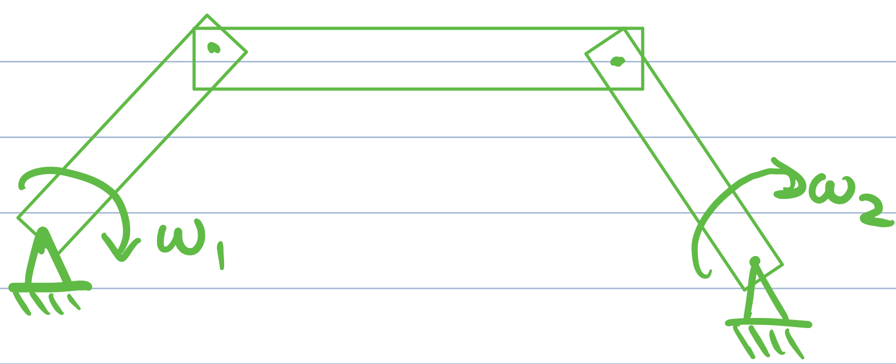
ME 104 Lec 1

The point of ME 104 is to be able to predict how systems move.

Two main ingredients needed:

① Kinematics: Kinematics is a mathematical description of the ways a system could move. Kinematics solves a geometry problem only; when you write out the kinematics of a problem you find a generic way to describe the ways a system could move that satisfy all constraints. What are common constraints?

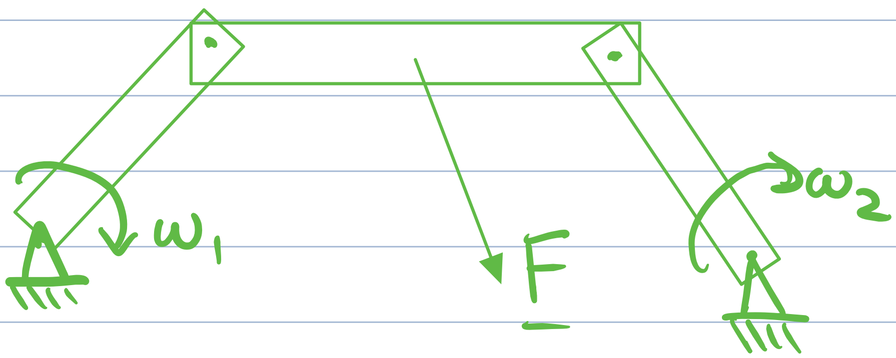
Ex:



Relate ω_1 to ω_2. This is a kinematics problem.

② **Dynamics**: Also known as "kinetics"; dynamics is the part where you actually <u>predict</u> how a system will move, given some initial conditions. This is where we use $F=ma$ or other laws of motion to determine <u>which</u> kinematically acceptable solution is the one the sytem will <u>actually take</u>.

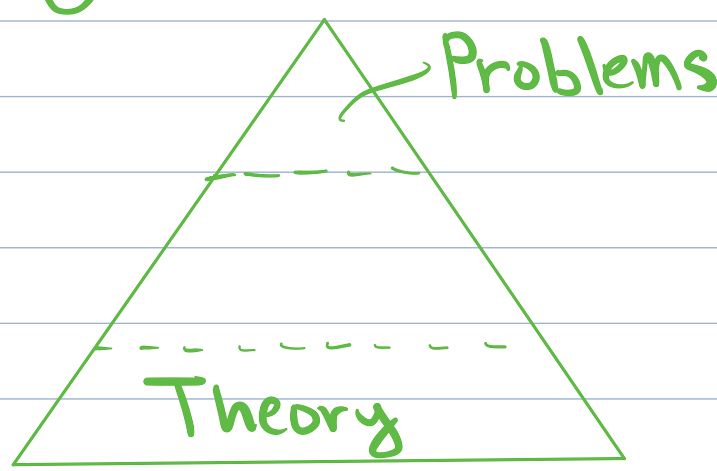
Ex:



Given the force $\underline{F}$ and initial setup, find $\omega_1(t)$ & $\omega_2(t)$. This is a dynamics problem.

My philosophy of ME 104:

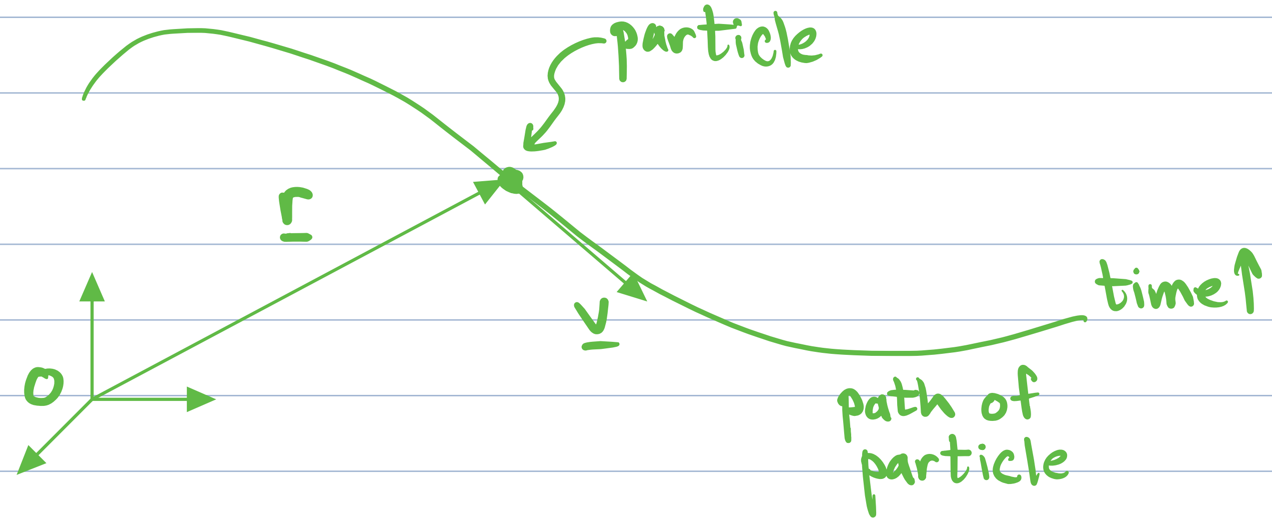
Pyramid of understanding



ME 104 is best learned by a merged bottom→top and top→bottom approach. Focus too much on theory and you won't recognize procedures for solving problems. Focus too much on problems and you will misuse the equations and get wrong answers. Need <u>both</u> to become a ME 104 master!

<u>Computation:</u> Because the dynamics of a system often take the form of ODE's, numerical ODE integration is a common solution method. Software survey...

One-particle kinematics:



"O" is a fixed origin.

$\underline{r} = \underline{r}(t)$ is the position vector relative to O.

$\underline{v} = \underline{v}(t) = \dfrac{d\underline{r}}{dt}$ is the velocity vector.

$v = v(t) = |\underline{v}(t)| = \sqrt{\underline{v}(t) \cdot \underline{v}(t)}$ is the speed, a scalar.

$\underline{a} = \underline{a}(t) = \dfrac{d\underline{v}}{dt} = \dfrac{d^2\underline{r}}{dt^2}$ is the acceleration vector.

$s = s(t) =$ the distance the particle has travelled since $t = 0$.

$$\Rightarrow \frac{ds}{dt} = |\underline{v}| .$$

$\Rightarrow$ Distance travelled between time t_1 & t_2 $= s(t_2) - s(t_1) = \int_{t_1}^{t_2} \frac{ds}{d\tau}\, d\tau$

$$= \int_{t_1}^{t_2} |\underline{v}(\tau)|\, d\tau .$$

We often will use a dot to denote the time derivative, i.e. $\underline{v} = \dot{\underline{r}}$.

Sometimes we want to write the position in terms of s rather than t:

$$\underline{r} = \tilde{\underline{r}}(s) \Rightarrow \underline{r} = \underline{r}(t) \equiv \tilde{\underline{r}}(s(t))$$

$$\Rightarrow \underline{v} = \dot{\underline{r}} = \frac{d\tilde{\underline{r}}}{ds}\frac{ds}{dt} = \frac{d\tilde{\underline{r}}}{ds} v \quad \{\text{chain rule}\}$$

$$\underline{a} = \ddot{\underline{r}} = \frac{d}{dt}\left(\frac{d\tilde{\underline{r}}}{ds}\frac{ds}{dt}\right)$$

$$= \left(\frac{d^2\tilde{\underline{r}}}{ds^2}\frac{ds}{dt}\right)\frac{ds}{dt} + \frac{d\tilde{\underline{r}}}{ds}\frac{d^2 s}{dt^2} \quad \{\text{product rule}\}$$

$$= \frac{d^2\underset{\sim}{r}}{ds^2} v^2 + \frac{d\underset{\sim}{r}}{ds} \dot{v} .$$

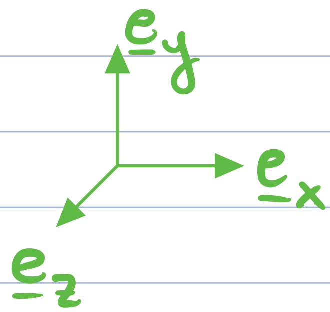
Note: Even if speed is const, the particle still has non-zero acceleration as long as $v \neq 0$ and its path is not straight (i.e. $\frac{d^2\underset{\sim}{r}}{ds^2} \neq \underline{0}$).

Where have you seen this before?

Circular motion.

So far, we have not used a particular coordinate system to express the components of these vectors, so the results are valid in any coordinate system.

Cartesian coordinate system:



$\{\underline{e}_x, \underline{e}_y, \underline{e}_z\}$ is an orthonormal basis.

That is, $|\underline{e}_x| = |\underline{e}_y| = |\underline{e}_z| = 1$ and

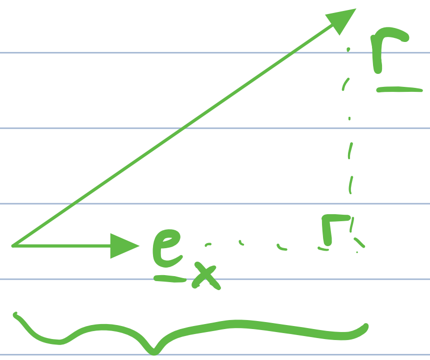
$\underline{e}_x \cdot \underline{e}_y = \underline{e}_y \cdot \underline{e}_z = \underline{e}_z \cdot \underline{e}_x = 0$.

We will also be using right-handed bases only in that $\underline{e}_x \times \underline{e}_y = \underline{e}_z$.

Given a (right-handed orthonormal) basis $\{\underline{e}_x, \underline{e}_y, \underline{e}_z\}$, every vector in 3D can be expressed in terms of components. For example:

$\underline{r} = r_x \underline{e}_x + r_y \underline{e}_y + r_z \underline{e}_z$ where $r_x, r_y,$ and r_z are the components.

Given a vector $\underline{r}$ (as an arrow only) you can extract its components geometrically using the dot product:



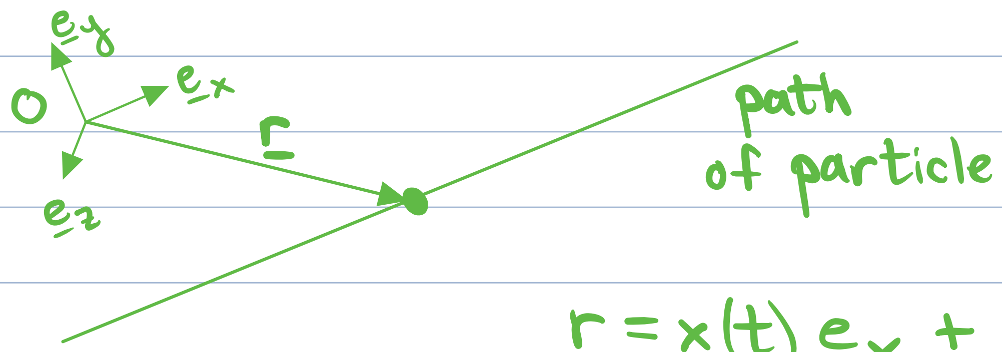
$$r_x = \underline{r} \cdot \underline{e}_x$$

$$= \{\text{length of } \underline{r}\} \times \{\text{Cosine of angle between } \underline{r} \text{ and } \underline{e}_x\}.$$

= Length of "shadow" $\underline{r}$ casts onto the $\underline{e}_x$ direction if sun directly overhead.

When the basis is <u>fixed</u> we call the components "cartesian".

Rectilinear "straight line" motion:



$$\underline{r} = x(t)\,\underline{e}_x + \underline{c},$$

$$\underline{v} = \dot{x}\,\underline{e}_x, \quad \underline{a} = \ddot{x}\,\underline{e}_x.$$

Is $s(t) = x(t)$?

$\dot{s} = v = |\underline{v}| = |\dot{x}|$ which $= \dot{x}$ only if $\dot{x} > 0$ always.

So, in general, $\boxed{\text{No}}$.