Esercizio 1

$q_1 = q_2 = 1 \mu C$, $r = 1$ cm

$$|F_e| = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2} = 89.88 \text{ N}$$

Esercizio 2

$q_1 = 9 q_2$ $\quad |F_e| = 18 N$

$$|F_e| = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2} \Rightarrow \overset{2}{\cancel{18}}[N] = \frac{1}{4\pi\varepsilon_0} \frac{\overset{1}{\cancel{9}} q_2^2}{r^2}$$

$$\Rightarrow q_2 = \pm\sqrt{8\pi\varepsilon_0}\, r \ [C] \ , \ q_1 = \pm 9 \sqrt{8\pi\varepsilon_0}\, r$$

Dall'informazione sulla repulsività della forza, sappiamo soltanto che le due cariche hanno lo stesso segno, ma non se siano positive o negative.

Esercizio 3

$q = 4 \mu C$, $r = 20$ cm

$$|E(r)| = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} = 8.99 \cdot 10^7 \frac{N}{C}$$

$$|E(2r)| = \frac{1}{4\pi\varepsilon_0} \frac{q}{(2r)^2} = \frac{1}{4\pi\varepsilon_0} \frac{q}{4r^2} = \frac{1}{4} |E(r)| = 2{,}25 \cdot 10^7 \frac{N}{C}$$

Esercizio 6

$r = 0{,}53 \cdot 10^{-10}$ m , $e = -1{,}6 \cdot 10^{-19}$ C , $m_e = 9.1 \cdot 10^{-31}$ kg , $m_p = 1.67 \cdot 10^{-27}$ kg

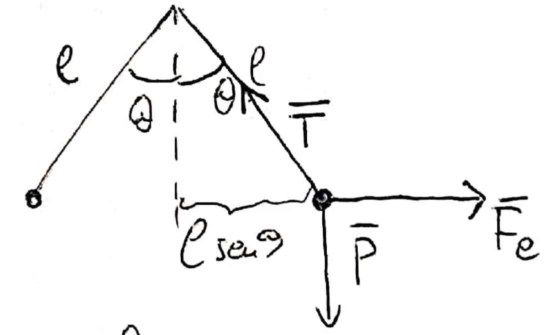
$$|F_e| = \frac{1}{4\pi\varepsilon_0} \frac{e^2}{r^2} = 8.19 \cdot 10^{-8} N$$

$$|F_g| = G \frac{m_e m_p}{r^2} \ , \text{ con } G = 6.67 \cdot 10^{-11} \frac{N m^2}{kg^2}$$

$$\Rightarrow |F_g| = 3.62 \cdot 10^{-47}$$

(l'interazione gravitazionale tra protone e elettrone è trascurabile, rispetto a quella elettromagnetica)

Esercizio 4



$m = 63\ mg$, $\ell = 12\ cm$, $\theta = 13°$

Condizione di equilibrio:

$$\bar{T} + \bar{F}_e + \bar{P} = 0$$

$$\Rightarrow \begin{cases} T_x + F_{e,x} = 0 \\ T_y + P_y = 0 \end{cases}$$

$T_x = -T \, \text{sen}\,\theta$, $T_y = T\cos\theta$

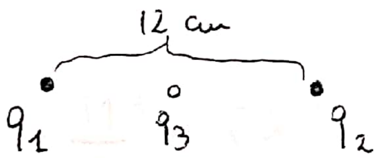
$P_y = -mg$ $\quad F_{e,x} = \dfrac{1}{4\pi\varepsilon_0} \dfrac{q^2}{(2\ell \, \text{sen}\,\theta)^2}$

$$\Rightarrow \begin{cases} \dfrac{1}{4\pi\varepsilon_0} \dfrac{q^2}{(2\ell\,\text{sen}\,\theta)^2} = T\,\text{sen}\,\theta \\ T\cos\theta = mg \Rightarrow T = \dfrac{mg}{\cos\theta} \end{cases} \Rightarrow$$

$$\Rightarrow q^2 = 4\pi\varepsilon_0 (2\ell\,\text{sen}\,\theta)^2 mg \tan\theta \Rightarrow$$

$$\Rightarrow q = \pm 1{,}94 \cdot 10^{-8}\ C$$

Esercizio 5



$q_1 = 8\ nC$, $q_2 = 12\ nC$, $q_3 = 20\ nC$

$r = 6\ cm$

$E_{p,1}(r) = \dfrac{1}{4\pi\varepsilon_0} \dfrac{q_1 q_3}{r}$ $\qquad E_{p,2}(r) = \dfrac{1}{4\pi\varepsilon_0} \dfrac{q_2 q_3}{r}$

$$W = \Delta E_p = E_p(r) - E_p(\infty) = E_p(r) = (E_{p,1} + E_{p,2})(r) =$$

$$= \frac{1}{4\pi\varepsilon_0} \frac{q_3}{r} (q_1 + q_2) = 5.99 \cdot 10^{-5}\ J$$

c) Campo elettrico generato da carica q in $\bar{r}'$:

$$\bar{E}(\bar{r}) = \frac{q}{4\pi\varepsilon_0} \frac{\bar{r} - \bar{r}'}{|\bar{r} - \bar{r}'|^3}$$

$$E_{1,x}(0,y_0) = \frac{q}{4\pi\varepsilon_0} \frac{(0-A)}{((0-A)^2 + (A-3A)^2)^{3/2}} = \frac{-q}{4\pi\varepsilon_0} \frac{\cancel{A}}{(5)^{3/2} A^{\cancel{3}2}} =$$

$$= \frac{28}{9} \frac{E_0}{(5)^{3/2}}$$

$$E_{1,y}(0,y_0) = \frac{q}{4\pi\varepsilon_0} \frac{(A-3A)}{(5)^{3/2} A^3} = \frac{-2q}{4\pi\varepsilon_0 (5)^{3/2} A^2} = \frac{56}{9} \frac{E_0}{(5)^{3/2}}$$

$$E_{2,x}(0,y_0) = \frac{q}{4\pi\varepsilon_0} \frac{(0-A)}{((0-A)^2 + (A-A)^2)^{3/2}} = \frac{-q}{4\pi\varepsilon_0 A^2} = \frac{28}{9} E_0$$

$$E_{2,y}(0,y_0) = 0$$

$$E_{3,x}(0,y_0) = \frac{-2q}{4\pi\varepsilon_0} \frac{(0-A)}{((0-A)^2 + (A-(-A))^2)} = \frac{2q}{4\pi\varepsilon_0 (5)^{3/2} A^2} =$$

$$= -\frac{56}{9} \frac{E_0}{(5)^{3/2}}$$

$$E_{3,y}(0,y_0) = -\frac{2q}{4\pi\varepsilon_0} \frac{(A-(-A))}{(5)^{3/2} A^3} = \frac{-4q}{4\pi\varepsilon_0 (5)^{3/2} A^2} =$$

$$= -\frac{112}{9} \frac{E_0}{(5)^{3/2}}$$

$$E_{TOT,x} = E_{1,x} + E_{2,x} + E_{3,x}$$

$$E_{TOT,y} = E_{1,y} + E_{2,y} + E_{3,y}$$

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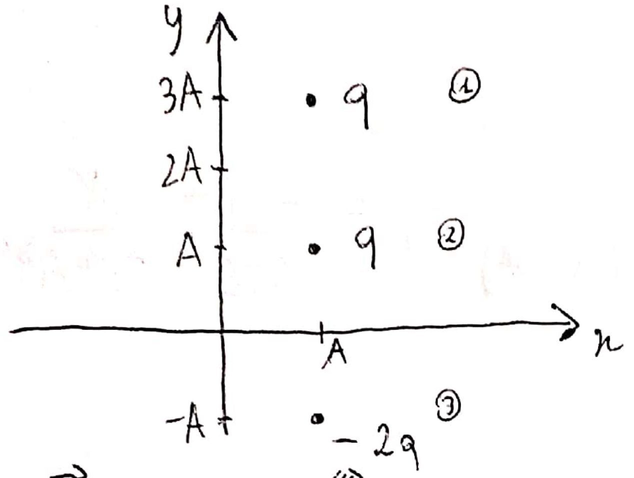
Esercizio 7



a) $\vec{E}(x_0, 0) = E_0 \cdot \vec{j}$

$$\vec{E}_1(x_0,0) = -\frac{1}{4\pi\varepsilon_0}\frac{q}{9A^2}\vec{j}$$

$$\vec{E}_2(x_0,0) = -\frac{1}{4\pi\varepsilon_0}\frac{q}{A^2}\vec{j}$$

$$\vec{E}_3(x_0,0) = -\frac{1}{4\pi\varepsilon_0}\frac{2q}{A^2}\vec{j}$$

$$\vec{E}(x_0,0) = (\vec{E}_1 + \vec{E}_2 + \vec{E}_3)(x_0,0) = -\frac{1}{4\pi\varepsilon_0}\frac{q}{A^2}\left(\frac{1}{9}+1+2\right)\vec{j} =$$

$$= -\frac{1}{4\pi\varepsilon_0}\frac{28}{9}\frac{q}{A^2}\vec{j} \Rightarrow$$

$$\Rightarrow q = -\left(\frac{9}{28}\right)4\pi\varepsilon_0 E_0 A^2$$

b) $V(r) = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$

$$V(x_0,0) = V_1(x_0,0) + V_2(x_0,0) + V_3(x_0,0) =$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{3A} + \frac{1}{4\pi\varepsilon_0}\frac{q}{A} + \frac{1}{4\pi\varepsilon_0}\frac{(-2q)}{A} = \frac{1}{4\pi\varepsilon_0}\frac{q}{A}\left(\frac{1}{3}+1-2\right) =$$

$$= -\frac{2}{3}\left(\frac{1}{4\pi\varepsilon_0}\frac{q}{A}\right) = -\frac{\cancel{2}^{1}}{\cancel{3}_{1}}\cdot\left(-\frac{\cancel{9}^{3}}{\cancel{28}_{14}}E_0 A\right) = \frac{3}{14}E_0 A$$

d) $W = \Delta E_p = E_p(0, y_0) - E_p(x_0, 0)$

$$E_p(x_0, 0) = (E_{p,1} + E_{p,2} + E_{p,3})(x_0, 0) =$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{Qq}{3A} + \frac{1}{4\pi\varepsilon_0}\frac{Qq}{A} + \frac{1}{4\pi\varepsilon_0}\frac{Q(-2q)}{A} =$$

$$= \frac{Qq}{4\pi\varepsilon_0 A}\left(\frac{1}{3} + 1 - 2\right) = \frac{3}{14} E_0 A Q$$

$$E_p(0, y_0) = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{\sqrt{5}A} + \frac{1}{4\pi\varepsilon_0}\frac{Qq}{A} + \frac{1}{4\pi\varepsilon_0}\frac{Q(-2q)}{\sqrt{5}A} =$$

$$= \frac{Qq}{4\pi\varepsilon_0 A}\left(\frac{1}{\sqrt{5}} + 1 - \frac{2}{\sqrt{5}}\right) = \frac{Qq}{4\pi\varepsilon_0 A}\,\frac{5-\sqrt{5}}{5} =$$

$$= Q\cdot\left(-\frac{9}{28}\right)\left(\frac{5-\sqrt{5}}{5}\right)E_0 A = -\frac{9(5-\sqrt{5})}{140}E_0 A Q$$

$$\Rightarrow W = -\left(\frac{9(5-\sqrt{5})}{140} + \frac{3}{14}\right)E_0 A Q$$

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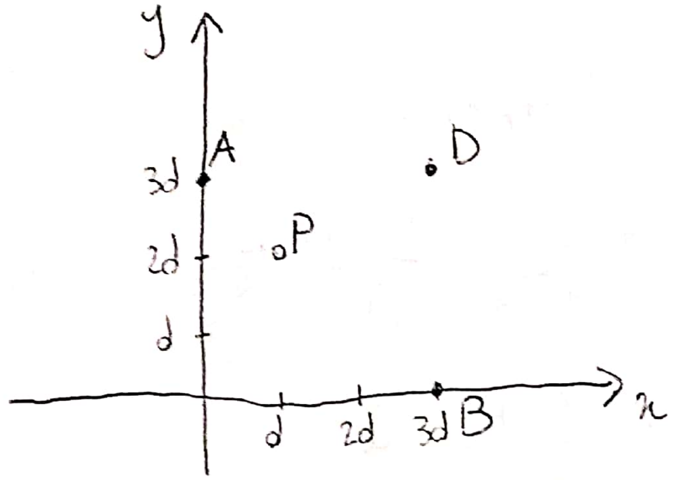
ESERCIZIO 8



$Q_A = 4q_0$

$Q_D = -q_0$

a) Nel punto P, la carica Q_D si trova sulla stessa retta di Q_A e Q_B, quindi i due campi elettrici hanno stessa direzione, e, per essere forza nulla deve essere:

$|\bar{E}_A| = |\bar{E}_B|$ nel punto P

$$\frac{1}{4\pi\varepsilon_0}\frac{Q_A}{r_A} = \frac{1}{4\pi\varepsilon_0}\frac{Q_B}{r_B}$$

$r_A = \sqrt{2}\,d$

$r_B = 2\sqrt{2}\,d$

$$\Rightarrow \frac{Q_A}{\cancel{\sqrt{2}}} = \frac{Q_B}{2\cancel{\sqrt{2}}} \Rightarrow Q_B = 2Q_A = 8q_0$$

b) $$F_x(D) = \frac{1}{4\pi\varepsilon_0}\frac{Q_D Q_A}{(3d)^2} = \frac{-4q_0^2}{4\pi\varepsilon_0 9d^2}$$

$$F_y(D) = \frac{1}{4\pi\varepsilon_0}\frac{Q_D Q_B}{(3d)^2} = \frac{-8q_0^2}{4\pi\varepsilon_0 9d^2}$$

$$\bar{F}(D) = \left(\frac{1}{4\pi\varepsilon_0}\frac{-4q_0^2}{9d^2}, \frac{1}{4\pi\varepsilon_0}\frac{-8q_0^2}{9d^2}\right)$$

c) $W = E_{P,D} - E_{P,P}$

$$E_{P,D} = \frac{1}{4\pi\varepsilon_0}\frac{Q_D Q_A}{3d} + \frac{1}{4\pi\varepsilon_0}\frac{Q_D Q_B}{3d} = \frac{-q_0}{4\pi\varepsilon_0 3d}(4q_0 + 8q_0) =$$

$$= \frac{-4q_0^2}{4\pi\varepsilon_0 d}$$

$$E_{P,P} = \frac{1}{4\pi\varepsilon_0}\frac{Q_D Q_A}{\sqrt{2}\,d} + \frac{1}{4\pi\varepsilon_0}\frac{Q_D Q_B}{2\sqrt{2}\,d} = \frac{1}{4\pi\varepsilon_0}\frac{-q_0}{\sqrt{2}\,d}\left(4q_0 + \frac{\cancel{8}^{\,4} q_0}{\cancel{2}_{\,1}}\right) =$$

$$= \frac{-8q_0^2}{4\pi\varepsilon_0\sqrt{2}\,d}$$

$$W = \frac{-4q_0^2}{4\pi\varepsilon_0 d} + \frac{8q_0^2}{4\pi\varepsilon_0\sqrt{2}\,d} = \frac{q_0^2}{4\pi\varepsilon_0 d}\left(\frac{8\sqrt{2}}{2} - 4\right) =$$

$$= \frac{\left(\frac{8\sqrt{2}-8}{2}\right) q_0^2}{4\pi\varepsilon_0 d}$$

d) Si assume che Q_D abbia massa m_d.

Per la conservazione dell'energia meccanica:

$$\underbrace{\frac{1}{2} m_D v_D^2}_{=0} + E_{P,D} = \frac{1}{2} m_D v_P^2 + E_{P,P} \Rightarrow$$

$$\Rightarrow \frac{1}{2} m_D v_P^2 = E_{P,D} - E_{P,P} = W \Rightarrow$$

$$\Rightarrow v_P = \sqrt{\frac{2W}{m_D}} = \sqrt{\frac{(8\sqrt{2}-8)}{4\pi\varepsilon_0 m_D d}}\; q_0$$

L'accelerazione a_P è nulla, poiché la carica Q_D si trova in equilibrio, nel punto P.