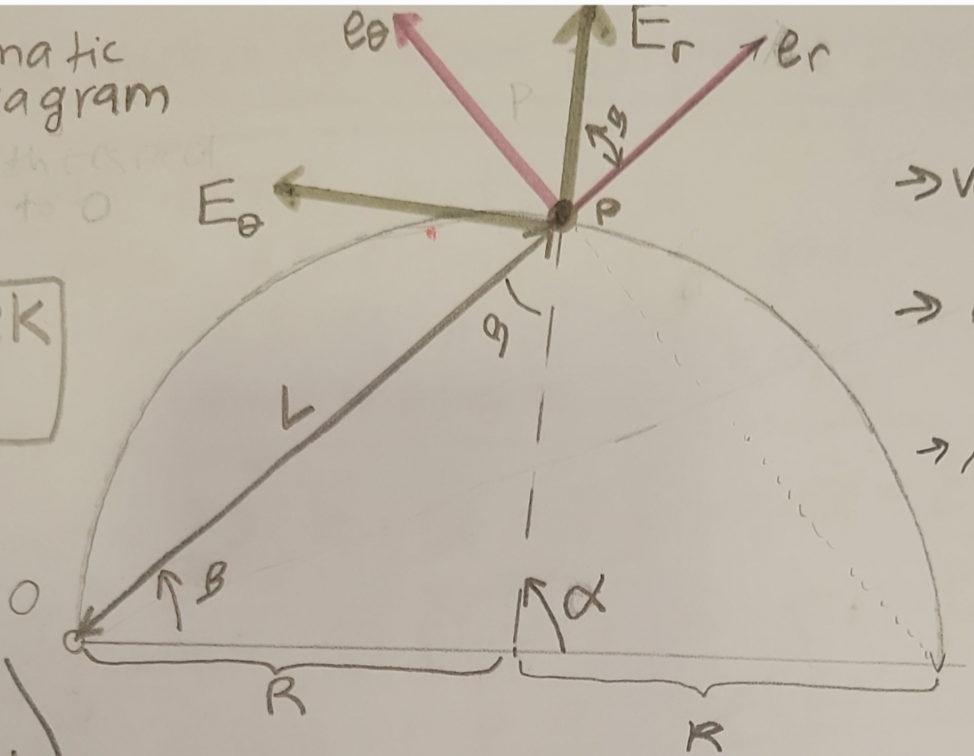
Kinematic Diagram

Week 4



Given

→ vertical plane (gravity

→ constant $\dot{\beta}$ ($\Rightarrow \ddot{\beta} = 0$)

→ Asks for 2 Normal Forces

$\boxed{\alpha = 2\beta}$

(Look at general form to see to what to solve for)

General form cylindrical coordinates

$$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\vec{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\vec{e}_\theta$$

With respect to the arm kinematics: $\vec{e}_r$, $\vec{e}_\theta$

$\theta = \beta$ | $r = L = 2R\cos\beta$

$\dot{\theta} = \dot{\beta}$ | $\dot{r} = \dot{L} = -2R\sin\beta\dot{\beta}$

$\ddot{\theta} = 0$ | $\ddot{r} = \ddot{L} = \underline{(-2R)\dot{\beta}^2\cos\beta} + \cancel{(-2R)\sin\beta\ddot{\beta}}$ 0

! L is getting longer & shorter ⇒ solve for it in terms of known

Sub into general ⇒

$$\vec{a}_A = (\ddot{L} - L\dot{\beta}^2)\vec{e}_r + (2\dot{L}\dot{\beta})\vec{e}_\theta$$

$$= \left((-2R)\dot{\beta}^2\cos\beta - (2R\cos\beta)\dot{\beta}^2\right)\vec{e}_r + \left(2(-2R\sin\beta\,\dot{\beta})\dot{\beta}\right)\vec{e}_\theta$$

1 $$= (-4Rc(\beta)\dot{\beta}^2)\vec{e}_r + (-4Rs(\beta)\dot{\beta}^2)\vec{e}_\theta$$

With respect to slot (circular) kinematics:

$\vec{E}_r$ & $\vec{E}_\theta$

r is fixed, but α is now different: hence get derivatives of α wrt to known parameters

$$\theta = \alpha = 2\beta \Rightarrow \dot{\theta} = \dot{\alpha} = 2\dot{\beta} \Rightarrow \ddot{\theta} = \ddot{\alpha} = 2\ddot{\beta} = 0$$

$$r = R \Rightarrow \dot{r} = 0 \Rightarrow \ddot{r} = 0$$

sub into general form

$$\vec{a}_C = \left(-R(2\dot{\beta})^2\right)\vec{E}_r + (0+0)\vec{E}_\theta$$

$$\boxed{\vec{a}_C = -4R\dot{\beta}^2\vec{E}_r}$$

observe & compare with $\vec{a}_A$

$$\boxed{\begin{aligned}\vec{a}_A = &-4R\dot{\beta}^2\cos(\beta)\vec{e}_r \\ &-4R\dot{\beta}^2\sin(\beta)\vec{e}_\theta\end{aligned}}$$

what is projection of $\vec{a}_C$ on to $\vec{e}_r$ & $\vec{e}_\theta$ coordinate system.

Importance of keeping terms together

Get intuition

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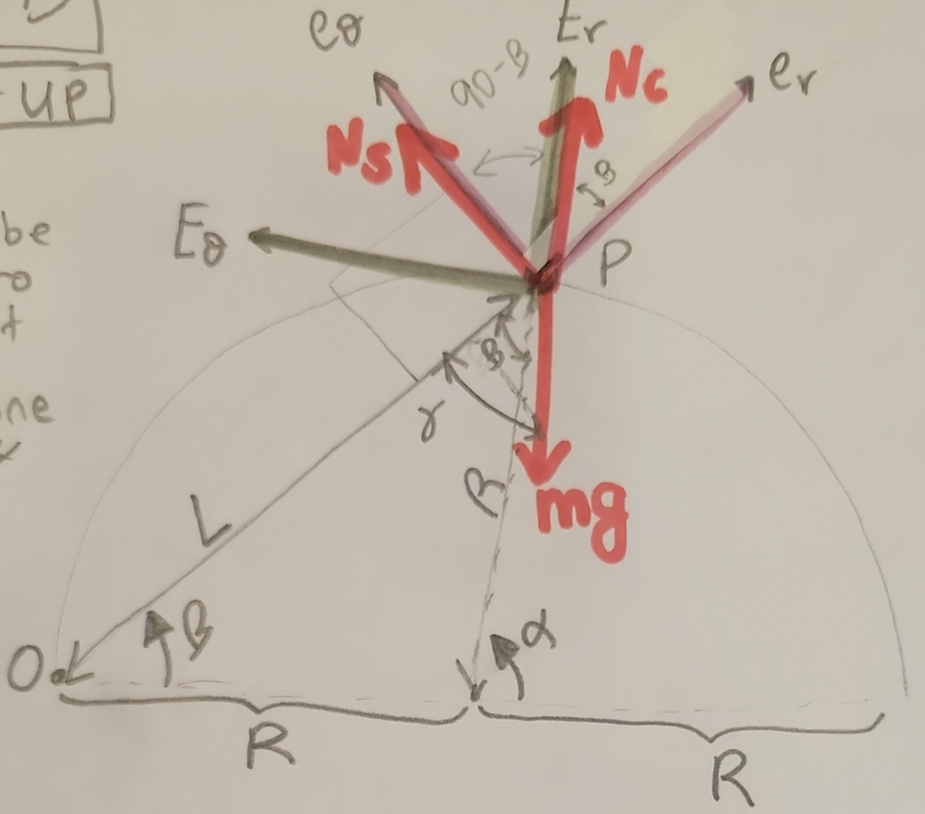
FBD

set up

Should be able to give it to someone else & they can write Equil eqns from it.



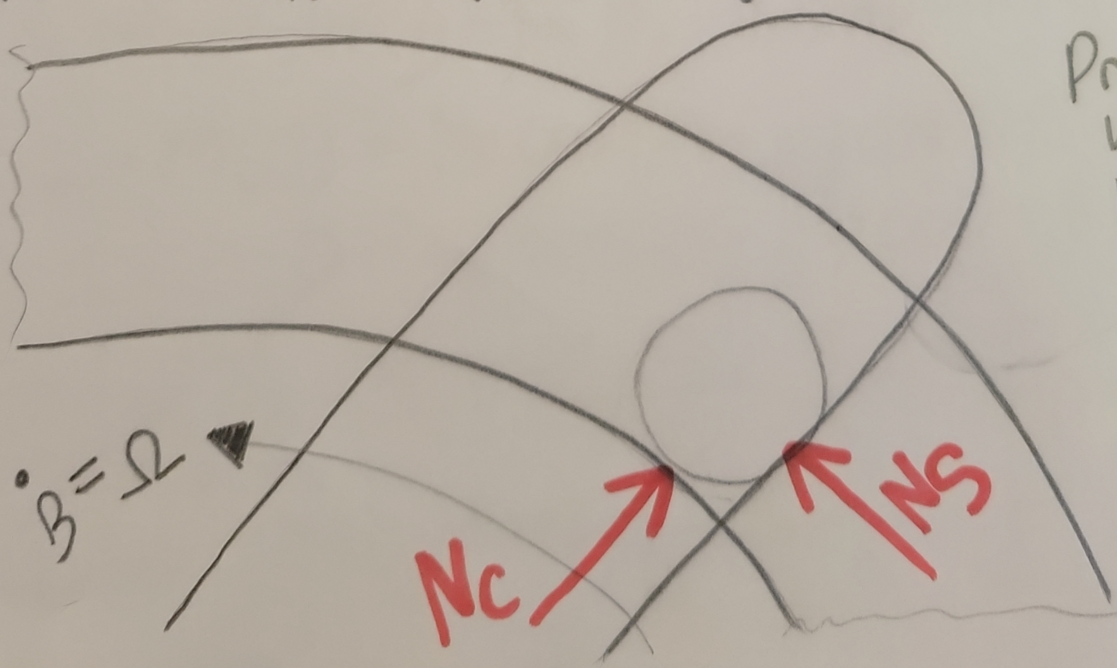
N_c: ⊥ to circular slot

N_s ⊥ to arm

$\boxed{\gamma = 90 - \beta}$

→ Before continuing to draw forces ask about actual physical meaning of normal forces?

→ How many? → what signs mean?



Predict where particle is (which corner)?

Equilibrium equations: $\vec{e}_r$ & $\vec{e}_\theta$

$$\sum F_{\vec{e}_r} = m a_A^r = N_C \cos\beta - mg\sin\beta \qquad (1)$$

$$= m(-4R\dot{\beta}^2\cos\beta) = N_C\cos\beta - mg\sin\beta \qquad (2)$$

$$\sum F_{\vec{e}_\theta} = m a_A^\theta = N_S + N_C\sin\beta - mg\cos\beta \qquad (3)$$

$$= m(-4R\dot{\beta}^2\sin\beta) = N_S + N_C\sin\beta - mg\cos\beta \qquad (4)$$

$$(1) \Rightarrow N_C = mg\tan(\beta) - 4mR\dot{\beta}^2 \; (= -0.80\,N)$$

$$(1)+(4) \Rightarrow N_S = mg\frac{\cos(2\beta)}{\cos(\beta)} \; (= 1.521\,N)$$

Equilibrium equations: E_r & E_θ

$$\sum F_{\vec{E}_r} = N_C + N_S\sin(\beta) - mg\sin(2\beta) = m a_C^r \qquad (5)$$

$$= m(-4R\dot{\beta}^2) = N_C + N_S\sin\beta - mg\sin(2\beta) \qquad (6)$$

$$\sum F_{\vec{E}_\theta} = m(0) = N_S\cos(\beta) - mg\cos(2\beta) \qquad (7)$$

$$0 = N_S\cos(\beta) - mg\cos(2\beta) \qquad (8)$$

$$(8) \Rightarrow N_S = mg\frac{\cos(2\beta)}{\cos(\beta)}$$

$$(8)+(6) \Rightarrow N_C = mg\tan(\beta) - 4mR\dot{\beta}^2$$

$\boxed{\text{match !}}$

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