ME 104 Engineering Mechanics II

FALL 2024

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Discussion Section Week 3: Curvilinear motion, one-particle dynamics, coupled motion

Week 3/14 (09/09/2024 - 09/13/2024)

PSET 1 due *this week* on Friday, September 13 (midnight)

MIDTERM 1 in *4 weeks* on Thursday, October 10

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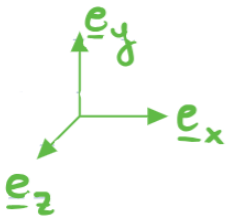
Cartesian Coordinate System

- **Position**: $\boldsymbol{r} = \boldsymbol{r}(t) = x(t)\boldsymbol{e}_x + y(t)\boldsymbol{e}_y + z(t)\boldsymbol{e}_z$

  Note that $[\boldsymbol{e}_x, \boldsymbol{e}_y, \boldsymbol{e}_z]$ is a fixed coordinate system, meaning that $\frac{d}{dt}[\boldsymbol{e}_x, \boldsymbol{e}_y, \boldsymbol{e}_z] = [0, 0, 0]$
- **Velocity**: $\boldsymbol{v} = \boldsymbol{v}(t) = \dot{\boldsymbol{r}}(t) = \frac{d\boldsymbol{r}(t)}{dt}$
- **Acceleration**: $\boldsymbol{a} = \boldsymbol{a}(t) = \dot{\boldsymbol{v}}(t) = \ddot{\boldsymbol{r}}(t)$
- **Speed**: $v(t) = ||\boldsymbol{v}(t)|| = \sqrt{\boldsymbol{v}(t) \cdot \boldsymbol{v}(t)} > 0$
- **Distance**: $s = s(t) = s_0 + \int_{t_0}^{t} v(\tau)d\tau > 0$ (increasing only)



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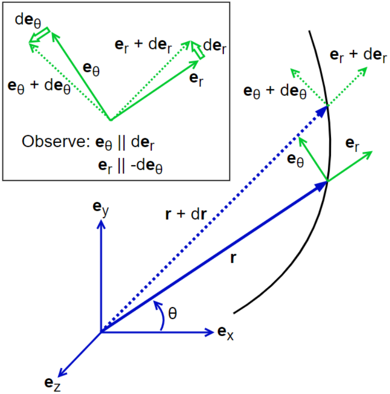
Cylindrical Coordinate System

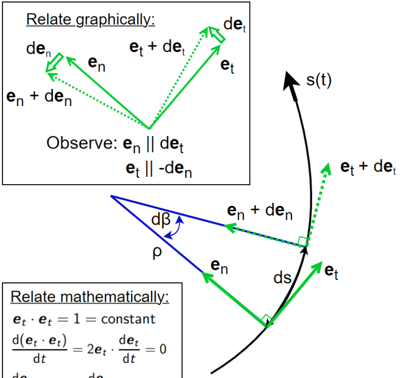
- **Cylindrical basis**: calculate derivatives using chain rule and Cartesian basis
  $\boldsymbol{e}_r = \cos\theta\, \boldsymbol{e}_x + \sin\theta\, \boldsymbol{e}_y, \qquad \dot{\boldsymbol{e}}_r = \dot\theta\, \boldsymbol{e}_\theta$
  $\boldsymbol{e}_\theta = -\sin\theta\, \boldsymbol{e}_x + \cos\theta\, \boldsymbol{e}_y, \quad \dot{\boldsymbol{e}}_\theta = -\dot\theta\, \boldsymbol{e}_r$
  $\dot{\boldsymbol{e}}_z = 0, \qquad x = r\cos\theta, \qquad y = r\sin\theta$
- **Position**: $\boldsymbol{r} = r\, \boldsymbol{e}_r + z\, \boldsymbol{e}_z$

  Calculate velocity and acceleration using product rule
- **Velocity**: $\boldsymbol{v} = \dot{\boldsymbol{r}} = \dot r\, \boldsymbol{e}_r + r\dot\theta\, \boldsymbol{e}_\theta + \dot z\, \boldsymbol{e}_z$
- **Acceleration**:
  $\boldsymbol{a} = \dot{\boldsymbol{v}} = (\ddot r - r\dot\theta^2)\, \boldsymbol{e}_r + (r\ddot\theta + 2\dot r\dot\theta)\, \boldsymbol{e}_\theta + \ddot z\, \boldsymbol{e}_z$



Normal-Tangent Coordinate System

- **Radius (ρ) of local curvature (κ)** approximated for sufficiently small angles:
  $$\kappa = \frac{1}{\rho} = \frac{\left|\frac{d^2y}{dx^2}\right|}{[1+(\frac{dy}{dx})^2]^{3/2}} \approx \frac{d^2y}{dx^2}$$
  (Recall slope: $m = \frac{\text{rise}}{\text{run}} = \frac{dy}{dx}$)
- **Infinitesimal path traveled:** $ds = \rho\beta$
- **Speed**: $v = \frac{ds}{dt} = \rho\frac{d\beta}{dt} = \rho\dot{\beta}$
- **Velocty**: $\boldsymbol{v} = v\boldsymbol{e}_t = \rho\dot{\beta}\boldsymbol{e}_t$
- **Acceleration**: using product rule
  $\boldsymbol{a} = \frac{d\boldsymbol{v}}{dt} = \frac{dv\boldsymbol{e}_t}{dt} = \frac{dv}{dt}\boldsymbol{e}_t + v\frac{d\boldsymbol{e}_t}{dt}$,
  where $\frac{d\boldsymbol{e}_t}{dt} = \frac{d\beta}{dt}\boldsymbol{e}_n = \dot{\beta}\boldsymbol{e}_n = \frac{v}{\rho}\boldsymbol{e}_n$,
  thus $\boldsymbol{a} = \dot{v}\boldsymbol{e}_t + \frac{v^2}{\rho}\boldsymbol{e}_n$



Relate mathematically:

$\boldsymbol{e}_t \cdot \boldsymbol{e}_t = 1 = \text{constant}$

$\frac{d(\boldsymbol{e}_t \cdot \boldsymbol{e}_t)}{dt} = 2\boldsymbol{e}_t \cdot \frac{d\boldsymbol{e}_t}{dt} = 0$

$\frac{d\boldsymbol{e}_t}{dt} \perp \boldsymbol{e}_t \rightarrow \frac{d\boldsymbol{e}_t}{dt} \| \boldsymbol{e}_n$

One-Particle Dynamics Problems

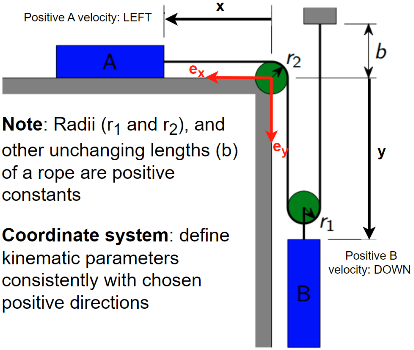
Four steps to solve:

1. **Write out the kinematics:** origin, coordinate system, $\boldsymbol{r}$, $\boldsymbol{v}$, and $\boldsymbol{a}$
2. **Draw a free body diagram (FBD):**
   - Draw the object
   - Draw the force vectors
3. **Write out** $\sum \boldsymbol{F} = m\boldsymbol{a}$
4. **Perform the analysis**: $\boldsymbol{a}(t) \rightarrow \boldsymbol{v}(t) \rightarrow \boldsymbol{r}(t)$ and/or unknown constraint forces

Coupled Motion

- Set *fixed* position of the coordinate system and choose positive directions
- Describe geometrical constraints mathematically: Length $L = x + 2y + \pi r_1 + \frac{1}{2}\pi r_2 + b$
- Take the derivative of the constraint to determine kinematic relationships (constants cancel out): $\dot{L} = 0$

$$\dot{x} + 2\dot{y} = 0 \;\rightarrow\; v_A = -2v_B$$
$$\ddot{x} + 2\ddot{y} = 0 \;\rightarrow\; a_A = -2a_B$$



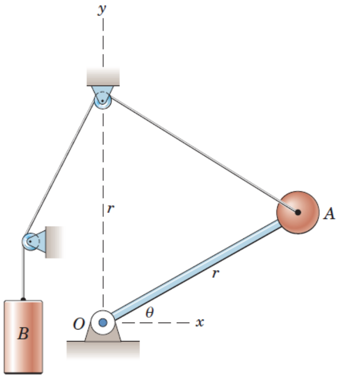
Note: Radii (r_1 and r_2), and other unchanging lengths (b) of a rope are positive constants

Coordinate system: define kinematic parameters consistently with chosen positive directions

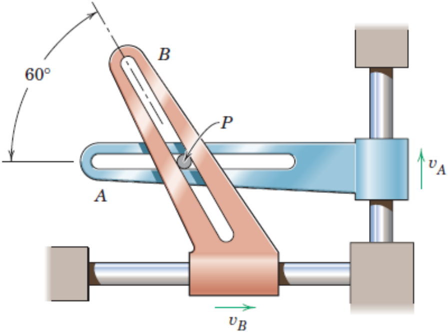
Conceptual question: How would solution change if $\boldsymbol{e}_x$ is chosen positive to the RIGHT? (Hint: length is constant regardless of the coordinate system)

Problems

1. The particle A is mounted on a light rod pivoted at O and therefore is constrained to move in a circular arc of radius r. Determine the velocity of A in terms of the downward velocity v_B of the counterweight for any angle θ.



2. The motion of pin P is controlled by the two moving slots A and B in which the pin slides. If B has a velocity $v_B = 3$ m/s to the right while A has an upward velocity $v_A = 2$ m/s, determine the magnitude of the velocity v_p of the pin.



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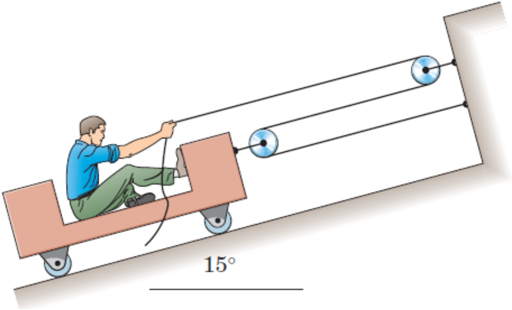
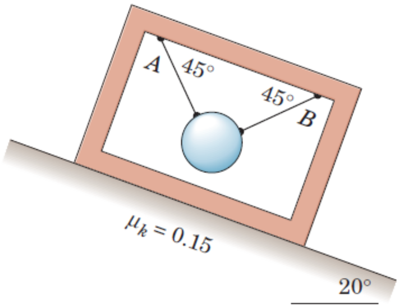
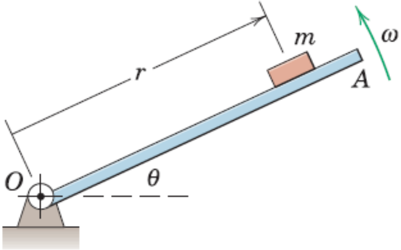
3. A man pulls himself up the incline by the method shown. If the combined mass of the man and cart is 100 kg, determine the acceleration of the cart if the man exerts a pull of 250 N on the rope. Neglect all friction and the mass of the rope, pulleys, and wheels.



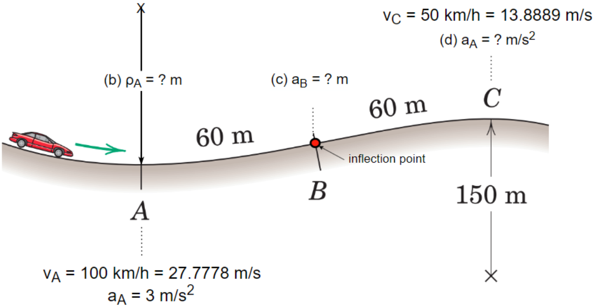
4. The 10-kg steel sphere is suspended from the 15-kg frame which slides down the incline. If the coefficient of kinetic friction between the frame and incline is 0.15, compute the tension in each of the supporting wires A and B.



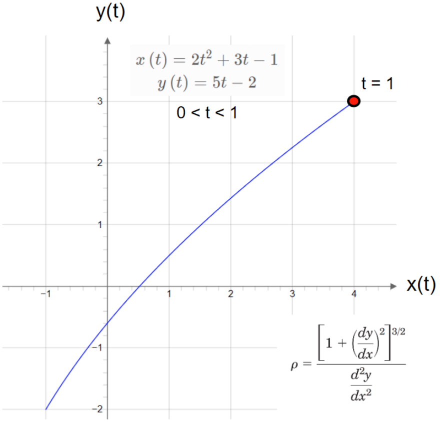
5. The member OA rotates about a horizontal axis through O with a constant counterclockwise angular velocity $\omega = 3$ rad/sec. As it passes the position $\theta = 0°$, a small block of mass m is placed on it at a radial distance $r = 45$ cm. If the block is observed to slip at $\theta = 50°$ determine the coefficient of static friction μ_s between the block and the member.



6. To anticipate the dip and hump in the road, the driver of a car applies her brakes to produce a *uniform deceleration*. Her speed is 100 km/h at the bottom A of the dip and 50 km/h at the top C of the hump, which is 120 m along the road from A. If the passengers experience a total acceleration of 3 m/s^2 at A and if the radius of curvature of the hump at C is 150 m, calculate
   (a) the deceleration along the path,
   (b) the radius of curvature ρ at A,
   (c) the acceleration at the inflection point B, and
   (d) the total acceleration at C.



7. A particle which moves in two-dimensional curvilinear motion has coordinates in meters which vary with time t in seconds according to $x(t) = 2t^2 + 3t - 1$ and $y(t) = 5t - 2$. For time $t = 1$, determine
   (a) the radius of curvature of the particle path,
   (b) the magnitude of the normal acceleration,
   (c) the magnitude of the tangential acceleration, and
   (d) position of the center of the radius of curvature.



8. The particle of mass m = 0.2 kg travels with constant speed v in a circular path around the conical body. Determine the tension T in the cord. Neglect all friction, and use the values h = 0.8 m and v = 0.6 m/s. For what value of v does the normal force go to zero?