ATMOS 5500/6500, Fall 2019
Numerical Weather Prediction
Dr. Zhaoxia Pu

Homework/Lab Assignment IV
Numerical methods to solve PDEs

Assigned Date: Monday, October 14, 2019
Due on Monday, October 21, 2019

Goals: 1) Practice numerical methods to solve simple PDEs with finite difference equations;
2) Understand the linear and nonlinear computational stabilities.

Problem # 1 [Continue with Lab Assignment II]

Integrate the linear wave equation using values typical of large-scale models. You can write your own MATLAB program or modify a sample program I offered to you with the periodic boundary condition (see the attachment).

$$\frac{\partial u}{\partial t} = -c\frac{\partial u}{\partial x}$$

Boundary conditions: periodic
Initial conditions:

$$u(x,0) = c + A\sin(kx)$$

$$c = 20ms^{-1}, A = 10ms^{-1}, \Delta x = 200km, k = 2\pi / L$$

$$with\_ L = 10\Delta x$$

(a) Use leapfrog scheme. Choose two time steps: one satisfies the CFL condition and the other violates it. 1) How long does it take to "blow up"; 2) Calculate the total kinetic energy of the solution u ($0.5\ u^2$) at each time step; write a simple MATLAB program and draw a curve to show the variation of total kinetic energy as function of the number of time steps.

(b) Compare the results from (a) with 1) static boundary condition and 2) periodic boundary condition; discuss the influence of the boundary condition on numerical results

Problem #2

Modify the equation and program in problem #1 to integrate a nonlinear wave equation using values typical of large scale models:

$$\frac{\partial u}{\partial t} = -(u + c)\frac{\partial u}{\partial x}$$

Boundary conditions: periodic
Initial conditions:

$$u(x,0) = c + A\sin(kx)$$
$$c = 20ms^{-1}, A = 10ms^{-1}, \Delta x = 200km, k = 2\pi / L$$
$$with\_L = 10\Delta x$$

(a) Choose two time steps: one satisfies the CFL condition and the other violates it. 1) how long does it take to "blow up"? Compare with the linear equation results. 2). Calculate the total kinetic energy of the solution u (0.5 u^2) at each time step, write a simple MATLAB program and draw a curve to show the variation of total kinetic energy as function of the number of time steps.

(b) Repeat with A=25 m/s
Repeat with L = 4Dx .
Computer a nonlinear solution with high resolution, taking it as "truth", and then prepare a table to summarize R and RE.

(c) Repeat (a) with new initial conditions

$$u(x,0) = A\sin(kx)$$
$$c = 20ms^{-1}, A = 10ms^{-1}, \Delta x = 200km, k = 2\pi / L$$
$$with\_L = 10\Delta x$$

Lab Report:

Based your results to solve above problems, write a report (minimum 1 to 2 pages of text; 12pt Times New Roman; doubled space) to discuss the linear and non-linear computational stability problems. You can attach figures to support your conclusion.

Please submit or hand in your lab report, figures and programs at beginning of the class on due day. I will also ask some of you to send your program electronically after you submitted the report.

Policy:

Lab assignment should finish independently. Although group work and discussion are encouraged, *each student must write report by him/her own*. In addition, any report after due day is not acceptable (unless the permission is given by the instructor).

Optional problems:

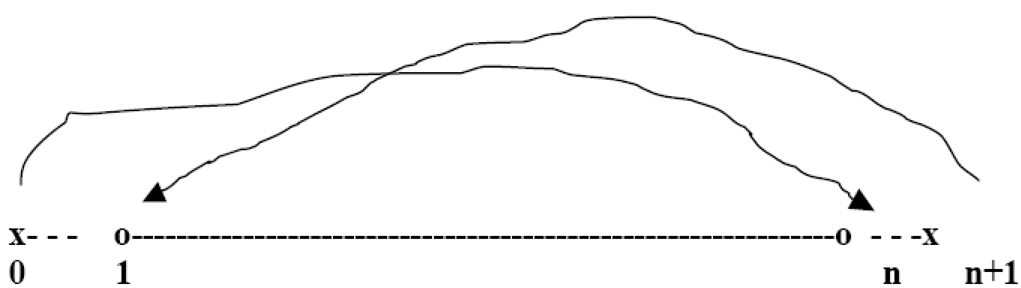
Repeat problem 2 using one or two additional time schemes listed in Table 3.2.1 on P.83;

Practice to control non-linear computation instability. Please write a separate report if you choose to do this.

Hint and an additional question:

How to implement a periodic boundary?

Periodic boundaries require that the value of the function be calculated as if the ends were tied together. The wave at one endpoint is calculated as if had the other endpoint as its nearest neighbor. This boundary condition is coded by using two “extra” grid points that have x coordinates just outside the medium. These extra points are updated to the values at the opposite boundary after every time step but they are not displayed on the screen.



u(0)=u(n)
u(1)=u(n+1)

Question: For any problem above, pick one experiment and change the periodic boundary condition into a static boundary condition; repeat your experiment again. How does this change impact on your results? Does boundary condition influence the numerical results of PDEs? Write a brief paragraph to discuss your results.