习题三解答

1.(1) 由 $F(-\infty,-\infty)=0$ 得 $a(b-\frac{\pi}{2})(c-\frac{\pi}{2})=0$

$F(+\infty,+\infty)=1$ 得 $a(b+\frac{\pi}{2})(c+\frac{\pi}{2})=1$

$F(x,-\infty)=0$ 得 $a(b+\arctan x)(c-\frac{\pi}{2})=0$

$F(-\infty,y)=0$ 得 $a(b-\frac{\pi}{2})(c+\arctan\frac{y}{2})=0$

$\Rightarrow$ $a\neq 0$, $\Rightarrow b=c=\frac{\pi}{2}$, $\Rightarrow a=\frac{1}{\pi^2}$

(2) $P\{X>0,Y>0\}=F(+\infty,+\infty)-F(0,+\infty)-F(+\infty,0)+F(0,0)$

$=1-\frac{1}{2}-\frac{1}{2}+\frac{1}{4}=\frac{1}{4}$

2. (1) 由 $F(+\infty,+\infty)=1$ 得 $ab=1$

由题意 $F(0,0)=0$ 得 $(a-1)(b-1)=0$

$\Rightarrow a=b=1$

(2) $P\{X>0,Y\le 2\}=F(+\infty,2)=\cancel{0}\ 1-e^{-2}$

3. 利用乘法公式 $P(AB)=P(B|A)P(A)$

Y \ X	1	2	3
1	0	$\frac{1}{5}\cdot\frac{2}{6}=\frac{2}{30}$	$\frac{1}{5}\cdot\frac{3}{6}=\frac{3}{30}$
2	$\frac{2}{5}\cdot\frac{1}{6}=\frac{2}{30}$	$\frac{1}{5}\cdot\frac{2}{6}=\frac{2}{30}$	$\frac{2}{5}\cdot\frac{3}{6}=\frac{6}{30}$
3	$\frac{3}{5}\cdot\frac{1}{6}=\frac{3}{30}$	$\frac{3}{5}\cdot\frac{2}{6}=\frac{6}{30}$	$\frac{2}{5}\cdot\frac{3}{6}=\frac{6}{30}$

4.

Y \ X	1	2	3	4	5
1	$\frac{1}{5}$	$\frac{1}{2}\cdot\frac{1}{5}$	$\frac{1}{3}\cdot\frac{1}{5}$	$\frac{1}{4}\cdot\frac{1}{5}$	$\frac{1}{5}\cdot\frac{1}{5}$
2	0	$\frac{1}{2}\cdot\frac{1}{5}$	$\frac{1}{3}\cdot\frac{1}{5}$	$\frac{1}{4}\cdot\frac{1}{5}$	$\frac{1}{5}\cdot\frac{1}{5}$
3	0	0	$\frac{1}{3}\cdot\frac{1}{5}$	$\frac{1}{4}\cdot\frac{1}{5}$	$\frac{1}{5}\cdot\frac{1}{5}$
4	0	0	0	$\frac{1}{4}\cdot\frac{1}{5}$	$\frac{1}{5}\cdot\frac{1}{5}$
5	0	0	0	0	$\frac{1}{5}\cdot\frac{1}{5}$

5.

Y \ X	1	2	3	4	·	-
2	$(0.1)^2$	0	0	0		
3	$(0.1)^2\,0.9$	$0.9\cdot(0.1)^2$	0	0		
4	$(0.1)^2\cdot 0.9^2$	$(0.1)^2\,0.9^2$	$(0.1)^2\,0.9^2$	0		
5						
⋮						

$P\{X=2,Y=4\}$

$=0.9\times 0.1\times 0.9\times 0.1$

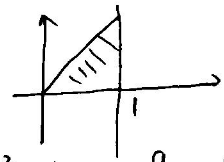
第一次 第三次 第二、四次

6. X 取值为 $1, 2, \cdots$　　Y 取值为 $3, 4, 5, 6$

则 $P\{X=m, Y=j\} = \frac{1}{6} \times \left(\frac{1}{3}\right)^{m-1}$　（第m次点数为3.4.5.6之一，前m-1次点数为1.2）

每次点数≤2 的概率为 $\frac{1}{3}$

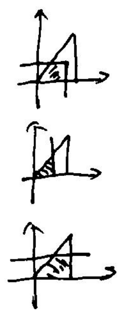
7. (1) 如图，由 $\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(x,y)\,dx\,dy = 1$



得 $\int_0^1 dx \int_0^x a(x+y)\,dy = a\int_0^1 \left(x^2 + \frac{x^2}{2}\right) dx = \frac{a}{2} = 1$　得 $a=2$

(2)

$$F(x,y) = \begin{cases} 0 & x<0 \text{ 或 } y<0 \\ \iint_D f(x,y)\,dx\,dy & 0<x\le 1,\ y\le x \\ & 0<x\le 1,\ y\ge x \\ & x>1,\ 0\le y\le 1 \\ & x>1,\ y>1 \end{cases} = \begin{cases} 0 \\ \int_0^y dy \int_y^x 2(x+y)\,dx \\ \int_0^x dx \int_0^x 2(x+y)\,dy \\ \int_0^y dy \int_y^1 2(x+y)\,dx \\ 1 \end{cases}$$



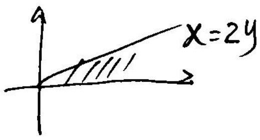
$$= \begin{cases} 0 & x<0 \text{ 或 } y<0 \\ x^2y + xy^2 - y^3 & 0\le x\le 1,\ 0\le y\le x \\ x^3 & 0\le x\le 1,\ y\ge x \\ y + y^2 - y^3 & 0\le x\le 1,\ y\ge 1 \\ 1 & x>1,\ y>1 \end{cases}$$

8. (1)

$$F(x,y) = \begin{cases} 0 & x<0 \text{ 或 } y<0 \\ \int_0^x dx \int_0^y f(x,y)\,dy & x>0,\ y>0 \end{cases}$$

$$= \begin{cases} 0 & x<0 \text{ 或 } y<0 \\ \int_0^x 3e^{-3x}dx \int_0^y 2e^{-2y}dy = (1-e^{-3x})(1-e^{-2y}) & x>0,\ y>0 \end{cases}$$

(2) $P\{2Y-X\le 0\} = \int_0^{+\infty} dx \int_0^{\frac{x}{2}} 6e^{-(3x+2y)}\,dy$



$= \int_0^{+\infty} 3e^{-3x}\left[-e^{-2y}\right]_0^{\frac{x}{2}} dx$

$= \int_0^{+\infty} 3(e^{-3x} - e^{-4x})\,dx$

$= 3\left[-\frac{1}{3}e^{-3x} + \frac{1}{4}e^{-4x}\right]_0^{+\infty} = 3\left(\frac{1}{3} - \frac{1}{4}\right) = \frac{1}{4}$

9. (1) 由 $\int_{-\infty}^{+\infty}dx\int_{-\infty}^{+\infty}f(x,y)dy=1$ 得 $a\int_0^1dx\int_0^2(x+y)dy=1$

即 $a\int_0^1(2x+2)dx=1 \Rightarrow 3a=1 \Rightarrow a=\frac{1}{3}$

(2) $P\{X\le\frac{1}{2}, Y\ge 1\}=\int_0^{\frac{1}{2}}dx\int_1^2\frac{1}{3}(x+y)dy=\int_0^{\frac{1}{2}}(\frac{x}{3}+\frac{3}{6})dx=\frac{7}{24}$

$P\{X\ge Y\}=\int_0^1dx\int_0^x\frac{1}{3}(x+y)dy=\frac{1}{3}\int_0^1(x^2+\frac{x^2}{2})dy=\frac{1}{3}\int_0^1\frac{3}{2}x^2dx=\frac{1}{6}$

10 由题1. $F(x,y)=\frac{1}{\pi^2}(\frac{\pi}{2}+\arctan x)(\frac{\pi}{2}+\arctan\frac{y}{2})$ $-\infty<x<+\infty, -\infty<y<+\infty,$

$\therefore f(x,y)=\frac{\partial^2F}{\partial x\partial y}=\frac{1}{\pi^2}\frac{1}{1+x^2}\frac{\frac{1}{2}}{1+(\frac{y}{2})^2}=\frac{2}{\pi^2}\frac{1}{(1+x^2)(4+y^2)}$ $\begin{matrix}-\infty<x<+\infty\\-\infty<y<+\infty\end{matrix}$

由题2. $F(x,y)=\begin{cases}(1-e^{-2x})(1-e^{-y}) & x>0,\ y>0\\ 0 & 其他\end{cases}$

$\therefore f(x,y)=\begin{cases}e^{-(2x+y)} & x>0,\ y>0\\ 0 & 其他\end{cases}$

11. 由题3 X的边缘分布律为

X	1	2	3
P	$\frac{5}{30}$	$\frac{10}{30}$	$\frac{15}{30}$

Y的边缘分布律为

Y	1	2	3
P	$\frac{5}{30}$	$\frac{10}{30}$	$\frac{15}{30}$

由题5. 参考题5表格

$P\{Y=j\}=(j-1)\times(0.1)^2\times(0.9)^{j-2}$ $j=2, 3, \cdots$

$P\{X=i\}=(0.1)^2\frac{(0.9)^{i-1}}{1-0.9}=0.1\times0.9^{i-1}$ $i=1, 2, \cdots$

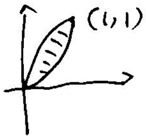
12.

X	1	2	3
P	$\frac{8}{20}$	$\frac{7}{20}$	$\frac{5}{20}$

Y	1	2	3	4
P	$\frac{5}{20}$	$\frac{5}{20}$	$\frac{4}{20}$	$\frac{6}{20}$

Y	1	2	3	4
$P\{Y=i\mid X=1\}$	$\frac{1}{8}$	$\frac{2}{8}$	$\frac{2}{8}$	$\frac{3}{8}$

X	1	2	3
$P\{X=i\mid Y=4\}$	$\frac{3}{6}$	$\frac{1}{6}$	$\frac{2}{6}$

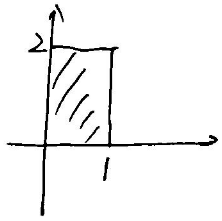
13. 如图. 

联合概率密度 $f(x,y)=\begin{cases} 6 & x^2\le y\le\sqrt{x} \\ 0 & 其他 \end{cases}$

$f_X(x)=\int_{-\infty}^{+\infty}f(x,y)dy=\begin{cases}\int_{x^2}^{\sqrt{x}}6dxdy & 0\le x\le 1\\ 0 & 其他\end{cases}=\begin{cases}6(\sqrt{x}-x^2) & x\in(0,1)\\ 0 & 其他\end{cases}$

同理 $f_Y(y)=\begin{cases}6(\sqrt{y}-y^2) & 0\le y\le 1\\ 0 & 其他\end{cases}$

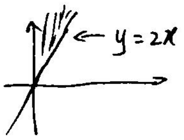
14.

(1) $f_X(x)=\int_{-\infty}^{+\infty}f(x,y)dy=\begin{cases}\int_0^2(x^2+\frac{1}{3}xy)dy & 0\le x\le 1\\ 0 & 其他\end{cases}$ 

$=\begin{cases}2x^2+\frac{2}{3}x & 0\le x\le 1\\ 0 & 其他\end{cases}$

$f_Y(y)=\int_{-\infty}^{+\infty}f(x,y)dx=\begin{cases}\int_0^1(x^2+\frac{1}{3}xy)dx & 0\le y\le 2\\ 0 & 其他\end{cases}=\begin{cases}\frac{1}{3}+\frac{1}{6}y & 0\le y\le 2\\ 0 & 其他\end{cases}$

(2) $P\{X+Y\le 1\}=\int_0^1dx\int_0^{1-x}(x^2+\frac{1}{3}xy)dy=\int_0^1[x^2(1-x)+\frac{1}{6}x(1-x)^2]dx=\frac{7}{72}$

15. $0<2x<y<+\infty$ 如图 

(1) 由 $\int_{-\infty}^{+\infty}dx\int_{-\infty}^{+\infty}f(x,y)dy=\int_0^{+\infty}dx\int_{2x}^{+\infty}ae^{-(x+y)}dy=1$

得 $a\int_0^{+\infty}e^{-x}[-e^{-y}]_{2x}^{+\infty}dx=a\int_0^{+\infty}e^{-3x}dx=a[-\frac{1}{3}e^{-3x}]_0^{+\infty}=\frac{a}{3}=1$

$\therefore a=3$

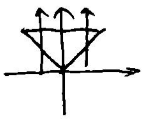
(2) $f(x,y)=\begin{cases}3e^{-(x+y)} & 0<2x<y<+\infty\\ 0 & 其他\end{cases}$

$\therefore f_X(x)=\int_{-\infty}^{+\infty}f(x,y)dy=\begin{cases}\int_{2x}^{+\infty}3e^{-(x+y)}dy & x>0\\ 0 & 其他\end{cases}=\begin{cases}3e^{-3x} & x>0\\ 0 & 其他\end{cases}$

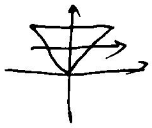
$f_Y(y)=\int_{-\infty}^{+\infty}f(x,y)dx=\begin{cases}\int_0^{\frac{y}{2}}3e^{-(x+y)}dx & y>0\\ 0 & 其他\end{cases}=\begin{cases}3e^{-y}(1-e^{-\frac{y}{2}}) & y>0\\ 0 & 其他\end{cases}$

(3) $P\{X\ge 1, Y\ge 2\}=\int_1^{+\infty}dx\int_{2x}^{+\infty}3e^{-(x+y)}dy=\int_1^{+\infty}3e^{-x}[-e^{-y}]_{2x}^{+\infty}dx=\int_1^{+\infty}3e^{-3x}dx$

$=[-e^{-3x}]_1^{+\infty}=e^{-3}$

16. (1) 

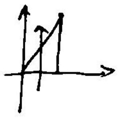
$$f_X(x)=\begin{cases}\int_{-x}^{1}\frac{15}{2}x^2y\,dy & -1\le x<0\\ \int_{x}^{1}\frac{15}{2}x^2y\,dy & 0\le x\le 1\\ 0 & 其他\end{cases}=\begin{cases}\frac{15}{4}x^2(1-x^2) & -1\le x\le 1\\ 0 & 其他\end{cases}$$



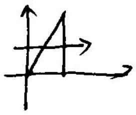
$$f_Y(y)=\begin{cases}\int_{-y}^{y}\frac{15}{2}x^2y\,dx & 0\le y\le 1\\ 0 & 其他\end{cases}=\begin{cases}5y^4 & 0\le y\le 1\\ 0 & 其他\end{cases}$$

17. 由题意 $f(x,y)=\begin{cases}1 & 0\le x\le 1\quad 0\le y\le 2x\\ 0 & 其他\end{cases}$

(1) $f_X(x)=\begin{cases}\int_0^{2x}1\,dy=2x & 0\le x\le 1\\ 0 & 其他\end{cases}$

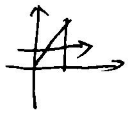


$f_Y(y)=\begin{cases}\int_{\frac{y}{2}}^{1}1\,dx=1-\frac{y}{2} & 0\le y\le 2\\ 0 & 其他\end{cases}$



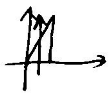
$0<y<2$时

$$f_{X|Y}(x|y)=\frac{f(x,y)}{f_Y(y)}=\begin{cases}\frac{1}{1-\frac{y}{2}}=\frac{2}{2-y} & \frac{y}{2}\le x\le 1\\ 0 & 其他\end{cases}$$



$0<x<1$时

$$f_{Y|X}(y|x)=\frac{f(x,y)}{f_X(x)}=\begin{cases}\frac{1}{2x} & 0\le y\le 2x\\ 0 & 其他\end{cases}$$



(2) 由(1) $f_{Y|X}(y|x)\Big|_{x=\frac{3}{4}}=\begin{cases}\frac{2}{3} & 0\le y\le\frac{3}{2}\\ 0 & 其他\end{cases}$

$$P\{Y>1|X=\tfrac{3}{4}\}=\int_1^{\frac{3}{2}}\frac{2}{3}\,dx=\frac{1}{3}$$

18. $$f_X(x)=\begin{cases}\int_0^1\frac{1}{2}x(3x+4y)\,dy & 0\le x\le 1\\ 0 & 其他\end{cases}=\begin{cases}\frac{3}{2}x^2+x & 0\le x\le 1\\ 0 & 其他\end{cases}$$

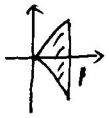
$$f_Y(y)=\begin{cases}\int_0^1\frac{1}{2}x(3x+4y)\,dx & 0\le y\le 1\\ 0 & 其他\end{cases}=\begin{cases}\frac{1}{2}+y & 0\le y\le 1\\ 0 & 其他\end{cases}$$

$0<y<1$时

$$f_{X|Y}(x|y)=\frac{f(x,y)}{f_Y(y)}=\begin{cases}\frac{\frac{1}{2}x(3x+4y)}{\frac{1}{2}+y} & 0\le x\le 1\\ 0 & 其他\end{cases}=\begin{cases}\frac{x(3x+4y)}{1+2y} & 0\le x\le 1\\ 0 & 其他\end{cases}$$

$0<x<1$时

$$f_{Y|X}(y|x)=\frac{f(x,y)}{f_X(x)}=\begin{cases}\frac{\frac{1}{2}x(3x+4y)}{\frac{3}{2}x^2+x} & 0\le y\le 1\\ 0 & 其他\end{cases}=\begin{cases}\frac{3x+4y}{3x+2} & 0\le y\le 1\\ 0 & 其他\end{cases}$$

19. 

$$f_X(x)=\begin{cases}\int_{-x}^{x}1\,dy=2x & 0\le x\le 1\\ 0 & \text{其他}\end{cases}$$

$$f_Y(y)=\begin{cases}\int_{-y}^{1}1\,dx=1+y & -1\le y\le 0\\ \int_{y}^{1}dx=1-y & 0<y\le 1\\ 0 & \text{其他}\end{cases}$$

$-1<y<1$时

$$\therefore\ f_{X|Y}(x|y)=\frac{f(x,y)}{f_Y(y)}=\begin{cases}\frac{1}{1+y} & -y\le x\le 1\\ \frac{1}{1-y} & y\le x\le 1\\ 0 & \text{其他}\end{cases}$$

$0<x<1$时

$$f_{Y|X}(y|x)=\frac{f(x,y)}{f_X(x)}=\begin{cases}\frac{1}{2x} & -x\le y\le x\\ 0 & \text{其他}\end{cases}$$

20 (1) $$f(x,y)=\frac{\partial^2F}{\partial x\partial y}=\begin{cases}\frac{xe^{-x}}{(1+y)^2} & x>0\quad y>0\\ 0 & \text{其他}\end{cases}$$

(2) $$F_X(x)=F(x,+\infty)=\begin{cases}\lim\limits_{y\to+\infty}\frac{y}{1+y}[1-(x+1)e^{-x}] & x>0\\ 0\end{cases}=\begin{cases}1-(x+1)e^{-x} & x>0\\ 0\end{cases}$$

$$F_Y(y)=F(+\infty,y)=\begin{cases}\lim\limits_{x\to+\infty}[1-(x+1)e^{-x}]\frac{y}{1+y}=\frac{y}{1+y} & y>0\\ 0 & \text{其他}\end{cases}$$

$\therefore F(x,y)=F_X(x)\cdot F_Y(y)$ 即X与Y独立.

21. 利用，由$F(x,y)$在(x,y)连续时，$f(x,y)=\frac{\partial^2F}{\partial x\partial y}$

得 $$f(x,y)=\begin{cases}1 & 0\le x\le 1,\ 0\le y\le 2x\\ 0 & \text{其他}\end{cases}$$

由7题 $$f_X(x)=\begin{cases}2x & 0\le x\le 1\\ 0 & \text{其他}\end{cases}\qquad f_Y(y)=\begin{cases}1-\frac{y}{2} & 0\le y\le 2\\ 0 & \text{其他}\end{cases}$$

$\therefore f(x,y)\ne f_X(x)\cdot f_Y(y)$ 即X,Y不独立.

22. (1) 题3. $P\{X=1,Y=1\}=0\ne P\{X=1\}\cdot P\{Y=1\}\Longrightarrow$ 不独立.

(2) 由6, $P\{X=m,Y=j\}=\frac{1}{6}(\frac{1}{3})^{m-1}$

$P\{X=m\}=\frac{2}{3}(\frac{1}{3})^{m-1}\ [\sum_{3,4,5,6}]$ $\quad m=1,2,\cdots$

$P\{Y=j\}=\sum_{m=1}^{\infty}\frac{1}{6}(\frac{1}{3})^{m-1}=\frac{1}{6}\frac{1}{1-\frac{1}{3}}=\frac{1}{4}\qquad j=3,4,5,6$

$\therefore P\{X=m\}\cdot P\{Y=j\}=\frac{2}{3}(\frac{1}{3})^{m-1}\cdot\frac{1}{4}=\frac{1}{6}(\frac{1}{3})^{m-1}=P\{X=m,Y=j\}$

$\therefore$ X,Y相互独立.

23 (1)

第7题中 $f(x,y)=\begin{cases}2(x+y) & 0\le x\le 1\quad 0\le y\le x\\ 0 & 其他\end{cases}$

得 $f_X(x)=\begin{cases}\int_0^x 2(x+y)\,dy = 2x^2+x^2=3x^2, & 0\le \cancel{y}\,x\le 1\\ 0 & 其他\end{cases}$

$f_Y(y)=\begin{cases}\int_y^1 2(x+y)\,dx = 1-y^2+2y(1-y)=1+2y-3y^2 & 0\le y\le 1\\ 0 & 其他\end{cases}$

$\Rightarrow f(x,y)\ne f_X(x)\cdot f_Y(y)$

不独立.

(2) 题8 $F(x,y)=\begin{cases}(1-e^{-3x})(1-e^{-2y}) & x>0,y>0\\ 0 & 其他\end{cases}$

$F_X(x)=\begin{cases}1-e^{-3x} & x>0\\ 0 & 其他\end{cases}$ $\quad F_Y(y)=\begin{cases}1-e^{-2y} & y>0\\ 0 & 其他\end{cases}$

$\therefore X, Y$ 相互独立.

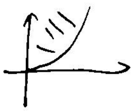
24. (1) $f_X(x)=\begin{cases}\int_0^{+\infty}\frac{x}{2}e^{-\frac{x^2}{4}-y}\,dy\\ 0 & 其他\end{cases} = \begin{cases}\frac{x}{2}e^{-\frac{x^2}{4}}\left[-e^{-y}\right]_0^{+\infty} = \frac{x}{2}e^{-\frac{x^2}{4}} & x>0\\ 0 & 其他\end{cases}$

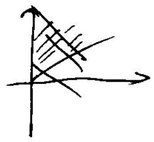
$f_Y(y)=\begin{cases}\int_0^{+\infty}\frac{x}{2}e^{-\frac{x^2}{4}-y}\,dx & y>0\\ 0 & 其他\end{cases} = \begin{cases}e^{-y}\left[-e^{-\frac{x^2}{4}}\right]_0^{+\infty}=e^{-y} & y>0\\ 0 & 其他\end{cases}$

$\therefore f(x,y)=f_X(x)\cdot f_Y(y) \quad \therefore X, Y$ 独立

(2)

$$P\{X^2-4Y^2\le 0\} = \iint\limits_{x^2-4y\le 0}\frac{x}{2}e^{-\frac{x^2}{4}-y}\,dx\,dy$$





$$= \int_0^{+\infty}dx\int_{\frac{x^2}{4}}^{+\infty}\frac{x}{2}e^{-\frac{x^2}{4}-y}\,dy$$

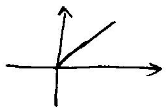
$$= \int_0^{+\infty}\frac{x}{2}e^{-\frac{x^2}{4}}\left[-e^{-y}\right]_{\frac{x^2}{4}}^{+\infty}dx = \int_0^{+\infty}\frac{x}{2}e^{-\frac{x^2}{2}}\,dx$$

$$= \frac{1}{2}\left[e^{-\frac{x^2}{2}}\right]_0^{+\infty} = \frac{1}{2}$$

25. (1) $f(x)=\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}} \quad -\infty<x<+\infty$ $\qquad f_Y(y)=\begin{cases}\sqrt{\frac{2}{\pi}}\,e^{-\frac{y^2}{2}} & y>0\\ 0 & 其他\end{cases}$

$$\therefore f(x,y)=f(x)\cdot f(y)=\begin{cases}\frac{1}{\pi}e^{-\frac{x^2+y^2}{2}} & -\infty<x<+\infty,\ y>0\\ 0 & y\le 0\end{cases}$$

(2) $P\{方程有实根\} = P\{Y-X\ge 0\} = \iint\limits_{y\ge x}\frac{1}{\pi}e^{-\frac{x^2+y^2}{2}}\,dx\,dy$



$$= \frac{1}{\pi}\int_{\frac{\pi}{4}}^{\pi}d\theta\int_0^{+\infty}e^{-\frac{r^2}{2}}r\,dr = \frac{3}{4}\left[-e^{-\frac{r^2}{2}}\right]_0^{+\infty} = \frac{3}{4}$$

26 (1) $f_X(x)=\begin{cases}\frac{1}{2} & 0<x<2\\ 0 & \text{其他}\end{cases}$ $\quad f_Y(y)=\begin{cases}2e^{-2y} & y>0\\ 0 & \text{其他}\end{cases}$

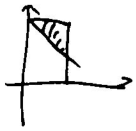
$\therefore f(x,y)=\begin{cases}e^{-2y} & 0<x<2,\ y>0\\ 0 & \text{其他}\end{cases}$

(2) $P\{X<2Y\}=\int_0^2 dx\int_{\frac{x}{2}}^{+\infty}e^{-2y}dy=\int_0^2\left[-\frac{1}{2}e^{-2y}\right]_{\frac{x}{2}}^{+\infty}dx$

$=\frac{1}{2}\int_0^2 e^{-x}dx=\frac{1}{2}\left[-e^{-x}\right]_0^2=\frac{1}{2}(1-e^{-2})$

27. (1) $f(x,y)=f(x)\cdot f(y)=\begin{cases}xy & 0\le x\le 1,\ 0\le y\le 2\\ 0 & \text{其他}\end{cases}$

(2) $P\{X+Y\geq 2\}=\int_0^1 dx\int_{2-x}^{2}xy\,dy$

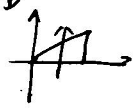


$=\frac{1}{2}\int_0^1 x[4-(2-x)^2]\,dx=\frac{1}{2}\int_0^1(4x^2-x^3)\,dx=\frac{13}{24}$

28. (1) 由 $\int_{-\infty}^{+\infty}dx\int_{-\infty}^{+\infty}f(x,y)\,dy=1$ 得 $a\int_0^2 dx\int_0^{\frac{x}{2}}(x+y)dy=a\int_0^2(\frac{x^2}{2}+\frac{x^2}{8})dx=\frac{5}{3}a$

得 $a=\frac{3}{5}$

(2) $f_X(x)=\begin{cases}\int_0^{\frac{x}{2}}\frac{3}{5}(x+y)dy=\frac{3}{8}x^2 & 0\le x\le 2\\ 0 & \text{其他}\end{cases}$



$f_Y(x)=\begin{cases}\int_{2y}^{2}\frac{3}{5}(x+y)dx=\frac{6}{5}(1+y-2y^2) & 0<y<1\\ 0 & \text{其他}\end{cases}$ $\Rightarrow X,Y$ 不独立.

(3) $P\{Y\geq\frac{3}{4}|X\geq 1\}=\frac{P\{Y\geq\frac{3}{4},X\geq 1\}}{P\{X\geq 1\}}=\frac{\int_{\frac{3}{2}}^{2}dx\int_{\frac{3}{4}}^{\frac{x}{2}}\frac{3}{5}(x+y)dy}{\int_1^2\frac{3}{8}x^2dx}=$

$=\frac{\frac{3}{5}\int_{\frac{3}{2}}^{2}\left[x(\frac{x}{2}-\frac{3}{4})+\frac{1}{2}(\frac{x^2}{4}-\frac{9}{16})\right]dx}{\frac{7}{8}}=\frac{\frac{1}{10}}{\frac{7}{8}}=\frac{4}{35}$

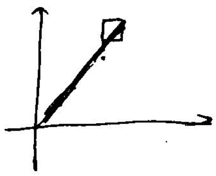
29. (1) $P\{X>120,Y>120\}=P\{120<X<+\infty,120<Y<+\infty\}=F(+\infty,+\infty)-F(120,+\infty)-F(+\infty,120)+F(120,120)$

$=1-(1-e^{-1.2})-(1-e^{-1.2})+(1-e^{-1.2}-e^{-1.2}+e^{-2.4})$

$=e^{-2.4}$

(2) $F_X(x)=\begin{cases}1-e^{-0.01x} & x>0\\ 0 & \text{其他}\end{cases}$ $\quad F_Y(y)=\begin{cases}1-e^{-0.01y} & y>0\\ 0 & \text{其他}\end{cases}$ 独立.(同例1)

30. $P\{\text{相对}\}=P\{0.004<Y-X<0.036\}=\cancel{P\{X+0.004<Y<X+0.036\}}$



$=\cancel{\frac{S_A}{S_B}=\frac{24}{25}}$

$=\iint\limits_{0.004<y-x<0.036}2500\,dxdy=\frac{24}{25}$