ME 104 Engineering Mechanics II

FALL 2024

Ekaterina Antimirova, Prashant Pujari

Discussion Section - Week 2

Berkeley
UNIVERSITY OF CALIFORNIA

TAs

1. **Ekaterina Antimirova**
   – Discussion Section: DIS 102 (5pm - 6pm, Tuesdays, Tan 180)
   – Office Hours:
     - 10am - 11am, Mondays (Etcheverry 1165)
     - 11am - 12pm, Thursdays (Etcheverry 1165)
2. **Prashant Pujari**
   – Discussion Section: DIS 101 (5pm - 6pm, Wednesdays, Tan 180)
   – Office Hours:
     - 1pm - 2pm, Wednesdays (Etcheverry 3117B)
     - 11am - 12pm, Fridays (Etcheverry 3117B)

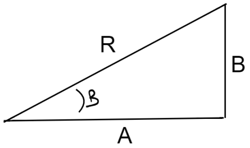
Contact

- For general inquiries, class or assignments related questions: **Ed-Discussion**
- For any personal or emergency related questions (NO technical questions will be answered over email): prashant_pujari@berkeley.edu
- DSP students, please sign up for midterms and exams via DSP portal.

Math Review

- **Dot product** (scalar output): $\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$. For $\vec{A} \cdot \vec{B} = 0$, $\vec{A} \perp \vec{B}$.
- **Orthonormal basis**: (1) Orthogonal set (every vector pair is $\perp$); (2) Every vector is a unit vector (ex. $|\vec{u}| = 1$).
- **Cross product** (vector output, where $(\vec{A} \times \vec{B}) \perp \vec{A}$ and $(\vec{A} \times \vec{B}) \perp \vec{B}$): $\vec{A} \times \vec{B} = \vec{C} = [(A_y B_z - A_z B_y), (A_z B_x - A_x B_z), (A_x B_y - A_y B_x)]$. For $\vec{A} \times \vec{B} = 0$, $\vec{A} \parallel \vec{B}$.
- **Composite functions**: let $f(x) = x^2 + 5$ and $g(x) = x^2$, composite $\tilde{f}(g(x))$ formulation would be $\tilde{f}(g) = g + 5$. With (helper) tilde added to distinguish fs.
- **Chain rule**: $\frac{d}{dx}[f(g(x))] = \frac{df(g)}{dg} \frac{dg(x)}{dx}$
- **Product rule**: $\frac{d}{dx}[f(x)g(x)] = f(x)\frac{dg(x)}{dx} + \frac{df(x)}{dx}g(x)$
- **Trigonometry**

- $\cos(\beta) = \frac{A}{R}$, $\sin(\beta) = \frac{B}{R}$, $\tan(\beta) = \frac{B}{A}$, $A^2 + B^2 = R^2$
- $\cos(\beta)^2 + \sin(\beta)^2 = 1$



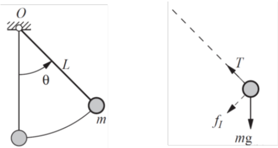
One-Particle Kinematics

- **Position**: $\boldsymbol{r} = \boldsymbol{r}(t) = x(t)\boldsymbol{e}_x + y(t)\boldsymbol{e}_y + z(t)\boldsymbol{e}_z$
- **Absolute velocity**: $\boldsymbol{v} = \boldsymbol{v}(t) = \frac{d\boldsymbol{r}}{dt}$
- **Absolute acceleration**: $\boldsymbol{a} = \boldsymbol{a}(t) = \frac{d\boldsymbol{v}}{dt}$
- **Speed**: $v(t) = ||\boldsymbol{v}(t)|| = \sqrt{\boldsymbol{v}(t) \cdot \boldsymbol{v}(t)} > 0$
- **Distance**: $s = s(t) = s_0 + \int_{t_0}^{t} v(\tau)d\tau > 0$ (increasing only)

Berkeley
UNIVERSITY OF CALIFORNIA

ODE Solver (MATLAB)

Consider a simple pendulum system with length L and mass m, as shown below:



$f_I = mL\ddot{\theta}$

The governing equation for the motion of this pendulum, which is given by the 2nd-order ODE:

$$\ddot{\theta} + \frac{g}{L}\sin\theta = 0$$

where $\theta(t)$ is the angle the pendulum makes with the vertical axis. Solve this simple pendulum problem for a given set of initial conditions θ_0 and ω_0 in the time domain $t = [t_0, t_f]$ and plot the solution.

Given $\ddot{\theta} + \frac{g}{L}\sin\theta = 0$, for large θ, we need to solve this 2nd-order ODE using an ODE solver. First, reduce this 2nd-order ODE as a system of two first order ODEs (left). Second, vectorize the system (right).

$$\begin{cases} \frac{d\theta}{dt} = \omega \\ \frac{d\omega}{dt} = \frac{-g}{L}\sin(\theta) \end{cases}$$

$$\frac{d}{dt}\begin{bmatrix} \theta \\ \omega \end{bmatrix} = \begin{bmatrix} \omega \\ -\frac{g}{L}\sin\theta \end{bmatrix}$$

In MATLAB, define: $y_1 = \theta$ and $y_2 = \omega = \frac{d\theta}{dt} = \frac{dy_1}{dt}$. Then:

$$\begin{cases} \frac{dy_1}{dt} = y_2 \\ \frac{dy_2}{dt} = \frac{-g}{L}\sin(y_1) \end{cases}$$

Define global vector $\boldsymbol{y} = [y_1; y_2]$

$$\frac{d\boldsymbol{y}}{dt} = \left[y(2); -\frac{g}{L}\sin(y(1))\right]$$

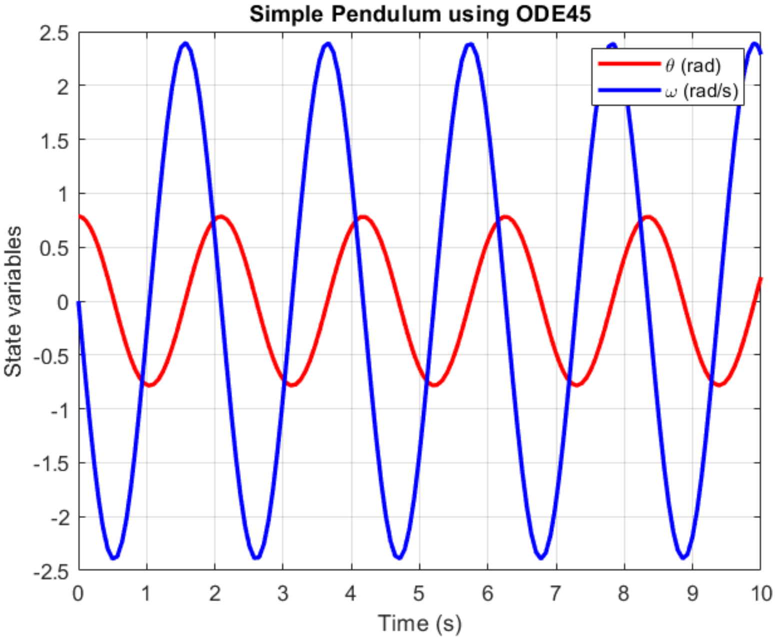
ODE45 Solver: $[t, y] = $ ode $45\left(@(t, \boldsymbol{y})\frac{d\boldsymbol{y}}{dt},\ \text{tspan}, y_0\right)$

Initial condition: $\boldsymbol{y}_0 = [y_0(1), y_0(2)] = [\theta_0, \omega_0]$

Time range: tspan $= [t_0, t_f]$

```matlab
%% ODE Solver Demo (Simple Pendulum)

clear; clc; close all

% Parameters
g = 9.81; % acceleration due to gravity (m/s^2)
L = 1.0;  % length of the pendulum (m)

% Initial conditions
theta0 = pi/4;  % initial angular displacement (radians)
omega0 = 0;     % initial angular velocity (rad/s)
y0 = [theta0; omega0]; % initial state vector

% Time span for the simulation
tspan = [0, 10]; % time range for the solution (seconds)

```

```matlab
%% Method 1

% Define the system of ODEs as a function
pendulumODEs = @(t, y) [y(2); -g/L * sin(y(1))];

% Solve the ODE using ode45
[t, y] = ode45(pendulumODEs, tspan, y0);

% Plot the results
figure;
plot(t, y(:,1), '-r', 'LineWidth', 2); % plot theta (angular displacement)
hold on;
plot(t, y(:,2), '-b', 'LineWidth', 2); % plot omega (angular velocity)
xlabel('Time (s)');
ylabel('State variables');
legend('\theta (rad)', '\omega (rad/s)');
title('Simple Pendulum using ODE45');
grid on;
```


Simple Pendulum using ODE45
θ (rad)
ω (rad/s)
State variables
Time (s)
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
0
1
2
3
4
5
6
7
8
9
10

Problems

1. The motion of a particle is such that its position vector $\mathbf{r}(t) = 3t\ \mathbf{e}_x + 4t\ \mathbf{e}_y + 10\ \mathbf{e}_z$ (meters). Show that the path of the particle is a straight line and that the particle moves along this line at a constant speed.

2. The motion of a particle is such that its position vector

   $\mathbf{r}(t) = 10 \cos(n\pi t)\ \mathbf{e}_x + 10 \sin(n\pi t)\ \mathbf{e}_y$ (meters)

   Show that the particle is moving on a circle of radius 10 meters and find the time period T.

3. A truck is moving at a speed of 20 m/s and its engine is suddenly stopped. If it takes 10 seconds for the truck to reduce its speed to 5 m/s, determine the distance s in meters moved by the truck and its speed v in m/s as functions of the time t during this interval. The deceleration of the truck is proportional to the square of its speed i.e. $a = -kv^2$.