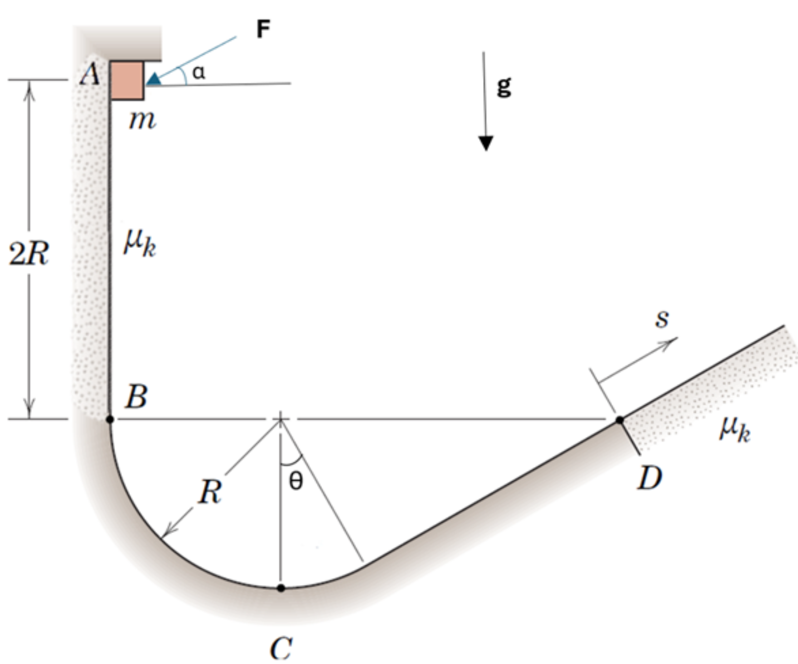
Problem Set 2
ME 104, Fall 2024

Due: Sept 27, 2024, 11:59pm

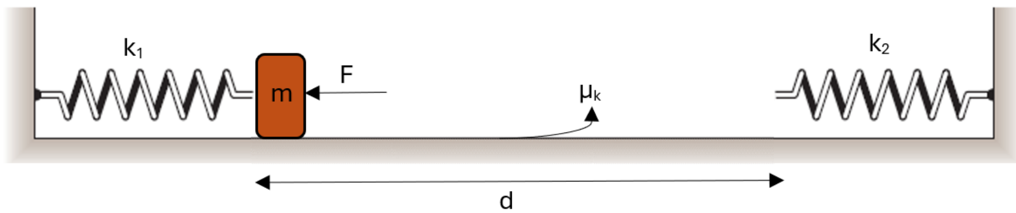
Instructions:

- Submit your homework on gradescope only.
- Please do each problem on a new page.
- Make sure to include the problem number at the top for each problem.
- **Show each of the 4 steps in every dynamics problem.**
- **Draw clear FBD's when using Newton's laws.**
- **Do not substitute values into your work until the very end.**
- Please write legibly!!! If we can't read your solution, there will be lost points.
- Please put a box around your final answer(s).

1) The small slider of mass m is released from rest while in position A and then slides along the vertical-plane track. The block is held against the rough track (A to B) with a force $\boldsymbol{F}$. The track is smooth from B to D and rough (coefficient of dynamic friction μ_k) from A to B and from point D onward. Determine (a) the normal force F_n^B exerted by the track on the slider just *after* it passes point B, (b) the normal force F_n^C exerted by the track on the slider as it passes the bottom point C, and (c) the distance s traveled along the incline past point D before the slider stops.



2) A block of mass $m = 1$ kg is pressed against a spring with a spring constant $k_1 = 100$ N/m by a force $F = 100$ N. Upon release, the block travels along a straight path and impacts another spring with a spring constant $k_2 = 200$ N/m. The block moves back and forth between the two springs, which are initially separated by a distance of 5 m. The coefficient of kinetic friction between the block and the path is $\mu_k = 0.05$. Find the number of oscillations N after which the block no longer hits either of the springs. Assume friction is only present along the distance d between the two springs.



3) A square platform of dimensions $L \times L$ is designed (see figure 1(a) below) with a rotating rigid arm AB with length R holding a mass m at its end. Point A is fixed on the platform, while B rotates over π radians (180°) to point C, following the path s(ϕ) (red dashed line). The arm AB can only hold up to some given maximum tensile force in the axial direction before failure. Therefore, a redesign is proposed, as shown in figure 1(b): an extendable arm is used, such that the length of the arm is given as a function of the angle ϕ by the relation

$$r(\varphi) = \frac{2}{3} R \left(1 + \frac{\varphi}{\pi}\right)$$

where R is as in the original design and ϕ is in radians. The initial velocity equals zero, while the angular acceleration is constant and such that point C is reached after t_{tot} seconds in both designs. Note that B and C remain in the same position for both cases but A has been moved in the second case.

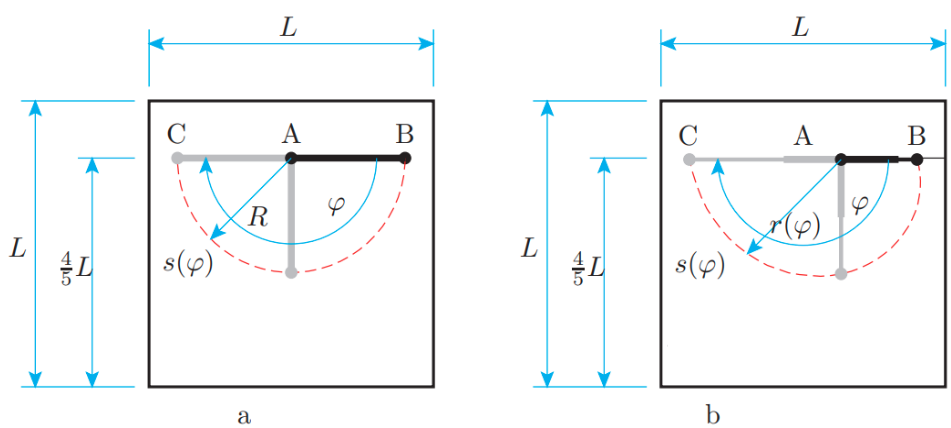
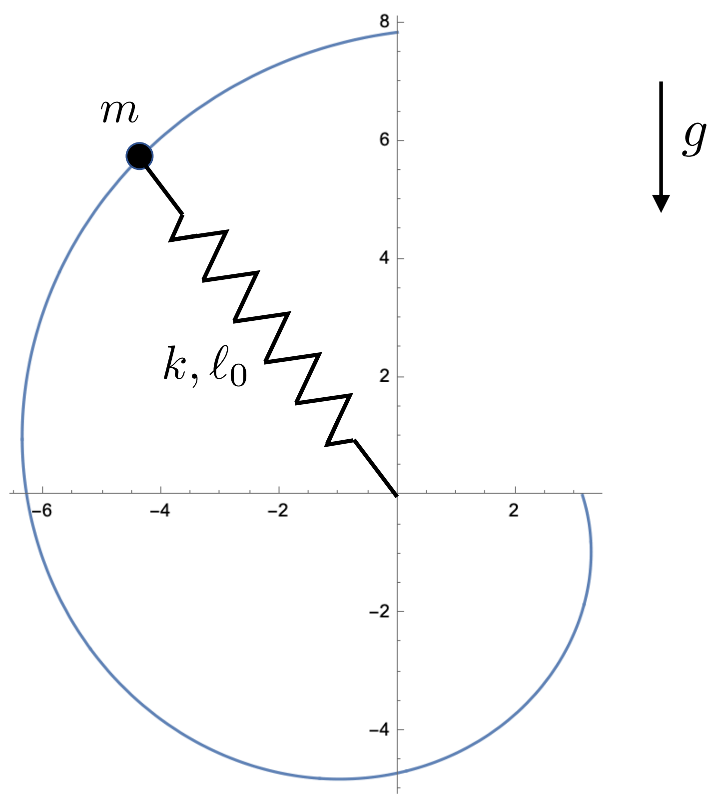


Figure 1: *Platform with arm AB, rotating about A to position C; a: original design, b: proposed redesign.*

In both cases, compute the axial tensile force $T(t)$ in the arm, from time 0 to t_{tot}, by clearly carrying out the "four steps" of dynamics problems. Do not neglect gravity. Given the objective (limiting the maximum tensile force in an arm), judge if the redesign is an improvement. While tension in the arm is the internal force we are currently worried about, does the arm only exert a tensile force on the mass or does it also exert other forces? Leave your answers in terms of m, R, g, t_{tot}. It may advantageous to look up a curvature formula in polar coordinates.

4) A bead of mass m slides along a frictionless curved guide rod as shown below. The shape of the guide is given by the polar equation $r(\theta) = L_0\,(3\pi - \theta)$ from $\theta = \frac{\pi}{2}$ to $\theta = 2\pi$. The bead feels non-negligible downward gravity g. Furthermore, the bead is attached to a spring, whose other end is at the origin. The spring has given properties k, ℓ_0 with $\ell_0 < \pi$. In terms of the variables given, and assuming the bead begins with no velocity at the top of the guide, how fast is it going when it exits the guide at $\theta = 2\pi$? Use energy principles. When the bead is at $\theta = \frac{3\pi}{2}$, what is the magnitude of force the shaft is applying to the bead?



5) Computer Problem: Using Python or MATLAB, in this problem you will simulate the formation of a hurricane! A hurricane forms when a low-pressure zone develops at a point

on the earth, which then attracts the air to it. But due to the earth's rotation, the pseudoforces that influence the air particles as they attract to the low-pressure zone cause the air to swirl.

We will make this problem more tractable by assuming the earth is a flat disk (no conspiracy theories please!) spinning at a rate of one rotation per day. The radius of the disk is the earth's radius, roughly 4000 miles. Let us assume the low-pressure center has a position vector $\boldsymbol{r}_0(t)$ that is fixed to a particular location on the earth r_0 away from the "North pole," which is the center of the disk. Suppose the low-pressure zone establishes a force field that we can imagine pulls on parcels of air with a force of

$$\boldsymbol{F}_{low}(\boldsymbol{r}, t) = -F_0\,(\boldsymbol{r} - \boldsymbol{r}_{\boldsymbol{0}}(t))$$

where F_0 has units of force per length. Here, a "parcel" of air is a 1 kg chunk of air that moves as a single particle. All air is initially moving with the earth. Air parcels also experience a drag force when they move relative to the earth's surface:

$$\boldsymbol{F}_{drag} = -c_s\,(\boldsymbol{v} - \boldsymbol{v}_{earth}).$$

In a frame rotating with the earth, carry out the four steps of solving dynamics to obtain ODE's for the motion of air parcels. Using about 10 air parcels initially scattered uniformly across the earth, compute trajectories of the parcels for $F_0 = 1000$ kg/day^2, $c_s = 15$ kg/day, and a low-pressure zone located $r_0 = 1500$ miles from the North pole. Plot your trajectories together on one plot. In what direction does your hurricane spin? That is, is its rotation rate vector in the same direction as that of the earth's spin or are they in opposing directions? Explain physically! You can neglect gravity on our flat earth. Please include your code and your plot along with your four steps and explanations.