Indian Institute of Technology
Madras

DESIGN OF REAR WHEEL TRANSMISSION

ME3201

DESIGN OF MACHINE ELEMENTS

Group 18

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Chapter 1

Introduction

1.1 Background of the Project

In automotive engineering, the rear wheel transmission system plays a crucial role in transferring power from the engine to the rear wheels efficiently and reliably. This system typically consists of a propeller shaft, a differential unit, and axle shafts. Together, these components ensure the effective transmission of torque while accommodating variations in wheel speed and load distribution during vehicle motion.

The design of the rear wheel transmission system requires careful consideration of various parameters such as torque transmission capacity, material selection, fatigue strength, and overall structural reliability. Each element—whether it is the propeller shaft transmitting power from the gearbox to the differential, or the axle shafts delivering torque to the wheels—must be optimized for both performance and durability.

1.2 Motivation

The motivation behind this project lies in understanding and applying the theoretical concepts of machine design to a practical engineering system. The rear wheel transmission system provides an excellent case study to analyze multiple mechanical elements working in unison under complex loading conditions. Designing each component from first principles allows for a deeper comprehension of material behavior, endurance limits, and failure criteria such as fatigue and yielding.

Chapter 2

Force Analysis

2.1 Introduction

In the rear wheel transmission system, the **propeller shaft** and **axle shafts** are subjected to a combination of tangential, radial, and axial forces during torque transmission. The propeller shaft transmits power from the gearbox to the differential and experiences a *tangential driving force* (F_{tp}) responsible for torque transmission, a *radial force* (F_{rp}) directed towards the gear center due to the pressure angle of the gear teeth, and an *axial or thrust force* (F_{ap}) acting along the shaft axis because of bevel gear engagement. These forces are balanced by the *bearing reactions* (R_{B1x}, R_{B1y}, R_{B1z}), ensuring static equilibrium.

In the **axle shafts**, which deliver torque from the differential to the wheels, similar forces act. The *axial force* (F_{al} or F_{ar}) is transmitted from the bevel gears, the *tangential or traction force* (F_{tl} or F_{tr}) produces the driving torque at the wheels, and the *radial force* (F_{rl} or F_{rr}) acts perpendicular to the shaft due to wheel load distribution. The *bearing reactions* (R_{B2} and R_{B3}) support these loads, while the *normal reactions at the wheels* (N_L and N_R) balance the vertical forces from the ground. Together, these forces maintain both linear and torque equilibrium in the left and right axle assemblies, ensuring smooth and efficient power transmission to the wheels.

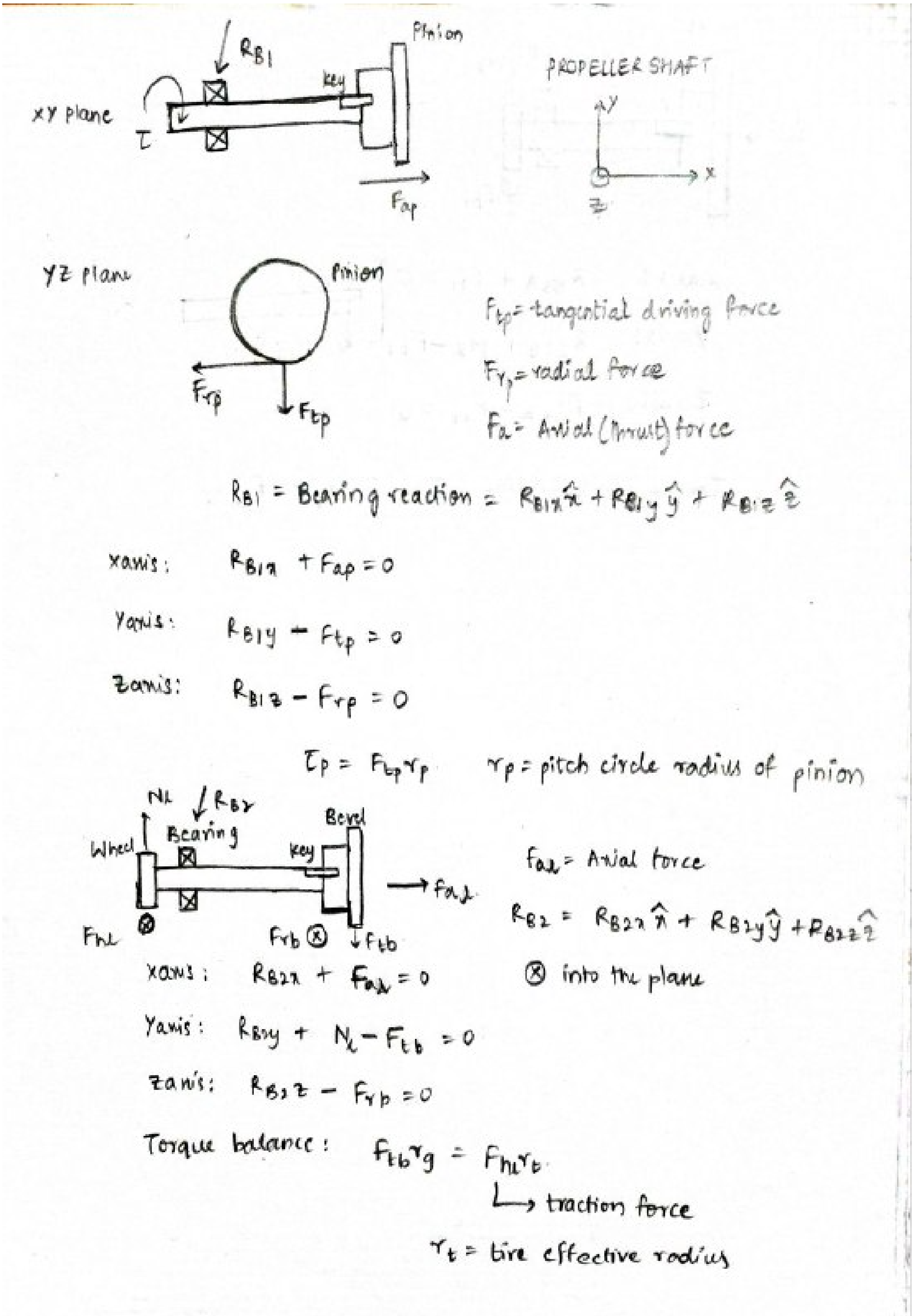


Figure 2.1: Force analysis of the propeller and the left axle shaft

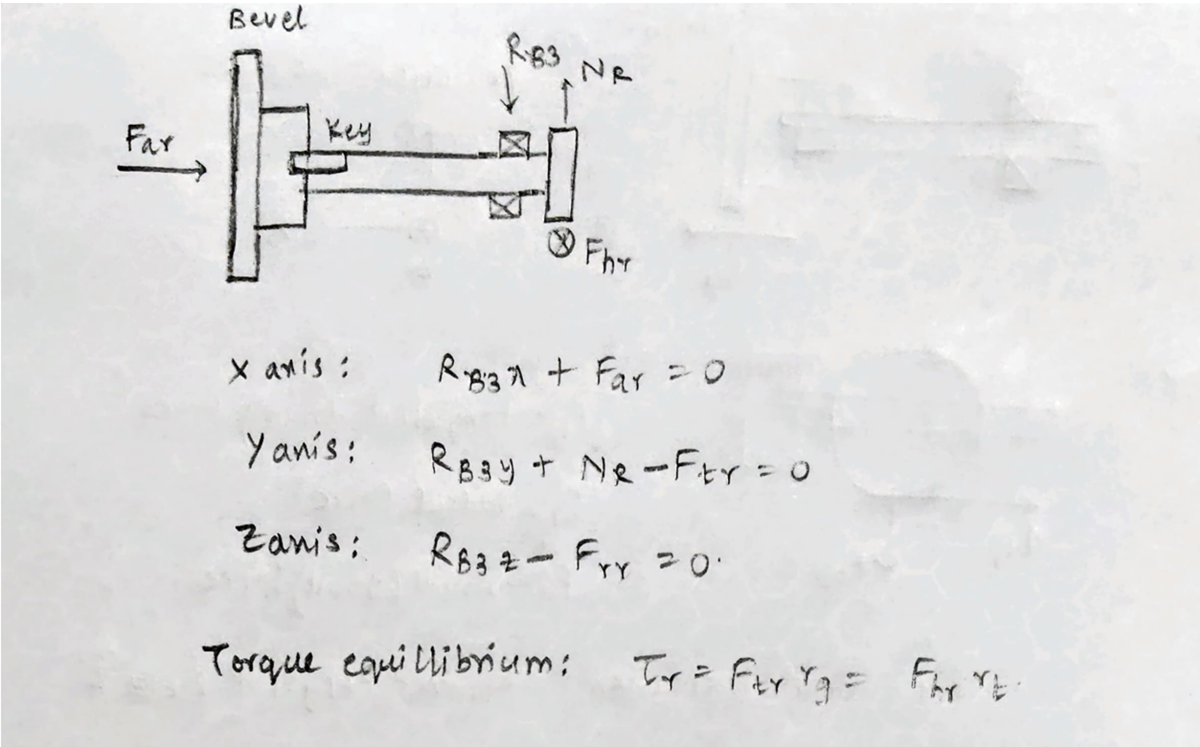


Figure 2.2: Force analysis of the right axle shaft

Chapter 3

Design of Components

Input Data

The input power and speed at the engine crankshaft are given as:

$$P = 182 \text{ bhp} = 135.717 \text{ kW}, \quad N = 3500 \text{ rpm} = 366.52 \text{ rad/s}$$

From this, the engine torque is calculated:

$$T_{\text{eng}} = \frac{P}{N} = \frac{135717 \text{ W}}{366.52 \text{ rad/s}} = 370.28 \text{ Nm}$$

This analysis considers the maximum torque condition, which occurs in the lowest gear.

- Gearbox first gear ratio: $i_g = 4.1$
- Final drive ratio: $i_f = 2.433$

The propeller shaft transmits the torque from the gearbox:

$$T_{\text{prop}} = T_{\text{eng}} \times i_g = 370.28 \times 4.1 = 1518.15 \text{ Nm}$$

The torque at each axle is split in two by the differential and multiplied by the final drive ratio:

$$T_{\text{axle}} = \frac{T_{\text{prop}} \times i_f}{2} = \frac{1518.15 \times 2.433}{2} = 1846.93 \text{ Nm}$$

3.1 Propeller Shaft

Assumptions

- Material: AISI 1040 HR Steel, $S_{ut} = 520$ MPa, $S_y = 290$ MPa

Reliability factor k_e

Reliability factor = $k_e = 1 - 0.08\, z_a$

Table 6–5

Reliability Factors k_e Corresponding to 8 Percent Standard Deviation of the Endurance Limit

Reliability, %	Transformation Variate z_a	Reliability Factor k_e
50	0	1.000
90	1.288	0.897
95	1.645	0.868
99	2.326	0.814
99.9	3.091	0.753
99.99	3.719	0.702
99.999	4.265	0.659
99.9999	4.753	0.620

Figure 3.1: Reliability Factor

- Hollow shaft: $d_i = 0.7d_o$
- Length = 1.5 m
- Factor of Safety, $n_f = 2$

The loading is fluctuating from zero to maximum.

$$T_{\max} = 1518.15 \text{ Nm}, \quad T_{\min} = 0$$

$$T_a = T_m = \frac{T_{\max} - T_{\min}}{2} = 759.075 \text{ Nm}$$

Assuming reliability = 99%, $k_e = 0.868$

Shear Endurance Limit

$$S_e' = 0.5S_{ut} = 260 \text{ MPa}$$

Surface factor (hot-rolled):

$$k_a = 57.7S_{ut}^{-0.718} = 57.7(520)^{-0.718} = 0.647$$

Size factor (for 51mm $< d_o \leq$ 256mm):

$$k_b = 1.51d_o^{-0.157}$$

Approximate endurance limit

$$S'_e = \begin{cases} 0.5S_{ut} & S_{ut} \le 200 \text{ kpsi (1400 MPa)} \\ 100 \text{ kpsi} & S_{ut} > 200 \text{ kpsi} \\ 700 \text{ MPa} & S_{ut} > 1400 \text{ MPa} \end{cases}$$

Two separate notations are used for endurance limit, viz, (S'_e) and (S_e) where,

S'_e = endurance limit stress of a rotating beam specimen subjected to reversed bending stress (N/mm^2)

S_e = endurance limit stress of a particular mechanical component subjected to reversed bending stress (N/mm^2)

Size factor k_b

The size factor has been evaluated using 133 sets of data points.[15] The results for bending and torsion may be expressed as

$$k_b = \begin{cases} (d/0.3)^{-0.107} = 0.879d^{-0.107} & 0.11 \le d \le 2 \text{ in} \\ 0.91d^{-0.157} & 2 < d \le 10 \text{ in} \\ (d/7.62)^{-0.107} = 1.24d^{-0.107} & 2.79 \le d \le 51 \text{ mm} \\ 1.51d^{-0.157} & 51 < d \le 254 \text{ mm} \end{cases} \quad (6\text{–}20)$$

For axial loading there is no size effect, so

$$k_b = 1 \quad (6\text{–}21)$$

Loading factor (torsion):

$$k_c = 0.59$$

$$k_d = 1, \quad S_e = k_a k_b k_c k_d k_e S'_e$$

$$S_e = (0.647)(1.51d_o^{-0.157})(0.59)(1)(0.868)(260)$$

$$S_e = 132.89d_o^{-0.157}$$

This is the bending endurance limit. The shear endurance limit is:

$$S_{se} = 0.577S_e = 76.68d_o^{-0.157}$$

Diameter Calculation

Assuming $K_{fs} = 2.2$ for keys.

$$J = \frac{\pi}{32}(d_o^4 - d_i^4) = \frac{\pi}{32}(d_o^4 - (0.7d_o)^4) = 0.0746d_o^4$$

$$\tau_a = \tau_m = K_{fs}\frac{T_a(d_o/2)}{J}$$

Using Goodman criterion (in Pa):

$$\frac{\tau_a}{S_{se}} + \frac{\tau_m}{S_{su}} = \frac{1}{n_f}$$

where $S_{su} = 0.67S_{ut} = 0.67(520) = 348.4$ MPa.

Using python code to calculate the diameter of the propeller shaft, the final diameter comes appx. 86.67 mm

Note: The propeller shaft is more likely limited by critical speed (whirling) which would require a much larger, stiffer diameter.

3.2 Axle Shaft (with Keyway)

Assumptions

- Material: AISI 1040 HR Steel
- $S_{ut} = 520$ MPa, $S_y = 290$ MPa
- $T_{\text{axle}} = 1846.93$ Nm
- $T_a = T_m = 923.465$ Nm
- Gross weight = 2500 kg, per wheel load = 625 kg
- Factor of Safety, $n_f = 2$

Vertical and lateral forces due to weight and cornering:

$$F_{\text{vertical}} = 625 \times 9.81 = 6131.25 \text{ N}$$

$$F_{\text{lateral}} \approx 625g = 6131.25 \text{ N}$$

Assuming moment arm $L = 0.8$ m (from hub to bearing):

$$M_{\max} = L\sqrt{F_{\text{lat}}^2 + F_{\text{vert}}^2}$$

The bending is completely reversed (0 to max and back).

$$M_a = M_{\max}, \quad M_m = 0$$

Endurance Limit

$$S'_e = 0.5 S_{ut} = 260 \text{ MPa}$$

Assuming machined finish:

$$k_a = 4.51 S_{ut}^{-0.265} = 4.51(520)^{-0.265} = 0.833$$

Size factor (for $d > 51$mm):

k$_b = 1.51 d^{-0.157}$

$$k_c = 1 \text{ (for bending)}, \quad k_d = 1, \quad k_e = 0.868 \text{ (99\% Rel.)}$$

$$S_e = (0.833)(1.51 d^{-0.157})(1)(1)(0.868)(260)$$

Stress Concentrations (End-mill Keyseat)

$$r/d = 0.02 \Rightarrow K_t = 2.14, \; K_{ts} = 3$$

$$q = 0.85, \quad q_{\text{shear}} = 0.9 \text{ (Assumed from charts)}$$

$$K_f = 1 + q(K_t - 1) = 1.969, \quad K_{fs} = 1 + q_{\text{shear}}(K_{ts} - 1) = 2.8$$

$$\sigma_a = K_f \frac{M_a c}{I}$$

$$\tau_a = \tau_m = K_{fs} \frac{T_m c}{J}$$

$$\sigma'_a = \sqrt{\sigma_a^2 + 3\tau_a^2}$$

$$\sigma'_m = \sqrt{3\tau_m^2}$$

Using ASME criterion with $n_f = 2$:

$$\left(\frac{n_f \sigma'_a}{S_e}\right)^2 + \left(\frac{n_f \sigma'_m}{S_y}\right)^2 = 1$$

Using python code to calculate diameter. **Final Axle Diameter $d = 83.63$ mm**.

Chapter 4

Key Design Calculations

Standard Dimensions and Formulae For a rectangular sunk key transmitting torque T through a shaft of diameter d:

$$b = \frac{d}{4}, \qquad h = \frac{d}{4}, \qquad l = 1.5d \tag{4.1}$$

The transmitted tangential force on the key at the shaft–hub interface is:

$$F = \frac{2T}{d} \tag{4.2}$$

The key is checked for:

$$\text{Shear stress:} \quad \tau = \frac{F}{b\,l} \tag{4.3}$$

$$\text{Crushing (bearing) stress:} \quad \sigma_c = \frac{2F}{h\,l} \tag{4.4}$$

4.1 Propeller Shaft Key

$$d = d_o = 83.63 \text{ mm}, \quad T = 1518.15 \text{ Nm} = 1.51815 \times 10^6 \text{ Nmm}$$

$$\begin{aligned} b &= \frac{83.63}{4} = 20.91 \text{ mm}, \\ h &= \frac{83.63}{4} = 20.91 \text{ mm}, \\ l &= 1.5 \times 83.63 = 125.45 \text{ mm} \end{aligned}$$

Force on key

$$F = \frac{2T}{d} = \frac{2 \times 1.51815 \times 10^6}{83.63} = 36{,}304.9 \text{ N}$$

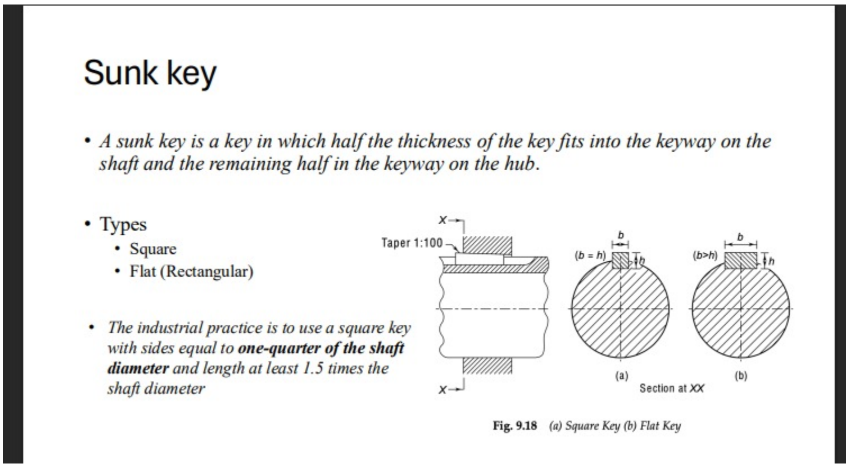


Figure 4.1: This is the caption for my image.

Stresses in key

$$\tau = \frac{F}{b\,l} = \frac{36{,}304.9}{20.91 \times 125.45} = 13.85 \text{ MPa}$$

$$\sigma_c = \frac{2F}{h\,l} = \frac{2 \times 36{,}304.9}{20.91 \times 125.45} = 27.6 \text{ MPa}$$

Result: $\tau = 13.85$ MPa, $\sigma_c = 55.4$ MPa

For a standard key material (e.g., C45 steel):

$$\tau_{\text{allow}} = 40 \text{ MPa}, \quad \sigma_{c,\text{allow}} = 80 \text{ MPa}$$

Hence, the design is **safe** in both shear and crushing.

4.2 Axle Shaft Key

$$d = 86.67 \text{ mm}, \quad T = 1846.93 \text{ Nm} = 1.84693 \times 10^6 \text{ Nmm}$$

$$b = \frac{86.67}{4} = 21.67 \text{ mm},$$
$$h = \frac{86.67}{4} = 21.67 \text{ mm},$$
$$l = 1.5 \times 86.67 = 130.01 \text{ mm}$$

Force on key

$$F = \frac{2T}{d} = \frac{2 \times 1.84693 \times 10^6}{86.67} = 42{,}612.5 \text{ N}$$

Stresses in key

$$\tau = \frac{F}{b\,l} = \frac{42{,}612.5}{21.67 \times 130.01} = 15.13 \text{ MPa}$$

$$\sigma_c = \frac{2F}{h\,l} = \frac{2 \times 42{,}612.5}{21.67 \times 130.01} = 30.25 \text{ MPa}$$

Result: $\tau = 15.13$ MPa, $\sigma_c = 30.25$ MPa

For the same key steel (C45):

$$\tau_{\text{allow}} = 40 \text{ MPa}, \quad \sigma_{c,\text{allow}} = 80 \text{ MPa}$$

Hence, the axle-shaft key is also **safe** in both shear and crushing.

Chapter 5

Bearing Design

5.1 Deep Groove Ball Bearing Calculations

5.1.1 Given Data

- Pressure angle: $\phi = 20°$
- Cone angles: $\delta_p = 22.363°$ (pinion), $\delta_r = 67.637°$ (ring)
- Torques: $T_{\text{prop}} = 1518.15$ Nm, $T_{\text{axle}} = 1846.93$ Nm
- Ring pitch radius: $r_{\text{ring}} = 62.937$ mm
- Pinion pitch radius: $r_{\text{pin}} = 25.9035$ mm
- Shaft geometry:
  - Axle: solid, $d_{\text{axle}} = 86.67$ mm
  - Propeller: hollow, $d_o = 83.63$ mm, $d_i = 58.54$ mm
- Life target: $L_{10} = 100 \times 10^6$ revs
- Bearing life exponent for ball bearings: $p = 3$
- Life multiplier: $(L_{10}/10^6)^{1/p} = (100)^{1/3} = 4.6416$
- When $F_a/F_r > e$, use conservative factors $X = 0.56$, $Y = 1.6$ (*SKF General Catalogue, ISO 281*).

5.1.2 1. Axle Bearings (Two Bearings, Gear Centered)

Gear Forces

$$F_t = \frac{T_{\text{axle}}}{r_{\text{ring}}} = \frac{1846.93}{0.062937} = 29{,}345.69 \text{ N}.$$

$$F_r = F_t \tan\phi \cos\delta_r = 29{,}345.69 \times \tan 20^\circ \times \cos 67.637^\circ = 4{,}063.82 \text{ N}.$$

$$F_a = F_t \tan\phi \sin\delta_r = 29{,}345.69 \times \tan 20^\circ \times \sin 67.637^\circ = 9{,}877.67 \text{ N}.$$

Each of the two bearings shares the load:

$$F_{r,\text{per}} = \frac{F_r}{2} = 2{,}031.91 \text{ N}, \qquad F_{a,\text{per}} = \frac{F_a}{2} = 4{,}938.84 \text{ N}.$$

Equivalent Dynamic Load

Since $\frac{F_a}{F_r} = 2.43 > e$, use $X = 0.56,\ Y = 1.6,\ V = 1.0.$

$$P_{\text{axle,per}} = XVF_{r,\text{per}} + YF_{a,\text{per}} = 0.56(2031.91) + 1.6(4938.84) = 9{,}040.01 \text{ N}.$$

Required Dynamic Load Rating

$$C_{\text{req,axle,per}} = P_{\text{axle,per}}(100)^{1/3} = 9{,}040.01 \times 4.6416 = 41{,}960 \text{ N} \approx \mathbf{41.96\ kN}.$$

Bearing Selection and Safety Check

- Selected Bearing: **SKF 6317** (Deep Groove Ball Bearing, 85 mm bore)
- Catalogue Dynamic Capacity: $C = 96$ kN
- Safety Factor: FOS $= \frac{C_{\text{actual}}}{C_{\text{req}}} = \frac{96}{41.96} = 2.29$

Result: Bearing selection satisfies load capacity with FOS > 2, hence safe.

5.1.3 2. Ring Gear–Shaft Bearing (Single, Both Races Rotating)

$$F_r = 4{,}063.82 \text{ N}, \quad F_a = 9{,}877.67 \text{ N}.$$

$$V = 1.2, \quad X = 0.56, \quad Y = 1.6.$$

$$P_{\text{ring}} = 0.56(1.2 \times 4063.82) + 1.6(9877.67) = 18{,}080 \text{ N}.$$

$$C_{\text{req,ring}} = 18{,}080 \times 4.6416 = 86{,}033 \text{ N} \approx \mathbf{86.03\ kN}.$$

Bearing Selection and Safety Check

- Selected Bearing: **SKF 6413** (Deep Groove Ball Bearing, 65 mm bore)
- Catalogue Dynamic Capacity: $C = 96.2$ kN
- Safety Factor: FOS $= \frac{C_{\text{actual}}}{C_{\text{req}}} = \frac{96.2}{86.03} = 1.12$

Result: Bearing satisfies load capacity with FOS = 1.12 (acceptable). If continuous high axial loads occur, we can use a paired angular-contact or tapered roller bearing (e.g., SKF 33217).

5.1.4 3. Propeller Shaft (Pinion End, Single Bearing)

$$F_t = \frac{T_{\text{prop}}}{r_{\text{pin}}} = \frac{1518.15}{0.0259035} = 58{,}607.91 \text{ N}.$$

$$F_r = F_t \tan\phi \cos\delta_p = 58{,}607.91 \times \tan 20^\circ \times \cos 22.363^\circ = 19{,}727.23 \text{ N}.$$

$$F_a = F_t \tan\phi \sin\delta_p = 58{,}607.91 \times \tan 20^\circ \times \sin 22.363^\circ = 8{,}116.08 \text{ N}.$$

$$P_{\text{pinion}} = 0.56(19{,}727.23) + 1.6(8{,}116.08) = 24{,}033 \text{ N}.$$

$$C_{\text{req,pinion}} = 24{,}033 \times 4.6416 = 111{,}551 \text{ N} \approx \mathbf{111.55\ kN}.$$

Bearing Selection and Safety Check

- Selected Bearing: **SKF 6414** (Deep Groove Ball Bearing, 70 mm bore)
- Catalogue Dynamic Capacity: $C = 116$ kN
- Safety Factor: FOS $= \frac{C_{\text{actual}}}{C_{\text{req}}} = \frac{116}{111.55} = 1.04$

Result: Bearing meets minimum load capacity (FOS = 1.04). For greater reliability under combined axial–radial loading, a tapered roller bearing or double-row angular contact bearing can also be used.

Chapter 6

Bevel Gear Design

6.1 Introduction

Bevel gears are used in the differential to transmit power between the propeller shaft and the axle shafts at a 90-degree angle. The design of bevel gears involves determining the appropriate gear ratio, tooth geometry, and material selection to ensure efficient torque transmission and durability under operating conditions.

6.2 Tire and Wheel Specifications

6.2.1 Assumed Tire Size and Rim Diameter

- Tire size: 235/65 R17
- Rim diameter: R17 = 17 inch = 431.8 mm
- Tire width: 235 mm
- Sidewall height: $0.65 \times 235 = 152.75$ mm

6.2.2 Total Wheel Diameter Calculation

The total wheel diameter includes the rim diameter and twice the sidewall height:

$$D_{total} = (\text{Sidewall Height}) \times 2 + \text{Rim Diameter}$$

$$D_{total} = 152.75 \times 2 + 431.8 = 737.3 \text{ mm} = 0.7373 \text{ m}$$

6.2.3 Wheel Circumference

The circumference of the wheel is:

$$C = \pi D_{total} = \pi \times 0.7373 = 2.315 \text{ m}$$

6.3 Vehicle Speed and Output RPM

6.3.1 Maximum Vehicle Speed

The maximum vehicle speed is given as:

$$v_{max} = 200 \text{ km/h}$$

Converting to m/min:

$$v_{max} = 200 \times \frac{1000}{60} = 3333.3 \text{ m/min}$$

Converting to m/s:

$$v_{max} = 200 \times \frac{1000}{3600} = 55.56 \text{ m/s}$$

6.3.2 Output Speed (Wheel RPM)

The output speed (wheel rotational speed) is calculated as:

$$N_{output} = \frac{\text{Speed (m/min)}}{\text{Circumference (m)}} = \frac{3333.3}{2.315} = 1440 \text{ rpm}$$

6.4 Overall Gear Ratio

6.4.1 Input and Output Speeds

- Engine speed (5th gear): $N_{input} = 3500$ rpm
- Wheel output speed: $N_{output} = 1440$ rpm

6.4.2 Gear Ratio Calculation

The overall gear ratio (including gearbox 5th gear and final drive) is:

$$i_{total} = \frac{N_{input}}{N_{output}} = \frac{3500}{1440} = 2.43$$

This overall gear ratio of 2.43:1 represents the combined effect of the transmission's 5th gear and the differential's final drive ratio. It indicates that the engine completes 2.43 revolutions for every single wheel revolution when the vehicle is traveling at 200 km/h in 5th gear.

6.5 Bevel Pinion and Gear

The number of teeth in pinion is assumed to be 20.

```
        "H": Power(179.5, PowerUnit.HP), # 182BHP converted to HP
        "n_P": AngularVelocity(3500, AngularVelocityUnit.RPM), #
           Bevel Pinion Speed
        "n_G": AngularVelocity(1440, AngularVelocityUnit.RPM), #
           Bevel Ring Speed
        "S_F": 1.0,  # Factor of safety for bending
        "S_H": np.sqrt(1.0),  # Factor of safety for pitting
        "t": Temperature(203, TemperatureUnit.FAHRENHEIT), #
           Operating Temperature
        "phi": Angle(20, AngleUnit.DEGREE), # Pressure Angle
        "N_P": 20,  # Number of pinion teeth
        "R": 0.99, # Reliability
        "L_P": 1e7, # Pinion Life in cycles
        "Q_v": 7,  # Quality number
        "grade": 3, # Material Grade
        "load": "medium_shock", # Load Type (For engine driven
           cases)
        "is_crowned": True,
        "is_through_hardened": False,
        "is_case_hardened": True, # Case Hardened
        "no_of_straddling_members": 0,
```

From the solver we obtain these values

```
    d_P 128.47399987102875 mm
    F 50.00448911990134 mm
```

Thus the values of Pitch Diameter and Face Width (constrained using max face width) were founded as summarized as below:

Property	Bevel Pinion	Ring Gear (GR=2.43)
Number of Teeth	20	48
Pitch Diameter	128.474 mm	312.192 mm
Face Width	50 mm	50 mm
Cone Angle	22.363 deg	67.637 deg

6.6 Turning Analysis

6.6.1 Given Parameters

For turning analysis, the following conditions are considered:

- Speed of car: $v = 80$ km/h
- Radius of turn: $R = 7.079$ m (calculated roughly)
- Power during turning: $P_{\text{turn}} = 23$ BHP

6.6.2 Angular Velocity and Wheel Speeds

Vehicle Angular Velocity

The angular velocity of the vehicle about the turning center is:

$$\omega = \frac{v}{R} = \frac{80 \times \frac{1000}{3600}}{7.079} = \frac{22.22}{7.079} = 15.024 \text{ rpm}$$

Differential Geometry

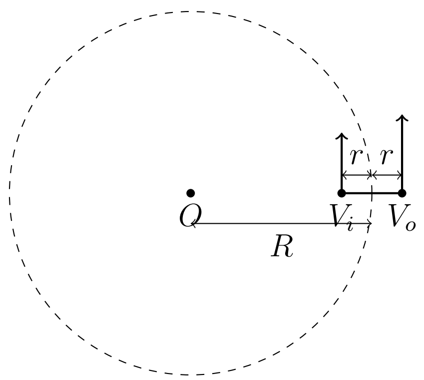


Figure 6.1: Differential action during turning

Let:

- r = shaft length + allowance = 0.925 m
- r_w = wheel radius = $\frac{17 \text{ inch}}{2} = \frac{0.432}{2} = 0.216$ m

Inner Wheel Speed

The inner wheel travels at radius $(R - r)$:

$$\omega_i = \frac{v_i}{r_w} = \frac{(R - r)\omega}{r_w} = \frac{(7.079 - 0.925) \times 15.024}{0.216} = 214.122 \text{ rpm}$$

Outer Wheel Speed

The outer wheel travels at radius $(R + r)$:

$$\omega_o = \frac{v_o}{r_w} = \frac{(R + r)\omega}{r_w} = \frac{(7.079 + 0.925) \times 15.024}{0.216} = 278.49 \text{ rpm}$$

6.6.3 Speed Difference and Differential Action

The speed difference between outer and inner wheels is:

$$\Delta\omega = \omega_o - \omega_i = 278.49 - 214.122 = 64.37 \text{ rpm}$$

This speed difference is accommodated by the differential gears, which allow the two wheels to rotate at different speeds while transmitting equal torque to both wheels. During turning:

- The outer wheel rotates approximately 30% faster than the inner wheel
- The spider gears within the differential rotate to distribute power accordingly
- The propeller shaft continues to rotate at the average speed of the two wheels

6.7 Spider and Side Gear

```
        "H": Power(23, PowerUnit.HP), # Assumed HP
        "n_P": AngularVelocity(278.49, AngularVelocityUnit.RPM),
           # Side Gear Speed
        "n_G": AngularVelocity(278.49, AngularVelocityUnit.RPM),
           # Assume GR is 1:1
        "S_F": 1.0,  # Factor of safety for bending
        "S_H": np.sqrt(1.0),  # Factor of safety for pitting
        "t": Temperature(203, TemperatureUnit.FAHRENHEIT), #
           Operating Temperature
        "phi": Angle(20, AngleUnit.DEGREE), # Pressure Angle
        "N_P": 18,  # Number of pinion teeth
        "R": 0.99, # Reliability
        "L_P": 1e7, # Pinion Life in cycles
```

```
        "Q_v": 7,  # Quality number
        "grade": 3, # Material Grade
        "load": "medium_shock", # Load Type (For engine driven
           cases)
        "is_crowned": True,
        "is_through_hardened": False,
        "is_case_hardened": True, # Case Hardened
        "no_of_straddling_members": 0,
```

From the solver we obtain these values

```
d_P 164.73659218194226 mm
F 33.989079370802216 mm
```

Thus the values of Pitch Diameter and Face Width (constrained using max face width) were founded as summarized as below:

Property	**Spider Gear**	**Side Gear**
Number of Teeth	18	18
Pitch Diameter	164.736 mm	164.736 mm
Face Width	33.989 mm	33.989 mm
Cone Angle	45 deg	45 deg

6.8 Code and Data Sources

The code with all tables and data are provided in below repository which includes shaft analysis, spur, epicyclic designs and extended to bevel gears by Team 18:

https://github.com/r3kste/pydome/blob/main/pydome/gears/bevel.py

Chapter 7

Material Selection of Propeller and Axle shafts

7.1 Overview of the Approach

To complement the manual design calculations, a comprehensive Python-based computational analysis was developed to automate the design and comparison of propeller and axle shafts across multiple materials and factors of safety. This approach enables systematic evaluation of design trade-offs between mechanical strength and economic efficiency.

The computational framework performs the following tasks:

- Iterative diameter calculations using Goodman criterion for propeller shafts and ASME elliptical criterion for axle shafts
- Parametric analysis across three materials (AISI 1040 HR, AISI 4140, AISI 4340 Q&T) and five FOS values (1.5 to 2.5)
- Weight and material cost estimation for each design configuration
- Generation of decision matrices to identify the most cost-effective material selections

7.2 Key Implementation Steps

7.2.1 Step 1: Torque Calculations

The drivetrain torque distribution is calculated from engine specifications through the transmission system:

```
P_w = P_kw * 1000
omega = 2 * np.pi * N_rpm / 60
Teng = P_w / omega
```

```
Tprop = Teng * ig
Taxle = (Tprop * ifinal) / 2
```

Explanation: Power is converted to watts, angular velocity calculated, and torque determined at each drivetrain stage. The final drive ratio distributes torque equally to both axles.

7.2.2 Step 2: Material Properties

Three steel grades with progressively higher strength are evaluated:

```
materials = {
    'AISI 1040 HR': {'Sut': 520, 'Sy': 290},
    'AISI 4140': {'Sut': 700, 'Sy': 450},
    'AISI 4340 Q&T': {'Sut': 800, 'Sy': 500}
}

Ta_prop = (Tmax_prop - Tmin_prop) / 2
Tm_prop = (Tmax_prop - Tmin_prop) / 2
Kfs_prop = 2.2  # Stress concentration for splines
```

Explanation: Material ultimate and yield strengths define fatigue limits. For fluctuating torque, alternating and mean components are equal. Stress concentration factor accounts for spline geometry effects.

7.2.3 Step 3: Propeller Shaft Design

An iterative solver determines the required diameter satisfying the Goodman fatigue criterion:

```
def prop_required_diameter(Sut, Sy, n):
    Se_prime = 0.5 * Sut
    ka = 57.7 * (Sut ** -0.718)
    Ssu = 0.67 * Sut

    def goodman_equation(d_m):
        kb = 1.51 * (d_m ** -0.157)
        Se = ka * kb * kc_prop * kd * ke * Se_prime
        Sse = 0.577 * Se
        tau = tau_num / (d_m ** 3)
        return tau / (Sse * 1e6) + tau / (Ssu * 1e6) - 1/n

    do = fsolve(goodman_equation, 0.052)[0]
```

```
    return do * 1000
```

Explanation: The endurance limit incorporates Marin factors (surface finish, size, reliability). The Goodman equation balances alternating and mean shear stresses against material fatigue limits, solved iteratively for the minimum safe diameter.

7.2.4 Step 4: Axle Shaft with Combined Loading

The axle experiences combined bending and torsion, requiring the ASME elliptical failure criterion:

```
Ma_axle = L_moment_arm * np.sqrt(F_lateral**2 + F_vertical**2)

def axle_required_diameter(Sut, Sy, n):
    def asme_equation(d_m):
        Se = ka * kb * kc_axle * kd * ke * Se_prime
        sigma_a = Kf * Ma_axle * (d_m/2) / I
        tau_a = Kfs_axle * Ta_axle * (d_m/2) / J
        sigma_a_prime = np.sqrt(sigma_a**2 + 3*tau_a**2)
        sigma_m_prime = np.sqrt(3*tau_m**2)
        term1 = (n * sigma_a_prime / (Se * 1e6)) ** 2
        term2 = (n * sigma_m_prime / (Sy * 1e6)) ** 2
        return term1 + term2 - 1

    d = fsolve(asme_equation, 0.065)[0]
    return d * 1000
```

Explanation: The resultant bending moment combines lateral and vertical forces. Von Mises equivalent stresses account for multiaxial loading. The ASME criterion creates a failure ellipse in stress space, ensuring safe operation.

7.2.5 Step 5: Parametric Iteration

Nested loops generate comprehensive design tables across all material-FOS combinations:

```
prop_results = []
for mat, props in materials.items():
    Sut, Sy = props['Sut'], props['Sy']
    for n in FOS_targets:
        d_mm = prop_required_diameter(Sut, Sy, n)
        prop_results.append([mat, n, d_mm])

df_prop = pd.DataFrame(prop_results,
```

```
    columns=["Material","FOS","Required Outer Dia (mm)"])
```

Explanation: All material-FOS combinations are systematically evaluated. Results are organized in pandas DataFrames for efficient analysis and visualization.

7.2.6 Step 6: Cost Analysis

Material costs are computed based on shaft geometry and material pricing:

```
def compute_weight_cost(df, L, is_hollow=False):
    for _, row in df.iterrows():
        if is_hollow:
            do = d_m
            di = di_do_ratio * do
            volume = (np.pi / 4) * (do**2 - di**2) * L
        else:
            volume = (np.pi / 4) * (d_m ** 2) * L

        mass_kg = rho * volume
        cost_rs = mass_kg * cost_per_kg
```

Explanation: For hollow propeller shafts, the inner diameter reduces material volume. Mass is calculated from material density, then multiplied by unit cost to determine total material expense.

7.3 Design Formulas

7.3.1 Propeller Shaft (Goodman Criterion)

For fluctuating torsional loading in a hollow shaft:

$$\frac{\tau_a}{S_{se}} + \frac{\tau_m}{S_{su}} = \frac{1}{n}$$

where:

$$\tau = \frac{K_{fs} \cdot T \cdot (d_o/2)}{J}, \quad J = \frac{\pi}{32}(d_o^4 - d_i^4), \quad S_{se} = 0.577 \cdot S_e$$

7.3.2 Axle Shaft (ASME Elliptical Criterion)

For combined bending and torsion:

$$\left(\frac{n \cdot \sigma'_a}{S_e}\right)^2 + \left(\frac{n \cdot \sigma'_m}{S_y}\right)^2 = 1$$

where von Mises equivalent stresses are:

$$\sigma'_a = \sqrt{\sigma_a^2 + 3\tau_a^2}, \quad \sigma'_m = \sqrt{\sigma_m^2 + 3\tau_m^2}$$

7.3.3 Weight and Cost

$$W = \rho\pi \left(\frac{d_o^2 - d_i^2}{4} \right) L, \quad \text{Cost} = W \times \text{Rate per kg}$$

7.4 Parametric Analysis Results

7.4.1 Propeller Shaft Diameter vs Factor of Safety

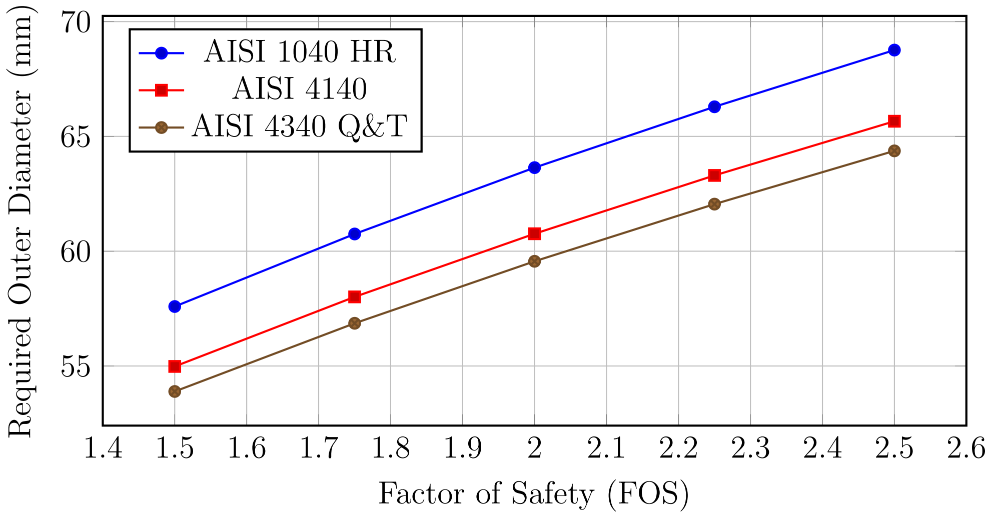


Figure 7.1: Variation of Propeller Shaft Diameter with Factor of Safety

The graph demonstrates that higher-strength materials (AISI 4340 Q&T) allow smaller diameters for equivalent safety factors. The relationship is nonlinear due to the diameter-dependent size factor in the endurance limit calculation.

7.4.2 Axle Shaft Diameter vs Factor of Safety

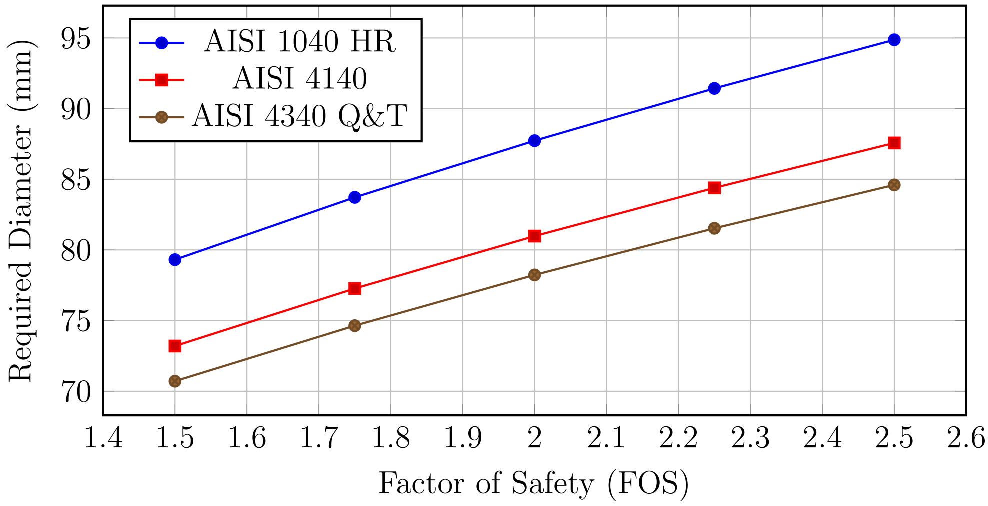


Figure 7.2: Variation of Axle Shaft Diameter with Factor of Safety

Axle shafts require larger diameters than propeller shafts due to the additional bending loads from wheel forces. The material strength advantage is more pronounced here, with AISI 4340 Q&T requiring 10.9% smaller diameter than AISI 1040 HR at FOS=2.0.

7.5 Material Selection at FOS = 2.0

7.5.1 Propeller Shaft Comparison

Table 7.1: Propeller Shaft Material Comparison (FOS = 2.0)

Material	Outer Diameter (mm)	Weight (kg)	Material Cost (Rs.)
AISI 1040 HR	86.67	19.10	1241.48
AISI 4140	83.54	17.41	1567.20
AISI 4340 Q&T	81.53	16.73	1840.60

Selected Material: AISI 1040 HR provides the most economical solution at Rs. 1241.48. Despite requiring 6.9% larger diameter than AISI 4340 Q&T, the lower material cost (Rs. 65/kg vs Rs. 110/kg) results in cost savings of Rs. 325.72 over AISI 4140 and Rs. 599.12 over AISI 4340 Q&T. The hollow shaft design with $d_i/d_o = 0.7$ minimizes weight while maintaining torsional rigidity.

7.5.2 Axle Shaft Comparison

Table 7.2: Axle Shaft Material Comparison (FOS = 2.0)

Material	Diameter (mm)	Weight (kg)	Material Cost (Rs.)
AISI 1040 HR	87.73	37.96	2467.62
AISI 4140	80.98	32.34	2910.77
AISI 4340 Q&T	78.23	30.18	3320.11

Selected Material: AISI 1040 HR remains the most cost-effective choice at Rs. 2467.62. Although it requires 12.1% larger diameter than AISI 4340 Q&T, substantial cost savings of Rs. 443.15 over AISI 4140 and Rs. 852.49 over AISI 4340 Q&T justify the selection. The increased diameter may actually provide benefits in terms of bearing journal size and keyway strength.

7.5.3 Final Design Recommendations

Based on comprehensive cost-benefit analysis at FOS = 2.0:

- **Propeller Shaft:** AISI 1040 HR with $d_o = 63.64$ mm (hollow: $d_i = 44.55$ mm)
- **Axle Shaft:** AISI 1040 HR with $d = 87.73$ mm (solid shaft)
- **Total Material Cost:** Rs. 3709.10

This unified material selection simplifies procurement, inventory management, and heat treatment processes. The total system cost is 17% lower than AISI 4140 (Rs. 4477.97) and 28% lower than AISI 4340 Q&T (Rs. 5160.71), representing significant economic advantage while maintaining full structural integrity.

7.6 Design Trade-off Analysis

7.6.1 Material Cost vs Performance

The analysis reveals a clear inverse relationship between material cost and required shaft diameter. While higher-strength steels enable more compact designs, the cost premium often exceeds the material savings from reduced volume:

- AISI 4340 Q&T reduces propeller shaft weight by 12.4% vs AISI 1040 HR, but increases cost by 48.2%
- AISI 4340 Q&T reduces axle shaft weight by 20.5% vs AISI 1040 HR, but increases cost by 34.5%

For mass-production automotive applications where material costs dominate over machining costs, AISI 1040 HR provides optimal value. Premium materials like AISI 4340 Q&T may be justified only in weight-critical applications (racing, aerospace) or space-constrained designs.

7.6.2 Factor of Safety Sensitivity

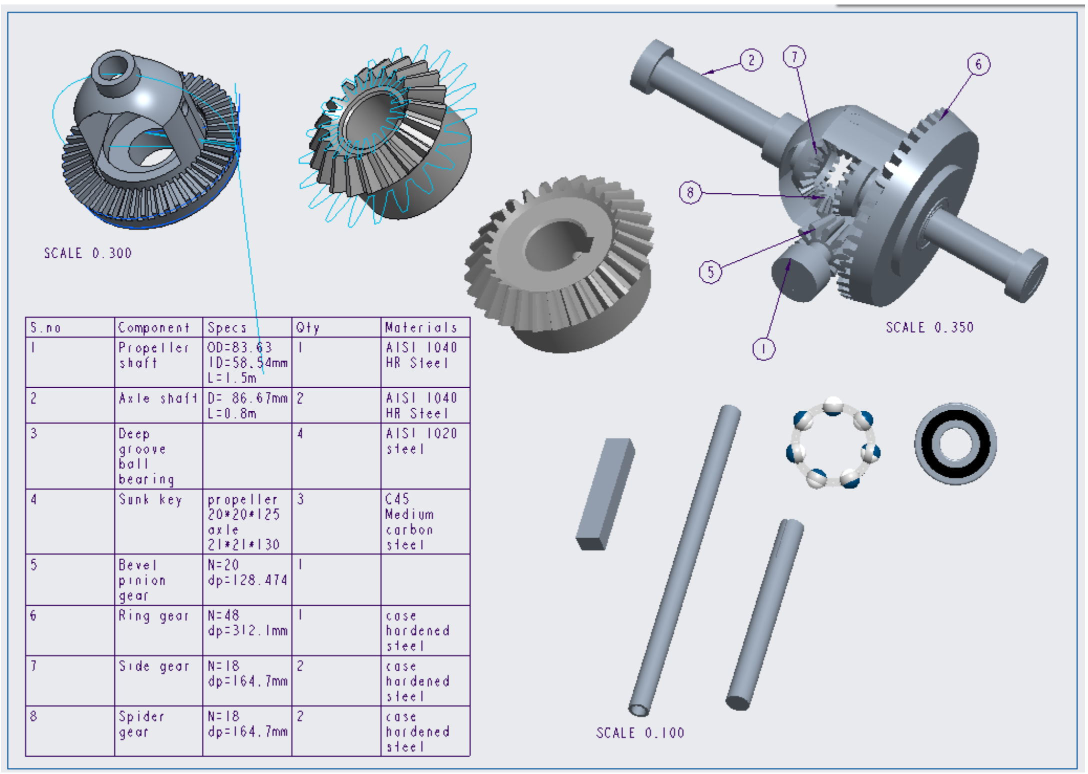
Diameter requirements scale approximately as $\text{FOS}^{1/3}$ due to the cubic relationship between diameter and stress. Increasing from FOS = 1.5 to 2.5 increases:

- Propeller shaft diameter by 19.4% (AISI 1040 HR)
- Axle shaft diameter by 19.6% (AISI 1040 HR)
- Material costs by 43% due to combined length and diameter effects

The selected FOS = 2.0 balances reliability requirements against material economy.

Chapter 8

Drawings



S.no	Component	Specs	Qty	Materials
1	Propeller shaft	OD=83.63 ID=58.54mm L=1.5m	1	AISI 1040 HR Steel
2	Axle shaft	D= 86.67mm L=0.8m	2	AISI 1040 HR Steel
3	Deep groove ball bearing		4	AISI 1020 steel
4	Sunk key	propeller 20*20*125 axle 21*21*130	3	C45 Medium carbon steel
5	Bevel pinion gear	N=20 dp=128.474	1	
6	Ring gear	N=48 dp=312.1mm	1	case hardened steel
7	Side gear	N=18 dp=164.7mm	2	case hardened steel
8	Spider gear	N=18 dp=164.7mm	2	case hardened steel

Figure 8.1: Assembly drawing with bill of materials

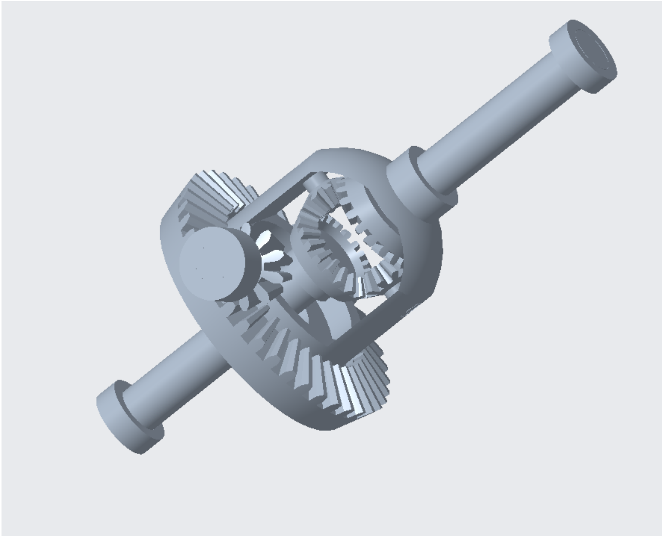


Figure 8.2: Differential assembly

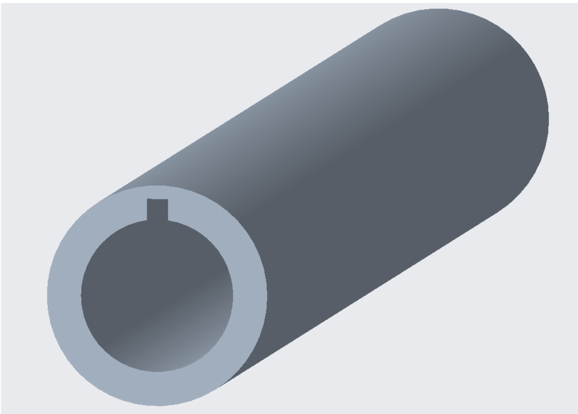


Figure 8.3: Propeller shaft design

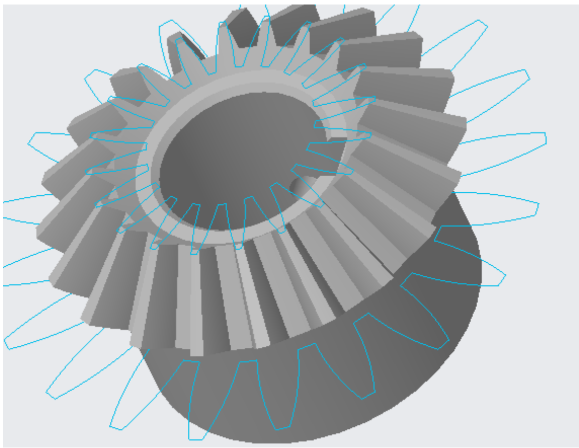


Figure 8.4: Bevel gear

Chapter 9

Conclusion

This report presents a comprehensive design of the rear wheel transmission system combining classical machine design principles with modern computational optimization. The final design parameters are:

Manual Design Results (Chapter 3)

- Propeller Shaft: $d_o = 83.63$ mm, $d_i = 58.471$ mm
- Axle Shaft: $d = 86.67$ mm
- Axle Key: $20.91 \times 125.45 \times 20.91$ mm

Key Findings

1. **Material Selection:** AISI 1040 HR emerged as the optimal material, providing 17-28% cost savings over higher-strength alternatives while meeting all structural requirements at FOS = 2.0.
2. **Design Methodology:** The computational approach validated manual calculations while enabling systematic exploration of the design space across multiple materials and safety factors.
3. **Cost-Performance Trade-offs:** Higher-strength materials (AISI 4140, AISI 4340 Q&T) reduce shaft diameters by 6-12% but increase material costs by 26-48%, making them economically unfavorable for standard automotive applications.
4. **Fatigue Considerations:** The Goodman criterion for propeller shafts and ASME elliptical criterion for axle shafts ensure infinite fatigue life under fluctuating loads with appropriate stress concentration factors for manufacturing features.

5. **Bearing Selection:** Required dynamic load capacities of 467 kN (propeller shaft) and 71.3 kN (axle shaft) guide bearing selection from standard manufacturer catalogs.

Recommendations

For production implementation, the following additional considerations are recommended:

- Critical speed analysis of the 1.5 m propeller shaft to ensure operation below first natural frequency
- Finite element analysis to validate stress concentrations at spline and keyway features
- Surface treatment (induction hardening, shot peening) to enhance fatigue resistance at critical locations
- Prototype testing to verify analytical predictions under actual operating conditions

The integrated analytical-computational approach demonstrates the power of modern design optimization while maintaining rigorous adherence to fundamental machine design principles. The resulting transmission system provides reliable, cost-effective power delivery suitable for volume automotive production.

Bibliography

[1] J.E. Shigley, C.R. Mischke, R.G. Budynas, *Mechanical Engineering Design*, McGraw Hill.

[2] R.L. Norton, *Machine Design: An Integrated Approach*, Pearson.

[3] Thomas D. Gillespie, *Fundamentals of Vehicle Dynamics*

[4] https://youtu.be/RCHpdU4uxDg?si=3nupni2b6yJgXqyo

[5] https://youtu.be/y8QaD8NJLxM?si=mNKvBf5H7v1pFVkw

[6] https://youtu.be/nC6fsNXdcMQ?si=LSe-F66W6479oReh