EC313-Fall 2012

Midterm

1:10-2:10pm, October 23, 2012

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You will have 50 minutes to complete this exam. Anyone still working on their exam after this time expires is subject to an automatic penalty of not less than 5 points. No notes or books are allowed, but you may use a calculator. All cell phones and pagers must be off.

When you write up your answers, your goal should be to (1) be correct, and (2) convince your reader that your answer is correct. Answers which do not achieve these goals will not be awarded full credit. To accomplish the second objective, it is helpful if your work is legible and if all steps are presented, possibly with a line of explanation. Total points =100/Share of total grade =35% .

1. This question asks you to think about the estimation of the cost of climate change conducted in Nordhaus and Mendelsohn (1995) that we discussed in class.

   (a) (20) Using the notation from lecture, suppose that

   $$R = A_0 + A_1 T + A_3 S + \epsilon$$

   where R is unit land rent, T is a scalar mean annual temperature, $A_1 > 0$ is the parameter we care about, S is the farmer's skill, and ϵ is unobserved determinants of land rent.

   Suppose that skillful farmers choose places with the best climate, but that we don't observe the skill of a farmer. We do know, however, that skill depends on climate according to $S = B_0 T + B_1 T^2$ for $B_0 > 0$ and $B_1 < 0$.

   Suppose we estimate the model

   $$R = \hat{A}_0 + \hat{A}_1 T + \hat{A}_2 T^2 + \hat{\epsilon}.$$

   Will our estimated coefficients of $\hat{A}_1$ and $\hat{A}_2$ measure what we want them too? Explain briefly.

   (b) (10) Can we determine if this approach to the problem leads us to overestimate or underestimate the effects of climate on agricultural land rents? If so, do we over or under estimate the effect of climate?

2. (30) Let $t = 0,1,2,...$ index years. Suppose that one ton of CO_2 emissions today causes 0\$ of damage for $t < 100$ and 100\$ of damage for $t \geq 50$. Let M denote the amount spent on mitigation at $t = 0$. If the interest rate is r how much will a planner who maximizes the discount present value of consumption be willing to spend on abatement to reduce future damage to zero.

3. You have two data sets. The first describes the change in CO2 concentration in the atmosphere over three years in parts per million. This data set is called `concentration.dta` and looks like this:

   Thus, in year 1 the atmospheric concentration of CO2 was 1ppm higher than in the previous year.

Year	D_CO2_concentration
1	1
2	2
3	3

CO2_Gt	Year
14.13	1
28.26	2
42.39	3

The second file contains data describing CO2 emissions in Gigatons in each of three years. This data set is called `emmissions.dta` and looks like this:

That is, in year 1 world emissions were 14.13 Gt of CO2.

Consider the following Stata program:

```
use emissions,clear;
merge 1:1 year using concentration;
gen calc_D_conc=CO2_Gt * 0.55*(1/2.12);
twoway scatter calc_D_conc D_CO2_concentration;
```

(a) (8)The output from this program is a graph. Draw the graph. Be sure to label the axes. Recall that the second variable in the `twoway` command ends up on the horizontal axis.

(b) (7)Does this graph suggest that the calculation performed in the program is correct? If not, why not and what is the error?

4. (25) 'Measuring temperature: the 170 year Thermometer-based record' suggests that we ought to be sceptical of measured temperature data showing an increase in temperature because individual instruments are subject to measurement error. While there are problems with the measured temperature record, this turns out not to be one of the important ones. This exercise asks you to work out the math to demonstrate this.

Suppose we have two thermometers which each produce a reading T_i for $i = 1,2$. For each reading, this error consists of the true temperature and an error e_i. e_i is random and takes the values x and $-x$ with equal probability. Let T denote the average recorded temperature across the two locations, that is, $T = (T_1 + T_2)/2$. Let $E(T_i)$ denote the expected value of T_i, $var(T_i)$ denote variance and $sd(T_i) = \sqrt{var(T_i)}$ the standard deviation. Assume the the errors are independent of each other and the true temperature is the same for both thermometers/locations.

(a) (5) Calculate $E(T_i)$.

(b) (5) Calculate $SD(T_i)$.

(c) (5) Calculate the expected or average value of T.

(d) (10) Calculate the average error (or standard deviation) for T.

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1. (a) We have

$$
\begin{aligned}
R &= A_0 + A_1 T + A_2 S + \epsilon \\
S &= B_0 T + B_1 T^2
\end{aligned}
$$

so the true model is

$$
\begin{aligned}
R &= A_0 + A_1 T + A_2 (B_0 T + B_1 T^2) + \epsilon \\
&= A_0 + (A_1 + A_2 B_0) T + A_2 B_1 T^2 + \epsilon
\end{aligned}
$$

If we estimate

$$R = \hat{A}_0 + \hat{A}_1 T + \hat{A}_2 T^2 + \hat{\epsilon}.$$

we will end up with

$$
\begin{aligned}
\hat{A}_0 &= A_0 \\
\hat{A}_1 &= A_1 + A_2 B_0 \\
\hat{A}_2 &= A_2 B_1.
\end{aligned}
$$

That is, except for the intercept, our estimates will confound the effects of temperature with the effects of skill.

(b) For the true model we have that

$$\frac{\partial R}{\partial T} = A_1.$$

For the estimated model we have that

$$
\begin{aligned}
\frac{\partial \hat{R}}{\partial T} &= \frac{\partial}{\partial T}\left[\hat{A}_0 + \hat{A}_1 T + \hat{A}_2 T^2 + \hat{\epsilon}\right] \\
&= \hat{A}_1 + 2\hat{A}_2 T \\
&= (A_1 + A_2 B_0) + 2 A_2 B_1 T.
\end{aligned}
$$

Since $B_1 < 0$ and $B_0 > 0$ we can't figure out if

$$\frac{\partial R}{\partial T} > \frac{\partial \hat{R}}{\partial T}$$

without knowing the value of T.

2. The discounted present value of damage is:

$$\begin{aligned}
&\sum_{t=50}^{\infty} \delta^t 100 \\
&= \delta^{50} \sum_{t=0}^{\infty} \delta^t 100 \\
&= 100 \frac{\delta^{50}}{1-\delta}
\end{aligned}$$

But $\delta = 1/(1+r)$, so this equals

$$\begin{aligned}
&100 \frac{(\frac{1}{1+r})^{50}}{1-(\frac{1}{1+r})} \\
&= 100 \frac{1}{r} \left(\frac{1}{1+r}\right)^{49}
\end{aligned}$$

3. (a) At the end of the program, the data in memory is going to look like this:

CO2_Gt	Year	D_CO2_concentration	calc_D_conc
14.13	1	1	3.67
28.26	2	2	7.33
42.39	3	3	11.00

So your graph is going to be of the three points $\{(1,3.67),(2,7.33),(3,11.00)\}$.

(b) If we were calculating concentration from emissions correctly, the line in your graph should have slope one, or close to it. In fact, it has slope 3.67 or 1/3.67 depending on how you drew it.

The problem arises because the third line of the program does not convert CO2 emissions to C emissions.

The third line of the program should be

```
gen calc_D_conc=CO2_Gt * 0.55*(1/2.12)*12/44;
```

4. (a) $T_i = (\frac{1}{2},\frac{1}{2}; t+x, t-x)$ so that $E(T_i) = \frac{1}{2}(t+x) + \frac{1}{2}(t-x) = t$

(b) $var(T_i) = \frac{1}{2}[(t+x) - E(T_i)]^2 + \frac{1}{2}[(t-x) - E(T_i)]^2 = x^2$. So $SD(T_i) = \sqrt{var(T_i)} = x$.

(c) $T = (\frac{1}{4},\frac{1}{4},\frac{1}{4},\frac{1}{4}; t+x, t, t, t-x)$ so that $E(T) = \frac{1}{4}(t+x) + \frac{1}{2}(x) + \frac{1}{4}(t-x) = t$

(d) $var(T) = \frac{1}{4}[(t+x) - E(T)]^2 + \frac{1}{2}[(t) - E(T)]^2 + \frac{1}{4}[(t-x) - E(T)]^2 = \frac{1}{2}x^2$. So $SD(T_i) = \sqrt{var(T)} = x/\sqrt{2}$.