Productive demand and sectoral capacity utilization

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Introduction

1. Perennial question: what drives business cycles?
2. Debate of technology vs. demand shocks closely tied to role of economic slack/capacity utilization and endogeneity of Solow residual
   - Evans (1992) shows that money, interest rates, and government expenditure Granger cause the Solow residual
   - Basu, Fernald, and Kimball (2006) construct a measure of technological change using structural estimation and finds that it behaves very differently than Solow residual
   - Fernald (2014) extend BFK approach into a quarterly TFP series adjusted for utilization of labor and capital

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Questions and key contribution

- Bai, Rios-Rull, and Storesletten (2025) use goods market frictions to impart productive role for demand in estimated model
- However, framework relies on shopping time data both for calibration and time series properties, and does not make use of capacity utilization data
- Problems with using shopping-time data to proxy for effort
  - Shocks to goods market frictions can also rise from fluctuations in matching efficiency, which cannot be separately identified
  - Shopping time can be contaminated with leisure
  - Limited time series: only annual post-2003
  - Limited portability to cross-country setting or government expenditure
    International measures of capacity utilization

Key contribution

Use capacity utilization jointly with sectoral data to identify key goods-market frictions parameters and investigate contribution of demand shocks to forecast error variance

Key findings

We estimate a multisector model with goods market frictions using Bayesian techniques and show

1. Key novel parameters associated with the transmission mechanism are economically significant and well-identified
2. Demand shocks explain a majority of the variation of output, the Solow residual, and utilization
3. Model fits the data reasonably well, including most sectoral variables (sectoral comovement)
   $\Rightarrow$ search demand shocks uniquely generate three-way comovement of utilization rates and Solow residual

Demand shocks and effect on measured productivity

- In a standard neoclassical model, prices adjust so that all produced output is sold
  $\Rightarrow$ output is just a function of capital and labor

- Under goods market frictions, output depends on arrival rate of customers

- Increases in shopping effort—whether exogenous or in response to other economic shocks—generate more matches and higher capacity utilization

- Raises TFP and output

- Mechanism reflects Keynes' idea of demand-determined output without relying on nominal rigidity

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Capacity utilization data

- Total capacity utilization from Federal Reserve Board is the ratio of an output index to a capacity index
- Capacity index designed to measure greatest level of output each plant can maintain within realistic work schedule
- Coverage
  - 89 detailed industries (71 manufacturing, 16 mining, 2 utilities)
  - Primarily correspond to industries at the 3 or 4-digit NAICS
  - Estimates are available for various groups (durables and non-durables, total manufacturing, mining, utilities, and total industry)
  - Not available for services
- Responses to the Bureau of the Census's Quarterly Survey of Plant Capacity (QSPC) at establishment level
- Compared to Fernald utilization measure, does not require assuming constant returns to scale/zero profits
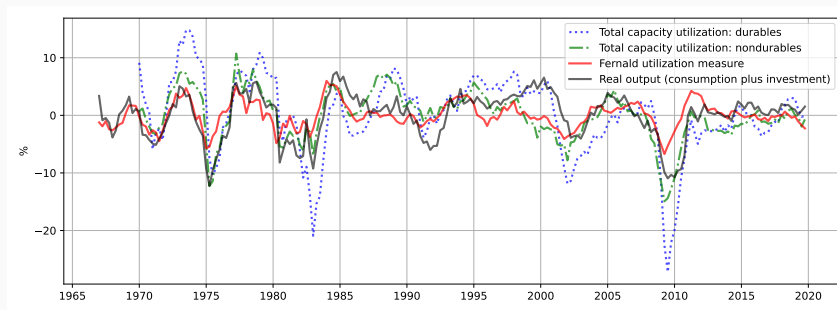
Figure 1: Total capacity utilization (non-durable and durable goods), Fernald utilization, and output. Each underlying series is detrended via the Hamilton regression filter ($p = 4, h = 8$).

Novel facts

- Novel facts: utilization measures comove and are procyclical
- Utilization in durables is highly volatile

Second moments (growth rates)

Variable	Symbol	SD(x)	STD(x)/STD(Y)	Cor(x, I)	Cor(x, n_I)	Cor(x,x$_{-1}$)
Real GDP	Y	0.87	1.00	0.94	0.70	0.47
Real Consumption	C	0.44	0.51	0.54	0.44	0.48
Real Investment	I	2.14	2.46	1.00	0.73	0.41
Labor in Consumption	n_c	0.57	0.66	0.66	0.87	0.67
Labor in Investment	n_i	1.94	2.23	0.73	1.00	0.64
Labor productivity	Y/N	0.64	0.73	0.36	-0.28	0.10
Relative price of investment	p_i	0.51	0.58	-0.28	-0.22	0.44
Utilization in Durables	$util_d$	2.27	2.61	0.69	0.84	0.55
Utilization in Non-durables	$util_{nd}$	1.26	1.45	0.61	0.65	0.51

Table 1: Time range: $1964Q1 - 2019Q4$. Each underlying series is expressed in 100 quarterly log deviations. Here output is defined as the sum of consumption and investment. Following Katayama and Kim (2018), we construct labor in consumption as sum of labor in nondurables and services and labor in investment as sum of labor in construction and durables.

Related literature

1. Purifying Solow residual:
   Basu, Fernald, and Kimball (2006), Fernald (2014)

2. Goods market frictions and firm productivity
   Moen (1997), Michaillat and Saez (2015), Bai, Rios-Rull, and Storesletten (2025), Huo and Ríos-Rull (2018), Qiu and Ríos-Rull (2022), Petrosky-Nadeau and Wasmer (2015), Bethune, Rocheteau, and Rupert (2015), Borys, Doligalski, and Kopiec (2021), Sun (2024)

3. Sectoral comovement and imperfect intersectoral factor mobility
   Long and Plosser (1983), Christiano and Fitzgerald (1998), Horvath (2000), Katayama and Kim (2018)

4. Total capacity utilization
   Corrado and Mattey (1997), Christiano, Eichenbaum, and Trabandt (2016), Qiu and Ríos-Rull (2022)

Production model with shocks and dynamics

Production technology

- 2 consumption sectors (nondurable goods mc and services sc) and an investment sector
- Each uses capital k and labor n to produce output
- Stochastic trend to technology X: growth rate $g_t = X_t/X_{t-1}$ is stationary
- Stationary technology component z_j
- Potential output given capital utilization rate h and fixed cost $\nu_j X$.

$$F_j = z_j f(h_j k_j, n_j) - \nu_j X, \quad j \in \{mc, sc, i\}, \quad (z_{mc} = z_{sc} \equiv z_c)$$

for

$$f(hk, n) = (hk)^{\alpha_k} n^{\alpha_n} X^{1-\alpha_k}, \ \alpha_k + \alpha_n \leq 1$$

- Set $z_i = z_c z_I$, where z_I is independent of z_c

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Investment

- Households shop for investment goods, accumulate capital in each sector, and collect rental income
- Capital law of motion in each sector

$$k'_j = (1 - \delta_j(h_j))k_j + [1 - S(i_j/i_{j,-1})]i_j, \quad j \in \{mc, sc, i\}$$

where $i = i_{mc} + i_{sc} + i_i$
- Endogenous capital depreciation (Christiano, Eichenbaum, and Trabandt (2016))

$$\delta^j(h) = \delta^K + \sigma_b(h - 1) + \frac{\sigma_{aj}\sigma_b}{2}(h - 1)^2, \quad j \in \{mc, sc, i\}, \sigma_{ac} \equiv \sigma_{amc} = \sigma_{asc}$$

$\Rightarrow \sigma_{aj} = \delta''_j(1)/\delta'_j(1)$ is the elasticity of marginal utilization cost wrt h at $h = 1$
- Investment adjustment cost (Christiano, Eichenbaum, and Evans (2005))

$$S(x) = \frac{\Psi_K}{2}(x - 1)^2$$

Matching technology

- **Competitive search**: households shop in markets indexed by price, market tightness, and quantity
- Each market is subject to Cobb-Douglas matching function

$$M_j(D, T) = A_j D^\phi T^{1-\phi}$$

where D is aggregate shopping effort and T is the measure of firms (normalize $T = 1$)

- Implied matching rates:

$$\Psi_{jd}(D) = M/D = A_j D^{\phi-1}, \quad \Psi_{jT}(D) = M/T = A_j D^\phi$$

so that D describes market tightness

Matching technology

- Once a match is formed, goods are traded at the price $p_j, j \in \{mc, sc, i\}$
- The real quantity of goods purchased given search effort d_j in sector j

Value added

$$y_j = \overbrace{d_j}^{\text{search intensity}} \times \underbrace{\Psi_{jd}(D)}_{\text{matching probability}} \times \overbrace{F_j}^{\text{potential output}} \qquad j \in \{mc, sc, i\}$$

Preferences

- Households have GHH preferences over search effort, consumption, and a labor composite

$$u(c, d, n^a, \theta) = \frac{\Gamma^{1-\sigma} - 1}{1 - \sigma}$$

where Γ is a composite parameter with external habit formation:

$$\Gamma = c - haC_{-1} - \theta_d \frac{d^{1+1/\eta}}{1 + 1/\eta} - \theta_n \frac{(n^a)^{1+1/\zeta}}{1 + 1/\zeta}$$

- Aggregate consumption C and total search effort $d = d_{mc} + d_{sc} + \theta_i d_i$
- Preference shifters $\theta = \{\theta_b, \theta_d, \theta_i, \theta_n\}$
  $\Rightarrow$ level consumption shock θ_c could be replicated by a proportional decrease in θ_d and θ_n and change in β

Consumption aggregator

- Consumption is bundle of non-durable goods y_{mc} and services y_{sc}

$$c = [\omega_{mc}^{1-\rho_c} y_{mc}^{\rho_c} + \omega_{sc}^{1-\rho_c} y_{sc}^{\rho_c}]^{1/\rho_c} \tag{1}$$

  such that $\omega_{mc} + \omega_{sc} = 1$

- Price index

$$p_c = \left(\omega_{mc} p_{mc}^{-\rho_c/(1-\rho_c)} + \omega_{sc} p_{sc}^{-\rho_c/(1-\rho_c)}\right)^{-\frac{1-\rho_c}{\rho_c}} \tag{2}$$

- Normalize $p_c = 1$

Imperfect labor mobility across sectors

- Assume imperfect substitutability between labor used in consumption and investment sectors (Horvath (2000) and Katayama and Kim (2018))

$$n^a = \left[\omega^{-\varepsilon} n_c^{1+\varepsilon} + (1-\omega)^{-\varepsilon} n_i^{1+\varepsilon}\right]^{\frac{1}{1+\varepsilon}}$$

- Labor composite n^a measures *perceived* labor hours by worker
- Elasticity of substitution $1/\varepsilon$ measures intersectoral labor mobility
- Induces wage dispersion
- As $\varepsilon \to 0$, $n^a \to n_c + n_i = n$ (perfect mobility benchmark)
- For ε fixed, if $\omega = n_c/n$, then $n^a = n_c + n_i = n$

- Continuum of monopolistically competitive labor unions in sector j provide services to firms
- Total labor is a CES aggregate of specialized types

$$n_j = \left(\int_0^1 n_j(s)^{1/\mu_j} ds \right)^{\mu_j}$$

- Pay workers W^* per unit and rent to firms at rate $W(s)$
- Wage paid by firm is a markup of (variable) wage received by workers

$$W_j = \mu_j W_j^*$$

with difference $W_j - W_j^*$ rebated to HH as fixed wage

Role of different ingredients

Ingredient	Role	Reference
Capital utilization	Control/amplification of technology shocks	Christiano, Eichenbaum, and Evans (2005)
Imp. factor mobility	Sectoral comovement	Horvath (2000), Katayama and Kim (2018)
Habit formation	Smooth consumption response	Christiano, Eichenbaum, and Evans (2005)
Inv. Adjustment costs	Hump-shaped investment responses	Christiano, Eichenbaum, and Evans (2005)
Fixed costs	Control (procyclical productivity)	Christiano, Eichenbaum, and Trabandt (2016)
Labor unions	Wage markups	Schmitt-Grohé and Uribe (2012)

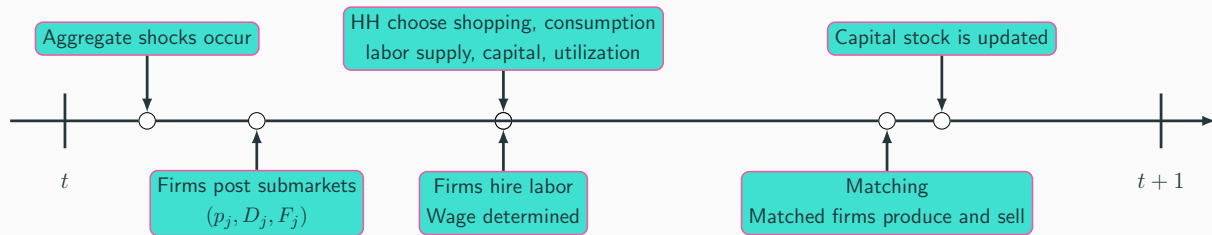**Table 2:** Compact overview of ingredients and their roles

Figure 2: Timing

Households' problem

- Households choose search effort, labor hours, consumption, capital, and utilization rates given markets $(p_j, D_j, F_j), j \in \{mc, sc, i\}$ and the aggregate state of the economy Λ

$$\widehat{V}(\Lambda, \{k_j\}, p, D, F) = \max_{d_j, n_c, n_i, y_j, i_j, k_j', h_j'} u(y_{mc}, y_{sc}, d, n^a, \theta) + \beta \theta_b \mathbb{E}\{V(\Lambda', \{k_j'\}) | \Lambda\}$$

s.t.

$$y_j = d_j \Psi_{jd}(D_j) F_j, \quad j \in \{mc, sc, i\}$$

$$\sum_{j \in \{mc, sc, i\}} y_j p_j = \pi + \sum_{j \in \{mc, sc, i\}} k_j h_j R_j + n_c W_c^* + n_i W_i^*$$

$$k_j' = (1 - \delta_j(h_j))k_j + [1 - S(i_j/i_{j,-1})]i_j, \quad j \in \{mc, sc, i\}$$

subject to endogenous depreciation δ_j, investment adjustment cost S_j, and consumption and labor aggregators (1) and (2)

- The value function is determined by the best market:

$$V(\Lambda, \{k_j\}) = \max_{\{p, D, F\} \in \Phi} \widehat{V}(\Lambda, \{k_j\}, p, D, F)$$

Details of household problem

Optimal shopping effort and demand

Optimal shopping effort

$$\overbrace{-\frac{u_d}{u_j}}^{\text{MRS}} = \overbrace{\phi\Psi_{jd}(D_j)}^{\Psi'_{jT}(D)} F_j \quad j \in \{mc, sc\}$$

- Two interpretations
  1. MRS equals HH matching probability multiplied by quantity of output sold and the shopping wedge ϕ
  2. MRS equals MRT (increase firm matching probability $\Psi'_{jT}(D)\times$ output sold)
- Under GHH preferences, $-u_d/u_j = \theta_d d^{1/\eta}$; aggregation yields

$$\theta_d D^{1/\eta} = \phi A_j D_j^{\phi-1} F_j, j \in \{mc, sc\}$$

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Optimal shopping effort and demand

- The matching elasticity ϕ equals relative wedge between marginal utility and price in equilibrium:

Elasticity as shopping wedge

$$\phi = \frac{u_j - \lambda p_j}{u_j}$$

- Express value of investment shopping by converting into consumption units using relative price

$$-\frac{u_d}{u_{mc}}\theta_i = \frac{p_i}{p_{mc}}\phi A_i D_i^{\phi-1} F_i$$

- Demand curves for non-durables and services

$$y_j = p_j^{-\xi}\omega_j C \quad j \in \{mc, sc\}$$

given elasticity of substitution $\xi = 1/(1 - \rho_c)$

Firms' problem

- A representative firm in sector $j \in \{mc, sc, i\}$ rents capital and hires labor in spot markets
- Firm chooses labor, capital inputs and submarket (p_j, D_j, F_j)
- Submarket must satisfy participation constraint of household

$$\max_{k_j, n_j, p_j, D_j, F_j} p_j A_j D_j^\phi F_j - \int_0^1 W_j(s) n_j(s) ds - R_j h_j k_j \quad \text{s.t.}$$

$$z_j f(h_j k_j, n_j) - \nu_j X \geq F_j$$

$$\widehat{V}(\Lambda, \{k_j\}, p_j, D_j, F_j) \geq V(\Lambda, \{k_j\})$$

$$n_j = \left(\int_0^1 n_j(s)^{1/\mu_j} ds \right)^{\mu_j}$$

Details of firm problem

Firm factor demands

$$(1-\phi)\frac{W_j}{p_j} = \alpha_n \frac{A_j D_j^{\phi} z_j f(h_j k_j, n_j)}{n_j} \quad j \in \{mc, sc, i\} \quad W_{mc} = W_{sc}$$

$$(1-\phi)\frac{R_j}{p_j} = \alpha_k \frac{A_j D_j^{\phi} z_j f(h_j k_j, n_j)}{h_j k_j} \quad j \in \{mc, sc, i\}$$

1. Input demand depends positively on shopping effort

2. Matching function elasticity ϕ appears as separate factor (from competitive search)
   $\Rightarrow$ Additional output relaxes participation constraint of households and effectively reduces input cost

Relative price of investment (apply ratio of labor factor demands for different sectors)

$$\frac{p_i}{p_j} = \frac{n_i W_i}{n_j W_j} \frac{A_j}{A_i} \left(\frac{D_j}{D_i}\right)^{\phi} \frac{z_j f(h_j k_j, n_j)}{z_i f(h_i k_i, n_i)}$$

Relative shopping effort under zero fixed costs

- Under zero fixed costs, combining firm investment relative price with HH optimal shopping yields

$$\frac{D_i}{D_j} = \frac{1}{\theta_i}\frac{n_i W_i}{n_j W_j}$$

- Relative shopping effort closely tied to relative labor hours
  $\Rightarrow$ reduces to $D_i/D_c = n_i/n_c$ in BRS

- But also need to match relative capacity utilization

Labor share of income

- Labor share of income is key component to constructing Solow residual
- Define fixed cost share $\nu_j^R = \nu_j X/(z_j f - \nu_j X)$, so that $Y_j = A_j D_j^\phi z_j f/(1 + \nu_j^R)$
- Write sectoral labor share of income as

$$\frac{W_j n_j}{p_j Y_j} = \frac{\alpha_n (1 + \nu_j^R)}{1 - \phi}$$

- Provided $\nu_j^R = \nu^R$ for all j, overall steady-state labor share of income is

$$\tau \equiv \frac{Wn}{Y} = \frac{\alpha_n (1 + \nu^R)}{1 - \phi}$$

A simple static model

A simple static model

- Consider simple static model with no investment; homogeneous labor as only input, $F = zn^{\alpha_n}$

Shopping	$\theta_d D^{1/\eta} = \phi C/D$
Consumption	$C = AD^\phi F$
Labor demand	$(1-\phi)W = \dfrac{\alpha_n C}{n}$
Labor supply	$\theta_n n^{1/\zeta} = (1-\phi)W$

- Labor share $\tau \equiv Wn/C = \alpha_n/(1-\phi)$ used for computing the Solow residual

$$SR \equiv \frac{C}{n^\tau} = AD^\phi z n^{\alpha_n - \tau} = AD^\phi z n^{-\alpha_n \phi/(1-\phi)}$$
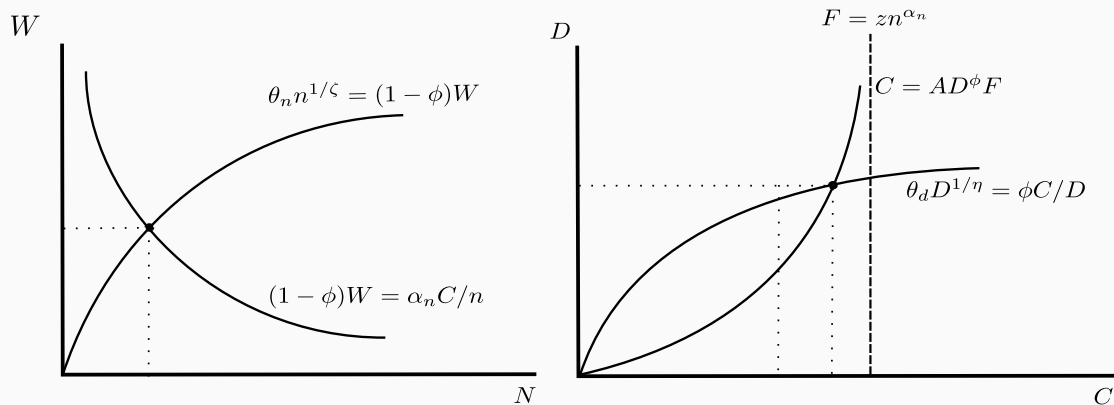
Equilibrium in static setting

Figure 3: Equilibrium of static model

$$D_1 = [C/(AF)]^{1/\phi} \quad \text{(Consumption)}, \quad D_2 = [\phi C/(\theta_d)]^{\eta/(\eta+1)} \quad \text{(Shopping)}$$
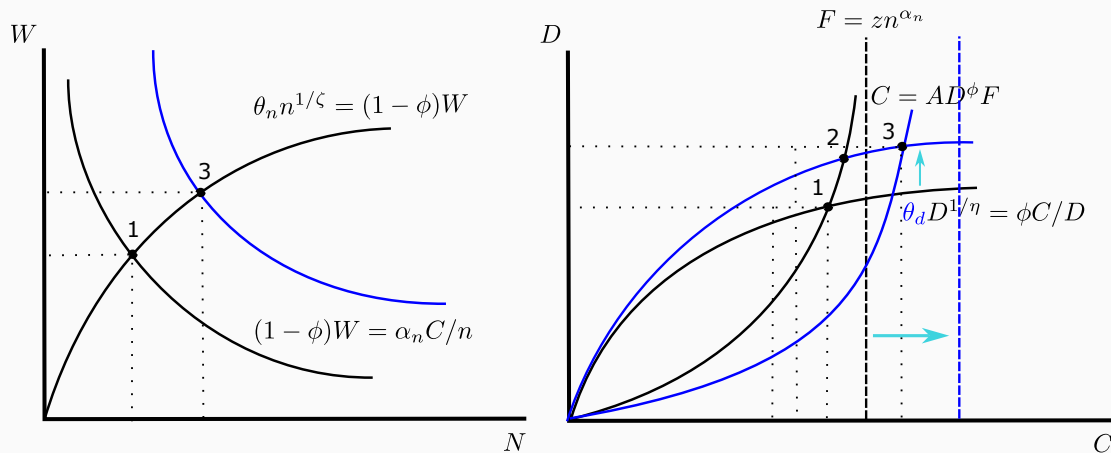
Figure 4: Reduction of shopping disutility in static model

1. Shopping curve shifts upward from lower marginal shopping cost
2. Induces movement along consumption curve from (1) to (2)
3. Firm labor demand shifts rightward, boosting hours and wages
4. More labor hours expand firm potential output, so consumption curve shifts rightward
5. Induces movement along shopping curve from (2) to (3)

Growth rates of Solow residual and capacity utilization

- Write sectoral Solow residual as

$$SR_{jt} \equiv \frac{Y_{jt}}{k_{jt}^{1-\tau} n_{jt}^{\tau}} = \frac{A_j D_{jt}^{\phi}(z_{jt} h_{jt}^{\alpha_k} X_t^{1-\alpha_k} k_{jt}^{\alpha_k-1+\tau} n_{jt}^{\alpha_n-\tau})}{1+\nu_{jt}^R}$$

given steady-state labor income share τ

- Rewrite using growth rates $dx_t = \Delta \log x_t$

$$dSR_{jt} = \overbrace{\phi dD_{jt}}^{\text{Shopping}} + \underbrace{\alpha_k dh_{jt}}_{\text{Capital utilization}} + \overbrace{dz_{jt} + (1-\alpha_k)dX_t}^{\text{Technology}}$$

$$+ \underbrace{(\alpha_k - 1 + \tau)dk_{jt} + (\alpha_n - \tau)dn_{jt}}_{\text{Input share mismeasurement}} + \overbrace{d(1+\nu_{jt}^R)}^{\text{Fixed costs}}$$

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- Define capacity in sector j following Qiu and Ríos-Rull (2022)

$$cap_j = z_j k_j^{\alpha_k} n_j^{\alpha_n} X^{1-\alpha_k} - \nu_j X$$

- Capacity utilization in sector j is the ratio of output to capacity (stationary measure):

$$util_j \equiv \frac{Y_j}{cap_j} = \frac{A_j D_j^{\phi}(z_j h_j^{\alpha_k} X^{1-\alpha_k} k_j^{\alpha_k} n_j^{\alpha_n} - \nu_j X)}{z_j k_j^{\alpha_k} n_j^{\alpha_n} X^{1-\alpha_k} - \nu_j X}$$

- Capacity utilization in growth rates

$$dutil_{jt} = \phi dD_{jt} + (1 + \nu_{ss}^R)\alpha_k dh_{jt}$$

- If $\nu_j = 0$, then Solow residual growth rate simplifies to

$$dSR_{jt}|_{\nu_j=0} = \overbrace{dutil_{jt}}^{\text{Utilization}} + \underbrace{dz_{jt} + (1-\alpha_k)dX_t}_{\text{Technology}} + \overbrace{(\alpha_k - 1 + \tau)dk_{jt} + (\alpha_n - \tau)dn_{jt}}^{\text{Input share mismeasurement}}$$

Capacity utilization and connection to Solow residual

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Aggregate measures

- Output

$$Y = C + p_i^{ss} I$$

- Using base-year prices makes results independent of numeraire choice (Explanation)

- Solow residual and capacity utilization

$$SR = \prod_j SR_j^{\frac{Y_j}{Y}}, \quad util = \prod_j util_j^{\frac{Y_j}{Y}} \tag{3}$$

- Applying logs and first differencing to (3) immediately implies

$$dSR = \sum_j \frac{Y_j}{Y} dSR_j, \quad dutil = \sum_j \frac{Y_j}{Y} dutil_j$$

(Equilibrium)

Quantitative analysis of general model

- Model nests BRS by shutting down additional frictions
  $(ha = 0, \rho_c = 1, \nu^R = 0, \sigma_b \to \infty, \Psi_K = 0, \varepsilon = 0)$
- Absent fixed costs and variable capital utilization

$$util_j = A_j D_j^\phi, \quad util = (C/Y)util_c + (I/Y)util_i$$

- Estimating special case of BRS on aggregate data implies
  - Shopping-related parameters are more precisely estimated, and demand channel is stronger
  - Capacity utilization volatility rises by 10 times, much closer to empirical value
  - Forecast error variance contribution of θ_d rises dramatically

Proof of concept using BRS model

Stochastic processes

- The growth rate of the stochastic trend $g_t = X_t/X_{t-1}$ follows an AR(1) process in logs as BRS

$$\log g_t = (1 - \rho_g) \log \overline{g} + \rho_g \log g_{t-1} + e_{g,t}$$

  where $e_{g,t} \sim N(0, \sigma_g)$

- Each stationary shock in the set $v = \{\theta_b, \theta_d, \theta_n, \theta_i, z_c, z_I, \mu_c, \mu_i\}$ follows an AR(1) process

$$\log v_t = \rho_v \log v_{t-1} + e_{v,t}$$

  where $e_{v,t} \sim N(0, \sigma_v)$

- Stationarize trending variable by dividing by X_t (X_{t-1} in case of predetermined capital stock K_{jt})

Observation equations

Bayesian estimation and choice of observable variables

- Parameter space Θ and data Y
- Bayesian estimate model subject to steady-state targeting
- Sample from posterior distribution combining likelihood and prior

$$P(\Theta|Y) = \frac{L(Y|\Theta)P(\Theta)}{P(Y)}, \quad P(Y) = \int L(Y|\Theta)P(\Theta)d\theta$$

- Use random-walk MH to sample posterior

Seven observables in growth rates: $1964Q1 - 2019Q4$

$$\mathcal{Y}_t = \begin{bmatrix} dC_t & dI_t & dn_{ct} & dn_{it} & dutil_{ND,t} & dutil_{D,t} & dp_{it} \end{bmatrix}'$$

- Use sectoral data on output and labor following Katayama and Kim (2018)
- Construct output from sum of private consumption and private investment (as BRS)
- Note that sectoral dataset implicitly targets labor productivity in each sector

Estimation procedure

Calibration: exogenous and estimated parameters

Targets	Target value	Parameter	Calibrated value/posterior mean
First group: parameters set exogenously			
Discount factor	0.99	β	0.99
Average annual growth rate	1.8%	$\overline{g}$	0.45%
Gross wage markup	1.15	μ	1.15
Labor share in consumption	0.8	ω	0.8
Share of services in consumption	0.65	ω_{sc}	0.65
Second group: estimated parameters used for calibration			
Risk aversion	—	σ	1.58
Frisch elasticity	—	ζ	1.24
Elasticity of matching function	—	ϕ	0.92
Elasticity of shopping effort cost	—	η	0.22
Fixed cost share of capacity	—	ν^R	0.094
Habit persistence	—	ha	0.74

Table 3: Calibration: exogenous and estimated parameters

Calibration: normalizations and standard targets

Targets	Target value	Parameter	Calibrated value/posterior mean
Third group: normalizations			
SS output	1	z_{mc}	0.30
Relative price of services	1	z_{sc}	0.45
Relative price of investment	1	z_i	0.25
Fraction time spent working	0.30	θ_n	0.45
Capacity utilization of nondurables	0.81	A_{mc}	2.6
Capacity utilization of services	0.81	A_{sc}	1.5
Capacity utilization of investment sector	0.81	A_i	3.6
Capital utilization rate	1	σ_b	0.031
Fourth group: standard targets			
Investment share of output	0.20	δ	1.37%
Physical capital to annual output ratio	2.75	α_k	0.31
Labor share of income	0.67	α_n	0.053

Table 4: Calibration: normalization and standard targets. Use posterior mean of the estimated parameters $\sigma, \zeta, \phi, \eta, \nu^R$ and ha.

Posterior estimates: structural parameters

Par	Interpretation	Prior			Posterior			
		Dist.	Mean	Stdev.	Mean	Stdev.	HPD inf	HPD sup
σ	Risk aversion	Beta	1.50	0.250	1.59	0.240	1.23	1.98
ha	Habit formation	Beta	0.500	0.200	0.739	0.0322	0.692	0.792
ζ	Frisch elast. of labor supply	Gamma	0.720	0.250	1.26	0.215	0.947	1.60
ϕ	Elast. of matching	Beta	0.320	0.200	0.917	0.0531	0.834	0.991
η	Elast. shopping disutility	Gamma	0.200	0.150	0.221	0.0388	0.159	0.277
ξ	Cons. elast. of subs.	Gamma	0.850	0.100	0.880	0.0826	0.753	1.02
ν_R	Fixed cost share	Beta	0.200	0.100	0.090	0.0400	0.028	0.152
σ_{ac}	Depreciation elast:cons	Inv. Gamma	1.00	1.00	1.74	0.328	1.20	2.23
σ_{ai}	Depreciation elast:inv	Inv. Gamma	1.00	1.00	0.440	0.0893	0.298	0.580
Ψ_K	Inv. adj. cost	Gamma	4.00	1.00	12.6	1.60	10.5	15.5
ε	Inv. elast. of labor mobility	Gamma	1.00	0.500	1.46	0.218	1.10	1.82

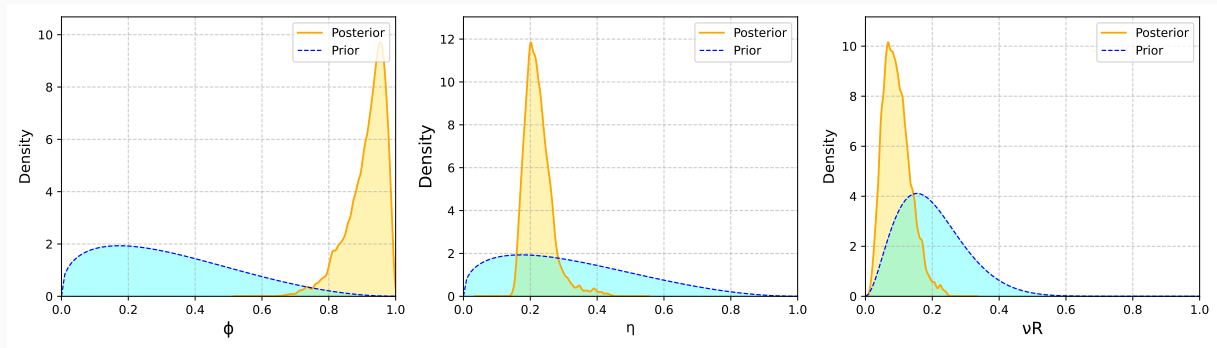**Table 5:** Posterior estimates: structural parameters

Figure 5: Posterior and prior distributions for matching function elasticity ϕ, the shopping disutility parameter η, and the fixed cost share ν^R.

Estimation on artificial data (parameter values set to posterior mean)

Parameter	True value	Posterior distribution		
		Median	5%	95%
σ	1.58	1.70	1.24	1.85
ha	0.736	0.714	0.689	0.746
ζ	1.24	1.29	1.09	1.50
ϕ	0.913	0.911	0.872	0.955
η	0.224	0.241	0.203	0.276
ξ	0.882	0.809	0.717	0.892
ν^R	0.0943	0.118	0.0694	0.21
σ_{ac}	1.76	2.04	1.69	2.47
σ_{ai}	0.441	0.289	0.216	0.374
Ψ_K	12.5	7.78	7.26	8.30
ε	1.46	1.53	1.38	1.69

Table 6: Parameters well identified in exercise using artificial data generated from model evaluated at posterior mean (Schmitt-Grohé and Uribe (2012))

Unconditional forecast error variance decomposition: grouped shocks

	Technology	Labor Supply	Shopping Effort	Discount Factor	Wage Markup
Y	28.6	0.01	70.5	0.92	0.02
SR	44.3	5.23	46.0	0.57	3.9
I	31.2	0.01	64.1	4.69	0.01
p_i	65	0.00	34.8	0.18	0.05
n_c	7.78	27.2	58.4	4.42	2.19
n_i	18.2	2.27	52.9	1.76	24.8
$util$	39.3	0.01	60.1	0.64	0.01
D	0.17	0	99.8	0.01	0
h	17.7	0.01	82.1	0.18	0

Table 7: Unconditional forecast error variance decomposition for variables in growth rates. Shocks are grouped in respective categories.

	Data	Baseline	Perfect labor mobility	Common wage markup	Remove			
					Fixed cost	VCU	SDS	SDS and utilization data
LML	—	4570.7	4548.9	3136.5	4573.4	4,568.1	2564.9	—
Δ LML	—	0	-21.9	-1434.2	2.71	-2.6	-2006	—
Posterior mean ϕ	—	0.91	0.43	0.95	0.96	0.27	0.71	0.52
FEVD(SR, SDS)	—	46.0	55.0	6.17	45.71	48.07	—	—
Var$(util)$/Var(SR)	—	1.95	1.21	0.40	2.36	0.56	2.21	0.19
std(Y)	0.87	1.38	1.70	5.11	1.36	1.99	207.71	0.64
std$(util_{ND})$	1.26	1.21	1.30	3.88	1.22	1.21	161.65	0.35
std$(util_D)$	2.27	3.65	2.60	9.72	3.90	2.37	266.65	1.14
std(n_c)	0.57	0.66	0.68	2.27	0.68	0.69	71.31	0.56
std(n_i)	1.94	2.35	3.33	8.99	2.36	1.80	344.8	1.87
Cor(C, I)	0.54	0.53	0.67	0.05	0.50	0.53	0.999	0.24
Cor$(util_{ND}, util_D)$	0.75	0.27	0.61	-0.31	0.26	0.60	0.999	-0.60
Cor(n_c, n_i)	0.87	0.67	0.24	-0.88	0.69	0.35	0.986	0.83
Cor$(util_{ND}, util_{ND,-1})$	0.51	0.21	0.31	0.53	0.18	-0.02	0.999	0.27
Cor$(util_D, util_{D,-1})$	0.55	0.48	0.52	0.51	0.48	0.03	0.999	0.26

Table 8: Comparison of model specification

Takeaway

1. Baseline model overstates output volatility and understates utilization comovement but otherwise fits sectoral data well
2. Sectoral wage-markup shocks are essential $(\Delta LML = -1434)$
   $\Rightarrow$ shopping-effort ratio is otherwise much more directly tied to labor ratio, and loses flexibility in fitting comovement of utilization
3. Search demand shocks are essential $(\Delta LML = -2006)$
   $\Rightarrow$ Capacity utilization data roughly pins down sectoral shopping efforts–model lacks freedom to jointly match sectoral labor and output and the relative price of investment
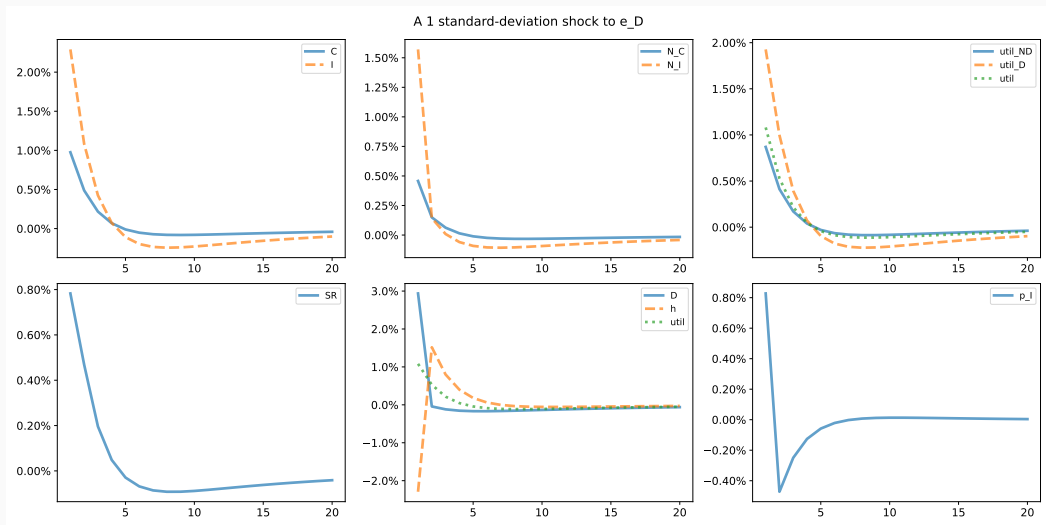
Figure 6: The vertical axis measures response in growth rates.
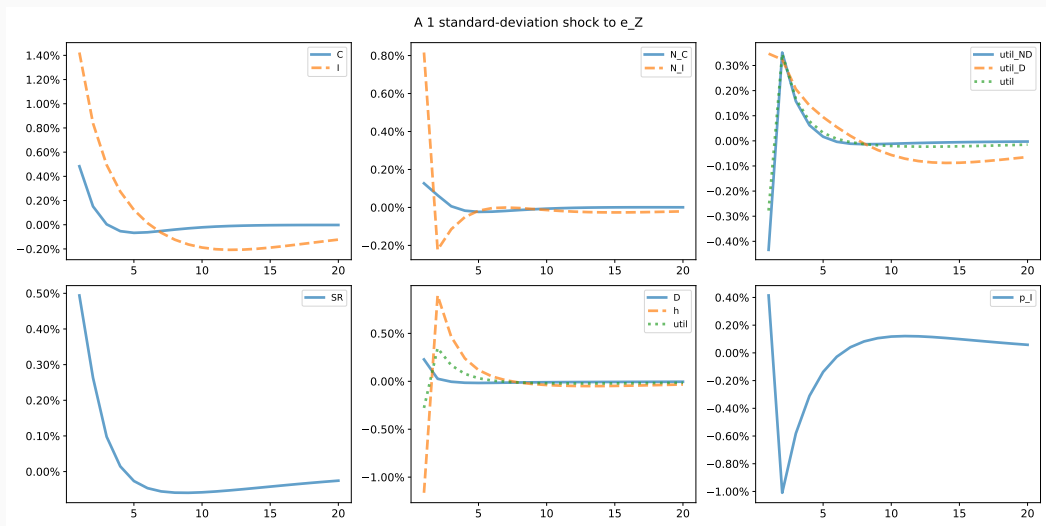
Figure 7: The vertical axis measures response in growth rates.

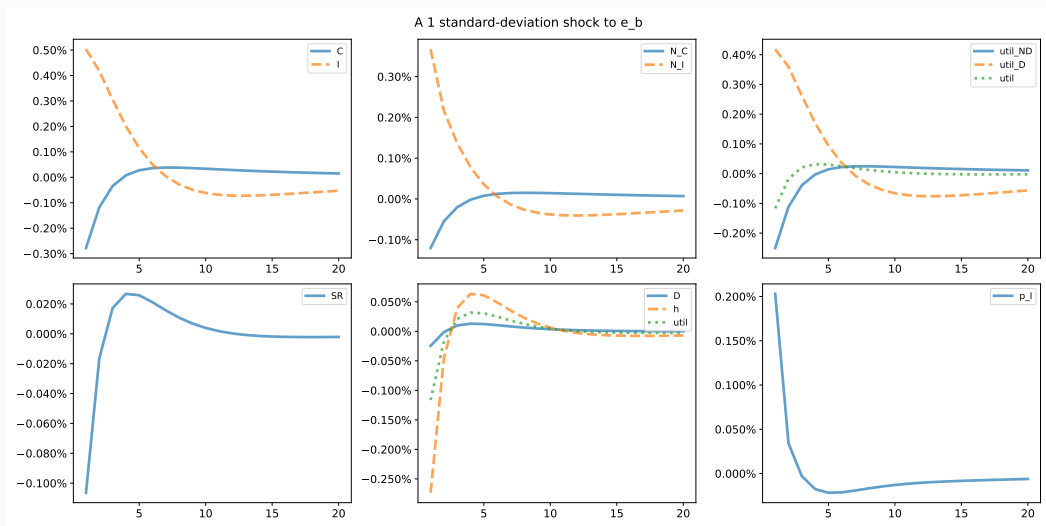**Figure 8:** The vertical axis measures response in growth rates.

Key findings

1. Develop multisector model linking shopping effort, capacity, utilization, and the Solow residual
2. Estimate precise, high value of key parameters ϕ, η, and shopping-effort shocks without relying on shopping time data
   $\Rightarrow$ can recover parameter values by estimating model on artificial data
3. Search demand shocks explain most of forecast error variance of output, TFP, relative price of investment, hours, and utilization
4. Baseline model fits second moments well other than output
5. Search demand shocks and sector-specific wage markups are essential to fit data

Extensions

- Approach adaptable to other countries: need to disaggregate utilization into durables and non-durables
- Relax competitive search–implies efficiency and has strong implication for labor share of income
- Link shopping activity to confidence shocks (Angeletos, Collard, and Dellas (2018)) or liquidity shocks

Conclusion

Key findings

1. Develop multisector model linking shopping effort, capacity, utilization, and the Solow residual
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Extensions

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Appendix

Data series

ID	Description	Source
PCND	Personal consumption: non-durable	BEA
PCESV	Personal consumption: services	BEA
HOANBS	Nonfarm business hours worked	BLS
CPIAUCSL	Consumer price index	BLS
GDPC1	Real GDP	BEA
GDPIC1	Real gross private domestic investment	BEA
COMPRNFB	Wages (real compensation per hour)	BLS
CNP160V	Civilian non-institutional population	BLS
GDPDEF	GDP Deflator	BEA
SR	Solow residual	Fernald (2014), FRB of San Francisco
Util	Total capacity utilization	Federal Reserve Board of Governors
SR_{util}	Utilization-adjusted Solow residual	Fernald (2014), FRB of San Francisco

Construction of variables

Symbol	Description	Construction
C	Nominal consumption	$PCEND + PCESV$
I	Nominal gross private domestic investment	GPDI
Deflator	GDP Deflator	GDPDEF
Pop	Civilian non-institutional population	CNP160V
c	Real per capita consumption	$\frac{C}{Pop * P_c}$
i	Investment	$\frac{I}{Pop * P_i}$
y	Real per capita output	$c + i$
N_c	Labor in consumption sector	Labor in nondurables and services, BLS
N_i	Labor in investment sector	Labor in construction and durables, BLS
N	Aggregate labor	$N_c + N_i$
P_i	Price index: investment goods	$A006RD3Q086SBEA$
P_c	Price index: consumption goods	$DPCERD3Q086SBEA$
p_i	Relative price of investment	P_i/P_c
$util_{ND}$	Total capacity utilization: non-durables	Federal Reserve Board
$util_D$	Total capacity utilization: durables	Federal Reserve Board
SR	Solow residual	Fernald (2014), FRB of San Francisco
SR_{util}	Utilization-adjusted Solow residual	Fernald (2014), FRB of San Francisco

More details on construction of sectoral data

- Closely follows Katayama and Kim (2018)

- Construct consumption and investment as follows

$$C_t = \left(\frac{Nondurable(PCND) + Services(PCESV)}{P_c \times CivilianNonstitutionalPopulation(CNP160V)} \right)$$

$$I_t = \left(\frac{Durable(PCDG) + NoresidentialInvestment(PNFI) + ResidentialInvestment(PRFI)}{P_i \times CivilianNoninstitutionalPopulation(CNP160V)} \right)$$

- Use HP-filtered trend for population ($\lambda = 10,000$) to eliminate jumps around census dates

- P_c: combine price indices of nondurable goods (DNDGRG3Q086SBEA) and services (DSERRG3Q086SBEA)

- P_i: use quality-adjusted investment deflator (INVDEV)

More details on construction of sectoral data

- BLS Current Employment Statistics (https://www.bls.gov/ces/data)
- BLS Table B6 contains the number of production and non-supervisory employees by industry
- BLS Table B7 contains average weekly hours of each sector
- We compute total hours for non-durables, services, construction, and durables by multiplying the relevant components of each table
- Construct labor in consumption as sum of non-durables and services
- Construct labor in investment as sum of construction and durables

Related international measures of capacity utilization

Country	Sectors Covered	Survey/Calc.?	How Question is Framed / Method	Source
Canada	Manufacturing, mining, utilities, construction	Survey	"At what percentage of your production capacity are you currently operating?"	Statistics Canada
Euro Area	Manufacturing	Survey	"At what capacity is your company currently operating (as a percentage of full capacity)?"	European Commission
UK	Manufacturing, services	Survey	"What is the current rate of capacity utilization in your business?" (firms give a percentage)	Office for National Statistics / CBI
Japan	Manufacturing, mining	Calculated	Based on indices of industrial production and capacity, using statistical/engineering estimates	Ministry of Economy, Trade and Industry (METI)
South Korea	Manufacturing	Survey	Firms are surveyed: "At what % of capacity are you currently operating?"	Statistics Korea (KOSTAT)
Russia	Manufacturing	Survey	Firms report their current use of production capacity as % of "normal/full" capacity	Rosstat
China	Manufacturing	Survey	Firms asked: "What is the current utilization rate of your production capacity?"	National Bureau of Statistics of China (NBSC)

Households' problem

- Households choose search effort, labor hours, consumption, capital, and utilization rates taking markets $(p_j, D_j, y_j), j \in \{c, i\}$ and the aggregate state of the economy $\Lambda = (\theta, Z, K)$ as given.

$$\widehat{V}(\Lambda, k_{mc}, k_{sc}, k_i, p, D, F) = \max_{d_j, n_c, n_i, y_j, i_j, k'_j, h'_j} u(y_{mc}, y_{sc}, d, n^a, \theta) + \beta \theta_b \mathbb{E}\{V(\Lambda', k'_c, k'_i)|\Lambda\} \quad s.t.$$

$$y_j = d_j A_j D_j^{\phi-1} F_j, \quad j \in \{mc, sc, i\}$$

$$\sum_j y_j p_j = \pi + \sum_{j \in \{mc, sc, i\}} k_j h_j R_j + n_c W_c + n_i W_i$$

$$k'_j = (1 - \delta_j(h_j))k_j + [1 - S_j(i_j/i_{j,-1})]i_j, \quad j \in \{mc, sc, i\}$$

and the consumption and labor aggregators

- The value function is determined by the best market:

$$V(\Lambda, k_{mc}, k_{sc}, k_i) = \max_{\{p, D, y\} \in \Omega} \widehat{V}(\Lambda, k_{mc}, k_{sc}, k_i, p, D, y)$$

First order conditions

- Let $\gamma_{mc}, \gamma_{sc}, \gamma_i, \lambda, \mu_c, \mu_i$ be the respective Lagrangian multipliers on the constraints
- FOC

$$
\begin{aligned}
[y_{mc}]: \quad & u_{mc} = \gamma_{mc} + \lambda p_{mc} \\
[y_{sc}]: \quad & u_{sc} = \gamma_{sc} + \lambda p_{sc} \\
[i_c]: \quad & -\gamma_i - \lambda p_i + \mu_c \left(1 - S'(x_c)x_c - S(x_c)\right) + \beta\theta_b \mathbb{E}\mu_c' S'(x')(x')^2 = 0 \\
[i_i]: \quad & -\gamma_i - \lambda p_i + \mu_i \left(1 - S'(x_i)x_i - S(x_i)\right) + \beta\theta_b \mathbb{E}\mu' S'(x_i')(x_i')^2 = 0 \\
[d_j]: \quad & u_d = -A_j D_j^{\phi-1} F_j \gamma_j, \quad j \in \{mc, sc\} \\
[d_i]: \quad & u_d \theta_i = -A_i D_i^{\phi-1} F_i \gamma_i \\
[n_c]: \quad & u_n \frac{\partial n^a}{\partial n_c} = -\lambda W_c^* \\
[n_i]: \quad & u_n \frac{\partial n^a}{\partial n_i} = -\lambda W_i^* \\
[h_j] \quad & \delta_h(h_j)\mu_j = \lambda R_j \quad j \in \{mc, sc, i\} \\
[k_j']: \quad & \mu_j = \beta\theta_b \mathbb{E}\left\{\lambda' R_j' h_j' + (1 - \delta_j(h_j'))\mu_j'\right\} \quad j \in \{mc, sc, i\}
\end{aligned}
$$

Envelope conditions

- Consumption

$$\frac{\partial V^j}{\partial p_j} = -\lambda j = -\lambda d_j A_j D_j^{\phi-1} F_j \quad j \in \{mc, sc\} \tag{4}$$

$$\frac{\partial V^j}{\partial D_j} = (\phi - 1) d_j A_j D_j^{\phi-2} F_j (u_j - \lambda p_j) \quad j \in \{mc, sc\} \tag{5}$$

$$\frac{\partial V^j}{\partial F_j} = d_j A_j D_j^{\phi-1} (u_j - \lambda p_j) \quad j \in \{mc, sc\}$$

- Investment

$$\frac{\partial V^i}{\partial p_i} = -\lambda i = -\lambda (d_i A_i D_i^{\phi-1} F_i) \tag{6}$$

$$\frac{\partial V^i}{\partial D_i} = -(\phi - 1) d_i A_i D_i^{\phi-2} F_i \gamma_i \tag{7}$$

$$\frac{\partial V^i}{\partial F_i} = d_i A_i D_i^{\phi-1} \gamma_i$$

Price-tightness tradeoff

- Take ratio of (4) and (5):

$$\frac{\frac{\partial V^j}{\partial p_j}}{\frac{\partial V^j}{\partial D_j}} = -\frac{\lambda D_j}{(\phi - 1)(u_j - \lambda p_j)} \tag{8}$$

- Take ratio of (6) and (7)

$$\frac{\frac{\partial V^i}{\partial p_i}}{\frac{\partial V^i}{\partial D_i}} = -\frac{\lambda D_i}{(\phi - 1)\gamma_i} \tag{9}$$

Firms' problem

- A representative firm in sector $j \in \{mc, s, i\}$ rents capital and hires labor in spot markets
- Continuum of monopolistically competitive labor unions in sector j sell differentiated services
- Firm chooses inputs and market bundle (p_j, D_j, F_j)
- Submarket must satisfy participation constraint of household

$$\max_{k_j, n_j, p_j, D_j, y_j} p_j A_j D_j^\phi F_j - \int_0^1 W_j(s) n_j(s) ds - R_j h_j k_j \quad \text{s.t.}$$

$$\widehat{V}(K, p_j, D_j, F_j) \geq V(K)$$

$$z_j f(h_j k_j, n_j) - \nu_j \geq F_j$$

$$n_j = \left(\int_0^1 n_j(s)^{1/\mu_j} ds \right)^{\mu_j}$$

Conditional labor demand and wage index

- Consider labor cost minimization problem

$$\min_{n_j(s)} \int_0^1 W_j(s) n_j(s) ds \quad \text{s.t.}$$

$$\left(\int_0^1 n_j(s)^{1/\mu_j} dj \right)^{\mu_j} \geq \overline{n}$$

- Take FOC and recognize W_j as Lagrangian multiplier on constraint

$$n_j(s) = \left(\frac{W_j(s)}{W_j} \right)^{-\frac{\mu_j}{\mu_j - 1}} n_j \tag{10}$$

- Wage index for composite labor input in sector j

$$W_j = \left[\int_0^1 W_j(s)^{1/(\mu_j - 1)} ds \right]^{\mu_j - 1}$$

Optimal wage choice of labor union and aggregation

- Problem of labor union

$$\max_{W_j(s)} (W_j(s) - W_j^*) n_j(s) \quad \text{s.t.} \quad (10) \Leftrightarrow$$

$$\max_{W_j(s)} (W_j(s) - W_j^*) \left(\frac{W_j(s)}{W_j} \right)^{-\frac{\mu_j}{\mu_j - 1}} n_j$$

- Labor union in each sector choose

$$W_j(s) = \mu_j W_j^*$$

- Labor unions pay same wage and firms choose identical quantities of labor within j

$$W_j(s) = W_j, n_j(s) = n_j$$

- Labor unions rebate earnings to HH in lump-sum fashion (regard as fixed component to wage)

Firm first order conditions

- Let ι_j and ∇_j be the multipliers on participation constraint and production technology

$$
\begin{aligned}
[F_j] \quad & \nabla_j = p_j A_j D_j^\phi + \iota_j \frac{\partial V^j}{\partial F^j} \\
[n_j] \quad & W_j = \nabla_j z_j f_n \\
[k] \quad & h_j R_j = \nabla_j z_j f_k \\
[p_j] \quad & A_j D_j^\phi F_j + \iota_j \frac{\partial V^j}{\partial p_j} = 0 \\
[D_j] \quad & \phi A_j D_j^{\phi-1} p_j F_j + \iota_j \frac{\partial V^j}{\partial D^j} = 0
\end{aligned}
$$

$$(11)$$

$$(12)$$

Firm problem: finding λ and γ_j

- Take ratio of first order conditions for (11) and (12)

$$\frac{D_j}{\phi p_j} = \frac{\frac{\partial V^j}{\partial p_j}}{\frac{\partial V^j}{\partial D_j}}$$

- Plug in (8)

$$\frac{D_j}{\phi p_j} = -\frac{\lambda D_j}{(\phi - 1)(u_j - \lambda p_j)}$$

- Simplify

$$\lambda \phi p_j = (1 - \phi)(u_j - \lambda p_j) \Rightarrow$$

$$\lambda = u_j(1 - \phi)/p_j$$

so that

$$\gamma_j = \phi u_j$$

Firm problem: finding γ_i

- Take ratio of first order conditions for (11) and (12) for $j = i$:

$$\frac{D_i}{\phi p_i} = \frac{\frac{\partial V^i}{\partial p_i}}{\frac{\partial V^i}{\partial D_i}}$$

- Plug in (9)

$$\frac{D_i}{\phi p_i} = -\frac{\lambda D_i}{(\phi - 1)\gamma_i}$$

- Simplify

$$\gamma_i = \frac{\phi}{1 - \phi}\lambda p_i$$
$$= \phi \frac{u_j}{p_j} p_i$$

Simplifying shopping conditions

- Plug in values of γ_j to find

$$-u_d = \phi u_j A_j D_j^{\phi-1}[z_j f(h_j k_j, n_j) - \nu_j] \quad j \in \{m_c, s_c\}$$

$$-u_d \theta_i = \phi \frac{u_{mc} p_i}{p_{mc}} A_i D_i^{\phi-1}[z_i f(h_i k_i, n_i) - \nu_i]$$

- Plug in $\lambda = u_{mc}(1-\phi)/p_{mc}$ to simplify labor-leisure tradeoff

$$u_n \frac{\partial n^a}{\partial n_j} = -\frac{u_{mc}(1-\phi)}{p_{mc}} W_j^* \quad j \in \{c, i\}$$

Demand for non-durables and services

- From the expression for λ we have

$$\frac{u_{mc}}{p_{mc}} = \frac{u_{sc}}{p_{sc}} \Rightarrow \phi = (u_j - \lambda p_j)/u_j$$

- Combine with consumption aggregation and price index to find demand curves

$$Y_j = p_j^{-\xi} \omega_j C \quad j \in \{m_c, s_c\}$$

where $\xi = 1/(1 - \rho_c)$ is the elasticity of substitution.

Tobin's Q

- Solve for value of investment: $j \in \{c, i\}$

$$\lambda p_i + \gamma_i = \mu_j(1 - S'(x_j)x_j - S(x_j)) + \beta\theta_b\mathbb{E}\mu'_j(S'(x'_j)(x'_j)^2)$$

$$\lambda p_i + \frac{\phi}{1-\phi}\lambda p_i = \mu_j(1 - S'(x_j)x_j - S(x_j)) + \beta\theta_b\mathbb{E}\mu'_j(S'(x'_j)(x'_j)^2)$$

$$\frac{\lambda p_i}{1-\phi} = \mu_j(1 - S'(x_j)x_j - S(x_j)) + \beta\theta_b\mathbb{E}\mu'_j(S'(x'_j)(x'_j)^2)$$

- Let $Q_j = \mu_j/\lambda$: relative price of capital in sector j in terms of consumption

- We can rearrange as

$$\frac{p_i}{1-\phi} = Q_j[1 - S'(x_j)x_j - S(x_j)] + \beta\theta_b\mathbb{E}\frac{\lambda'}{\lambda}Q'_jS'(x'_j)(x'_j)^2$$

Tobin's Q

- Rewrite optimal choice of utilization: $j \in \{mc, sc, i\}$

$$\delta_h(h_j)Q_j = R_j$$

- Euler equation

$$Q_j = \beta\theta_b\mathbb{E}\frac{\lambda'}{\lambda}\left[(1 - \delta(h'_j))Q'_j + R'_j h'_j\right] \quad j \in \{mc, sc, i\}$$

Solving for firm multipliers

$$\iota_j = \frac{A_j q_j^\phi F_j}{\frac{\partial V^j}{\partial p_j}} = \frac{1}{\lambda}$$

$$\nabla_j = p_j A_j D_j^\phi + \iota_j \frac{\partial V^j}{\partial F^j}$$

$$= p_j A_j D_j^\phi + \frac{A_j D_j^\phi \gamma_j}{\lambda}$$

$$= p_j A_j D_j^\phi + A_j D_j^\phi \frac{\phi}{1-\phi} p_j$$

$$= A_j D_j^\phi \left(p_j + \frac{\phi}{1-\phi} p_j \right)$$

$$= \frac{p_j A_j D_j^\phi}{1-\phi}$$

Simplified optimality conditions for firm

$$(1 - \phi)\frac{W_c}{p_j} = A_j(D_j)^\phi z_c f_{N_j} \quad j \in \{m_c, s_c\}$$

$$\frac{W_c}{R_j} = \frac{f_{N_c}}{f_{K_c}}$$

$$(1 - \phi)\frac{W_i}{p_i} = A_i(D_i)^\phi z_i f_{N_i}$$

$$\frac{W_i}{R_i} = \frac{f_{N_i}}{f_{K_i}}$$

Firm factor demands

$$(1 - \phi)\frac{W_c}{p_j} = \alpha_n \frac{Y_j + A_j D_j^\phi \nu_j}{N_j} \quad j \in \{mc, sc, i\}$$

$$(1 - \phi)\frac{R_j}{p_j} = \alpha_k \frac{Y_j + A_j D_j^\phi \nu_j}{h_j K_j} \quad j \in \{mc, sc, i\}$$

Summary of equilibrium conditions

$$\theta_n (n^a)^{1/\nu} \left(\frac{n_c}{n^a}\right)^{\theta} \omega^{-\theta} = (1-\phi)\frac{W_c}{\mu_c \zeta}$$

$$\theta_n (n^a)^{1/\nu} \left(\frac{n_i}{n^a}\right)^{\theta} (1-\omega)^{-\theta} = (1-\phi)\frac{W_i}{\mu_i \zeta}$$

$$n^a = \left[\omega^{-\theta} n_c^{1+\theta} + (1-\omega)^{-\theta} n_i^{1+\theta}\right]^{\frac{1}{1+\theta}}$$

$$\theta_d D^{1/\eta} = \phi p_j \frac{Y_j}{D_j} \quad j \in \{mc, sc\}$$

$$\theta_i \theta_d D^{1/\eta} = \phi p_i \frac{I}{D_i}$$

$$\frac{p_i}{1-\phi} = Q_j [1 - S_j'(x_j) x_j - S(x_j)] + \beta \theta_b \mathbb{E}\frac{\lambda'}{\lambda} Q_j' S'(x_j')(x_j')^2$$

$$Q_j = \beta \theta_b \mathbb{E}\frac{\lambda'}{\lambda} \left[(1-\delta_j(h_j'))Q_j' + R_j' h_j'\right] \quad j \in \{mc, sc, i\}$$

Summary of equilibrium conditions

$$C = [\omega_c^{1-\rho_c} Y_{mc}^{\rho_c} + (1 - \omega_c)^{1-\rho_c} Y_{sc}^{\rho_c}]^{1/\rho_c}$$
$$Y_j = p_j^{-1/(1-\rho_c)} \omega_j C \quad j \in \{mc, sc\}$$
$$C = p_{mc} Y_{mc} + p_{sc} Y_{sc}$$
$$\lambda = \Gamma^{-\sigma} (1 - \phi)$$

Summary of equilibrium conditions

$$\delta_h(h_j)Q_j = R_j, \quad j \in \{mc, sc, i\}$$

$$Y_j = A_j(D_j)^\phi(z_j(h_jK_j)^{\alpha_k}(n_j)^{\alpha_n} - \nu_j) \quad j \in \{mc, sc, i\}$$

$$I = I_c + I_i$$

$$K'_j = (1 - \delta_j(h_j))k_j + [1 - S(x_j)]I_j \quad j \in \{mc, sc, i\}$$

$$(1-\phi)\frac{W_j}{p_j} = \alpha_n \frac{Y_j + A_j D_j^\phi \nu_j}{N_j} \quad j \in \{mc, sc, i\}$$

$$\frac{W_j}{R_j} = \frac{\alpha_n}{\alpha_k}\frac{h_jK_j}{n_j} \quad j \in \{mc, sc, i\}$$

Explanation of numeraire dependence

- Quantity movements may depend on the numeraire in a multisector model

- Consider positive shock to Z^C: relative price of consumption goods falls

- In terms of the investment good, consumption may fall even though actual units purchased rises

- However, if the consumption good were the numeraire, the investment good instead rises in price, so output rises by more

- Reasoning is symmetric with a positive Z^I shock

- Using base-year prices eliminates dependence as by Bai, Rios-Rull, and Storesletten (2024)

- Fisher index also eliminates dependence on base year, but it is equivalent in the case of a first-order approximation.

- See Duernecker, Herrendorf, Valentinyi et al. (2017) for a detailed discussion

Calibration

Details: depreciation

- Over sample, the average annual growth rate of output is 1.8%

- Set $\overline{g} = 0.45\%$ (1.8% annual growth)

- Capital accumulation (ignoring adjustment costs)

$$g\widehat{K}' = (1-\delta)\widehat{K} + g\widehat{I}$$

  so that in steady state

$$\delta = 1 - \overline{g} + \frac{I}{K}$$

- Let investment share $\kappa = p_i I/Y = 0.2$ and $p_i K/Y = 2.75(4) = 11$

- Hence, $\delta = 0.2/11 - 0.0045 = 1.37\%$

Details: labor share α_n

- Rearrange FOC for labor demand

$$p_j = (1-\phi)\frac{W_j n_j}{\alpha_n A_j (D_j)^\phi F_j}$$

Hence,

$$W_j n_j = \frac{\alpha_n}{1-\phi} p_j Y^j (1+\nu^R)$$

where $\nu^R = \nu_j/(F_j)$ and thus labor share is

$$\frac{\sum W_j n_j}{Y} = \frac{\alpha_n}{1-\phi}\frac{C + p_i I}{Y}(1+\nu^R) = \frac{\alpha_n}{1-\phi}(1+\nu^R)$$

so that $\alpha_n = (1-\phi)$labor share$/(1+\nu^R)$

Details: capital share α_k and deprecation parameter σ_b

- $R_j = R$ in steady state
- Note $\beta(\overline{g})^{-\sigma} = 1/(1+r) \Rightarrow \overline{g} - 1 \approx (r - \rho)/\gamma$
- Implies $\rho \approx r - \gamma\overline{g}$ (so we must have $r \geq \gamma\overline{g}$)
- Steady-state Euler

$$Q = \beta\overline{g}^{-\gamma}[(1-\delta)Q + R] \Rightarrow$$
$$(1+r)Q = (1-\delta)Q + R$$
$$(r+\delta)Q = R$$

- Steady-state optimal utilization

$$\sigma_b = \frac{R}{Q} = r + \delta$$

- Combine with steady state Tobin's Q: $p_i/(1-\phi) = Q$ and we find

$$(1-\phi)\frac{R}{p_i} = r + \delta$$

Details: capital share α_k and deprecation parameter σ_b

- Firm optimization yields

$$(1 - \phi)\frac{R_j}{p_j} = \alpha_k \frac{Y_j}{K_j}(1 + \nu^R)$$

- Note

$$\frac{Y_j}{K_j} = \frac{Y}{K} \quad \forall K$$

and hence

$$r + \delta = \alpha_k \frac{Y}{K}(1 + \nu^R)$$

so that

$$\alpha_k = \frac{r + \delta}{1 + \nu^R} \frac{K}{Y}$$

Using $r, \delta, K/Y, \nu^R$, we recover α_k

Details: weight of services ω_{sc}

- We pin down the weight of services ω_{sc} as the empirical measure $S_c = Y_{sc}/C$ and set $S_c = 0.65$.

- The ratio of demand in consumption subsectors implies

$$\frac{Y_{mc}}{Y_{sc}} = \left(\frac{p_{mc}}{p_{sc}}\right)^{-\xi} \frac{\omega_{mc}}{\omega_{sc}}$$

  Multiply each side by p_{mc}/p_{sc}, so that

$$\frac{p_{mc}Y_{mc}}{p_{sc}Y_{sc}} = \left(\frac{p_{mc}}{p_{sc}}\right)^{1-\xi} \frac{\omega_{mc}}{\omega_{sc}}$$

  and plug in S_c, using $\omega_{sc} = S_c$:

$$\left(\frac{1-S_c}{S_c}\right) = \left(\frac{p_{mc}}{p_{sc}}\right)^{1-\xi} \frac{1-S_c}{S_c}$$

  so that $p_{mc} = p_{sc}$

- Given normalization $p_{sc} = 1$, all consumption goods prices equal unity.

Details: matching technology coefficient A_j

- Given $\Psi_j = A_j D_j^\phi$, the matching technology coefficient satisfies

$$A_j = \frac{\Psi_j}{D_j^\phi}$$

- Need to find D_j for each j

- We first solve for D. Let us sum each side of the shopping optimality condition across sectors:

$$\sum_j D^{1/\eta} D_j = \sum_j \phi p_j Y_j \rightarrow$$

$$D^{\frac{\eta+1}{\eta}} = \phi Y$$

- Given that we choose technology coefficients such that $Y = 1$, we obtain $D = \phi^{\frac{\eta}{\eta+1}}$.

Details: matching technology coefficient A_j

- Consider ratio in shopping optimality conditions between m_c and i:

$$\frac{D_{mc}}{D_i} = \frac{p_{mc}}{p_i}\frac{Y_{mc}}{Y_i}$$
$$= (1 - \omega_{sc})\frac{1 - I/Y}{I/Y}$$

- Hence,

$$D_{mc} = (1 - S_c)(1 - I/Y)D$$
$$D_{sc} = S_c(1 - I/Y)D$$
$$D_i = (I/Y)D$$

Estimation

Balanced growth and transformation of variables

- Output, consumption, investment, wages, and capital grow at common rate g_t

- Transform each trending variable y_t determined at time t

$$\widehat{y}_t = \frac{y_t}{X_t}$$

  so that $\log \widehat{y}_t$ represents log deviation from stochastic trend

- Capital stock K_t is determined at $t-1$, so we deflate by X_{t-1}

$$\widehat{K}_t = \frac{K_t}{X_{t-1}}$$

- Transform preferences to make shopping stationary

$$\Gamma_t = c_t - haC_{t,-1} - X_t\theta_{dt}\frac{d^{1+1/\eta}}{1+1/\eta} - \theta_{nt}X_t\frac{(n_t^a)^{1+1/\nu}}{1+1/\nu}$$

Equations modified by growth

Observation equations

- Match demeaned growth rates in model to those of data
- Nonstationary series

$$C_t^{obs} = \log C_t - \log C_{t-1} + g_t - \overline{g}$$
$$I_t^{obs} = \log I_t - \log I_{t-1} + g_t - \overline{g}$$

- Stationary series

$$N_{jt}^{obs} = \log N_{jt} - \log N_{j,t-1}, \quad j \in \{c, i\}$$
$$p_{i,t}^{obs} = \log p_{i,t} - \log p_{i,t-1}$$
$$util_{j,t}^{obs} = \log util_{j,t} - \log util_{j,t-1}$$

Vector of observable variables

Vector of observables

$$
= \begin{bmatrix} \Delta \log(C_t) \\ \Delta \log(I_t) \\ \Delta \log(N_{ct}) \\ \Delta \log(N_{it}) \\ \Delta \log(util_{ND,t}) \\ \Delta \log(util_{D,t}) \\ \Delta \log(p_{it}) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}
$$

Observation equations

- Match demeaned growth rates in model to those of data

- Nonstationary series

$$C_t^{obs} = \log C_t - \log C_{t-1} + g_t - \overline{g}$$
$$I_t^{obs} = \log I_t - \log I_{t-1} + g_t - \overline{g}$$

- Stationary series

$$N_{jt}^{obs} = \log N_{jt} - \log N_{j,t-1}, \quad j \in \{c, i\}$$
$$p_{i,t}^{obs} = \log p_{i,t} - \log p_{i,t-1}$$
$$util_{j,t}^{obs} = \log util_{j,t} - \log util_{j,t-1}, \quad j \in \{mc, i\}$$

Estimation procedure

- Estimate mode of posterior distribution by maximizing log posterior function (combines priors and likelihood)

- Use Metropolis-Hastings algorithm to sample posterior distribution and to evaluate marginal likelihood of the model

- Mode is used to initialize Markov chain

- Sample over 1 million draws (discard first 30%)

- Hessian defines transition probability that generates new proposed draw

- Check convergence and identification (trace plots)

On the use of growth rates for estimation

- Many macroeconomic series are difference-stationary

- Growth rates are a model-consistent way to render observables stationary

- Kalman filter uses same transformation on model and data

- Other filters (such as HP filter/Hamilton filter) extract specific frequencies of time series

- Latter may be reasonable for *description* depending on the notion of business cycle

FEVD: breakdown of search demand shocks

Table 9: Forecast error variance decomposition

	e_d	e_{di}
Y	97.23	2.77
SR	94.26	5.74
I	88.83	11.17
p_i	46.65	53.35
n_c	99.67	0.33
n_i	96.38	3.62
$util$	96.92	3.08
D	99.97	0.03
h	98.77	1.23

Table 9: Contribution of components to forecast error variance decomposition of search shocks.

FEVD: breakdown of technology shocks

Table 10: Forecast error variance decomposition

	e_g	e_Z	e_{zI}
Y	31.68	63.30	5.02
SR	48.24	43.87	7.90
I	3.25	74.14	22.62
p_i	0.14	43.91	55.95
n_c	22.23	75.51	2.26
n_i	6.20	61.70	32.10
$util$	0.64	83.26	16.10
D	10.20	76.28	13.52
h	1.34	89.29	9.36

Table 10: Contribution of components to forecast error variance decomposition of technology shocks.

BRS as special case

- Model nests BRS by shutting down additional frictions:
  - $ha = 0$
  - $\rho_c = 1$
  - $\nu^R = 0$
  - $\sigma_b \to \infty$
  - $\Psi_K = 0$
  - $\theta = 0$

- Absent fixed costs and variable capital utilization, $util_j = A_j D_j^\phi$ and $util = (C/Y)util_c + (I/Y)util_i$

Exercise: role of capacity utilization data in BRS special case

- Fix $\beta = 0.99, \sigma = 2.0$ and Frisch elasticity $\zeta = 0.72$
- Estimate model with same observables as BRS $(Y, I, Y/L, p_i)$ and also with capacity utilization
- Total shock processes $\{\theta_d, \theta_n, g, z, z_I\}$
- In contrast to BRS, estimate ϕ and η
- Otherwise use same prior distributions

Table 11: Prior distributions

Parameter	Distribution	Mean	Std
ϕ	Beta	0.32	0.20
η	Gamma	0.20	0.15
σ_{e_g}	Inv. Gamma	0.010	0.10
σ_x	Inv. Gamma	0.010	0.10
ρ_g	Beta	0.10	0.050
ρ_x	Beta	0.60	0.20

Table 12: Role of capacity utilization on parameter estimates

Parameter	BRS dataset		Add capacity utilization	
	Post. mean	90% HPD interval	Post. mean	90% HPD interval
ϕ	0.0978	[0.0001, 0.205]	0.883	[0.863, 0.906]
η	0.412	[0.282, 0.572]	1.87	[1.86, 1.90]
ρ_d	0.871	[0.775, 0.961]	0.928	[0.914, 0.941]
σ_d	0.0484	[0.0024, 0.0987]	0.0075	[0.0068, 0.0081]

Table 12: Estimation of baseline BRS model with two sets of observable series. The first considers growth rates of output, investment, labor productivity, and the relative price of investment. The second specification also considers total capacity utilization growth.

Comparison of volatility and variance decomposition

Table 13: Comparison of volatility and variance decomposition

Variable	BRS dataset	Add capacity utilization
Std. dev.		
D	1.54	1.69
$util$	0.15	1.49
FEVD of demand shocks θ_d		
Y	7.73	63.6
Y/N	2.49	27.0
SR	6.14	54.1

Table 13: The first sub-table documents standard deviations of shopping-related variables under two sets of observables. The BRS dataset includes growth rates of output, investment, labor productivity, and the relative price of investment. The second column adds variable total capacity utilization. The second sub-table shows the fraction of the variance decomposition attributable to the demand shock θ_d. See Table 12.

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