ME104: Engineering Mechanics II Discussion Week 5 of 15

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Topics: Rotating Frames, Work, Energy, and Power

PSET 2 due this week on Friday, September 27 (midnight)

Midterm I is *in three weeks* on October 10, 2024 (in class)

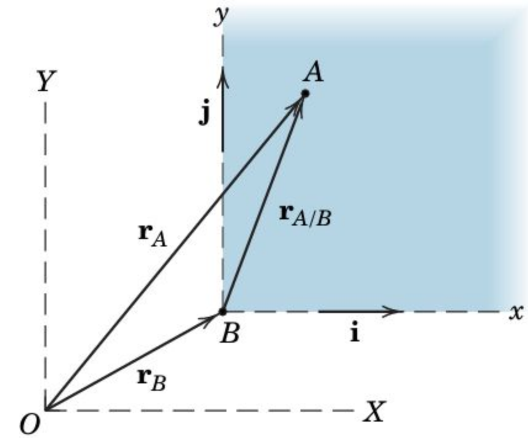
Relative motion in *non-rotating* frame

Attach non-rotating **i-j** coordinate system to point B and take derivatives to calculate relative velocity and acceleration

$$\boldsymbol{r}_A = \boldsymbol{r}_B + \boldsymbol{r}_{A/B} \longrightarrow \mathbf{r}_{A/B} = x\mathbf{i} + y\mathbf{j}$$

$$\dot{\boldsymbol{r}}_A = \dot{\boldsymbol{r}}_B + \dot{\boldsymbol{r}}_{A/B} = \boldsymbol{v}_B + \boldsymbol{v}_{A/B} = \boldsymbol{v}_A$$

$$\ddot{\boldsymbol{r}}_A = \ddot{\boldsymbol{r}}_B + \ddot{\boldsymbol{r}}_{A/B} = \boldsymbol{a}_B + \boldsymbol{a}_{A/B} = \boldsymbol{a}_A$$



Note, that within this **i-j** plane, same logic applies as before, for example:

- Cartesian motion: $\mathbf{a}_{A/B} = (\mathbf{a}_{A/B})_x + (\mathbf{a}_{A/B})_y$
- Normal-tangential: $\boldsymbol{a}_{A/B} = \left(\boldsymbol{a}_{A/B}\right)_t + \left(\boldsymbol{a}_{A/B}\right)_n$

$$\left(\boldsymbol{a}_{A/B}\right)_t = \boldsymbol{\alpha} \times \boldsymbol{r} \qquad \left(\boldsymbol{a}_{A/B}\right)_n = \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \boldsymbol{r})$$

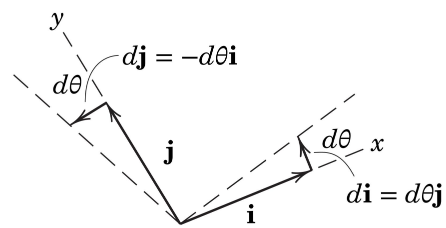
Relative motion in *rotating* frame: recap (simpler) derivation

1. Attach now rotating **i-j** coordinate system to point B

$$\boldsymbol{r}_A = \boldsymbol{r}_B + \boldsymbol{r}_{A/B} = \boldsymbol{r}_B + (x\boldsymbol{i} + y\boldsymbol{j})$$

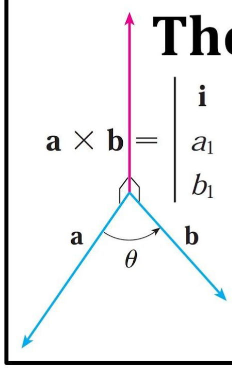
2. Take derivative $\dot{\boldsymbol{r}}_A = \dot{\boldsymbol{r}}_B + \frac{\mathrm{d}}{\mathrm{d}t}(x\boldsymbol{i} + y\boldsymbol{j})$ (chain rule) $= \dot{\boldsymbol{r}}_B + (\dot{x}\boldsymbol{i} + \dot{y}\boldsymbol{j}) + \left(x\dot{\boldsymbol{i}} + y\dot{\boldsymbol{j}}\right)$

3. Rotate of **i** and **j** through d(theta) and divide with with dt



$$\boxed{\dot{\mathbf{i}} = \omega \times \mathbf{i} \quad \text{and} \quad \dot{\mathbf{j}} = \omega \times \mathbf{j}}$$

The Cross Product



$$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = (a_2b_3 - a_3b_2)\,\mathbf{i} - (a_1b_3 - a_3b_1)\,\mathbf{j} + (a_1b_2 - a_2b_1)\,\mathbf{k}$$

Determinant Form

4. Calculate velocity and acceleration using chain rule, product rule, and derivatives of **i-j** direction vectors

$$\boxed{\mathbf{v}_A = \mathbf{v}_B + \omega \times \mathbf{r}_{\text{rel}} + \mathbf{v}_{\text{rel}}}$$

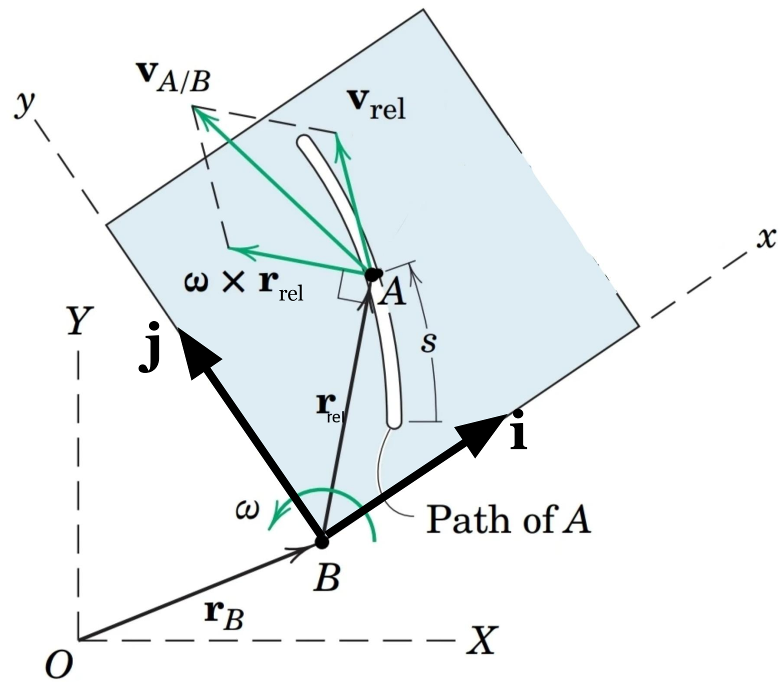
$$\boxed{\mathbf{a}_A = \mathbf{a}_B + \dot{\omega} \times \mathbf{r}_{\text{rel}} + \omega \times (\omega \times \mathbf{r}_{\text{rel}}) + 2\omega \times \mathbf{v}_{\text{rel}} + \mathbf{a}_{\text{rel}}}$$

Relative motion in *rotating* frame: kinematic diagrams

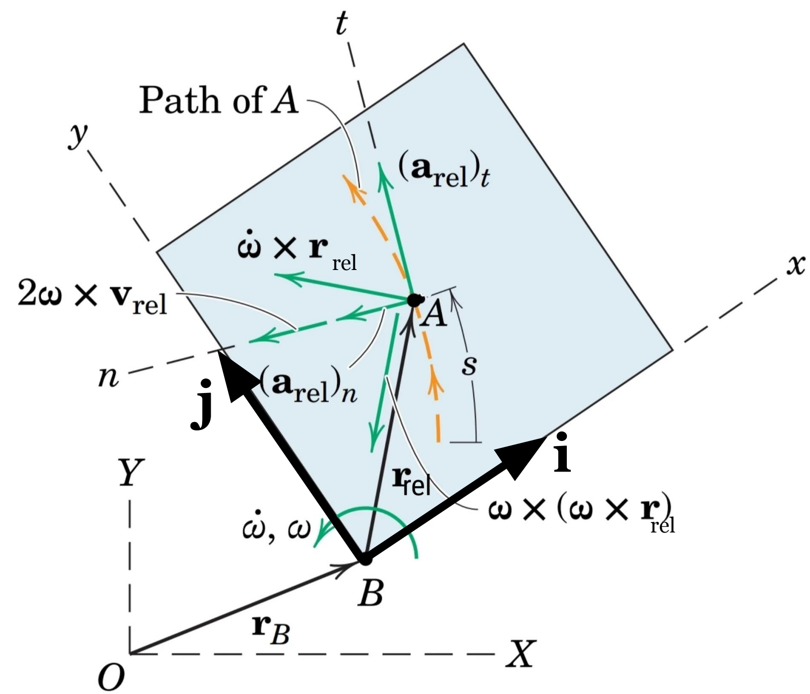
$$\mathbf{r}_A = \mathbf{r}_B + \mathbf{r}_{\text{rel}} = \mathbf{r}_B + (x\mathbf{i} + y\mathbf{j})$$

$$\boxed{\mathbf{v}_A = \mathbf{v}_B + \boldsymbol{\omega} \times \mathbf{r}_{\text{rel}} + \mathbf{v}_{\text{rel}}}$$

$$\boxed{\mathbf{a}_A = \mathbf{a}_B + \dot{\boldsymbol{\omega}} \times \mathbf{r}_{\text{rel}} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_{\text{rel}}) + 2\boldsymbol{\omega} \times \mathbf{v}_{\text{rel}} + \mathbf{a}_{\text{rel}}}$$



Velocity diagram



Acceleration diagram

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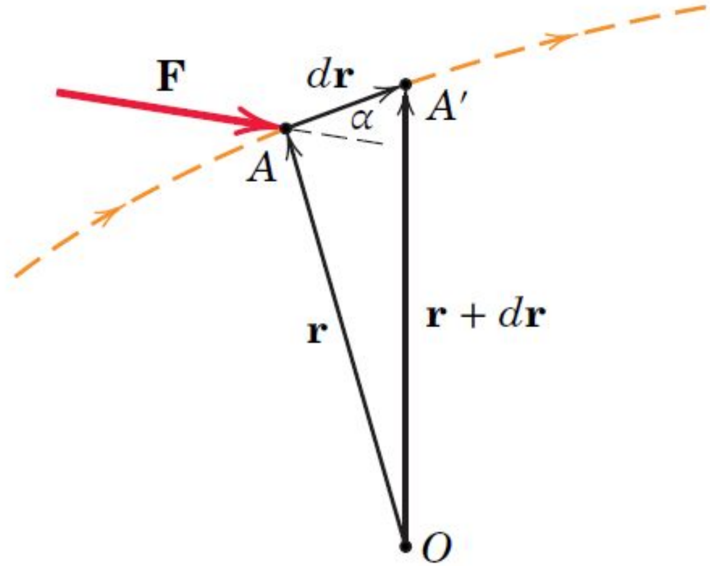
Guide questions: (1) What is orientation of all acceleration components? (2) What new components are added compared to non-rotating relative motion? (3) What about rotation about a fixed axis/polar coordinates? (4) Which components are considered to be pseudo-forces? (5) How do equations reduce when various components are held constant?

Work, Energy, and Power

Power: $P = \mathbf{F} \cdot \mathbf{v}$

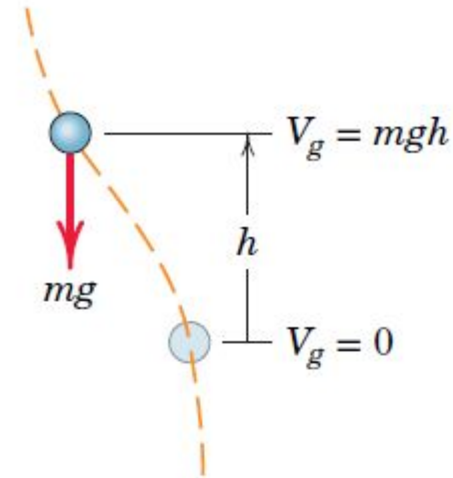


Work: $W_{AB} = \int_{\mathbf{r}_A}^{\mathbf{r}_B} P\,dt = \int_{\mathbf{r}_A}^{\mathbf{r}_B} \mathbf{F} \cdot \mathbf{v}\,dt$

$$= \int_{\mathbf{r}_A}^{\mathbf{r}_B} \mathbf{F} \cdot d\mathbf{r}$$

Kinetic Energy: $K = \frac{1}{2} m\mathbf{v} \cdot \mathbf{v} = \frac{1}{2} mv^2$

Gravitational Potential Energy: $U_g = mgh$

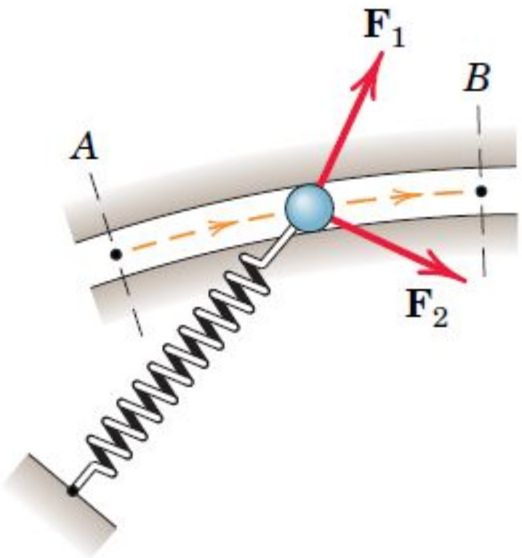


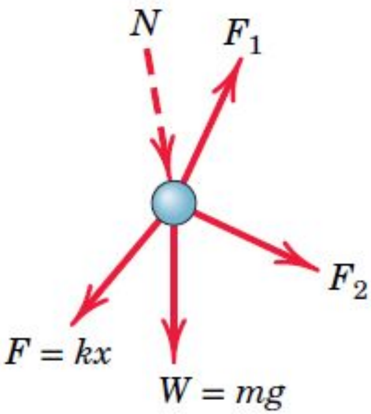
Elastic Potential Energy: $U_e = \frac{1}{2} kx^2$

Work - Energy Theorem

The work-energy theorem relates the change in kinetic energy to the total work done by all the forces acting on the particle.

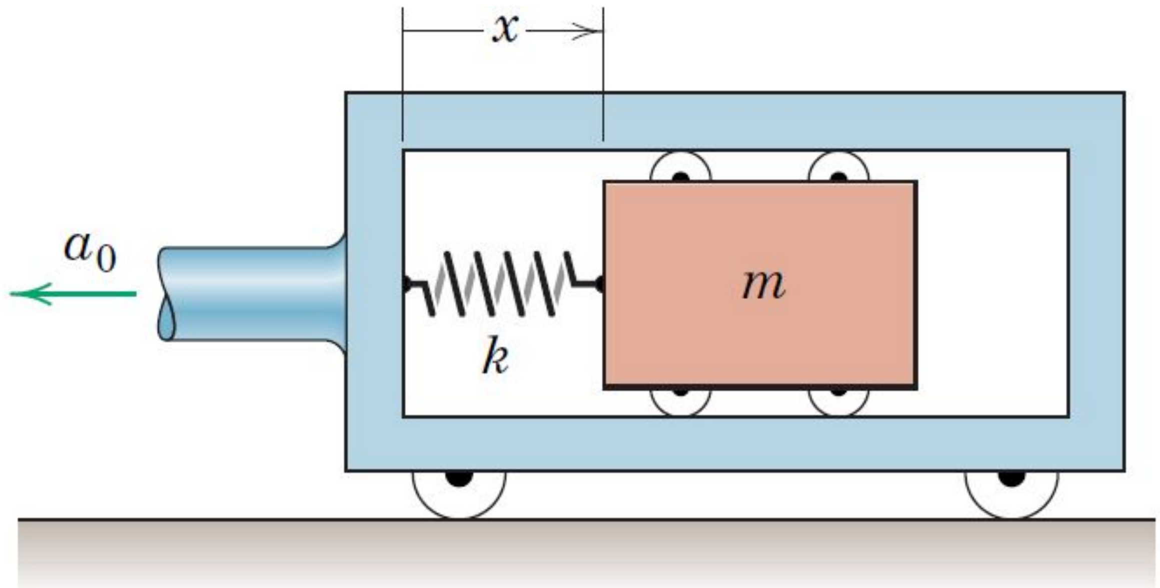
$$W_{AB} = \Delta K = \frac{1}{2}mv_B^2 - \frac{1}{2}mv_A^2$$





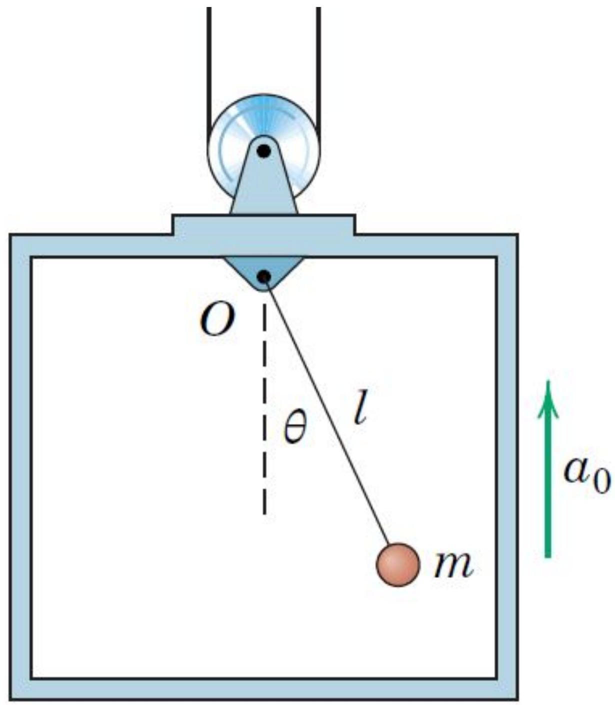
Practice Problem 1: non-rotating relative motion

The block of mass m is attached to the frame by the spring of stiffness k and moves horizontally with negligible friction within the frame. The frame and block are initially at rest with $x = x_0$, the uncompressed length of the spring. If the frame is given a constant acceleration a_0, determine the maximum velocity $\dot{x}_{max} = (v_{rel})_{max}$ of the block relative to the frame.



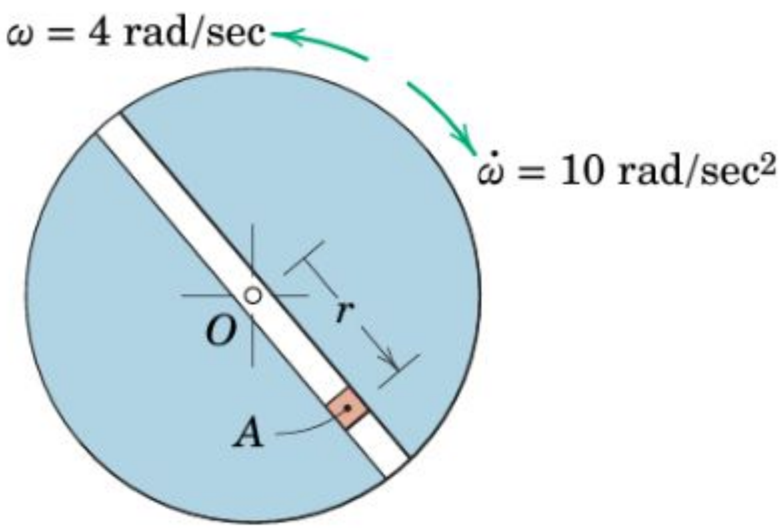
Practice Problem 2: non-rotating relative problem

A simple pendulum is placed on an elevator, which accelerates upward as shown. If the pendulum is displaced an amount θ_0 and released from rest relative to the elevator, find the tension T_0 in the supporting light rod when $\theta = 0$. Evaluate your result for $\theta_0 = \pi/2$.

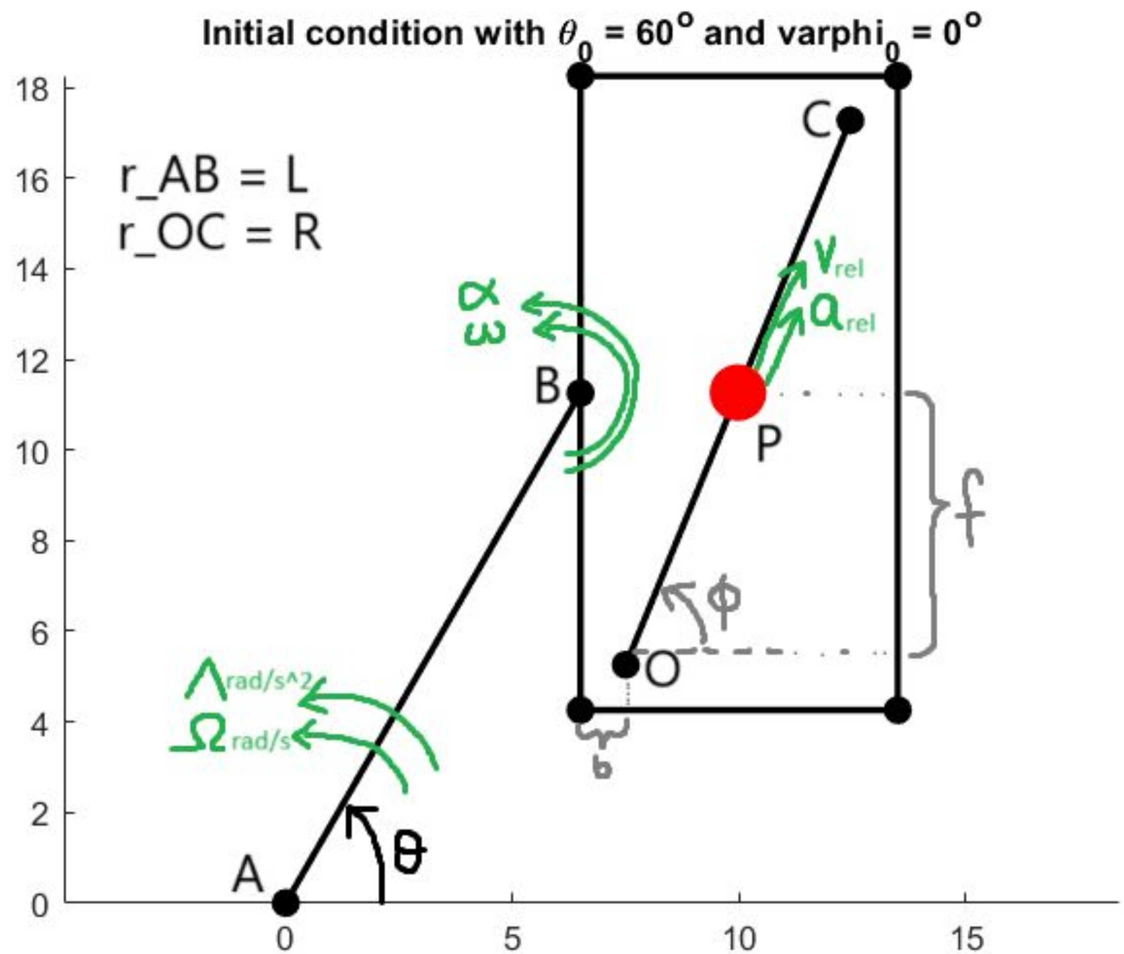


Practice Problem 3: rotating frame

At the instant represented, the disk with the radial slot is rotating about O with a counterclockwise angular velocity of 4 rad/sec which is decreasing at the rate of 10 rad/sec^2. The motion of slider A is separately controlled, and at this instant, $r = 6$ in., $\dot{r} = 5$ in./sec, and $\ddot{r} = 81$ in./sec^2. Determine the absolute velocity and acceleration of A for this position.



Practice Problem 4: rotating frame demo

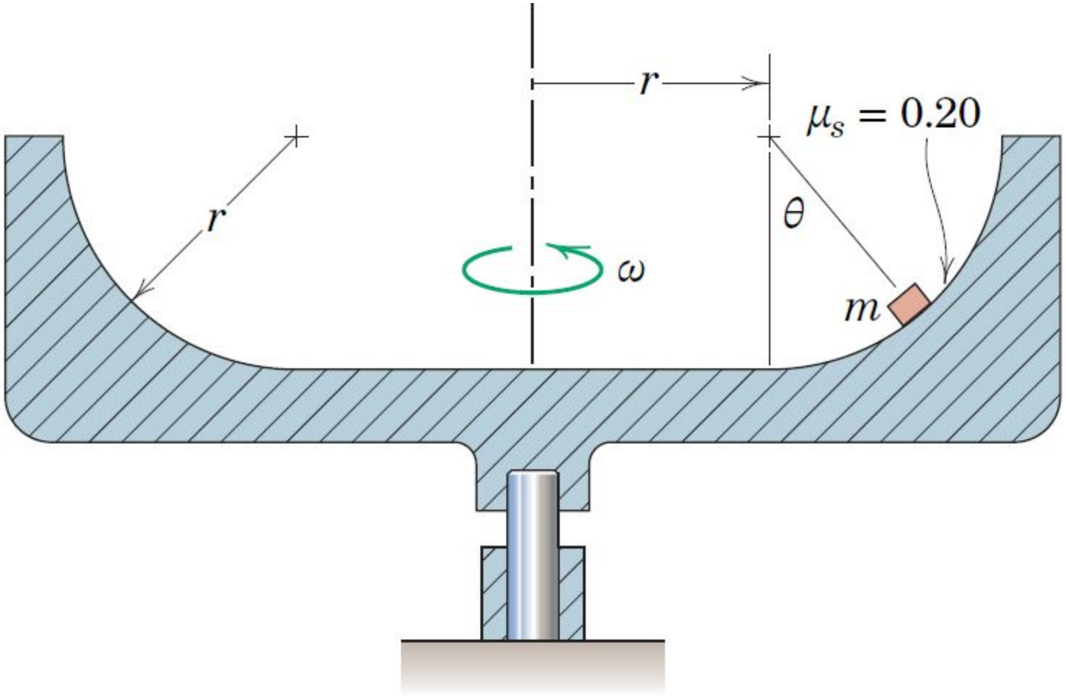


Particle P sits in a slot. Platform and arm are rotating, describe resulting motion along the slot. Don't neglect gravity.

Start with sketching relative position, velocity, and acceleration diagrams in order. See if you can derive accelerations yourself!

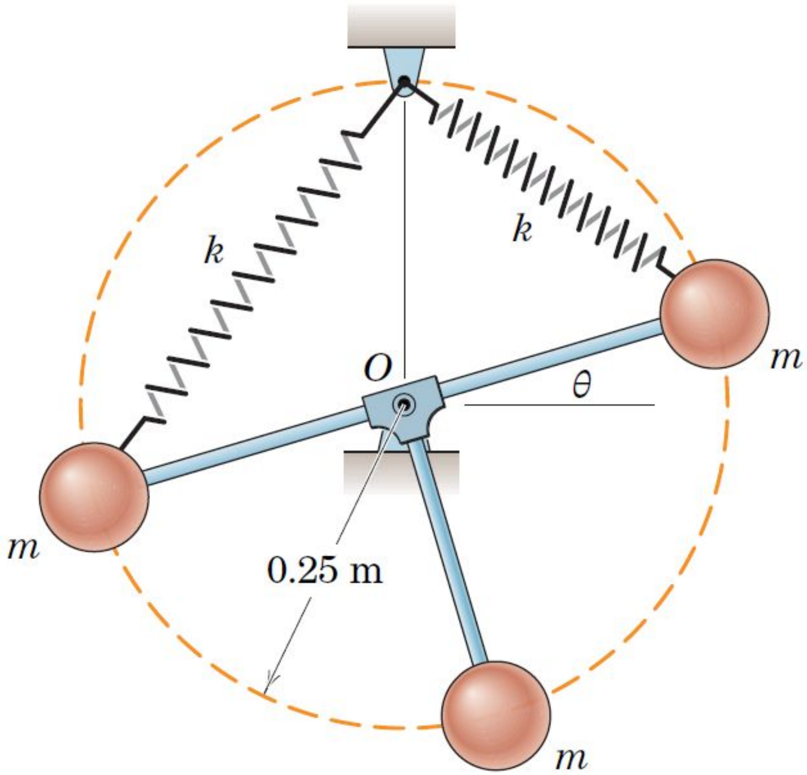
Practice Problem 5:

The bowl-shaped device rotates about a vertical axis with a constant angular velocity $\omega = 6$ rad/s. The value of r is 0.2 m. Determine the range of the position angle θ for which a stationary value is possible if the coefficient of static friction between the particle and the surface is $\mu_s = 0.20$.



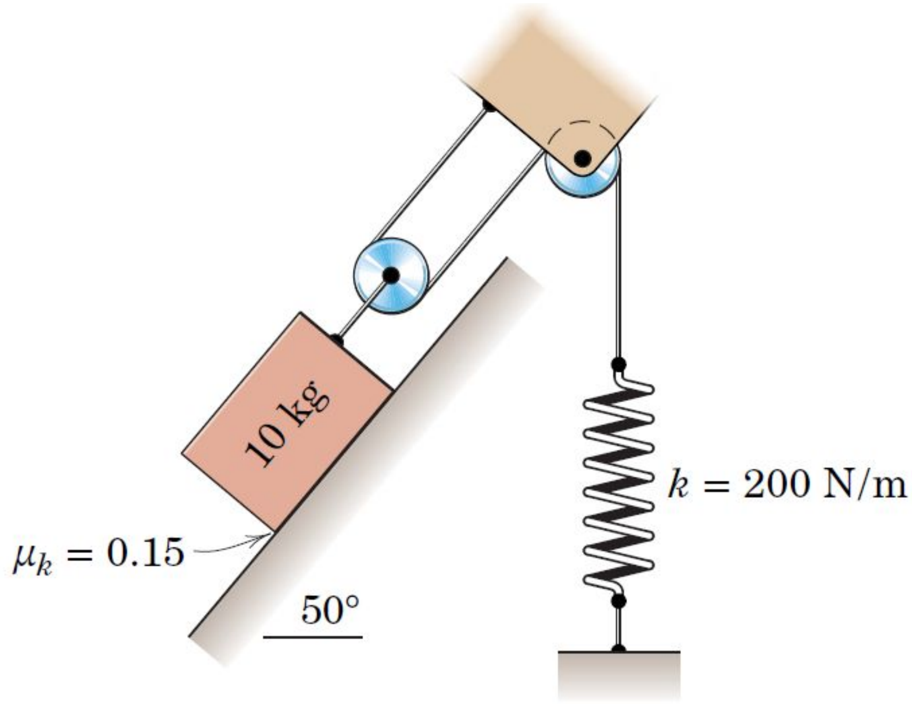
Practice Problem 6:

The two springs, each of stiffness $k = 1.2$ kN/m, are of equal length and undeformed when $\theta = 0$. If the mechanism is released from rest in the position $\theta = 20°$, determine its angular velocity $\dot{\theta}$ when $\theta = 0$. The mass m of each sphere is 3 kg. Treat the spheres as particles and neglect the masses of the light rods and springs.



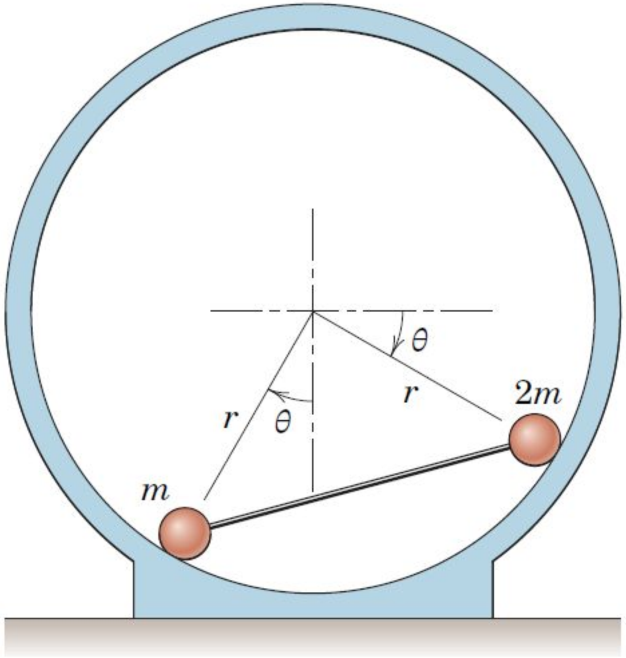
Practice Problem 7:

The system is initially moving with the cable taut, the 10-kg block moving down the rough incline with a speed of 0.3 m/s, and the spring stretched 25 mm. By the method of this article, (*a*) determine the velocity v of the block after it has traveled 100 mm, and (*b*) calculate the distance traveled by the block before it comes to rest.



Practice Problem 8:

The two particles of mass m and $2m$, respectively, are connected by a rigid rod of negligible mass and slide with negligible friction in a circular path of radius r on the inside of the vertical circular ring. If the unit is released from rest at $\theta = 0$, determine (a) the velocity v of the particles when the rod passes the horizontal position, (b) the maximum velocity $v_{\max}$ of the particles, and (c) the maximum value of θ.



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