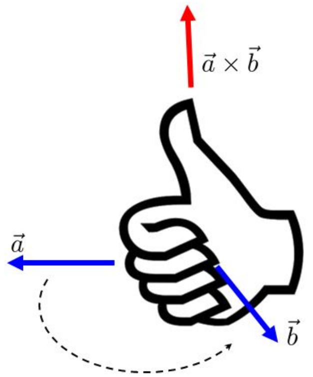
Midterm review session

Friction, work, FBDs, work-energy, relative motion

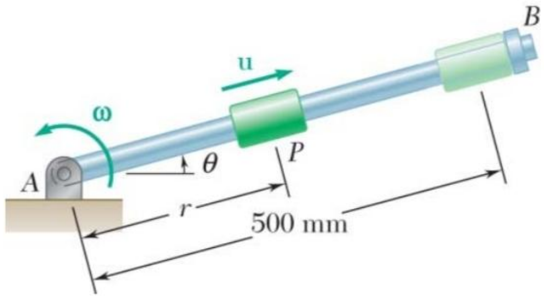
Relative motion

$$= \underline{\tilde{a}} + \ddot{\phi}\, \underline{e}_z \times \underline{\tilde{r}} - \dot{\phi}^2 \underline{\tilde{r}} + 2\dot{\phi}\, \underline{e}_z \times \underline{\tilde{v}} \; .$$

$$\Rightarrow \boxed{m\underline{\tilde{a}} = \underline{\tilde{F}} \underbrace{- m\ddot{\phi}\, \underline{e}_z \times \underline{\tilde{r}}}_{\text{"Euler force"}} \underbrace{+ m\dot{\phi}^2 \underline{\tilde{r}}}_{\text{"Centrifugal force"}} \underbrace{- 2m\dot{\phi}\, \underline{e}_z \times \underline{\tilde{v}}}_{\text{"Coriolis force"}}} \; .$$



These are called <u>pseudoforces</u>. To use Newton's Law in a rotating frame, we just add these "fictitious forces" and otherwise proceed as usual.

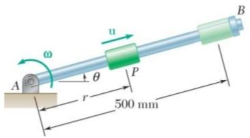


PROBLEM 15.172

The collar P slides outward at a constant relative speed u along rod AB, which rotates counterclockwise with a constant angular velocity of 20 rpm. Knowing that $r = 250$ mm when $\theta = 0$ and that the collar reaches B when $\theta = 90°$, determine the magnitude of the acceleration of the collar P just as it reaches B.

$$= \underline{\tilde{a}} + \ddot{\phi}\, \underline{e}_z \times \underline{\tilde{r}} - \dot{\phi}^2 \underline{\tilde{r}} + 2\dot{\phi}\, \underline{e}_z \times \underline{\tilde{v}} .$$

$$\Rightarrow \boxed{m\underline{\tilde{a}} = \underline{\tilde{F}} \underbrace{- m\ddot{\phi}\, \underline{e}_z \times \underline{\tilde{r}}}_{\text{"Euler force"}} + \underbrace{m\dot{\phi}^2 \underline{\tilde{r}}}_{\text{"Centrifugal force"}} \underbrace{- 2m\dot{\phi}\, \underline{e}_z \times \underline{\tilde{v}}}_{\text{"Coriolis force"}}}$$



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SOLUTION

$$\omega = 20 \text{ rpm} = \frac{(20)(2\pi)}{60} = \frac{2\pi}{3} \text{ rad/s}$$

$$\alpha = 0$$

$$\theta = 90° = \frac{\pi}{2} \text{ radians}$$

Uniform rotational motion.

$$\theta = \theta_0 + \omega t$$

$$t = \frac{\theta - \theta_0}{\omega} = \frac{\frac{\pi}{2}}{\frac{2\pi}{3}} = 0.75 \text{ s}$$

Uniform motion along rod.

$$r = r_0 + ut$$

$$u = \frac{r - r_0}{t} = \frac{0.5 - 0.25}{0.75} = \frac{1}{3} \text{ m/s},$$

$$\mathbf{v}_{P/AB} = \frac{1}{3} \text{ m/s} \uparrow$$

Acceleration of coinciding Point P' on the rod. $(r = 0.5 \text{ m})$

$$\mathbf{a}_{P'} = r\omega^2$$

$$= (0.5)\left(\frac{2\pi}{3}\right)^2$$

$$= \frac{2\pi^2}{9} \text{ m/s}^2 \downarrow$$

$$= 2.1932 \text{ m/s}^2 \downarrow$$

Acceleration of collar P relative to the rod. $\mathbf{a}_{P/AB} = 0$

Coriolis acceleration. $2\boldsymbol{\omega} \times \mathbf{v}_{P/AB} = 2\omega u = (2)\left(\frac{2\pi}{3}\right)\left(\frac{1}{3}\right) = 1.3963 \text{ m/s}^2 \longleftarrow$

Acceleration of collar P.

$$\mathbf{a}_P = \mathbf{a}_{P'} + \mathbf{a}_{P/AB} + 2\boldsymbol{\omega} \times \mathbf{v}_{P/AB}$$

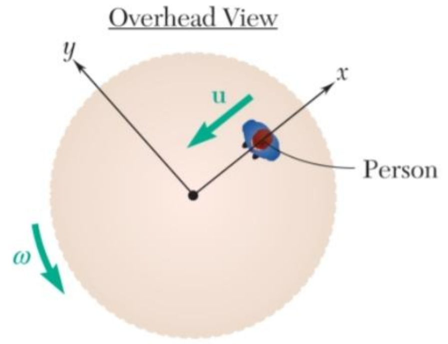
$$\mathbf{a}_P = [2.1932 \text{ m/s}^2 \downarrow] + [1.3963 \text{ m/s}^2 \longleftarrow]$$

$$\mathbf{a}_P = 2.60 \text{ m/s}^2 \nearrow 57.5°$$

$a_P = 2.60 \text{ m/s}^2$ ◀

PROBLEM 15.CQ8

A person walks radially inward on a platform that is rotating counterclockwise about its center. Knowing that the platform has a constant angular velocity **ω** and the person walks with a constant speed **u** relative to the platform, what is the direction of the acceleration of the person at the instant shown?



$$= \tilde{\underline{a}} + \ddot{\phi}\, \underline{e}_z \times \tilde{\underline{r}} - \dot{\phi}^2 \tilde{\underline{r}} + 2\dot{\phi}\, \underline{e}_z \times \tilde{\underline{v}} \,.$$

$$\Rightarrow \boxed{m\tilde{\underline{a}} = \tilde{\underline{F}} \underbrace{- m\ddot{\phi}\, \underline{e}_z \times \tilde{\underline{r}}}_{\text{"Euler force"}} + \underbrace{m\dot{\phi}^2 \tilde{\underline{r}}}_{\text{"Centrifugal force"}} \underbrace{- 2m\dot{\phi}\, \underline{e}_z \times \tilde{\underline{v}}}_{\text{"Coriolis force"}}}$$

PROBLEM 15.CQ8

A person walks radially inward on a platform that is rotating counterclockwise about its center. Knowing that the platform has a constant angular velocity $\boldsymbol{\omega}$ and the person walks with a constant speed $\mathbf{u}$ relative to the platform, what is the direction of the acceleration of the person at the instant shown?

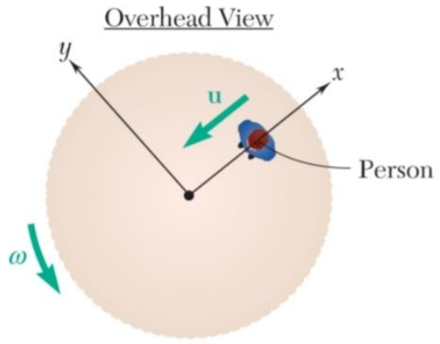
(*a*) Negative x

(*b*) Negative y

(*c*) Negative x and positive y

(*d*) Positive x and positive y

(*e*) Negative x and negative y



SOLUTION

The $\omega^2 r$ term will be in the negative x-direction and the Coriolis acceleration will be in the negative y-direction.

Answer: (*e*) ◀

Forces arising from potential energy:

Suppose a force varies with position according to a formula of the form:

$$\boxed{\underline{F}(\underline{r}) = -\nabla U(\underline{r}) \equiv -\left[\frac{\partial U}{\partial x}\underline{e}_x + \frac{\partial U}{\partial y}\underline{e}_y + \frac{\partial U}{\partial z}\underline{e}_z\right]}$$

where the scalar function $U(\underline{r})$ is called the "potential energy". Such forces are called "conservative". Not all forces are conservative! But many are.

Conservative force examples:

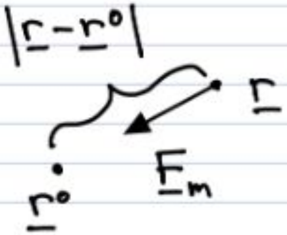
Weight: $\underline{W} = -mg\,\underline{e}_y = -\nabla(mgy). \Rightarrow U_g(\underline{r}) = mgy.$

Spring: $\underline{F}_s(\underline{r}) = -k(|\underline{r}-\underline{r}^\circ| - l_o)\dfrac{\underline{r}-\underline{r}}{|\underline{r}-\underline{r}\ |} = -\nabla\left(\frac{k}{2}(|\underline{r}-\underline{r}^\circ| - l_o)^2\right).$

$\Rightarrow U_s(\underline{r}) = \frac{k}{2}(|\underline{r}-\underline{r}^\circ| - l_o)^2.$

Constant force: $\underline{F}_o = -\nabla(-\underline{F}_o \cdot \underline{r}). \Rightarrow U_o(\underline{r}) = -\underline{F}_o \cdot \underline{r}.$

Power-law force: $\underline{F}_m(\underline{r}) = -c\,|\underline{r}-\underline{r}^\circ|^m \dfrac{\underline{r}-\underline{r}^\circ}{|\underline{r}-\underline{r}^\circ|}$



$= -\nabla\, c \cdot \frac{1}{m+1}|\underline{r}-\underline{r}^\circ|^{m+1}.$

$\Longrightarrow U_m(\underline{r}) = \frac{c}{m+1}|\underline{r}-\underline{r}^\circ|^{m+1}.$

PROBLEM 13.79*

Prove that a force $F(x, y, z)$ is conservative if, and only if, the following relations are satisfied:

$$\frac{\partial F_x}{\partial y} = \frac{\partial F_y}{\partial x} \quad \frac{\partial F_y}{\partial z} = \frac{\partial F_z}{\partial y} \quad \frac{\partial F_z}{\partial x} = \frac{\partial F_x}{\partial z}$$

$$\boxed{\underline{F}(\underline{r}) = -\nabla U(\underline{r}) \equiv -\left[\frac{\partial U}{\partial x}\underline{e}_x + \frac{\partial U}{\partial y}\underline{e}_y + \frac{\partial U}{\partial z}\underline{e}_z\right]}$$

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SOLUTION

For a conservative force, Equation (13.22) must be satisfied.

$$F_x = -\frac{\partial V}{\partial x} \quad F_y = -\frac{\partial V}{\partial y} \quad F_z = -\frac{\partial V}{\partial z}$$

We now write

$$\frac{\partial F_x}{\partial y} = -\frac{\partial^2 V}{\partial x \partial y} \quad \frac{\partial F_y}{\partial x} = -\frac{\partial^2 V}{\partial y \partial x}$$

Since $\dfrac{\partial^2 V}{\partial x \partial y} = \dfrac{\partial^2 V}{\partial y \partial x}$:

$$\frac{\partial F_x}{\partial y} = \frac{\partial F_y}{\partial x} \blacktriangleleft$$

We obtain in a similar way

$$\frac{\partial F_y}{\partial z} = \frac{\partial F_z}{\partial y} \quad \frac{\partial F_z}{\partial x} = \frac{\partial F_x}{\partial z} \blacktriangleleft$$

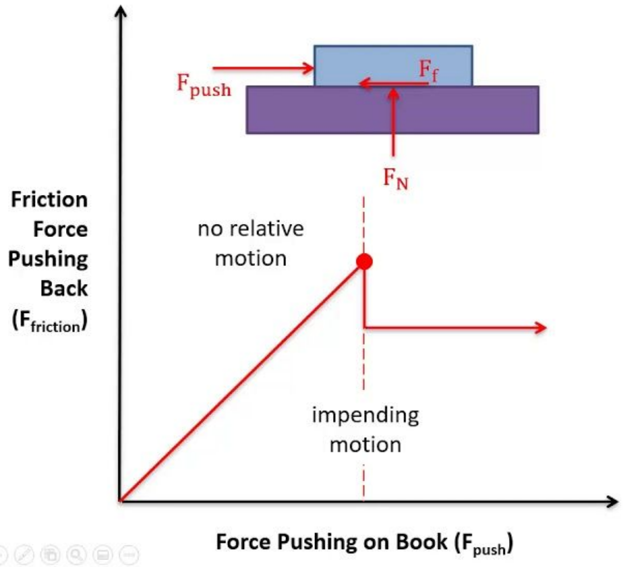
Friction

- Define $\underline{F}^*$ as sum of all forces except for $\underline{F}_t$ on a body.
- $\underline{F}_t^* \equiv \underline{F}^* - (\underline{F}^* \cdot \underline{e}_n)\underline{e}_n$ is the part of $\underline{F}^*$ tangent to the plane.
- $\underline{F}_{trial} \equiv m[\dot{\underline{v}}_{wall} - (\underline{e}_n \cdot \dot{\underline{v}}_{wall})\underline{e}_n] - \underline{F}_t^*$ is the force the friction would have to supply to keep the object from sliding.

The force of friction can thus be expressed by:

$$\underline{F}_t = \begin{cases} \underline{F}_{trial} & \text{if } \underline{v}_s = \underline{0} \ \& \ |\underline{F}_{trial}| \le \mu_s F_n \\ \mu_s F_n \left(\underline{F}_{trial} / |\underline{F}_{trial}|\right) & \text{if } \underline{v}_s = \underline{0} \ \& \ |\underline{F}_{trial}| > \mu_s F_n \\ -\mu_d F_n \left(\underline{v}_s / |\underline{v}_s|\right) & \text{if } \underline{v}_s \neq \underline{0}. \end{cases}$$

Static and Kinetic Friction



$$F_f = F_{push}$$

Until the point where...

$$F_f = \mu_s * F_N$$

After that point the friction force often drops down to and stays at...

$$F_f = \mu_k * F_N$$

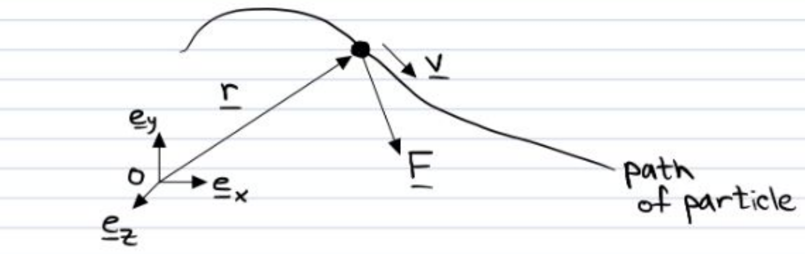
Work-Energy

① $$\boxed{W_{AB}^{Tot} = \sum_{\substack{\text{All}\\ \text{forces}}} \int_{\substack{\text{Path}\\ \underline{r}_A\to\underline{r}_B}} \underline{F}^i\cdot d\underline{r} = \frac{1}{2}mv_B^2 - \frac{1}{2}mv_A^2}.$$

② $$\boxed{W_{AB}^{nc} = \sum_{\substack{\text{Non-}\\ \text{conservative}\\ \text{forces}}} \int_{\substack{\text{Path}\\ \underline{r}_A\to\underline{r}_B}} \underline{F}^{nc}\cdot d\underline{r} = E_B - E_A}$$

where $E = \frac{1}{2}mv^2 + \sum_c U^c$

Total potential energy of all conservative forces. $(\underline{F}^c = -\nabla U^c)$

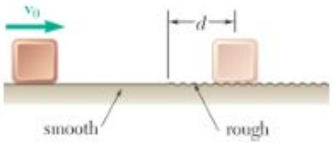


Power

$$\boxed{\text{Power} \equiv \underline{F}\cdot\underline{v}}$$

= Force • velocity

Q: Powerless force?



PROBLEM 13.CQ1

Block A is traveling with a speed v_0 on a smooth surface when the surface suddenly becomes rough with a coefficient of friction of μ causing the block to stop after a distance d. If block A were traveling twice as fast, that is, at a speed $2v_0$, how far will it travel on the rough surface before stopping?

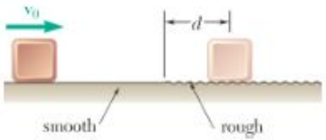
(*a*) $d/2$

(*b*) d

(*c*) $\sqrt{2}d$

(*d*) $2d$

(*e*) $4d$



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(*d*) $2d$

(*e*) $4d$

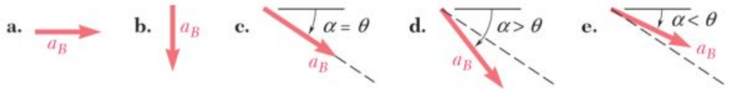
SOLUTION

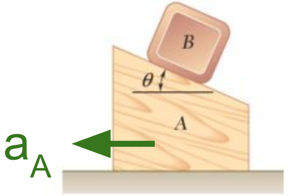
If the block were traveling twice as fast, its kinetic energy would be 4x as much ($T = \frac{1}{2}mv^2$). The work done by friction to stop the block is $U = Fd$. This work must equal the amount of kinetic energy before the block hits the rough patch. Since the kinetic energy is 4x as much it will take 4x the distance to stop.

Answer: (*e*)

PROBLEM 11.CQ7

Blocks A and B are released from rest in the positions shown. Neglecting friction between all surfaces, which figure below best indicates the direction α of the acceleration of block B?





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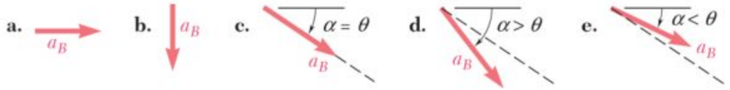
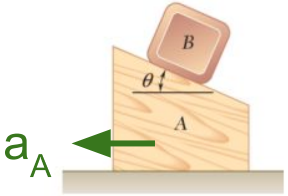
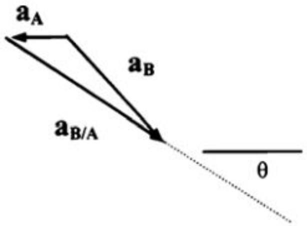


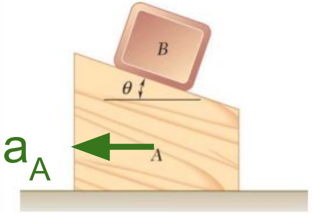


SOLUTION

Since $\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$ we get

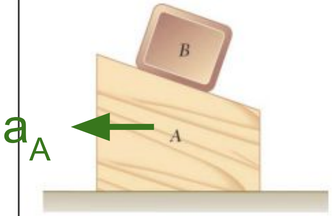


Answer: (d) ◀



PROBLEM 12.F4

Blocks A and B have masses m_A and m_B respectively. Neglecting friction between all surfaces, draw the FBD and KD for each mass.



PROBLEM 12.CQ4

The system shown is released from rest in the position shown. Neglecting friction, the normal force between block A and the ground is

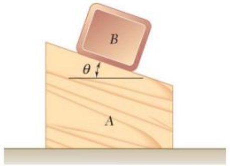
(a) less than the weight of A plus the weight of B

(b) equal to the weight of A plus the weight of B

(c) greater than the weight of A plus the weight of B

FBD with friction?

PROBLEM 12.F4



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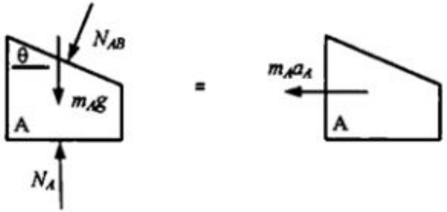
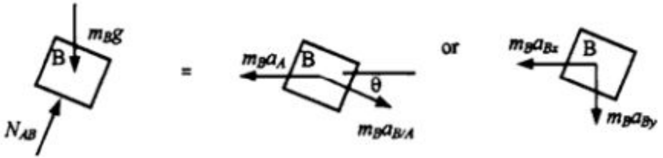
SOLUTION

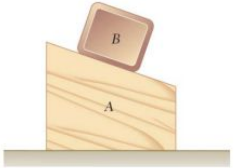
Block A



Block B



PROBLEM 12.CQ4



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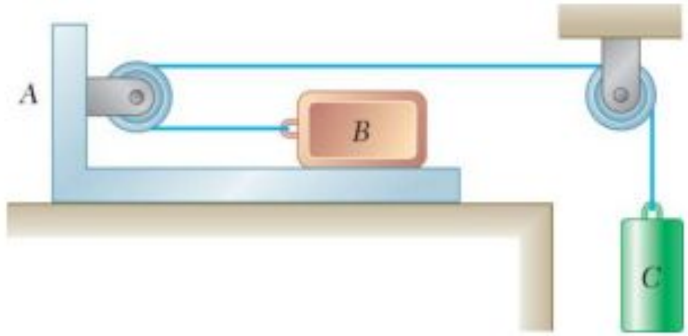
(*a*) less than the weight of A plus the weight of B

(*b*) equal to the weight of A plus the weight of B

(*c*) greater than the weight of A plus the weight of B

SOLUTION

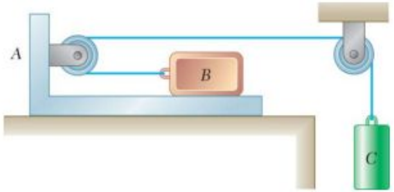
Answer: (*a*) Since B has an acceleration component downward the normal force between A and the ground will be less than the sum of the weights. To see this, you can draw an FBD of both block A and B. From block A you will find that the normal force on the ground is the sum of the weight of block A plus the vertical component of the normal force of block B on A since block A does not accelerate in the vertical direction. From the FBD of block B and using F=ma, you will see that the vertical component of the normal force of B on A is less than the weight of block B since it is accelerating downwards.



PROBLEM 12.F3

Objects A, B, and C have masses m_A, m_B, and m_C respectively. The coefficient of kinetic friction between A and B is μ_k, and the friction between A and the ground is negligible and the pulleys are massless and frictionless. Assuming B slides on A draw the FBD and KD for each of the three masses A, B and C.

PROBLEM 12.F3



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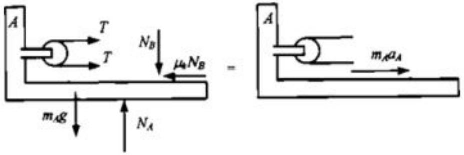
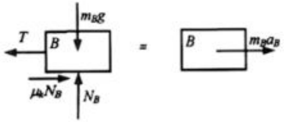
SOLUTION

Answer:

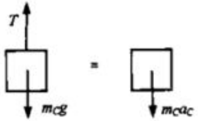
Block A



Block B

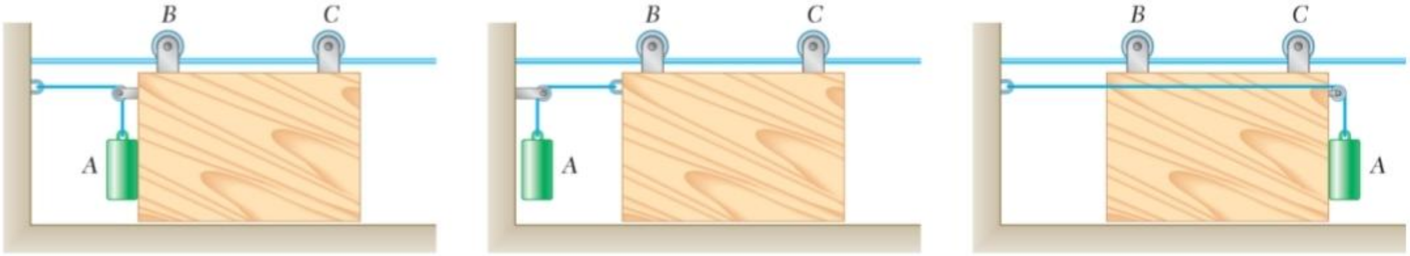


Block C



PROBLEM 12.29

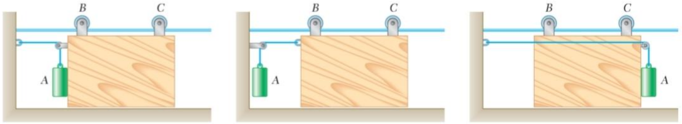
A 40-lb sliding panel is supported by rollers at *B* and *C*. A 25-lb counterweight *A* is attached to a cable as shown and, in cases *a* and *c*, is initially in contact with a vertical edge of the panel. Neglecting friction, determine in each case shown the acceleration of the panel and the tension in the cord immediately after the system is released from rest.



(*a*) (*b*) (*c*)

PROBLEM 12.29

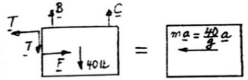
A 40-lb sliding panel is supported by rollers at B and C. A 25-lb counterweight A is attached to a cable as shown and, in cases a and c, is initially in contact with a vertical edge of the panel. Neglecting friction, determine in each case shown the acceleration of the panel and the tension in the cord immediately after the system is released from rest.



(*a*) (*b*) (*c*)

SOLUTION

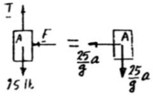
(*a*) <u>Panel</u>: F = Force exerted by counterweight



$\overset{+}{\leftarrow} \Sigma F_x = ma$: $\qquad T - F = \dfrac{40}{g}a \qquad (1)$

<u>Counterweight A</u>: Its acceleration has two components

$$\mathbf{a}_A = \mathbf{a}_P + \mathbf{a}_{A/P} = a \rightarrow + a \downarrow$$



$\overset{+}{\leftarrow} \Sigma F_x = ma_x$: $\quad F = \dfrac{25}{g}a \qquad (2)$

$+\!\downarrow \Sigma F_g = ma_g$: $\quad 25 - T = \dfrac{25}{g}a \qquad (3)$

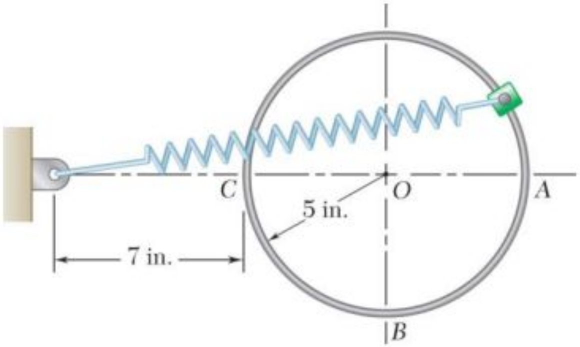
Adding (1), (2), and (3):

$$\cancel{T} - F + F + 25 - \cancel{T} = \frac{40+25+25}{g}a$$

$$a = \frac{25}{90}g = \frac{25}{90}(32.2) \qquad \mathbf{a} = 8.94 \text{ ft/s}^2 \longleftarrow \blacktriangleleft$$

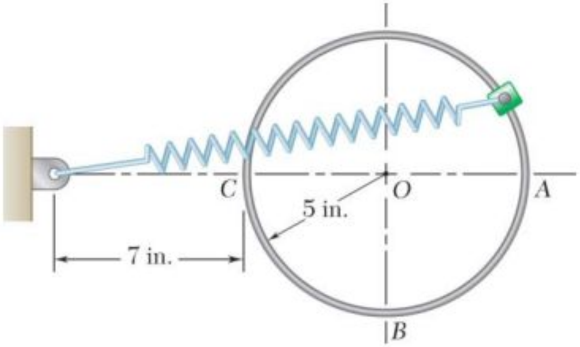
Substituting for a into (3):

$$25 - T = \frac{25}{g}\left(\frac{25}{90}g\right) \qquad T = 25 - \frac{625}{90} \qquad T = 18.06 \text{ lb} \blacktriangleleft$$



PROBLEM 12.F8

A collar of mass m is attached to a spring and slides without friction along a circular rod in a vertical plane. The spring has an undeformed length of 5 in. and a constant k. Knowing that the collar has a speed v at Point C, draw the FBD and KD of the collar at this point.



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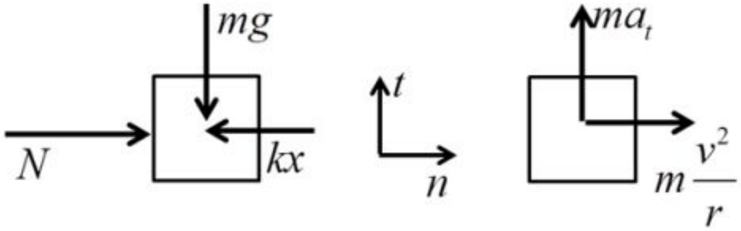
SOLUTION



where $x = 2/12$ ft and $r = 5/12$ ft.