{T_H}$
$Cost^{T_D}$
$Cost^{UGB}$
$Cost^{T_G}$