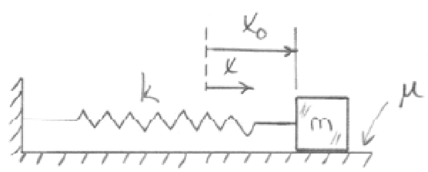
Week 4 Solutions

Monday, September 16, 2024 6:56 PM

Q1



$m = 5\ kg$
$\mu = 0.40$
$k = 150\ N/m$
$x_0 = 200\ mm$

MINIMUM STRETCH TO OVERCOME STATIC FRICTION.

$$a = 0 = \mu g - \frac{k}{m} x_{min} \longrightarrow x_{min} = \frac{\mu m g}{k} = \frac{0.40(5)(9.81)}{150}$$

$x_{min} = 0.1308\ m$ OR $130.8\ mm$

(MASS WILL MOVE LEFT)

$$a = \mu g - \frac{k}{m} x = \frac{v\,dv}{dx} \longrightarrow \int_{x_0}^{x} \left(\mu g - \frac{k}{m} x\right) dx = \int_{v_0=0}^{v} v\,dv$$

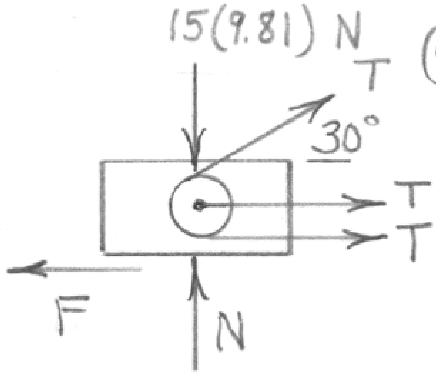
SO $\frac{1}{2} v^2 = \mu g (x - x_0) - \frac{k}{2m}(x^2 - x_0^2)$

FOR COMPLETE STOP... $v = 0$, AND SOLVE FOR $x = x_f$

$$x_f = x_0 \quad \text{OR} \quad x_f = \frac{2 m g \mu - k x_0}{k} = \frac{2(5)(9.81)(0.40) - 150(0.2)}{150}$$

$\underline{x_f = 0.0616\ m \quad \text{OR} \quad 61.6\ mm}$

Q2



(a) Assume static equilibrium:

$$\begin{cases} \Sigma F_x = 0: -F + T(2+\cos 30^\circ) = 0 \\ \Sigma F_y = 0: T\sin 30^\circ + N - 15(9.81) = 0 \end{cases}$$

Solution (for T = 23 N): $\begin{cases} N = 135.6 \text{ N} \\ F = 65.9 \text{ N} \end{cases}$

$F_{max} = \mu_s N = 0.50(135.6) = 67.8 \text{ N} > F$

So assumption is valid and $\underline{a = 0}$

(b) With $T = 26$ N, $\begin{cases} N = 134.2 \text{ N} \\ F = 74.5 \text{ N} \end{cases}$

$F_{max} = \mu_s N = 0.50(134.2) = 67.1 \text{ N} < F$

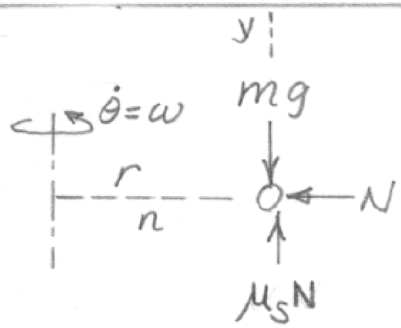
So motion occurs and $F = F_k = \mu_k N$

$= 0.40(134.2) = 53.7 \text{ N}$

$\Sigma F_x = ma_x : -53.7 + 26(2 + \cos 30^\circ) = 15a$

$\underline{a = 1.390 \text{ m/s}^2 (\rightarrow)}$

Q3



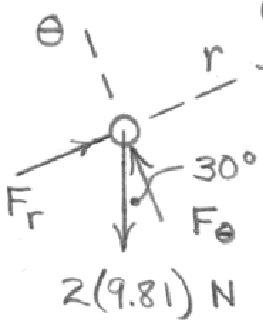
$\Sigma F_n = ma_n;\ N = mr\omega^2$

$\Sigma F_y = 0;\ \mu_s(mr\omega^2) = mg$

$\omega^2 = \dfrac{g}{\mu_s r},\quad \underline{\omega = \sqrt{\dfrac{g}{\mu_s r}}}$

Q4

F_r and F_θ are the r- and θ-components of the total friction force F.



$\Sigma F_r = ma_r = m(\ddot{r} - r\dot{\theta}^2):$

$F_r - 19.62 \sin 30° = 2[0 - 1(-0.873)^2]$

$F_r = 8.29 \text{ N}$

$\Sigma F_\theta = ma_\theta = m(r\ddot{\theta} + 2\dot{r}\dot{\theta})$

$F_\theta - 19.62 \cos 30° = 2[(1)(3.49) + 2(-0.5)(-0.873)]$

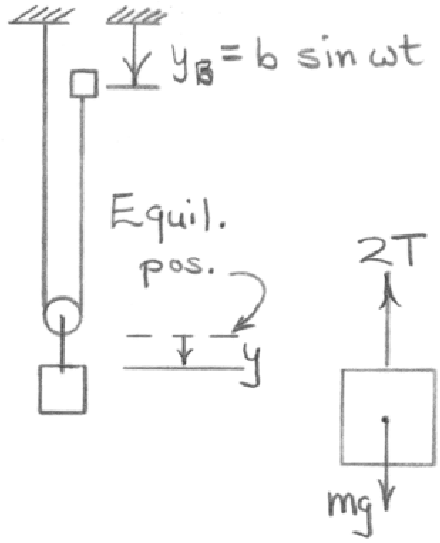
$F_\theta = 25.7 \text{ N}$

$F = \sqrt{F_r^2 + F_\theta^2} = 27.0 \text{ N}$

$P = \dfrac{F/2}{\mu_s} = \dfrac{27.0/2}{0.5} = \underline{27.0 \text{ N}}$

(Static gripping force $= \underline{19.62 \text{ N}}$)

Q5



In equilibrium position, the spring tension is $T_0 = \frac{1}{2} mg$

In displaced position, spring is stretched $2y - y_B$, so spring force is

$$T = \tfrac{1}{2} mg + k(2y - y_B)$$

For $\Sigma F_y = m\ddot{y}$ on m:

$$mg - 2\left[\tfrac{1}{2} mg + k(2y - y_B)\right] = m\ddot{y}$$

or $$\underline{\ddot{y} + \frac{4k}{m} y = \frac{2k}{m} b \sin \omega t}$$

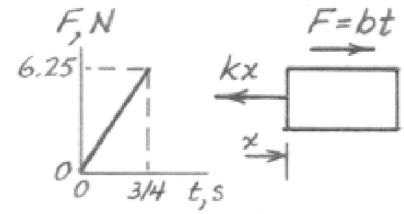
For particular solution, assume $y = Y \sin \omega t$ and obtain $Y = \dfrac{b/2}{1 - (\omega/\omega_n)^2}$ where $\omega_n = 2\sqrt{k/m}$

Thus $\underline{\omega_c = 2\sqrt{k/m}}$

Q6

$\Sigma F_x = m\ddot{x}$: $bt - kx = m\ddot{x}$,

$\ddot{x} + \frac{k}{m}x = \frac{bt}{m}$



Sol. is $x = x_c + x_p$ where

$x_c = C_1 \sin\omega_n t + C_2 \cos\omega_n t$, $x_p = C_3 t$ with $C_3 = \frac{b}{k}$ $\quad b = \frac{6.25}{3/4} = 8.33$ N/s

so $x = C_1 \sin\omega_n t + C_2\cos\omega_n t + \frac{b}{k}t$ $\quad k = 90$ N/m

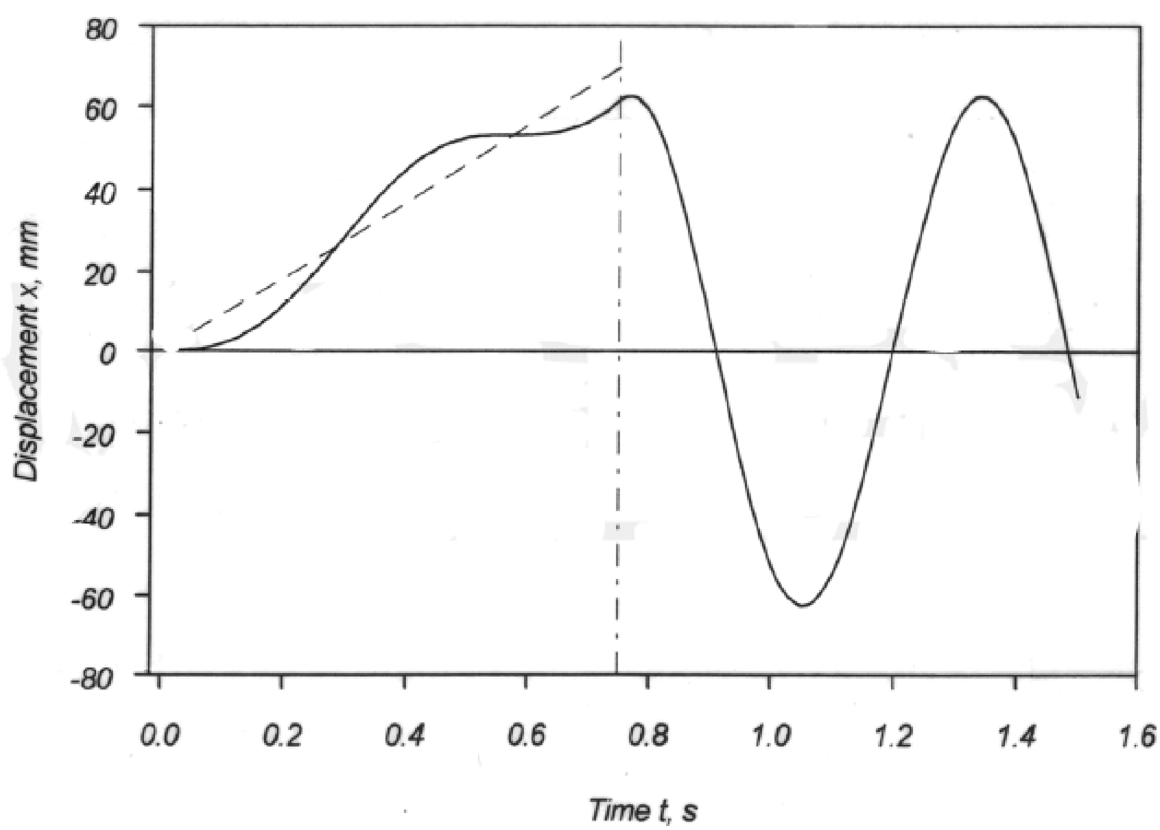
When $t=0$, $\dot{x}=0$ & $x=0$ giving $C_1 = -\frac{b}{\omega_n k}$, $\quad b/k = \frac{8.33}{90} = 0.0926$ m/s

$C_2 = 0$

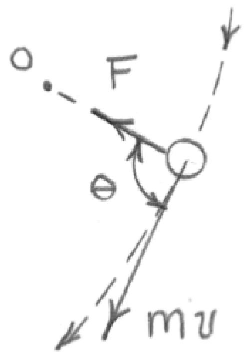
& $x = -\frac{b}{\omega_n k}\sin\omega_n t + \frac{b}{k}t = \frac{b}{k}\left(t - \frac{1}{\omega_n}\sin\omega_n t\right)$

where $\omega_n = \sqrt{k/m} = \sqrt{90/0.75} = 10.95$ rad/s, $\frac{1}{\omega_n} = 0.0913$ s

Thus $\underline{x = 0.0926(t - 0.0913\sin 10.95t)\text{ m}}$ for first 3/4 s



Q7



$\sum M_o = \dot{H}_o = 0$, so $H_o = \text{const.}$

$H_{O_A} = H_{O_B}$

$m(4)(0.350 \sin 54°) =$

$m v_B \ (0.230 \sin 65°)$

$\underline{v_B = 5.43 \ \text{m/s}}$