Midterm review session

Friction, work, FBDs, work-energy, relative motion

Relative motion

let this be $\vec{a}_{pseudo}$

Newton's Law:

$$m\vec{a}_{tot} = \tilde{\underline{F}}$$

Σ Forces with respect to the rotating frame

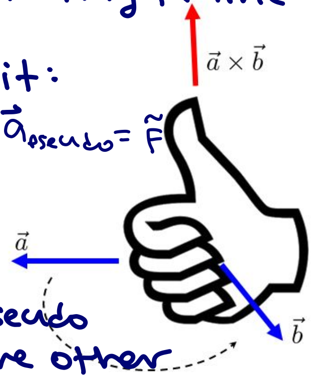
$$\vec{a}_{tot} = \tilde{\underline{a}} + \left[\ddot{\phi}\,\underline{e}_z \times \tilde{\underline{r}} - \dot{\phi}^2 \tilde{\underline{r}} + 2\dot{\phi}\,\underline{e}_z \times \tilde{\underline{v}}\right].$$

Split:

$$m\tilde{a} + m\vec{a}_{pseudo} = \tilde{F}.$$

put $m\vec{a}_{pseudo}$ to the other side

$$\Rightarrow \boxed{m\tilde{\underline{a}} = \tilde{\underline{F}} \underbrace{- m\ddot{\phi}\,\underline{e}_z \times \tilde{\underline{r}}}_{\text{"Euler force"}} \underbrace{+ m\dot{\phi}^2 \tilde{\underline{r}}}_{\text{"Centrifugal force"}} \underbrace{- 2m\dot{\phi}\,\underline{e}_z \times \tilde{\underline{v}}}_{\text{"Coriolis force"}}}$$

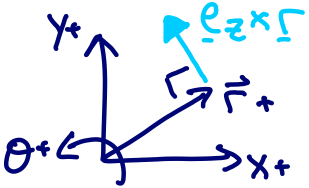


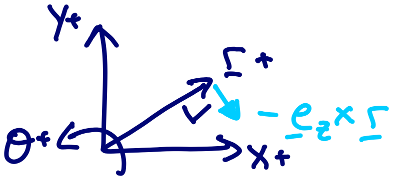
These are called <u>pseudoforces</u>. To use Newton's Law in a rotating frame, we just add these "fictitious forces" and otherwise proceed as usual.

To Draw use right hand rule:

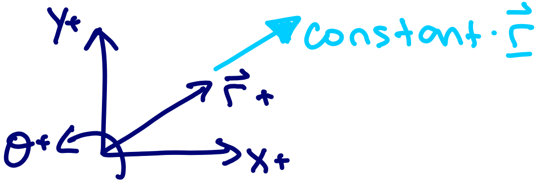
★ Tips to draw term by term: use Right Hand Basis

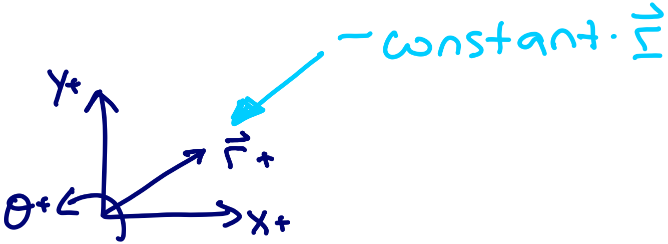
- $\underline{e}_z \times \underline{r}$ term will be $\perp$ to $\underline{r}$



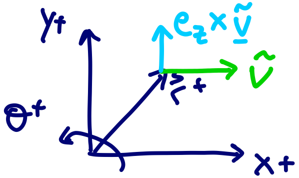


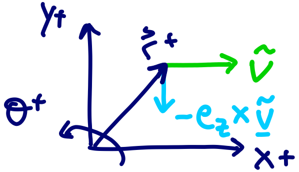
- $\underline{r}$ // constant $\cdot\, \underline{r}$

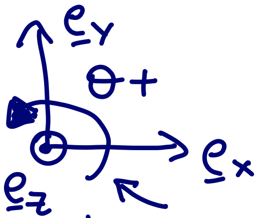




- $\underline{e}_z \times \underline{\tilde{v}}$ [$\underline{\tilde{v}}$ tangent to the path]







(out of page)

positive (counter-clockwise rotation)

Ex:

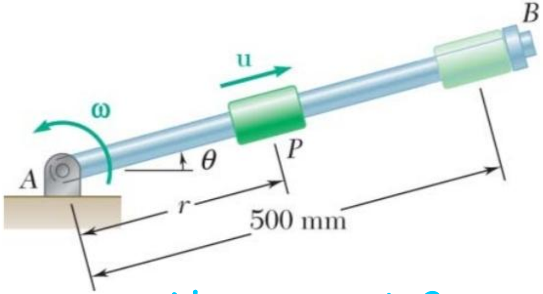
$$\underline{e}_z \times \underline{e}_x = \underline{e}_y$$

where $\underline{e}_x \perp \underline{e}_y$

(we rotated $\underline{e}_x$ 90° ccw to get $\underline{e}_y$)

$\therefore$ Use the same logic with other vectors 90° counter clockwise for positive vectors & Flip if there are negatives!

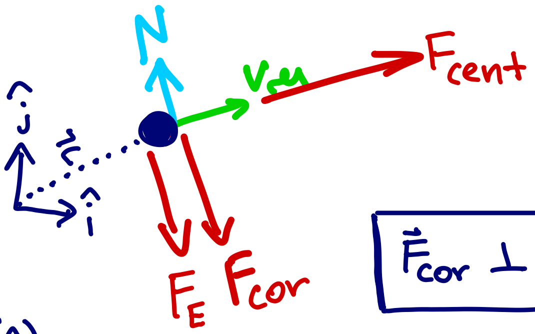
★ During discussion talked about FBD in local frame:



PROBLEM 15.172

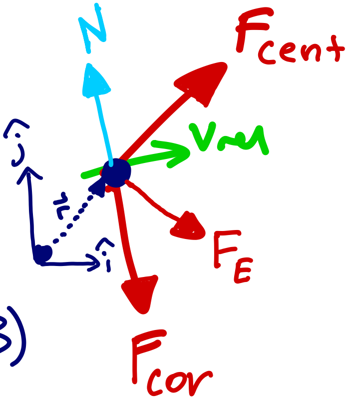
The collar P slides outward at a constant relative speed u along rod AB, which rotates counterclockwise with a constant angular velocity of 20 rpm. Knowing that $r = 250$ mm when $\theta = 0$ and that the collar reaches B when $\theta = 90°$, determine the magnitude of the acceleration of the collar P just as it reaches B. (no gravity here)

N: normal force (direction assumed but ⊥ to wall)



(A)

$\boxed{\vec{F}_{cor} \perp \vec{v}_{rel}\,!}$



(B)

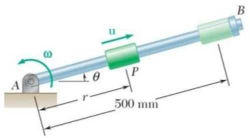
2 examples:

(A) $\vec{v}_{rel}$ is in the same direction as $\vec{r}$

(B) $\vec{v}_{rel}$ is <u>not</u> in the same direction as $\vec{r}$

$$= \tilde{a} + \ddot{\phi}\, e_z \times \tilde{r} - \dot{\phi}^2 \tilde{r} + 2\dot{\phi}\, e_z \times \tilde{v} \,.$$

$$\Rightarrow \boxed{m\tilde{a} = \tilde{F} \underbrace{- m\ddot{\phi}\, e_z \times \tilde{r}}_{\text{"Euler force" } F_E} + \underbrace{m\dot{\phi}^2 \tilde{r}}_{\text{"Centrifugal force" } F_{cent}} \underbrace{- 2m\dot{\phi}\, e_z \times \tilde{v}}_{\text{"Coriolis force" } F_{cor}}}$$



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SOLUTION

$$\omega = 20 \text{ rpm} = \frac{(20)(2\pi)}{60} = \frac{2\pi}{3} \text{ rad/s}$$

$$\alpha = 0$$

$$\theta = 90° = \frac{\pi}{2} \text{ radians}$$

Uniform rotational motion.

$$\theta = \theta_0 + \omega t$$

$$t = \frac{\theta - \theta_0}{\omega} = \frac{\frac{\pi}{2}}{\frac{2\pi}{3}} = 0.75 \text{ s}$$

Uniform motion along rod.

$$r = r_0 + ut$$

$$u = \frac{r - r_0}{t} = \frac{0.5 - 0.25}{0.75} = \frac{1}{3} \text{ m/s},$$

$$\mathbf{v}_{P/AB} = \frac{1}{3} \text{ m/s} \uparrow$$

Acceleration of coinciding Point P′ on the rod. $(r = 0.5 \text{ m})$

$$\mathbf{a}_{P'} = r\omega^2$$

$$= (0.5)\left(\frac{2\pi}{3}\right)^2$$

$$= \frac{2\pi^2}{9} \text{ m/s}^2 \downarrow$$

$$= 2.1932 \text{ m/s}^2 \downarrow$$

Acceleration of collar P relative to the rod. $\mathbf{a}_{P/AB} = 0$

Coriolis acceleration. $2\boldsymbol{\omega} \times \mathbf{v}_{P/AB} = 2\omega u = (2)\left(\frac{2\pi}{3}\right)\left(\frac{1}{3}\right) = 1.3963 \text{ m/s}^2 \longleftarrow$

Acceleration of collar P.

$$\mathbf{a}_P = \mathbf{a}_{P'} + \mathbf{a}_{P/AB} + 2\boldsymbol{\omega} \times \mathbf{v}_{P/AB}$$

$$\mathbf{a}_P = [2.1932 \text{ m/s}^2 \downarrow] + [1.3963 \text{ m/s}^2 \longleftarrow]$$

$$\mathbf{a}_P = 2.60 \text{ m/s}^2 \measuredangle 57.5°$$

$a_P = 2.60 \text{ m/s}^2$ ◀

Forces arising from potential energy:

Suppose a force varies with position according to a formula of the form:

$$\boxed{\underline{F}(\underline{r}) = -\nabla U(\underline{r}) \equiv -\left[\frac{\partial U}{\partial x}\underline{e}_x + \frac{\partial U}{\partial y}\underline{e}_y + \frac{\partial U}{\partial z}\underline{e}_z\right]}$$

$$W = \int F \cdot dr = \int -\nabla U \cdot dr$$
$$= -(U_2 - U_1)$$
$$= U_1 - U_2$$

where the scalar function $U(\underline{r})$ is called the "potential energy". Such forces are called "conservative". Not all forces are conservative! But many are.

Conservative force examples:

Weight: $\mathbf{W} = -mg\,\mathbf{e}_y = -\nabla(mgy). \Rightarrow U_g(\mathbf{r}) = mgy.$

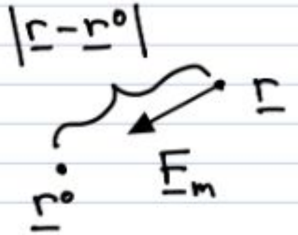
Spring: $\mathbf{F}_s(\mathbf{r}) = -k(|\mathbf{r}-\mathbf{r}^\circ| - l_0)\dfrac{\mathbf{r}-\mathbf{r}}{|\mathbf{r}-\mathbf{r}\ |} = -\nabla\left(\frac{k}{2}(|\mathbf{r}-\mathbf{r}^\circ|-l_0)^2\right).$ $\Rightarrow F_s = kx$ & $U_s = \frac{1}{2}kx^2$

$$\Rightarrow U_s(\mathbf{r}) = \frac{k}{2}(|\mathbf{r}-\mathbf{r}^\circ|-l_0)^2.$$

Constant force: $\mathbf{F}_0 = -\nabla(-\mathbf{F}_0\cdot\mathbf{r}). \Rightarrow U_0(\mathbf{r}) = -\mathbf{F}_0\cdot\mathbf{r}.$

More general form

Power-law force: $\mathbf{F}_m(\mathbf{r}) = -C|\mathbf{r}-\mathbf{r}^\circ|^m \dfrac{\mathbf{r}-\mathbf{r}^\circ}{|\mathbf{r}-\mathbf{r}^\circ|}$



$$= -\nabla\, C\cdot\frac{1}{m+1}|\mathbf{r}-\mathbf{r}^\circ|^{m+1}.$$

$$\Rightarrow U_m(\mathbf{r}) = \frac{C}{m+1}|\mathbf{r}-\mathbf{r}^\circ|^{m+1}.$$

exercise: check how above relate to power-law form

as in what is m for constant F, spring F, weight?

PROBLEM 13.79*

Prove that a force $F(x, y, z)$ is conservative if, and only if, the following relations are satisfied:

$$\frac{\partial F_x}{\partial y} = \frac{\partial F_y}{\partial x} \quad \frac{\partial F_y}{\partial z} = \frac{\partial F_z}{\partial y} \quad \frac{\partial F_z}{\partial x} = \frac{\partial F_x}{\partial z}$$

$$\boxed{\underline{F}(\underline{r}) = -\nabla U(\underline{r}) \equiv -\left[\frac{\partial U}{\partial x}\underline{e}_x + \frac{\partial U}{\partial y}\underline{e}_y + \frac{\partial U}{\partial z}\underline{e}_z\right]}$$

PROBLEM 13.79*

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SOLUTION

For a conservative force, Equation (13.22) must be satisfied.

$$F_x = -\frac{\partial V}{\partial x} \quad F_y = -\frac{\partial V}{\partial y} \quad F_z = -\frac{\partial V}{\partial z}$$

We now write

$$\frac{\partial F_x}{\partial y} = -\frac{\partial^2 V}{\partial x \partial y} \quad \frac{\partial F_y}{\partial x} = -\frac{\partial^2 V}{\partial y \partial x}$$

Since $\dfrac{\partial^2 V}{\partial x \partial y} = \dfrac{\partial^2 V}{\partial y \partial x}$:

$$\frac{\partial F_x}{\partial y} = \frac{\partial F_y}{\partial x} \blacktriangleleft$$

We obtain in a similar way

$$\frac{\partial F_y}{\partial z} = \frac{\partial F_z}{\partial y} \quad \frac{\partial F_z}{\partial x} = \frac{\partial F_x}{\partial z} \blacktriangleleft$$

$F_f^* = \mu_s F_N$ is NOT general static friction. $F_f^* = F_f$ only at critical transition moment! Before that $F_f = -[$what ever total tangent force there F_t^* + (if wall is moving) tangent of pseudo force from the moving wall $(a_w - (a_w \cdot e_n)e_n)m]$

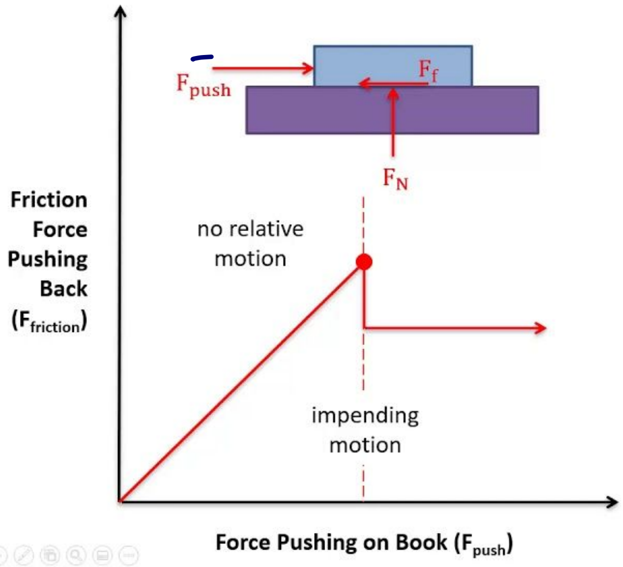
Friction

- Define $\underline{F}^*$ as sum of all forces except for $\underline{F}_t$ on a body.
- $\underline{F}_t^* \equiv \underline{F}^* - (\underline{F}^* \cdot \underline{e}_n)\underline{e}_n$ is the part of $\underline{F}^*$ tangent to the plane.
- $\underline{F}_{trial} \equiv m[\dot{\underline{v}}_{wall} - (\underline{e}_n \cdot \dot{\underline{v}}_{wall})\underline{e}_n] - \underline{F}_t^*$ is the force the friction would have to supply to keep the object from sliding.

The force of friction can thus be expressed by:

$$\underline{F}_t = \begin{cases} \underline{F}_{trial} & \text{if } \underline{v}_s = \underline{0} \;\&\; |\underline{F}_{trial}| \le \mu_s F_n \\ \mu_s F_n \left(\underline{F}_{trial} / |\underline{F}_{trial}|\right) & \text{if } \underline{v}_s = \underline{0} \;\&\; |\underline{F}_{trial}| > \mu_s F_n \\ -\mu_d F_n \left(\underline{v}_s / |\underline{v}_s|\right) & \text{if } \underline{v}_s \neq \underline{0}\,. \end{cases}$$

Static and Kinetic Friction



$F_f = F_{push}$

Until the point where...

$F_f = \mu_s * F_N$

After that point the friction force often drops down to and stays at...

$F_f = \mu_k * F_N$

$\boxed{\underline{a}_{wall} = \dot{\underline{v}}_{wall}}$

★ Note on question about the moving wall:

⇒ treat the same way as we treat pseudo forces in the rotating frame!

$$\underline{F}^* = m(\underline{a} + \underline{a}_{wall}) \quad \text{general vector description of a system}$$

move to other side

$$\underline{F}^* - \underbrace{m\underline{a}_{wall}}_{\text{pseudo force!}} = m\underline{a} \quad \Rightarrow \quad \text{Define } \underline{F}_{tot} = \underline{F}^* - m\underline{a}_{wall}$$

⇒ Now take tangent of $\underline{F}_{tot}$ to get effective tangential force that we can compare against maximum static friction of the system.

To get tangent (if only normal direction is known) subtract normal component of the $\underline{F}_{tot}$ from $\underline{F}_{tot}$

- Scalar along normal : $\underline{F}_{tot} \cdot \underline{e}_n = F^n_{tot}$
- Vector along normal : $(\underline{F}_{tot} \cdot \underline{e}_n)\underline{e}_n = \underline{F}^n_{tot}$
- resolve $\underline{F}^n_{tot} = (\underline{F}^* \cdot \underline{e}_n)\underline{e}_n - m(\underline{a}_{wall} \cdot \underline{e}_n)\underline{e}_n$
- resolve $\underline{F}^t_{tot} = \underbrace{\underline{F}^* - (\underline{F}^* \cdot \underline{e}_n)\underline{e}_n}_{\underline{F}^*_t \text{ from lecture}} - m(\underline{a}_{wall} - (\underline{a}_{wall} \cdot \underline{e}_n)\underline{e}_n)$

friction will act in the opposite direction

$$\text{So} \quad \underline{F}_{trial} = -\underline{F}^t_{tot} = -\underline{F}^*_t + m(\underline{a}_{wall} - (\underline{a}_{wall} \cdot \underline{e}_n)\underline{e}_n)$$

Yay :)

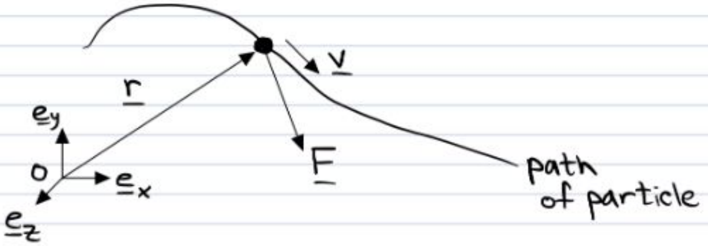
Matches ♥

Work-Energy

① $$\boxed{W_{AB}^{Tot} = \sum_{\substack{\text{All} \\ \text{forces}}} \int_{\substack{\text{Path} \\ \underline{r}_A \to \underline{r}_B}} \underline{F}^i \cdot \underline{dr} = \frac{1}{2} m v_B^2 - \frac{1}{2} m v_A^2}.$$

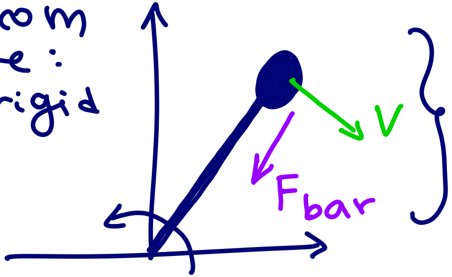
② $$\boxed{W_{AB}^{nc} = \sum_{\substack{\text{Non-} \\ \text{conservative} \\ \text{forces}}} \int_{\substack{\text{Path} \\ \underline{r}_A \to \underline{r}_B}} \underline{F}^{nc} \cdot \underline{dr} = E_B - E_A}$$

where $E = \frac{1}{2} m v^2 + \underbrace{\sum_c U^c}_{}$ → Total potential energy of all conservative forces. $(\underline{F}^c = -\nabla U^c)$



If dot product $= 0$ then Force ⊥ velocity

Recall from lecture: rotating rigid bar



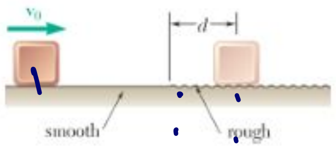
$F_{bar} \perp V$

Power

$$\boxed{\text{Power} \equiv \underline{F} \cdot \underline{v}}$$

= Force • velocity

Q: Powerless force?



PROBLEM 13.CQ1

Block A is traveling with a speed v_0 on a smooth surface when the surface suddenly becomes rough with a coefficient of friction of μ causing the block to stop after a distance d. If block A were traveling twice as fast, that is, at a speed $2v_0$, how far will it travel on the rough surface before stopping?

(a) $d/2$

(b) d

(c) $\sqrt{2}d$

(d) $2d$

(e) $4d$

$$\frac{1}{2}mv^2 = F_f \cdot d \Rightarrow v^2 \propto d \quad \text{(proportional)}$$

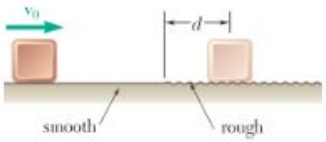
$$(2v)^2$$

$$\downarrow$$

$$4v^2 \propto 4d$$

must mult. d with 4 to maintain $v^2 \propto d$ proportionality

PROBLEM 13.CQ1



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(*a*) $d/2$

(*b*) d

(*c*) $\sqrt{2}d$

(*d*) $2d$

(*e*) $4d$

SOLUTION

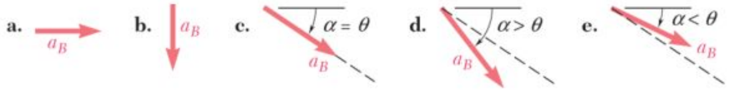
If the block were traveling twice as fast, its kinetic energy would be 4x as much ($T = \frac{1}{2}mv^2$). The work done by friction to stop the block is $U = Fd$. This work must equal the amount of kinetic energy before the block hits the rough patch. Since the kinetic energy is 4x as much it will take 4x the distance to stop.

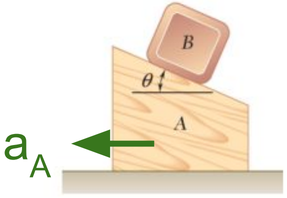
Answer: (*e*)

PROBLEM 11.CQ7

Bad wording: basically just ignore dynamics & do kinematics: what's the direction of a_B? Given a_A to the left?

~~Blocks A and B are released from rest in the positions shown. Neglecting friction between all surfaces, which figure below best indicates the direction α of the acceleration of block B~~?





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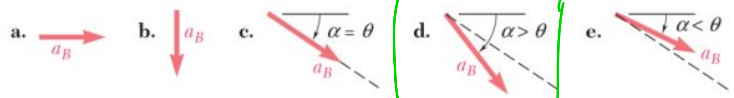
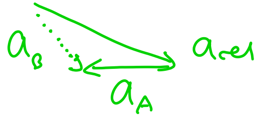
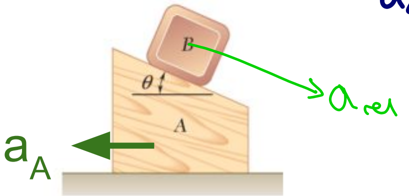
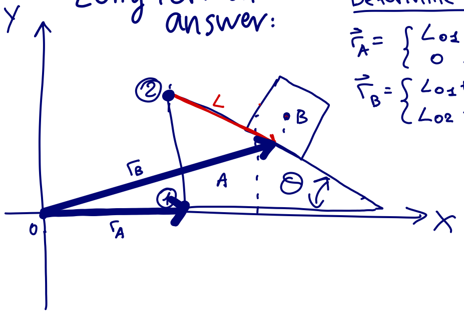


Quick Answer:





Long Formal answer:



Determine the constraint! (using geometry)

$$\vec r_A = \begin{Bmatrix} L_{01} \\ 0 \end{Bmatrix}$$

$$\vec r_B = \begin{Bmatrix} L_{01} + L\cos\theta \\ L_{02} - L\sin\theta \end{Bmatrix}$$

↓ derive twice

$$\vec a_A = \begin{Bmatrix} \ddot L_{01} \\ 0 \end{Bmatrix} = \begin{Bmatrix} a_A^x \\ a_A^y \end{Bmatrix} \quad \text{where } a_A^y = 0$$

$$\vec a_B = \begin{Bmatrix} \ddot L_{01} + \ddot L\cos\theta \\ -\ddot L\sin\theta \end{Bmatrix} = \begin{Bmatrix} \ddot L_{01} \\ 0 \end{Bmatrix} + \begin{Bmatrix} \ddot L\cos\theta \\ -\ddot L\sin\theta \end{Bmatrix} = \begin{Bmatrix} a_B^x \\ a_B^y \end{Bmatrix}$$

$$\begin{Bmatrix} a_A^x \\ 0 \end{Bmatrix} + \begin{Bmatrix} \ddot L\cos\theta \\ -\ddot L\sin\theta \end{Bmatrix} = \begin{Bmatrix} a_B^x \\ a_B^y \end{Bmatrix}$$

$$\left.\begin{aligned} a_A^x + \ddot L\cos\theta &= a_B^x \\ -\ddot L\sin\theta &= a_B^y \end{aligned}\right\}$$

which in case if a_A^x is negative, as asked, gives us diagram (d)

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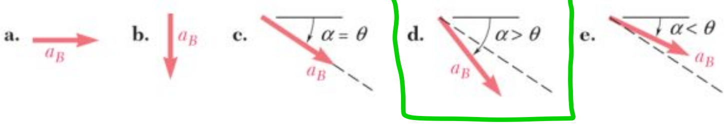
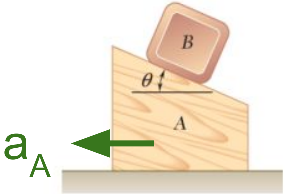
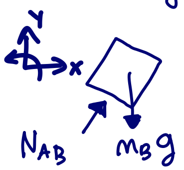


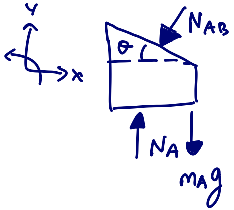


Note on question about gravity:



$\sum F_x = N_{AB}\sin\theta = m a_B^x$

$\sum F_y = N_{AB}\cos\theta - m_B g = m a_B^y$



$\sum F_x = m_A a_A$

$-N_{AB}\sin\theta = m_A a_A^x$

$\sum F_y = m_A a_A^y = 0$

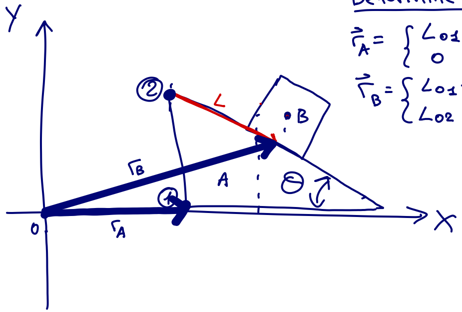
$N_A - N_{AB}\cos\theta - m_A g = 0$

$N_A = N_{AB}\cos\theta + m_A g$

#unknowns: N_A, N_{AB}, a_A^x, a_B^x, a_B^y

equations: 4 ⇒ need one more

Determine the constraint!



$\vec{r}_A = \begin{Bmatrix} L_{01} \\ 0 \end{Bmatrix}$

$\vec{r}_B = \begin{Bmatrix} L_{01} + L\cos\theta \\ L_{02} - L\sin\theta \end{Bmatrix}$

derive twice

$\vec{a}_A = \begin{Bmatrix} \ddot{L}_{01} \\ 0 \end{Bmatrix} = \begin{Bmatrix} a_A^x \\ a_A^y \end{Bmatrix}$ where $a_A^y = 0$

$\vec{a}_B = \begin{Bmatrix} \ddot{L}_{01} + \ddot{L}\cos\theta \\ -\ddot{L}\sin\theta \end{Bmatrix} = \begin{Bmatrix} \ddot{L}_{01} \\ 0 \end{Bmatrix} + \begin{Bmatrix} \ddot{L}\cos\theta \\ -\ddot{L}\sin\theta \end{Bmatrix} = \begin{Bmatrix} a_B^x \\ a_B^y \end{Bmatrix}$

$\begin{Bmatrix} a_A^x \\ 0 \end{Bmatrix} + \begin{Bmatrix} \ddot{L}\cos\theta \\ -\ddot{L}\sin\theta \end{Bmatrix} = \begin{Bmatrix} a_B^x \\ a_B^y \end{Bmatrix}$

$a_A^x + \ddot{L}\cos\theta = a_B^x$

$-\ddot{L}\sin\theta = a_B^y$

now # unk N_A, N_{AB}, a_A^x, a_B^x, a_B^y, $\ddot{L} = 6$

but # eqn : 6 (4 $\sum F = ma$ & 2 kinematic)

or

unk : N_A, N_{AB}, a_A^x, a_A^y, a_B^x, a_B^y, $\ddot{L} = 7$

eqn : 7 (4 $\sum F = ma$ & 3 kinematic (include $a_A^y = 0$ as constraint)

Solvable!

actual solving is beyond scope (too tedious)

Total System: solution included for those interested

$$a_A^x + \ddot{L}\cos\theta = a_B^x \quad (1)$$
$$-\ddot{L}\sin\theta = a_B^y \quad (2)$$

Get rid of $\ddot{L}$

$(2) \rightarrow \ddot{L} = \frac{a_B^y}{-\sin\theta}$ plug into (1) $a_A^x - \frac{a_B^y}{\sin\theta}\cos\theta = a_B^x \Leftrightarrow a_A^x\sin\theta - a_B^y\cos\theta = a_B^x\sin\theta \Leftrightarrow a_B^y = (a_A^x - a_B^x)\frac{\sin\theta}{\cos\theta}$ (7)

$$N_{AB}\sin\theta = m_B a_B^x \quad (3)$$
$$N_{AB}\cos\theta - m_B g = m_B a_B^y \quad (4)$$
$$-N_{AB}\sin\theta = m_A a_A^x \quad (5)$$

Get rid of N_{AB} & simplify:

$(3)+(5) \Rightarrow$

$(3) \Leftrightarrow N_{AB} = m_B a_B^x/\sin\theta \Rightarrow$ plug into (4) $\Rightarrow \frac{\cancel{m_B} a_B^x}{\sin\theta}\cos\theta - \cancel{m_B} g = \cancel{m_B} a_B^y \Leftrightarrow a_B^x\cos\theta - g\sin\theta = a_B^y\sin\theta$ (9)

cancel m_B

$$N_A - N_{AB}\cos\theta - m_A g = 0 \quad (6)$$

---

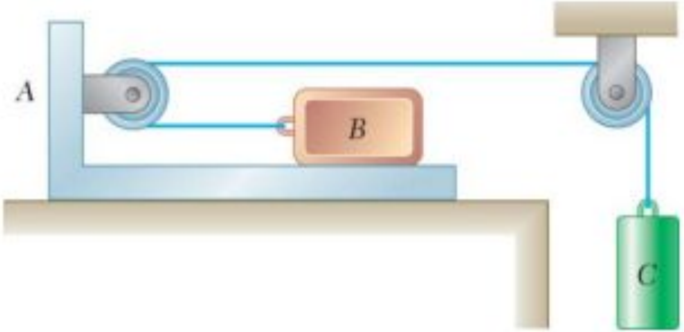
3 eqn & 3 unk!

$$a_B^y = (a_A^x - a_B^x)\frac{\sin\theta}{\cos\theta} \quad (7)$$
$$m_B a_B^x + m_A a_A^x = 0 \quad (8) \Leftrightarrow a_B^x = \frac{-m_A a_A^x}{m_B}$$
$$a_B^x\cos\theta - g\sin\theta = a_B^y\sin\theta \quad (9)$$

$$\Rightarrow a_B^x = \frac{m_A g\sin\theta\cos\theta}{m_A + m_B\sin^2\theta} \quad \& \quad a_A^x = \frac{-m_B g\sin\theta\cos\theta}{m_A + m_B\sin^2\theta}$$

$$\Rightarrow a_B^y = \frac{-(m_B + m_A)g\sin\theta^2}{(m_A + m_B\sin^2\theta)}$$

$$\ddot{L} = \frac{(m_B + m_A)g\sin\theta}{(m_A + m_B\sin^2\theta)}$$ (acceleration of B relative to A, defined positive downwards)

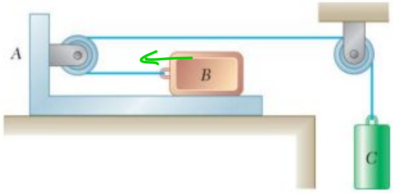


PROBLEM 12.F3

Objects A, B, and C have masses m_A, m_B, and m_C respectively. The coefficient of kinetic friction between A and B is μ_k, and the friction between A and the ground is negligible and the pulleys are massless and frictionless. Assuming B slides on A draw the FBD and KD for each of the three masses A, B and C.

+ <u>perform kinematics!</u>

PROBLEM 12.F3



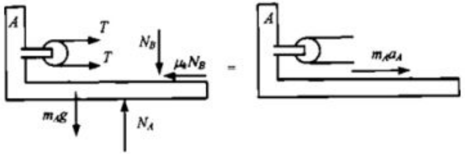
motion

Objects A, B, and C have masses m_A, m_B, and m_C respectively. The coefficient of kinetic friction between A and B is μ_k, and the friction between A and the ground is negligible and the pulleys are massless and frictionless. Assuming B slides on A draw the FBD and KD for each of the three masses A, B and C.

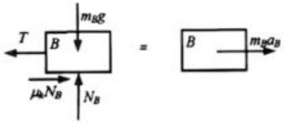
SOLUTION

Answer:

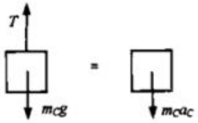
Block A

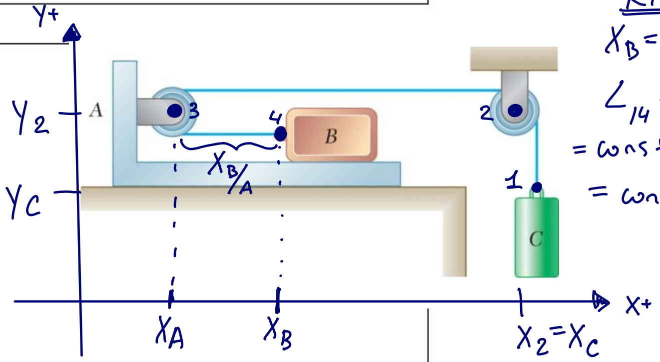


Block B



Block C





<u>Kinematics</u>

$X_B = X_4$ & $X_A = X_3$ $\qquad X_C = X_1$

$L_{14} = L_{12} + L_{23} + L_{34} + \text{const}$

$= \text{const} + (Y_2 - Y_c) + (X_2 - X_A) + (X_B - X_A)$

$= \text{const} - Y_c - 2X_A + X_B$

$$\begin{cases} L_{14} = -Y_c - 2X_A + X_B + \text{const} \\ X_B = X_A + X_{B/A} \end{cases}$$

$$\begin{cases} 0 = -\dot{Y}_c - 2\dot{X}_A + \dot{X}_B \\ \dot{X}_B = \dot{X}_A + \dot{X}_{B/A} \end{cases}$$

$\Rightarrow$ If no relative motion then $\dot{X}_{B/A} = 0$ $\quad \dot{X}_B = \dot{X}_A$

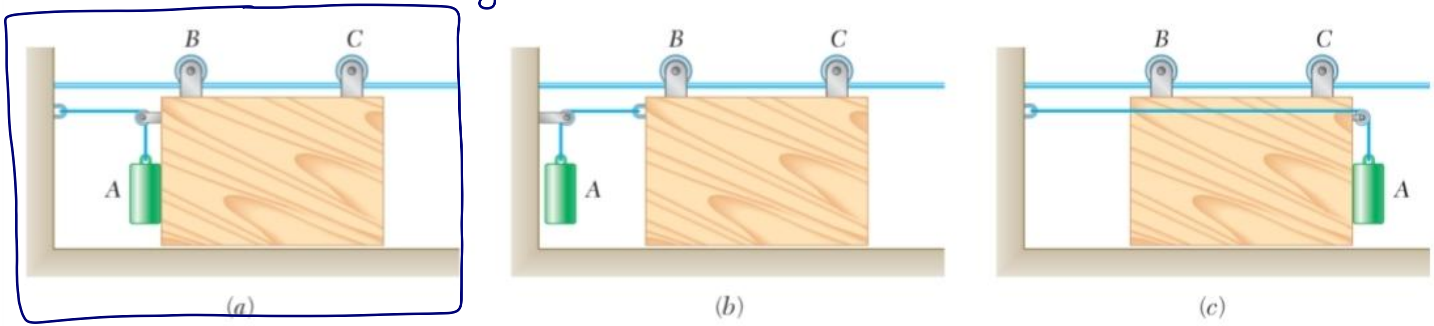
$$\Rightarrow -\dot{Y}_c - 2\dot{X}_A + \dot{X}_A = 0$$

$$-\dot{Y}_c - \dot{X}_A = 0$$

$$\boxed{-\dot{Y}_c = \dot{X}_A}$$

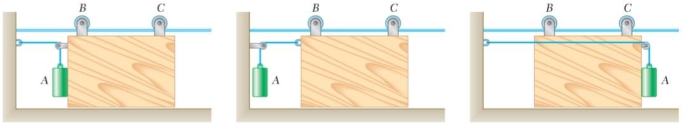
PROBLEM 12.29

A 40-lb sliding panel is supported by rollers at *B* and *C*. A 25-lb counterweight *A* is attached to a cable as shown and, in cases *a* and *c*, is initially in contact with a vertical edge of the panel. Neglecting friction, determine in each case shown the acceleration of the panel and the tension in the cord immediately after the system is released from rest. Only!



PROBLEM 12.29

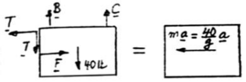
A 40-lb sliding panel is supported by rollers at B and C. A 25-lb counterweight A is attached to a cable as shown and, in cases a and c, is initially in contact with a vertical edge of the panel. Neglecting friction, determine in each case shown the acceleration of the panel and the tension in the cord immediately after the system is released from rest.



(a) (b) (c)

SOLUTION

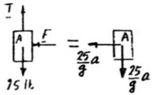
(*a*) Panel: F = Force exerted by counterweight



$\xleftarrow{+} \Sigma F_x = ma$: $\quad T - F = \frac{40}{g}a \quad (1)$

Counterweight A: Its acceleration has two components

$$\mathbf{a}_A = \mathbf{a}_P + \mathbf{a}_{A/P} = a \rightarrow + a \downarrow$$



$\xleftarrow{+} \Sigma F_x = ma_x$: $\quad F = \frac{25}{g}a \quad (2)$

$+\!\downarrow \Sigma F_g = ma_g$: $\quad 25 - T = \frac{25}{g}a \quad (3)$

Adding (1), (2), and (3):

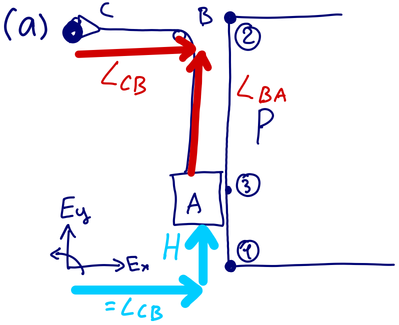
$$\cancel{T} - F + F + 25 - \cancel{T} = \frac{40 + 25 + 25}{g}a$$

$$a = \frac{25}{90}g = \frac{25}{90}(32.2) \qquad \mathbf{a} = 8.94 \text{ ft/s}^2 \longleftarrow ◀$$

Substituting for a into (3):

$$25 - T = \frac{25}{g}\left(\frac{25}{90}g\right) \qquad T = 25 - \frac{625}{90} \qquad T = 18.06 \text{ lb} ◀$$

Kinematics



$L_{AC} = L_{CB} + L_{BA}$

$\vec{r}_P = \begin{Bmatrix} L_{CB} \\ 0 \end{Bmatrix}$ + constants

$\vec{r}_A = \begin{Bmatrix} L_{CB} \\ 0 \end{Bmatrix} + \begin{Bmatrix} 0 \\ H \end{Bmatrix}$ + constants

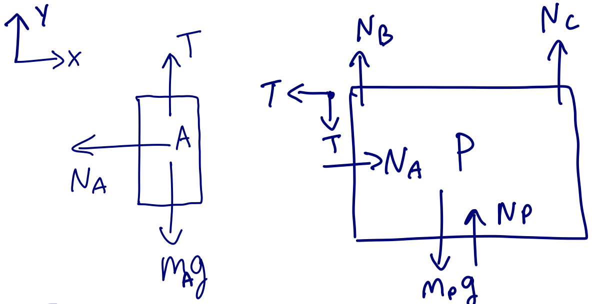
$0 = \dot{L}_{CB} + \dot{L}_{BA} \Rightarrow \quad \underbrace{\dot{L}_{CB} = -\dot{L}_{BA}}$

$\vec{V}_P = \begin{Bmatrix} \dot{L}_{CB} \\ 0 \end{Bmatrix} \qquad \dot{H} = \dot{L}_{BA}$

$\vec{V}_A = \begin{Bmatrix} \dot{L}_{CB} \\ \dot{H} \end{Bmatrix} = \begin{Bmatrix} \dot{L}_{CB} \\ -\dot{L}_{CB} \end{Bmatrix}$

Where $\dot{L}_{CB}$ is V_P

Dynamics



Equilibrium equations

$\sum F_x = m_A a_A^x = -N_A \quad (1) \qquad \sum F_x = m_P a_P^x = -T + N_A$

$\sum F_y = m_A a_A^y = -m_A g + T \quad (2) \qquad \sum F_y = -m_P g + N_P + N_B + N_C$

where

$a_A^x = a_P^x$

&

$a_A^y = -a_P^x$

Solve

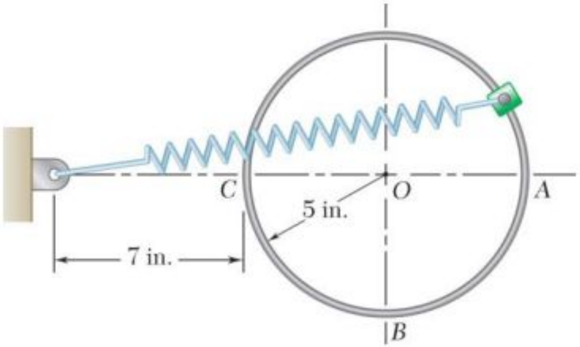
(1) $\quad -m_A a_P^x = N_A$

(2) $\quad -m_A a_P^x = -m_A g + T$

$\Rightarrow T = m_A g - m_A a_P^x$

$\Rightarrow m_P a_P^x = -(m_A g - m_A a_P^x) + (-m_A a_P^x)$

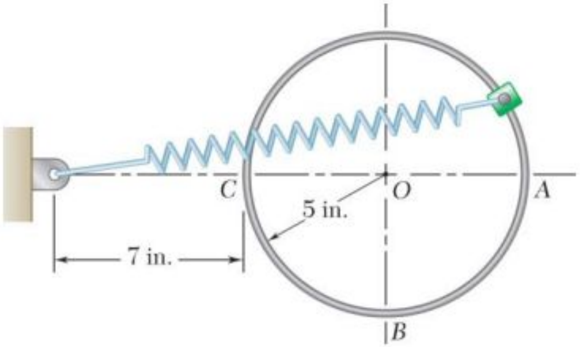
$\Rightarrow \boxed{a_P^x = -\frac{m_A g}{m_P}}$



PROBLEM 12.F8

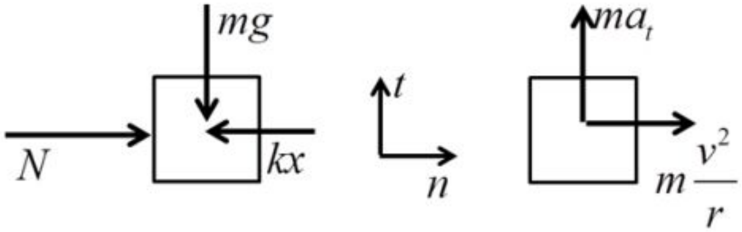
A collar of mass *m* is attached to a spring and slides without friction along a circular rod in a vertical plane. The spring has an undeformed length of 5 in. and a constant *k*. Knowing that the collar has a speed *v* at Point *C*, draw the FBD and KD of the collar at this point.

PROBLEM 12.F8



A collar of mass m is attached to a spring and slides without friction along a circular rod in a vertical plane. The spring has an undeformed length of 5 in. and a constant k. Knowing that the collar has a speed v at Point C, draw the FBD and KD of the collar at this point.

SOLUTION



where $x = 2/12$ ft and $r = 5/12$ ft.