Navidi Section 2.6

1. In a certain community, levels of air pollution may exceed federal standards for ozone or for particulate matter on some days. For a particular summer season, let X be the number of days on which the ozone standard is exceeded and let Y be the number of days on which the particulate matter standard is exceeded. Assume that the joint probability mass function of X and Y is given in the following table:

	y		
x	**0**	**1**	**2**
0	0.10	0.11	0.05
1	0.17	0.23	0.08
2	0.06	0.14	0.06

a. Find $P(X = 1 \text{and } Y = 0)$.

b. Find $P(X \geq 1 \text{and } Y < 2)$.

c. Find $P(X < 1)$.

d. Find $P(Y \geq 1)$.

e. Find the probability that the standard for ozone is exceeded at least once.

f. Find the probability that the standard for particulate matter is never exceeded.

g. Find the probability that neither standard is ever exceeded.

Navidi Section 2.6

For the joint PDF given by the last problem:

a. Find the marginal probability mass function $p_X(x)$.

b. Find the marginal probability mass function $p_Y(y)$.

c. Find μ_X.

d. Find μ_Y.

e. Find σ_X.

f. Find σ_Y.

g. Find Cov(X, Y). TOO TEDIOUS.

h. Find $\rho_{X,Y}$. SAME

i. Are X and Y independent? Explain.

j. Find the conditional probability mass function $P_{Y|X=0}$

k. Find the conditional probability mass function $P_{X|Y=1}$

l. Find the conditional expectation $E(Y \mid X = 0)$.

m. Find the conditional expectation $E(X \mid Y = 1)$.

21. The lifetime of a certain component, in years, has probability density function

$$p_X(x) = \begin{cases} e^{-x} & x > 0 \\ 0 & x \le 0 \end{cases}$$

Two such components, whose lifetimes are independent, are available. As soon as the first component fails, it is replaced with the second component. Let X denote the lifetime of the first component, and let Y denote the lifetime of the second component.

a. Find the joint probability density function of X and Y.

b. Find $P(X \le 1 \text{and } Y > 1)$.

c. Find μ_X.

d. Find μ_{X+Y}.

e. Find $P(X + Y \le 2)$. (*Hint:* Sketch the region of the plane where $x + y \le 2$, and then integrate the joint probability density function over that region.)

2. Let A and B be events with $P(A) = 0.5$ and $P(A \cap B^c) = 0.4$. For what value of $P(B)$ will A and B be independent?

Use $P(A \cap B^c) + P(A \cap B) = P(A)$

3. A box contains 15 resistors. Ten of them are labeled 50 Ω and the other five are labeled 100 Ω.

a. What is the probability that the first resistor is 100 Ω?

b. What is the probability that the second resistor is 100 Ω, given that the first resistor is 50 Ω?

c. What is the probability that the second resistor is 100 Ω, given that the first resistor is 100 Ω?

Navidi Section 2.3

5. On graduation day at a large university, one graduate is selected at random. Let A represent the event that the student is an engineering major, and let B represent the event that the student took a calculus course in college. Which probability is greater, $P(A|B)$ or $P(B|A)$? Explain.

Navidi Section 2.3

10. At a certain college, 30% of the students major in engineering, 20% play club sports, and 10% both major in engineering and play club sports. A student is selected at random.

a. What is the probability that the student is majoring in engineering?

b. What is the probability that the student plays club sports?

c. Given that the student is majoring in engineering, what is the probability that the student plays club sports?

d. Given that the student plays club sports, what is the probability that the student is majoring in engineering?

e. Given that the student is majoring in engineering, what is the probability that the student does not play club sports?

f. Given that the student plays club sports, what is the probability that the student is not majoring in engineering?

36. A system contains two components, A and B, connected in series, as shown in the diagram.

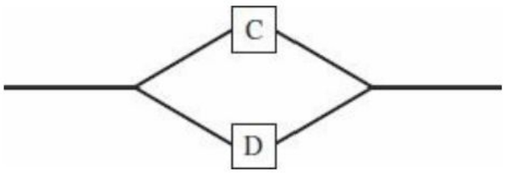
Assume A and B function independently. For the system to function, both components must function.

a. If the probability that A fails is 0.05, and the probability that B fails is 0.03, find the probability that the system functions.

b. If both A and B have probability p of failing, what must the value of p be so that the probability that the system functions is 0.90?

c. If three components are connected in series, and each has probability p of failing, what must the value of p be so that the probability that the system functions is 0.90?

37. A system contains two components, C and D, connected in parallel as shown in the diagram.



Assume C and D function independently. For the system to function, either C or D must function.

a. If the probability that C fails is 0.08 and the probability that D fails is 0.12, find the probability that the system functions.

b. If both C and D have probability p of failing, what must the value of p be so that the probability that the system functions is 0.99?

c. If three components are connected in parallel, function independently, and each has probability p of failing, what must the value of p be so that the probability that the system functions is 0.99?

d. If components function independently, and each component has probability 0.5 of failing, what is the minimum number of components that must be connected in parallel so that the probability that the system functions is at least 0.99?

Carlton Section 5.1

Consider a random sample $Y_1, Y_2, \ldots, Y_n$ from the probability density function

$$p(y;\theta) = \frac{1}{2}(1+\theta y) \qquad -1 \leq y \leq 1,$$

where $-1 \leq \theta \leq 1$ (this distribution arises in particle physics). Show that $\hat{\theta} = 3\bar{Y}$ is an unbiased estimator of θ. [*Hint:* First determine $\mu = E[Y] = E[\bar{Y}]$]

Carlton Section 5.3

Data in ACT_score.txt

39. Here is a sample of ACT scores (average of the Math, English, Social Science, and Natural Science scores) for students taking college freshman calculus:

24.00	28.00	27.75	27.00	24.25	23.50	26.25
24.00	25.00	30.00	23.25	26.25	21.50	26.00
28.00	24.50	22.50	28.25	21.25	19.75	

(b) Calculate a 95% confidence interval for the population mean.

(c) The university ACT average for entering freshmen that year was about 21. Are the calculus students better than average, as measured by the ACT?

Carlton Section 5.3

Data: Escape_time.txt

41. A sample of 26 offshore oil workers took part in a simulated escape exercise, resulting in the accompanying data on time (seconds) to complete the escape ("Oxygen Consumption and Ventilation During Escape from an Offshore Platform," *Ergonomics*, 1997: 281–292):

389	356	359	363	375	424	325	394	402
373	373	370	364	366	364	325	339	393
392	369	374	359	356	403	334	397	

(a) Calculate a 99% confidence interval for the population mean escape time.
(b) Would a 90% CI based on the same data be wider or narrower? Explain.

55. For which of the given P-values would the null hypothesis be rejected when performing a level .05 test?
    (a) .001
    (b) .021
    (c) .078
    (d) .047
    (e) .148

59. Let μ denote the mean reaction time to a certain stimulus. For a large-sample z test of H_0: $\mu = 5$ versus H_a: $\mu > 5$, find the P-value associated with each of the given values of the z test statistic.

(a) 1.42
(b) .90
(c) 1.96
(d) 2.48
(e) $-.11$

Clarification: Here the z test statistic is our $\dfrac{\hat{\mu}_N - 5}{\sigma_X/\sqrt{N}}$

Carlton Section 5.5

Data: Train_repair_time.txt

65. The article "Uncertainty Estimation in Railway Track Life-Cycle Cost" (*J. of Rail and Rapid Transit*, 2009) presented the following data on time to repair (min) a rail break in the high rail on a curved track of a certain railway line.

159	120	480	149	270	547	340	43	228	202	240	218

A normal probability plot of the data shows a reasonably linear pattern, so it is plausible that the population distribution of repair time is at least approximately normal. The sample mean and standard deviation are 249.7 and 145.1, respectively. Is there compelling evidence for concluding that true average repair time exceeds 200 min? Carry out a test of hypotheses using a significance level of .05.