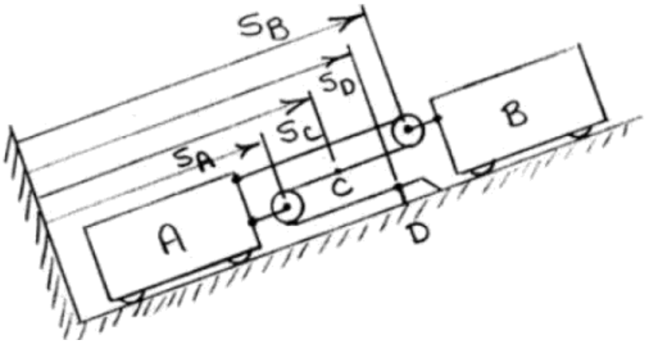
Problem Set 1 Solutions

Monday, September 9, 2024 11:55 AM

Q1



The cable length is :

$$L = 2(S_B - S_A) + S_D - S_A + \text{constants}$$

Differentiating :

$$0 = 2(\dot{S}_B - \dot{S}_A) + 0 - \dot{S}_A$$

$$\dot{S}_A = \frac{2}{3}\dot{S}_B$$

$$\Rightarrow \quad V_A = \frac{2}{3}\dot{V}_B$$

Differentiating again :

$$\Rightarrow \quad a_A = \frac{2}{3}a_B$$

Given : $V_B = 2\ ft/s$

$a_B = 3\ ft/s^2$

$$\Rightarrow \quad V_A = \frac{2}{3}V_B = \frac{2}{3} \times 2$$

$$\boxed{V_A = \frac{4}{3}\ ft/s}$$

$$\Rightarrow \quad a_A = \frac{2}{3}a_A = \frac{2}{3} \times 3$$

$$\boxed{a_A = 2\ ft/s^2}$$

The length of cable between A and C is :

$$L_{AC} = (S_B - S_A) + (S_B - S_C) + \text{constants}$$

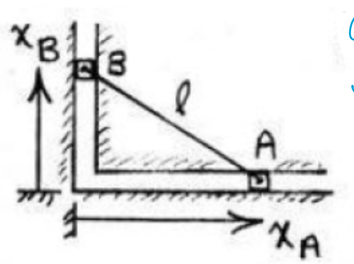
Differentiating :

$$0 = (\dot{S}_B - \dot{S}_A) + (\dot{S}_B - \dot{S}_C)$$

$$V_C = 2V_B - V_A = 2 \times 2 - \frac{4}{3}$$

$$\boxed{V_C = \frac{8}{3}\ ft/s}$$

Q2



Given :

$l = 0.5\,m$
$x_A = 0.4\,m$ } $x_B = 0.3\,m$
$V_A = 2\,m/s$

$$x_A^2 + x_B^2 = l^2$$

Differentiate :

$$2x_A\dot{x}_A + 2x_B\dot{x}_B = 0$$

$$\Rightarrow \quad x_A\dot{x}_A + x_B\dot{x}_B = 0 \quad — (1)$$

Differentiate :

$$x_A\ddot{x}_A + \dot{x}_A^2 + x_B\ddot{x}_B + \dot{x}_B^2 = 0 \quad — (2)$$

From (1) :

$$\dot{x}_B = -\frac{x_A\dot{x}_A}{x_B} = \frac{-x_A V_A}{x_B} = \frac{-0.4 \times 2}{0.3} = -\frac{8}{3}\,m/s$$

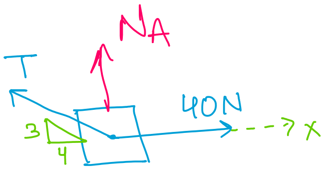
From (2) :

$$\ddot{x}_B = \frac{-\dot{x}_B^2 - \dot{x}_A^2 - x_A\ddot{x}_A}{x_B} = \frac{-(-8/3)^2 - (2)^2 - 0.4\ddot{x}_A}{0.3}$$

$$a_B = -\frac{1000}{27} - \frac{4}{3} a_A \quad \text{——} \; (3)$$

FBD:

A:

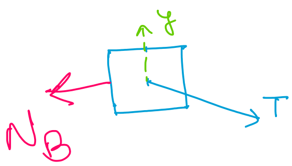


(Normal forces from the walls are balanced by the other component of T)

$$\Sigma \underline{F}_x = m \underline{a}_x$$

$$\Rightarrow \quad 40 - \frac{4}{5} T = 2 \underline{a}_A \quad \text{——} \; (4)$$

B:



$$\Sigma \underline{F}_y = m \underline{a}_y$$

$$-\frac{3}{5} T = 3 \underline{a}_B \quad \text{——} \; (5)$$

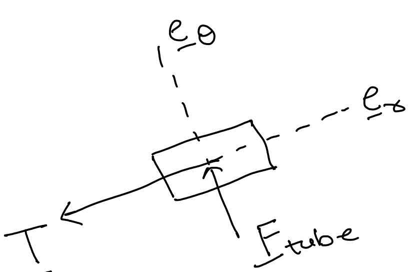
Solve (3), (4), and (5) simultaneously:

$$\boxed{a_A = -14.75 \text{ m/s}^2}$$

$$\boxed{a_B = -17.37 \text{ m/s}^2}$$

$$\boxed{T = 86.87 \text{ N}}$$

Q3



Given:

$\dot{\theta} = 4$ rad/s

$\ddot{\theta} = -2$ rad/s^2

$m = 0.2$ kg

$r = 1$ m, $\dot{r} = -3$ m/s

$\ddot{r} = 2$ m/s^2

e_r: $\quad \Sigma F_r = m a_r$

$$-T = m(\ddot{r} - r\dot{\theta}^2)$$

$$T = -0.2(2 - 1\times 4^2)$$

$$\Rightarrow \boxed{T = 2.8\,N} \text{ or } \boxed{\underline{T} = -2.8\,\underline{e}_r\,N}$$

$\underline{e}_\theta$:

$$\Sigma F_\theta = m a_\theta$$

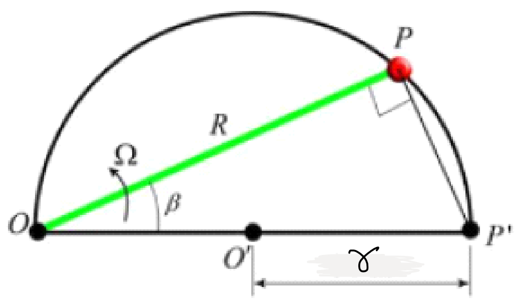
$$F_{tube} = m(r\ddot{\theta} + 2\dot{r}\dot{\theta})$$

$$F_{tube} = 0.2(1\times(-2) + 2\times(-3)\times 4)$$

$$\Rightarrow \boxed{\underline{F}_{tube} = -5.2\,\underline{e}_\theta\,N} \text{ or } \boxed{F_{tube} = 5.2\,N}$$

and arrow pointing downwards

Q4



Radius (R) of particle P from $\Delta OPP'$:

$$R = 2r\cos\beta \quad , \quad r = 1m \text{ (constant)}$$

$$\Rightarrow \quad R = 2\cos\beta$$

Differentiating:

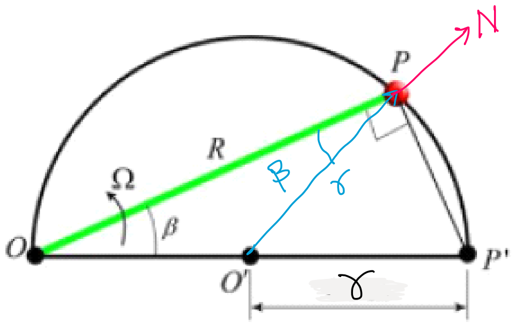
$$\dot{R} = -2\sin\beta\cdot\dot{\beta} \quad , \quad \ddot{R} = -2(\sin\beta\cdot\ddot{\beta} + \cos\beta\cdot\dot{\beta}^2)$$

Given: $m = 0.2\,kg$, $\beta = 22^\circ$, $\Omega = \dot{\beta} = 2\,rad/s$,

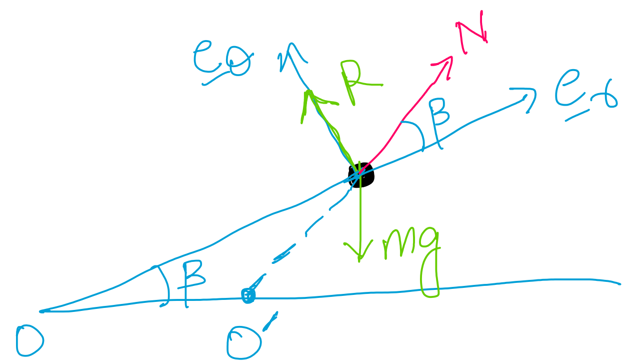
$R = 2\cos 22^\circ = 1.854\,m$

$\dot{R} = -2 \times \sin 22^\circ \times 2 = -1.498\,m/s$

$\ddot{R} = -2(\sin 22^\circ \times 0 + \cos 22^\circ \times 2^2) = -7.417\,m/s^2$



FBD



In e_r direction:

$\Sigma F_r = m a_r$

$\Rightarrow \quad N\cos 22^\circ - mg\sin 22^\circ = m(\ddot{R} - R\dot{\beta}^2)$

$$\boxed{N = -2.407\,N}$$ (reverse direction)

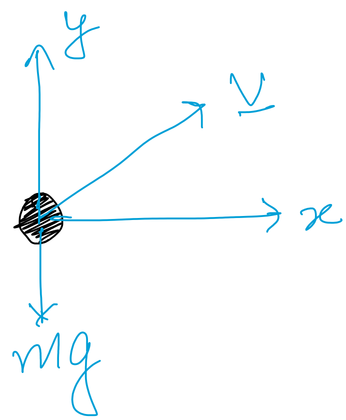
In e_θ direction:

$\Sigma F_\theta = m a_\theta$

$\Rightarrow \quad R + N\sin 22^\circ - mg\cos 22^\circ = m(R\ddot{\beta} + 2\dot{R}\dot{\beta})$

$$\boxed{R = 1.522\,N}$$

Q5



Given: $|\underline{V}| = 10$ m/s

$\underline{F}_{Drag} = -C_s \underline{V}$

$m = 0.01$ kg

$$\sum \underline{F} = m\underline{a}$$

$$\Rightarrow \quad m\underline{g} + \underline{F}_{Drag} = m\frac{d\underline{V}}{dt}$$

$$m\frac{d\underline{V}}{dt} = m\underline{g} - C_s\underline{V}$$

components in x and y directions:

$$m\frac{dV_x}{dt} = -C_s V_x \quad , \quad m\frac{dV_y}{dt} = -mg - C_s V_y$$

$$\Rightarrow \quad \frac{dV_x}{dt} = -\frac{C_s}{m} V_x \;,$$

$$\Rightarrow \quad \frac{dV_y}{dt} = -g - \frac{C_s}{m} V_y$$

We need to plot the trajectories,

$$\left.\begin{aligned} \frac{dx}{dt} &= V_x \\ \frac{dV_x}{dt} &= -\frac{C_s}{m} V_x \\ \frac{dy}{dt} &= V_y \\ \frac{dV_y}{dt} &= -g - \frac{C_s}{m} V_y \end{aligned}\right\}$$

Solve this ODE system using MATLAB or Python for two different values of C_s.

let $$\underline{y} = \begin{bmatrix} x \\ V_x \\ y \\ V_y \end{bmatrix}$$

$\Rightarrow$ ODE system:

$$\frac{d}{dt}\underline{y} = \begin{bmatrix} y(2) \\ -\frac{C_s}{m} y(2) \\ y(4) \\ -g - \frac{C_s}{m} y(4) \end{bmatrix}$$

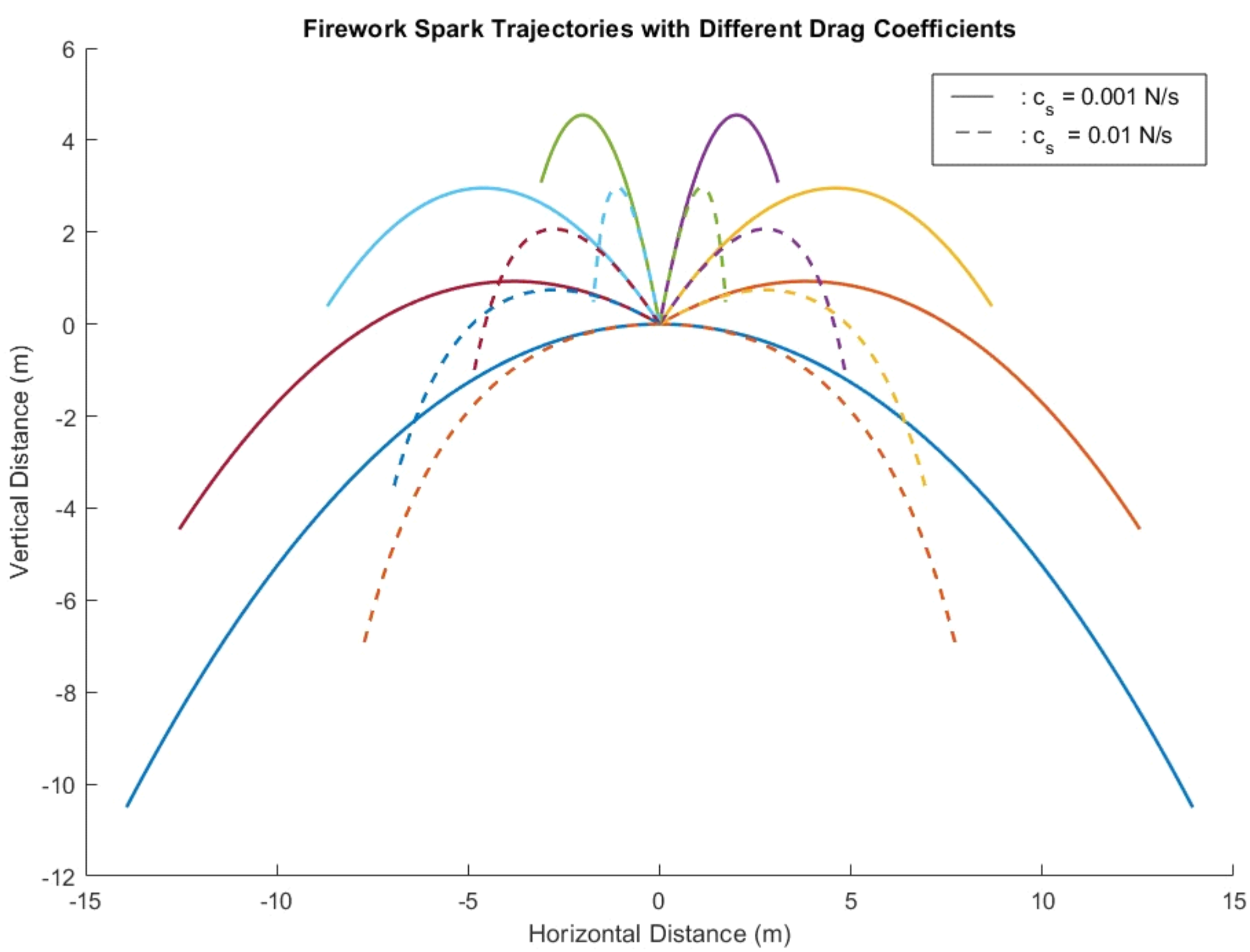
Solve for eight sparks for different values of θ with initial condition:

At $t=0$, $x=0$, $y=0$

$$V_x = |\underline{V}| \cos\theta = 10 \cos\theta$$

$$V_y = |\underline{V}| \sin\theta = 10 \sin\theta$$

In a single plot show the trajectories of eight sparks for two different values of drag coefficient c_s.



As the drag coefficient (C_s) increases, the sparks experience greater air resistance, which shortens their trajectories.