Доделки: ~~N9.21~~, ~~T2~~, ~~N4.51(1)~~, T5, T6.

N9.21 V параллелепипеда?

$$\begin{cases} a_1x+b_1y+c_1z=\pm d_1 \\ a_2x+b_2y+c_2z=\pm d_2 \\ a_3x+b_3y+c_3z=\pm d_3 \end{cases} \quad \text{замена: } \begin{cases} x'=a_1x+b_1y+c_1z \\ y'=a_2x+b_2y+c_2z \\ z'=a_3x+b_3y+c_3z \end{cases} \Rightarrow J^{-1}=\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}, \det J^{-1}\neq 0$$

$$\Rightarrow V=\int_{-d_1}^{d_1}dx'\int_{-d_2}^{d_2}dy'\int_{-d_3}^{d_3}dz'\cdot \det(J^{-1}) = \frac{8d_1d_2d_3}{\det(J^{-1})}$$

T2 a?: $\forall x\in U(x_0=0)\ \exists$ непр. дифф. $y(x)$: $e^y=1+ay+x$: $y(0)=0$.

$F(x,y)=e^y-1-ay-x$ непрерывна в окр. $(0,0)$: $F(0,0)=0$

$\frac{\partial F}{\partial y}=e^y-a$ непрерывна в окр. $(0,0)$

по Т.о неявной ф-ии при условии, что $\frac{\partial F}{\partial y}(0,0)\neq 0$

т.е. при $a\neq 1$

Ответ: $a\in\mathbb{R}\setminus\{1\}$

N4.51(1) $y=r\sin\varphi$, $x=r\cos\varphi$, $u(x,y)\longrightarrow u(r,\varphi)$ $\quad \frac{\partial^2u}{\partial x^2}+\frac{\partial^2u}{\partial y^2}=0$

$x=r\cos\varphi$, $y=r\sin\varphi$, $u(x,y)=\tilde{z}(r(x,y),r(x,y))$

$$u'_x=u'_r\cdot\underbrace{r'_x}_{\text{найдем из}}+u'_\varphi\cdot\underbrace{\varphi'_x}_{\text{замены}}=u'_r\cos\varphi-u'_\varphi\frac{\sin\varphi}{r},\quad u'_y=u'_r\cdot r'_y+u'_\varphi\cdot\varphi'_y=u'_r\sin\varphi+u'_\varphi\frac{\cos\varphi}{r}$$

при вычислении z'_x, r'_x, φ'_x y фикс.

$$\begin{cases} x'_x(r,\varphi):\ 1=r'_x\cos\varphi-r\sin\varphi\cdot\varphi'_x \quad (1) \\ y'_x(r,\varphi):\ 0=r'_x\sin\varphi+r\cos\varphi\cdot\varphi'_x \quad (2) \end{cases} \quad \begin{cases} (1)\sin\varphi-(2)\cos\varphi:\ \sin\varphi=-r\cdot\varphi'_x\Rightarrow\varphi'_x=-\frac{\sin\varphi}{r} \\ (1)\cos\varphi+(2)\sin\varphi:\ \cos\varphi=r'_x\Rightarrow r'_x=\cos\varphi \end{cases}$$

аналогично $\varphi'_y=\frac{\cos\varphi}{r}$, $r'_y=\sin\varphi$

— Ошибка заключалась в том, что я считал так: $\varphi'_x=\frac{1}{x'_\varphi}$

$$\frac{\partial^2u}{\partial x^2}=\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial r}\cdot\cos\varphi-\frac{\partial u}{\partial\varphi}\cdot\frac{\sin\varphi}{r}\right)=\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial r}\right)\cos\varphi-\frac{\partial u}{\partial r}\frac{\partial\varphi}{\partial x}\sin\varphi-\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial\varphi}\right)\frac{\sin\varphi}{r}-$$

$$-\frac{\partial u}{\partial\varphi}\,\frac{r\frac{\partial\varphi}{\partial x}\cos\varphi-\sin\varphi\frac{\partial r}{\partial x}}{r^2}=\frac{\partial^2u}{\partial r^2}\cos^2\varphi+\frac{\partial u}{\partial r}\frac{\sin^2\varphi}{r}+\frac{\partial^2u}{\partial\varphi^2}\cdot\frac{\sin^2\varphi}{r^2}+2\frac{\partial u}{\partial\varphi}\frac{\sin\varphi\cos\varphi}{r^2}$$

$$\frac{\partial^2u}{\partial y^2}=\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial r}\sin\varphi+\frac{\partial u}{\partial\varphi}\frac{\cos\varphi}{r}\right)=\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial r}\right)\sin\varphi+\frac{\partial u}{\partial r}\cdot\frac{\partial\varphi}{\partial y}\cdot\cos\varphi+\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial\varphi}\right)\cdot\frac{\cos\varphi}{r}+$$

$$+\frac{\partial u}{\partial\varphi}\cdot\frac{-\sin\varphi\frac{\partial\varphi}{\partial y}r-\frac{\partial r}{\partial y}\cos\varphi}{r^2}=\frac{\partial^2u}{\partial r^2}\cdot\sin^2\varphi+\frac{\partial u}{\partial r}\cdot\frac{\cos^2\varphi}{r}+\frac{\partial^2u}{\partial\varphi^2}\cdot\frac{\cos^2\varphi}{r^2}-2\frac{\partial u}{\partial\varphi}\frac{\sin\varphi\cos\varphi}{r^2}$$

$$\frac{\partial^2 u}{\partial r^2}\cos^2\varphi + \frac{\partial u}{\partial r}\frac{\sin^2\varphi}{r} + \frac{\partial^2 u}{\partial \varphi^2}\cdot\frac{\sin^2\varphi}{r^2} + \cancel{2\frac{\partial u}{\partial \varphi}\frac{\sin\varphi\cos\varphi}{r^2}} +$$

$$+\frac{\partial^2 u}{\partial r^2}\cdot\sin^2\varphi + \frac{\partial u}{\partial r}\cdot\frac{\cos^2\varphi}{r} + \frac{\partial^2 u}{\partial \varphi^2}\cdot\frac{\cos^2\varphi}{r^2} - \cancel{2\frac{\partial u}{\partial \varphi}\frac{\sin\varphi\cos\varphi}{r^2}} = 0$$

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \varphi^2} = 0$$