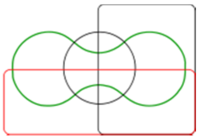
SE3IA11/SEMIP12
Image Analysis
Morphological Image Processing



Chapter 9 in the Gonzalez & Woods book

Contents in this topic

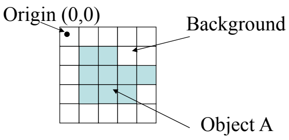
- Basic concepts from set theory
- Logic operations in binary images
- Morphological operations
  - Dilation and erosion
  - Opening and closing
  - Hit-or-miss transform
- Basic morphological algorithms
- Some applications
- Extension to grey-level images

General Introduction to Morphology

- Morphology is an important branch in biology to deal with objects, shapes and structures of animals and plants.
- Mathematical morphology is used to denote similar methodology adopted in image analysis.
- The language of mathematical morphology is “set theory”.
- In the following discussion, we shall concentrate on binary images where object pixels are binary 1, and background pixels are binary 0.

Object representation by a set

- Object A can be represented by set A with all pixel coordinates of A in a 2D integer space Z^2.



$$A = \{(1,1),(2,1),(3,1),(1,2),(2,2),(3,2),(2,3),(3,3),(4,2)\}$$

Basics of Set Theory

- As stated before, set theory is the foundation of mathematical morphology.
- Let A be a set in a 2D integer space Z^2.
  - If $a = (a_1, a_2)$ is an element of A, we have $a \in A$
  - If a is not an element of A, it says $a \notin A$
- The null or empty set $Ø$: no element in the set.
- A set is specified by the contents of two braces: $\{\bullet\}$
  - For the expression of $C = \{c \mid c = -d, \text{for } d \in D\}$, it means that set C is formed by elements c, which equal to elements d of set D multiplied by -1.

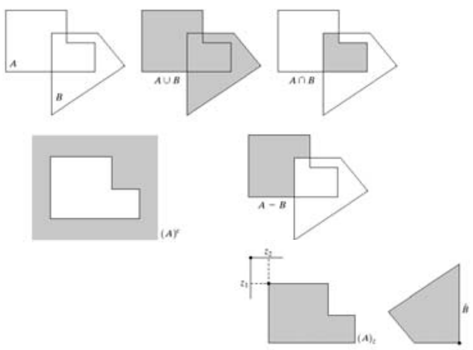
Definitions of sets (1)

- $A \subseteq B$: all elements in set A are also elements of set B, *i.e.* set A is a subset of B.
- The *union* of two sets A and B
  - $C = A \cup B$: Set C has all elements belonging to either A, B, or both.
- The *intersection* of two sets A and B
  - $D = A \cap B$: Set D is formed by elements belonging to both A and B.
- Two sets A and B are *disjoint* or *mutually exclusive*: $A \cap B = Ø$

Definitions of sets (2)

- ***Translation*** of a set A by $z = (z_1, z_2)$ is denoted by $(A)_z$ and defined as
  $$(A)_z = \{c \mid c = a + z, \text{ for } a \in A\}$$
- ***Reflection*** of A is denoted by $\hat{A}$, and defined as
  $$\hat{A} = \{c \mid c = -a, \text{ for } a \in A\}$$
- ***Complement*** of A is defined as
  $$A^c = \{c \mid c \notin A\}$$
- The ***difference*** of two sets A and B is denoted by A-B, and defined as
  $$A - B = \{w \mid w \in A, w \notin B\} = A \cap B^c$$

Diagrams showing operations on sets

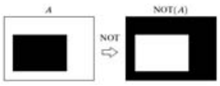


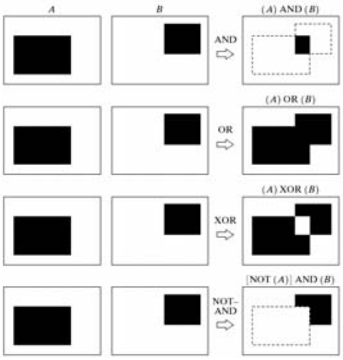
Binary images: logic operations

- Logic operations provide a powerful complement to implementation of binary image processing algorithms.
- There are three main logic operations used in image processing, *i.e.* AND, OR, and NOT.

p	q	p AND q (also $p \cdot q$)	p OR q (also $p + q$)	NOT (p) (also $\bar{p}$)
0	0	0	0	1
0	1	0	1	1
1	0	0	1	0
1	1	1	1	0

Diagrams showing logic operations on binary images





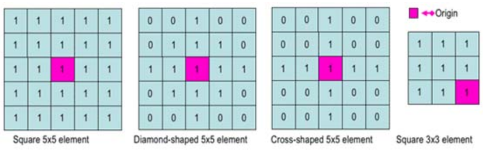
NOT operation only involves one image, whilst other operations involve more than one image.

Morphological operation: dilation

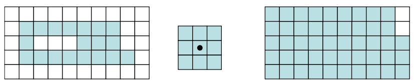
- Dilation is a very basic operation in mathematical morphology.
- Dilation of *A* by *B* is denoted by $A \oplus B$, and defined as $A \oplus B = \{c | c = a + b \text{ for } a \in A \text{ and } b \in B\}$
- Dilation is based on addition so that it is commutative, *i.e.* $A \oplus B = B \oplus A$.
- In practice, set *A* represents an image under processing, and set *B* is smaller and referred to as the *structuring element*.
- Dilation may be represented as a union of translations of the structuring element, *i.e.* $A \oplus B = \bigcup_{a \in A} (B)_a$

Structuring element

- The structuring element is a small binary image. Its pixel values are of 0 or 1.
  - The pattern of 1s and 0s specifies the shape of the structuring element.
  - The origin of the structuring element is usually one of its pixels, although this is not necessary.



An example of dilation



A B $A \oplus B$

- Some important points are illustrated.
  - Dilation expands the original image.
  - Original image is contained in the dilated image, *i.e.* $A \subset A \oplus B$.
  - Dilation fill up small holes in objects.

Morphological operation: erosion

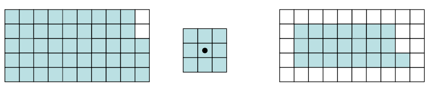
- Erosion is another very basic operation in mathematical morphology, opposite to dilation.
- Set A eroded by *structuring element* B is defined as

$$A \Theta B = \{c \mid \text{for each } b \in B \text{ there exists an } a \in A \text{ so that } c = a - b\}$$

- Erosion is based on subtraction so that it is not commutative, *i.e.* $A \Theta B \neq B \Theta A$.
- Erosion may be described as an intersection of the negative translations of the image set.

$$A \Theta B = \bigcap_{b \in B} (A)_{-b}$$

An example of erosion

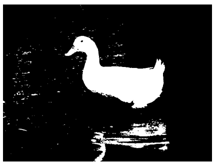


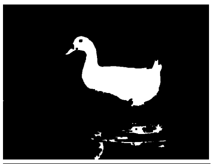
A B $A \Theta B$

- Erosion has effects on an image set opposite to dilation.
  - Erosion operation shrinks the original image.
  - The eroded image is contained in the original image.
  - Erosion removes or breaks narrow “bridges”.
  - It has been shown that $(A \oplus B) \Theta B \neq A$

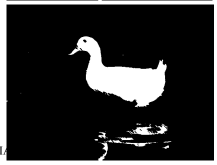
An example of erosion (2)

- The shape of the structuring element matters.





1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1



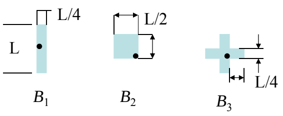
1	0	0	0	0
0	1	0	0	0
0	0	1	0	0
0	0	0	1	0
0	0	0	0	1

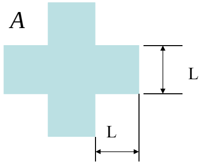
Group discussions

- A is a set to be operated.
- There are three different types of structuring elements as denoted by B_1, B_2, and B_3.
- Sketch the result of the following morphological operations

$(A \Theta B_1) \oplus B_2$

$(A \oplus B_2) \Theta B_3$





Morphological operation: opening

- Opening is a combined morphological operation, and defined as

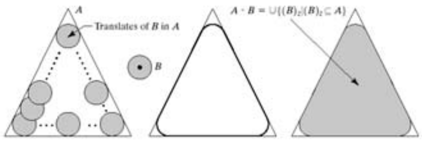
$$A \circ B = (A \Theta B) \oplus B$$

- The opening of A by B selects all those points of A each of which can be covered by some translation of the structuring element B while the translated structuring element is itself entirely contained in A.
- In other words, the opening of A by B is obtained by taking the union of all translations of B that fit into A.

$$A \circ B = \cup \{ (B)_z \mid (B)_z \subset A \}$$

Geometric interpretation of opening

- Imagine the structuring element B as a rolling ball.
- The boundary of $A \circ B$ is given by the points on the boundary of B that are closest to the boundary of A as B is rolled around from the **inside** of A.



Example of opening operation (1)

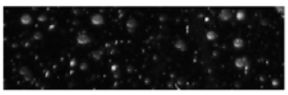
- Opening can open up a gap between objects connected by a thin bridge of pixels.
- Regions that have survived the erosion are partially restored to their original size by the dilation.

Binary image A | $A \circ B$ (5×5 square) | $A \circ B$ (9×9 square)

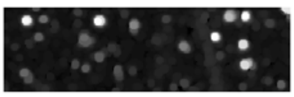
Example of opening operation (2)

- A disk-shaped structuring element with a radius of 5 pixels is applied to the original image.
- Snowflakes with a radius less than 5 pixels have been removed by the opening operation.

Original image



After opening operation



Morphological operation: closing

- Closing is another combined morphological operation, and defined as

$$A \bullet B = (A \oplus B) \Theta B$$

- The closing of A by B includes all points satisfying the condition that each time one of these points is covered by a translation of the reflected structuring element $\hat{B}$, there must be at least one point in common between A and the translation of $\hat{B}$.

$$A \bullet B = \{x \mid x \in (\hat{B})_t \text{ implies } (\hat{B})_t \cap A \neq \Phi\}$$

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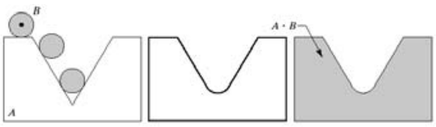
Geometric interpretation of closing

- Imagine the structuring element B as a rolling ball.
- The boundary of A•B is similarly obtained, except that we now roll B from the **outside** of A.
- Closing smoothes the boundary, and eliminates small holes and fills gaps in the boundary



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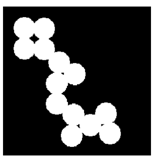
Example of closing operation

- A disk-shaped structuring element with a radius of 10 pixels is applied to the original image.
- The gaps are closed.



Original image

After closing operation

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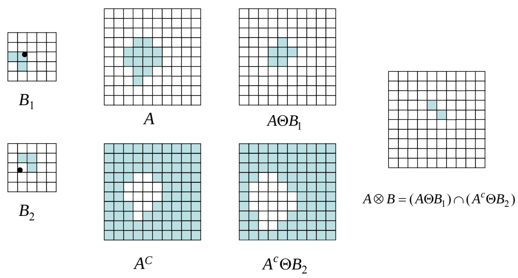
Morphological operation: hit-or-miss transform

- The morphological hit-or-miss transform is used to extract various image features from an object. It is defined as

$$A \otimes B = (A \Theta B_1) \cap (A^c \Theta B_2)$$

- Where A is the set representing the object, and B=(B_1,B_2) is called a composite structuring element, satisfying $B_1 \cap B_2 = \Phi$.
- In order to extract features from object A, it is necessary to carefully design or choose the composite structuring element $B(B_1B_2)$.

Example of the hit-or-miss transform



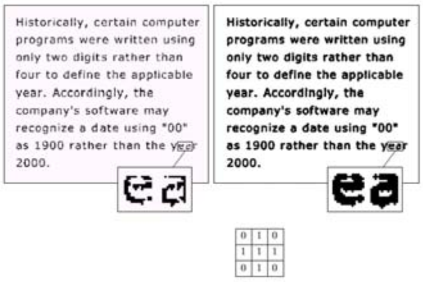
The upper right-hand corner points of object A are found by hit-or-miss transform.

Morphological algorithms

- Based on the basic morphological operations, *e.g.* erosion, dilation, opening, closing, and hit-or-miss, many morphological algorithms can be generated.
- Some morphological algorithms are listed here
  - Boundary extraction
  - Region filling
  - Extraction of connected components
  - Thinning and thickening
  - Convex hull
  - Skeletons: sketching frame
  - Pruning: removing parasitic components

Selected applications (1)

- Enhancement of extracted characters



Selected applications (2)

- Enhancement of fingerprint

Selected applications (3)

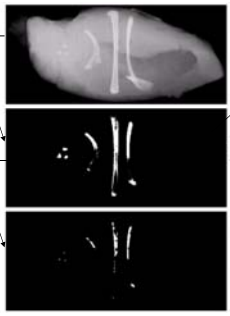
- Boundary extraction

$$\beta(A) = A - (A \Theta B)$$

Selected applications (4)

- Extraction of connected components

Thresholding



A

Set A eroded by a 5×5 structuring element

Connected component	No. of pixels in connected comp
01	11
02	9
03	9
04	39
05	133
06	1
07	1
08	743
09	7
10	11
11	11
12	9
13	9
14	674
15	85

Grey-scale morphology

- Grey-scale image $f(x,y)$, structuring element $b(x,y)$
- Dilation: enhance brightness

$$(f \oplus b)(s,t) = \max\{f(s-x, t-y) + b(x,y) \mid (s-x),(t-y) \in D_f; (x,y) \in D_b\}$$

- Erosion: enhance darkness

$$(f \Theta b)(s,t) = \min\{f(s+x, t+y) - b(x,y) \mid (s+x),(t+y) \in D_f; (x,y) \in D_b\}$$

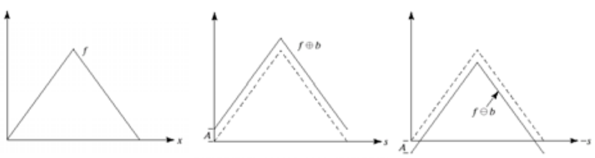
- Opening: remove bright details

$$f \circ b = (f \Theta b) \oplus b$$

- Closing: remove dark details

$$f \bullet b = (f \oplus b) \Theta b$$

Geometric interpretation of grey-scale dilation and erosion

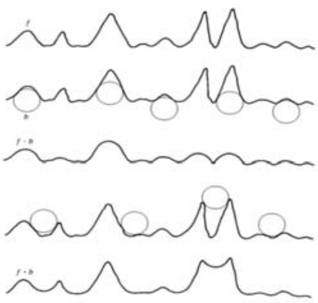


Original function

Dilated by structuring element $b = A$

Eroded by structuring element $b = A$

Rolling ball analogy of opening and closing



A gray-scale scan line

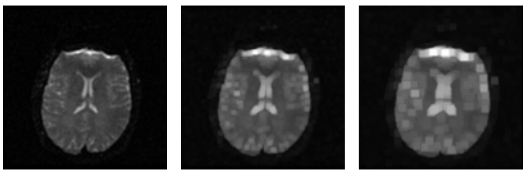
Positions of rolling ball for opening

Result of opening

Positions of rolling ball for closing

Result of closing

Example of grey-scale dilation

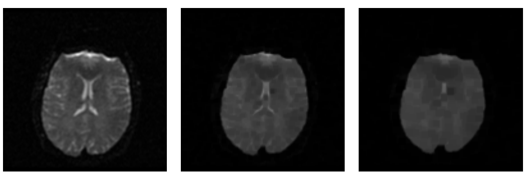


Original function

Dilated by 3×3 unit structuring element

Dilated by 5×5 unit structuring element

Example of grey-scale erosion



Original function

Eroded by 3×3 unit structuring element

Eroded by 5×5 unit structuring element

Example of grey-scale dilation and erosion

Original function

Dilated by 5×5 unit structuring element (enhance brightness)

Eroded by 5×5 unit structuring element (enhance darkness)

Example of grey-scale opening and closing

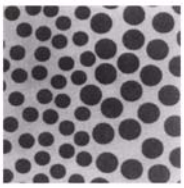
Original function

Opening by 5×5 unit structuring element (remove bright details)

Closing by 5×5 unit structuring element (remove dark details)

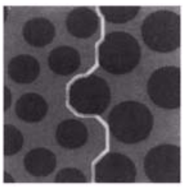
Application of grey-scale morphology: textural segmentation

Objective: separate two texture regions



Original image

1. Closing the input image by a structuring element having size of the small blobs – light left;
2. Opening the output image by a structuring element with size larger than gaps between larger blobs – dark right;
3. A simple thresholding to separate light left and dark right regions.



Final segmentation

End of the two lectures

- Summary what you have learned in the two lectures.
- Think about to what applications mathematical morphology can be used (many – parallel to and associated with other image processing techniques).