

Lab Assignment II

Numerical methods to solve PDEs

Assigned Date: Wednesday, September 4, 2019
Due on Monday, September 9, 2019

Goals: 1) Practice numerical methods to solve simple PDEs with finite difference equations;
2) Understand the linear computational stability.

Problem # 1

Integrate the linear wave equation using values typical of large-scale models. You can write your own MATLAB program or modify a sample program I offered to you (see the attachment).

$$\frac{\partial u}{\partial t} = -c \frac{\partial u}{\partial x}$$

Boundary conditions: periodic

Initial conditions:

$$u(x,0) = c + A \sin(kx)$$

$$c = 20ms^{-1}, A = 10ms^{-1}, \Delta x = 200km, k = 2\pi / L$$

$$with_L = 10\Delta x$$

(a) Use leapfrog scheme. Choose two time steps: one satisfies the CFL condition and the other violates it. 1) How long does it take to “blow up”; 2) Calculate the total kinetic energy of the solution u ($0.5 u^2$) at each time step; write a simple MATLAB (or IDL) program and draw a curve to show the variation of total kinetic energy as function of the number of time steps.

(b) Compare with the exact solution, computer the root-mean-square (RMS) error R and the relative error RE . Then,

Repeat with $A=25$ m/s

Repeat with $L = 4\Delta x$.

Prepare a table to summarize R and RE .

(c) (*For ATMOS 6500 students only*) Modify the programs; use the upstream scheme (instead of leapfrog scheme) and repeat (a)-(b).

Lab Report:

Based on your results to solve above problem, write a report (minimum 1 to 2 pages of text; 12pt Times New Roman; doubled space) to discuss the linear computational stability problem. You can attach figures to support your conclusion.

Please submit or hand in your lab report, figures and programs at beginning of the class on due day. I may also ask some of you (in ATMOS 6500) to send your program electronically after you submitted the report.

Policy:

Lab assignment should finish independently. Although group work and discussion are encouraged, *each student must write report by him/her own*. In addition, any report after due day is not acceptable (unless the permission is given by the instructor).