

- Kepler's Laws:
- According to Newton's law of gravitation the pull of the Sun on the Earth is contained in:

$$F_G = \frac{GM_s M_E}{r^2}$$

- Expanding on this we get from Newton's second law: $\frac{d^2 x}{dt^2} = \frac{F_{G,x}}{M_E}$, $\frac{d^2 y}{dt^2} = \frac{F_{G,y}}{M_E}$
- We then have to expand further to cover the X and Y components of the force such as :

$$F_{G,x} = -\frac{GM_s M_E}{r^2} \cos(\theta) = -\frac{GM_s M_E x}{r^3}$$

- The difference between the X and Y components are the cosine versus sine components.
- The rest of this section discusses in further detail the other planets in the solar system and their relation to the sun in their orbits.
- Inverse Square Law:
- The orbital trajectory of a body of reduced mass in polar coordinates is given by:

$$\frac{d^2}{d\theta^2} \left(\frac{1}{r} \right) + \frac{1}{r} = -\frac{\mu r^2}{L^2} F(r) \text{ where } \mu \text{ is the reduced mass of the body.}$$

$$\rightarrow r = \left(\frac{L^2}{\mu GM_s M_p} \right) \frac{1}{1-e \cdot \cos(\theta)}$$

$$v_{max} = \sqrt{GM_s} \cdot \sqrt{\frac{(1+e)}{a(1+e)}} \left(1 + \frac{M_p}{M_s} \right)$$

$$v_{min} = \sqrt{GM_s} \cdot \sqrt{\frac{(1-e)}{a(1-e)}} \left(1 + \frac{M_p}{M_s} \right)$$

- When examining the equation $F_G = \frac{GM_s M_E}{r^\beta}$ we obtain the Inverse Square Law for a beta value of 2. Any change in this beta value results in a mathematical shift in the elliptical orbit.

- Gravitational force predicted by general relativity is given by: $F_G \approx \frac{GM_s M_M}{r^2} \left(1 + \frac{\alpha}{r^2} \right)$ where M_M is the Mass of mercury and $\alpha = 1.1E-8 \text{ AU}^2$

$$\text{Three Body Problem: } F_{E,J} = \frac{GM_J M_E}{r_{E,J}^2}$$

- Once again breaking this equation down to its components works roughly the same as the equation above for the relation between the Earth and the Sun.
- The graphs of the orbits in the three body problem interact and change based primarily on the masses of the planets involved in the observation.