20/10/20

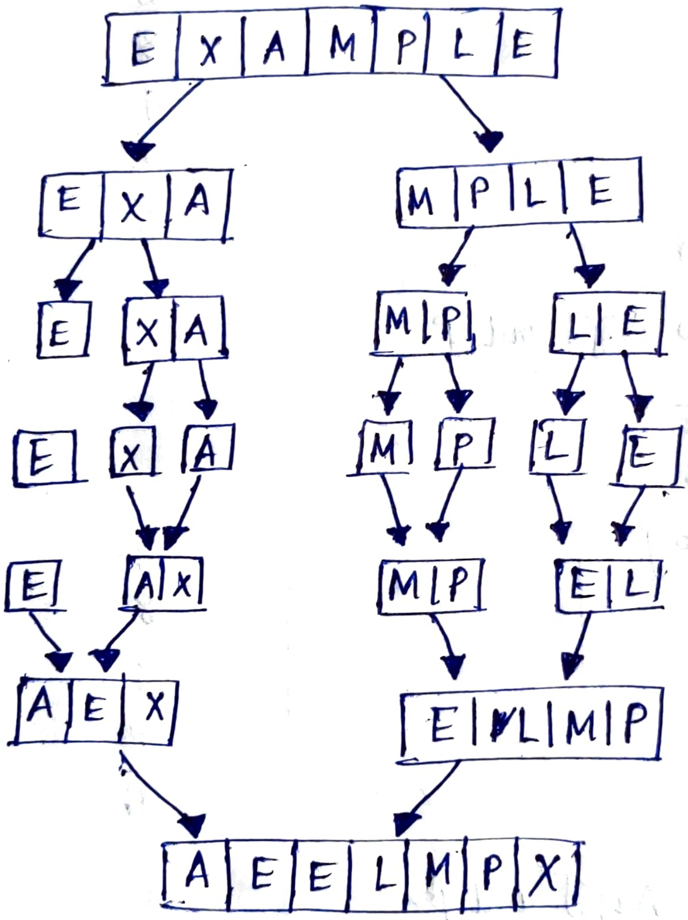
Q No.

HOMEWOR – (Divide and conquer, Greedy approach)

1. Apply merge sort to sort the list E, X, A, M, P, L, E in alphabetical order. Check if the algorithm is stable.

Ans

In place- doesn't use extra space.



The algorithm is stable because the 1st E in EXAMPLE is before the 2nd E (EXAMPLE)

A sorting algorithm is said to be stable if two objects with equal keys appear in the same order in sorted output as they appear in the input array to be sorted.

2. Apply quick sort to sort the list A, B, C, D, E in alphabetical order. Check the number of comparisons by choosing (i) the first element as the pivot (ii) last element as the pivot.

Ans (i) A B C D E

No of comparisions

$p = A$ $i = 0$ $j = 4$

$i = 1$	$B > A$ go to j	1
$j = 4$	$E > A$	2
$j = 3$	$D > A$	3
$j = 2$	$C > A$	4
$j = 1$	$B > A$	5
$j = 0$	$A > A$ (false)	6

Since $i > j$ swap $A[j]$ and P

B C D E

$p = B$ $i = 0$ $j = 3$

$i = 1$	$C > B$ go to j	7
$j = 3$	$E > B$	8
$j = 2$	$D > B$	9
$j = \cancel{3}1$	$C > B$	10
$j = 0$	$B \leq B$	11

Since $i > j$ Swap $A[j]$ and P

C D E

$p = C$ $i = 0$ $j = 2$

$i = 1$	$D > C$ go to j	12
$j = 2$	$E > C$	13
$j = 1$	$D > C$	14
$j = 0$	$C \leq C$	15

Since $i > j$ Swap $A[j]$ and P.

D E

$p = D$ $i = 0$ $j = 1$

$i=1$ $E > D$ go to j 16

$j=1$ $E > D$ 17

$j=0$ $D \leq D$ 18

swap $A[j]$ and P

E – no comparision (1 element)

No. of comparisions = 18.

(ii) A B C D E

$P=E$ $i=0$ $j=4$ — No. of comparisions

$i=0$ $A \leq E$ 1

$i=1$ $B \leq E$ 2

$i=2$ $C \leq E$ 3

$i=3$ $D \leq E$ 4

$i=4$ $E \leq E$ 5

i out of bounds

Swap $A[j]$ and P.

A B C D

$P=D$ $i=0$ $j=3$.

$i=0$ $A \leq D$ 6

$i=1$ $B \leq D$ 7

$i=2$ $C \leq D$ 8

$i=3$ $D \leq D$ 9

i out of bounds

swap $A[j]$ and P.

A B C

$P=C$ $i=0$ $j=2$

$i=0$ $A \leq C$ 10

$i=1$ $B \leq C$ 11

$i=2$ $C \leq C$ 12

i out of bounds
Swap A[j] and P
A B
p = B i=0 j=1

i=0 A ≤ ~~A~~B

i=1 B ≤ B

i out of bounds
Swap A[j] and P.
~~B~~ - A - single element (no comparision)
No. of comparisions = 14.

13
14.

3. Apply Strassen's algorithm to compute

$$\begin{bmatrix} 1 & 0 & 2 & 1 \\ 4 & 1 & 1 & 0 \\ 0 & 1 & 3 & 0 \\ 5 & 0 & 2 & 1 \end{bmatrix} * \begin{bmatrix} 0 & 1 & 0 & 1 \\ 2 & 1 & 0 & 4 \\ 2 & 0 & 1 & 1 \\ 1 & 3 & 5 & 0 \end{bmatrix}$$

Ans

$a_{00} = \begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix}$ $a_{01} = \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}$ $a_{10} = \begin{bmatrix} 0 & 1 \\ 5 & 0 \end{bmatrix}$ $a_{11} = \begin{bmatrix} 3 & 0 \\ 2 & 1 \end{bmatrix}$

$b_{00} = \begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix}$ $b_{01} = \begin{bmatrix} 0 & 1 \\ 0 & 4 \end{bmatrix}$ $b_{10} = \begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix}$ $b_{11} = \begin{bmatrix} 1 & 1 \\ 5 & 0 \end{bmatrix}$

$\boxed{m_1} = (a_{00} + a_{11}) * (b_{00} + b_{11})$

$= \begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 2 & 1 \end{bmatrix} * \begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 5 & 0 \end{bmatrix}$

$= \begin{bmatrix} 4 & 0 \\ 6 & 2 \end{bmatrix} * \begin{bmatrix} 1 & 2 \\ 7 & 1 \end{bmatrix}$

$m_1 = (a_{00} + a_{11}) * (b_{00} + b_{11})$
$= 6 * 2$
$= 12$

$m_2 = (a_{10} + a_{11}) * b_{00}$
$= 8 * 1 = 8$

$m_3 = a_{00} * (b_{01} - b_{11})$
$= 4 * 1 = 4$

$m_4 = a_{11} * (b_{10} - b_{00})$
$= 2 * 6 = 12$

$m_5 = (a_{00} + a_{01}) * b_{11}$
$= 4 * 1 = 4$

$m_6 = (a_{10} - a_{00}) * (b_{00} + b_{01})$
$= 2 * 3 = 6$

$m_7 = (a_{01} - a_{11}) * (b_{10} + b_{11})$
$= -2 * 8 = -16$

$m_1 + m_4 - m_5 + m_7 = 12 + 12 - 4 - 16$
$= 4$

$m_3 + m_5 = 8$

$m_2 + m_4 = 20$

$m_1 + m_3 - m_2 + m_6 = 14$

$$\boxed{m_1} = \begin{bmatrix} 4 & 8 \\ 20 & 14 \end{bmatrix}$$

$m_2 = (a_{10} + a_{11}) * b_{00}$

$$= \begin{bmatrix} 0 & 1 \\ 5 & 0 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 2 & 1 \end{bmatrix} * \begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix}$$

$$= \begin{array}{c} a_{00} \quad a_{01} \\ \begin{bmatrix} 3 & 1 \\ 7 & 1 \end{bmatrix} \\ a_{10} \quad a_{11} \end{array} * \begin{array}{c} b_{00} \quad b_{01} \\ \begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix} \\ b_{10} \quad b_{11} \end{array}$$

$m_1 = (a_{00} + a_{11}) * (b_{00} + b_{11})$
$= 4 * 1 = 4$

$m_2 = (a_{10} + a_{11}) * b_{00}$
$= 8 * 0 = 0$

$m_3 = a_{00} * (b_{01} - b_{11})$
$= 3 * (0) = 0$

$m_4 = a_{11} * (b_{10} - b_{00})$
$= 1 * (2) = 2$

$m_5 = (a_{00} + a_{01}) * b_{11}$
$= 4 * 1 = 4$

$m_6 = (a_{10} - a_{00}) * (b_{00} + b_{01})$
$= 4 * 1 = 4$

$m_7 = (a_{01} - a_{11}) * (b_{10} + b_{11})$
$= 0 * 3 = 0$

$m_1 + m_4 - m_5 + m_7 = 4 + 2 - 4 + 0 = 2$

$m_3 + m_5 = 0 + 4 = 4$

$m_2 + m_4 = 0 + 2 = 2$

$m_1 + m_3 - m_2 + m_6 = 4 + 0 - 0 + 4 = 8$

$$\boxed{m_2} = \begin{bmatrix} 2 & 4 \\ 2 & 8 \end{bmatrix}$$

$m_3 = a_{00} * (b_{01} - b_{11})$

$$= \begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix} * \begin{bmatrix} 0 & 1 \\ 0 & 4 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ 5 & 0 \end{bmatrix}$$

$$= \begin{array}{c} a_{00} \quad a_{01} \\ \begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix} \\ a_{10} \quad a_{11} \end{array} * \begin{array}{c} b_{00} \quad b_{01} \\ \begin{bmatrix} -1 & 0 \\ -5 & 4 \end{bmatrix} \\ b_{10} \quad b_{11} \end{array}$$

$m_1 = (a_{00} + a_{11}) * (b_{00} + b_{11})$
$= 2 * 3 = 6$

$m_2 = (a_{10} + a_{11}) * b_{00}$
$= 5 * -1 = -5$

$m_3 = a_{00} * (b_{01} - b_{11})$
$= 1 * (-4) = -4$

$m_4 = a_{11} * (b_{10} - b_{00})$
$= 1 * (-4) = -4$

$m_5 = (a_{00} + a_{01}) * b_{11}$
$= 1 * 4 = 4$

$m_6 = (a_{10} - a_{00}) * (b_{00} + b_{01})$
$= 3 * -1 = -3$

$m_7 = (a_{01} - a_{11}) * (b_{10} + b_{11})$
$= -1 * -1 = 1$

$m_1 + m_4 - m_5 + m_7 = 6 + -4 - 4 + 1 = -1$
$m_3 + m_5 = -4 + 4 = 0$
$m_2 + m_4 = -5 + -4 = -9$
$m_1 + m_3 - m_2 + m_6 = 6 + -4 - (-5) + (-3) = 4$

$$\boxed{m_3} = \begin{bmatrix} -1 & 0 \\ -9 & 4 \end{bmatrix}$$

$m_4 = a_{11} * (b_{10} - b_{00})$

$$= \begin{bmatrix} 3 & 0 \\ 2 & 1 \end{bmatrix} * \begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 0 \\ 2 & 1 \end{bmatrix} * \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

$m_1 = (a_{00} + a_{11}) * (b_{00} + b_{11})$
$= 4 \times 4 = 16$

$m_2 = (a_{10} + a_{11}) * b_{00}$
$= 3 \times 2 = 6$

$m_3 = a_{00} * (b_{01} - b_{11})$
$= 3 \times -3 = -9$

$m_4 = a_{11} * (b_{10} - b_{00})$
$= 1 \times (-3) = -3$

$m_5 = (a_{00} + a_{01}) \times b_{11}$
$= 3 \times 2 = 6$

$m_6 = (a_{10} - a_{00}) * (b_{00} + b_{01})$
$= -1 * 1 = -1$

$m_7 = (a_{01} - a_{11}) * (b_{10} + b_{11})$
$= -1 \times 1 = -1$

$m_1 + m_4 - m_5 + m_7 = 16 - 3 - 6 - 1 = 6$
$m_3 + m_5 = -3$
$m_2 + m_4 = 3$
$m_1 + m_3 - m_2 + m_6 = 0$

$$\boxed{m_4} = \begin{bmatrix} 6 & -3 \\ 3 & 0 \end{bmatrix}$$

$m_5 = (a_{00} + a_{01}) * b_{11}$

$$= \left(\begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}\right) * \begin{bmatrix} 1 & 1 \\ 5 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 1 \\ 5 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 1 \\ 5 & 0 \end{bmatrix}$$

$m_1 = (a_{00} + a_{11}) * (b_{00} + b_{11})$
$= 4 \times 1 = 4$

$m_2 = (a_{10} + a_{11}) \times b_{00}$
$= 6 \times 1 = 6$

$m_3 = a_{00} \times (b_{01} - b_{11})$
$= 3 \times 1 = 3$

$m_4 = a_{11} \times (b_{10} - b_{00})$
$= 1 \times 4 = 4$

$m_5 = (a_{00} + a_{01}) \times b_{11}$
$= 4 \times 0 = 0$

$m_6 = (a_{10} - a_{00}) \times (b_{00} + b_{01})$
$= 2 \times 2 = 4$

$m_7 = (a_{01} - a_{11}) \times (b_{10} + b_{11})$
$= 0 \times 5 = 0$

$m_1 + m_4 - m_5 + m_7 = 4 + 4 + 0 - 0 = 8$
$m_3 + m_5 = 3 + 0 = 3$
$m_2 + m_4 = 6 + 4 = 10$
$m_1 + m_3 - m_2 + m_6 = 4 + 3 - 6 + 4 = 5$

$$\boxed{m_5} = \begin{bmatrix} 8 & 3 \\ 10 & 5 \end{bmatrix}$$

$m_6 = (a_{10} - a_{10}) \times (b_{00} + b_{01})$

$$= \left(\begin{bmatrix} 0 & 1 \\ 5 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix}\right) \times \left(\begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 0 & 4 \end{bmatrix}\right)$$

$$= \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \times \begin{bmatrix} 0 & 2 \\ 2 & 5 \end{bmatrix}$$

$m_1 = (a_{00} + a_{11}) \times (b_{00} + b_{11})$
$= -2 \times 5 = -10$
$m_2 = (a_{10} + a_{11}) \times b_{00}$
$= 0 \times 0 = 0$
$m_3 = a_{00} \times (b_{01} - b_{11})$
$= -1 \times -3 = 3$
$m_4 = a_{11} \times (b_{10} - b_{00})$
$= -1 \times 2 = -2$
$m_5 = (a_{00} + a_{01}) \times b_{11}$
$= 0 \times 5 = 0$
$m_6 = (a_{10} - a_{00}) \times (b_{00} + b_{01})$
$= 2 \times 2 = 4$
$m_7 = (a_{01} - a_{11}) \times (b_{10} + b_{11})$
$= 2 \times 7 = 14$

$m_1 + m_4 - m_5 + m_7 = -10 - 2 - 0 + 14 = 2$
$m_3 + m_5 = 3 + 0 = 3$
$m_2 + m_4 = 0 + -2 = -2$
$m_1 + m_3 - m_2 + m_6 = -10 + 3 - 0 + 4 = -3$

$$\boxed{M6} = \begin{bmatrix} 2 & 3 \\ -2 & -3 \end{bmatrix}$$

$m_7 = (a_{01} - a_{11}) \times (b_{10} + b_{11})$
$$= \left(\begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 2 & 1 \end{bmatrix}\right) \times \left(\begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 5 & 0 \end{bmatrix}\right)$$
$$= \begin{bmatrix} -1 & 1 \\ -1 & -1 \end{bmatrix} \times \begin{bmatrix} 3 & 1 \\ 6 & 3 \end{bmatrix}$$

$m_1 = (a_{00} + a_{11}) \times (b_{00} + b_{11})$
$= \cancel{0}^{-2} \times 6 = \cancel{0} -12$
$m_2 = (a_{10} + a_{11}) \times b_{00}$
$= -2 \times 3 = -6$
$m_3 = a_{00} \times (b_{01} - b_{11})$
$= -1 \times -2 = 2$
$m_4 = a_{11} \times (b_{10} - b_{00})$
$= -1 \times (3) = -3$
$m_5 = (a_{00} + a_{01}) \times b_{11}$
$= 0 \times 3 = 0$
$m_6 = (a_{10} - a_{00}) \times (b_{00} + b_{01})$
$= 0 \times 4 = 0$

$m_7 = (a_{01} - a_{11}) \times (b_{10} + b_{11})$
$= 2 \times 9 = 18$

$m_1 + m_4 - m_5 + m_7 = -12 - 3 + 18 = 3$
$m_3 + m_5 = 2 + 0 = 2$
$m_2 + m_4 = -6 + -3 = -9$
$m_1 + m_3 - m_2 + m_6 = -12 + 2 + 6 + 0 = -4$

$$\boxed{M7} = \begin{bmatrix} 3 & 2 \\ -9 & -4 \end{bmatrix}$$

$m_1 + m_4 - m_5 + m_7$
$$= \begin{bmatrix} 4 & 8 \\ 20 & 14 \end{bmatrix} + \begin{bmatrix} 6 & -3 \\ 3 & 0 \end{bmatrix} - \begin{bmatrix} 8 & 3 \\ 10 & 5 \end{bmatrix} + \begin{bmatrix} 3 & 2 \\ -9 & -4 \end{bmatrix}$$
$$= \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix}$$

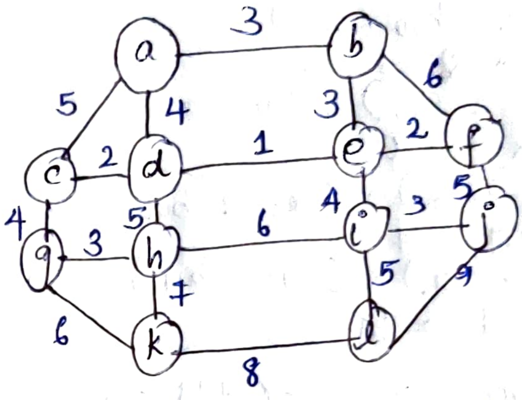
$$m_3 + m_5 = \begin{bmatrix} -1 & 0 \\ -9 & 4 \end{bmatrix} + \begin{bmatrix} 8 & 3 \\ 10 & 5 \end{bmatrix} = \begin{bmatrix} 7 & 3 \\ 1 & 9 \end{bmatrix}$$

$$m_2 + m_4 = \begin{bmatrix} 2 & 4 \\ 2 & 8 \end{bmatrix} + \begin{bmatrix} 6 & -3 \\ 3 & 0 \end{bmatrix} = \begin{bmatrix} 8 & 1 \\ 5 & 8 \end{bmatrix}$$

$m_1 + m_3 - m_2 + m_6$
$$= \begin{bmatrix} 4 & 8 \\ 20 & 14 \end{bmatrix} + \begin{bmatrix} -1 & 0 \\ -9 & 4 \end{bmatrix} - \begin{bmatrix} 2 & 4 \\ 2 & 8 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ -2 & -3 \end{bmatrix}$$
$$= \begin{bmatrix} 3 & 7 \\ 7 & 7 \end{bmatrix}$$

Ans:
$$\begin{bmatrix} 5 & 4 & 7 & 3 \\ 4 & 5 & 1 & 9 \\ 8 & 1 & 3 & 7 \\ 5 & 8 & 7 & 7 \end{bmatrix}$$

4. Find the minimum spanning tree of the graph given below using
(i) Prims algorithm

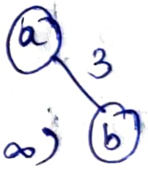


a

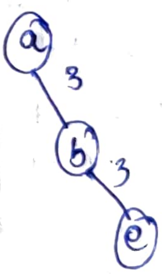
1) a(_, _)

b (a, 3)
c (a, 5)
d (a, 4)
e (_, ∞)
f (_, ∞)
g (_, ∞)
h (_, ∞) k (_, ∞)
i (_, ∞) l (_, ∞)
j (_, ∞)

2) b(a, 3) c (a, 5) d (a, 4) e (b, 3)
f (b, 6) g (_, ∞) h (_, ∞)
i (_, ∞) j (_, ∞) k (_, ∞) l (_, ∞)



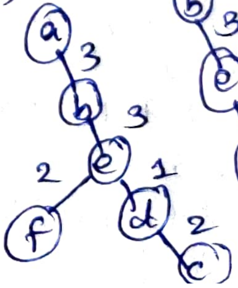
3) e(b, 3) c (a, 5) d (e, 1) f (e, 2)
g (_, ∞) h (_, ∞) i (e, 4) j (_, ∞)
k (_, ∞) l (_, ∞)



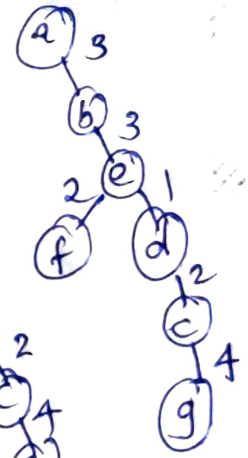
4) d(e, 1) c (d, 2) f (~~f~~ e, 2) g (_, ∞) h (d, 5)
i (e, 4) j (_, ∞) k (_, ∞) l (_, ∞)

5) c (d, 2) f(e, 2) g(c, 4) h(d, 5) i (e, 4)
j (_, ∞) k (_, ∞) l (_, ∞)

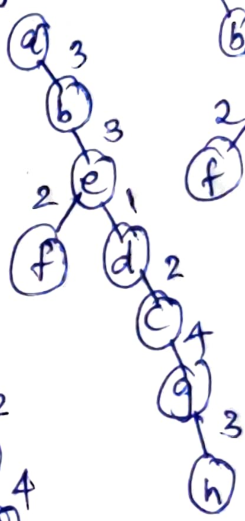
6) f (e, 2) g (c, 4) h (d, 5) i (e, 4)
j (f, 5) k (_, ∞) l (_, ∞)



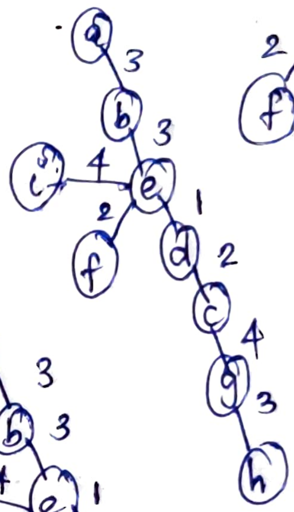
7) g(c, 4) h(g, 3) i(e, 4) j(f, 5) k(g, 6)
l(–, ∞)



8) h(g, 3) i(e, 4) j(f, 5) k(g, $\cancel{7}$ 6)
l(–, ∞)



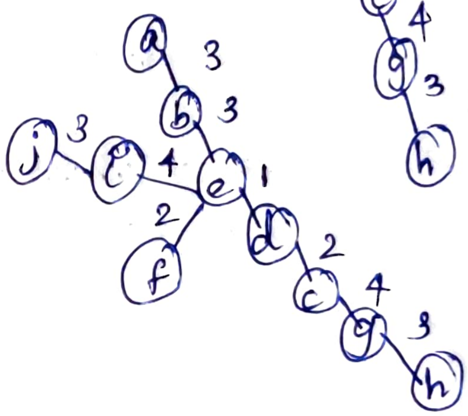
9) i(e, 4) j(i, 3) k(g, 6)
l(i, 5)



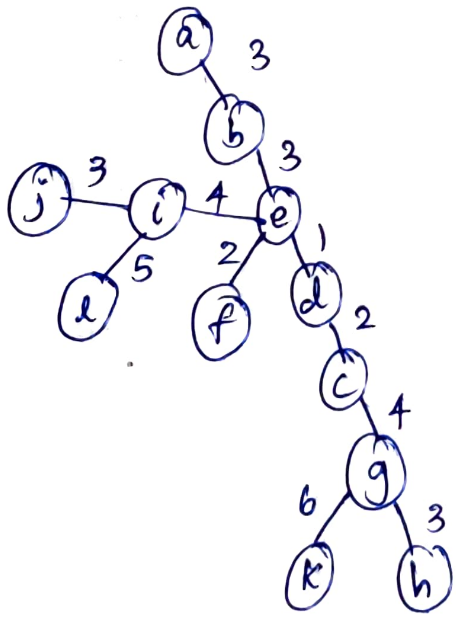
10) j(i, 3) k(g, 6) l(i, 5)

11) l(i, 5) k(g, 6)

12) k(g, 6) — empty



Final tree

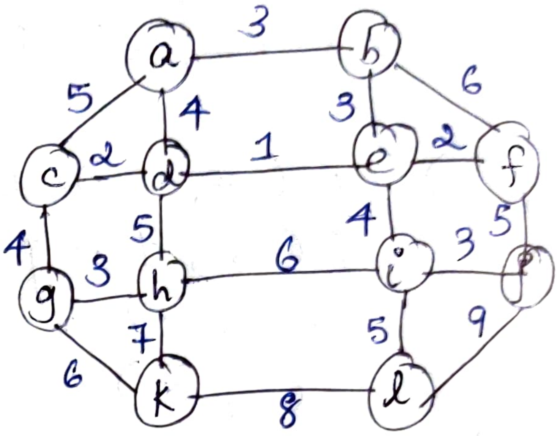


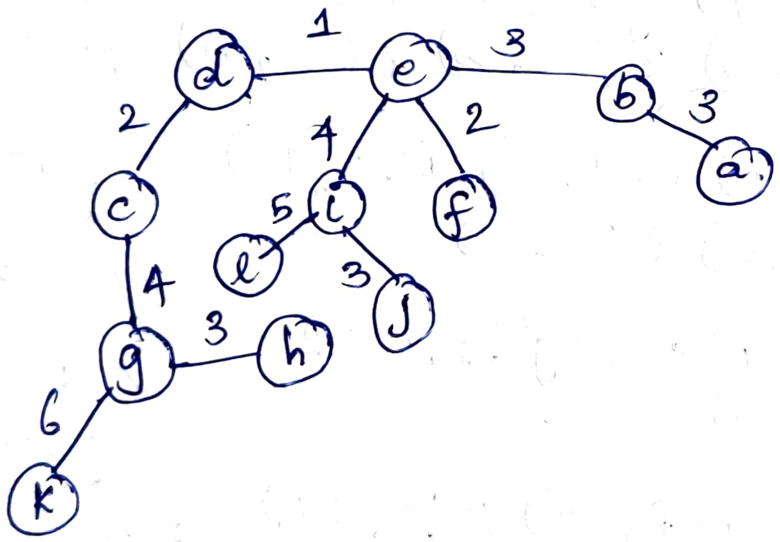
Sum of weights = 36.

WS-4 Continuation

4. (ii) Kruskal's algorithm

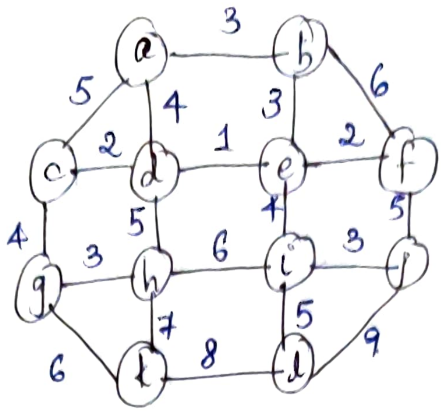
1	2	2	3	3		
de	cd	ef	ab	eb		
3	3	4	4	4		
gh	ij	ad	cg	ei		
5	5	5	5	6	6	6
ac	dh	fj	li	bf	hi	gk
7	8	9				
hk	kl	lf				





The sum of weights = 36

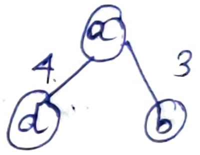
5. Find the shortest path in the graph given below considering the source vertex as 'a' using dijkstra's algorithm.



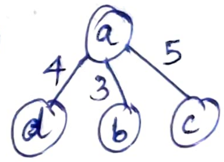
a(–, 0) b(a, 3) c(a, 5) d(a, 4) e(–, ∞) f(–, ∞) g(–, ∞) h(–, ∞) i(–, ∞) j(–, ∞) k(–, ∞) l(–, ∞)

b(a, 3) c(a, 5) d(a, 4) e(b, 6) f(b, 9) g(–, ∞) h(–, ∞) i(–, ∞) j(–, ∞) k(–, ∞) l(–, ∞)

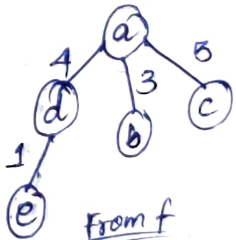
d(a, 4) c(a, 5) e(d, 5) f(b, 9) g(–, ∞) h(d, 9) i(–, ∞) j(–, ∞) k(–, ∞) l(–, ∞)



c(a, 5) e(d, 5) f(b, 9) g(c, 9) h(d, 9) i(–, ∞) j(–, ∞) k(–, ∞) l(–, ∞)

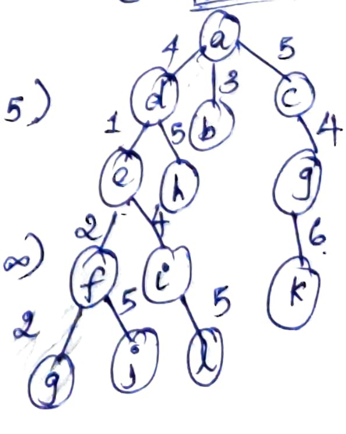


e(d, 5) f(e, 7) g(c, 9) h(d, 9) i(e, 9) j(–, ∞) k(–, ∞) l(–, ∞)



f(e, 7) g(c, 9) h(d, 9) i(e, 9) j(f, 12) k(–, ∞) l(–, ∞)

From f



g(c, 9) h(d, 9) i(e, 9) j(f, 12) k(g, 15) l(–, ∞)

h(d, 9) i(e, 9) j(f, 12) k(g, 15) l(–, ∞)

i(e, 9) j(f, 12) k(g, 15) l(i, 14)

j(f, 12) k(g, 15) l(i, 14)

l(i, 14) k(g, 15)

k(g, 15)

Homework - 5 Dynamic Programming.

1. Apply the bottom-up dynamic programming algorithm to the following instance of the knapsack problem.

item	weight	value
1	3	\$25
2	2	\$20
3	1	\$15
4	4	\$40
5	5	\$50

capacity $W = 6$.

Ans

i/j	0	1	2	3	4	5	6
0	0	0	0	0	0	0	0
1	0	0	0	25	25	25	25
2	0	0	20	25	25	45	45
3	0	15	20	35	40	45	60
4	0	15	20	35	40	55	60
5	0	15	20	35	40	55	65

$$F(i,j) = \begin{cases} \max[F(i-1,j), v_i + F(i-1, j-w_i)) & \text{if } j - w_i \geq 0 \\ F(i-1,j) & \text{if } j - w_i < 0 \end{cases}$$

$F(1,1)$ $1-3=-2<0$ $F(0,1)$

$F(1,2)$ $2-3=-1<0$ $F(0,2)$

$F(1,3)$ $3-3 \geq 0$ $\max(F(0,3), 25+F(0,0)) = 25$

$F(1,4)$ $\max(F(0,4), 25+F(0,1)) = 25$

$F(1,5)$ $\max(F(0,5), 25+F(0,2)) = 25$

$F(1,6)$ $\max(F(0,6), 25+F(0,3)) = 25$

$F(2,1)$ $1-2=-1<0$ $F(1,1)=0$

$F(2,2)$ $\max(F(1,2), 20+F(1,0)) = 20$

$F(2,3)$ $\max(F(1,3), 20+F(1,1)) = 25$

$F(2,4)$ $\max(F(1,4), 20+F(1,2)) = 25$

$F(2,5)$ $\max(F(1,5), 20+F(1,3)) = 45$

$F(2,6)$ $\max(F(1,6), 20+F(1,4)) = 45$

$F(3,1)$ max $(F(2,1),\ 15+F(2,0)) = 15$
$F(3,2)$ max $(F(2,2),\ 15+F(2,1)) = 20$
$F(3,3)$ max $(F(2,3),\ 15+F(2,2)) = 35$
$F(3,4)$ max $(F(2,4),\ 15+F(2,3)) = 40$
$F(3,5)$ max $(F(2,5),\ 15+F(2,4)) = 45$
$F(3,6)$ max $(F(2,6),\ 15+F(2,5)) = 60$

$F(4,1)$ $F(3,1) = 15$
$F(4,2)$ $F(3,2) = 20$
$F(4,3)$ $F(3,3) = 35$
$F(4,4)$ max $(F(3,4),\ 40+F(3,0)) = 40$
$F(4,5)$ max $(F(3,5),\ 40+F(3,1)) = 55$
$F(4,6)$ max $(F(3,6),\ 40+F(3,2)) = 60$

$F(5,1)$ $F(4,1) = 15$
$F(5,2)$ $F(4,2) = 20$
$F(5,3)$ $F(4,3) = 35$
$F(5,4)$ $F(4,4) = 40$
$F(5,5)$ max $(F(4,5),\ 50+F(4,0)) = 55$
$F(5,6)$ max $(F(4,6),\ 50+F(4,1)) = 65$

⇒ Weight = 6 . Value = 65.
Comparing $F(5,6)$ & $F(4,6)$ $65 > 60$ ⇒ 5^{th} element included

⇒ Weight = 1 Value = 15.
Comparing $F(4,1)$ & $F(3,1)$ $15 = 15$ ⇒ 4^{th} element not included.
Comparing $F(3,1)$ & $F(2,1)$ $15 > 0$ ⇒ 3^{rd} element included

⇒ Weight = 0 Value = 0

Ans: Value = 65 Items: 3, 5.

2. Apply Warshall's algorithm to find the transitive closure of the digraph defined by the following adjacency matrix:

$$R^0 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R^{(k)}[i, j] \leftarrow R^{(k-1)}[i, j]$ or $(R^{(k-1)}[i, k]$ and $R^{(k-1)}[k, j])$

$$R^1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$R^3 = \begin{bmatrix} 0 & 1 & 1 & \underline{1} \\ 0 & 0 & 1 & \underline{1} \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$R^2 = \begin{bmatrix} 0 & 1 & \underline{1} & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$R^4 = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

3. Solve the all-pairs shortest-path problem for the digraph with the following weight matrix

$$D^0 = \begin{bmatrix} 0 & 2 & \infty & 1 & 8 \\ 6 & 0 & 3 & 2 & \infty \\ \infty & \infty & 0 & 4 & \infty \\ \infty & \infty & 2 & 0 & 3 \\ 3 & \infty & \infty & \infty & 0 \end{bmatrix}$$

$D[i, j] \leftarrow \min \{ D[i, j], D[i, k] + D[k, j] \}$

$$D^4 = \begin{bmatrix} 0 & 2 & \underline{3} & 1 & \underline{4} \\ 6 & 0 & 3 & 2 & \underline{5} \\ \infty & \infty & 0 & 4 & \underline{7} \\ \infty & \infty & 2 & 0 & 3 \\ 3 & 5 & \underline{6} & 4 & 0 \end{bmatrix}$$

$$D^1 = \begin{bmatrix} 0 & 2 & \underline{9} & 1 & 8 \\ 6 & 0 & 3 & 2 & \underline{14} \\ \infty & \infty & 0 & 4 & \infty \\ \infty & \infty & 2 & 0 & 3 \\ 3 & 5 & \infty & 4 & 0 \end{bmatrix}$$

$$D^5 = \begin{bmatrix} 0 & 2 & 3 & 1 & 4 \\ 6 & 0 & 3 & 2 & 5 \\ \underline{10} & \underline{12} & 0 & 4 & 7 \\ \underline{6} & \underline{8} & 2 & 0 & 3 \\ 3 & 5 & 6 & 4 & 0 \end{bmatrix}$$

$$D^2 = \begin{bmatrix} 0 & 2 & \underline{5} & 1 & 8 \\ 6 & 0 & 3 & 2 & 14 \\ \infty & \infty & 0 & 4 & \infty \\ \infty & \infty & 2 & 0 & 3 \\ 3 & 5 & \underline{8} & 4 & 0 \end{bmatrix}$$

$$D^3 = \begin{bmatrix} 0 & 2 & 5 & 1 & 8 \\ 6 & 0 & 3 & 2 & 14 \\ \infty & \infty & 0 & 4 & \infty \\ \infty & \infty & 2 & 0 & 3 \\ 3 & 5 & 8 & 4 & 0 \end{bmatrix}$$