Productive demand and sectoral capacity utilization

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Abstract

The Solow residual, often used to measure technological progress, also reflects fluctuations in input utilization and related factors. We develop a multisector model decomposing capacity utilization into shopping-effort and variable capital intensity components, attributing sectoral Solow residual variation to utilization, technology, and input share mismeasurement. Using Bayesian estimation with capacity utilization data from nondurable and durable goods sectors, we identify key parameters governing goods market frictions. We find that search demand shocks explain most forecast error variance in the Solow residual, output, and utilization. Together with matching frictions, these shocks are essential for replicating observed sectoral dynamics, including volatility, correlations, and autocorrelations of utilization rates. Impulse response analysis reveals that demand shocks uniquely generate three-way comovement among utilization rates and the Solow residual.

Keywords: identifying goods market frictions, capacity utilization, sectoral comovement, endogeneity of Solow residual, Bayesian estimation, demand shocks

JEL Classification: E32; O47; D24; D83; E24; C11

1. Introduction

Macroeconometric research shows that the Solow residual does not purely measure technology. **?** finds that variables like money, interest rates, and government spending Granger-cause movements in the Solow residual, with demand fluctuations accounting for up to half of its

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variance. Similarly, ? use structural estimation to impute technological change and find it behaves quite differently from the measured Solow residual.

Goods market frictions, as introduced by ?, offer a clear explanation for this endogeneity. Imagine a traditional pizzeria. In a neoclassical model, output depends only on inputs—staff, tables, wood fire ovens—regardless of demand, because prices adjust to clear markets. In practice, however, production ramps up when customers arrive; sales spike at meal times and inputs are more greatly utilized. This means that higher search effort by consumers influences output even if inputs are held fixed, thereby raising TFP. While firms eventually adjust inputs or capacity in response to sustained demand, such adjustments are absent during short-run cyclical fluctuations.[2]

Our research problem is to quantify the extent to which movements in TFP are driven by demand-side economic fluctuations rather than purely technological innovations. To do this, we build a multisector model with goods market frictions, where consumers' shopping behavior influences aggregate demand and, in turn, firms' capacity utilization. Capacity utilization then affects both the measurement of the Solow residual and the dynamics of the business cycle, allowing demand shocks to directly impact measured productivity. This logic reflects Keynes' idea that demand shocks drive business cycles, but differs from the New Keynesian literature by not relying on nominal rigidity.[3]

We estimate the model using Bayesian methods and find that search demand shocks explain nearly two-thirds of the variation in output, as well as a majority of the variation in the Solow residual and capacity utilization. These shocks also have significant effects on labor supply and the relative price of investment. The key parameters governing the transmission mechanism are well identified. Moreover, the model fits major sectoral data reasonably well. Importantly, search demand shocks uniquely generate the observed comovement between utilization measures and the Solow residual.

Our work addresses a central question in macroeconomics: What fundamentally drives business cycles? The ongoing debate over the roles of technology and demand shocks is closely linked to capacity utilization—or economic slack. For example, ? argue that variable factor utilization allows modest technology shocks to produce realistic cycles, while ? emphasizes

[2]A simple static version of our model in Section 4 illustrates both the immediate impact of increased search effort on utilization and the subsequent adjustment of firm capacity.

[3]The requirement that goods are found by a shopper creates a wedge between actual and potential output. The gap stems from omitting consumer search effort as an input. In contrast, firms' search effort (i.e., advertising) would not contribute to this mismeasurement since these inputs appear in output measures.

preference shocks, particularly for explaining labor under-utilization in recessions. **?** shows that demand shocks can generate procyclical investment, and that variable capital utilization reduces the degree of persistence required for such shocks. More recent studies, such as **?** and **?**, incorporate capacity constraints and frictions in goods and labor markets, finding that demand shocks are the primary drivers of business cycles.

Despite the fundamental role of capacity utilization in transmitting shopping effort to observed productivity, its value for identification has been largely overlooked. We address this by decomposing the growth rate of the Solow residual into contributions from capacity utilization, technology, and mismeasurement of input shares.[4] Specifically, the growth rate of capacity utilization is a weighted sum of the growth rates of shopping effort and variable capital utilization. Incorporating capacity utilization data enables us to identify novel parameters related to goods market frictions—such as matching technology and the disutility of shopping effort—as well as shocks to shopping disutility.

Following **?**, we define sectoral capacity utilization in our model as the ratio of an output index to a capacity index, mirroring the Federal Reserve Board's empirical approach. This sectoral definition is necessary, as capacity utilization is not defined economy-wide—particularly for services. Accordingly, we develop a multisector model in which the consumption sector comprises nondurable goods and services, while the investment sector consists of durables. We then map capacity utilization from the model's nondurable and durable sectors to empirical data, which further disciplines the model with sectoral information. The strong comovement in these two utilization series provides a stringent test, akin to the comovement of labor hours across sectors. Sectoral comovement—a stylized fact underpinning the NBER's business cycle definition—persists even at a fine sectoral level (**?**), posing a challenge that many models fail to meet. Our framework, however, successfully captures the volatility, cross-correlation, and autocorrelation of capacity utilization.

Search-based demand shocks are essential for matching these empirical moments. To our knowledge, no other dynamic general equilibrium model has disaggregated capacity utilization and matched these sectoral patterns. Existing research typically treats capacity utilization as an aggregate measure and struggles to capture its volatility (e.g., **?**; **?**).

Our formulation closely follows **?** (hereafter BRS), where output depends on firms' tech-

[4] The analytic expression for the Solow residual and its relationship to utilization appears in equation (**??**) in Section 5.4. The mismeasurement term arises from incorrectly imposing constant returns to scale and perfectly competitive labor markets; it vanishes if the production technology is specified correctly. This section also derives the analytic link between capacity utilization and search effort.

nology, inputs, and their efficiency in matching with customers. However, our identification strategy differs from that of BRS. For clarity, let ϕ denote the elasticity of the matching function, and η the elasticity of disutility. BRS calibrate ϕ and η using cross-sectional price dispersion for identical goods and the elasticity of shopping time with respect to expenditure, treating shopping time as a proxy for effort. They employ two sets of observables: one uses shopping time from the American Time Use Survey as a proxy for effort, while the other relies on the relative price of investment. Both approaches also use data on output, investment, and labor productivity.

While leveraging identified micro moments to estimate ϕ and η is generally appealing, using shopping time as a proxy for effort raises two key concerns. First, as BRS note, fluctuations in shopping effort should be interpreted broadly to include changes in match efficiency, not just time spent shopping. Second, shopping time may be contaminated by leisure activities—time spent browsing could reflect window shopping rather than genuine effort. For example, a stronger desire to find a specific item may shift behavior from casual browsing to active searching. Thus, changes in shopping time may reflect multiple factors. Moreover, even if these measurement issues could be managed, shopping time data from the American Time Use Survey are available only annually since 2003.

The model incorporates several features to better match business cycle moments and the role of demand shocks. First, variable capital intensity with endogenous depreciation separates goods market frictions from intensive margin adjustments. Investment adjustment costs further moderate investment volatility and generate hump-shaped impulse responses. Second, external habits and limited factor mobility improve persistence and allow technology shocks to generate positive labor comovement. Third, fixed costs offer an alternative explanation for output–productivity comovement and shape the link between capital intensity and capacity utilization. Finally, we introduce new stochastic processes—wage-markup shocks and investment-specific shopping disutility—which help the model fit sectoral data on hours, utilization, and the relative price of investment. Despite these additions, search-based demand shocks remain central in the variance decomposition.

The model is specified to avoid favoring demand or technology shocks a priori. The additional components help match second moments and allow technology shocks to more flexibly account for comovement patterns. For example, incorporating external habits and limited factor mobility into a standard RBC framework enables technology shocks to generate labor comovement. This addresses the sectoral comovement puzzle of **?**. Similarly, demand shocks help explain capacity utilization and the Solow residual. Omitting variable capital utilization would bias results toward search effort as the main driver of capacity utilization fluctuations.

The observables used for Bayesian estimation are demeaned growth rates of consumption, investment, labor hours in consumption, labor hours in investment, utilization in nondurable goods, utilization in durable goods, and the relative price of investment to consumption.[5] This set extends **?** with the utilization measures but drops aggregate wages.

Along with standard macroeconomic series and capacity utilization, we include sectoral labor hours and the relative price of investment to help identify shock transmission. Specifically, we show that the ratio of labor inputs across sectors is closely related to the ratio of shopping effort across sectors.[6] In addition, combining sectoral labor hours with output data requires the model to match sectoral labor productivity. The relative price of investment is also a key target in a multisector model. In this context, it provides information about capital intensity through Tobin's Q.

The stochastic processes encompass shocks to the trend in technology, stationary neutral technology, investment-specific technology, neutral shopping effort cost, investment-specific shopping effort cost, discount-factor, and wage markups.[7] The wage markup shocks capture unexpected spreads between the marginal product of labor and the wage paid by firms, serving as a proxy for changes in labor market conditions and bargaining power. The model's components and shock structure build on the framework introduced by BRS, while integrating key elements from **?** and **?**.

While our work is most closely related to **?**, it is also inspired by **?**, who highlight the impact of aggregate demand on unemployment and idle time through goods market frictions. Like our approach, they treat economic operation rates and their business cycle comovement as fundamental outcomes, though they model matching costs as expenditures rather than effort. In Online Appendix G, we compare both specifications. Although the core transmission mechanism is unchanged, the labor share of income—which is key for the Solow residual—differs between them. **?** apply this framework to show that the effectiveness of fiscal policy depends on whether fluctuations are supply- or demand-driven. **?** extend the model to a dynamic, stochastic setting, estimating demand and technology shocks using unemployment

[5]BRS also estimate an extension with variable capital intensity using the same series as the baseline. By contrast, we incorporate data on sectoral capacity utilization.

[6]The precise relationship is given by Equation (**??**).

[7]The discount-factor shock affects the consumption Euler equation, similarly to the risk-premium shock in **?**. However, unlike the latter, it does not mechanically help explain the comovement of consumption and investment.

and labor productivity data; they find demand shocks explain most data variability, especially with endogenous job separation. While their model allows capacity utilization to vary only under fixed prices, ours produces significant variation in capacity utilization under competitive search. Additionally, we estimate a richer model with endogenous capital intensity and investment, disciplined by a broader dataset that includes capacity utilization.

Section ?? reviews key facts on capacity utilization and sectoral comovement, placing it in the context of related empirical measures. Section ?? presents the model. Section ?? illustrates the impact of demand shocks on utilization and the Solow residual in a static setting. Section ?? characterizes equilibrium and decomposes Solow residual growth. Section ?? estimates the full model and highlights the role of goods market frictions and shocks. This section also presents the variance decomposition, investigates the contribution of different ingredients, and plots impulse responses. Section ?? concludes. The appendices detail data, derivations, calibration, identification, and supporting estimation exercises. Time indices are sometimes omitted for clarity.

2. Background and stylized facts on utilization and sectoral comovement

The Federal Reserve Board constructs total capacity utilization as the ratio of an output index to a capacity index for manufacturing, mining, and electric and gas utilities. This measure of capacity aims to quantify a plant's maximum sustainable output given its resource constraints. Here, 'sustainable output' refers to the greatest level of output each plant can maintain, given a realistic work schedule and normal downtime. Though subject to some definitional ambiguity, capacity utilization has, in practice, performed well as a measure of economic slack and as a predictor of short-to-medium-term inflation (?).

This measure covers 89 detailed industries (71 in manufacturing, 16 in mining, and 2 in utilities).[8] These industries primarily correspond to the 3- or 4-digit North American Industry Classification System (NAICS) codes. Importantly, estimates are available for both durable and nondurable goods. In manufacturing, most capacity indices are based on responses to the Census Bureau's Quarterly Survey of Plant Capacity. The survey is conducted quarterly at the establishment level. Prior to 2007, it was conducted annually, so it was necessary to interpolate measures of capacity. The probability that each establishment is selected is proportional to the value of shipments within each industry.

[8]The methodology underlying the Survey of Plant Capacity is available from https://www.census.gov/programs-surveys/qpc/technical-documentation/methodology.html.

An alternative measure of capacity utilization can be derived using the utilization-adjusted Solow residual developed by **?**. However, unlike Fernald's measure, the total capacity utilization metric accommodates non-constant returns to scale, profits, and fixed costs. This flexibility is advantageous, as goods market frictions and competitive search models typically imply decreasing returns to scale, and fixed costs provide a realistic link between output and productivity.

We decompose total capacity utilization into subcomponents for nondurables and durables. Figure **??** compares cyclical capacity utilization in durables and nondurables alongside real output and a Fernald-based utilization measure, which we construct as the difference between cyclical TFP and the utilization-adjusted counterpart from **?**. The capacity utilization series comove closely with each other and with the Fernald measure, and all are procyclical. Notably, total capacity utilization in durables exhibits greater volatility.

utilization_output.pdf

Figure 1: Total capacity utilization in non-durable and durable goods and output, here defined as consumption plus investment. Each underlying series is detrended via the Hamilton regression filter with the four most recent observations 8 quarters in the past ($p = 4, h = 8$).

Lastly, we examine business cycle statistics of the sectoral and utilization data. Table **??** presents the second moments of the series expressed in growth rates from 1964Q1-2019Q4. The use of growth rates aligns with the treatment of data in estimation, a standard practice since **?**, and eases comparison with other studies. Hours are constructed using the BLS Current Employment Statistics, following **?**. The data appendix provides details, and Figure **??** shows the detrended time series of hours in each sector alongside the aggregate measure. Following BRS, we define output as the sum of consumption and investment, consistent with our model framework. The findings indicate a strong correlation of 0.87 between labor hours across sectors, and a moderate correlation of 0.54 between consumption and investment. Each utilization measure also has robust comovement with investment and labor hours in investment. Additionally, all series exhibit significant autocorrelation, except for labor productivity. Notably, investment, labor hours in investment, and utilization in durables display substantial volatility compared to consumption, labor hours in consumption, and utilization in nondurables.

	SD(x)	STD(x)/STD(Y)	Cor(x, I)	Cor(x, n_i)	Cor(x, x_{-1})
Y	0.87	1.00	0.94	0.70	0.47
C	0.44	0.51	0.54	0.44	0.48
I	2.14	2.46	1.00	0.73	0.41
n_c	0.57	0.66	0.66	0.87	0.67
n_i	1.94	2.23	0.73	1.00	0.64
Y/n	0.64	0.73	0.36	-0.28	0.10
p_i	0.51	0.58	-0.28	-0.22	0.44
$util_D$	2.27	2.61	0.69	0.84	0.55
$util_{ND}$	1.26	1.45	0.61	0.65	0.51

Table 1: Time range: $1964Q1 - 2019Q4$. Each underlying series is expressed in 100 quarterly log deviations. Here output is defined as the sum of consumption and investment. We use the symbols Y for output, C for consumption, I for investment, n_c for labor supply, in consumption, n_i for labor supply in investment, Y/n for labor productivity, p_i for the relative price of investment, and $util_D$ and $util_{ND}$ for the utilization of durables and nondurables, respectively. **??** describes the construction of variables in detail.

Researchers have used various methods to measure capacity utilization across countries. Online Appendix H summarizes indicators conceptually aligned with the Federal Reserve Board's measure for several economies, though alternative approaches are also common. For example, **?**, following **?**, uses a cost-minimization framework with both variable and quasi-fixed inputs to construct a utilization measure for Canada—an approach that also extends to the service sector.[9] The World Bank's Enterprise Surveys provide another resource, compiling data from over 135,000 firms in 111 mostly emerging economies, based on interviews with business owners and managers. Using this dataset, **?** show that factors such as water shortages, severe electricity constraints, and corruption significantly limit firms' capacity utilization, with notable variation by firm size, age, ownership, and managerial experience. While valuable for cross-country comparisons, this dataset generally lacks the time series dimension needed for analyzing business cycle fluctuations.

[9]However, our method does not assume price-taking in input and output markets—an important distinction given the competitive search environment—and explicitly incorporates goods market frictions.

3. Model environment

3.1. Technology and markets

There is a unit mass of households and a unit mass of firms in each production sector. There are three sectors: two for consumption (goods mc and services sc), and one for investment (i). Each sector j produces output using capital and labor, with capital utilized at a rate h_j. Production in each sector involves a fixed cost ν_j, which is proportional to the stochastic trend X to ensure that the fixed cost share of output remains stationary along the balanced growth path.[10]

The economy grows with a stochastic trend X such that its growth rate $g_t = X_t/X_{t-1}$ is a stationary process with steady state $\overline{g}$. The production technology in sector j is given by

$$\begin{aligned} F_j &= z_j f(h_j k_j, n_j) - \nu_j X, \quad j \in \{mc, sc, i\} \\ f(hk, n) &= (hk)^{\alpha_k} n^{\alpha_n} X^{1-\alpha_k} \end{aligned}$$

Formulation (**??**) says that F_j is the remaining output available to be sold after taking into account dissipation from fixed costs. Higher utilization of capital raises depreciation according to an increasing and convex function $\delta(\cdot)$. We assume the form

$$\delta^j(h) = \delta^K + \sigma_b(h-1) + \frac{\sigma_{aj}\sigma_b}{2}(h-1)^2, \quad j \in \{mc, sc, i\}, \sigma_{ac} \equiv \sigma_{amc} = \sigma_{asc}$$

where δ^K is an exogenous depreciation rate, σ_b is the marginal cost of utilization at $h = 1$, and $\sigma_{aj} = (1)\delta^j_{hh}(1)/\delta^j_h(1)$ is the elasticity of the marginal utilization cost at the steady state. As $\delta^j(1) = \delta^K$, δ^K is the economy-wide steady-state depreciation rate of capital. Alternatively, $1/\sigma_{aj}$ is the sector j elasticity of capital utilization with respect to the rental rate. The parameterization of σ_b ensures that the capital utilization rate is unity ($h = 1$) in all sectors. For parsimony, we also restrict the depreciation function to be the same within each subsector of consumption.

Investment is sector-specific and subject to both endogenous depreciation and quadratic adjustment costs as in **?**:

$$\begin{aligned} k'_j &= (1 - \delta_j(h_j))k_j + [1 - S(i_j/i_{j,-1})]i_j, \quad j \in \{mc, sc, i\} \\ S(x) &= \frac{\Psi_K}{2}(x-1)^2 \end{aligned}$$

[10]By 'fixed' we mean that the cost does not vary with the choices of inputs.

so that aggregate investment is $i = i_{mc} + i_{sc} + i_i$. We also use a common adjustment term Ψ_K for parsimony.[11]

In the spirit of **?**, there is a competitive search protocol in which each submarket is indexed by price, market tightness, and potential output (p, q, F). The measure of matches in each submarket is given by a constant returns to scale matching function

$$M_j(D,T) = A_j D^{\phi} T^{1-\phi}, \quad 0 < \phi < 1, \quad j \in \{mc, sc, i\}$$

where D is aggregate shopping (search) effort, T is the measure of firms, and A_j is sector-specific matching efficiency. Market tightness is defined as search effort per firm location, $q = D/T$. We set $T = 1$, so that D measures market tightness. The probability that a unit of shopping effort is matched with a firm is $\Psi_{jd} = A_j D^{\phi-1}$ and the probability that a firm location matches with a customer is $\Psi_{jT} = A_j D^{\phi}$.

Once a match is formed, goods are traded at the posted price p_j. A household exerting search d_j purchases a real quantity of goods given by

$$y_j = d_j \Psi_{jd}(D) F_j, \quad j \in \{mc, sc, i\}$$

3.2. Households and firms

Households have preferences over search effort, consumption, and a labor composite following **?**. However, we also accommodate external habit formation, which is important to fit the data. Letting $\boldsymbol{\theta} = (\theta_d, \theta_n, \theta_i)$ be a vector of preference shifters, household utility is given by

$$u(c, d, n^a, \boldsymbol{\theta}) = \frac{\Gamma^{1-\sigma} - 1}{1 - \sigma}$$

where Γ is a composite variable

$$\Gamma = c - haC_{-1} - \theta_d \frac{d^{1+1/\eta}}{1 + 1/\eta} - \theta_n \frac{(n^a)^{1+1/\zeta}}{1 + 1/\zeta}$$

where C is aggregate consumption, ha is habit stock, and $d = d_{mc} + d_{sc} + \theta_i d_i$ is total search effort. Thus, θ_i is an exogenous wedge in the search cost of investment goods relative

[11]We have also estimated the model with sector-specific investment adjustment cost functions and have not found significant differences in the results.

to consumption goods. The parameter η is the elasticity of shopping effort, and ζ is the Frisch elasticity of labor supply.

Household consumption is a constant-elasticity-of-substitution aggregator of a bundle of goods y_{mc} and services y_{sc} with the associated price index:

$$c = [\omega_{mc}^{1-\rho_c} y_{mc}^{\rho_c} + \omega_{sc}^{1-\rho_c} y_{sc}^{\rho_c}]^{1/\rho_c}$$
$$p_c = \left(\omega_{mc} p_{mc}^{-\rho_c/(1-\rho_c)} + \omega_{sc} p_{sc}^{-\rho_c/(1-\rho_c)}\right)^{-\frac{1-\rho_c}{\rho_c}}$$

such that $\omega_{mc} + \omega_{sc} = 1$ and the elasticity of substitution is given by $\xi = 1/(1-\rho_c)$. Thus, p_{mc}/p_c and p_{sc}/p_c are the relative prices of nondurables and services to consumption overall.

Households have preferences over the composition of labor they supply across sectors, following **?** and **?**. Specifically, the labor composite n^a is

$$n^a = \left[\omega^{-\varepsilon} n_c^{1+\varepsilon} + (1-\omega)^{-\varepsilon} n_i^{1+\varepsilon}\right]^{\frac{1}{1+\varepsilon}}$$

where elasticity of substitution $1/\varepsilon$ measures intersectoral labor mobility. In the limiting case as $\varepsilon \to 0$, the marginal rate of substitution becomes infinite and labor is perfectly mobile: $n^a = n_c + n_i = n$.

A representative firm in sector $j \in \{mc, sc, i\}$ offers market bundle (p_j, D_j, F_j) and employs capital at rental rate R_j and labor at wage W_j in competitive spot markets to maximize profits. We introduce exogenous time-varying wage markups following the approach by **?**, where a continuum of monopolistically competitive labor unions in each sector sell differentiated labor services.

Figure **??** summarizes the timing of the economy. First, aggregate shocks occur at the beginning of each period. Second, in each sector j, firms post submarket offers (p_j, D_j, F_j). Third, given the submarket choices, households choose shopping effort, consumption, labor supply, and capital utilization. Firms simultaneously hire labor in a competitive spot market, which determines the wage. Fourth, matching takes place, and matched firms produce and sell. Fifth, the capital stock is updated in each sector, reflecting investment adjustment costs and endogenous depreciation.

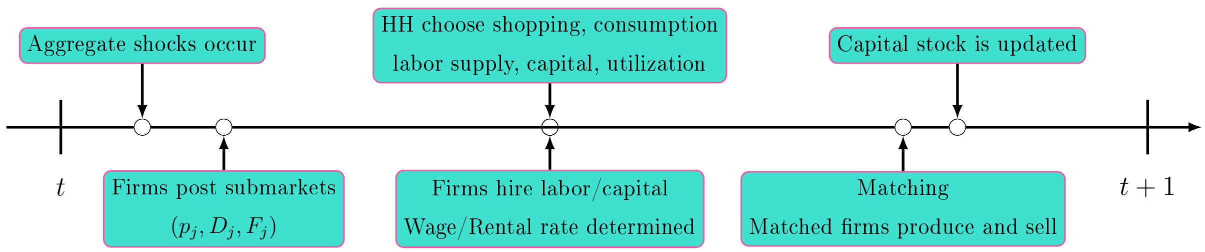

Figure 2: Timing

4. Demand shocks and the role of capacity utilization in a static setting

We first highlight the productive role of demand and show that capacity utilization data can be used to discipline the key parameters underlying transmission. Consider the baseline model by **?**. This formulation is a special case of our general environment without habit formation ($ha = 0$); perfectly mobile labor ($\varepsilon = 0$); a single consumption sector ($\rho_c \to 1$); no fixed costs in production ($\nu_j = 0$ for all j); fixed capital intensity ($\sigma_b \to \infty$); and no investment adjustment costs ($\Psi_K = 0$). In addition to demonstrating the importance of using capacity utilization data, we also show that sectoral comovement patterns, besides being important business cycle moments in their own right, inform the transmission of demand shocks in our environment.

To show how demand shocks can influence measured productivity, first consider a static version of BRS. The consumption good is produced using only labor ($\alpha_k \to 0$), so that (**??**) is simply $f(n) = n^{\alpha_n}$. A household who shops in submarket (p, D, F) chooses consumption, search effort, and labor supply in order to maximize their period utility:

$$\hat{V}(p, D, F) = \max_{d,c,n} u(c, d, n, \boldsymbol{\theta})$$
$$\text{s.t.} \quad c \leq d\Psi_d(D)F$$
$$pc \leq nW$$

Let $V = \max_{p,D,F} \hat{V}(p, D, F)$ be the value of the best submarket. Firms must provide households with value V to ensure their participation. The value V is an equilibrium object but is taken as given by firms. A firm chooses which market bundle (p, D, F) to offer and the amount of labor n to employ to maximize period profits:

$$\max_{p,D,F,n} p\Psi_T(D)F - Wn$$
$$\text{s.t.} \quad \hat{V}(p, D, F) \geq V$$

$$zn^{\alpha_n} \geq F$$

Applying matching function (??), preferences (??), and aggregating shows that an equilibrium can be characterized as a tuple (C, D, W, n) satisfying optimal shopping, consistency of output, labor supply, and labor demand:

$$\theta_d D^{\frac{1}{\eta}} = \phi \frac{C}{D}$$
$$C = AD^{\phi} zn^{\alpha_n}$$
$$(1-\phi)W = \alpha_n \frac{C}{n}$$
$$\theta_n N^{\frac{1}{\zeta}} = (1-\phi)W$$

The GHH structure of preferences between consumption and shopping effort in (??) implies that the marginal rate of substitution is an increasing function of shopping effort: $-u_d/u_c = \theta_d d^{1/\eta}$. Equation (??) equates this marginal rate of substitution to the new matches induced by greater shopping effort—the product of $\partial M/\partial D = \phi \Psi_d$ and firm capacity F, which simplifies to $\phi C/D$. Equation (??) is a standard labor demand condition which equates the cost of labor to its value marginal product. Here the marginal product includes the probability of a firm finding a customer, $\Psi_T z f'(n) = z\alpha_n n^{\alpha_n - 1} AD^{\phi}$, so that labor demand is increasing in aggregate search effort. Equation (??) is a GHH labor supply condition: the marginal rate of substitution between consumption and labor, $-u_n/u_d = \theta_n n^{1/\zeta}$ equals the wage rate scaled by $(1-\phi)$. Moreover, the cost of labor is scaled by $(1-\phi)$. This feature arises from competitive search: increased output relaxes the household's participation constraint and thereby effectively lowers the input cost for the firm.

The labor share of income is $\tau \equiv Wn/C = \alpha_n/(1-\phi)$ using (??). Hence, the Solow residual is

$$SR \equiv C/n^{\tau} = AD^{\phi} zn^{\alpha_n - \tau} = AD^{\phi} zn^{-\alpha_n \phi/(1-\phi)}$$

Total factor productivity thus depends on technology, shopping effort, and mismeasurement of labor component. Capacity utilization is defined as the ratio of actual output (??) to capacity F:

$$util \equiv C/F = AD^{\phi}$$

Capacity utilization (??) measures how far realized output is from potential output. In the absence of any shocks to matching efficiency, the growth rate of capacity utilization is simply shopping effort scaled by the matching elasticity ϕ.

Figure **??** depicts the equilibrium using two graphs. The figure on the right shows the determination of search effort and consumption, for a given level of capacity F, as the intersection between (**??**) and (**??**). The figure on the left illustrates the determination of hours and wages, given consumption C, as the intersection between (**??**) and (**??**).

`static_model_rev.pdf`

Figure 3: Equilibrium of static model

Now, let us consider a negative shock to the shopping disutility θ_d (Figure **??**). The marginal cost of exerting shopping effort falls, inducing households to shop more intensely, represented by the shopping curve shifting rightward. More shopping effort increases firms' matching rate and therefore boosts total production. This effect constitutes movement along the consumption curve from point 1 to point 2. To satisfy higher production levels, firms demand more workers, shifting the labor demand curve rightward and boosting labor hours and wages. Finally, more labor hours expands the productive capacity of firms, so the consumption curve shifts upward. This higher capacity further spurs shopping effort, represented by movement along the shopping curve from point 2 to point 3. The Solow residual therefore reflects both the initial increase in shopping effort from the demand shock followed by a further increase in shopping effort as households respond to increased capacity of firms. However, the rise of the Solow residual is slightly dampened by the mismeasurement of input shares. Notice that the demand shock to θ_d induces positive comovement across all variables in the economy and therefore resembles a standard technology shock to z.

static_model_shock_rev.pdf

Figure 4: Reduction of shopping disutility in static model

Similarly, we examine the impact of a fall in labor disutility θ_n. This shocks shifts the labor supply curve rightward and increases capacity. The consumption curve shifts rightward and triggers a movement along the shopping curve, as before.[12]

?? builds on this simple setting by estimating a dynamic version with capital accumulation. The exercise follows **?**, who investigates how observable variable selection affects estimated parameters in a rich New Keynesian model. We use the same dataset as BRS, other than shopping time, and estimate ϕ and η directly. We find that the posterior 90% probability band of ϕ ranges from 0.00 to 0.21, and the importance of shopping-disutility shocks in the variance decomposition drops significantly relative to BRS.

Next, we estimate the same model but include capacity utilization as an observable series. Remarkably, the posterior probability band of ϕ changes to $(0.86, 0.91)$, and the contribution of demand shocks to the variance decomposition rises dramatically. Additionally, the standard deviation of capacity utilization increases ten-fold in this case compared to the former, aligning with empirical values. Second, we show that the estimated model generates sectoral comovement of labor and output consistent with the data, in contrast to a standard two-sector RBC model driven solely by technology shocks.

5. Equilibrium

5.1. Households

Let $(p, D, F) = \{(p_j, D_j, F_j) | j \in \{mc, sc, i\}\}$ be the set of submarkets available to a household. Let Λ be the aggregate state and let $\hat{V}(\Lambda, k_{mc}, k_{sc}, k_i, p, D, F)$ be the value of the household conditional on these submarkets. Letting Φ be the set of available submarkets, then the value function is determined by the best combination of submarkets: $V(\Lambda, k_{mc}, k_{sc}, k_i) = \max_{\{p,D,F\}\in\Phi} \widehat{V}(\Lambda, k_{mc}, k_{sc}, k_i, p, D, F)$. The household chooses search effort, labor hours, consumption, future capital, and utilization rates to solve:

$$\widehat{V}(\Lambda, k_{mc}, k_{sc}, k_i, p, D, F) = \max_{d_j, n_c, n_i, y_j, i_j, k'_j, h'_j} u(y_{mc}, y_{sc}, d, n^a, \boldsymbol{\theta}) + \beta\theta_b \mathbb{E}\{V(\Lambda', k'_{mc}, k'_{sc}, k'_i)|\Lambda\}$$

$$\text{s.t.} \quad y_j = d_j A_j D_j^{\phi-1} F_j, \quad j \in \{mc, sc, i\}$$

$$\sum_j y_j p_j = \pi + \sum_{j\in\{mc,sc,i\}} k_j h_j R_j + n_c W_c^* + n_i W_i^*$$

$$k'_j = (1 - \delta_j(h_j))k_j + [1 - S(i_j/i_{j,-1})]i_j, \quad j \in \{mc, sc, i\}$$

[12] In Online Appendix G, we also examine equilibrium in a static setting in which matching costs arise from expenditure à la **?**. The causal effect of demand on output and productivity is essentially the same, but the labor share of income is α_n. Hence, there is no input share mismeasurement in the Solow residual.

and the consumption and labor aggregators (??) and (??).

?? derives each step of the household and firm problem. Here we focus on central and innovative features of equilibrium. Under goods market frictions, households trade off the marginal disutility of shopping with the marginal benefit of consumption and investment:

$$
\begin{aligned}
-u_d &= u_j \phi A_j D_j^{\phi-1} F_j \quad j \in \{mc, sc\} \\
-u_d \theta_i &= \frac{u_{mc} p_i}{p_{mc}} \phi A_i D_i^{\phi-1} F_i
\end{aligned}
$$

Equation(??) gives optimal shopping in each consumption subsector. The friction creates a wedge between marginal utility and price, which depends only on ϕ:

$$
\frac{u_j}{\lambda p_j} = \frac{1}{1-\phi} \Rightarrow \frac{u_{mc}}{p_{mc}} = \frac{u_{sc}}{p_{sc}}
$$

where λ is the marginal utility of wealth. Multiplying the price by λ converts it into units of utility. Equivalently, $\phi = (u_j - \lambda p_j)/u_j$.

Recall from GHH preferences that $-u_d/u_j = \theta_d d^{1/\eta}$ is an increasing function of shopping effort alone. Combining this with equation (??), we conclude that households increase their shopping effort in response to higher firm capacity and matching probability, as well as a lower disutility of shopping effort. The condition for investment goods in equation (??) is similar, but with the marginal disutility adjusted by θ_i and the value of output computed in consumption units, accounting for the relative price.

Given (??), households optimally divide their labor hours between consumption and investment sectors:

$$
\frac{n_c}{n_i} = \frac{\omega}{1-\omega} \left(\frac{W_c^*}{W_i^*} \right)^{1/\varepsilon}
$$

so that $1/\varepsilon$ is the elasticity of substitution.

Taking the first order condition with respect to y_{mc} and y_{sc} and combining it with (??), we derive the demand curves for nondurables and services

$$
y_j = p_j^{-\xi} \omega_j C \quad j \in \{mc, sc\}
$$

where $\xi = 1/(1-\rho_c)$ represents the elasticity of substitution. By using (??) together with (??), we find that $\lambda = \Gamma^{-\sigma}(1-\phi)$. Here, the term $\Gamma^{-\sigma}$ captures the direct influence from the marginal utility of consumption, while the goods market frictions introduce a wedge represented by ϕ.

Furthermore, the ratio of (??) and (??) provides insight into the relative price of investment:

$$\frac{p_i}{p_j} = \theta_i \frac{A_j}{A_i} \left(\frac{D_j}{D_i}\right)^{\phi-1} \frac{z_j}{z_i} \frac{f(h_j k_j, n_j) - \nu_j X}{f(h_i k_i, n_i) - \nu_i X}$$

If the price p_i increases compared to p_j, with capacity held constant, it implies that investment goods become more valuable in terms of consumption, leading to an increase in D_i/D_j. Additionally, equation (??) reflects the typical mechanism where an increase in investment capacity results in a decrease in the relative price p_i/p_j.

5.2. Firms and labor unions

A representative firm in sector $j \in \{mc, sc, i\}$ rents capital and hires labor in spot markets. We introduce exogenous time-varying wage markups following the approach by **?**. A continuum of monopolistically competitive labor unions in sector j sell differentiated services, indexed by type s. The firm chooses inputs and market bundle (p_j, D_j, F_j) to maximize profits given the household participation constraint, technology, and differentiated labor. The problem is

$$\begin{aligned}
\max_{k_j, n_j, p_j, D_j, F_j} \quad & p_j A_j D_j^{\phi} F_j - \int_0^1 W_j(s) n_j(s) ds - R_j h_j k_j \\
\text{s.t.} \quad & \widehat{V}(\Lambda, k_{mc}, k_{sc}, k_i, p_j, D_j, F_j) \geq V(\Lambda, k_{mc}, k_{sc}, k_i) \\
& z_j f(h_j k_j, n_j) - \nu_j X \geq F_j \\
& n_j = \left(\int_0^1 n_j(s)^{1/\mu_j} ds\right)^{\mu_j}
\end{aligned}$$

The conditional demand for labor type s in sector j and corresponding wage index are

$$n_j(s) = \left(\frac{W_j(s)}{W_j}\right)^{-\frac{\mu_j}{\mu_j - 1}} n_j, \quad W_j = \left[\int_0^1 w_j(s)^{1/(1-\mu_j)} ds\right]^{1-\mu_j}$$

The labor union charges the firm a wage $W_j(s)$ and pays W_j^* to its members. It maximizes earnings subject to the conditional labor demand of the firm. The problem of the union is thus

$$\max_{W_j(s)} (W_j(s) - W_j^*) \left(\frac{W_j(s)}{W_j}\right)^{-\frac{\mu_j}{\mu_j - 1}} n_j$$

The solution to (??) is $W_j(s) = \mu_j W_j^*$. Within sector j, labor unions pay the same wage and firms choose identical quantities of labor within j: $W_j(s) = W_j, n_j(s) = n_j$ for all s. Labor unions provide additional earnings to households in the form of a wage rebate. Consequently, $W_j(s) - W_j^* = (\mu_j - 1)W_j^*$ represents a fixed component of the wage from the perspective of workers.[13]

The factor demand curves for the firm are

$$
\begin{aligned}
(1-\phi)\frac{W_j}{p_j} &= \alpha_n \frac{A_j D_j^{\phi} z_j f(h_j k_j, n_j)}{n_j} \quad j \in \{mc, sc, i\} \quad W_{mc} = W_{sc} \\
(1-\phi)\frac{R_j}{p_j} &= \alpha_k \frac{A_j D_j^{\phi} z_j f(h_j k_j, n_j)}{h_j k_j} \quad j \in \{mc, sc, i\}
\end{aligned}
$$

To provide an alternative characterization of the relative price of investment, we take the ratio of (??) for sectors i and $j \in \{mc, sc\}$:

$$
\frac{p_i}{p_j} = \frac{n_i W_i}{n_j W_j} \frac{A_j}{A_i} \left(\frac{D_j}{D_i}\right)^{\phi} \frac{z_j f(h_j k_j, n_j)}{z_i f(h_i k_i, n_i)}
$$

When D_j/D_i increases, while holding inputs and technology constant, it becomes easier to sell nondurables or services to customers, resulting in an increase in p_i/p_j. Equation (??) also takes into account the standard relationship where p_i/p_j decreases as investment-specific technology z_i/z_j rises.

Relationships (??) and (??) represent distinct curves that connect the relative price of investment p_i/p_j to relative shopping effort D_i/D_j. However, a direct comparison is complicated by the fact that fixed costs are present in (??) but not in (??). In the case of zero fixed costs, mutual consistency requires the following relationship:

$$
\frac{D_i}{D_j} = \frac{1}{\theta_i} \frac{n_i W_i}{n_j W_j}
$$

Relative shopping effort is determined by relative labor income and the variation in shopping disutility. Over the business cycle, the degree of sectoral comovement influences n_i/n_j and thus provides information about relative shopping effort. However, compared with (??), in which the ratios of shopping effort and labor supply perfectly coincide, (??) is significantly more flexible. Limited factor mobility and wage markup shocks allow for additional fluctuations in relative wages, and the exogenous wedge θ_i also helps explain fluctuations in relative

[13]Labor unions here are a mechanism here designed entirely for the benefit of workers. Thus, the earnings rebated to the workers count as labor income, which matters for the mapping between model and data. Note that wages remain flexible even though there is market power in wage setting.

shopping effort.

The final three equilibrium conditions encompass Tobin's Q, optimal capital utilization, and Euler equations pertaining to the selection of future capital. These conditions incorporate investment adjustment costs and depreciation resulting from capital utilization:

$$\begin{aligned}
\frac{p_i}{1-\phi} &= Q_j[1 - S'(x_j)x_j - S(x_j)] + \beta\theta_b\mathbb{E}\frac{\lambda'}{\lambda}Q'_jS'(x'_j)(x'_j)^2 \quad j \in \{mc, sc, i\} \\
\delta^j_h(h_j)Q_j &= R_j \quad j \in \{mc, sc, i\} \\
Q_j &= \beta\theta_b\mathbb{E}\frac{\lambda'}{\lambda}\left[(1-\delta^j(h'_j))Q'_j + R'_jh'_j\right] \quad j \in \{mc, sc, i\}
\end{aligned}$$

The variable Q_j represents the relative price of capital in sector j in terms of consumption. The presence of investment adjustment costs introduces a disparity between Q_j and $p_i/(1-\phi)$. Households determine the level of utilization such that the value of depreciated capital, $\delta_h(h_j)Q_j$, is equal to the marginal product of capital, R_j. Finally, households choose the capital level that equates the marginal cost of foregone consumption Q_j to the anticipated discounted return. The expected return comprises the marginal product of capital in addition to the value of undepreciated capital, and the stochastic discount factor $\beta\theta_b\mathbb{E}\lambda'/\lambda$ transforms returns into current marginal utility.

5.3. Inducing stationarity

The specification of technology (**??**) implies that output, consumption, wages, and capital have the same stochastic trend as technology X_t, characterized by the gross growth rate $g_t = X_t/X_{t-1}$. The next section shows that the trend growth rate of the Solow residual is g_t^τ, given labor share τ. Preferences regarding labor supply imply zero long-run wealth effects and hence ensure its stationarity. We also adjust GHH preference weights to ensure stationarity of shopping effort. To analyze fluctuations around trend, we detrend all variables (except the Solow residual and capital) by dividing by X_t. Capital is instead divided by X_{t-1} to preserve its predetermined status, while the Solow residual is detrended by X_t^τ.

5.4. The sector-specific Solow residual and capacity utilization

We construct the Solow residual for a specific sector in the model and relate it to capacity utilization and other structurally interesting components. Begin by expressing sectoral output as follows:

$$Y_{jt} = A_jD^\phi_{jt}(z_{jt}h^{\alpha_k}_{jt}X^{1-\alpha_k}_t k^{\alpha_k}_{jt}n^{\alpha_n}_{jt} - \nu_jX_t)$$

Let $\nu_j^R = \nu_j X/F_j$ be the fixed cost share of capacity. Then note that $\nu_j X/(z_j f(h_j k_j, n_j)) = \nu_j^R/(1+\nu_j^R)$, so that

$$Y_{jt} = \frac{A_j D_j^{\phi}(z_{jt} h_{jt}^{\alpha_k} X_t^{1-\alpha_k} k_{jt}^{\alpha_k} n_{jt}^{\alpha_n})}{1+\nu_{jt}^R}$$

? constructs the sectoral Solow residual under the assumptions of constant returns to scale Cobb-Douglas technology in capital and labor, no fixed costs, and perfectly competitive factor markets. Analogously, define the Solow residual in sector j as

$$SR_{jt} \equiv \frac{Y_{jt}}{k_{jt}^{1-\tau} n_{jt}^{\tau}} = \frac{A_j D_{jt}^{\phi}(z_{jt} h_{jt}^{\alpha_k} X_t^{1-\alpha_k} k_{jt}^{\alpha_k - 1+\tau} n_{jt}^{\alpha_n - \tau})}{1+\nu_{jt}^R}$$

where τ represents the steady-state labor income share. To express (**??**) in terms of growth rates, we introduce the symbol $dx_t = \Delta \log x_t$ and rewrite as

$$\begin{aligned} dSR_{jt} &= \phi dD_{jt} + dz_{jt} + \alpha_k dh_{jt} + (1-\alpha_k)dX_t + (\alpha_k - 1 + \tau)dk_{jt} \\ &\quad + (\alpha_n - \tau)dn_{jt} - d(1+\nu_{jt}^R) \end{aligned}$$

From (**??**) we note that the trend net growth rate of the Solow residual is

$$(1-\alpha_k)dX_t + (\alpha_k - 1 + \tau)dX_t = \tau \log g_t$$

which implies that the Solow residual grows at the rate of output multiplied by the labor share of income τ. By introducing the log deviation $\tilde{\nu}_j^R = \log(\nu_j^R/\nu_{ss}^R)$, we can rewrite (**??**) as[14]:

$$dSR_{jt} = \phi dD_{jt} + dz_{jt} + \alpha_k dh_{jt} + (1-\alpha_k)dX_t + (\alpha_k - 1+\tau)dk_{jt} + (\alpha_n - \tau)dn_{jt} - \frac{\nu_{ss}^R}{1+\nu_{ss}^R}\Delta\tilde{\nu}_{jt}^R$$

Expression (**??**) decomposes the growth rate of the Solow residual into structural forces. It comprises a demand component ϕdD_{jt}, a capital utilization component $\alpha_k dh_{jt}$, a technology

[14]Calculate

$$\log(1+\nu_j^R) \approx \log(1+\nu_{ss}^R) + \frac{1}{1+\nu_{ss}^R}(\nu_{jt}^R - \nu_{ss}^R) \approx \log(1+\nu_{ss}^R) + \frac{\nu_{ss}^R}{1+\nu_{ss}^R}\tilde{\nu}_{jt}^R$$

Hence, $d(1+\nu_{jt}^R) = \Delta\log(1+\nu_{jt}^R) \approx \frac{\nu_{ss}^R}{1+\nu_{ss}^R}\Delta\tilde{\nu}_{jt}^R$

component $dz_{jt} + (1 - \alpha_k)dX_t$, an input share mismeasurement component $(\alpha_k - 1 + \tau)dk_{jt} + (\alpha_n - \tau)dn_{jt}$, and a change in the fixed cost share component $[\nu_{ss}^R/(1 + \nu_{ss}^R)]\Delta\tilde{\nu}_{jt}^R$. The first component reflects the direct effect of goods market frictions, and there is also a general equilibrium feedback between higher shopping effort and the other components. Additionally, the calibration strategy establishes a relationship between the coefficients α_k and α_n in relation to ϕ. It is worth noting that the growth rate of cyclical labor productivity $d(Y_{jt}/n_{jt})$ has the same expression as (**??**), except that τ is replaced by 1. Therefore, $d(Y_{jt}/n_{jt}) = dSR_{jt} + (1 - \tau)(dk_{jt} - dn_{jt})$. In general, we find that the Solow residual and labor productivity behave similarly in cyclical terms, and choose to emphasize the former because of its significance in the literature.

We next turn to capacity utilization and relate it to the Solow residual. Following **?**, we define capacity in sector j as

$$cap_j = z_j k_j^{\alpha_k} n_j^{\alpha_n} X^{1-\alpha_k} - \nu_j X$$

Consistent with the definition from the Federal Reserve Board, capacity utilization in sector j is the ratio of output to capacity:

$$\begin{aligned} util_{jt} \equiv \frac{Y_{jt}}{cap_{jt}} &= \frac{A_j D_{jt}^{\phi}(z_{jt} h_{jt}^{\alpha_k} X_t^{1-\alpha_k} k_{jt}^{\alpha_k} n_{jt}^{\alpha_n} - \nu_{jt} X_t)}{z_{jt} k_{jt}^{\alpha_k} n_{jt}^{\alpha_n} X_t^{1-\alpha_k} - \nu_{jt} X_t} \\ &= \frac{A_j D_{jt}^{\phi}(z_{jt} h_{jt}^{\alpha_k} (k_{jt}/X_t)^{\alpha_k} n_{jt}^{\alpha_n} - \nu_{jt})}{z_{jt}(k_{jt}/X_t)^{\alpha_k} n_{jt}^{\alpha_n} - \nu_{jt}} \end{aligned}$$

Capacity utilization is stationary since k_j grows at the same rate g as X on the balanced growth path. Expressing (**??**) in growth rates yields

$$dutil_{jt} = \phi dD_{jt} + (1 + \nu_{ss}^R)\alpha_k dh_{jt}$$

The growth rate of utilization equals that of shopping effort scaled by ϕ and capital utilization scaled by $(1 + \nu_{ss}^R)\alpha_k$. Therefore, higher fixed costs amplify the weight of capital utilization relative to shopping effort.

By comparing (**??**) and (**??**), we see that shopping effort enters with the same weight ϕ but that the weight of capital utilization differs due to the presence of fixed costs. In the special case of zero fixed costs, the Solow residual growth rate simplifies to the sum of growth rates of utilization, technology, and mismeasurement of input shares:

$$dSR_{jt}|_{\nu_j=0} = dutil_{jt} + dz_{jt} + (1-\alpha_k)dX_t + (\alpha_k - 1 + \tau)dk_{jt} + (\alpha_n - \tau)dn_{jt}$$

Our sectoral definition of the Solow residual, following the methodology outlined by **?**, mitigates composition bias that may arise from employing an aggregate production technology. Furthermore, it aligns sensibly with the FRB measure of capacity utilization, which applies to specific industries.

We define the aggregate Solow residual and capacity utilization as the output-weighted average of sectoral values:

$$SR = \prod_j SR_j^{\left(\frac{Y_j}{Y}\right)}, \quad util = \prod_j util_j^{\left(\frac{Y_j}{Y}\right)}$$

Applying logs and first differencing to (**??**) immediately implies

$$dSR = \sum_j \frac{Y_j}{Y} dSR_j, \quad dutil = \sum_j \frac{Y_j}{Y} dutil_j$$

Using the geometric average in (**??**) enables us to express the average growth rate as the weighted average of sectoral growth rates exactly, whereas it is an approximation in the case of the arithmetic average. Hence, we can quantify the proportion of Solow residual variance explained by the utilization component, $Var(dutil)/Var(dSR)$.

We have discussed the Solow residual and capacity utilization in terms of growth rates to facilitate comparison with empirical practice (e.g., **?**) and to maintain consistency with the form of variables used in the observation equations and for business cycle statistics. In **??**, we provide a similar comparison between the cyclical deviations of the Solow residual and capacity utilization.

6. Main quantitative analysis

We now describe the stochastic processes of the model and estimate it using Bayesian techniques.

6.1. Stochastic processes

The growth rate of the stochastic trend $g_t = X_t/X_{t-1}$ follows an AR(1) process in logs, as in **?**:

$$\log g_t = (1-\rho_g)\log \overline{g} + \rho_g \log g_{t-1} + e_{g,t}$$

where $e_{g,t} \sim N(0, \sigma_g)$. Here, $\log X_t$.

We also consider a stationary neutral shock z_c and an investment-specific shock z_i. We let $z_i \equiv z_c z_I$ where z_I is independent of z_c. Finally, there are disturbances to general shopping disutility θ_d, investment-specific shopping disutility θ_i, the discount factor θ_b, labor supply θ_n, and wage markups μ_c and μ_i. We do not include consumption preference shocks because they can be replicated by sequences of labor supply, shopping disutility, and discount-factor shocks. Note that nondurables and services are subject to the same technology shocks.

Each stationary shock in the set $v = \{\theta_b, \theta_d, \theta_n, \theta_i, z_c, z_I, \mu_c, \mu_i\}$ follows an AR(1) process:

$$\log v_t = \rho_v \log v_{t-1} + e_{v,t}, \quad e_{v,t} \sim N(0, \sigma_v)$$

6.2. Bayesian estimation

The Bayesian framework enables us to incorporate prior information—such as microeconomic evidence—quantify parameter uncertainty, decompose the forecast error variance attributable to each shock, and compare model fit through the marginal likelihood. Notably, the marginal likelihood automatically penalizes unnecessary parameter complexity: expanding the parameter space without improving model fit dilutes the prior probability assigned to effective parameter values, thereby reducing the marginal likelihood.

Along these lines, we estimate the general model using Bayesian techniques with quarterly data from 1964Q1 to 2019Q4. The likelihood of the data sample $\mathcal{Y}$ given the estimated parameters Θ is denoted as $L(\mathcal{Y}|\Theta)$. By incorporating the prior parameter distribution $P(\Theta)$, the posterior density is proportional to $L(\mathcal{Y}|\Theta)P(\Theta)$. We employ the random walk Metropolis-Hastings algorithm, which is standard practice for drawing from the posterior distribution of Θ. To sample the posterior distribution, we draw more than 1 million parameter sets and discard the first 30% as burn-in. We use the mode of the posterior distribution as the initial value for the chain and the Hessian as the proposal covariance matrix.

We use the following observables expressed in growth rates: consumption C, investment I, labor hours n_c and n_i, sectoral utilization $util_{ND}$ and $util_D$, and the relative price of investment

p_i. This dataset is similar to ?, but we include the utilization variables and exclude wages. Formally, the vector of observables $\mathcal{Y}_t$ is

$$\mathcal{Y}_t = \begin{bmatrix} dC_t & dI_t & dn_{ct} & dn_{it} & dutil_{ND,t} & dutil_{D,t} & dp_{it} \end{bmatrix}'$$

The vector of estimated parameters Θ consists of the persistence and conditional standard deviations for shocks, the risk aversion parameter σ, the habit formation parameter ha, the parameter ζ (closely related to the Frisch elasticity of labor supply), the fixed cost share parameter of potential output ν^R, the elasticity of depreciation with respect to capital utilization (σ_{ac} and σ_{ai}), the investment adjustment cost parameter Ψ_K, the inverse of the intersectoral elasticity of labor supply ε, and the elasticity of substitution between nondurables and services ξ. Our primary focus is on the elasticity of the matching function with respect to shopping effort ϕ and the shopping disutility parameter η.

To calibrate the remaining parameters, we use long-run targets, normalizations, and a subset Θ_R of the estimated parameters. Table **??** presents the results. The fixed exogenous parameters include the discount factor β, average growth rate $\overline{g}$, gross wage markup μ, the share ω of labor hours in consumption, and the share of services in consumption. Following the approach of **?** and standard practice, we set $\beta = 0.99$, $\overline{g} = 0.45\%$, $\mu = 1.15$, and $\omega = 0.8$. We pin down the weight of services ω_{sc} in the consumption aggregator as the average share of services in consumption, $\omega_{sc} = p_{sc}y_{sc}/C = 0.65$ over the sample. Consequently, the share of services in output is $0.65 \times 0.80 = 0.52$, implying that capacity utilization is unobserved for about half of the economy.

The second set of parameters Θ_R is estimated and used to calibrate other parameters. These are the parameters of risk aversion σ, labor supply ζ, elasticity of the matching function ϕ, elasticity of shopping effort cost η, fixed cost share ν^R, and habit persistence ha.

The third set of parameters determines the choice of units but does not impact the cyclical behavior of the economy. We normalize output and the relative price of services and investment to unity, effectively determining the level parameters of technology for each sector. Additionally, we set the fraction of time allocated to work as 30%, which, in conjunction with other parameters, specifies the value of θ_n. To achieve a target capacity utilization of 81% in each sector, we adjust the level parameters A_j of the matching function accordingly. Finally, by setting the capital intensity to 1, we obtain the value for σ_b.

The fourth set of parameters are determined through long-run targets and the estimated parameters in the second group. The long-run targets include those chosen by **?**. These are an investment share of output of 20%, an annual capital-to-output ratio of 2.75, and a labor share of income of 67%. These in turn pin down the parameters δ, α_k and α_n. **??** discusses the calibration in detail. Note that, at the posterior mean, $\phi = 0.92$, $\nu^R = 0.094$, and $\alpha_k = 0.31$. Hence, from (**??**), $dutil_t \approx 0.92dD_t + 0.34dh_t$.

Targets	Target value	Parameter	Calibrated value/posterior mean
First group: parameters set exogenously			
Discount factor	0.99	β	0.99
Average growth rate	1.8%	$\bar{g}$	0.45%
Gross wage markup	1.15	μ	1.15
Labor share in consumption	0.8	ω	0.8
Share of services in consumption	0.65	ω_{sc}	0.65
Second group: estimated parameters used for calibration			
Risk aversion	–	σ	1.58
Frisch elasticity	–	ζ	1.24
Elasticity of matching function	–	ϕ	0.92
Elasticity of shopping effort cost	–	η	0.22
Fixed cost share of capacity	–	ν^R	0.094
Habit persistence	–	ha	0.74
Third group: normalizations			
SS output	1	z_{mc}	0.30
Relative price of services	1	z_{sc}	0.45
Relative price of investment	1	z_i	0.25
Fraction time spent working	0.30	θ_n	0.45
Capacity utilization of nondurables	0.81	A_{mc}	2.6
Capacity utilization of services	0.81	A_{sc}	1.5
Capacity utilization of investment sector	0.81	A_i	3.6
Capital utilization rate	1	σ_b	0.031
Fourth group: standard targets			
Investment share of output	0.20	δ	1.37%
Physical capital to output ratio	2.75	α_k	0.31
Labor share of income	0.67	α_n	0.053

Table 2: Calibration targets and parameter values. Here we calibrate a subset of parameters using long-run targets and the posterior mean of the estimated parameters $\sigma, \zeta, \phi, \eta, \nu^R$ and ha.

Table ?? presents the posterior estimates alongside their prior distributions. The parameters ϕ and η are fundamental to the transmission mechanism and uncommon in the DSGE literature, so it is especially important to assess their identification. Figure ?? plots the densities of the posterior and prior distributions for these two parameters, as well as for the fixed cost share ν^R, which provides an alternative channel through which the model can generate procyclical productivity.

posterior_prior_plots.pdf

Figure 5: Posterior and prior distributions for matching function elasticity ϕ, the shopping disutility parameter η, and the fixed cost share ν^R.

The posterior mean of the matching function elasticity ϕ is estimated to be 0.92, suggesting that the search-based demand channel plays a significant role in the model. Moreover, the influence of the data on updating ϕ is especially stark: at the posterior mean of 0.92, the prior density is minimal, underscoring how the likelihood dominates the shape of the posterior distribution. This sharp divergence indicates that ϕ is well identified, and the narrow HPD interval further suggests that the data provide precise information about its value. Although the posterior mean of η is estimated to be 0.22—close to its prior mean of 0.2—the posterior distribution is markedly tighter than the prior. The 95% HPD interval of $[0.16, 0.28]$, compared to the prior's diffuse distribution, reflects a significant gain in precision. This tightening suggests that while the data largely confirm prior beliefs about the magnitude of η, they also provide informative evidence that sharply reduces uncertainty about its true value.

Unlike the search-based demand channel—captured by ϕ and η—fixed costs can generate procyclical productivity by affecting average production efficiency over the business cycle. The posterior mean of ν^R is 0.09, substantially lower than the prior mean of 0.20, suggesting that the data favor a very limited role for fixed costs. Moreover, ν^R is well identified: the posterior distribution has a single, well-defined peak that diverges noticeably from the more diffuse prior. The joint posterior estimates thus support the search-based demand channel as a key driver of procyclical productivity.

We turn to the other parameters. The posterior mean values of σ (1.59) and ha (0.74) are consistent with previous findings in the literature. The inverse of the elasticity of substitution of labor, ε, has a posterior mean of 1.46, substantially less than the value 2.57 estimated by **?**. This difference can be attributed to the use of search demand shocks and absence of wealth effects, which naturally induce complementarity. The elasticity of substitution ξ between nondurables and services has a posterior mean of 0.88, which is fairly close to the prior mean, and is somewhat more concentrated compared to the prior distribution.

We estimate a high posterior mean of 12.6 for the investment adjustment cost parameter Ψ_K. Intuitively, large investment adjustment costs are necessary to permit a high volatility of utilization without triggering excessively high volatility of investment. The estimated elasticities of the marginal cost of capital utilization are higher for consumption than for investment, which aligns with the greater volatility of investment and capacity utilization in durable goods. However, the estimated values are lower than those reported in **?**, in order to fit the volatility of the utilization series.

We estimate generally high values for the persistence parameters. This is notably the case for the shopping-effort shocks, with posterior means of 0.90 and 0.98, respectively. The mean persistence of the neutral shopping-effort is very close to the value of 0.928 obtained in Section **??** and BRS's own estimate in Table 3. The posterior mean of ρ_g is 0.51, which indicative of moderate peristence of shocks to the stochastic trend, is moderately lower than the value 0.60 reported by BRS in Table 3. We also find greater persistence of wage markup shocks in investment (0.98) compared to consumption (0.74), a feature which seems necessary to fit the utilization data in conjunction with hours and the relative price of investment. The investment wage markup shock also has a far greater conditional standard deviation. Online Appendix F assesses the identifiability of these parameters by estimating the model on artificial data generated from the model evaluated at the posterior mean. Most parameters, in particular ϕ, η, ν^R, and most of the shock parameters are well-identified.[15]

[15] In the main text we focus on the standard criterion for identification that the observed data update the prior beliefs to yield a different posterior distribution. The test using artificial data in the Online Appendix instead focuses on the model structure rather than the actual data.

Table **??**: Bayesian estimation of baseline model

	Prior			Posterior			
	Dist.	Mean	Stdev.	Mean	Stdev.	5%	HPD 95%
σ	Beta	1.50	0.250	1.59	0.240	1.23	1.98
ha	Beta	0.500	0.200	0.739	0.0322	0.692	0.792
ζ	Gamma	0.720	0.250	1.26	0.215	0.947	1.60
ϕ	Beta	0.320	0.200	0.917	0.0531	0.834	0.991
η	Gamma	0.200	0.150	0.221	0.0388	0.159	0.277
ξ	Gamma	0.850	0.100	0.880	0.0826	0.753	1.02
ν^R	Beta	0.200	0.100	0.090	0.0400	0.028	0.152
σ_{ac}	Inv Gamma	1.00	1.00	1.74	0.328	1.20	2.23
σ_{ai}	Inv Gamma	1.00	1.00	0.440	0.0893	0.298	0.580
Ψ_K	Gamma	4.00	1.00	12.6	1.60	10.5	15.5
ε	Gamma	1.00	0.500	1.46	0.218	1.10	1.82
ρ_g	Beta	0.100	0.0500	0.507	0.0722	0.385	0.617
ρ_z	Beta	0.600	0.200	0.794	0.0376	0.735	0.856
ρ_{zI}	Beta	0.600	0.200	0.847	0.0300	0.797	0.896
ρ_n	Beta	0.600	0.200	0.989	0.0079	0.977	1.00
ρ_d	Beta	0.600	0.200	0.904	0.0257	0.863	0.947
ρ_{di}	Beta	0.600	0.200	0.982	0.0084	0.969	0.996
ρ_b	Beta	0.600	0.200	0.911	0.0234	0.873	0.949
$\rho_{\mu c}$	Beta	0.600	0.200	0.743	0.277	0.267	0.998
$\rho_{\mu i}$	Beta	0.600	0.200	0.977	0.0225	0.943	1.00
e_g	Gamma	0.010	0.010	0.004	0.0005	0.0037	0.0049
e_z	Gamma	0.010	0.010	0.009	0.0007	0.0083	0.0107
e_{zI}	Gamma	0.010	0.010	0.020	0.0021	0.0161	0.0230
e_n	Gamma	0.010	0.010	0.006	0.0009	0.0046	0.0075
e_d	Gamma	0.010	0.010	0.138	0.0188	0.1097	0.1706
e_{di}	Gamma	0.010	0.010	0.015	0.0010	0.0135	0.0167
e_b	Gamma	0.010	0.010	0.015	0.0059	0.0056	0.0236
$e_{\mu c}$	Gamma	0.010	0.010	0.001	0.0006	0.0001	0.0015
$e_{\mu i}$	Gamma	0.010	0.010	0.027	0.0030	0.0226	0.0326

Table 3: Prior and posterior distribution.

Table **??** documents the unconditional forecast error variance decomposition of the model. Technology shocks and shopping-effort shocks are the primary drivers of forecast error variance in output, the Solow residual, variable capital intensity, investment and its relative price, and variable capital intensity. Shopping-effort shocks have a particularly significant impact on utilization and output. The only significant contribution of discount-factor, wage markup, and labor supply shocks lies in explaining portions of labor in consumption and investment. However, the fraction of consumption-sector labor explained by labor supply shocks (27.2%) is second only to shopping-effort shocks.

Table **??**: Unconditional forecast error variance decomposition

	Technology	Labor Supply	Shopping Effort	Discount Factor	Wage Markup
Y	28.6	0.01	70.5	0.92	0.02
SR	44.3	5.23	46	0.57	3.9
I	31.2	0.01	64.1	4.69	0.01
p_i	65	0.00	34.8	0.18	0.05
n_c	7.78	27.2	58.4	4.42	2.19
n_i	18.2	2.27	52.9	1.76	24.8
$util$	39.3	0.01	60.1	0.64	0.01
D	0.17	0	99.8	0.01	0
h	17.7	0.01	82.1	0.18	0

Table 4: Unconditional forecast error variance decomposition for variables in growth rates. Shocks are grouped in respective categories.

Here our primary focus is on the Solow residual and utilization. Shopping-effort and technology shocks play similarly important roles for the former, but the search demand shocks explain over 60% of utilization. Hence, the evidence strongly supports a powerful causal channel of demand shocks into productivity. It is sensible to compare our results to Table 3 in BRS, which consider an estimation of the model without shopping-time data. They find that search demand shocks account for about 58% of the forecast error variance of the Solow residual, compared to 46% in our specification. However, this result relies on calibrating ϕ and η using shopping time and price dispersion information, whereas we instead estimate these parameters using capacity utilization data.

Our results also show that search demand shocks explain the majority of fluctuations in output, investment, and sectoral labor. This suggests that demand shocks play a greater role than technology shocks in driving business cycles, consistent with **?** and subsequent studies arguing that demand shocks are essential for generating strong comovement between hours worked and consumption. The dominance of shopping effort shocks over discount factor shocks supports a similar interpretation of business cycles as in **?**: during recessions, people spend less effort shopping for goods, consume fewer goods and services, and work fewer hours.

Table **??** compares the log marginal likelihood, posterior mean of ϕ, variance decomposition, and second moments for various specifications of the model. We calculate the log marginal data density using the modified harmonic mean estimator. The posterior mean of ϕ is significant in most specifications, but falls significantly under perfect labor mobility or absence of variable capital intensity. In the baseline model, search-based demand shocks account for nearly half of the Solow residual. The relative variance of utilization to the Solow residual is 1.95, but falls substantially under perfect labor mobility and the common wage markup.

Table ??: Comparison of model specification

	Data	Baseline	Perfect labor mobility	Common wage markup	Remove			
					Fixed cost	VCU	SDS	SDS and utilization data
LML	–	4570.7	4548.9	3136.5	4573.4	4,568.1	2564.9	–
Δ LML	–	0	-21.9	-1434.2	2.71	-2.6	-2006	–
Posterior mean ϕ	–	0.91	0.43	0.95	0.96	0.27	0.71	0.52
FEVD(SR, SDS)	–	46.0	55.0	6.17	45.71	48.07	–	–
Var$(util)$/Var(SR)	–	1.95	1.21	0.40	2.36	0.56	2.21	0.19
std(Y)	0.87	1.38	1.70	5.11	1.36	1.99	207.71	0.64
std$(util_{ND})$	1.26	1.21	1.30	3.88	1.22	1.21	161.65	0.35
std$(util_D)$	2.27	3.65	2.60	9.72	3.90	2.37	266.65	1.14
std(n_c)	0.57	0.66	0.68	2.27	0.68	0.69	71.31	0.56
std(n_i)	1.94	2.35	3.33	8.99	2.36	1.80	344.8	1.87
Cor(C, I)	0.54	0.53	0.67	0.05	0.50	0.53	0.999	0.24
Cor$(util_{ND}, util_D)$	0.75	0.27	0.61	-0.31	0.26	0.60	0.999	-0.60
Cor(n_c, n_i)	0.87	0.67	0.24	-0.88	0.69	0.35	0.986	0.83
Cor$(util_{ND}, util_{ND,-1})$	0.51	0.21	0.31	0.53	0.18	-0.02	0.999	0.27
Cor$(util_D, util_{D,-1})$	0.55	0.48	0.52	0.51	0.48	0.03	0.999	0.26

Table 5: Comparison of log marginal likelihood, posterior mean of ϕ, variance decomposition, and second moments for various specifications of the model. The log marginal likelihood (LML) is calculated using the modified harmonic mean. The first column describes relevant empirical moments, and the second column corresponds to the baseline model. The third and fourth columns present estimates of the model with perfect labor mobility ($\varepsilon = 0$) and only a common wage markup shock, respectively. The fifth and sixth columns present estimates in which fixed costs and variable capital utilization are removed. The seventh column removes search-based demand shocks, and the eighth column also removes the utilization series from the set of observables.

We next provide more context and probe more deeply into model fit by examining the second moments. The baseline model tends to overestimate the volatility of output but fits the volatility of the utilization series and labor hours quite well. It captures the correlation between consumption and investment well and reasonably fits the comovement of labor hours. It qualitatively captures the comovement of the utilization series but undershoots the magnitude. Finally, the model also matches the autocorrelation of the utilization series reasonably well, especially for durables.

The third column shows the results after estimating the model with perfect labor mobility ($\varepsilon = 0$). Even though the posterior mean of ϕ decreases to 0.43, search-based demand shocks continue to have an outsized role in the variance decomposition. The model fits data worse overall but better captures the correlation of the utilization variables. The fourth column reestimates the model under common wage-markup shocks. As expected from our discussion of the shopping ratio (??), this omission dramatically worsens model fit. The reason is that the shopping-effort ratio is now much more directly tied to the labor ratio, and it loses flexibility in fitting the comovement of utilization. Consequently, at the posterior mean, the correlations of utilization (-0.31) and especially labor hours (-0.88) become negative. The volatilities are dramatically higher as well. The substantial reduction in model fit is reflected in a 1,434.2 reduction in the log marginal likelihood compared to the baseline.

In the next two columns, we remove fixed costs and variable capital utilization, one-by-one. Both of these ingredients can be considered important robustness checks on the search-based demand channel. Removing fixed costs yields similar second moments as the baseline and slightly raises the marginal likelihood. Removing variable capital utilization leads to a mild reduction of the marginal likelihood (-2.6). Intuitively, the model loses flexibility in explaining utilization, output, and labor variables. The implied autocorrelation of the utilization variables collapses, and the comovement of labor inputs is too low (0.35). However, the comovement of utilization is actually better than the baseline model, and it compares similarly among other moments–hence the only mild reduction of the marginal likelihood.

The penultimate (seventh) column removes search-based demand shock. This specification resembles ?, but goods market frictions still operate through other shocks. It is immediately evident that this change completely prevents the model from fitting the data: the log marginal data density collapses by over $2,000$ points, the standard deviations exceed those of the data by two orders of magnitude, and the correlations and autocorrelations are nearly unity. Intuitively, the capacity utilization data roughly pins down the sectoral shopping efforts, and the model lacks freedom to fit sectoral labor and output and the relative price of investment jointly. The appendix makes this statement more precise by showing that the special case of a unitary consumption sector, no fixed costs, and no investment adjustment costs gives rise to stochastic singularity.

The final column also removes utilization data, making the set of observables similar to ?. Estimating this specification confirms that the model can fit non-utilization data reasonably well. The volatility of output (0.64) and labor hours $(0.56, 1.87)$ are close to the empirical values. The model also fits the labor comovement well (0.83), though the comovement of consumption and investment is too low (0.24). However, the volatility of the utilization variables–especially nondurables–is far below the data, and their comovement is sharply negative (-0.6). That is, absent search-based demand shocks, the model fits standard macro series well at the expense of matching the volatility and comovement of utilization. A corollary is that a multisector real business cycle model without the goods market frictions would cannot fit the data.

To better understand the baseline result, we examine impulse responses of consumption, investment, their respective labor inputs, utilization variables, and the relative price of investment from the baseline model. We set the parameters to the posterior mean and present the impulse responses in growth rates for ease of comparison. The utilization variables consist of the observable subcomponents, durables and nondurables, together with aggregate utilization. A large fraction of aggregate utilization reflects services and is thus unobservable. We also include shopping effort D and capital intensity h in reference to Equation (**??**): $dutil_t \approx 0.91 dD_t + 0.34 dh_t$.

Figure **??** plots the impulse response to a unit standard deviation reduction in the shopping effort e_d. This shock prompts households to increase their shopping effort, leading to a boost in matching and utilization. More matches raise demand for firms' goods, prompting them to expand production and hire additional workers. As a result, labor demand rises in both sectors, generating positive comovement in the growth rates of output, inputs, and utilization across durables and nondurables. As expected, the Solow residual rises on impact. Moreover, the relative price of investment is countercyclical as in the data. Variable capital intensity falls on impact, indicating that it is a substitute for higher shopping effort.

irf_e_D.pdf

Figure 6: A unit standard deviation negative shock e_d to shopping effort in baseline model with parameters set at the posterior mean. The outcome variables are presented in growth rates.

Figure **??** plots the impulse response to a positive unit standard deviation discount-factor shock e_b. Households are more patient, which raises the desire to consume in the future relative to the present. As a result, consumption falls while investment rises. Overall, given the predominant share of consumption in output, output falls, indicating a paradox of thrift. Additionally, there is an increase in utilization in the durables sector but a decrease in utilization in the nondurables sector. Limited factor mobility attenuates, but does not prevent, the fall in labor in the consumption sector. Contrary to the data, there is positive comovement of investment and its relative price.

irf_e_b.pdf

Figure 7: A unit standard positive shock e_b to the discount factor in baseline model with parameters set at the posterior mean. The outcome variables are presented in growth rates.

What about technology shocks? It may appear that technology shocks can generate all the comovement properties as search demand shocks. To that end, Figure **??** plots the impulse response to a positive standard deviation neutral stationary technology shock e_z. The Solow residual rises, but by a smaller amount than from the demand shock. The shock produces positive comovement in consumption, investment, and labor across both sectors—consistent with sectoral comovement in **?** and **?**. Limited factor mobility contributes to this feature. Moreover, the relative price of investment falls. The technology boost increases the expected return on investment, thereby incentivizing an immediate rise in utilization in the durable sector. Concurrently, utilization in nondurables initially declines due to sectoral reallocation, rising only after several periods as the technology shock subsides. Hence, search demand shocks are unique in producing positive comovement in the growth rates of all series.

irf_e_Z.pdf

Figure 8: A unit standard deviation positive shock e_z to technology in the baseline model with parameters set at the posterior mean. The outcome variables are presented in growth rates.

7. Conclusion

We investigate the contribution of demand shocks to business cycle fluctuations in a three-sector model using Bayesian techniques. In our framework, actual output falls short of potential output due to matching frictions. These frictions give rise to search-based demand shocks, which influence capacity utilization and, in turn, the Solow residual. Our estimation strategy is novel in its use of sectoral data, incorporating capacity utilization in both durable and nondurable goods sectors alongside labor hours and output data from consumption and investment sectors. This unique data combination incorporates information on sectoral productivity while subjecting the model to a rigorous test. In particular, we require the model to fit not only overall capacity utilization dynamics but also the volatility, comovement, and persistence of its subcomponents.

Our findings are threefold. First, we estimate high and precise values for the matching function elasticity ϕ and the shopping disutility η, indicating an important role for our search-based demand channel. By testing the model on artificial data drawn at the posterior mean, we demonstrate that parameter estimates cluster around true values, indicating robustness to the data-generating process. Second, shocks to shopping effort account for a large part of the forecast error variance of output, the Solow residual, the relative price of investment, hours, and utilization. Third, the model provides good empirical fit, capturing the comovement in labor input and output effectively, although it understates the comovement of utilization series. Impulse responses reveal that search demand shocks uniquely generate the three-way comovemment of utilization series and TFP.

We examine in detail the contribution of different model ingredients. Fixed costs are inessential, and the model still provides a major productive role for demand and reasonable empirical fit without them. Perfect labor mobility also retains demand as a key driver of productivity, but sacrifices overall empirical fit. Removing variable capital intensity reduces fit by only a small amount, primarily driven by an inability to match the utilization autocorrelations and greater output volatility. Sector-specific wage markups and search-based demand shocks, however, are crucial for accurately fitting sectoral data. Models with only common wage markup shocks overestimate volatility and fail to capture labor-utilization comovement. Without search-based demand shocks, shopping effort becomes overdetermined, constrained by both output and relative price variables as well as utilization measures. Omitting search-based demand shocks and utilization variables allows the model to fit standard macro series but leads to a counterfactual negative correlation of utilization and understates its volatility.

Our broader argument leverages sectoral data to support a demand-based explanation of the business cycle. Our demand shocks include a standard shock to the discount factor (θ_b) and two novel shocks related to goods market frictions (θ_d and θ_i), with the latter proving substantially more influential for business cycle fluctuations. We do not claim that these shocks literally represent fluctuations in shopping disutility. Rather, our framework emphasizes the unique ability of fluctuations in demand to replicate key empirical comovements—particularly in capacity utilization—as a critical test of the model's validity.

A promising avenue for future research is to incorporate confidence shocks—following

?—into a framework featuring goods market frictions and endogenous shopping effort. Linking autonomous shifts in confidence to shopping activity resonates with the spirit of ?, yet remains distinct from standard New Keynesian approaches. A related fruitful application is by ?, who demonstrates how a monetary model with search-driven capacity utilization can fit data relatively well. Introducing random search would naturally generate congestion inefficiencies, highlighting new channels for policy intervention. More broadly, macroeconomic models in which demand plays a productive role—empirically disciplined by capacity utilization data—can shed light on the state dependence of fiscal multipliers.

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Appendix A. Background on endogeneity of TFP

The early real business cycle literature treated the Solow residual as a pure measure of technology, but subsequent analysis found that it contained important components unrelated to technology. To address this issue, **?** purify the Solow residual by removing aggregation effects, variation in capital and labor utilization, non-constant returns to scale, and imperfect competition. They find that the purified technology process is about half as volatile as TFP, appears to be permanent, and is generally uncorrelated with output. Building on these findings, **?** constructs a quarterly measure of TFP adjusted for utilization. Figure **??** plots detrended utilization-adjusted TFP alongside standard TFP. The Fernald series not only leads the Solow residual but also exhibits less volatility. Moreover, these series occasionally diverge significantly, most notably during the pandemic shock in 2020Q1, the Great Recession, and the recession of the early 1980's. In the following, define Fernald utilization as the difference between cyclical TFP and its utilization-adjusted counterpart.

TFP_comparison.pdf

Figure A.9: Time series of the Solow residual and its utilization-adjusted counterpart, following the methodology in **?**. Each underlying series is detrended via the Hamilton regression filter with the four most recent observations 8 quarters in the past ($p = 4, h = 8$)

Appendix B. Data appendix

Table **??** provides the details on constructing the model variables, which are used for summary statistics and Bayesian estimation.

Symbol	Description	Construction
C	Nominal consumption	PCND+PCESV
I	Nominal gross private domestic investment	PCDG+PNFI+PRFI
$Deflator$	GDP Deflator	GDPDEF
Pop	Civilian non-institutional population	CNP160V
P_c	Price index: consumption	PCEPI
P_i	Price index: investment	INVDEV
c	Real per capita consumption	$\frac{C}{Pop*P_c}$
i	Real per capita investment	$\frac{I}{Pop*P_i}$
y	Real per capita output	$c+i$
n_c	Labor in consumption sector	Labor in nondurables and services
n_i	Labor in investment sector	Labor in construction and durables
n	Aggregate labor	$n_c + n_i$
p_i	Relative price of investment	P_i/P_c
$util_{ND}$	Total capacity utilization: nondurables	TCU
$util_D$	Total capacity utilization: durables	TCU
SR	Solow residual	Fernald (2014), FRB of San Francisco
SR_{util}	Utilization-adjusted Solow residual	Fernald (2014), FRB of San Francisco

Table B.6: Data sources used in motivating evidence and estimation.

The construction of sectoral data follows **?**. We calculate consumption and investment as follows:

$$C_t = \left(\frac{Nondurable(PCND) + Services(PCESV)}{P_c \times CivilianNonstitutionalPopulation(CNP160V)}\right)$$
$$I_t = \left(\frac{Durable(PCDG) + NoresidentialInvestment(PNFI) + ResidentialInvestment(PRFI)}{P_i \times CivilianNoninstitutionalPopulation(CNP160V)}\right)$$

We use an HP-filtered trend for population ($\lambda = 10,000$) to eliminate jumps around census dates.

For labor data, we make use of the BLS Current Employment Statistics (https://www.bls.gov/ces/data). BLS Table B6 contains the number of production and non-supervisory employees by industry, and BLS Table B7 contains average weekly hours of each sector. We compute total hours for nondurables, services, construction, and durables by multiplying the relevant components of each table. Then we impute labor in consumption as sum of labor in nondurables and services. Similarly, we construct labor in investment as sum of labor in construction and durables. Figure ?? plots labor hours in each sector. The close comovement and greater volatility of hours in investment is evident.

hours_comovement.pdf

Figure B.10: Sectoral and aggregate hours. Each underlying series is detrended via the Hamilton regression filter with the four most recent observations 8 quarters in the past ($p = 4, h = 8$).

We also make use of disaggregated data on total capacity utilization from the Federal Reserve Board. Estimates are available for 89 detailed industries (71 manufacturing, 16 mining, 2 utilities) and also for several industry groups. Our focus is on durables and nondurables.[16]

[16] Capacity utilization data can be downloaded at https://www.federalreserve.gov/datadownload/Choose.aspx?rel=G17 and is also available from the Federal Reserve Economic Database (FRED).

Appendix C. Analysis of simplified model

We consider a dynamic version of the static model, which includes capital accumulation.[17] We estimate the shock processes $\{\theta_d, \theta_n, g, z, z_I\}$, each AR(1) with persistence $\{\rho_d, \rho_n, \rho_g, \rho_z, \rho_i\}$ and conditional standard deviation $\{\sigma_d, \sigma_n, \sigma_g, \sigma_z, \sigma_i\}$. This approach extends the set of shocks used by BRS to include neutral stationary technology shocks. While generally adhering to the same calibration strategy and targets, we now fix the following parameters: risk aversion $\beta = 0.99$, $\sigma = 2.0$, and Frisch elasticity $\zeta = 0.72$.[18] We estimate the model by adding total capacity utilization as an observable series to the BRS specification, which includes output, investment, labor productivity, and the relative price of investment. We then compare the estimates with and without capacity utilization.

Table **??** reports the prior distributions used for both specifications. In addition to ϕ and η, we specify distributions for the persistence parameters of nonstationary neutral technology, stationary neutral technology, investment-specific technology, labor supply, and shopping effort. We apply identical prior distributions for the conditional standard deviation and persistence of the stationary shocks. These conditional standard deviations follow an inverse gamma distribution with a mean of 0.01 and a standard deviation of 0.1. The persistence parameters have a prior mean of 0.6 and a standard deviation of 0.2.

Table **??**: Prior distributions

Parameter	Distribution	Mean	Std
ϕ	Beta	0.32	0.20
η	Gamma	0.20	0.15
σ_{e_g}	Inv. Gamma	0.01	0.10
σ_x	Inv. Gamma	0.01	0.10
ρ_g	Beta	0.10	0.05
ρ_x	Beta	0.60	0.20

Table C.7: Prior distributions. We use the symbol x as a shorthand for a shock in the set $\{\theta_d, \theta_n, z, z_I\}$.

Table **??** compares the posterior means and 90% probability bands of the key shopping-related parameters. In the first panel, the parameter ϕ is imprecisely estimated with a lower

[17] Appendix G lists the full set of equilibrium conditions.

[18] BRS also fix $\zeta = 0.72$ but they use $\sigma = 1$ and $\beta = 0.997$. We have also estimated the model with $\phi = 0.32$ and $\eta = 0.2$ as by BRS and obtained a similar variance decomposition as that paper.

posterior mean. In fact, the 90% probability band includes essentially a null effect. By contrast, when we add total capacity utilization, the posterior mean increases substantially to 0.88 and the estimate is precise. Estimates for the shopping cost elasticity η are also significantly higher and more precisely estimated. Generally, estimates of ρ_d and σ_d are more precise as well, though the properties differ. With the use of utilization data, demand shocks exhibit greater persistence, but their innovations become less volatile.

Table **??**: Role of capacity utilization on parameter estimates

Parameter	BRS dataset		Add capacity utilization	
	Post. mean	90% HPD interval	Post. mean	90% HPD interval
ϕ	0.0978	[0.0001, 0.205]	0.883	[0.863, 0.906]
η	0.412	[0.282, 0.572]	1.87	[1.86, 1.90]
ρ_d	0.871	[0.775, 0.961]	0.928	[0.914, 0.941]
σ_d	0.0484	[0.0024, 0.0987]	0.0075	[0.0068, 0.0081]

Table C.8: Estimation of baseline BRS model with two sets of observable series. The first considers growth rates of output, investment, labor productivity, and the relative price of investment. The second specification also considers total capacity utilization growth.

The top panel of Table **??** compares the standard deviations at the posterior mean of shocks θ_d, shopping effort D, and utilization $util$, where the last two are expressed in growth rates. The main result is that total capacity utilization is ten times more volatile even though shopping-effort shocks are less volatile and shopping effort has similar volatility. The key difference lies in the transmission of shopping effort to utilization through ϕ. The bottom panel highlights the role of these varying parameter estimates for the forecast error variance fraction attributable to demand shocks. It is very small in the former case but large in the latter, accounting for about two thirds of output, almost a third of labor productivity, and about half the Solow residual.

Table ??: Comparison of volatility and variance decomposition

	BRS dataset	Add capacity utilization
Volatility		
θ_d	9.84	2.00
D	1.54	1.69
$util$	0.15	1.49
FEVD		
Y	7.73	63.6
Y/N	2.49	27.0
SR	6.14	54.1

Table C.9: The first sub-table documents standard deviations of shopping-related variables under two sets of observables. The second sub-table shows the fraction of the unconditional variance decomposition attributable to the demand shock θ_d. See Table ??.

These two exercises sharply illustrate the informative role of total capacity utilization. The shopping-related parameters more precisely estimated, demand shocks explain much more of the forecast error variance, and the volatility of total capacity utilization in the model rises ten-fold, much closer to the empirical value.[19]

Yet there are significant caveats to this analysis. First, in the absence of variable capital utilization, only shopping can influence total capacity utilization. Firms should also be able to select the intensity of capital use. To make the dynamic tradeoff more interesting and better fit investment data, there should also be investment adjustment costs. Then capacity utilization will reflect both shopping effort and intensity of capital use. Moreover, given the focus on productivity, it makes sense to incorporate fixed production costs. These empirically relevant costs help explain why productivity rises with output and also affect the contribution of capital intensity to capacity utilization.

Second, total capacity utilization is inappropriate as an economy-wide target since it is only constructed for specific industries. In particular, it is not measured for consumption services, a large part of the economy. Enriching the model to include multiple sectors allows

[19] On page 23, footnote 14, BRS state that, in the absence of cross-sectional evidence, 'we find that the parameter ϕ is not well identified by the aggregate data. In particular, the resulting estimates of ϕ vary widely across data sets, ranging from 0.09 to 0.44 depending on whether we include or omit shopping time data.' By contrast, we find that ϕ *is* well-identified from aggregate data given the inclusion of capacity utilization.

us to exploit dis-aggregated capacity utilization data in our estimation.

Third, the model struggles to fit aspects of sectoral data, which important for the transmission mechanism operating via goods market frictions. In the second specification, though the correlation of labor in each sector is not too far below the data (0.58), the autocorrelation is 0.18 for n_c and -0.01 for n_i. For consumption and investment, the respective autocorrelations are 0.28 and 0.20, well below the empirical values.

In the special case of BRS, relative shopping effort across sectors equals the relative labor allocation and the relative value of output:

$$\frac{D_c}{D_i} = \frac{n_c}{n_i} = \frac{C}{p_i I}$$

Equation (**??**) highlights the informative role of sectoral data: (1) the labor ratio pins down the ratio of shopping effort, (2) and labor inputs and sectoral output data provides information on sectoral labor productivity, which are in turn linked to the relative price of investment. Such data is especially relevant given our focus on a demand-based explanation of productivity.

Unfortunately, (**??**) raises the following challenge. The variables C, I, and p_i are observables in estimation and thus determine n_c/n_i. Trying to use n_c and n_i—or even just their ratio—as observables in estimation would induce stochastic singularity. The use of these series versus the relative price of investment becomes arbitrary.

The main text generalizes (**??**), breaking the one-for-one link between shopping effort and hours. The more general form arises from using sector-specific wage markup shocks, incorporating imperfect competition in the labor market in the vein of **?**. Additionally, limited factor mobility facilitates sectoral comovement and dampens excessive volatility, and fixed costs permit a more general relationship between output and augment the contribution of capital intensity to total capacity utilization.

Appendix D. Details of household and firm problem

Competitive search creates additional interconnections between the household and firm problems. A complete characterization requires solving both jointly. We start with the household problem. Let $\gamma_{mc}, \gamma_{sc}, \gamma_i, \lambda, \mu_{mc}, \mu_{sc}, \mu_i$ be the respective Lagrangian multipliers on the constraints. The first order conditions are

$$[y_{mc}]: \quad u_{mc} = \gamma_{mc} + \lambda p_{mc}$$

$$[y_{sc}]: \quad u_{sc} = \gamma_{sc} + \lambda p_{sc}$$

$$
\begin{aligned}
&[i_j]: \quad -\gamma_i - \lambda p_j + \mu_j \left(1 - S_j'(x_j)x_j - S_j(x_j)\right) + \beta\theta_b \mathbb{E}\mu_j' S_j'(x_j')(x_j')^2 = 0 \\
&[d_j]: \quad u_d = -A_j D_j^{\phi-1} F_j \gamma_j, \quad j \in \{mc, sc\} \\
&[d_i]: \quad u_d \theta_i = -A_i D_i^{\phi-1} F_i \gamma_i \\
&[n_c]: \quad u_n \frac{\partial n^a}{\partial n_c} = -\lambda W_c^* \\
&[n_i]: \quad u_n \frac{\partial n^a}{\partial n_i} = -\lambda W_i^* \\
&[h_j] \quad \delta_h(h_j)\mu_j = \lambda R_j \quad j \in \{mc, sc, i\} \\
&[k_j']: \quad \mu_j = \beta\theta_b \mathbb{E}\left\{\lambda' R_j' h_j' + (1 - \delta_j(h_j'))\mu_j'\right\} \quad j \in \{mc, sc, i\}
\end{aligned}
$$

The multipliers $\gamma_{mc}, \gamma_{sc}, \gamma_i$ reflect the value of an additional unit of traded output. In the consumption submarkets, these represent a wedge between the marginal utility of consumption and the marginal utility of wealth. For investment, the multiplier γ_i represents an analogous wedge between the marginal utility of wealth and value of the investment good. Equations (**??**) and (**??**) equate the marginal shopping disutility to the additional units obtained by search multiplied by the value of the unit. Equations (**??**) and (**??**) equate the marginal disutility of work in each sector to the (variable) wage multiplied by the marginal utility of wealth. Equation (**??**) equates the marginal cost of depreciated capital to the value of additional output generated in terms of consumption. Finally, (**??**) equates the marginal value of capital to the expected discounted rate of return, composed of the rental income and value of undepreciated capital.

We next characterize the envelope conditions:

$$
\begin{aligned}
\frac{\partial V^j}{\partial p_j} &= -\lambda j = -\lambda d_j A_j D_j^{\phi-1} F_j \quad j \in \{mc, sc, i\} \\
\frac{\partial V^j}{\partial D_j} &= (\phi - 1) d_j A_j D_j^{\phi-2} F_j \gamma_j \quad j \in \{mc, sc, i\} \\
\frac{\partial V^j}{\partial F_j} &= d_j A_j D_j^{\phi-1} \gamma_j \quad j \in \{mc, sc, i\}
\end{aligned}
$$

The ratio of (**??**) and (**??**) characterizes the indifference curve between price and tightness in a submarket:

$$
\frac{\frac{\partial V^j}{\partial p^j}}{\frac{\partial V^j}{\partial D^j}} = -\frac{\lambda D_j}{(\phi - 1)\gamma_j}
$$

We next turn to the firm's problem. The firm chooses labor type s in sector j so as to generate an effective labor bundle n_j at the lowest possible cost. The problem is

$$\min_{n_j(s)} \int_0^1 W_j(s) n_j(s) ds \quad \text{s.t.}$$
$$\left(\int_0^1 n_j(s)^{1/\mu_j} dj\right)^{\mu_j} \geq \overline{n}$$

Take the first order condition of (**??**) and recognize W_j as the Lagrangian multiplier on constraint (**??**). Rearrange as

$$n_j(s) = \left(\frac{W_j(s)}{W_j}\right)^{-\frac{\mu_j}{\mu_j - 1}} n_j$$

The corresponding wage index for composite labor input in sector j is

$$W_j = \left[\int_0^1 W_j(s)^{1/(\mu_j - 1)} ds\right]^{\mu_j - 1}$$

We can now examine the simplified firm problem. Let ι_j and ∇_j be the multipliers on participation constraint and production technology. The first order conditions are

$$[F_j] \quad \nabla_j = p_j A_j D_j^{\phi} + \iota_j \frac{\partial V^j}{\partial F^j}$$
$$[n_j] \quad W_j = \nabla_j z_j f_n$$
$$[k] \quad h_j R_j = \nabla_j z_j f_k$$
$$[p_j] \quad A_j D_j^{\phi} F_j + \iota_j \frac{\partial V^j}{\partial p_j} = 0$$
$$[D_j] \quad \phi A_j D_j^{\phi - 1} p_j F_j + \iota_j \frac{\partial V^j}{\partial D^j} = 0$$

Take the ratio of first order conditions (**??**) and (**??**) to alternately characterize the indifference curve between price and tightness:

$$\frac{\frac{\partial V^j}{\partial p^j}}{\frac{\partial V^j}{\partial D^j}} = \frac{D_j}{\phi p_j}$$

Plug in (**??**) to find

$$\frac{D_j}{\phi p_j} = -\frac{\lambda D_j}{(\phi - 1)\gamma_j}$$

which we rearrange as

$$\gamma_j = \frac{\phi}{1-\phi}\lambda p_j$$

Since $\gamma_j = u_j - \lambda p_j$ for $j = \{mc, sc\}$, we have

$$\lambda = (1-\phi)\frac{u_j}{p_j}$$

which allows us to characterize γ_i:

$$\gamma_i = \phi \frac{u_j}{p_j} p_i \quad j \in \{mc, sc\}$$

Note that (**??**) also implies that the marginal utility relative to the price is the same in each consumption subsector. The values of γ_{mc}, γ_{sc} and λ allows us to rewrite the shopping optimality conditions and labor leisure tradeoff:

$$\begin{aligned}
-u_d &= \phi u_j A_j D_j^{\phi-1}[z_j f(h_j k_j, n_j) - \nu_j] \quad j \in \{mc, sc\} \\
-u_d \theta_i &= \phi \frac{u_{mc} p_i}{p_{mc}} A_i D_i^{\phi-1}[z_i f(h_i k_i, n_i) - \nu_i] \\
u_n \frac{\partial n^a}{\partial n_j} &= -\frac{u_{mc}(1-\phi)}{p_{mc}} W_j^* \quad j \in \{c, i\}
\end{aligned}$$

We next revisit the investment first order condition (**??**) and characterize Tobin's Q. For sector $j \in \{mc, sc, i\}$ we have

$$\begin{aligned}
\lambda p_i + \gamma_i &= \mu_j(1 - S'(x_j)x_j - S(x_j)) + \beta\theta_b \mathbb{E}\mu_j'(S'(x_j')(x_j')^2) \\
\lambda p_i + \frac{\phi}{1-\phi}\lambda p_i &= \mu_j(1 - S'(x_j)x_j - S(x_j)) + \beta\theta_b \mathbb{E}\mu_j'(S'(x_j')(x_j')^2) \\
\frac{\lambda p_i}{1-\phi} &= \mu_j(1 - S'(x_j)x_j - S(x_j)) + \beta\theta_b \mathbb{E}\mu_j'(S'(x_j')(x_j')^2)
\end{aligned}$$

Let $Q_j = \mu_j/\lambda$: relative price of capital in sector j in terms of consumption. Using Q_j rewrite the choice of optimal investment as

$$\frac{p_i}{1-\phi} = Q_j[1 - S_j'(x_j)x_j - S_j(x_j)] + \beta\theta_b \mathbb{E}\frac{\lambda'}{\lambda} Q_j' S_j'(x_j')(x_j')^2$$

We also use Tobin's Q to rewrite the optimal utilization in $j \in \{mc, sc, i\}$ and the Euler equation:

$$
\begin{aligned}
\delta_h(h_j)Q_j &= R_j \\
Q_j &= \beta\theta_b\mathbb{E}\frac{\lambda'}{\lambda}\left[(1-\delta(h_j'))Q_j' + R_j'h_j'\right]
\end{aligned}
$$

It remains to solve for the Lagrangian multipliers ι_j and ∇_j on the firm problem. This is straightforward given λ and γ_j. First,

$$\iota_j = \frac{A_j q_j^\phi F_j}{\frac{\partial V^j}{\partial p_j}} = \frac{1}{\lambda}$$

Second,

$$
\begin{aligned}
\nabla_j &= p_j A_j D_j^\phi + \iota_j \frac{\partial V^j}{\partial F^j} \\
&= p_j A_j D_j^\phi + \frac{A_j D_j^\phi \gamma_j}{\lambda} \\
&= p_j A_j D_j^\phi + A_j D_j^\phi \frac{\phi}{1-\phi} p_j \\
&= A_j D_j^\phi \left(p_j + \frac{\phi}{1-\phi} p_j\right) \\
&= \frac{p_j A_j D_j^\phi}{1-\phi}
\end{aligned}
$$

The value of additional production capacity ∇_j exceeds the additional sales $p_j A_j D_j^\phi$. This is because the additional sales also relax the participation constraint of households. Finally, the value of these multipliers enables us to characterize the factor demands for the firms. Substitute for ∇_j in (**??**) to find

$$
\begin{aligned}
(1-\phi)\frac{W_j}{p_j} &= A_j(D_j)^\phi z_j \frac{\partial f(h_j k_j, n_j)}{\partial n} \\
&= \frac{\alpha_n}{n_j} A_j D_j^\phi z_j f(h_j k_j, n_j) \\
&= \frac{\alpha_n}{n_j} A_j D_j^\phi \left(\frac{y_j}{A_j D_j^\phi} + \nu_j\right) \\
&= \frac{\alpha_n}{n_c}(y_j + A_j D_j^\phi \nu_j) \\
&= \frac{\alpha}{n_c} y_j (1 + \nu^R)
\end{aligned}
$$

where we use $\nu_j^R = \nu_j \Psi_T / y_j$. We can simplify the capital demand (or rental rate) (**??**) using ratios as

$$\frac{W_j}{R_j} = \frac{\alpha_n}{\alpha_k} \frac{h_j k_j}{n_j}$$

Aggregating across sectors, the steady-state labor labor of income is $\alpha_n(1 + \nu^R)/(1 - \phi)$ and the capital share of income is $\alpha_k(1 + \nu^R)/(1 - \phi)$.

Appendix E. Calibration

In general, we determine some (fixed) parameters from long-run targets, estimate the parameter set Θ described in the main text, and back out the remaining (dependent parameters) given draws from Θ and long-run targets. The dependent parameters are thus random variables. Here we use the term calibration more broadly to characterize the determination of dependent parameters as a function of both estimated parameters and long-run targets.

Several key targets used for calibration are investment-to-output $p_i I/Y$, capital-to-output $p_i k/Y$, the labor share of income, the unconditional growth rate $\overline{g}$, and share of services S_c in consumption. In terms of model variables at quarterly frequency, we have

$$\kappa \equiv p_i I/Y = 20\%,\ p_i k/Y = 2.75(4) = 11,\ \overline{g} = 0.45\%,\ \tau \equiv \frac{nW}{Y} = 67\%,\ S_{sc} \equiv \frac{p_{sc} y_{sc}}{C} = 65\%$$

The first two targets are identical to **?**, and the third corresponds to 1.8% per capital annual growth, which is very close to the average over the data sample. The share of services to overall output is $0.65 * 0.80 = 0.52$. That is, at our calibrated steady state, capacity utilization is unobserved for about half the economy.

Capital accumulation (ignoring adjustment costs) in transformed variables [20] is given by

$$g\widehat{k}' = (1 - \delta)\widehat{k} + g\widehat{I}$$

Balanced growth, in terms of original variables, implies a steady state in terms of $\widehat{k}$, such that

$$\delta = 1 - \overline{g} + \frac{I}{k} \approx 1.37\%$$

[20] Investment is divided by the stochastic trend $\hat{I}_t = I_t / X_t$ while the capital stock is divided by the lagged stochastic trend $\hat{K}_t = K_t / X_{t-1}$ to maintain its status as a predetermined variable.

Next, we characterize α_n, α_k and σ_b. Labor demand (**??**) for each sector implies

$$W_j n_j = \frac{\alpha_n}{1-\phi} p_j Y^j (1+\nu_j^R)$$

where $\nu_j^R = \nu_j X/F_j$. The steady state labor share is thus

$$\frac{\sum W_j n_j}{Y} = \frac{\alpha_n}{1-\phi}\frac{C+p_i I}{Y}(1+\nu_{ss}^R) = \frac{\alpha_n}{1-\phi}(1+\nu_{ss}^R)$$

so that $\alpha_n = (1-\phi)\times$labor share$/(1+\nu_{ss}^R)$.

In steady state, the rate of return on capital in each sector is equal, so we let R denote the common value: $R = R_j$ for all j. It is helpful to use the interest rate r on an illiquid bond as the value which satisfies $\beta\overline{g}^{-\sigma} = 1/(1+r)$.

The Euler equations in the steady state imply

$$\begin{aligned} Q &= \beta\overline{g}^{-\sigma}[(1-\delta)Q + R] \Rightarrow \\ (1+r)Q &= (1-\delta)Q + R \\ (r+\delta)Q &= R \end{aligned}$$

Given that capital utilization $h_j = 1$ for all j in the steady state, the parameter σ_b satisfies

$$\sigma_b = \frac{R}{Q} = r + \delta$$

Combining with Tobin's Q, $p_i/(1-\phi) = Q$, we have

$$(1-\phi)\frac{R}{p_i} = r + \delta$$

Now, turn to the firm demand for capital (**??**):

$$(1-\phi)\frac{R_j}{p_j} = \alpha_k \frac{Y_j}{k_j}(1+\nu^R)$$

An immediate corollary is that $Y_j/k_j = Y/k$ for all k and hence

$$r + \delta = \alpha_k \frac{Y}{k}(1+\nu^R)$$

so that

$$\alpha_k = \frac{r+\delta}{1+\nu^R}\frac{k}{Y}$$

We pin down the weight of services ω_{sc} as the empirical measure $S_c = p_{sc}Y_{sc}/C$ and set $S_c = 0.65$. The ratio of demand in consumption subsectors implies

$$\frac{Y_{mc}}{Y_{sc}} = \left(\frac{p_{mc}}{p_{sc}}\right)^{-\xi}\frac{\omega_{mc}}{\omega_{sc}}$$

Multiply each side by p_{mc}/p_{sc}, so that

$$\frac{p_{mc}Y_{mc}}{p_{sc}Y_{sc}} = \left(\frac{p_{mc}}{p_{sc}}\right)^{1-\xi}\frac{\omega_{mc}}{\omega_{sc}}$$

and plug in S_c:

$$\left(\frac{1-S_c}{S_c}\right) = \left(\frac{p_{mc}}{p_{sc}}\right)^{1-\xi}\frac{1-S_c}{S_c}$$

so that $p_{mc} = p_{sc}$. Since we normalize $p_{sc} = 1$ and have also normalized the consumption price index to unity, we have $p_{mc} = p_{sc} = p_c = 1$.

Given the target for capacity utilization $\Psi_{T,j}$, we wish to find the corresponding level coefficient $A_j = \Psi_{T,j}/D_j^{\phi}$. This entails solving for each D_j. We first solve for D. Let us sum each side of the shopping optimality condition across sectors:

$$\sum_j D^{1/\eta}D_j = \sum_j \phi p_j Y_j$$
$$D^{\frac{\eta+1}{\eta}} = \phi Y$$

Given that we choose technology coefficients such that $Y = 1$, we obtain $D = \phi^{\frac{\eta}{\eta+1}}$.

Now, take the ratio of the shopping conditions rearrange for relative shopping effort:

$$\frac{D_{mc}}{D_{sc}} = \frac{p_{mc}}{p_{sc}}\frac{Y_{mc}}{Y_{sc}} = \frac{1-S_c}{S_c}$$

Similarly,

$$\frac{D_j}{D_i} = S_j\frac{1-I/Y}{I/Y}$$

Now, we put (**??**) and (**??**) together to characterize shopping effort in each sector:

$$\begin{aligned} D_{mc} &= (1 - S_c)(1 - I/Y)D \\ D_{sc} &= S_c(1 - I/Y)D \\ D_i &= (I/Y)D \end{aligned}$$

Appendix F. Cyclical deviations of Solow residual and total capacity utilization

In the main text we analyze the relationship between the Solow residual and capacity utilization in growth rates. Here we compare them in terms of cyclical deviations. Using (**??**), the cyclical component of the Solow residual is

$$\hat{SR}_j \equiv \frac{SR_j}{X^\tau} = \frac{A_j D_j^\phi z_j h_j^{\alpha_k} g^{1-\alpha_k-\tau} \hat{k}_j^{\alpha_k - 1 + \tau} n_j^{\alpha_n - \tau}}{1 + \nu_j^R} = g^{1-\tau} \frac{\hat{Y}_j}{\hat{k}_j^{1-\tau} n_j^\tau}$$

The log linear representation is

$$\widetilde{\hat{SR}}_j = \phi \widetilde{D}_j + \widetilde{z}_j + \alpha_k \widetilde{h}_j + (1 - \alpha_k - \tau)\widetilde{g} + (\alpha_k - 1 + \tau)\widetilde{\hat{k}}_j + (\alpha_n - \tau)\tilde{n}_j - \frac{\nu_{ss}^R}{1 + \nu_{ss}^R} \widetilde{\nu}_j^R$$

and note that $\widetilde{g}_t = \log g_t - \log \overline{g}$ which is first-order equivalent to X^{obs}. Log linearizing (**??**) yields

$$\widetilde{util}_j = \phi \widetilde{D}_j + (1 + \nu_{ss}^R)\alpha_k \widetilde{h}_j$$

Thus, in the absence of fixed costs, we have

$$\widetilde{\hat{SR}}_j|_{\nu_j=0} = \widetilde{util}_j + \tilde{z}_j + (1 - \alpha_k - \tau)(\log g_t - \log \overline{g}) + (\alpha_k - 1 + \tau)\widetilde{\hat{k}}_j + (\alpha_n - \tau)\tilde{n}_j$$

Given the detrending, the coefficient on nonstationary technology is $1 - \alpha_k - \tau$ rather than $1 - \alpha_k$. Otherwise, the relationship between cyclical components of the Solow residual and utilization has the same form as the one in growth rates.

The relationship between the cyclical form and growth rate form is

$$\begin{aligned} dSR_t &= \Delta \log SR_t \\ &= \log \hat{SR}_t + \tau \log X_t - (\log \hat{SR}_{t-1} + \tau \log X_{t-1}) \\ &= \Delta \widetilde{\hat{SR}}_{jt} + \tau \log g_t \end{aligned}$$

The growth rate of the Solow residual equals the growth rate of cyclical deviations plus the log deviation of the stochastic trend growth rate relative to the unconditional mean multiplied by the labor share.