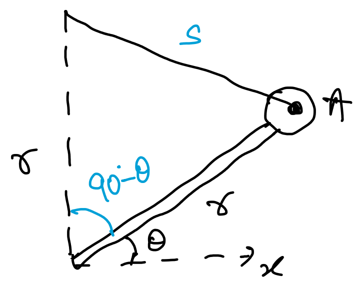
Week 3 Solutions

Sunday, September 8, 2024 6:13 PM

Q1



cosine rule:

$$\cos(90-\theta) = \frac{r^2+r^2-s^2}{2r\cdot r}$$

$$s^2 = 2r^2 - 2r^2\sin\theta$$

$$s^2 = 2r^2(1-\sin\theta)$$

$$\Rightarrow \quad s = r\sqrt{2(1-\sin\theta)}$$

$$\dot{s} = \frac{r}{2\sqrt{1-\sin\theta}}\cdot\sqrt{2}\cdot(-\cos\theta)\frac{d\theta}{dt}$$

$$r\frac{d\theta}{dt} = -\frac{\dot{s}\sqrt{2(1-\sin\theta)}}{\cos\theta}$$

here, $V_B = -\dot{s}$, and $V_A = r\dfrac{d\theta}{dt}$

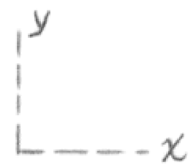
$$V_A = \frac{V_B\sqrt{2(1-\sin\theta)}}{\cos\theta}$$

Q2

Let B = point on slot B coincident with P.

$\underline{v}_P = \underline{v}_B + \underline{v}_{P/B}$ But $\underline{v}_P = \underline{v}_{P_x} + \underline{v}_{P_y}$

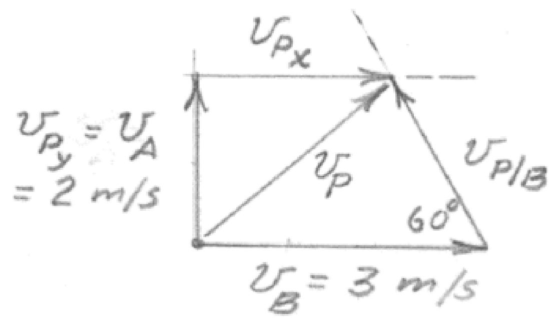
$= \underline{v}_{P_x} + \underline{v}_A$



So $\underline{v}_{P_x} + \underline{v}_A = \underline{v}_B + \underline{v}_{P/B}$

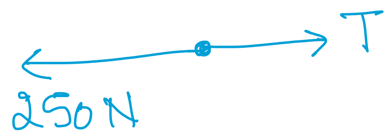
$$v_P = \sqrt{2^2 + \left(3 - \frac{2}{\sqrt{3}}\right)^2}$$

$$= \underline{2.72 \text{ m/s}}$$



Q3

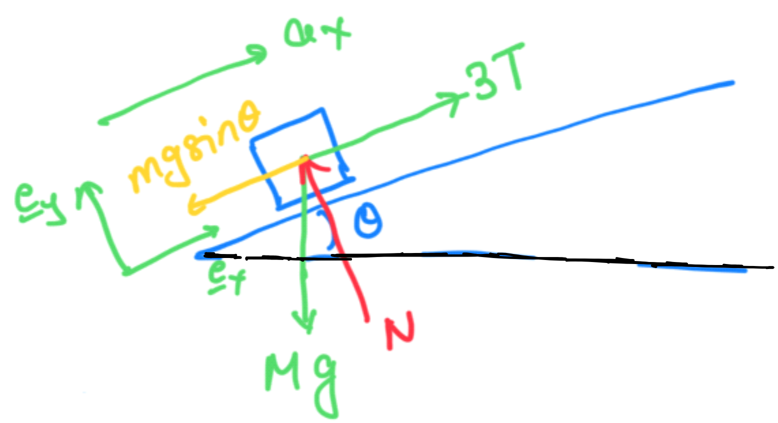
The man is pulling the rope:



$T = 250\,N$.

FBD:

$\theta = 15^\circ$



Motion in e_x direction only:

e_x: $\Sigma F = m a_x$

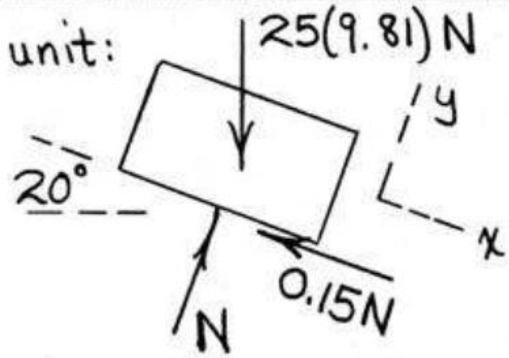
$$-mg\sin\theta + 3T = m a_x$$

$$a_x = \frac{3T - mg\sin\theta}{m} = \frac{750 - (100)(9.81)\sin 15^\circ}{100}$$

$$\underline{a_x = 4.96\ m/s^2}.$$

Q4

Frame & sphere as a unit:



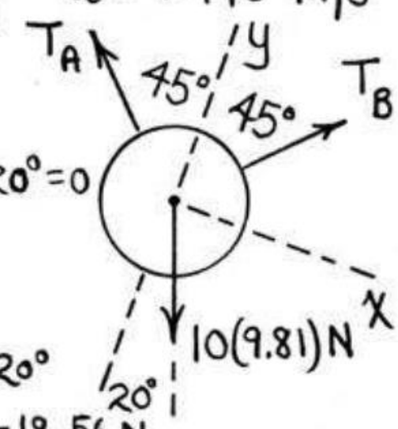
$\Sigma F_y = 0: \; N - 25(9.81)\cos 20° = 0$

$N = 230 \text{ N}$

$\Sigma F_y = ma_x:$

$25(9.81)\sin 20° - 0.15(230) = 25a, \quad a = 1.973 \text{ m/s}^2$

Sphere alone:



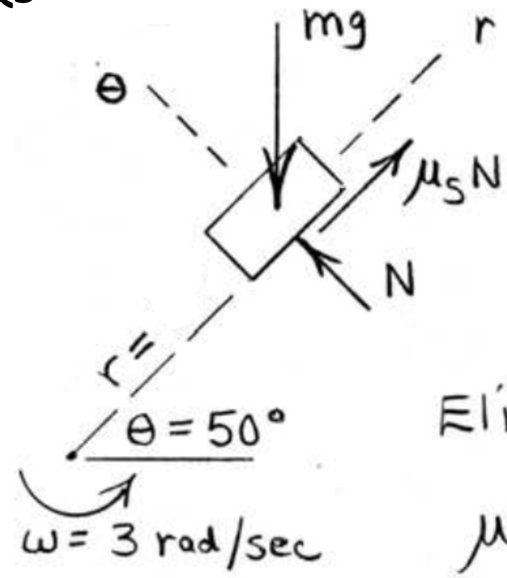
$\Sigma F_y = 0: \; (T_A + T_B)\cos 45° - 10(9.81)\cos 20° = 0$

$T_A + T_B = 130.4 \text{ N}$

$\Sigma F_x = ma_x: \; (T_B - T_A)\sin 45° + 98.1 \sin 20°$
$= 10(1.973), \text{ or } T_B - T_A = -19.56 \text{ N}$

Solution: $\underline{T_A = 75.0 \text{ N}}, \quad \underline{T_B = 55.4 \text{ N}}$

Q5



$$\sum F_\theta = m a_\theta : \; N - mg\cos\theta = 0$$

$$N = mg\cos\theta$$

$$\sum F_r = m a_r : \; \mu_s N - mg\sin\theta = m(0 - r\omega^2)$$

Eliminate N:

$$\mu_s = \tan\theta - \frac{r\omega^2}{g\cos\theta}$$

Numbers: $\mu_s = \tan 50° - \dfrac{(0.45)(3)^2}{9.81 \times \cos 50°} = \underline{0.55}$

Q6

The dimensions of the car are small compared with those of the path, so we will treat the car as a particle. ① The velocities are

$$v_A = \left(100\,\frac{\text{km}}{\text{h}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right)\left(1000\,\frac{\text{m}}{\text{km}}\right) = 27.8\text{ m/s}$$

$$v_C = 50\,\frac{1000}{3600} = 13.89\text{ m/s}$$

We find the constant deceleration along the path from

$$\left[\int v\,dv = \int a_t\,ds\right] \qquad \int_{v_A}^{v_C} v\,dv = a_t \int_0^s ds$$

$$a_t = \frac{1}{2s}\left(v_C{}^2 - v_A{}^2\right) = \frac{(13.89)^2 - (27.8)^2}{2(120)} = -2.41\text{ m/s}^2$$

(a) Condition at A With the total acceleration given and a_t determined, we can easily compute a_n and hence ρ from

$[a^2 = a_n{}^2 + a_t{}^2] \qquad a_n{}^2 = 3^2 - (2.41)^2 = 3.19 \qquad a_n = 1.785\text{ m/s}^2$

$[a_n = v^2/\rho] \qquad \rho = v^2/a_n = (27.8)^2/1.785 = 432\text{ m} \qquad$ *Ans.*

(b) Condition at B Since the radius of curvature is infinite at the inflection point, $a_n = 0$ and

$$a = a_t = -2.41\text{ m/s}^2 \qquad \textit{Ans.}$$

(c) Condition at C The normal acceleration becomes

$[a_n = v^2/\rho] \qquad a_n = (13.89)^2/150 = 1.286\text{ m/s}^2$

With unit vectors $\mathbf{e}_n$ and $\mathbf{e}_t$ in the n- and t-directions, the acceleration may be written

$$\mathbf{a} = 1.286\mathbf{e}_n - 2.41\mathbf{e}_t\text{ m/s}^2$$

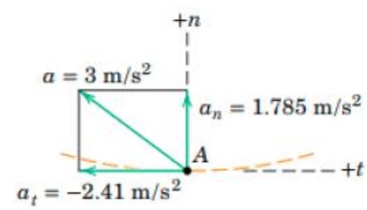
where the magnitude of $\mathbf{a}$ is

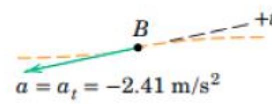
$[a = \sqrt{a_n{}^2 + a_t{}^2}] \qquad a = \sqrt{(1.286)^2 + (-2.41)^2} = 2.73\text{ m/s}^2 \qquad$ *Ans.*

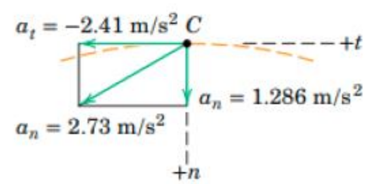
The acceleration vectors representing the conditions at each of the three points are shown for clarification.

HELPFUL HINT

① Actually, the radius of curvature to the road differs by about 1 m from that to the path followed by the center of mass of the passengers, but we have neglected this relatively small difference.







Q7

Write out equations and their derivatives as functions of x:

$$\begin{cases} x = 2t^2 + 3t - 1; & \dot{x} = 4t + 3; & \ddot{x} = 4 \\ y = 5t - 2; & \dot{y} = 5 & \ddot{y} = 0 \end{cases}$$

Determine position, velocity, and acceleration: At $t = 1$ s : $\begin{cases} x = 4 \text{ m}; & \dot{x} = 7 \text{ m/s}; & \ddot{x} = 4 \text{ m/s}^2 \\ y = 3\text{m}; & \dot{y} = 5 \text{ m/s}; & \ddot{y} = 0 \end{cases}$

Speed then is the magnitude of the velocity vector: $v = \sqrt{\dot{x}^2 + \dot{y}^2} = \sqrt{7^2 + 5^2} = 8.60$ m/s

Using velocity x and y components, we can determine tangential and normal directions to the path at t = 1s: $\theta = \tan^{-1}(\frac{\dot{y}}{\dot{x}}) = \tan^{-1}(\frac{5}{7}) = 35.5^o$

Tangential basis direction: $\boldsymbol{e}_t = \cos(\theta)\boldsymbol{e}_x + \sin(\theta)\boldsymbol{e}_y$

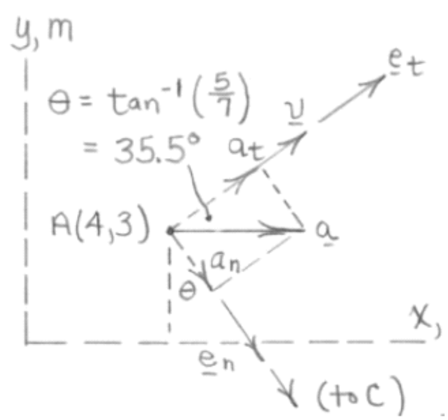
Rotate clockwise by 90^o to get normal basis direction (shortcut: switch x and y, flip sign on y) $\boldsymbol{e}_n = \sin(\theta)\boldsymbol{e}_x - \cos(\theta)\boldsymbol{e}_y$

Use dot product to project acceleration vector (determined from parametric equations) onto normal and tangential basis directions to find magnitudes normal and tangential acceleration: $a_n = \boldsymbol{a} \cdot \boldsymbol{e}_n = [4, 0] \cdot [\cos(\theta), \sin(\theta)] = 2.32\ m/s^2$

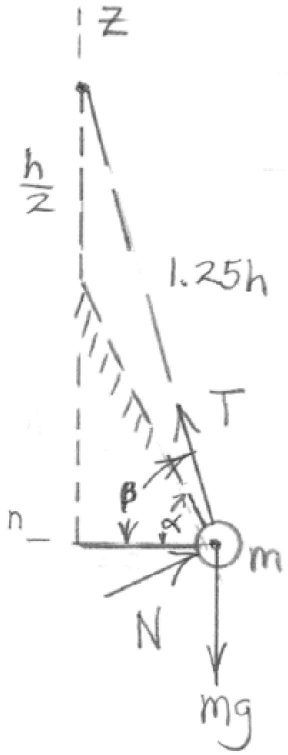
Tangential acceleration can be determined in the same way via $a_t = \boldsymbol{a} \cdot \boldsymbol{e}_t$ or using right-triangle relationship between total acceleration and its components: $a^2 = a_t^2 + a_n^2 = 4^2 = a_t^2 + (2.32)^2 \rightarrow a_t = 3.25\ m/s^2$

To find radius of curvature ρ: we can use relationship determined earlier for normal acceleration: $a_n = \frac{v^2}{\rho}$ since we have information about the kinematics. $\rho = \frac{v^2}{a_n} = \frac{8.60^2}{2.32} = 31.8m$

Lastly, position of the center of the radius of curvature will be located ρ distance away from the particle along the $\boldsymbol{e}_n$ direction: $\boldsymbol{x}_{center} = \boldsymbol{x}_{particle} + \rho \boldsymbol{e}_n = [4, 3] + 31.8[\sin(\theta), -\cos(\theta)] = [22.5, -22.9]m$



Q8



$$\alpha = \tan^{-1}\frac{h}{h/2} = 63.4^\circ$$

$$\frac{\sin(\beta-\alpha)}{h/2} = \frac{\sin(90^\circ+\alpha)}{1.25h} \Rightarrow \beta = 73.7^\circ$$

$$\begin{cases} \Sigma F_z = 0: \; T\sin\beta + N\cos\alpha - mg = 0 \\ \Sigma F_n = m\frac{v^2}{\rho}: \; T\cos\beta - N\sin\alpha = m\frac{v^2}{1.25h\cos\beta} \end{cases}$$

For $h = 0.8$ m & $v = 0.6$ m/s $\begin{cases} T = 1.901\text{ N} \\ N = 0.307\text{ N} \end{cases}$

Set $N = 0$ in the above governing equations to obtain $\underline{v = 0.895\text{ m/s}}$ & $T = 2.04$ N