Physical phenomena in a Thermoacoustic system

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Introduction

Thermoacoustic instability is a phenomena that occurs in combustion chambers that can lead to many catastrophic effects. It occurs due to mutual coupling between different subsystems, such as the acoustic pressure (set up in the duct/chamber), heat release rate (due to combustion of fuel) and hydrodynamic effects such as vortex shedding (if the reactive flow is turbulent).

First prominently observed in rocket engines, such as the F-1 engines used for the apollo mission, and also in combustors of gas turbines, aeroengines, domestic boilers, afterburners etc., this phenomenon has been found to lead to catastrophic effects as the acoustic pressure can reach a state of very high amplitude self-sustained oscillations. It can cause structural damage, excessive vibration and noise, damaging electrical components, flame flashback, blow-off and other deteriorating effects. Since the system consists of such a wide range of interacting elements, it exhibits rich physics that are attempted to be explained from classical theories of acoustics, combustion and hydrodynamics. Moreover, it has been found to exhibit rich nonlinear dynamical phenomena along with wide spatio-temporal patterns that emerge due to the interaction among the subsystems. Hence it is essential to study the underpinning mechanisms in order to devise methods for mitigating instability and to obtain computational predictions to inform the design and aid in the instability prevention process. This requires us to have an understanding of each of the subsystems and also of the interaction among the subsystems which give rise to this complex phenomenon.

The Reductionist Approach

The Reductionist approach to studying the system of interest is to zoom into it's constituent elements and analyze them in detail, in order to provide explanations from a classical theoretical framework. From an empirical point of view one can propose reduced order models to describe the same, mathematically. Expressions for the acoustic field and velocity field are obtained from the principles of mass and momentum conservation, leading to a set of nonlinear

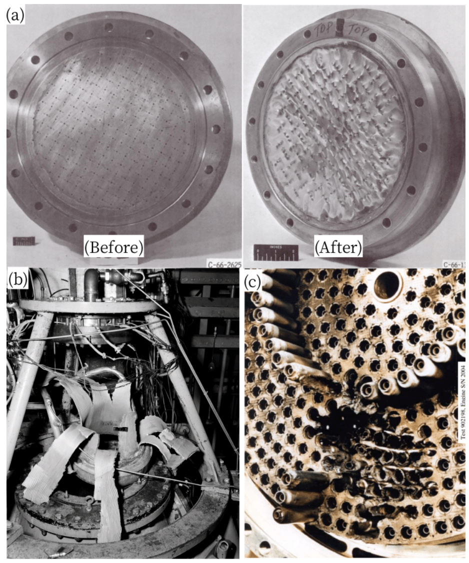


Figure 1: (a) Effect of thermoacoustic oscillations on an LPRE injector before and after a test. Bloomer et al. (1968). (b) Explosion of the thrust chamber in the rocket engine test facility of NASA in 1958 (Glenn Research Center, 2018). (c) Injector failure during the development of the space shuttle main engine. Goetz and Monk (2005).

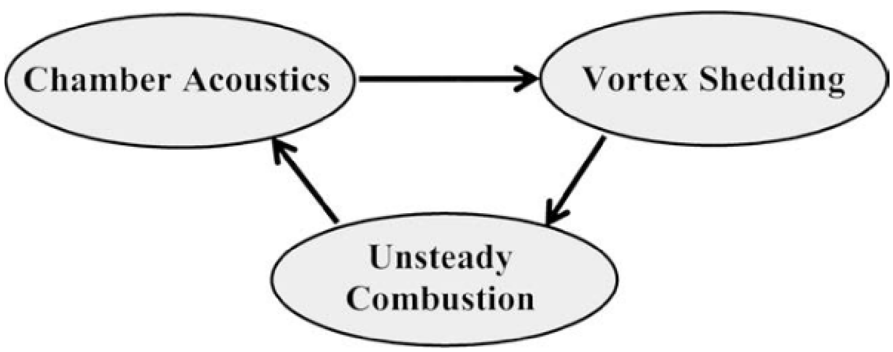


Figure 2: The Subsystems

coupled differential equations, which are linearized in order to obtain meaningful solutions.

The behavior of acoustic waves within a combustor, its length and time scales of propagation and reflection is dictated by the combustor's geometry, boundary conditions and temperature profile. Together with the combustor's acoustic modes and speed of sound field, we can get to know the frequency and acoustic mode shape corresponding to any Thermoacoustic instability. TAI is determined by the balance between acoustic energy gain and acoustic energy damping.

Rayleigh's Criteria: In 1878, Lord Rayleigh postulated that when heat is added in phase with unsteady pressure, sound is generated. When the maximum rate of heat released due to combustion coincides with the maximum acoustic pressure fluctuations at several locatios in the flow field, then then net acoustic driving becomes greater than the acoustic damping and the system attains thermoacoustic instability. Here the acoustic oscillations grow in amplitude until nonlinearities in the system cause the amplitude of oscillations to saturate. This positive feedback between heat release rate and acoustic field can result from a number of mechanisms like equivalence ratio fluctuations, flame surface area modulations, modulations in the mixing field, coherent structures in the flow, droplet and spray dynamics and the resulting unsteady evaporation and entropy fluctuations. On the other hand, viscous and thermal losses, transfer of energy to vorticity fluctuations, radiation from combustor exit or nozzle and wall vibrations contribute to damping in the system. Avoiding TAI requires us to establish stability boundaries of the system done by elaborate testing and measurement. For modeling, thermoacoustic systems have traditionally been treated as an acoustic system driven by combustion.

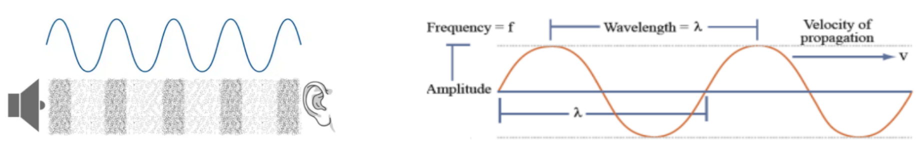


Figure 3: Acoustics

A phenomenological model, which describes the empirical relationship of phenomena to each other, is described in the work of Matveev and Culick.

In a bluff-body stabilized back-ward facing step combustor operating at turbulent Reynolds numbers, Vortices are formed at the dump plane, which roll up

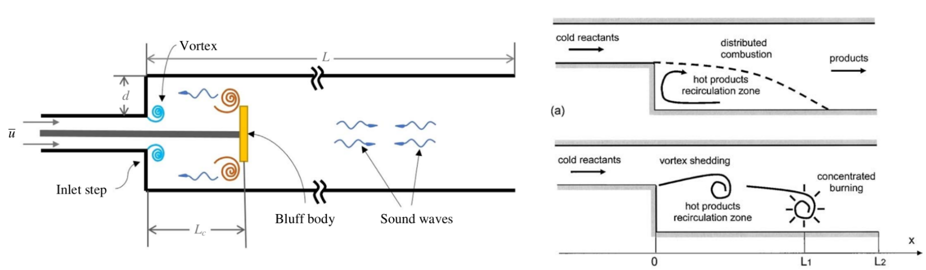


Figure 4: Vortex shedding in turbulent systems

to entrain reactants within, carry the flame and convect downstream to impinge on the bluff-body. This impingement leads to intense fine scale mixing and heat release. This vigorous burning of vortices, formed behind flame stabilizers, can drive significant pressure oscillations. Vortex burning is assumed to be localized in space and time and a "kicked" oscillator model is utilized for deriving the appropriate dynamical system. The acoustic field is proposed to get a kick of energy when and where then vortex combusts. The moment of burning and the corresponding vortex location are dependent on the chamber geometry, velocity field, and characteristic chemical and hydrodynamic times.

Unsteady combustion is an efficient acoustic source and in turn generates new acoustic waves which propagate away from the flame. These partially reflect from the boundaries of the combustor, arriving back at the flame where they further perturb the heat release rate.

Under some conditions, the resulting cycle will lead to oscillation of certain frequencies successively growing in amplitude. Thermoacoustic instability is typically linear, meaning that small perturbations grow in time, eventually reaching levels that introduce nonlinear effects that limit further growth.

The linearized equations of motion show that in a viscous heat-conducting compressible medium, three mode of fluctuations exist, namely the vorticity mode, entropy mode and sound mode. Perturbation theory is a method for solving complex problems by making small approximations to a known solution. Closed form solutions can be obtained for linear equations, and are extended for more complex scenarios by using a consistent higher order perturbation theory, where effects of a small perturbation on a system are calculated by expanding the solution as a series with terms beyond the first order. The non-linearity of the full Navier-Stokes equations can be interpreted as interaction between the three basic modes, classified as 'mass-like','force-like', and 'heat-like' effects and the theory is explained based on expansion of the disturbance fields in powers of an amplitude parameter. The equations show the interaction between all these three modes, which in turn enhance the generation/damping/interaction of each of the three modes, as they are all non-linearly coupled PDEs.

The key flow phenomena, acoustic waves, flame's unsteady heat release rate, are thus dependent upon a wide range of factors such as acoustic propagation, reflection, turbulent reacting flow, vortex dynamics, combustion chemistry, heat transfer etc.

The Complex and Dynamical Systems perspective

Analysis through experiments led to a series of very interesting observations from the data, that provide descriptions of the system physics from a nonlinear dynamics and complex systems perspective. The key ingredients of a complex system are: interacting elements, nonlinear feedback that affects the behavior of these systems and exchange of energy and mass with the surroundings. Our

Thermoacoustic system is an outstanding example of a complex system, with high level of sensitivity to parameters, comprising interactions of wide spatio-temporal scales that co-evolve under different conditions. A high level of coherence is observed in the system during the state of instability, that sometimes the combustor is said to be 'alive' or 'breathing'. The Self organization and Emergence of an ordered acoustic field and large coherent structures without the need for forcing has led to the conjecture that the intertwined and highly intricate interactions between the wide spatio-temporal scales in the flame, flow and acoustics are through pattern formation.[George *et al.*, 2018]

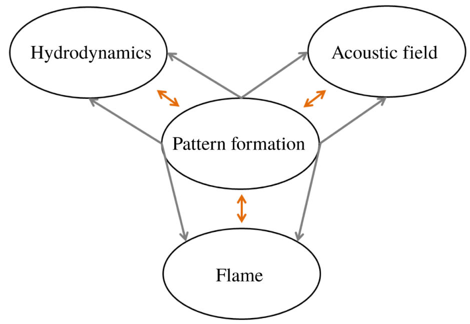
Examining the stability of perturbations close to the onset of thermoacoustic instability, we see that at a parameter value prior to onset of TAI, small perturbations in the system decay. Post TAI, small perturbations grow. i.e. Growth rate changes its sign from -ve to +ve, and there is a qualitative change in the behavior of the Thermoacoustic system. Such a qualitative change occurring when a parameter is changed smoothly is termed a bifurcation in dynamical systems theory. And this particular type of bifurcation is called a Hopf bifurcation, and the parameter value at which there is a change in the dynamical behavior is called a Hopf point.

By using nonlinear time series analysis and other measures, a wide and rich amount of dynamical behavior has been observed in our thermoacoustic system, like chaose, limit cycle oscillations and bifurcations. Thermoacoustic instability has been found to be the transition from a chaotic state of combustion noise to an ordered state of large amplitude self-sustained oscillations, through a dynamical state called intermittency. Intermittency is characterized by bursts of periodic oscillations amidst aperiodic fluctuations.

The presence of intermittency helps to forewarn the onset of instability, and hence is an important phenomena to study under different experimental conditions. In quasi-static conditions the control parameter is held fixed so that the system is allowed to settle onto a specific dynamical state and the physics of that state is studied. Under continuous rate of change of control parameter, the system undergoes a transition through all dynamical states, from combustion noise to thermoacoustic instability, sometimes even starting directly from intermittency if the parameter value is sufficient enough.

Further, application of synchronization theory to the system from natural observance of the acoustic pressure and heat release rate signals has led to interesting observations where the phases of acoustic pressure and heat release rate oscillations are locked during periodic epochs and unlocked during the aperiodic and relatively silent epochs.

During combustion noise the interactions are highly incoherent, and aperiodic epochs of intermittency have the flame distorted by incohrently shed structures and the acoustic field is also not in phase with the dominant modes of the other disturbances. As periodicity emerges, the small vortices shed start to interact, coherently rolling up to form larger structures, which perturb the flame, at the right frequency matching with the acoustic field, leading to an increase of amplitude. This leads to a fall of amplitude near the inlet region, thereby increasing the velocity of the flow field and causing more coherent shedding of



the vortices. Thus a positive feedback loop is established where the interactions within an individual subsytem enhances the interaction among different subsystems which in turn lead to the emergence of a phenomenon like thermoacoustic instability in the system.

Conclusion

Thus Thermoacoustic instability is an extremely rich and complex phenomenon which is highly beautiful to study from a physics point of view and also of high importance from an engineering safety perspective. All these frameworks of description portray the unique nature of the underlying science and show the existence of some fundamental universal behavior which can be used to establish a connection with different complex dynamical systems like climate, ecology, etc., with a description that unites the classical theory, with visualizations of dynamical systems and data driven analysis using tools from complex systems. It is thus essential to bridge the link between the descriptions provided by each of these perspectives.

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