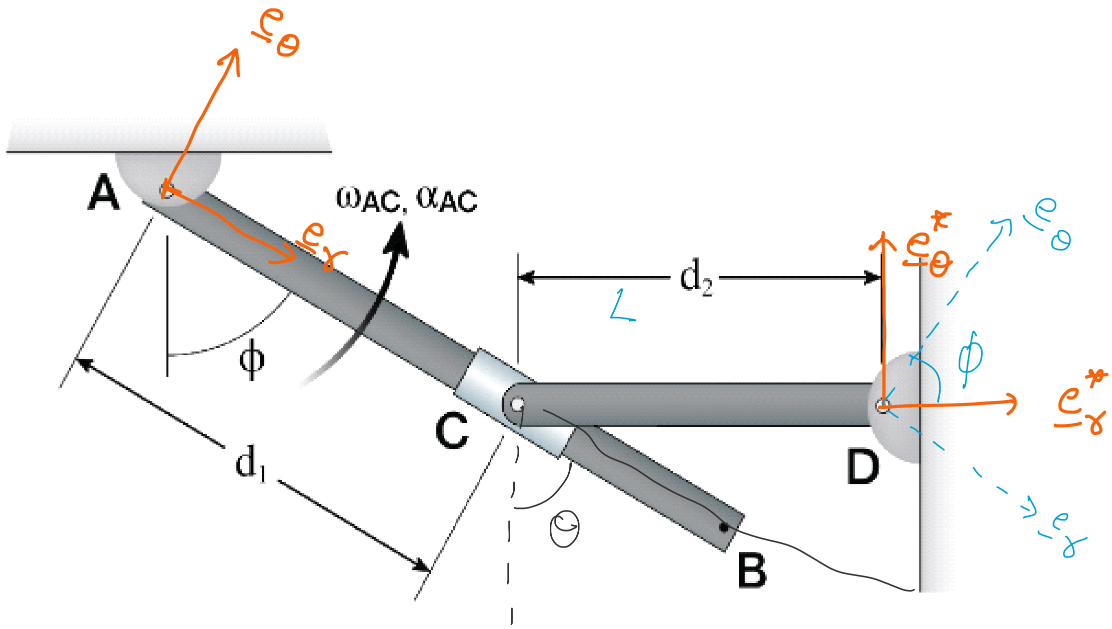
October 4th, 7th , 2024

Q. A collar which is pinned to rod *CD* slides along rod *AB*. At the instant shown, when $\phi = 60^\circ$, the angular velocity of rod *AB* is 2 rads/s in the direction shown. Find the rate at which *C* travels along rod *AB*, and the angular velocity of rod *CD*. Assume $d_1 = 0.3$ m and $d_2 = 0.2$ m.



$$\underline{r}_C = d_1 \underline{e}_r \Rightarrow \dot{\underline{r}}_C = \dot{d}_1 \underline{e}_r + d_1 \dot{\underline{e}}_r$$

$$\dot{\underline{e}}_r = \dot{\theta} \underline{e}_\theta$$

$$\dot{\underline{e}}_\theta = -\dot{\theta} \underline{e}_r$$

$$= \dot{d}_1 \underline{e}_r + d_1 \dot{\theta} \underline{e}_\theta$$

$$= \dot{d}_1 \underline{e}_r + d_1 \omega_{AC} \underline{e}_\theta \quad \text{— } (1)$$

$$\underline{r}_C = -d_2 \underline{e}_r^*$$

$$\underline{e}_\theta^* = -\cos\phi \, \underline{e}_r + \sin\phi \, \underline{e}_\theta$$

$$\dot{\underline{r}}_C = -d_2 \dot{\underline{e}}_r^* = d_2 \omega_{CD} \underline{e}_\theta^* \quad (\text{Assuming } \omega_{CD} \rightarrow CCW)$$

$$= d_2 \omega_{CD} (-\cos\phi \, \underline{e}_r + \sin\phi \, \underline{e}_\theta) \quad \text{— } (2)$$

$$\underline{\underline{(1) = (2)}} \, ;$$

$\underline{e}_r$: $\dot{d}_1 = -d_2 \omega_{CD} \cos\phi$

$\underline{e}_\theta$: $d_1 \omega_{AC} = d_2 \omega_{CD} \sin\phi$

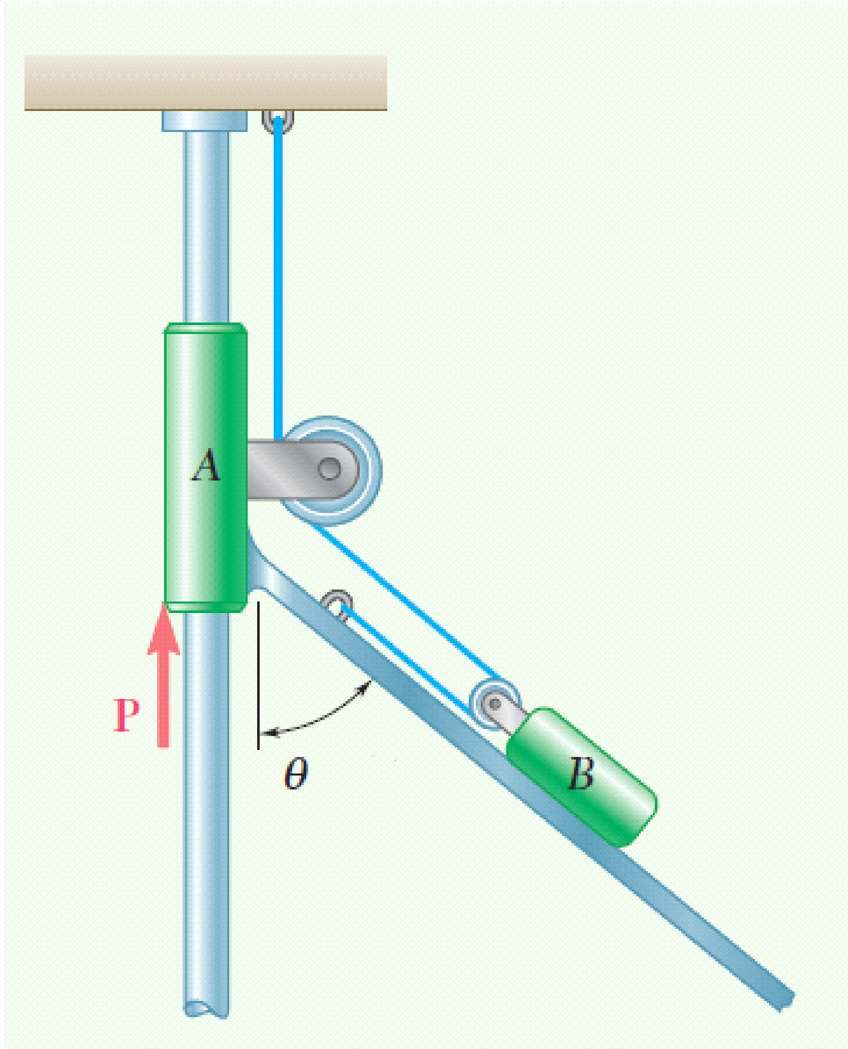
$$\Rightarrow \quad \omega_{CD} = \frac{d_1 \omega_{AC}}{d_2 \sin\phi}$$

$$\Rightarrow \quad \dot{d}_1 = -d_2 \omega_{CD} \cos\phi = \frac{-d_2 d_1 \omega_{AC}}{d_2 \sin\phi} \cos\phi$$

$$= -\frac{\omega_{AC} \cos\phi}{d_1 \sin\phi}$$

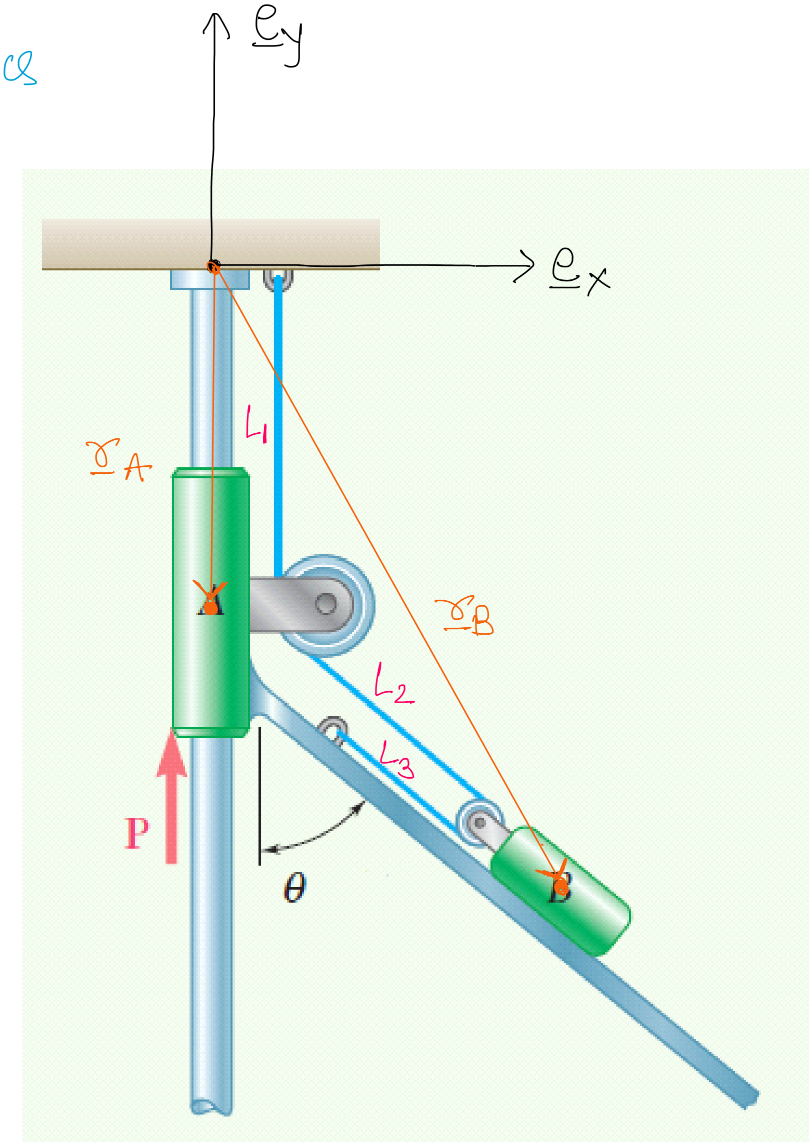
Question:

Collar A has a ramp that is welded to it and a force P applied as shown. Collar A and the ramp weigh W_A, and block B weighs W_B. Neglecting friction, determine the tension in the cable.


A
P
θ
B

Solution

- Kinematics



$$\underline{r}_A = -L_1 \underline{e}_y$$

$$\underline{r}_B = -L_1 \underline{e}_y + L_2 \sin\theta \, \underline{e}_x - L_2 \cos\theta \, \underline{e}_y$$

$$= L_2 \sin\theta \, \underline{e}_x - (L_1 + L_2 \cos\theta) \, \underline{e}_y$$

$$\underline{v}_A = -\dot{L}_1 \underline{e}_y$$

$$\underline{v}_B = \dot{L}_2 \sin\theta \, \underline{e}_x - (\dot{L}_1 + \dot{L}_2 \cos\theta) \, \underline{e}_y$$

$$\underline{a}_A = -\ddot{L}_1 \underline{e}_y$$

$$\underline{a}_B = \ddot{L}_2 \sin\theta \, \underline{e}_x - (\ddot{L}_1 + \ddot{L}_2 \cos\theta) \underline{e}_y$$

Constraint Relation:

(i) $L_1 + L_2 + L_3 = \text{constant}$

$$\Rightarrow \dot{L}_1 + \dot{L}_2 + \dot{L}_3 = 0$$

$$\Rightarrow \ddot{L}_1 + \ddot{L}_2 + \ddot{L}_3 = 0$$

(ii) $\dot{L}_2 = \dot{L}_3 \Rightarrow \ddot{L}_2 = \ddot{L}_3$

Using (i) & (ii):

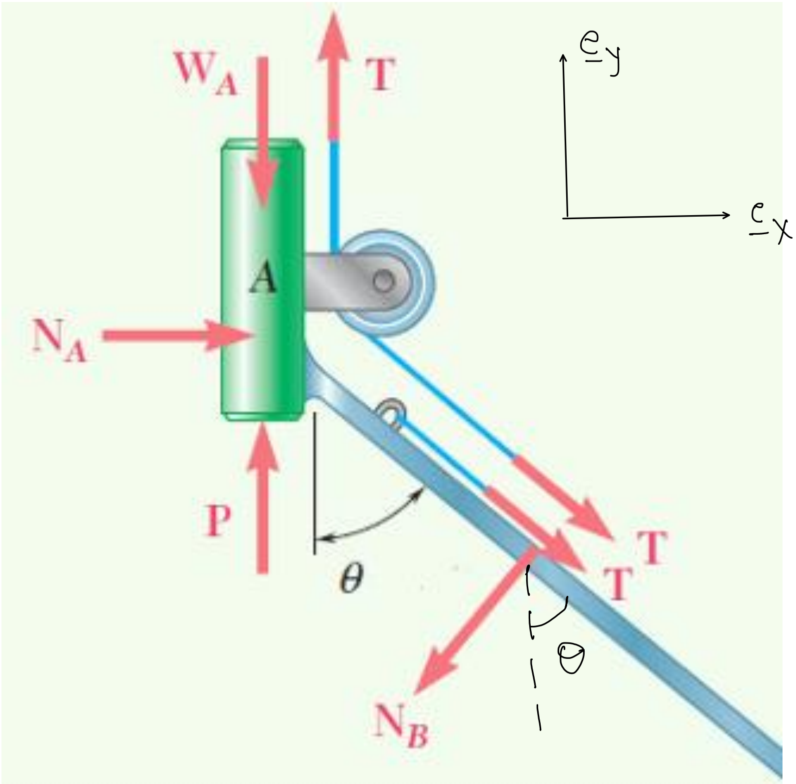
$$\ddot{L}_1 = -2\ddot{L}_2$$

$$\underline{a}_B = -\frac{1}{2}\ddot{L}_1 \sin\theta \, \underline{e}_x - \left(\ddot{L}_1 - \frac{1}{2}\ddot{L}_1 \cos\theta\right) \underline{e}_y$$

$$\underline{a}_A = -\ddot{L}_1 \underline{e}_y$$

- FBD and $\Sigma \underline{F} = m\underline{a}$

Collar A:



$$\sum \underline{F} = m\underline{a}$$

$$N_A\,\underline{e}_x + 2T\sin\theta\,\underline{e}_x - N_B\cos\theta\,\underline{e}_x - 2T\cos\theta\,\underline{e}_y - N_B\sin\theta\,\underline{e}_y - W_A\underline{e}_y + T\underline{e}_y + P\underline{e}_y = m_A(-\ddot{L}_1\,\underline{e}_y)$$

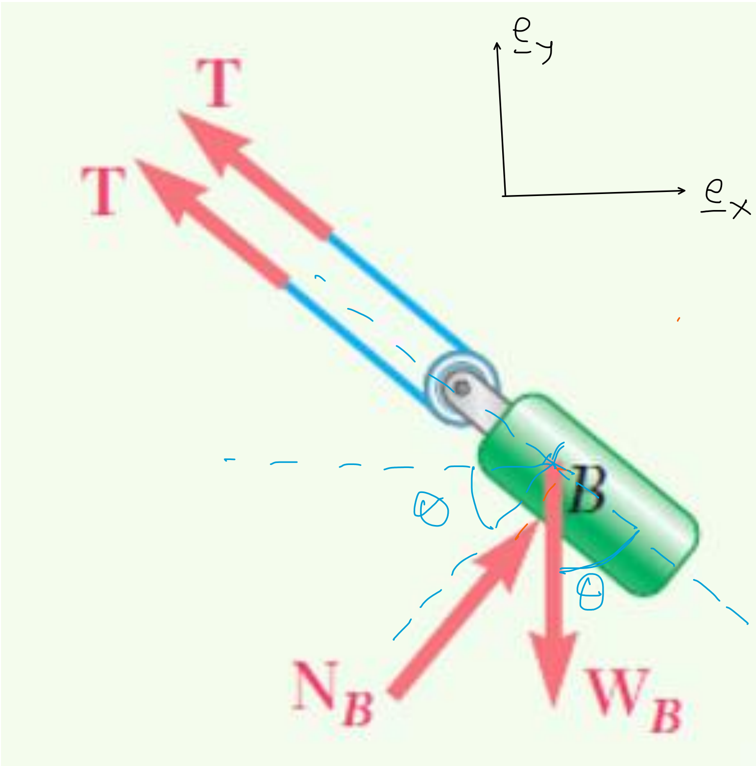
$\longrightarrow$ (1, 2)

↳ Two equations

→ $\underline{e}_x$

→ $\underline{e}_y$

<u>Block B</u> :



$$\sum \underline{F}_B = m\,\underline{a}_B$$

$$2T\cos\theta\,\underline{e}_y - 2T\sin\theta\,\underline{e}_x - W_B\,\underline{e}_y + N_B\cos\theta\,\underline{e}_x + N_B\sin\theta\,\underline{e}_y = m\left[-\frac{1}{2}\ddot{L}_1\sin\theta\,\underline{e}_x - \left(\ddot{L}_1 - \frac{1}{2}\ddot{L}_1\cos\theta\right)\underline{e}_y\right]$$

↳ (3, 4) Two eq^s.
→ $\underline{e}_x$
→ $\underline{e}_y$

$$\left.\begin{array}{ll} \text{Unknowns:} & T,\ N_A,\ N_B,\ \ddot{L}_1 \\ \text{Equations:} & \text{Four} \end{array}\right\} \text{solve for } T.$$