

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ -1 & -1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 3 & 2 & 0 \end{array} \right]$$

先搞定第1列，做初等行变换

$$\text{newLine2} = (\text{line1}) \times 1 + \text{line2}$$

$$\text{newLine4} = (\text{line1}) \times (-1) + \text{line4}$$

$$= \left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{array} \right]$$

再搞定第2列，做初等行变换

$$\text{newLine1} = (\text{line2}) \times (-2) + \text{line1}$$

$$\text{newLine3} = (\text{line2}) \times (-1) + \text{line3}$$

$$\text{newLine4} = (\text{line2}) \times (-1) + \text{line4}$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

再搞定第3列的第1行，做初等列变换

$$\text{newCol3} = (\text{Col1}) \times (1) + \text{Col3}$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

再搞定第3列的第二行，做初等列变换

$$\text{newCol3} = (\text{Col2}) \times (-1) + \text{Col3}$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

←就得到了“标准形”