

On the practical distance-diameter relationship for extracellular recruitment of spinal afferent and efferent axons

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Abstract

The relative recruitment of axons of differing caliber under extracellular stimulation is determined not by local membrane impedance but by the field gradients that develop along the cable. Here we derive a simple scaling law from the activating-function framework and illustrate how fiber diameter governs the distance from an electrode at which excitation occurs.

1 Intro to Cable Theory

The term *resistivity* (ρ) describes a material property—the ratio between the electric field E and current density J within a homogeneous conductor: $E = \rho J$. It is independent of geometry and reflects the ease with which charge carriers (e.g., ions in cytoplasm) move under an applied field. For intracellular fluid, $\rho_i \approx 70\text{--}100 \Omega\cdot\text{cm}$ has been measured empirically by inserting two microelectrodes into an axon of known diameter and length, injecting a small steady current, and using Ohm's law $R = \frac{\rho_i L}{A}$ to solve for ρ_i . Early work by Cole and Curtis [1939] and by Hodgkin and Rushton [1946] on the squid giant axon established these values.

In contrast, the *resistance* of a segment depends on its geometry:

$$R = \frac{\rho_i L}{A} = \rho_i \frac{L}{\pi a^2} \quad (1)$$

Larger diameters provide more parallel ionic pathways and therefore a lower axial resistance per unit length ($r_i = \frac{\rho_i}{\pi a^2}$). This geometric dependence—not a change in material resistivity—explains why larger axons conduct more rapidly and are more easily excited by extracellular fields.

The mathematical formalism was imported from telegraph-wire physics. Lord Kelvin and colleagues developed the telegrapher's equations to describe

voltage and current propagation along long under-sea cables as distributed RC lines (Thomson [1855]; for a modern description, see e.g. Hayt et al. [2011]).

In the 1940s, Hodgkin and Rushton recognized the same mathematics applied to nerve fibers: the axial core resistance r_i , membrane resistance r_m , and membrane capacitance c_m together form a continuous, lossy cable. The modern *cable equation* and its extracellular extension thus trace directly to this historical analogy, with axoplasm substituting for copper and membrane for insulation.

2 Intro to Activating Functions

For a point-source electrode delivering current I in a homogeneous medium of conductivity σ , the extracellular potential is

$$\phi(r) = \frac{I}{4\pi\sigma r}, \quad (2)$$

and the axial field along an axon with minimum distance ρ from the source is

$$E(x) = -\frac{\partial\phi}{\partial x} \quad (3)$$

Depolarization depends on the spatial curvature of the extracellular potential, quantified by the *activating function* [McNeal, 1976, Rattay, 1986]:

$$f(x) = -\frac{\partial^2\phi}{\partial x^2} \quad (4)$$

Near the closest axon-source approach ($x = 0$),

$$|f(0)| \propto \frac{I}{\sigma\rho^3}. \quad (5)$$

Cable theory introduces a proportionality between induced membrane potential and fiber diameter d ,

since larger axons have lower axial resistance ($R_a \propto 1/d^2$). Setting a constant threshold of depolarizing drive yields

$$d \frac{I}{\sigma \rho^3} = \text{constant}$$

$$\Rightarrow \rho \propto \left(\frac{dI}{\sigma} \right)^{1/3} \quad (6)$$

Equation 6 captures the well-known inverse relationship between recruitment threshold and diameter: larger myelinated fibers are recruited at greater distance for the same current.

3 Illustrative Examples

For equal-threshold recruitment distances of two fibers of diameters d_1 and d_2 ,

$$\frac{\rho_2}{\rho_1} = \left(\frac{d_2}{d_1} \right)^{1/3}. \quad (7)$$

Hence, as indicated in Table 1, a 20 μm fiber is activated at roughly $2^{1/3} \approx 1.26$ times the distance of a 10 μm fiber for a fixed monophasic 1 mA pulse.

4 In Dorsal-Root Stimulation

Greiner et al. [2021] used biophysical finite-element models of cervical epidural stimulation and found that large myelinated dorsal-root afferents (mean $d \approx 14 \mu\text{m}$) dominate recruitment, with action potentials initiating in dorsal-root branches near the epidural contact (Fig. 2b,c). The same scaling implies that the ascending branch of a bifurcating Ia afferent (often $\sim 2 \times$ thicker than its descending branch) can be excited at $\sim 26\%$ greater distance for identical field strength, consistent with their simulations.

5 Implications

Equation 6 provides an analytical shorthand linking geometric and biophysical parameters. Although absolute thresholds depend on pulse width, nodal spacing, and tissue inhomogeneity, the cube-root

Table 1: Because of the cube-root dependence, recruitment distance grows slowly with diameter.

Fiber pair	Diameter ratio	Distance ratio
8 vs. 16 μm	2.0	1.26
10 vs. 20 μm	2.0	1.26
12 vs. 18 μm	1.5	1.15

Type	Function	Width (μm)
Group Ia	muscle spindle	17–20
Group Ib	golgi tendon organ	16–18
A α	efferent	13–16
A β	cutaneous	6–12
A γ	efferent	4–8
A δ	pain/temperature	2–6

Table 2: Approximate diameters of major myelinated fiber classes (adapted from Lloyd and Chang [1948], Boyd and Kalu [1979], Arbuthnott et al. [1977], Greiner et al. [2021]).

scaling of recruitment distance with diameter is robust and explains why the largest myelinated afferents (group Ia/Ib and A β ; Table 2) are preferentially activated during dorsal epidural stimulation, whereas direct activation of motor efferents in the ventral roots typically requires closer proximity or higher current.

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