A Note of Firm Entry and Liquidity *

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Abstract

This paper explores the link between goods and labor markets in a New Monetarist model of liquidity. The framework integrates a model of money and credit into a Mortensen-Pissarides labor market in order to study the relationship between the availability of credit, firm entry, and unemployment. I show that there exists a non-monotone relationship between credit an-dunemployment even with a uniquely determined monetary equilibrium. Finally, I show that the modeler's choice of bargaining protocol can affect the qualitative relationship between unemployment and credit.

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JEL Classification: E41, E42, E52

A Note on Firm Entry and Liquidity

1 Introduction

This note is intended to provide additional insight toward the link between goods and labor markets in a New Monetarist model of liquidity as presented in Rocheteau and Nosal 2017 (RN). The framework integrates a model of money and credit into a Mortensen-Pissarides labor market to study the relationship between the availability of credit, firm entry, and unemployment.

First, RN show that the strategic complementarities between buyers' choice of real money balances and firms' entry decision generate multiple monetary equilibria. As expected, this multiplicity generates contrary comparative statics at the "high" equilibria (low unemployment and large trading volume) versus the "low" equilibria (high unemployment and low trading volume). As credit becomes more accessible there is higher unemployment at the "high" equilibrium, but

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lower unemployment at the "low" equilibrium. I show that the non-monotone relationship between credit and unemployment does not rely on multiplicity. I break the link between the measure of operative firms and retail trading frequency thereby producing a unique equilibrium. Even with a uniquely determined monetary equilibrium, there exists a non-monotone relationship between credit and unemployment dependent on the value of money.

Second, I show that the modeler's choice of bargaining protocol has a substantive impact on the qualitative relationship between unemployment and credit. Under the benchmark model considered by RN terms of trade are settled by proportional bargaining. I show that Nash bargaining reverses the response of unemployment to credit observed by RN. Specifically, more access to credit can decrease unemployment at the high equilibrium under Nash, whereas there is an increase in unemployment under proportional bargaining. The modeler's choice of bargaining protocol is not innocuous.

2 A Stripped Down RN Model

Time is discrete and continues forever. Each period contains three subperiods where different markets open sequentially. The first market is a labor market (LM) where firms hire workers that produce a consumption good denoted q. The second market is a decentralized goods market (DM) where consumers purchase q from firms in bilateral meetings. The final market is a Walrasian market (CM) where debts are settled and portfolio choices are made and each agent can produce the numeraire good at unit cost.

There are two types of agents, producers and consumers, who are completely characterized by idiosyncratic preferences and technology. I begin with the firm. Each firm possesses a technology to produce $\bar{q}$ units of a consumption good with one worker. To acquire a worker, a firm posts vacancies at fixed cost k prior to the opening of a labor market (LM) where workers and firms are randomly matched according to the matching function $H(U, V)$. Let $f(\tau) = H(U, V)/U = H(1, \tau)$ denote the probability that an unemployed worker finds a job and $f(\tau)/\tau$ the probability that a firm finds a worker. The matching function is strictly increasing and concave in both of its arguments, and exhibits constant returns to scale. Once a firm is matched with a worker, it produces $\bar{q}$ each period until the match is destroyed with exogenous probability δ. The firm realizes match creation and destruction at the beginning of the LM; therefore, a firm must wait one period after job destruction to search for a new worker. For now, I take the wage w_1 as exogenous and denote expected period profit by ρ. The value of an employed firm is thus $J = (\rho - w_1)/(1 - \beta(1-\delta))$. Free entry guarantees that the ex-ante value of an employed firm is driven to zero in equilibrium,

$$k\left(\frac{f(\tau)}{\tau}\right)^{-1} = \frac{\beta}{1-\beta(1-\delta)}(\rho - w_1) \tag{1}$$

which gives market tightness τ for any given expected revenue.

A firm's expected revenue is determined in a decentralized goods market (DM) where firms and buyers randomly meet in pairs to trade. In a fraction μ of matches there exists a perfect record keeping technology and enforcement mechanism permitting the use of credit. The remaining $1-\mu$ trades are unmonitored precluding the use of credit and generating a need for money.

Terms of trade in the DM will be settled according to proportional bargaining which promises the buyer a fraction θ of the total match surplus. The jointly efficient outcome is $q^* : u'(q^*) = 1$ with the corresponding issuance of debt $b = (1-\theta)u(q^*) + \theta q^*$. Without loss of generality, I assume that only credit is used in monitored matches. Since there are no debt limits, the first best quantity q^* will be traded in all monitored matched. In unmonitored matches, however, $q < q^*$ will be purchased so long as money is costly to hold where $d = z(q) = (1-\theta)u(q) + \theta q$ is the monetary payment. That is, a buyer will never carry more real money balances z then he expects to spend d to acquire q units of the consumption good.

A worker may be employed or unemployed in any period. However, since portfolio decisions are independent of current wealth, employment status has no effect on the buyer's choice of real money balances. The buyer's optimal portfolio choice equates the cost of holding money to the liquidity value it brings in the following DM,

$$\max_z -iz + (1-\mu)\theta(u(q(z)) - q(z))$$

where $i = (\gamma - \beta)/\beta$ is the nominal interest rate on an illiquid bond. To guarantee that the above problem is concave and admits an interior solution, we must have that $(1-\mu)\theta/(1-\theta) > i$. The cost of holding money must be low enough if money is to be valued in equilibrium. Given that an interior solution exists, optimal money holdings satisfy the following,

$$i = (1-\mu)\theta\left(\frac{u'(q)-1}{(1-\theta)u'(q)+\theta}\right). \tag{2}$$

Given the optimal choice of money holdings, a firm's expected revenue in the DM given by

$$\rho = (1-\theta)\left[\mu(u(q^*)-q^*)+(1-\mu)(u(q(z))-q(z))\right]+\bar{q}. \tag{3}$$

A firm will always receive a fraction $1-\theta$ of the joint surplus, where the value of the surplus will be $u(q^*)-q^*$ with probability μ and $u(q(z))-q(z)$ with probability $(1-\mu)$.

A stationary equilibrium is a tuple (q,τ) which satisfies (1)-(3). Notice that because there are no matching frictions in the retail market the buyer's portfolio choice can be solved independently of the entry of firms. The model is solved recursively where (2) determines the quantity traded in unmonitored matches and entry adjusts according to (1) and (3). Finally, the equilibrium level of *employment* is determined by the steady state condition,

$$n = \frac{f(\tau)}{f(\tau)+\delta}. \tag{4}$$

Since retail trading opportunities are independent of the level of employment, there is a unique monetary equilibrium.

First, I discuss the relationship between credit and unemployment. Notice that an increase in credit availability ($\uparrow \mu$) has two consequences: (i) there are fewer occasions where trade is unmonitored, and (ii) there is a lower quantity traded in unmonitored trades as households choose to hold fewer real balances. The first effect increases expected revenue since firms will more frequently find themselves in matches where the efficient surplus $u(q^*)-q^*$ is obtained. The second effect decreases expected revenue since firms that find themselves in unmonitored matches now receive less surplus. If expected revenue increases on net, there will be more vacancies posted and higher employment. If expected revenue decreases then employment decreases.

Figure 1 shows the equilibrium in (q,n). Notice that the quantity traded given by (2) is independent of employment. With q determined by the buyer's problem, (1) pins down the equilibrium level of employment. The dashed line indicates a higher level of μ representing greater access to

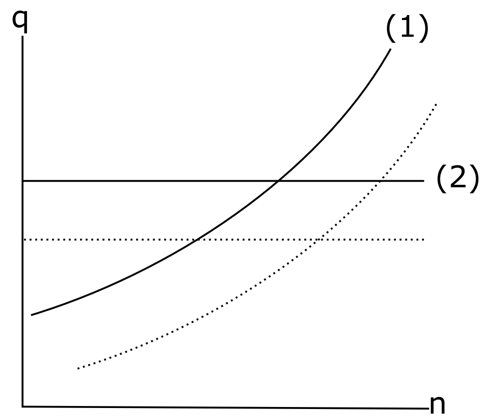


Figure 1: Monetary Equilibrium in (q, n)

credit. Note the two countervailing forces: movement along (1) indicates the negative impact on employment from the intensive margin while shifts of (1) indicate the positive impact on employment from the extensive margin. Which effect dominates will determine whether unemployment responds positively or negatively to increased access to credit.

To be more precise, letting $S(q) = u(q) - q$ we have that

$$\frac{\partial \rho}{\partial \mu} \propto S(q^*) - \Omega(q) \tag{5}$$

where $\Omega(q) = S(q) - \frac{(u'(q)-1)^2((1-\theta)u'(q)+\theta)}{u''(q)} > 0$.

If $S(q^*) - \Omega(q) > 0$ firms expect higher profits when credit is more available and will respond with greater entry and hence lower unemployment. If $S(q^*) - \Omega(q) < 0$ firms expect lower revenues, reduce entry, and higher unemployment results.

PROPOSITION 1 *For a given bargaining power, there exists a unique threshold $\hat{q}(\theta) \in [0, q^*]$: $\Omega(\hat{q}) = S(q^*)$ such that for all $q \leq \hat{q}(\theta)$ unemployment decreases as credit becomes more available whereas for all $q > \hat{q}$ unemployment increases as credit becomes more available.*

PROOF 1 *From* (3) *we have that,*

$$\frac{\partial \rho}{\partial \mu} \propto u(q^*) - q^* + (1-\mu)\frac{\partial S(q)}{\partial q}\frac{\partial q}{\partial \mu} - S(q)$$

The only term left to compute is $\partial q/\partial \mu$ which measures the degree to which consumers alter their real money balances given a small change in credit access. This term is computed from (2) *using a* *simple application of implicit differentiation,*

$$\frac{\partial q}{\partial \mu} = \frac{S'(q)[(1-\theta)u'(q)+\theta]}{(1-\mu)S''(q)} < 0$$

COROLLARY 1 *For a given bargaining power and nominal interest rate, there exists a unique threshold $\hat{\mu}(\theta, i) \in [0,1]$ such that for all $\mu \leq \hat{\mu}$ unemployment increases as credit becomes more available whereas for all $\mu > \hat{\mu}$ unemployment decreases as credit becomes more available.*

PROOF 2 *The buyer's portfolio decision given by (2) shows a one-to-one mapping between q and μ. Moreover, the relation is monotone decreasing for $q \in (0, q^*]$ as shown in Proposition 1 proof.*

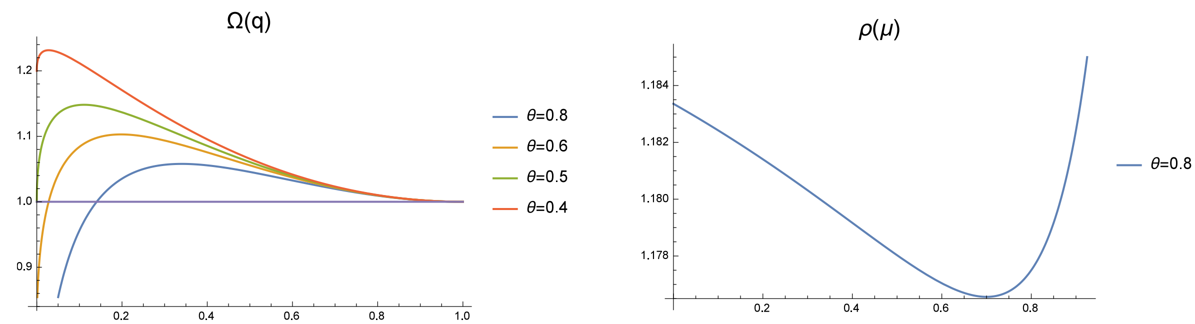
Figure 2 illustrates Proposition 1 with a numerical example setting $u(q) = 2\sqrt{q}$ and varying bargaining powers. Given this functional form for utility, the threshold value addressed in Proposition 1 can be solved closed form,

$$\begin{aligned} \hat{q} &= \left(\frac{2\theta - 1}{2\theta}\right)^2 \quad &&\text{for} \quad \theta \in (1/2, 1) \\ &= 0 &&\text{otherwise} \end{aligned}$$

The curves in Figure 2a represent $\Omega(q)$ and the jointly efficient surplus is $S(q^*) = 1$. Notice that for bargaining powers one-half or less we have that $\hat{q}(\theta \leq 0.5) = 0$ so that there is no set of trades which increase expected revenue. More credit access causes firms to reduce entry resulting in greater unemployment. However, for bargaining powers above one-half we have that $\hat{q}(0.8) \approx 0.1406$ and $\hat{q}(0.6) \approx 0.0278$ indicating that for small volume trades expected revenue increases as credit becomes more available; unemployment would in fact decline following more access to credit. Greater bargaining power to the buyer increases $\hat{q}(\theta)$.[1] Of course, the quantity traded is an equilibrium object given by (2). According to Corollary 1, for a given nominal interest rate there exists a threshold value $\hat{\mu}(\theta, i)$ corresponding to that of $\hat{q}(\theta)$. Given the functional form for utility, the locus of interest rates and credit access that corresponds to $\hat{q}$ takes a very simple form,

$$i = 1 - \mu.$$

[1]Giving all bargaining power to the buyer $\theta = 1$ would shut down the market. To have bounded entry it must be the case that $-k + \beta\bar{q} < 0$; if buyer's have all the bargaining power then the firm's expected revenue, $\bar{q}$, is less than the capitalized entry cost k/β and the retail market shuts down.



(a) Ω on q (b) ρ on μ

Figure 2: Response of Expected Revenue to Credit Access

One only needs to check that the locos of points given by $i = 1 - \mu$ satisfy the existence condition for a monetary equilibrium. Notice that for all $\theta > 1/2$ we have that $1 - \mu < (1 - \mu)\theta/(1 - \theta)$ so all combinations of nominal interest rate and credit availability satisfy a monetary equilibrium.

Figure **??** focuses on the case where $\theta = 0.8$ and $i = 0.3$ and therefore the level of credit access which correspond to $\hat{q}(0.8) \approx 0.1406$ is given by $\hat{\mu}(0.8, 0.3) = 0.7$. More access to credit decreases firm revenue and thus leads to greater unemployment up to $\hat{\mu}(0.8, 0.3) = 0.7$; then more access to credit increases firm revenue and lowers unemployment. Note that the monetary equilibrium is sustained up to $\mu = 0.925$ so there exists a region of the parameter space where a monetary equilibrium exists and is characterized by less unemployment following more access to credit. Although the level of firm revenue varies with bargaining power, the non-monotone relation and root are robust for all $\theta > 1/2$.

To test the sensitivity of the results on the bargaining protocol, I consider the same model but use Nash bargaining to determine the DM terms of trade. Assuming money is costly to hold, the monetary transfer to acquire q units of the consumption good is now given by,

$$z_\theta(q) = [1 - \Theta(q)]u(q) + \Theta(q)q \tag{6}$$

where $\Theta(q) = \frac{\theta u'(q)}{\theta u'(q) + (1-\theta)}$. The Nash solution exhibits a non-monotonicity that was absent under proportional bargaining: the buyer's surplus is not always increasing in his real balances. Although the match surplus $S(q)$ increases as $q \to q^*$, the buyer's share of the surplus decreases. Consequently, even if it is costless to hold real balances, $i \approx 0$, the buyer will not bring sufficient real balances into the DM to be able to purchase q^*.

The solution to the buyer's problem is now given by,

$$i = (1-\mu)\theta\left(\frac{u'(q)-1}{z'_\theta(q)}\right). \tag{7}$$

where $z'_\theta(q)$ is the first derivative of (6), and firm revenue is

$$\rho = \mu(1-\theta)S(q^*) + (1-\mu)(1-\Theta(q))(u(q)-q). \tag{8}$$

Since the portfolio decision and entry are still uncoupled, the system is solved recursively as before: (7) determines q and then (1) and (8) determine entry.

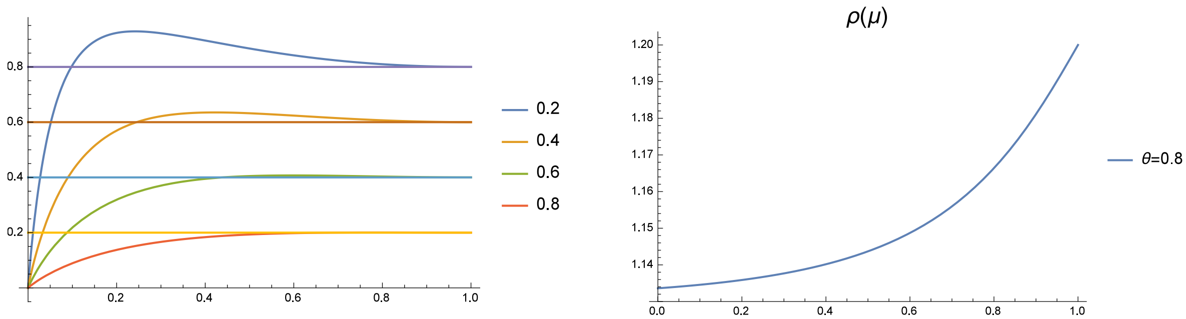
The response of firm revenue to credit access is now given by,

$$\frac{\partial\rho}{\partial\mu} = (1-\theta)S(q^*) - \Gamma(q) \tag{9}$$

where

$$\Gamma(q) = (1-\Theta(q))S(q) - \frac{S'(q)z'_\theta(q)}{S''(q)z'_\theta(q) - S'(q)z''_\theta(q)}\left[(1-\Theta(q))S'(q) - \Theta'(q)S(q)\right]$$

Under Nash similar results to Proposition 1 and Corollary 1 hold in that there exists thresholds $\hat{q}(\theta) : \Gamma(\hat{q}) = S(q^*)$ and $\hat{\mu}(\theta, i)$ defining where increases in credit availability decrease unemployment. Figure 3a illustrates threshold $\hat{q}(\theta)$ with varying bargaining powers. We see a similar pattern in that low volume trades are required to see an increase in expected revenue. However, the Nash solution generates a larger set of trades where expected revenue can increase. Notice the curve with bargaining power $\theta = 0.6$: whereas proportional bargaining had $\hat{q} \approx 0.0278$ under Nash we have that $\hat{q} \approx 0.4390$ suggesting a larger subset of the parameter space where trades increase firm revenue. Notice the curve with bargaining power $\theta = 0.4$: whereas the proportional solution predicted no trades where firm revenue increased, the Nash solution shows $\hat{q} \approx 0.3$. For comparability with Figure 2b, I focus on the case $\theta = 0.8$ and show the relationship between firm revenue and credit in Figure 3b. Notice that under Nash revenue is monotone increasing in credit availability. Although not shown, for any bargaining power this positive monotone relation is preserved. That is, the threshold $\hat{\mu}$ needed to generate high enough volume trades is infeasible ($\hat{\mu} < 0$). Compared



(a) Γ on q

(b) ρ on μ

Figure 3: Response of Expected Revenue to Credit Access

to the proportional solution, the relationship between credit and unemployment is reversed. The modeler's choice of the bargaining solution is not innocuous.

Finally, I consider the case where the level of unemployment affects the arrival rate of trading opportunities. Suppose that the retail market is subject to search and matching frictions where the probability trading opportunity is increasing the number of operative firms. Firms and buyers are matched in the DM according to a constant returns to scale matching function $M(B, F)$ whose arguments are the measure of buyers and operative firms respectively. Normalizing the measure of buyers to one, I have that the matching probabilities are summarized by the measure of operative firms n: $\sigma(n) = M(1, n)$ is the probability that a buyer meets a firm, and $\sigma(n)/n$ is the probability that a firm meets a buyer. Of course, the measure of operative firms will depend on labor market conditions. We may then write $\sigma(n(\tau))$ to make explicit the link between the labor market and goods market.[2] The solution to the buyer's problem is now given by,

$$i = \sigma(n(\tau))(1 - \mu)\theta \left(\frac{u'(q) - 1}{z'_\theta(q)} \right). \tag{10}$$

and firm revenue is scaled by the probability of a match $\sigma(n(\tau))/n(\tau)\rho$.

[2]Previously, there was still a link between the goods and labor market but it was uni-directional: a buyers portfolio choice was based on the liquidity value realized in the goods market, and this in turn affected firm revenue and their decision to enter the labor market. Now the link is bi-directional: labor market outcomes affect the liquidity value of real balances, and the buyer's choice of real balances affect firms labor market prospects.

3 An Example

I show the analytic properties of the model when the functional form for utility is $u(q) = 2q^{1/2}$. Immediately we have that the quantity trade that maximizes joint surplus is $q^* = 1$ which provides one unit of retail surplus to be split between sellers and buyers.

From Proposition 1, the threshold quantity traded is defined as $\Omega(\hat{q}) = 1$ which, given the functional form above, simplifies to

$$2\theta q^{3/2} + (1 - 6\theta)q - (2 - 6\theta)q^{1/2} + (2 - 2\theta) = 1$$

whose solution is

$$\begin{aligned} \hat{q} &= \left(\frac{2\theta - 1}{2\theta}\right)^2 \quad \text{for} \quad \theta \in (1/2, 1] \\ &= 0 \quad \text{otherwise} \end{aligned}$$

Given $\hat{q}$, the pairs (μ, i) which are consistent with buyers bringing in real money balances $z(\hat{q})$ are given by (2). If we take the nominal interest rate as given, then we retrieve the threshold value $\hat{\mu}$ from Corollary 1. I show here that (2) dramatically simplifies given the functional form on utility.

$$\begin{aligned} i &= (1 - \mu)\theta \left(\frac{u'(\hat{q}) - 1}{(1 - \theta)u'(\hat{q}) + \theta} \right) \\ &= (1 - \mu)\theta \left(\frac{\left(\frac{2\theta-1}{2\theta}\right)^{-1} - 1}{(1 - \theta)\left(\frac{2\theta-1}{2\theta}\right)^{-1} + \theta} \right) \\ &= 1 - \mu \end{aligned}$$

Given the function form for utility, there exists a simple linear relationship between credit availability and the nominal interest rate that implicitly defines the threshold $\hat{q}$.