ME 104 Lec 6

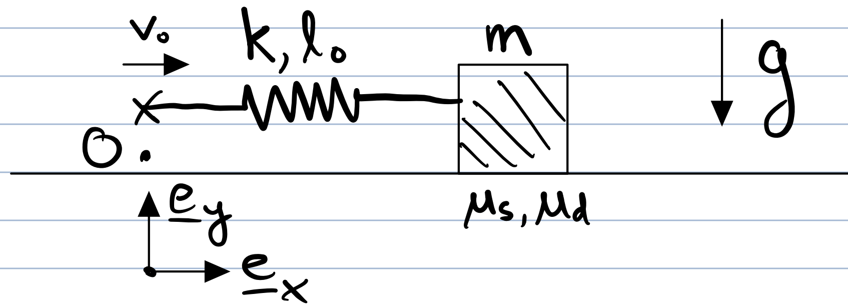
Last time: Friction

- Define $\underline{F}^*$ as sum of all forces <u>except for</u> $\underline{F}_t$ on a body.
- $\underline{F}_t^* \equiv \underline{F}^* - (\underline{F}^* \cdot \underline{e}_n)\underline{e}_n$ is the part of $\underline{F}^*$ tangent to the plane.
- $\underline{F}_{trial} \equiv m[\dot{\underline{v}}_{wall} - (\underline{e}_n \cdot \dot{\underline{v}}_{wall})\underline{e}_n] - \underline{F}_t^*$ is the force the friction would have to supply to keep the object from sliding.

The force of friction can thus be expressed by:

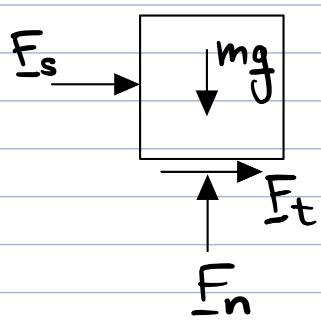
$$\underline{F}_t = \begin{cases} \underline{F}_{trial} & \text{if } \underline{v}_s = \underline{0} \ \& \ |\underline{F}_{trial}| \le \mu_s F_n \\ \mu_s F_n \left(\underline{F}_{trial} / |\underline{F}_{trial}|\right) & \text{if } \underline{v}_s = \underline{0} \ \& \ |\underline{F}_{trial}| > \mu_s F_n \\ -\mu_d F_n \left(\underline{v}_s / |\underline{v}_s|\right) & \text{if } \underline{v}_s \neq \underline{0}. \end{cases}$$

Example: (from Last time)



v_0 = Constant velocity of back of spring.

If the block starts at rest and spring begins unloaded, how does block move?



Last time, we found

$$\underline{F}_n = mg\,\underline{e}_y$$

$$\underline{F}_s = -k\left(|\underline{r}-\underline{r}^A| - l_0\right)\frac{\underline{r}-\underline{r}^A}{|\underline{r}-\underline{r}^A|}$$

Point of attachment of other side of spring.

$$= -k\left(|x-v_0t| - l_0\right)\operatorname{sign}(x-v_0t)\,\underline{e}_x .$$

$$\underline{F}_{trial} = -\underline{F}_s .$$

$$m\ddot{x} = \left(\underline{F}_s - \underline{F}_t\right)\cdot \underline{e}_x$$

Solve with Matlab...

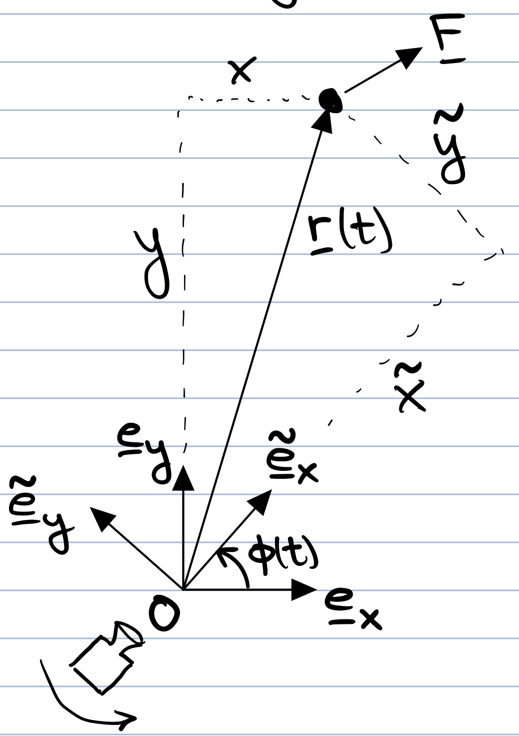
<u>Rotating Frames</u>: We have previously used polar coords, which ties a basis $\{\underline{e}_r, \underline{e}_\theta\}$ to a particle and rotates according to the particle's position.

Sometimes it's advantageous to tie a basis to a rotating frame different from $\{\underline{e}_r, \underline{e}_\theta\}$ for the particle, where the particle of interest moves relative to this frame.

Examples where convenient to use rotating frame of reference:

- Problems on rotating bodies (like earth)
- Problems viewed by a rotating observer.
- Problems where forces are easier to express in a rotating frame.

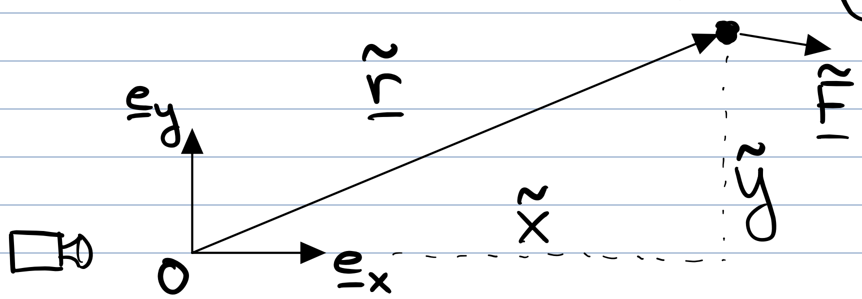
Suppose we view a particle's motion through a rotating camera whose angle relative to lab frame is $\phi(t)$.



$$\underline{r}(t) = x\,\underline{e}_x + y\,\underline{e}_y$$
$$= \tilde{x}\,\underline{\tilde{e}}_x + \tilde{y}\,\underline{\tilde{e}}_y$$

$\{\underline{e}_x, \underline{e}_y\}$ is non-moving lab frame.

A camera attached to the ~ frame does not know the frame is spinning. ~ frame sees:



$\underline{\tilde{r}} = \tilde{x}\,\underline{e}_x + \tilde{y}\,\underline{e}_y$ is the position vector in ~ frame.

Note that in ~ frame, a vector as seen along "$\underline{e}_x$" is actually along $\underline{\tilde{e}}_x$ in the lab frame. Similarly, since ~ frame does not see the bases are moving, it sees the velocity as

$\underline{\tilde{v}} = \dot{\tilde{x}}\underline{e}_x + \dot{\tilde{y}}\underline{e}_y$ and acceleration as $\underline{\tilde{a}} = \ddot{\tilde{x}}\underline{e}_x + \ddot{\tilde{y}}\underline{e}_y$.

$\underline{\text{But}}$, this is measured in an accelerating frame so the actual acceleration of the particle, the one that equals $\underline{F}/m$, is $\underline{a}$ not $\underline{\tilde{a}}$. Let's relate $\underline{a}$ and $\underline{\tilde{a}}$.

First, note that, according to the lab frame,

$$\underline{\tilde{e}}_x = \cos\phi\, \underline{e}_x + \sin\phi\, \underline{e}_y \,, \qquad \underline{\tilde{e}}_y = -\sin\phi\, \underline{\tilde{e}}_x + \cos\phi\, \underline{\tilde{e}}_y$$

$$\Rightarrow \dot{\underline{\tilde{e}}}_x = \dot{\phi}\, \underline{\tilde{e}}_y \,, \qquad \dot{\underline{\tilde{e}}}_y = -\dot{\phi}\, \underline{e}_x .$$

$$\Rightarrow \underline{v} = \dot{\underline{r}} = \frac{d}{dt}\left(\tilde{x}\underline{\tilde{e}}_x + \tilde{y}\underline{\tilde{e}}_y\right) = \dot{\tilde{x}}\underline{\tilde{e}}_x + \tilde{x}\dot{\phi}\underline{\tilde{e}}_y + \dot{\tilde{y}}\underline{\tilde{e}}_y - \tilde{y}\dot{\phi}\underline{\tilde{e}}_x .$$

$$\begin{aligned}
\Rightarrow \underline{a} = \dot{\underline{v}} &= \ddot{\tilde{x}}\underline{\tilde{e}}_x + \dot{\tilde{x}}\dot{\phi}\underline{\tilde{e}}_y + \dot{\tilde{x}}\dot{\phi}\underline{\tilde{e}}_y + \tilde{x}\ddot{\phi}\underline{e}_y - \tilde{x}\dot{\phi}^2\underline{\tilde{e}}_x \\
&\quad + \ddot{\tilde{y}}\underline{\tilde{e}}_y - \dot{\tilde{y}}\dot{\phi}\underline{\tilde{e}}_x - \dot{\tilde{y}}\dot{\phi}\underline{\tilde{e}}_x - \tilde{y}\ddot{\phi}\underline{\tilde{e}}_x - \tilde{y}\dot{\phi}^2\underline{\tilde{e}}_y \\
&= \left(\ddot{\tilde{x}}\underline{\tilde{e}}_x + \ddot{\tilde{y}}\underline{\tilde{e}}_y\right) + \ddot{\phi}\left(\tilde{x}\underline{\tilde{e}}_y - \tilde{y}\underline{\tilde{e}}_x\right) \\
&\quad - \dot{\phi}^2\left(\tilde{x}\underline{\tilde{e}}_x + \tilde{y}\underline{\tilde{e}}_y\right) + 2\dot{\phi}\left(\dot{\tilde{x}}\underline{\tilde{e}}_y - \dot{\tilde{y}}\underline{\tilde{e}}_x\right) .
\end{aligned}$$

Since $\underline{a} = \underline{F}/m$, we can equate the RHS above to $\underline{F}/m$ and then rotate both sides' vectors $\phi(t)$ clockwise. Such a rotation causes:

$$\underline{F} \to \underline{\tilde{F}}, \quad \underline{\tilde{e}}_x \to \underline{e}_x, \quad \underline{\tilde{e}}_y \to \underline{e}_y.$$

So: $$\underline{\tilde{F}}/m = (\ddot{\tilde{x}}\underline{e}_x + \ddot{\tilde{y}}\underline{e}_y) + \ddot{\phi}(\tilde{x}\underline{e}_y - \tilde{y}\underline{e}_x)$$
$$- \dot{\phi}^2(\tilde{x}\underline{e}_x + \tilde{y}\underline{e}_y) + 2\dot{\phi}(\dot{\tilde{x}}\underline{e}_y - \dot{\tilde{y}}\underline{e}_x)$$

$$= \underline{\tilde{a}} + \ddot{\phi}\,\underline{e}_z \times (\tilde{x}\underline{e}_x + \tilde{y}\underline{e}_y)$$
$$- \dot{\phi}^2(\tilde{x}\underline{e}_x + \tilde{y}\underline{e}_y) + 2\dot{\phi}\,\underline{e}_z \times (\dot{\tilde{x}}\underline{e}_x + \dot{\tilde{y}}\underline{e}_y)$$

$$= \underline{\tilde{a}} + \ddot{\phi}\,\underline{e}_z \times \underline{\tilde{r}} - \dot{\phi}^2\underline{\tilde{r}} + 2\dot{\phi}\,\underline{e}_z \times \underline{\tilde{v}}.$$

$$\Rightarrow \boxed{m\underline{\tilde{a}} = \underline{\tilde{F}} \underbrace{- m\ddot{\phi}\,\underline{e}_z \times \underline{\tilde{r}}}_{\text{"Euler force"}} + \underbrace{m\dot{\phi}^2\underline{\tilde{r}}}_{\text{"Centrifugal force"}} \underbrace{- 2m\dot{\phi}\,\underline{e}_z \times \underline{\tilde{v}}}_{\text{"Coriolis force"}}}.$$

These are called <u>pseudoforces</u>. To use Newton's Law in a rotating frame, we just add these "fictitious forces" and otherwise proceed as usual.