



## hilbert

Discrete-time analytic signal using Hilbert transform

### Syntax

```
x = hilbert(xr)
x = hilbert(xr,n)
```

### Description

`x = hilbert(xr)` returns a complex helical sequence, sometimes called the *analytic signal*, from a real data sequence. The analytic signal  $x = x_r + i \cdot x_i$  has a real part, `xr`, which is the original data, and an imaginary part, `xi`, which contains the Hilbert transform. The imaginary part is a version of the original real sequence with a 90° phase shift. Sines are therefore transformed to cosines and vice versa. The Hilbert transformed series has the same amplitude and frequency content as the original real data and includes phase information that depends on the phase of the original data.

If `xr` is a matrix, `x = hilbert(xr)` operates columnwise on the matrix, finding the Hilbert transform of each column.

`x = hilbert(xr,n)` uses an `n` point FFT to compute the Hilbert transform. The input data `xr` is zero-padded or truncated to length `n`, as appropriate.

The Hilbert transform is useful in calculating instantaneous attributes of a time series, especially the amplitude and frequency. The instantaneous amplitude is the amplitude of the complex Hilbert transform; the instantaneous frequency is the time rate of change of the instantaneous phase angle. For a pure sinusoid, the instantaneous amplitude and frequency are constant. The instantaneous phase, however, is a sawtooth, reflecting the way in which the local phase angle varies linearly over a single cycle. For mixtures of sinusoids, the attributes are short term, or local, averages spanning no more than two or three points.

Reference [1] describes the Kolmogorov method for minimum phase reconstruction, which involves taking the Hilbert transform of the logarithm of the spectral density of a time series. The toolbox function [rceps](#) performs this reconstruction.

For a discrete-time analytic signal `x`, the last half of `fft(x)` is zero, and the first (DC) and center (Nyquist) elements of `fft(x)` are purely real.

### Examples

```
xr = [1 2 3 4];
x = hilbert(xr)
x =
1.0000+1.0000i 2.0000-1.0000i 3.0000-1.0000i 4.0000+1.0000i
```

You can see that the imaginary part, `imag(x) = [1 -1 -1 1]`, is the Hilbert transform of `xr`, and the real part, `real(x) = [1 2 3 4]`, is simply `xr` itself. Note that the last half of `fft(x) = [10 -4+4i -2 0]` is zero (in this example, the last half is just the last element), and that the DC and Nyquist elements of `fft(x)`, 10 and -2 respectively, are purely real.

## Algorithm

The analytic signal for a sequence  $x$  has a *one-sided Fourier transform*, that is, negative frequencies are 0. To approximate the analytic signal, `hilbert` calculates the FFT of the input sequence, replaces those FFT coefficients that correspond to negative frequencies with zeros, and calculates the inverse FFT of the result.

In detail, `hilbert` uses a four-step algorithm:

1. It calculates the FFT of the input sequence, storing the result in a vector  $x$ .
2. It creates a vector  $h$  whose elements  $h(i)$  have the values:
  - 1 for  $i = 1, (n/2)+1$
  - 2 for  $i = 2, 3, \dots, (n/2)$
  - 0 for  $i = (n/2)+2, \dots, n$
3. It calculates the element-wise product of  $x$  and  $h$ .
4. It calculates the inverse FFT of the sequence obtained in step 3 and returns the first  $n$  elements of the result.

If the input data `xr` is a matrix, `hilbert` operates in a similar manner, extending each step above to handle the matrix case.

## References

- [1] Claerbout, J.F., *Fundamentals of Geophysical Data Processing*, McGraw-Hill, 1976, pp.59-62.
- [2] Marple, S.L., "Computing the discrete-time analytic signal via FFT," IEEE Transactions on Signal Processing, Vol. 47, No.9 (September 1999), pp.2600-2603.
- [3] Oppenheim, A.V., and R.W. Schaffer, *Discrete-Time Signal Processing*, 2nd ed., Prentice-Hall, 1998.

## See Also

[fft](#), [ifft](#), [rceps](#)

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icceps 

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