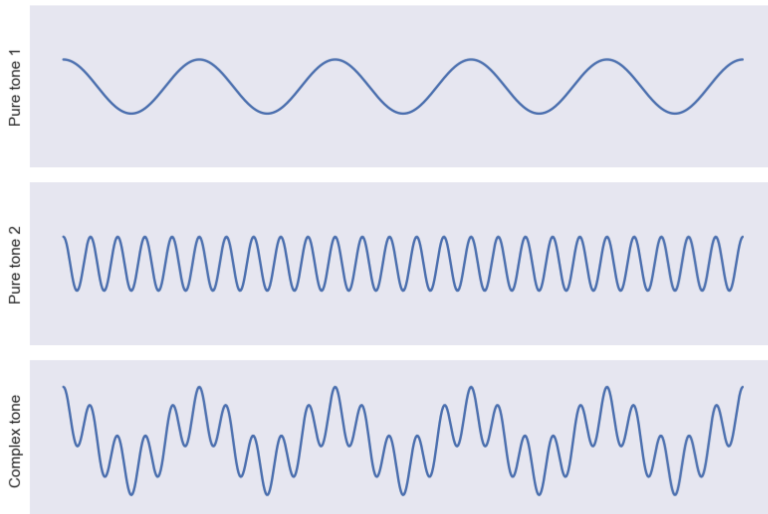
- The Fourier Transform:
- For a given (hypothetical) signal: $y(t) = \sum_{j=1}^{5} y_j sin(2\pi f_j t + \phi_j)$
- The fourier transform of the signal is given by:

$$y(t) = \int_{-\infty}^{\infty} Y(f) e^{-2\pi i f t} dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} Y(\omega/2\pi) e^{-i\omega t} d\omega$$



- Figure showing a similar example to the one in the book. The sum of these functions as shown in the third graph is the fourier transform.
- A forward transform followed by a backward transform returns the function to its original state as shown by the example: $\int_{-\infty}^{\infty} e^{i\omega(t-t')} d\omega = 2\pi\delta(t - t')$
- Discrete fourier transforms are far more applicable to real world problems we would encounter in physics.
- In the realm of discrete transforms we are still left with the idea that if we perform a forward transform followed by a backward transform the result will still be the original function. This method is fully reversible.
- Fast Fourier Transforms are specific algorithms which, given the specific input shape of a function, show exactly how to perform the operation of the transform. Examples of these FFTs are contained in the table below:

Sa (x) = sin(x) / x
sinc function

Table of Fourier Transform Pairs

Function, f(t)	Fourier Transform, F(ω)
Definition of Inverse Fourier Transform $f(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}F(\omega)e^{j\omega t}d\omega$	*Definition of Fourier Transform* $F(\omega)=\int_{-\infty}^{\infty}f(t)e^{-j\omega t}dt$
$f(t-t_0)$	$F(\omega)e^{-j\omega t_0}$
$f(t)e^{j\omega_0 t}$	$F(\omega-\omega_0)$
$f(\alpha t)$	$\frac{1}{\lvert\alpha\rvert}F(\frac{\omega}{\alpha})$
$F(t)$	$2\pi f(-\omega)$
$\frac{d^n f(t)}{dt^n}$	$(j\omega)^n F(\omega)$
$(-jt)^n f(t)$	$\frac{d^n F(\omega)}{d\omega^n}$
$\int_{-\infty}^{t} f(\tau)d\tau$	$\frac{F(\omega)}{j\omega}+\pi F(0)\delta(\omega)$
$\delta(t)$	1
$e^{j\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$
$\mathbf{sgn}(t)$	$\frac{2}{j\omega}$

Signals & Systems - Reference Tables 1

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- More examples of Fourier Transforms of signals were given graphically in order to show the frequency of the input signal.
- Examples were then shown where given an input signal, performing a transform allows us to examine the frequency versus the power of the specific signal similar to what we did in the fourier transform lab.