Introduction to Software Testing
(2nd edition)
Chapter 8

Logic Coverage

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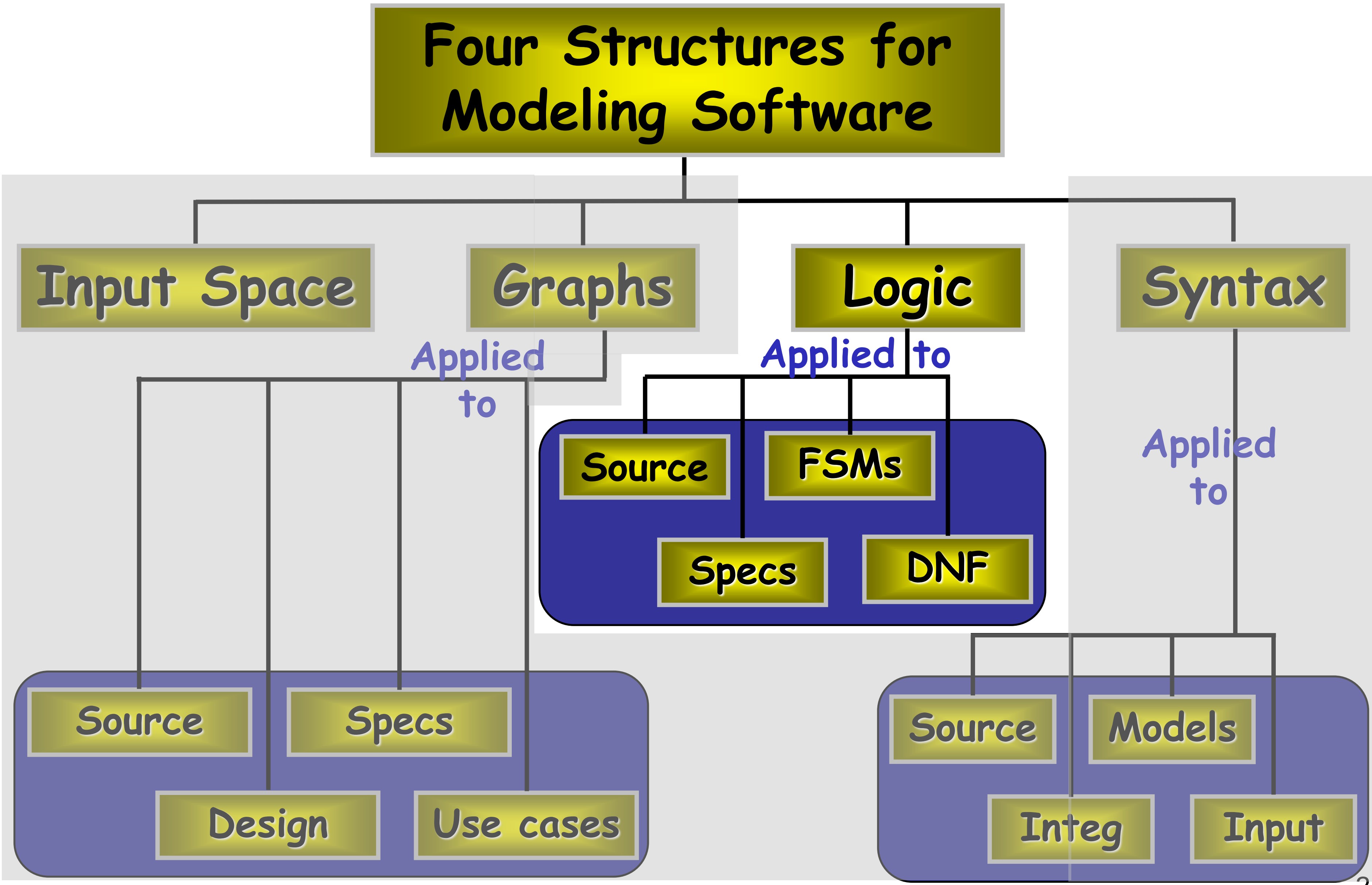
Ch. 8: Logic Coverage



Semantic Logic Criteria (8.1)

- Logic expressions show up in many situations
- Covering logic expressions is required by the US Federal Aviation Administration for **safety critical software**
- Logical expressions can come from many sources
  - Decisions in programs
  - FSMs and state charts
  - Requirements
- Tests are intended to choose some subset of the total number of truth assignments to the expressions

Logic Predicates and Clauses

- A *predicate* is an expression that evaluates to a boolean value
- Predicates can contain
  - boolean variables
  - non-boolean variables that contain >, <, ==, >=, <=, !=
  - boolean function calls
- Internal structure is created by logical operators
  - $\neg$: the *negation* operator
  - $\wedge$: the *and* operator
  - $\vee$: the *or* operator
  - $\rightarrow$: the *implication* operator
  - $\oplus$: the *exclusive or* operator
  - $\leftrightarrow$: the *equivalence* operator
- A ***clause*** is a predicate with no logical operators

Example and Facts

- Predicate `(a<b)` ∨ `f(z)` ∧ `D` ∧ `(m>=n*o)` has four **clauses**:
  1. (a < b) – relational expression
  2. f (z) – boolean-valued function
  3. D – boolean variable
  4. (m >= n*o) – relational expression
- Most predicates have few clauses **(Pareto principle)**
  - 88.5% have 1 clause
  - 9.5% have 2 clauses
  - 1.35% have 3 clauses
  - Only 0.65% have 4 or more!

from a study of 63 open source programs, > 400,000 predicates

- Sources of predicates
  - Decisions in programs
  - Guards in finite state machines
  - Decisions in UML activity graphs
  - Requirements, both formal and informal
  - SQL queries

Translating from English

- "I am interested in SWE 637 and CS 652"
- *course = swe637 OR course = cs652*

Humans have trouble translating from English to logic

- "If you leave before 6:30 AM, take Braddock to 495, if you leave after 7:00 AM, take Prosperity to 50, then 50 to 495"
- $(time < 6{:}30 \rightarrow path = Braddock) \wedge (time > 7{:}00 \rightarrow path = Prosperity)$
- Hmm … this is incomplete !
- $(time < 6{:}30 \rightarrow path = Braddock) \wedge (time >= 6{:}30 \rightarrow path = Prosperity)$

Logic Coverage Criteria (8.1.1)

- We use predicates in testing as follows:
  - Developing a model of the software as one or more predicates
  - Requiring tests to satisfy some combination of clauses

- Abbreviations:
  - P is the set of predicates
  - p is a single predicate in P
  - C is the set of clauses in P
  - C_p is the set of clauses in predicate p
  - c is a single clause in C

Predicate and Clause Coverage

- The first (and simplest) two criteria require that each predicate and each clause be evaluated to both true and false

<u>Predicate Coverage (PC)</u> : For each *p* in *P*, *TR* contains two requirements: *p* evaluates to true, and *p* evaluates to false.

- When predicates come from conditions on edges, this is equivalent to edge coverage
- PC does not evaluate all the clauses, so …

<u>Clause Coverage (CC)</u> : For each *c* in *C*, *TR* contains two requirements: *c* evaluates to true, and *c* evaluates to false.

Predicate Coverage Example

$((a < b) \lor D) \land (m >= n*o)$

predicate coverage

Predicate = true

a = 5, b = 10, D = true, m = 1, n = 1, o = 1

$= (5 < 10) \lor true \land (1 >= 1*1)$

$= true \lor true \land TRUE$

= true

Predicate = false

a = 10, b = 5, D = false, m = 1, n = 1, o = 1

$= (10 < 5) \lor false \land (1 >= 1*1)$

$= false \lor false \land TRUE$

= false

Clause Coverage Example

$((a < b) \lor D) \land (m >= n*o)$

Clause coverage

(a < b) = true	(a < b) = false	D = true	D = false
a = 5, b = 10	a = 10, b = 5	D = true	D = false

m >= n*o = true	m >= n*o = false
m = 1, n = 1, o = 1	m = 1, n = 2, o = 2

true cases

false cases

Two tests

1) a = 5, b = 10, D = true, m = 1, n = 1, o = 1

2) a = 10, b = 5, D = false, m = 1, n = 2, o = 2

Problems with PC and CC

- PC does not fully exercise all the clauses, especially in the presence of short circuit evaluation

- CC does not always ensure PC
  - That is, we can satisfy CC without causing the predicate to be both true and false
  - This is definitely <u>not</u> what we want!

- The simplest solution is to test all combinations …

Combinatorial Coverage

- CoC requires every possible combination
- Sometimes called **Multiple Condition Coverage**

Combinatorial Coverage (CoC) : For each p in P, TR has test requirements for the clauses in Cp to evaluate to each possible combination of truth values.

	a < b	D	m >= n*o	((a < b) ∨ D) ∧ (m >= n*o)
1	T	T	T	T
2	T	T	F	F
3	T	F	T	T
4	T	F	F	F
5	F	T	T	T
6	F	T	F	F
7	F	F	T	F
8	F	F	F	F

Combinatorial Coverage

- This is simple, neat, clean, and comprehensive …
- But quite expensive!
- 2^N tests, where N is the number of clauses
  - Impractical for predicates with more than 3 or 4 clauses
- The literature has lots of suggestions – some confusing
- The general idea is simple:

Test each clause independently from the other clauses

- Getting the details right is hard
- What exactly does "independently" mean ?
- The book presents this idea as "*making clauses active*" …

Active Clauses (8.1.2)

- Clause coverage has a weakness: The values do not always make a difference
- Consider the first test for clause coverage, which caused each clause to be true:
  - $(5 < 10) \vee true \wedge (1 >= 1*1)$
- Only the first clause *counts*!
- To really test the results of a clause, the clause should be the determining factor in the value of the predicate

Determination :	**A clause c_i in predicate *p*, called the *major clause*, *determines p* if and only if the values of the remaining *minor clauses* c_j are such that changing c_i changes the value of *p***

- This is considered to *make the clause active*

Determining Predicates

$P = A \vee B$

if B = *true*, p is always true.

so if B = *false*, A determines p.

if A = *false*, B determines p.

$P = A \wedge B$

if B = *false*, p is always false.

so if B = *true*, A determines p.

if A = *true*, B determines p.

- Goal: Find tests for each clause when the clause determines the value of the predicate
- This is formalized in a family of criteria that have subtle, but very important, differences

Active Clause Coverage

<u>Active Clause Coverage (ACC)</u> : For each *p* in *P* and each major clause c_i in *Cp*, choose minor clauses c_j, *j* != *i*, so that c_i determines *p*. TR has two requirements for each c_i: c_i evaluates to true and c_i evaluates to false.

<u>p = a $\vee$ b</u>

1) a = true, b = false
2) a = false, b = false

a is major clause

3) a = false, b = true
4) ~~a = false, b = false~~

b is major clause

Duplicate

- This is a form of MCDC, which is required by the FAA for safety critical software
- Ambiguity: Do the minor clauses have to have the same values when the major clause is true and false?

Resolving the Ambiguity

$p = a \vee (b \wedge c)$

Major clause: a

a = true, b = false, c = true

a = false, b = false, c = false

Is this allowed?

- This question caused confusion among testers for years
- Considering this carefully leads to three separate criteria:
  - Minor clauses do not need to be the same
  - Minor clauses do need to be the same
  - Minor clauses force the predicate to become both true and false

General Active Clause Coverage

<u>General Active Clause Coverage (GACC)</u>: For each *p* in *P* and each major clause c_i in *Cp*, choose minor clauses c_j, *j* != *i*, so that *ci* determines *p*. TR has two requirements for each *ci*: c_i evaluates to true and c_i evaluates to false. The values chosen for the minor clauses c_j do <u>not</u> need to be the same when c_i is true as when c_i is false, that is, $c_j(c_i = true) = c_j(c_i = false)$ for all c_j OR $c_j(c_i = true) \mathrel{!=} c_j(c_i = false)$ for all c_j.

- This is complicated!
- It is possible to satisfy GACC without satisfying predicate coverage.
- We really want to cause predicates to be both true and false.

Restricted Active Clause Coverage

<u>Restricted Active Clause Coverage (RACC)</u>: For each *p* in *P* and each major clause c_i in *Cp*, choose minor clauses c_j, j != i, so that c_i determines *p*. TR has two requirements for each c_i: c_i evaluates to true and c_i evaluates to false. The values chosen for the minor clauses *cj* <u>must be the same</u> when c_i is true as when c_i is false, that is, it is required that $c_j(c_i = true) = c_j(c_i = false)$ for all c_j.

- This has been a common interpretation by aviation developers
- RACC often leads to infeasible test requirements
- There is no logical reason for such a restriction

Correlated Active Clause Coverage

Correlated Active Clause Coverage (CACC): For each p in P and each major clause c_i in Cp, choose minor clauses c_j, j != i, so that c_i determines p. TR has two requirements for each c_i: c_i evaluates to true and c_i evaluates to false. The values chosen for the minor clauses c_j must cause p to be true for one value of the major clause c_i and false for the other, that is, it is required that $p(c_i = true)$!= $p(c_i = false)$.

- A more recent interpretation
- Implicitly allows minor clauses to have different values
- Explicitly satisfies (subsumes) predicate coverage

CACC and RACC

	a	b	c	$a \wedge (b \vee c)$
1	T	T	T	T
2	T	T	F	T
3	T	F	T	T
4	T	F	F	F
5	F	T	T	F
6	F	T	F	F
7	F	F	T	F
8	F	F	F	F

major clause

P_a : b=true or c = true

CACC can be satisfied by choosing any of rows 1, 2, 3 AND any of rows 5, 6, 7 – a total of *nine* pairs

	a	b	c	$a \wedge (b \vee c)$
1	T	T	T	T
2	T	T	F	T
3	T	F	T	T
4	T	F	F	F
5	F	T	T	F
6	F	T	F	F
7	F	F	T	F
8	F	F	F	F

RACC can only be satisfied by row pairs (1, 5), (2, 6), or (3, 7)

Only three pairs

Inactive Clause Coverage (8.1.3)

- The active clause coverage criteria ensure that "major" clauses do affect the predicates
- Inactive clause coverage takes the opposite approach – major clauses do not affect the predicates

<u>Inactive Clause Coverage (ICC)</u>: For each p in P and each major clause c_i in Cp, choose minor clauses c_j, j != i, so that c_i <u>does not</u> determine p. TR has <u>four</u> requirements for each c_i: (1) c_i evaluates to true with p true, (2) c_i evaluates to false with p true, (3) c_i evaluates to true with p false, and (4) c_i evaluates to false with p false.

General and Restricted ICC

- Unlike ACC, the notion of correlation is not relevant
  - – ci does not determine p, so cannot correlate with p
- Predicate coverage is always guaranteed

General Inactive Clause Coverage (GICC) : For each *p* in *P* and each major clause c_i in *Cp*, choose minor clauses c_j , *j != i*, so that c_i does not determine *p*. The values chosen for the minor clauses c_j do not need to be the same when c_i is true as when c_i is false, that is, $c_j(c_i = true) = c_j(c_i = false)$ for all c_j OR $c_j(c_i = true) \mathrel{!=} c_j(c_i = false)$ for all c_j.

Restricted Inactive Clause Coverage (RICC) : For each *p* in *P* and each major clause c_i in *Cp*, choose minor clauses c_j, *j != i*, so that c_i does not determine *p*. The values chosen for the minor clauses *cj* must be the same when c_i is true as when c_i is false, that is, it is required that $c_j(c_i = true) = c_j(c_i = false)$ for all c_j.

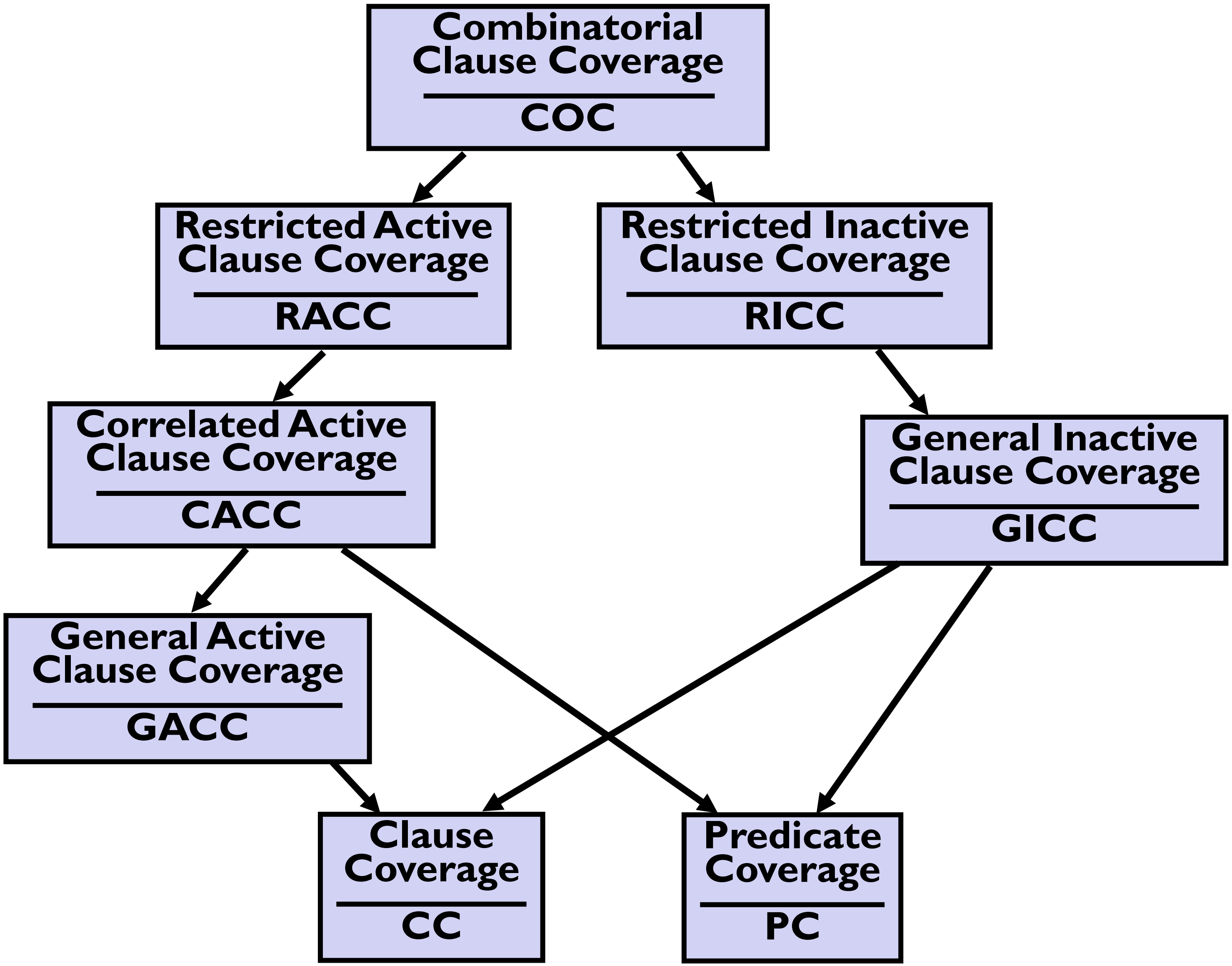
Infeasibility & Subsumption (8.1.4)

- Consider the predicate:

$$(a > b \wedge b > c) \vee c > a$$

- $(a > b) = true$, $(b > c) = true$, $(c > a) = true$ is infeasible
- As with graph-based criteria, infeasible test requirements have to be recognized and ignored
- Recognizing infeasible test requirements is hard, and in general, undecidable
- Software testing is inexact – engineering, not science

Logic Criteria Subsumption



Making Clauses Determine a Predicate

(8.1.5)

- Finding values for minor clauses c_j is easy for simple predicates
- But how to find values for more complicated predicates ?
- Definitional approach:
  - $p_{c=true}$ is predicate p with every occurrence of c replaced by *true*
  - $p_{c=false}$ is predicate p with every occurrence of c replaced by *false*
- To find values for the minor clauses, connect $p_{c=true}$ and $p_{c=false}$ with exclusive *OR*

$$p_c = p_{c=true} \oplus p_{c=false}$$

- After solving, p_c describes exactly the values needed for c to determine p

Examples

$p = a \vee b$

$$
\begin{aligned}
P_a &= P_{a=true} \oplus P_{a=false} \\
&= (true \vee b) \text{ XOR } (false \vee b) \\
&= true \text{ XOR } b \\
&= \neg b
\end{aligned}
$$

$p = a \wedge b$

$$
\begin{aligned}
P_a &= P_{a=true} \oplus P_{a=false} \\
&= (true \wedge b) \oplus (false \wedge b) \\
&= b \oplus false \\
&= b
\end{aligned}
$$

$p = a \vee (b \wedge c)$

$$
\begin{aligned}
P_a &= P_{a=true} \oplus P_{a=false} \\
&= (true \vee (b \wedge c)) \oplus (false \vee (b \wedge c)) \\
&= true \oplus (b \wedge c) \\
&= \neg (b \wedge c) \\
&= \neg b \vee \neg c
\end{aligned}
$$

- **"*NOT b* $\vee$ *NOT c*" means either *b* or *c* can be false**
- **RACC requires the same choice for both values of *a*, CACC does not**

XOR Identity Rules

Exclusive-OR (*xor*, $\oplus$) means both cannot be true

That is, A *xor* B means

"*A or B is true, but not both*"

$$p = A \oplus A \wedge b$$
$$= A \wedge \neg b$$

$$p = A \oplus A \vee b$$
$$= \neg A \wedge b$$

with fewer symbols …

p = A xor (A and b)
= A and !b

p = A xor (A or b)
= !A and b

Repeated Variables

- The definitions in this chapter yield the same tests no matter how the predicate is expressed

- $(a \vee b) \wedge (c \vee b) == (a \wedge c) \vee b$

- $(a \wedge b) \vee (b \wedge c) \vee (a \wedge c)$
  - Only has 8 possible tests, not 64

- Use the simplest form of the predicate, and ignore contradictory truth table assignments

A More Subtle Example

$$p = (a \land b) \lor (a \land \neg b)$$

$$\begin{aligned} p_a &= p_{a=true} \oplus p_{a=false} \\ &= ((true \land b) \lor (true \land \neg b)) \oplus ((false \land b) \lor (false \land \neg b)) \\ &= (b \lor \neg b) \oplus false \\ &= true \oplus false \\ &= true \end{aligned}$$

$$p = (a \land b) \lor (a \land \neg b)$$

$$\begin{aligned} p_b &= p_{b=true} \oplus p_{b=false} \\ &= ((a \land true) \lor (a \land \neg true)) \oplus ((a \land false) \lor (a \land \neg false)) \\ &= (a \lor false) \oplus (false \lor a) \\ &= a \oplus a \\ &= false \end{aligned}$$

- ***a*** always determines the value of this predicate.
- ***b*** never determines the value – *b* is irrelevant!

Tabular Method for Determination

- The math sometimes gets complicated
- A truth table can sometimes be simpler
- Example

	a	b	c	$a \wedge (b \vee c)$	p_a	p_b	p_c
1	T	T	T	T			
2	T	T	F	T			
3	T	F	T	T			
4	T	F	F	F			
5	F	T	T	F			
6	F	T	F	F			
7	F	F	T	F			
8	F	F	F	F			

Tabular Method for Determination

- The math sometimes gets complicated
- A truth table can sometimes be simpler
- Example

b* & *c* are the same, *a* differs, and *p* differs … thus TTT and FTT cause *a* to determine the value of *p

	a	b	c	$a \wedge (b \vee c)$	p_a	p_b	p_c
1	T	T	T	T			
2	T	T	F	T			
3	T	F	T	T			
4	T	F	F	F			
5	F	T	T	F			
6	F	T	F	F			
7	F	F	T	F			
8	F	F	F	F			

Tabular Method for Determination

- The math sometimes gets complicated
- A truth table can sometimes be simpler
- Example

***b* &**
diffe
to d

Again, *b* & *c* are the same, so TTF and FTF cause *a* to determine the value of *p*

	a	b	c	$a \wedge (b \vee c)$	p_a	p_b	p_c
1	T	T	T	T			
2	T	T	F	T			
3	T	F	T	T			
4	T	F	F	F			
5	F	T	T	F			
6	F	T	F	F			
7	F	F	T	F			
8	F	F	F	F			

Tabular Method for Determination

- The math sometimes gets complicated
- A truth table can sometimes be simpler
- Example

b & diffe to d

A ar va

Finally, this third pair, TFT and FFT, also cause *a* to determine the value of *p*

	a	b	c	$a \wedge (b \vee c)$	p_a	p_b	p_c
1	T	T	T	T			
2	T	T	F	T			
3	T	F	T	T			
4	T	F	F	F			
5	F	T	T	F			
6	F	T	F	F			
7	F	F	T	F			
8	F	F	F	F			

Tabular Method for Determination

- The math sometimes gets complicated
- A truth table can sometimes be simpler
- Example

b & diffe to d

A ar va

Fin als of *p*

For clause *b*, only one pair, TTF and TFF cause *b* to determine the value of *p*

	a	b	c	$a \wedge (b \vee c)$	p_a	p_b	p_c
1	T	T	T	T			
2	T	T	F	T			
3	T	F	T	T			
4	T	F	F	F			
5	F	T	T	F			
6	F	T	F	F			
7	F	F	T	F			
8	F	F	F	F			

Tabular Method for Determination

- The math sometimes gets complicated
- A truth table can sometimes be simpler
- Example

b **&**
diffe
to d

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Likewise, for clause *c*, only one pair, TFT and TFF, cause *c* to determine the value of *p*

	a	b	c	$a \wedge (b \vee c)$	p_a	p_b	p_c
1	T	T	T	T			
2	T	T	F	T			
3	T	F	T	T			
4	T	F	F	F			
5	F	T	T	F			
6	F	T	F	F			
7	F	F	T	F			
8	F	F	F	F			

Tabular Method for Determination

- The math sometimes gets complicated
- A truth table can sometimes be simpler
- Example

b & diffe to d

A ar va

Fi als of

For and valu

Likewise, for clause *c*, only one pair, TFT and TFF, cause *c* to determine the value of *p*

	a	b	c	$a \wedge (b \vee c)$	p_a	p_b	p_c
1	T	T	T	T			
2	T	T	F	T			
3	T	F	T	T			
4	T	F	F	F			
5	F	T	T	F			
6	F	T	F	F			
7	F	F	T	F			
8	F	F	F	F			

In sum, three separate pairs of rows can cause *a* to determine the value of *p*, and only one pair each for *b* and *c*

Logic Coverage Summary

- Predicates are often very simple—in practice, most have less than 3 clauses.
  - In fact, most predicates only have one clause!
  - With only clause, **PC** is enough
  - With 2 or 3 clauses, **CoC** is practical
  - Advantages of **ACC** and **ICC** criteria significant for large predicates
    - **CoC** is impractical for predicates with many clauses
- **Control software** such as Proportional–integral–derivative controller (PID), often has many complicated predicates, with lots of clauses.