

$$\text{已知} \begin{cases} P(A) = P(B) = P(C) = \frac{1}{4} \\ P(AB) = 0 \\ P(AC) = P(BC) = \frac{1}{16} \end{cases}, \text{求} \begin{cases} 1. A, B, C \text{ 至少一个发生的概率} \\ 2. A, B, C \text{ 都不发生的概率} \end{cases}$$

我们先来算 $P(ABC)$, 因为下面会用到.

$$\text{因为 } ABC \subset AB, \text{ 所以 } P(ABC) \leq \underbrace{P(AB)}_{=0}, \text{ 因此 } P(ABC) = 0$$

→ 至少一个发生, 就是用“并 \cup ”, 用加法:

$$\begin{aligned} \text{所以 } P(A + B + C) &= \underbrace{P(A)}_{=\frac{1}{4}} + \underbrace{P(B)}_{=\frac{1}{4}} + \underbrace{P(C)}_{=\frac{1}{4}} - \underbrace{P(AB)}_{=0} - \underbrace{P(AC)}_{=\frac{1}{16}} - \underbrace{P(BC)}_{=\frac{1}{16}} + \underbrace{P(ABC)}_{=0} \\ &= \frac{3}{4} - \frac{2}{16} = \frac{5}{8} = 0.625 \end{aligned}$$

$$\rightarrow \text{都不发生, 即 } \bar{A} \cap \bar{B} \cap \bar{C} = 1 - \underbrace{P(A + B + C)}_{\text{任意一个发生}} = 1 - \frac{5}{8} = \frac{3}{8} = 0.375$$