ME104: Engineering Mechanics II Discussion Week 4 of 15

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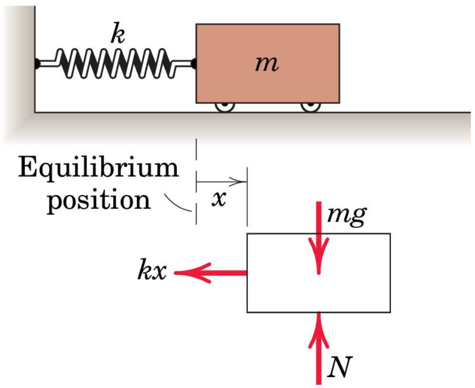
Topics: Vibration, spring, friction, and conservation of angular momentum

PSET 2 due *in two weeks* on September 27, 2024

Midterm I is *in three weeks* on October 10, 2024 (in class)

Free Vibration of Particles: Undampened



$$\Sigma F_x = m\ddot{x}$$

$$-kx = m\ddot{x} \qquad \text{or} \qquad m\ddot{x} + kx = 0$$

Rearrange to formulate: simple harmonic motion equation

$$\boxed{\ddot{x} + \omega_n{}^2 x = 0} \qquad \boxed{\omega_n = \sqrt{k/m}}$$

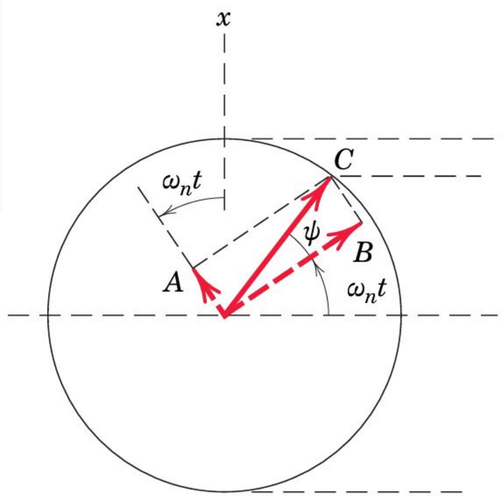
With solution of the form:

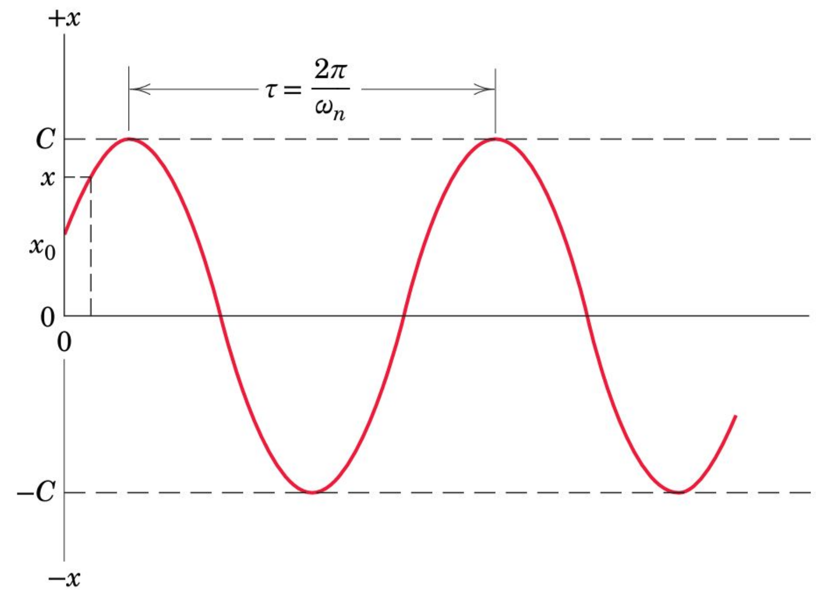
$$x = A\cos\omega_n t + B\sin\omega_n t \qquad \text{or} \qquad x = C\sin(\omega_n t + \psi)$$

What is natural frequency?

$$f_n = \omega_n / 2\pi$$

Units: 1 hertz (Hz) = 1 cycle per second



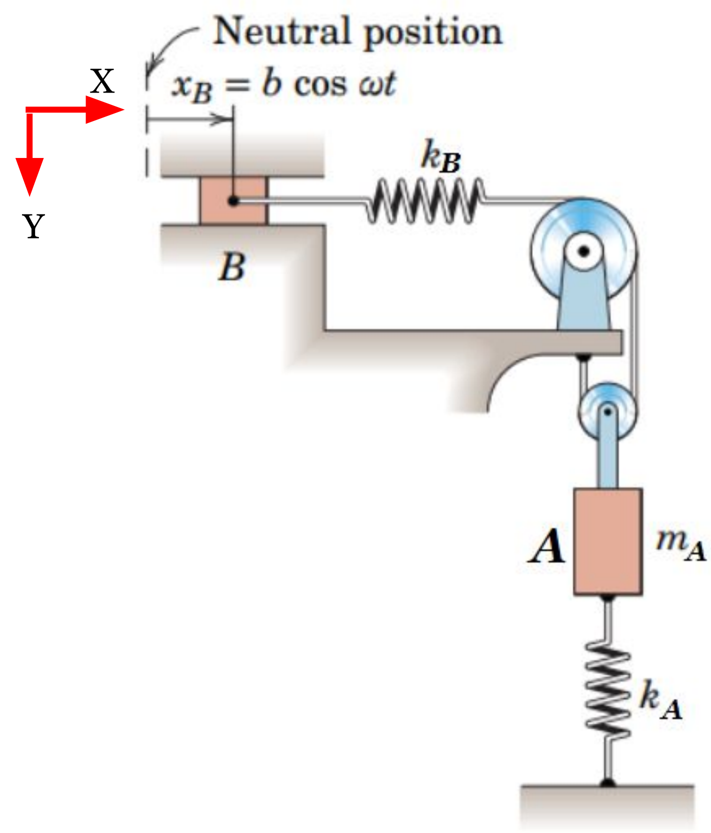


Free Vibration of Particles: Undamped Example

Formulate equations of motion for block A:

1. What is the equation for the length of the rope under equilibrium?
2. What is the length of the rope once the system is moving?
3. What is the tension T in the rope (dynamic tension)?
4. Which direction does the spring force act?

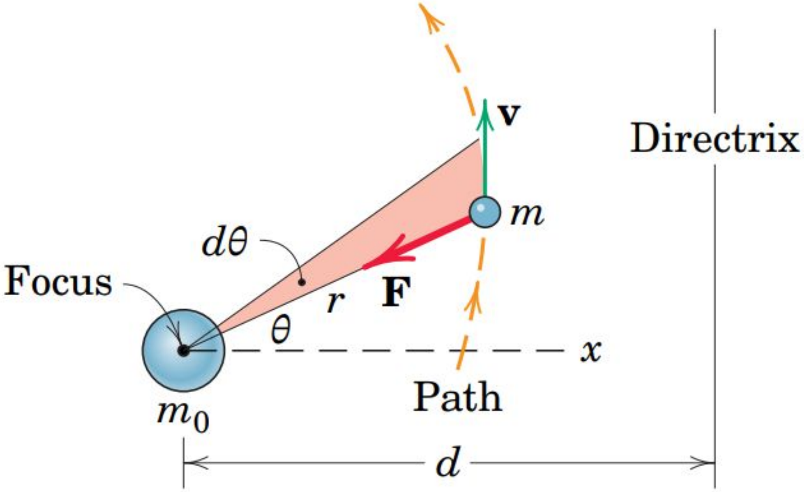
Bonus question: what frequency should be avoided when driving block B?



Conservation of angular momentum: central-force motion

$$-F = m(\ddot{r} - r\dot{\theta}^2)$$

$$0 = m(r\ddot{\theta} + 2\dot{r}\dot{\theta})$$



Multiply 2nd equation with: r/m

$$0 = r^2\ddot{\theta} + 2r\dot{r}\dot{\theta}$$

Observe that it is equal to:

$$0 = \frac{d}{dt}(r^2\dot{\theta})$$

$$\boxed{\mathbf{a} = (\ddot{r} - r\dot{\theta}^2)\mathbf{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\mathbf{e}_\theta}$$

Integrating zero gives us a constant:

$$\boxed{constant = r^2\dot{\theta}}$$

$$a_r = \ddot{r} - r\dot{\theta}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} \longrightarrow a_\theta = \frac{1}{r}\frac{d}{dt}(r^2\dot{\theta})$$

$$a = \sqrt{{a_r}^2 + {a_\theta}^2}$$

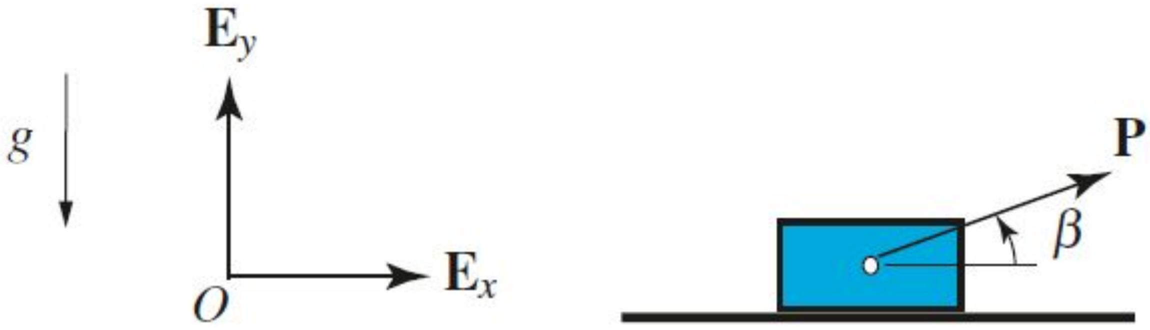
Friction: static and kinetic

- The amount of static friction available is limited by the coefficient of static friction:

$$\left\|\mathbf{F}_f\right\| \leq \mu_s \left\|\mathbf{N}\right\|$$

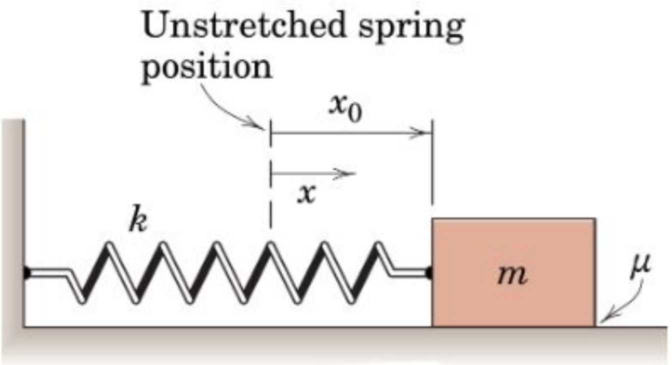
- If this criterion fails, then the particle will move relative to the surface. The friction force in this case is dynamic:

$$\mathbf{F}_f = \mathbf{F}_{f_{\text{dynamic}}} = -\mu_d \left\|\mathbf{N}\right\| \frac{\mathbf{v}_{\text{rel}}}{\left\|\mathbf{v}_{\text{rel}}\right\|}$$



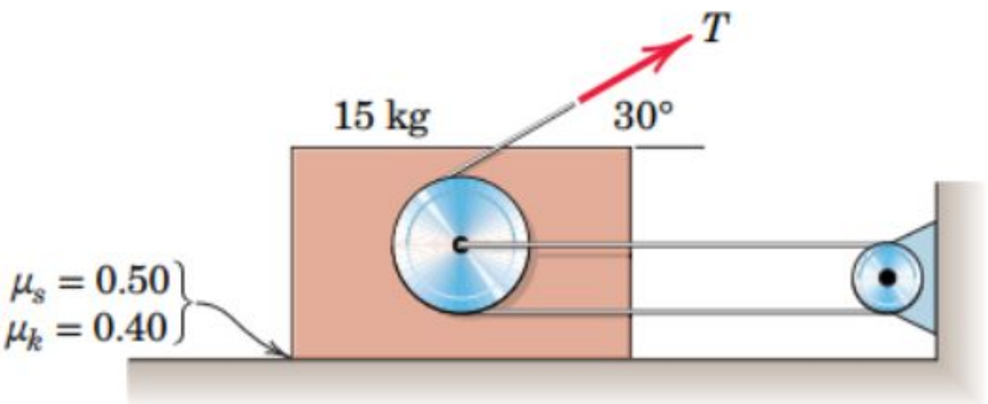
Practice problem 1: kinetic and static friction

A block of mass m rests on a rough horizontal surface and is attached to a spring of stiffness k. The coefficients of both static and kinetic friction are μ. The block is displaced a distance x_0 to the right of the unstretched position of the spring and released from rest. If the value of x_0 is large enough, the spring force will overcome the maximum available static friction force and the block will slide toward the unstretched position of the spring with an acceleration $a = \mu g - \frac{k}{m}x$, where x represents the amount of stretch (or compression) in the spring at any given location in the motion. Use the values $m = 5$ kg, $k = 150$ N/m, $\mu = 0.40$, and $x_0 = 200$ mm and determine the final spring stretch (or compression) x_f when the block comes to a complete stop.



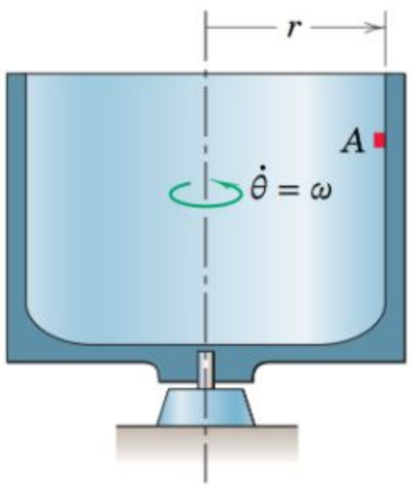
Practice problem 2: kinetic and static friction

Determine the initial acceleration of the 15-kg block if (*a*) $T = 23$ N and (*b*) $T = 26$ N. The system is initially at rest with no slack in the cable, and the mass and friction of the pulleys are negligible.



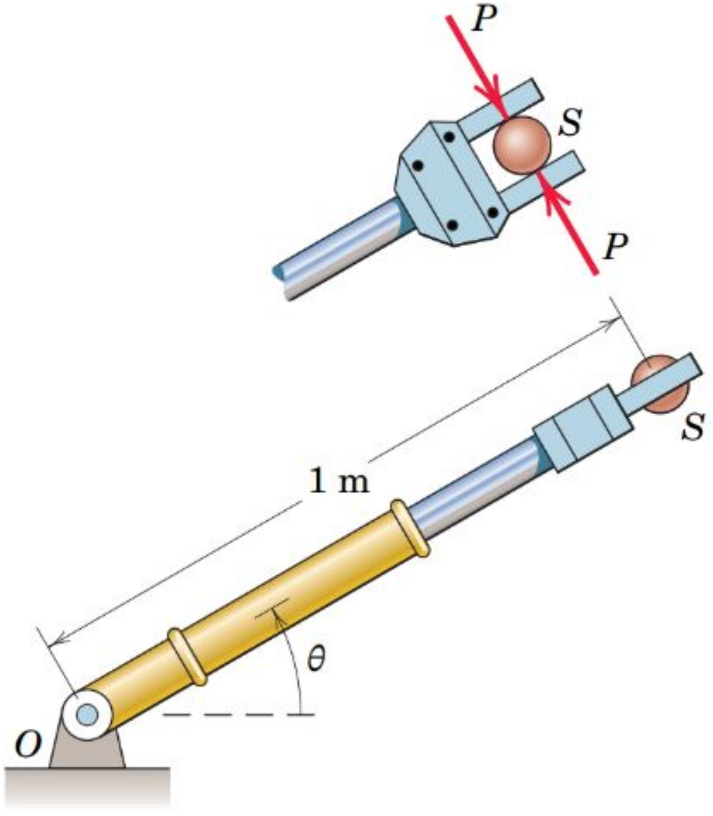
Practice problem 3: static friction and fixed R rotation

A small object A is held against the vertical side of the rotating cylindrical container of radius r by centrifugal action. If the coefficient of static friction between the object and the container is μ_s, determine the expression for the minimum rotational rate $\dot{\theta} = \omega$ of the container which will keep the object from slipping down the vertical side.



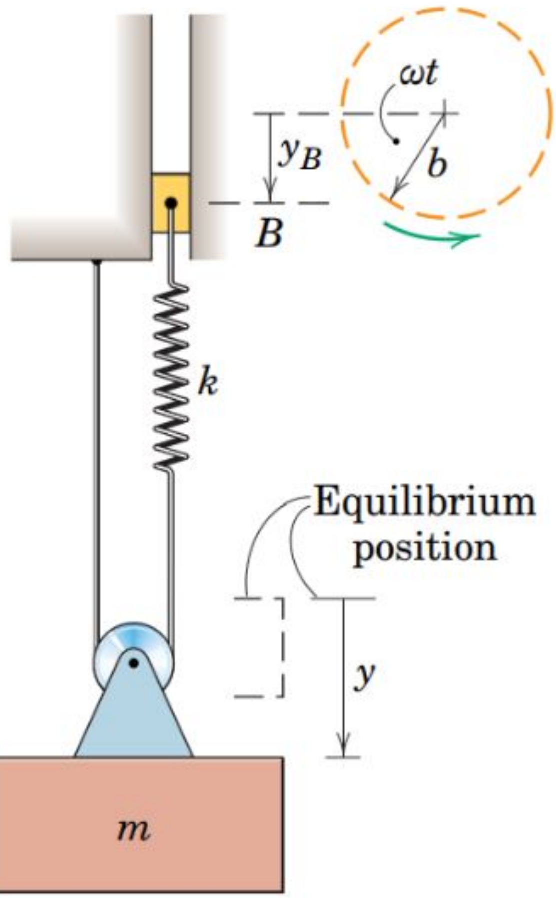
Practice problem 4: static friction and rotating arm

A 2-kg sphere S is being moved in a vertical plane by a robotic arm. When the angle θ is 30°, the angular velocity of the arm about a horizontal axis through O is 50 deg/s clockwise and its angular acceleration is 200 deg/s^2 counterclockwise. In addition, the hydraulic element is being shortened at the constant rate of 500 mm/s. Determine the necessary minimum gripping force P if the coefficient of static friction between the sphere and the gripping surfaces is 0.50. Compare P with the minimum gripping force P_s required to hold the sphere in static equilibrium in the 30° position.

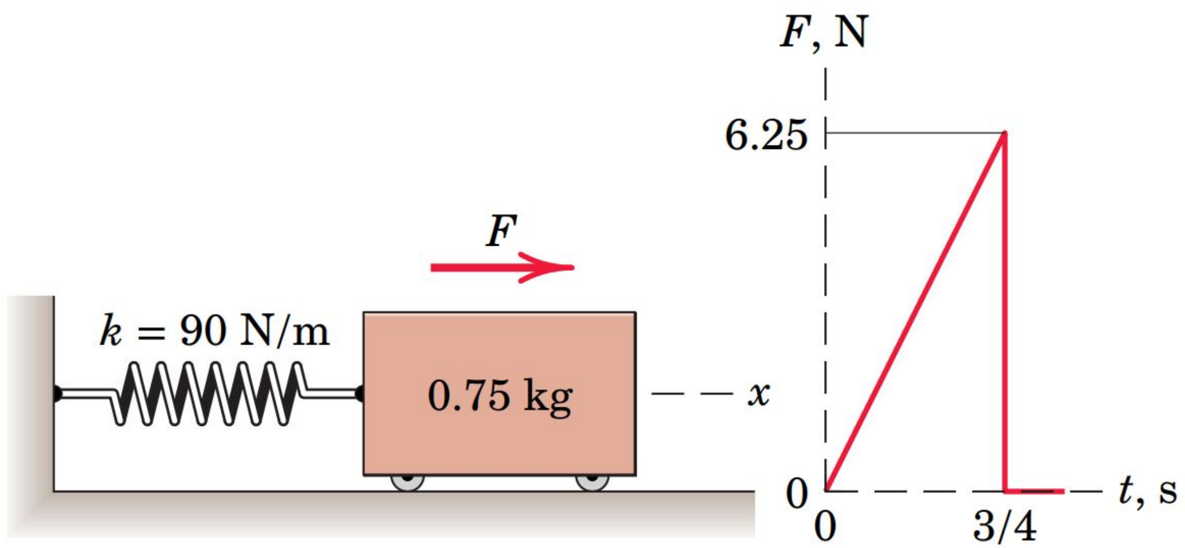


Practice problem 5: rope and springs

The equilibrium position of the mass m occurs where $y = 0$ and $y_B = 0$. When the attachment B is given a steady vertical motion $y_B = b \sin \omega t$, the mass m will acquire a steady vertical oscillation. Derive the differential equation of motion for m and specify the circular frequency ω_c for which the oscillations of m tend to become excessively large. The stiffness of the spring is k, and the mass and friction of the pulley are negligible.



Practice problem 6: computer implementation



Determine and plot the response x as a function of time for the undamped linear oscillator subjected to the force F which varies linearly with time for the first $\frac{3}{4}$ second as shown. The mass is initially at rest with $x = 0$ at time $t = 0$.

Practice problem 7: spring central motion

A particle of mass m moves with negligible friction on a horizontal surface and is connected to a light spring fastened at O. At position A the particle has the velocity $v_A = 4$ m/s. Determine the velocity v_B of the particle as it passes position B.