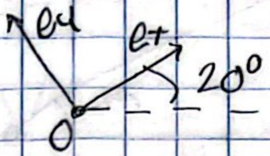
Problem Set #1

Question 1:

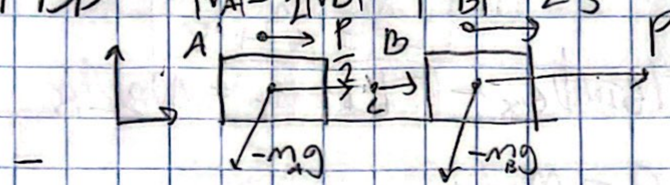
① Kinematics
- Cartesian coords



$$\underline{R} = x e_x + y e_y \qquad \underline{V} = \dot{\underline{R}} = \dot{x} e_x + \dot{y} e_y \qquad \underline{A} = \dot{\underline{V}} = \ddot{x} e_x + \ddot{y} e_y$$

$$y = \dot{y} = \ddot{y} = 0$$

② FBD $\quad |V_A| = \frac{1}{2}|V_B| \quad |V_B| = 2\frac{ft}{s}$



$$\underline{g} = -9.8\frac{m}{s}\sin\theta\, e_x$$

③ $\sum \underline{F}_B = m\underline{A}_B \longrightarrow -m_B\underline{g} + \underline{P} = m_B A_B \quad m_B = ? \quad A_B = 3\frac{ft}{s^2}$

$\sum \underline{F}_C = m_C\underline{A}_C \longrightarrow -m_C\underline{g} + c_1 \underline{P} = m_C A_C \quad c_1 c_2 = \text{CONST.}$

$\sum \underline{F}_A = m_A \underline{A}_A \longrightarrow -m_A\underline{g} + c_2 \underline{P} = m_A A_A$

$$\underline{P} = m_B A_B + m_B g = \frac{1}{c_1}(m_C A_C + m_C g) = \frac{1}{c_2}(m_A A_A + m_A g)$$

$$2R_A = R_B \rightarrow 2V_A = V_B \rightarrow 2A_A = A_B$$

$$V_C = \frac{V_A + V_B}{2} = \frac{V_B/2 + V_B}{2} = \frac{3V_B}{4} \qquad A_C = \frac{3A_B}{4}$$

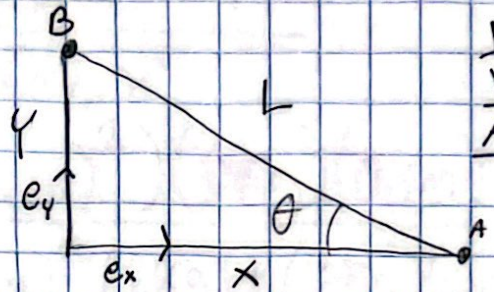
$$\boxed{V_A = \frac{2}{2}\frac{ft}{s} = 1\frac{ft}{s}} \qquad \boxed{A_A = \frac{3}{2}\frac{ft}{s^2}}$$

$$\boxed{V_C = \frac{3 \cdot 2}{4}\frac{ft}{s} = \frac{3}{2}\frac{ft}{s}}$$

Problem Set #1

Question 2

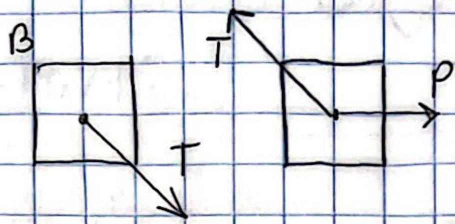
① Kinematics
- Cartesian



$\underline{r} = x e_x + y e_y$
$\underline{V} = \dot{x} e_x + \dot{y} e_y$
$\underline{A} = \ddot{x} e_x + \ddot{y} e_y$

$\sin\theta = y/L$

② FBD



③ $\Sigma F = M \underline{A}$

① $T = m A_B \Rightarrow \boxed{-T\frac{y}{L} = m_B \ddot{y}}$

② $\boxed{-T\frac{x}{L} + P = m_A \ddot{x}}$

③ $\boxed{L^2 = y^2 + x^2} \xrightarrow{\frac{d}{dt}} 0 = 2x\dot{x} + 2y\dot{y} \Rightarrow \boxed{x\dot{x} + y\dot{y} = 0}$ ④

$\xrightarrow{\frac{d}{dt}} \boxed{\dot{x}^2 + x\ddot{x} + \dot{y}^2 + y\ddot{y} = 0}$ ⑤

$x = 0.4m$
$L = 0.5m$
$y = 0.3m$
$\dot{x} = 2 m/s$
$\dot{y} = -8/3\ m/s$
$M_A = 2kg$
$M_B = 3kg$

③ $(0.5m)^2 = y^2 + (0.4m)^2 \quad y = 0.3m$

④ $(0.4m)(2 m/s) + (0.3m)\dot{y} = 0 \quad \dot{y} = -8/3\ m/s$

$T = \frac{-L m_B \ddot{y}}{y} \qquad T = \frac{L(P - m_A\ddot{x})}{x}$

$-\frac{m_B \ddot{y}}{y} = \frac{P - m_A\ddot{x}}{x} \qquad \ddot{y} = \frac{y(m_A\ddot{x} - P)}{m_B x}$

$\dot{x}^2 + x\ddot{x} + \dot{y}^2 + y^2(m_A\ddot{x} - P)\frac{1}{m_B x} = 0$

$\frac{y^2(m_A\ddot{x} - P)}{m_B x} + x\ddot{x} = -\dot{x}^2 - \dot{y}^2$

$y^2 m_A \ddot{x} + m_B x^2 \ddot{x} = (-\dot{x}^2 - \dot{y}^2) m_B x + y^2 P$

$\ddot{x}(y^2 m_A + m_B x^2) = (-\dot{x}^2 - \dot{y}^2) m_B x + y^2 P$

$\longrightarrow$

$$\ddot{x} = \frac{(-\dot{x}^2 - \dot{\varphi}^2) m_B x + \varphi^2 P}{\varphi^2 m_A + m_B x^2}$$

$$\ddot{x} = \frac{(-(2\frac{m}{s})^2 - (8/3 \frac{m}{s})^2)(3kg)(0.4m) + (0.3m)^2 (40N)}{(0.3m)^2 (2kg) + (3kg)(0.4m)^2}$$

$$\boxed{\ddot{x} = -14.74 \frac{m}{s^2}}$$

$$m_A \ddot{x} = P + \frac{m_B \ddot{\varphi} x}{\varphi}$$

$$11.11 + 0.4\ddot{x} + 0.3\ddot{\varphi} = 0$$

$$40 + \frac{3\ddot{\varphi} \cdot 0.4}{0.3} = 2\ddot{x}$$

$$20 + 2\ddot{\varphi} = \ddot{x}$$

$$\ddot{\varphi} = \frac{\ddot{x} - 20}{2} = \frac{-14.74 - 20}{2}$$

$$= \boxed{-17.37 \frac{m}{s}}$$

ACC OF BLOCK B

$$T = \frac{-(0.5m)(3kg)(-17.37\frac{m}{s^2})}{0.3m}$$

$$\boxed{T = 86.85 \, kg \frac{m}{s^2}}$$

PROBLEM SET #1

QUESTION 3

① KINEMATICS
- CYLINDRICAL

$\underline{r} = r e_r + z e_z$

$\underline{V} = \dot r e_r + r\dot\theta e_\theta + \dot z e_z$

$\underline{A} = (\ddot r - r\dot\theta^2)e_r + (r\ddot\theta + 2\dot r\dot\theta)e_\theta + \ddot z e_z$

$z = \dot z = \ddot z = 0$

② FBD

$F_c = m\omega^2 r(t)$

③ $\sum F_r = mA_r$ $A_{e_r} = (\ddot r - r\dot\theta^2)e_r$

$-T = m(\ddot r - r\dot\theta^2) \rightarrow T = (0.2)(2 - 1(4)^2)$

$\boxed{T = 2.8N}$ - STRING TENSION

$\sum F_\theta = mA_\theta$ $A_\theta e_\theta = (r\ddot\theta + 2\dot r\dot\theta)e_\theta$

$F_\theta = m(r\ddot\theta + 2\dot r\dot\theta) = (0.2)(-2 + 2(-3)(4))$

$\boxed{F_\theta = 5.2N}$ - FORCE EXERTED BY TUBE

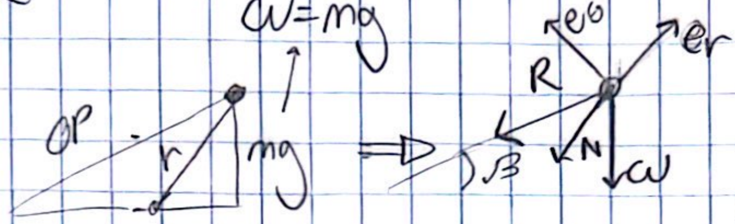
PROBLEM SET #1

QUESTION 4

① KINEMATICS
- CYLINDRICAL
$\{e_r, e_\theta\}$

$$\underline{r} = r e_r$$
$$\underline{V} = \dot{r} e_r + r\dot{\theta} e_\theta$$
$$\underline{A} = (\ddot{r} - r\dot{\theta}^2) e_r + (r\ddot{\theta} - 2\dot{r}\dot{\theta}) e_\theta$$

② FBD



$\ddot{r} = 0 \quad \dot{\theta} = \Omega$

③

$$\sum F_r = m A_{e_r} \qquad A_{e_r} = (\ddot{r} - r\dot{\theta}^2) e_r = -r\Omega^2$$

$$-N - R\cos\beta - W\cos\beta = -mr\Omega^2 \qquad \ddot{\theta} = 0 \quad \dot{r} = 0$$

$$\sum F_\theta = m A_{e_\theta} \qquad A_{e_\theta} = (r\ddot{\theta} - 2\dot{r}\dot{\theta}) e_r = 0$$

$$R\sin\beta - W\sin\beta = 0 \longrightarrow R = W$$

$$-N - 2W\cos\beta = -mr\Omega^2$$

$$-N = -mr\Omega^2 - 2mg\cos\beta = -(0.2\,kg)(1\,m)(2\,\tfrac{rad}{s})^2 - 2(0.2\,kg)(9.8\,\tfrac{m}{s^2})(0.92$$

$$\boxed{N = 4.434\,N}$$

$$\boxed{R = mg = (0.2\,kg)(9.8\,\tfrac{m}{s^2}) = 1.96\,N}$$