ME 104 Lec 4

Last time: <u>Dynamics</u>.

Dynamics problems are solved in four steps:

① Write out the kinematics: Pick an origin, pick a coord system, then write expressions for $\underline{r}$, $\underline{v}$, and $\underline{a}$.

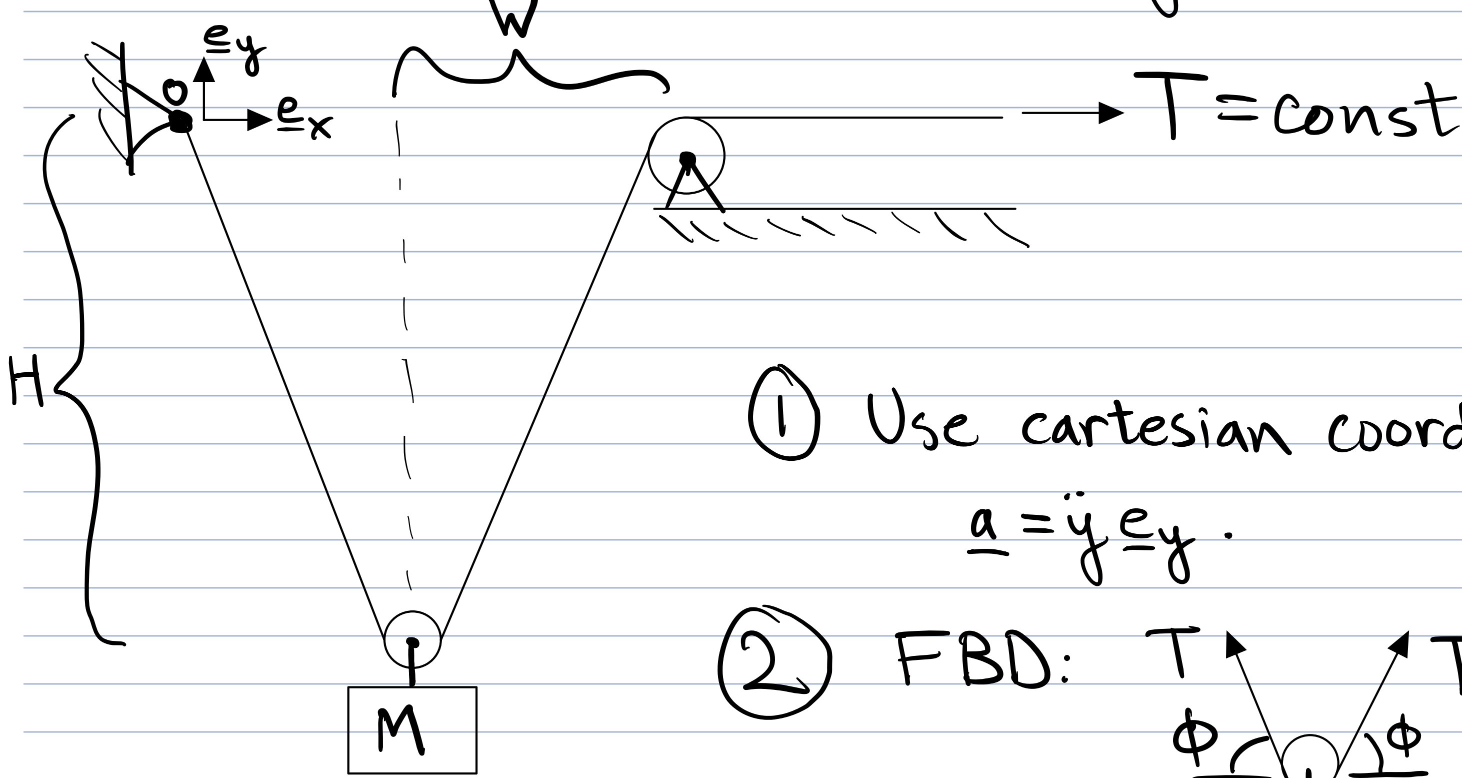
② Draw a <u>free body diagram</u> (FBD):

- Draw the object.
- Draw in force vectors. Hint: forces should appear everywhere the object is <u>touched</u> by something else, and <u>weight</u> should be added if gravity non-negligible.

③ Write out $\Sigma \underline{F} = m\underline{a}$. (Newton's 2nd Law)

④ Perform the analysis (analytic or numeric) to compute $\underline{a}(t) \Rightarrow \underline{v}(t) \Rightarrow \underline{x}(t)$. Often will also want to compute the unknown constraint forces.

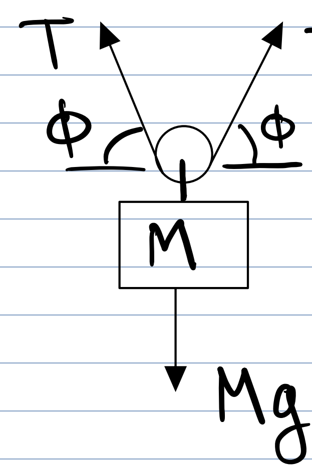
Ex: A weight hangs from a pulley as shown below, initially at rest H below the rope's attatchment point. If a constant tension T is applied as shown, what is the weight's speed when the rope is pulled straight?



① Use cartesian coords

$$\underline{a} = \ddot{y}\,\underline{e}_y .$$

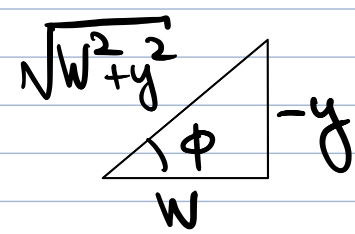
② FBD:



$\phi = \tan^{-1}(-y/w)$

③ $2T\sin\phi\,\underline{e}_y - Mg\,\underline{e}_y = M\ddot{y}\,\underline{e}_y$

④ Note: Use a triangle to recall $\sin(\tan^{-1}(\ ))$ rule:



$$\Rightarrow \sin(\tan^{-1}(-y/w)) = -y/\sqrt{W^2+y^2}.$$

So $\underline{e}_y$ component of $\Sigma F = ma$ gives:

$$\frac{-2yT}{\sqrt{W^2+y^2}} - Mg = M\ddot{y} \Rightarrow \ddot{y} = -g - \frac{2yT}{M\sqrt{W^2+y^2}}$$

Use integration rule from Lec 2 for $a(x) = h(x)$:

$$\Rightarrow v_y = \tilde{v}_y(y) = \oplus\sqrt{2\int_{-H}^{y}\left[-g - \frac{2yT}{M\sqrt{W^2+y^2}}\right]dy} \qquad \{\tilde{v}(-H)=0\}$$

$$= \left[2\left(-gy - \frac{2}{M}T\sqrt{W^2+y^2} - \left(+gH - \frac{2}{M}T\sqrt{W^2+H^2}\right)\right)\right]^{1/2}$$

$$\Rightarrow (v_y \text{ @ } y=0) = \tilde{v}_y(y=0)$$

$$= \boxed{\left[2\left(-\frac{2TW}{M} - gH + \frac{2T}{M}\sqrt{W^2+H^2}\right)\right]^{1/2}}$$

Q: What does it mean if the answer is not a real number?

Ans: T is not big enough to pull the weight up to $y=0$.

Q: Do you need more or less tension T to pull the rope straight if H is smaller?

Ans: For a given H, the least T is the one that gives $\tilde{v}_y(0) = 0$.

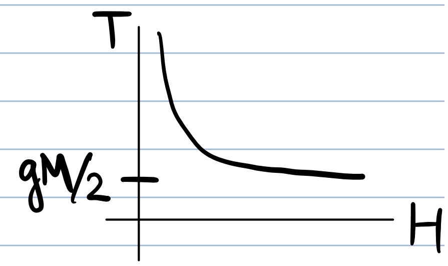
$$\Rightarrow -\frac{2TW}{M} - gH + \frac{2T}{M}\sqrt{W^2+H^2} = 0$$

$$\Rightarrow T = \left[gHM/2\right] \Big/ \left[\sqrt{W^2+H^2} - W\right]$$

Look at limitting cases to infer function behavior.

$$\Rightarrow T(H\to\infty) \overset{L'Hop}{=} \frac{gM/2}{\lim\limits_{H\to\infty} \frac{H}{\sqrt{W^2+H^2}}} = gM/2$$

$$T(H\to 0) \overset{L'Hop}{=} \frac{gM/2}{\lim\limits_{H\to 0} \frac{2H}{\sqrt{W^2+H^2}}} = \infty$$



As H shrinks, the required T grows. In fact as $H\to 0$, the required T explodes.

Types of forces:

Have already seen:

- Weight: $\underline{W} = m\underline{g}$
- Viscous drag: $\underline{F}_D = c_s \underline{v}$
- Wall normal: $\underline{F}_n = F_n \underline{e}_n$

- Tension: $\underline{T} = T\, \underline{e}_{rope}$

  $\underline{e}_{rope}$: ∥ to rope.

Other common forces:

- Friction: $\underline{F}_t = -\mu F_n\, \underline{v}_s / |\underline{v}_s|$

  μ: friction coefficient

  $\underline{v}_s$: Wall slip velocity $= \underline{v} - \underline{v}_{wall}$

  $\underbrace{\underline{v}_s/|\underline{v}_s|}_{\perp\, \underline{e}_n}$

  If $|\underline{v}_s| = 0$, $\quad |\underline{F}_t| < \mu F_n$.

- Spring: $\underline{F}_s = -k(\ell - \ell_0)\, \underline{\ell}/\ell$

  k: spring stiffness

  ℓ_0: Neutral length

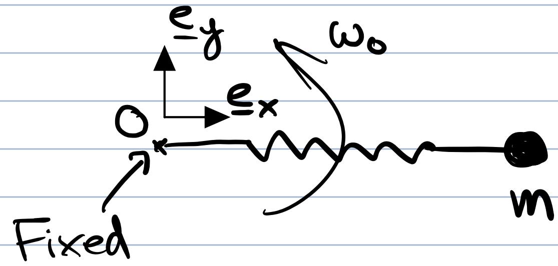
  where $\underline{\ell} = \underline{r} - \underline{r}^A$, $\ell = |\underline{\ell}|$.

  $\underline{r}^A$: Attachment point

- Inertial fluid drag: $\underline{F}_I = -c_I\, v\, \underline{v}$

  c_I: Inertial drag coeff

Ex:

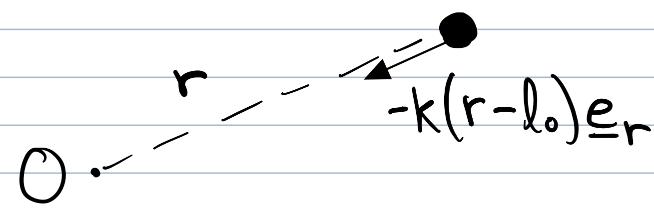


What's the frequency of small oscillations of the spring if m is perturbed outward a bit from a stable spin of ω_0?

① Use polar: $\underline{r} = r\underline{e}_r$, $\underline{v} = \dot{r}\underline{e}_r + r\dot{\theta}\underline{e}_\theta$

$$\underline{a} = (\ddot{r} - r\dot{\theta}^2)\underline{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\underline{e}_\theta$$

② FBD:



③ $\Sigma \underline{F} = m\underline{a} \Longrightarrow -k(r-l_0)\underline{e}_r = m(\ddot{r} - r\dot{\theta}^2)\underline{e}_r + m(r\ddot{\theta} + 2\dot{r}\dot{\theta})\underline{e}_\theta$

④ Solve. Match up components:

$$-k(r-l_0) = m(\ddot{r} - r\dot{\theta}^2) \quad \& \quad r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.$$

How to solve coupled system?

(i) Use a computer: $\underline{U} = [\alpha; r; \beta; \theta]$.

$$\dot{\underline{U}} \rightarrow \begin{bmatrix} \dot{\alpha} \\ \dot{r} \\ \dot{\beta} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} -\frac{k}{m}(r-l_0) + r\beta^2 \\ \alpha \\ -2\alpha\beta/r \\ \beta \end{bmatrix} \leftarrow \underline{f}(\underline{U})$$

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(ii) Try a "clever" change of variables.

Define $H = mr^2\dot{\theta}$. $\dot{H} = (2r\dot{r}\dot{\theta} + r^2\ddot{\theta})m$

$$= r\underbrace{(2\dot{r}\dot{\theta} + r\ddot{\theta})}_{=0 \text{ by Newton!}}m$$

$\Rightarrow \dot{H} = 0 \Rightarrow H = \text{const} =$ "Angular momentum"

Now, Newton in $\underline{e}_r$ can be rewritten as:

$$-k(r - l_0) = m(\ddot{r} - r\dot{\theta}^2)$$

$$= m(\ddot{r} - r\,H^2/m^2r^4) = m(\ddot{r} - H^2/m^2r^3)$$

$$\Rightarrow \boxed{\ddot{r} = -k(r - l_0)/m + H^2/m^2r^3}$$

This equation does not have an analytic sol'n. :(

But all we want is the freq of small oscillations about the initial r_0, which can be approximated in the limit of small deviations from $r = r_0$.