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Effects of diversification among assets in an agent-based market model

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ABSTRACT

We extend to the multi-asset case the framework of a discrete time model of a single asset financial market developed in Ghoulmié et al.[1] In particular, we focus on adaptive agents with threshold behavior allocating their resources among two assets. We explore numerically the effect of this diversification as an additional source of complexity in the financial market and we discuss its destabilizing role. We also point out the relevance of these studies for financial decision making.

Keywords: Agent-based model, complex systems, financial markets, stylized facts, multi-asset market model, diversification, stability, business and management

1. INTRODUCTION

In the agent-based approach, financial markets are modelled as systems of interacting agents and several examples of such models have been successful in reproducing the stylized facts that are common to a wide variety of markets, instruments and periods.[2] By finding economic explanations for the statistical signatures of these market fluctuations, agent-based models can inspire investors and regulators to conceive tools and policies for improved financial decisions and financial risk management. In this literature, herd behavior,[3,4] social interaction and mimicry,[3,5,6] heterogeneity,[7–9] investor inertia[1,3] and switching between "chartist" and "fundamentalist" behavior[10–14] have been invoked as possible mechanisms. The mechanism involved in the single asset financial market model introduced in Ghoulmié et al,[1] adds clarity on how the stylized facts are generated, and in particular how the volatility clustering phenomenon is linked to economic decision making as commented in Cont.[15] In this model, one type of agent interacts indirectly via the price, and heterogeneity appears endogenously due to an asynchronous updating scheme. The structure of the model allows one to trace back the behavior of the price to agents' decision-making rule based on a threshold behavior. Threshold response in the behavior of market participants can be seen either as resulting from trade friction in order to reduce transactions costs or, more generally, from the risk aversion of agents which leads them to be inactive if uncertain about their action. This agent-based model generically leads to an absence of autocorrelation in the returns, mean-reverting stochastic volatility, excess volatility, volatility clustering, and endogenous bursts of market activity that is not attributable to external noise. However, the model does not generate heavy-tails in the distribution of returns or the long memory effect in the volatility, and this can in itself motivate a further study by adding complexity to the model in order to get these facts. Diversification among multiple assets, following for example Markovitz's mean-variance portfolio optimization,[16] is crucial for investment decisions and may be the key ingredient to getting all the facts. Moreover, these stylized facts are so constraining that it is even surprising to be able to reproduce them in a convincing way with a single-asset financial market model. The study of the multi-asset case is also the road to understand the non trivial correlations between assets,[17,18] an area of research that is receiving recently an intense interest with the use of random matrix theory,[19,20] complex networks,[21,22] and multi-scaling.[23–26] Correlations among returns of different assets are highly unstable and this is a major challenge for portfolio optimization, for the pricing of multi-asset derivatives and

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for co-integration based trading strategies. Our approach aims also at bringing insights on the direct impact of trading strategies on speculative markets dynamics compared to the importance given to microstructure effects, e.g in the zero-intelligence agents model studied in Daniels et al[27] and to the importance given to the topology of interactions between agents in Cont and Bouchaud[3] and Iori.[5] We thus extend the agent-based model to the multi-asset case and, in order to obtain transparent pictures, we focus on agents diversifying their strategies among two assets.

The article is structured as follows. Section 2 summarizes the single asset model and discusses its main properties. We describe the multi-asset market model in section 3. We discuss the numerical effects of the diversification in section 4. Conclusions are drawn in the last section.

2. PROPERTIES OF THE SINGLE ASSET MARKET MODEL

The model describes a market where a single asset is traded by n agents. Trading takes place at discrete time steps t. Provided the parameters of the model are chosen in a certain range, these periods may be interpreted as "trading days". At each time period, every agent receives public news about the asset's performance. If the news is judged to be significant the agent places for a unit of asset a buy or sell order, depending on whether the news received is pessimistic or optimistic. Prices then move up or down according to excess demand. The model produces stochastic heterogeneity and sustains it through the updating of agents' strategies. Let us recall in a mathematical way the ingredients of the single asset model. At each time period:

• All the agents receive a common signal ϵ_t generated by a Gaussian distribution with 0 mean and standard deviation D, namely $N(0, D^2)$.
• Each agent i compares the signal to its threshold $\theta_i(t)$.
• If $|\epsilon_t| > \theta_i(t)$ the agent considers the signal as significant and generates an order $\phi_i(t)$ according to

$$\phi_i(t) = 1_{\epsilon_t > \theta_i} - 1_{\epsilon_t < -\theta_i}, \tag{1}$$

where $\phi_i(t) > 0$ is a buy order, $\phi_i(t) < 0$ is a sell order and $\phi_i(t) = 0$ is an order to remain inactive.
• The market price p_t is affected by the excess demand and moves according to

$$r_t = \ln\left(\frac{p_t}{p_{t-1}}\right) = g\left(\frac{\sum_i \phi_i(t)}{n}\right), \tag{2}$$

where r_t are the returns at time t and g is the price impact function.
• Each agent updates, with probability s, her threshold to $|r_t|$.

The evolution of the thresholds distribution is described with the following master equation:

$$f_{t+1}(\theta) = (1-s)\ f_t(\theta) + s\ \delta_{|r_t|,\theta}, \tag{3}$$

with $|r_t| = |g(\mathrm{sign}(\epsilon_t)\mathrm{F_t}(|\epsilon_t|))|$, F_t being the cumulative distribution of the thresholds.
The solution of Eq. 3 can be derived analytically and reads as

$$f_t(\theta) = (1-s)^t\ f_0 + s\sum_{j=1}^{t}(1-s)^{j-1}\delta_{|r_{t-j}|,\theta}. \tag{4}$$

Moreover, numerical tests confirm the validity of the former solution. Stationary solutions are the limiting cases: without feedback $s = 0$, and without heterogeneity $s = 1$. We specify now the range of parameters that leads to realistic price behaviors. First of all, we want a large number of agents in order to guarantee heterogeneity in the market. Indeed, when the number of agents is lowered, the distribution of returns becomes multi-modal with 3 local maxima, one at zero, one positive maximum and a negative one. This can be interpreted as a disequilibrium regime: the market moves either one way or the other. The updating frequency s should be chosen small, $s << 1$, in order to guarantee heterogeneity. When the amplitude of the noise is small, $D << g(1/n)$, the absolute value of the returns evolves through a series of periods characterized by "jumps" whose amplitudes decay exponentially in time. The sensitivity of the thresholds increases when the noise level increases and the behavior of the returns is closer to the Gaussian signal. On the other hand, when the amplitude of the news is too high, $D >> g(1)$, the returns distribution has two peaks: the maximum at $g(1)$ and the minimum at $g(-1)$. We thus want the following condition in order to get realistic returns dynamics:

$$g(1/n) << D << g(1). \tag{5}$$

If we consider now the linear price impact function, $g(x) = x/\lambda$, with λ the market depth characterizing how much the market moves when filling one unit of asset, the above condition leads to the parameter reduction $D_{\text{eff}} = D\lambda$ and $1/n << D_{\text{eff}} << 1$. We then get clusters of volatility of length $1/s$ consistent with the correlation structure suggested by the stationary solution. This slow feedback mechanism generates endogenous heterogeneity, excess volatility, volatility clustering and transforms Gaussian news into semi heavy-tailed price returns. When a majority of agents have a low value for their threshold a large price fluctuation becomes very probable. Because only a small fraction of agents increases its threshold response when a large fluctuation occurs the probability of also getting a large fluctuation at the next time step remains high. In other words, the slow feedback mechanism causes persistence in the fluctuations.

3. DESCRIPTION OF THE MULTI-ASSET MARKET MODEL

We now extend the previous model to a two-asset case that can in fact be directly generalized to a higher number of assets. The model describes a market where two assets, with prices denoted by $p_{1,t}$ and $p_{2,t}$ respectively, are traded by n agents at discrete time steps t. At each time step, the model is updated according to the following steps:

• Every agent receives a common signal $\epsilon_t \in N(0, D^2)$ for both assets.
• Each agent i compares the signal to her threshold for the first, $\theta_{1,i}(t)$, and for the second, $\theta_{2,i}(t)$, asset and then places orders of respectively $\omega_{1,i}(t)$ and $\omega_{2,i}(t)$ units of assets. The weights $\omega_{1,i}(t)$ and $\omega_{2,i}(t)$ are defined positive and bounded by the following constrains:

$$\omega_{1,i}(t) + \omega_{2,i}(t) = 1. \tag{6}$$

• If $|\epsilon_t| > \theta_{1,i}(t)$ the agent considers the signal as significant and generates for the first asset an order $\omega_{1,i}\phi_{1,i}(t)$ with

$$\phi_{1,i}(t) = 1_{\epsilon_t > \theta_{1,i}} - 1_{\epsilon_t < -\theta_{1,i}}. \tag{7}$$

• If $|\epsilon_t| > \theta_{2,i}(t)$ the agent considers the signal as significant and generates for the second asset an order $\omega_{2,i}\phi_{2,i}(t)$ with

$$\phi_{2,i}(t) = 1_{\epsilon_t > \theta_{2,i}} - 1_{\epsilon_t < -\theta_{2,i}}. \tag{8}$$

• The asset prices $p_{1,t}$ and $p_{2,t}$ are affected by the excess demand and move according to

$$r_{1,t} = \ln\left(\frac{p_{1,t}}{p_{1,t-1}}\right) = g\left(\frac{\sum_i \omega_{1,i}\phi_{1,i}(t)}{n}\right), \tag{9}$$

$$r_{2,t} = \ln\left(\frac{p_{2,t}}{p_{2,t-1}}\right) = g\left(\frac{\sum_i \omega_{2,i}\phi_{2,i}(t)}{n}\right), \tag{10}$$

where $r_{1,t}$ and $r_{2,t}$ are the returns and g is the price impact function.
• Each agent updates, with probability s, its threshold $\theta_{1,i}(t)$ to $|r_{1,t}|$ and $\theta_{2,i}(t)$ to $|r_{2,t}|$.

• Each agent updates, with probability s, the weights according to

$$\omega_{1,i}(t) = \frac{1}{1 + e^{\beta . V_t}}, \tag{11}$$

$$\omega_{2,i}(t) = 1 - \omega_{1,i}(t), \tag{12}$$

where the difference between assets absolute returns $V_t = |r_{2,t}| - |r_{1,t}|$ is the fitness function that determines the adaptive asset allocation and β is the parameter that controls the intensity of choice between the two assets. The asset allocation follows a discrete choice model that is popular among economists under the label *logit* choice model.[28] It has been used in various models of social choices[29] and it is an example of the intense and fertile exchange between physics and social sciences as explained and reviewed in Nadal et al.[30] The parameter β determines the rationality of agent's choice. For $\beta = 0$, agents keep the same equal weights on both assets independently of the fitness function, and the behavior of the returns is equivalent to the single asset market studied in the first section. For an infinite value of β, agents are fully rational and chose radically the most profitable asset, the asset with the highest absolute return.

4. NUMERICAL EFFECTS OF THE DIVERSIFICATION

4.1. Nonstationarity of assets returns

In this section we explore numerically the effects of the diversification on the complex behavior of asset returns. In particular, by setting the other parameters in ranges similar to those required to get realistic dynamics in the single asset case, we study the role of the parameter β that characterizes the asset allocation. The price impact function has been chosen to be linear, $g(x) = x/\lambda$, with the same market depth λ for both assets. The behavior of the returns generated by the present model is significantly different from the single asset case. In particular, the returns, reported in Fig. 1 for $n = 1500$, $s = 0.015$, $D = 0.001$, $\lambda = 2$ and $\beta = 1000$, produce fluctuations that are larger than those of the the single asset model. This behavior is emphasized by the fatter tails observed in the probability distribution function and reported in Fig. 2 where a Gaussian distribution is also plotted for visual comparison.

Fig. 3 illustrates that additional differences from the single asset model are observed in the behavior of the annualized volatilities. It is seen that the volatilities of the two-asset model switch intermittently, in a sort of mean reverting behavior, from high to low values in an anticorrelated fashion. These sudden changes in the volatility are observed in financial data and in order to capture this feature, extensions of GARCH models with regime-switching type of behavior have been proposed such as in Bauwens et al.[31] Compared to the single asset model, this new result brings us closer to the empirical facts observed in financial data. Moreover, these higher fluctuations occur without structural changes in the market or as a result of changes in the fundamentals and are simply related to the diversification process: the agents having very similar trading strategies allocate intermittently more weight on one of the asset and, therefore, leading to higher fluctuations in the traded volume.

We also perform an analysis of summary statistics for 5 consecutive periods of 10000 time steps for the time series generated by the multi-asset model and the single-asset model. These summary statistics are reported in Tabs. 1, 2 and 3 respectively. In the two-asset case we observe fluctuations of the kurtosis between 3 and 12, indicating nonstationarity of asset returns. In the single asset case, instead, this parameter is almost stationary. This result implies that the multi asset diversification of the proposed model promotes non-stationarity, an effective feature that often causes problems to policymakers. We must remark on the destabilizing role of this diversification in comparison with the benchmark single asset market case. This is an interesting case where diversification, which is usually seen as a way to reduce portfolio risk, increases market instability.

4.2. Nonlinear diagnostics

In the present section we estimate the autocorrelation function for the returns of the two assets as well as their absolute value. The results, reported in Fig. 4, show an absence of correlation in the returns time series (bottom). For the absolute value (top), the nonlinear diagnostics point out two different phases: the first, for

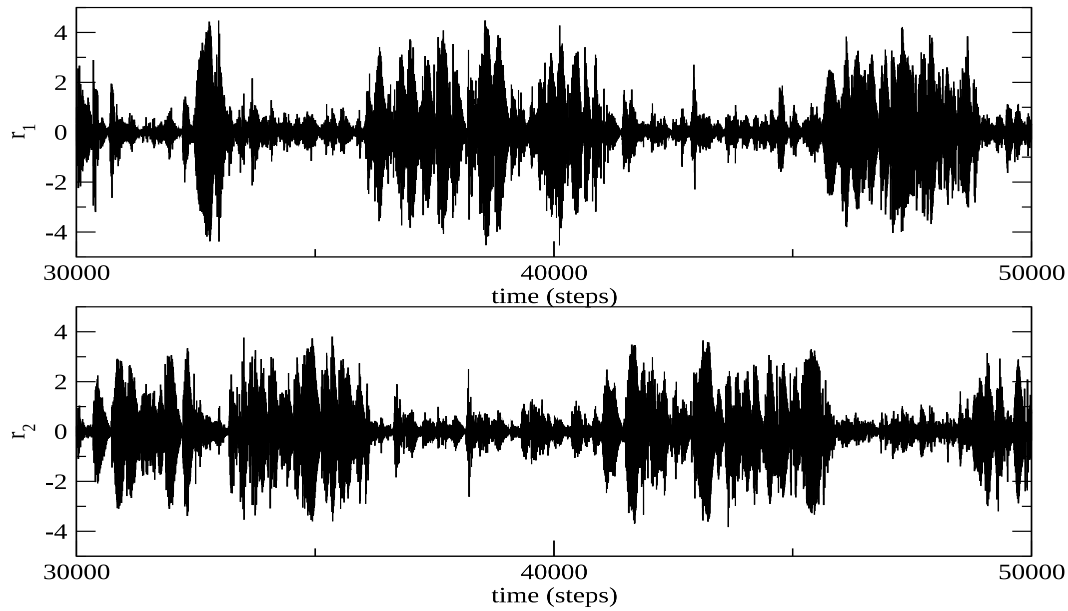

Figure 1. Time series plots of asset1 and asset2 returns, r_1 (top) and r_2 (bottom), generated numerically by the agent-based market model for $n = 1500$, $D = 0.001$, $\lambda = 2$, $s = 0.015$ and $\beta = 1000$. The time series exhibit regime-switching type of behaviors between a high and low volatility periods.

Period	**mean**(10^{-4})	**std**(10^{-2})	**skew**(10^{-2})	**kurtosis**	**max**(10^{-2})	**min**(10^{-2})
1	1.1	1.3	24	11.1	10.19	-9.64
2	7.3	2.3	7.5	7.6	12	-12.53
3	0.6	4	-1.9	4	13.44	-13.38
4	3.9	3.1	1.9	6.5	12.87	-12.94
5	-2.4	2.8	0.5	5.7	12.29	-12.89

Table 1. Summary statistics for the returns of the first asset in the two-asset model. The estimation is obtained over 10000 observations.

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Period	**mean**(10^{-4})	**std**(10^{-2})	**skew**(10^{-2})	**kurtosis**	**max**(10^{-2})	**min**(10^{-2})
1	2.2	4.3	8.3	3.5	13.37	-13.63
2	6.8	3.6	4	4.3	12.85	-12.89
3	0.6	1.8	-7	11.6	12.71	-12.33
4	3.8	3.1	2.5	5.7	12.61	-11.89
5	-0.1	3.2	3	5.6	12.1	12.63

Table 2. Summary statistics for the returns of the second asset in the two-asset model. The estimation is obtained over 10000 observations.

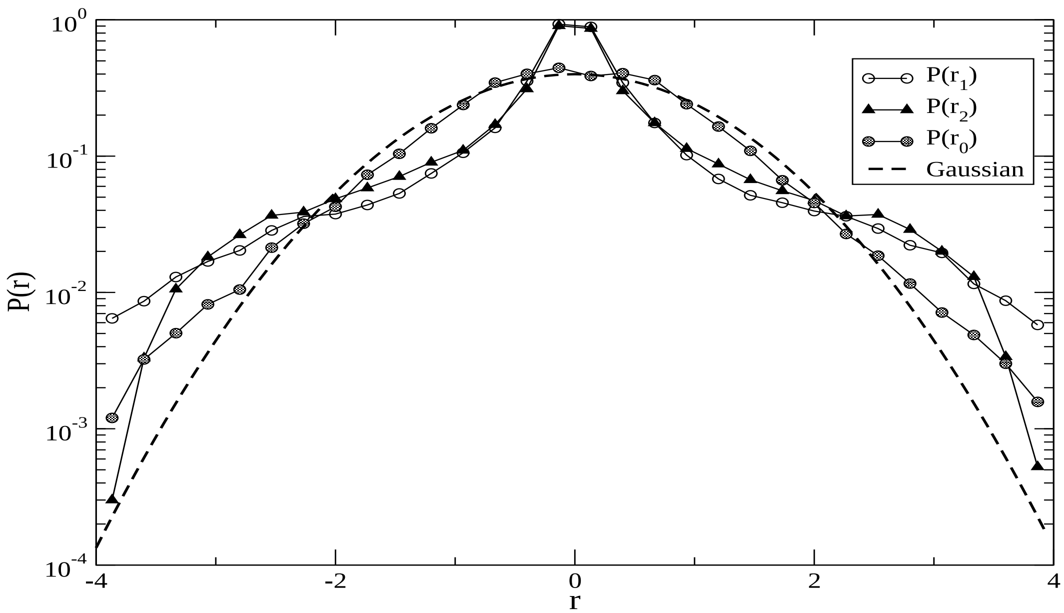

Figure 2. Empirical densities plots $P(r_1)$ and $P(r_2)$ for asset1 and asset2 time series returns, generated numerically by the agent-based market model for $n = 1500$, $D = 0.001$, $\lambda = 2$, $s = 0.015$ and $\beta = 1000$. These time series exhibit fatter tails than the Gaussian distribution. The Gaussian distribution case and the empirical density $P(r)$ for the returns generated by the single-asset model for $n = 1500$, $D = 0.001$, $\lambda = 10$ and $s = 0.015$ are also plotted for comparison.

Period	**mean**(10^{-4})	**std**(10^{-2})	**skew**(10^{-2})	**kurtosis**	**max**(10^{-2})	**min**(10^{-2})
1	0.05	1.05	4.3	4.3	6.7	-5.3
2	0.5	1.04	4.2	4.2	5.13	-5.4
3	0.03	1.05	3.6	3.6	4.4	-5.2
4	2.7	1.05	3	4.7	5.3	-5.6
5	0.4	1.01	-5	4	5.3	-6.7

Table 3. Summary statistics for the returns in the single asset model. The estimation is obtained over 10000 observations.

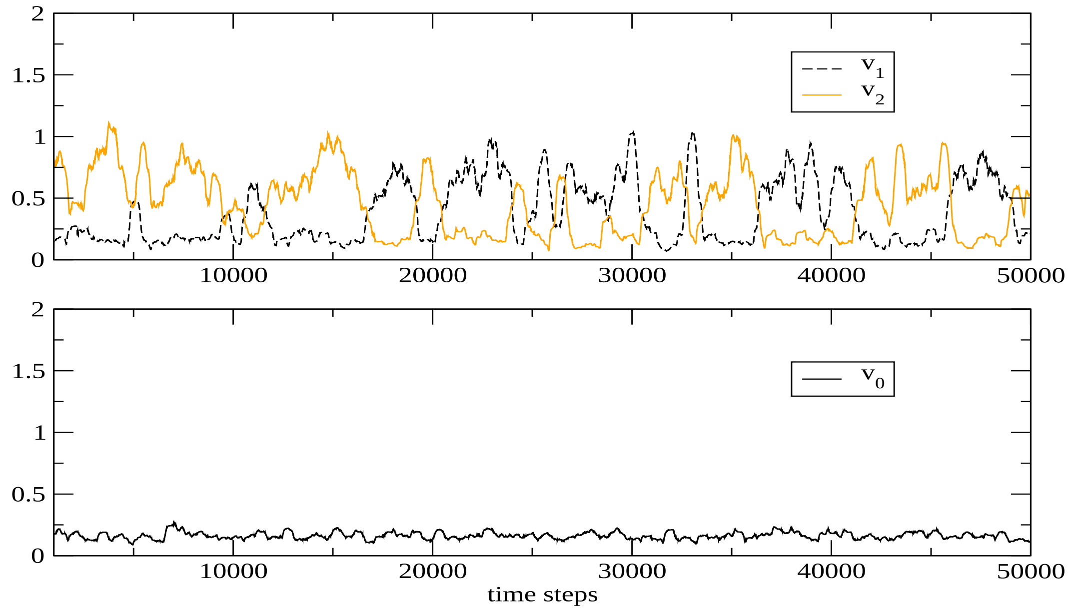

Figure 3. Top: Annualized volatilities v_1 and v_2 for the time series returns of the two assets generated numerically by the agent-based market model with $n = 1500$, $D = 0.001$, $\lambda = 2$, $s = 0.015$ and $\beta = 1000$. Bottom: Annualized volatility v_0 for the returns generated by the single-asset model for $n = 1500$, $D = 0.001$, $\lambda = 10$ and $s = 0.015$. The volatilities are calculated as the standard deviation of the returns on a moving window of 500 time steps multiplied by the annualization factor $\sqrt{250}$.

small lags, is related to the presence of volatility clustering, a feature noticed also in the single asset model, the second persistent phase has not previously been observed. The first phase, analogously to that seen in the one asset model, corresponds to the clusters of volatility of length $1/s$ and can be described by an exponential decay with characteristic time scale of 70 periods. At large lags we instead observe a slower decay while the autocorrelations remain significantly positive over a long period. We thus conclude that the fluctuations persist in the multi-asset case over a longer period than in the single asset case, Fig. 4. This behavior is definitely closer to that observed in financial time series than is the behavior exhibited by the single asset model.

4.3. Robustness and implications of the results

A parametric analysis of the two-asset model confirms the robustness of the results presented in the previous two sections: this is a desirable property for practitioners when building market models. Varying the updating frequency, s, affects the initial exponential decay of the autocorrelation function of the volatilities. The characteristic time of this decay is, indeed, roughly proportional to $1/s$. When increasing significantly the number of agents, n, we have to decrease the parameter D_{eff} in order to get an effective impact of the asset allocation on market moves and reproduce the results reported in the previous section. When increasing the intensity of choice, β, the diversification is more effective and the results are more pronounced: for $\beta = 10000$, the fluctuations in the kurtosis of the returns is higher as are the fluctuations in V_t. The autocorrelation function of the instantaneous volatility, instead, remains unchanged against variations of β for both assets, see Fig. 5. The limit case with infinite β exhibits two distinct behaviors for the volatilities of the two assets as a result of the radical choice: returns similar to the single asset case for the most often chosen asset and returns with lower amplitude but with higher variations and persistence in the volatility for the other. The autocorrelation

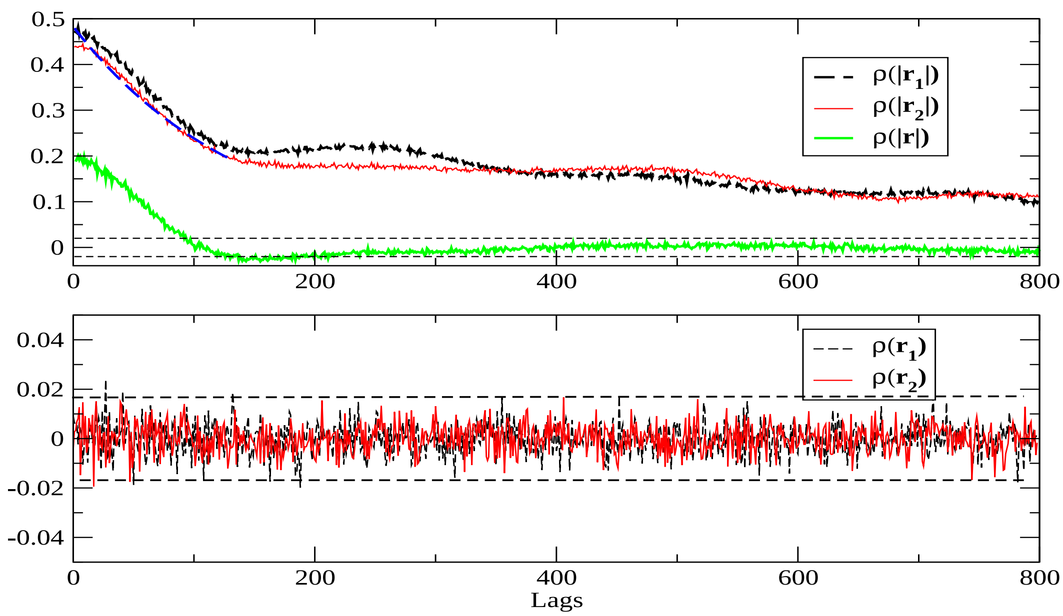

Figure 4. Top: Autocorrelation functions $\rho(|r_1|)$ and $\rho(|r_2|)$ of $|r_1|$ and $|r_2|$ generated numerically by the agent-based market model for $n = 1500$, $D = 0.001$, $\lambda = 2$, $s = 0.015$ and $\beta = 1000$. The volatility clustering phenomenon is reproduced: these functions remain significantly positive over a long period with an initial phase similar to the single asset case. An exponential fit (dashed line) of the initial decay has been plotted. We also plotted for comparison the autocorrelation function $\rho(|r|)$ for the absolute values of the returns generated by the single-asset model with $n = 1500$, $D = 0.001$, $\lambda = 10$ and $s = 0.015$. In the two-asset case, we observe at large lags a slower decay and the autocorrelations remain significantly positive. The persistence in the amplitude of the returns in the two-asset case is definitely closer to that observed in financial data than is the persistence generated in the one-asset case. Bottom: Corresponding autocorrelation functions $\rho(r_1)$ and $\rho(r_2)$ of r_1 and r_2. The multi-asset model generates an absence of autocorrelation in the returns.

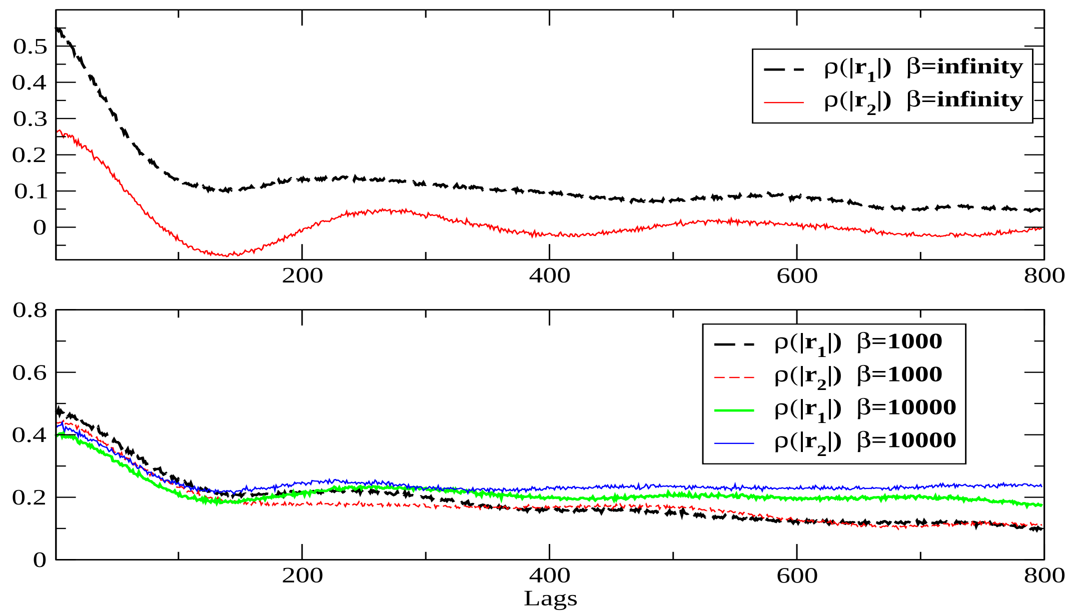

Figure 5. Top: Autocorrelation functions $\rho(|r_1|)$ and $\rho(|r_2|)$ of $|r_1|$ and $|r_2|$, absolute values of asset1 and asset2 time series returns generated numerically by the agent-based market model for $n = 1500$, $D = 0.001$, $\lambda = 2$, $s = 0.015$ and an infinite value of β. This limit case, which is the radical choice case, exhibit two distinct behaviors for the volatilities of the two assets returns. Bottom: Autocorrelation functions $\rho(|r_1|)$ and $\rho(|r_2|)$ of $|r_1|$ and $|r_2|$, absolute values of the two assets time series returns generated numerically by the agent-based market model for $n = 1500$, $D = 0.001$, $\lambda = 2$, $s = 0.015$, $\beta = 1000$ and $\beta = 10000$. When varying the intensity of choice, the model is robust in generating higher persistence in the fluctuations compared to the single asset case.

function for this particular case is reported in Fig. 5 where it is evident that the volatility behaves similarly to the single asset case which is plotted alongside.

By using a comprehensive and dynamic approach in the asset allocation decision this agent-based model also offers a plausible mechanism for generating correlations among returns of different assets. The model is successful in reproducing an observed phenomenon in speculative markets where traders crowd or flock from one volatile security to another. Moreover, we computed the correlation coefficient between assets returns for 5 consecutive periods of 10000 time steps generated by the multi-asset model. This correlation parameter fluctuates between 0.3 and 0.7 and we conclude that the correlations are unstable. This instability emerges endogenously as a result of trading strategies and not from the input signal that can be interpreted as the fundamentals of these assets. We note the relationship between volatilities and correlations. The correlation parameter is indeed positively correlated to the difference between volatilities which determines the asset allocation choice and its impact on market movements. The agent-based approach can thus lead to a better appreciation of the relationships among many asset classes, and this should improve the financial decision-making process.

5. DISCUSSIONS AND CONCLUSIONS

In the present work we have extended to the multi-asset case a parsimonious single asset agent-based market model capable of reproducing the main empirical stylized facts observed in the returns of financial assets. The model is based on three main ingredients:

- Threshold behavior of agents.
- Heterogeneity of agent strategies, generated endogenously through a feedback effect of recent price.
- Diversification among assets following a discrete choice model.

Compared to the single asset case, numerical simulations of the model produce higher variations of assets returns volatilities and an autocorrelation function of absolute returns that remains positive over a longer period. Moreover, the model mimics a regime switching type in assets dynamics along with an endogenous instability in assets correlations as the result of trading strategies and without any structural change. These results question some conclusions previously drawn from simulations of agent-based models regarding the origins of stylized properties of asset returns, e.g the long memory effect, and also question the importance given to microstructure effects and assets fundamentals in previous studies. Taking into account diversification among several assets, a key feature of financial decision making, when designing agent-based market models may be critical in generating all the stylized facts. The model is also successful at reproducing the crowd dynamics between volatile assets observed on trading floors. Traders do indeed flock from one volatile security to another one, a phenomenon that has not been extensively explored in agent-based modelling. These cross-market dynamics are not only the road to fully understand the instability of assets correlations, but may also explain the persistence in the returns volatilities and the heavy-tailed distribution of the returns. We will thus explore this path and other cross-market dynamics in further studies. For practitioners, the potential direct benefits from understanding these fluctuations are improved estimates of the quantities used for trading strategies, portfolio optimization, derivatives pricing and risk management. By shedding light on the origins of market movements, the agent-based approach can inspire new strategies and policies for traders and regulators. It is also to sophisticated investors a first class guide for understanding the ecology of financial markets.

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