CIBioFi
Centro de Investigación e Innovación en Bioinformática y Fotónica

QuanTIC
Tecnología, Información & Complejidad Cuántica

Universidad del Valle

STUDYING THE TWO-PHOTON INTERFERENCE

"A photon interferes only with itself" - Dirac

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AGENDA

1. Interference
2. Two-photon entangled state
3. Temporal two-photon interference
4. Gaussian-shape interference
5. Conclusions
6. Perspectives

INTERFERENCE

correlations

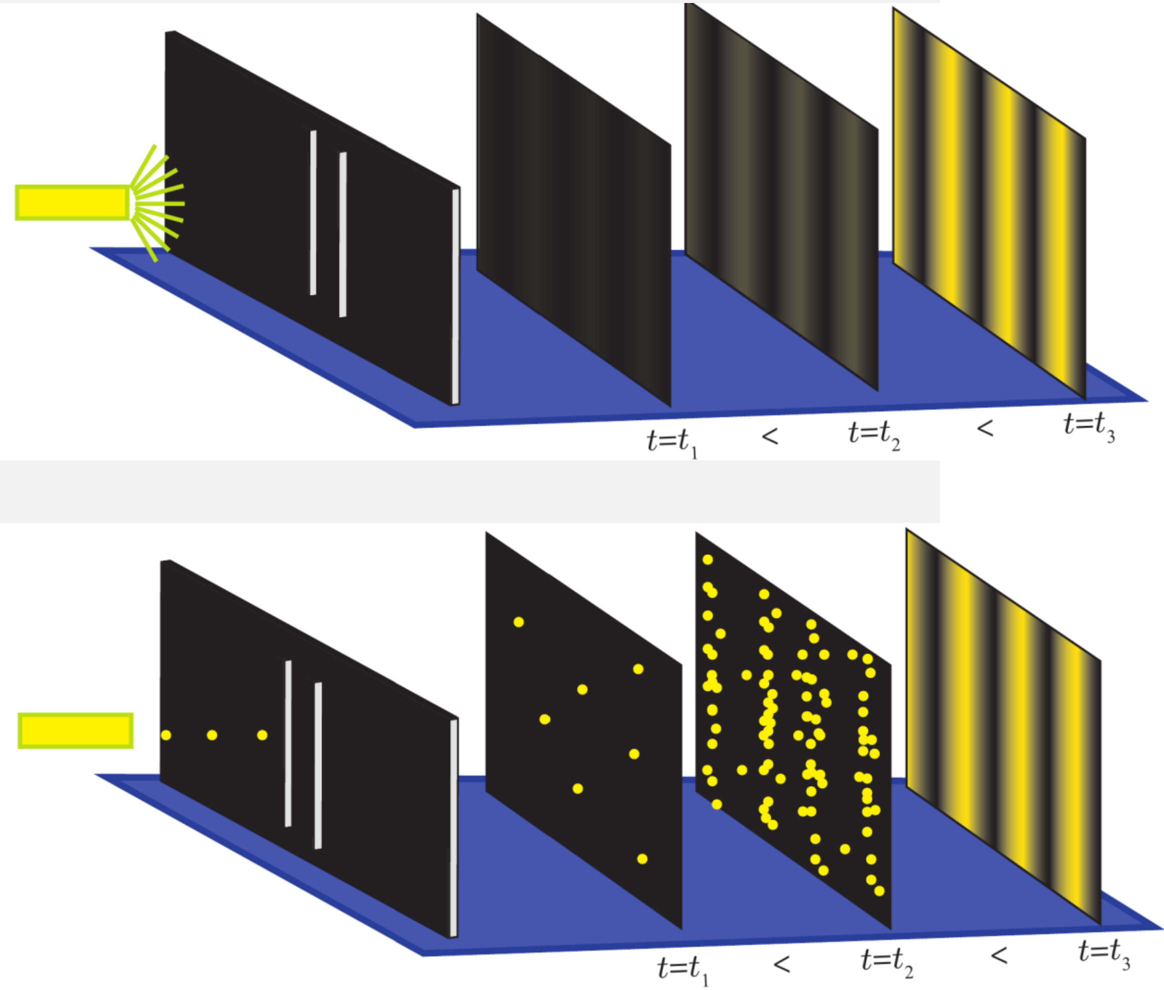
CORRELATION FUNCTION

Coherence is the degree of order in a random field.

- In the case of random fields, interference occurs to the extent that the fields at two different space-time points are mutually correlated.

- Interference is a periodic variation of the intensity as a function of a parameter. What conditions? (coherent, indistinguishable)

- The nature and the degree of this correlation is described through what is known as a correlation function.



Coherence and Quantum Entanglement lectures.
Anand Kumar Jha

CORRELATION FUNCTION

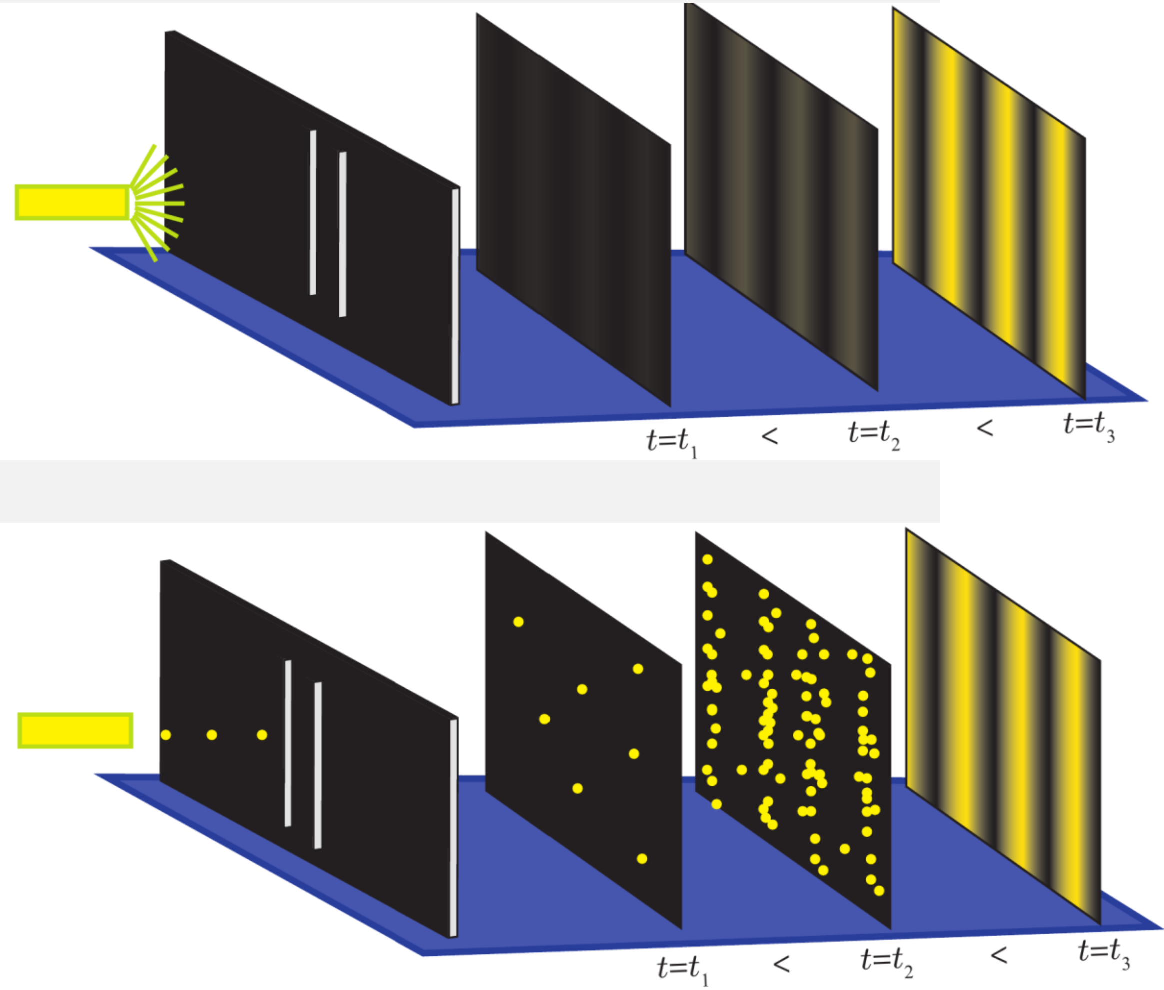
Coherence is the degree of order in a random field.

Two interfering field-amplitudes in the classical theory and the two interfering wave-functions in the quantum theory need to be mutually coherent for the interference to take place.

Physical amplitude → Probability amplitude

Types of light interference

- Spatial
- Polarization
- Temporal



Coherence and Quantum Entanglement lectures.
Anand Kumar Jha

QUANTIFYING TEMPORAL CORRELATIONS

Analogy with classical mechanics

Classical

$$V(\mathbf{r},t) = k_1 V(\mathbf{r_0}, t-t_1) + k_2 V(\mathbf{r_0}, t-t_2).$$

$$I(\mathbf{r},t) = \langle V^*(\mathbf{r},t) V(\mathbf{r},t)\rangle$$

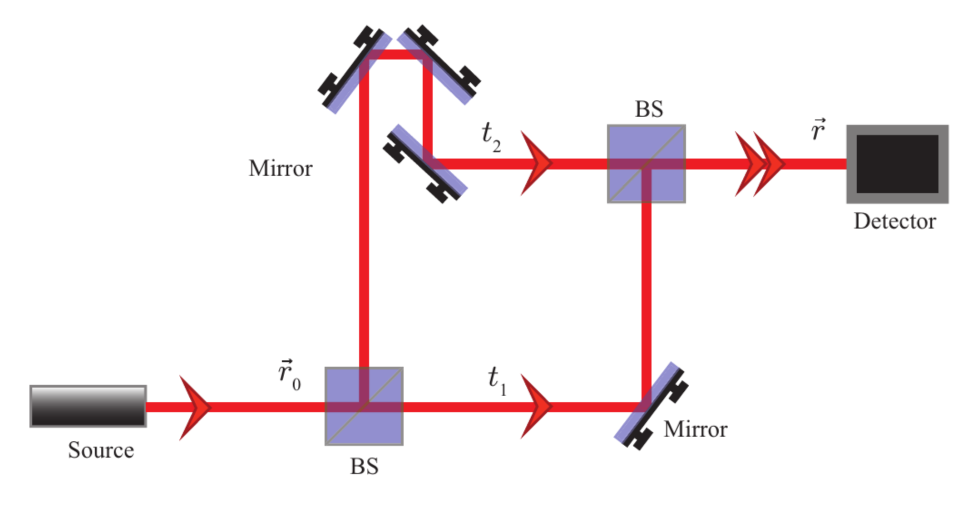
$$= |k_1|^2 I(t-t_1) + |k_2|^2 I(t-t_2) + k_1^* k_2 \Gamma(t-t_1, t-t_2) + \text{c.c.}$$

Quantum

$$\hat{E}^{(+)}(\mathbf{r},t) = k_1 \hat{E}^{(+)}(\mathbf{r_0}, t-t_1) + k_2 \hat{E}^{(+)}(\mathbf{r_0}, t-t_2)$$

$$P(\mathbf{r},t) = \langle\psi|\hat{E}^{(-)}(\mathbf{r},t)\hat{E}^{(+)}(\mathbf{r},t)|\psi\rangle$$

$$= |k_1|^2 P(\mathbf{r}_0, t-t_1) + |k_2|^2 P(\mathbf{r}_0, t-t_2) + 2|k_1||k_2| G^{(1)}(\mathbf{r}_0, t-t_1; \mathbf{r}_0, t-t_2)$$



Coherence and Quantum Entanglement lectures.
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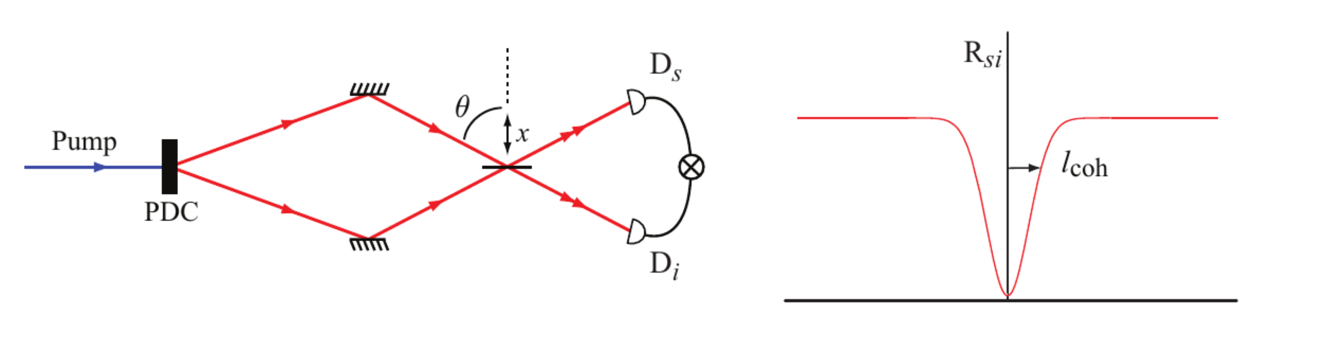
Γ, $G^{(1)}$ $\rightarrow$ Correlation function

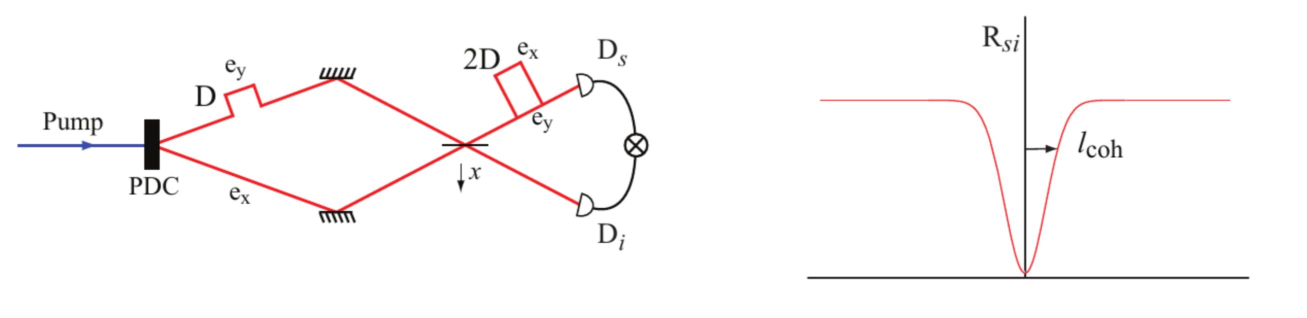
FAMOUS TWO-PHOTON INTERFERENCE

Hong-Ou-Mandel effect



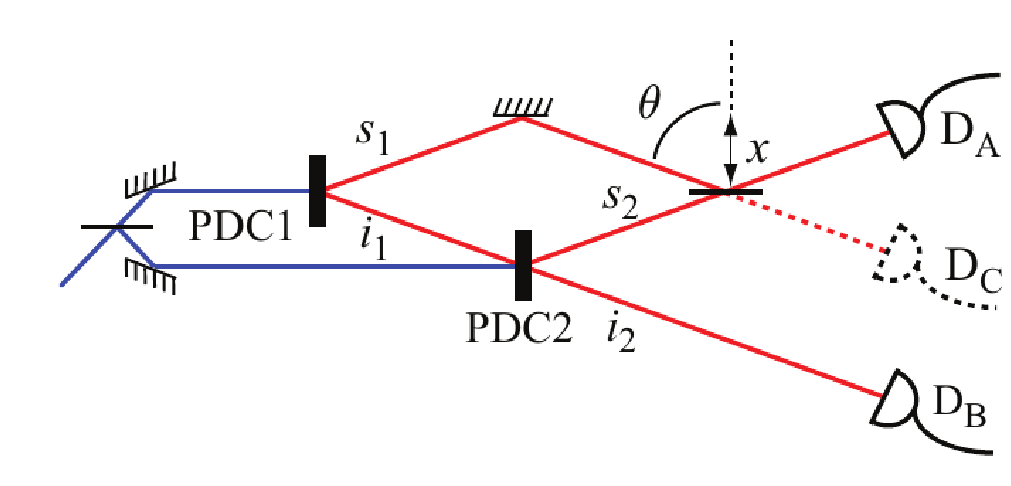
PRL 59, 2044 (1987)

Postponed compensation



PRL 77, 1917 (1996)

Induced coherence



PRL 67, 318 (1991)

IMPORTANCE OF TWO-PHOTON INTERFERENCE

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Quantum Spectroscopy and sensing

- Enables super-resolution imaging & Lidar.

  LSA 13:163 (2024)

- Used in astronomical measurements, direct imaging of black hole accretion discs. OE 31, 26, 44246 (2023)

Applications in OCT & LiDAR

- Enhances quantum optical coherence tomography.

  PRR 5, 023170 (2023)

- Improves depth sensing in Quantum LiDAR.

  PRL 131, 033603 (2023)

Quantum Information & Computing

- Key for entanglement & Linear Optical Quantum Computing (LOQC).

  JH APL 25, 2 (2004)

- Essential for Boson Sampling

  PRA 104, 032204 (2021)

Quantum Communication & Cryptography

- Secures Quantum Key Distribution (QKD).

  arXiv 2411.07884 (2024)

- Fundamental for quantum teleportation & Bell tests.

  OE 15, 16, 10188 (2007)

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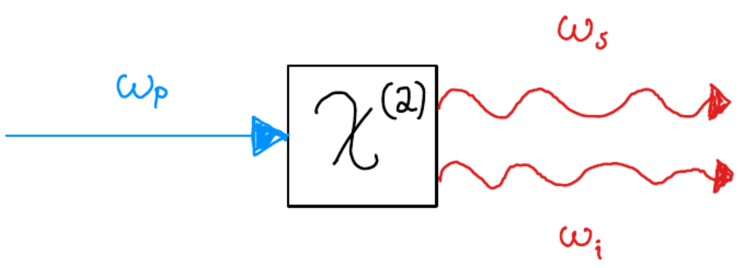
ENTANGLED TWO-PHOTON STATE

Spontaneous Parametric Down Conversion

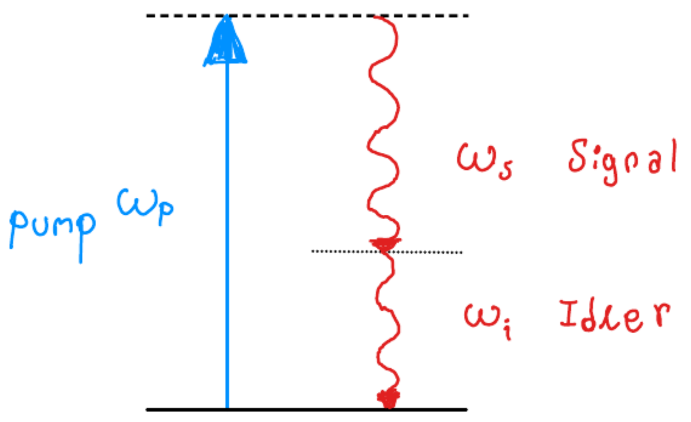
SPDC

Process able to produce two-photon entangled states

- Inherently random
- Due to second-order effects in a non-linear environment
- Generated photon pairs can be entangled in various degrees of freedom



$$\hat{H}(t) = \epsilon_0 \int_{-L}^{0} dz\, \chi^{(2)} \hat{E}_p^{(+)}(z,t) \hat{E}_s^{(-)}(z,t) \hat{E}_i^{(-)}(z,t) + \text{H.c.}$$



$$\hat{E}_p^{(+)}(z,t) = \int_0^\infty A_p d\omega_p V(\omega_p) e^{i[k_{pz}(\omega_p)z - \omega_p t]} e^{i(\omega_p \tau_p + \phi_p)},$$

$$\hat{E}_s^{(-)}(z,t) = \int_0^\infty A_s^* d\omega_s \hat{a}_s^\dagger(\omega_s) e^{i[\omega_s t - k_{sz}(\omega_s)z]}.$$

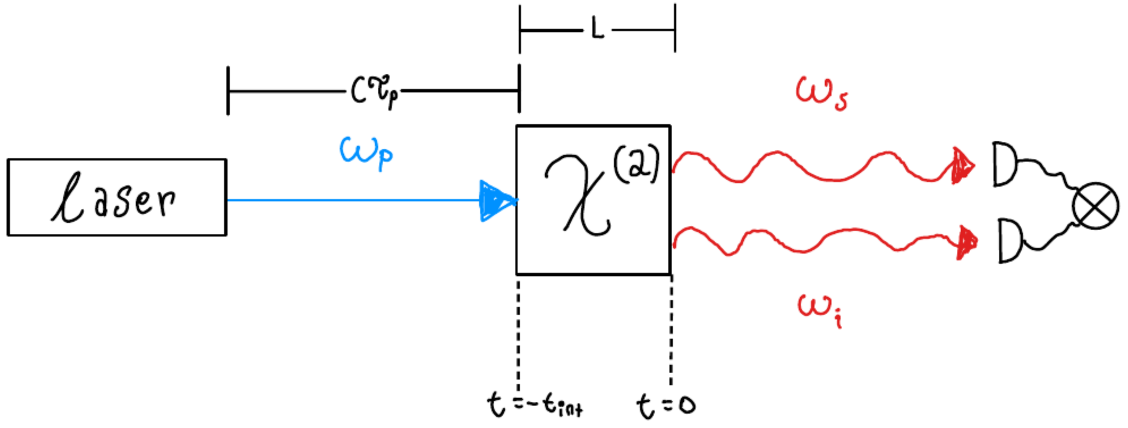
TIME EVOLUTION

Obtaining an entangled state

$$\hat{H}(t) = \epsilon_0 \int_{-L}^{0} dz\, \chi^{(2)} \hat{E}_p^{(+)}(z,t) \hat{E}_s^{(-)}(z,t) \hat{E}_i^{(-)}(z,t) + \text{H.c.}$$

$$|\psi(-t_{\text{int}})\rangle = |\text{vac}\rangle_s |\text{vac}\rangle_i$$

$$|\psi(0)\rangle = \exp\left[\frac{1}{i\hbar} \int_{-t_{\text{int}}}^{0} dt\, \hat{H}(t)\right] |\psi(-t_{\text{int}})\rangle$$



$$|\psi_{t_p}\rangle = A \iint_0^{\infty} d\omega_s d\omega_i\, V(\omega_s + \omega_i) \Phi(\omega_s, \omega_i) e^{i[(\omega_s+\omega_i)\tau_p + \phi_p]} |\omega_s\rangle_s |\omega_i\rangle_i \,,$$

$$\Phi(\omega_s, \omega_i) = \int_{-L}^{0} dz\, e^{i[k_{pz}(\omega_p) - k_{sz}(\omega_s) - k_{iz}(\omega_i)]z}.$$

ENTANGLED STATES

Phase matching function and entanglement

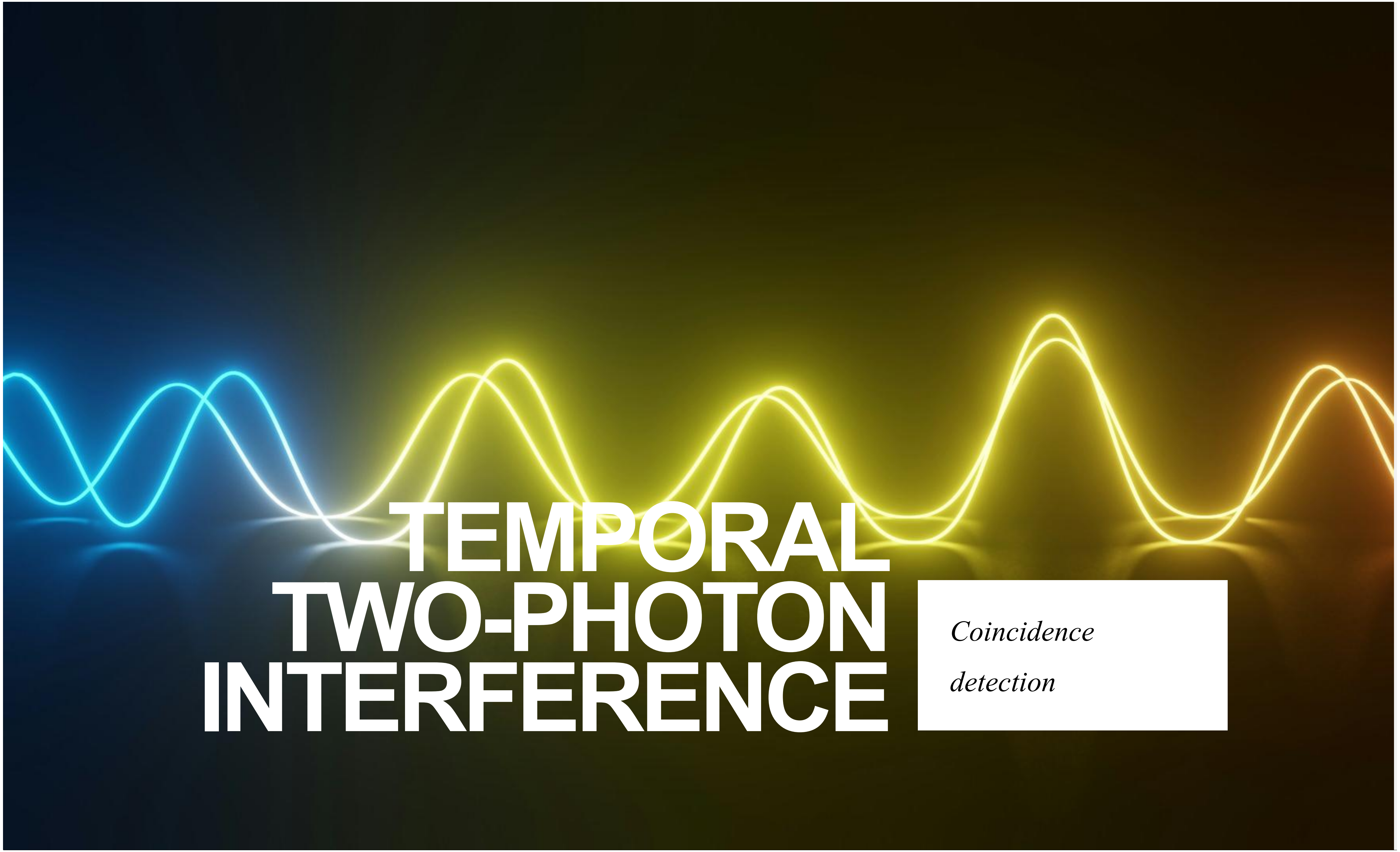
$$\left|\psi_{t_p}\right\rangle = A \iint_0^\infty d\omega_s d\omega_i \, V(\omega_s + \omega_i)\Phi(\omega_s, \omega_i) e^{i[(\omega_s+\omega_i)\tau_p + \phi_p]} \left|\omega_s\right\rangle_s \left|\omega_i\right\rangle_i$$

$$\omega_p = \omega_s + \omega_i \quad \text{and} \quad \omega_d = \frac{\omega_s - \omega_i}{2} \quad \text{such that} \quad d\omega_s d\omega_i \to d\omega_p d\omega_d.$$

$$\left|\psi_{t_p}\right\rangle = A \iint_0^\infty d\omega_p d\omega_d V(\omega_p)\Phi\left(\frac{\omega_p}{2} + \omega_d, \frac{\omega_p}{2} - \omega_d\right) e^{i(\omega_p\tau_p + \phi_p)} \left|\frac{\omega_p}{2} + \omega_d\right\rangle_s \left|\frac{\omega_p}{2} - \omega_d\right\rangle_i$$

$$V(\omega_p) = V_0\delta(\omega_0 - \omega_p)$$

$$\left|\psi_{t_p}\right\rangle = AV_0 e^{i(\omega_0\tau_p + \phi_p)} \int_0^\infty d\omega_d \Phi(\omega_d) \left|\frac{\omega_0}{2} + \omega_d\right\rangle_s \left|\frac{\omega_0}{2} - \omega_d\right\rangle_i$$



TEMPORAL TWO-PHOTON INTERFERENCE

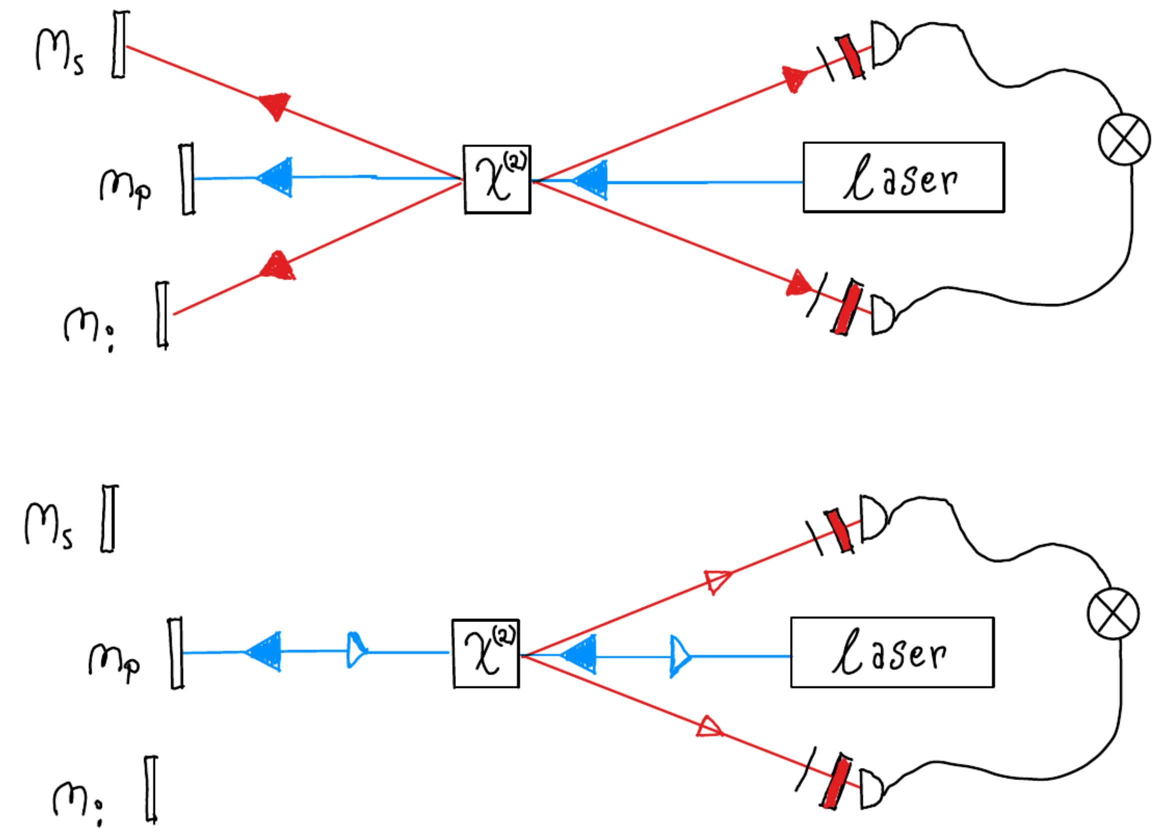
Coincidence detection

SUPERPOSITION OF TWO-PHOTON STATES

Interference of two possibilities

$$|\psi_{tp\mathbf{a}}\rangle = A \iint_0^\infty d\omega_p d\omega_d V_{\mathbf{a}}(\omega_p)\Phi_{\mathbf{a}}\left(\frac{\omega_p}{2}+\omega_d, \frac{\omega_p}{2}-\omega_d\right) e^{i(\omega_p\tau_p+\phi_p)} \left|\frac{\omega_p}{2}+\omega_d\right\rangle_{s\mathbf{a}} \left|\frac{\omega_p}{2}-\omega_d\right\rangle_{i\mathbf{a}}$$

$$|\psi\rangle = \sum_{\mathbf{a}=1}^{2} |\psi_{tp\mathbf{a}}\rangle = |\psi_{tp1}\rangle + |\psi_{tp2}\rangle \,.$$



- Perfect spatial coherence
- Quasi-monochromatic approximation

$$\omega_0 >> \Delta\omega$$

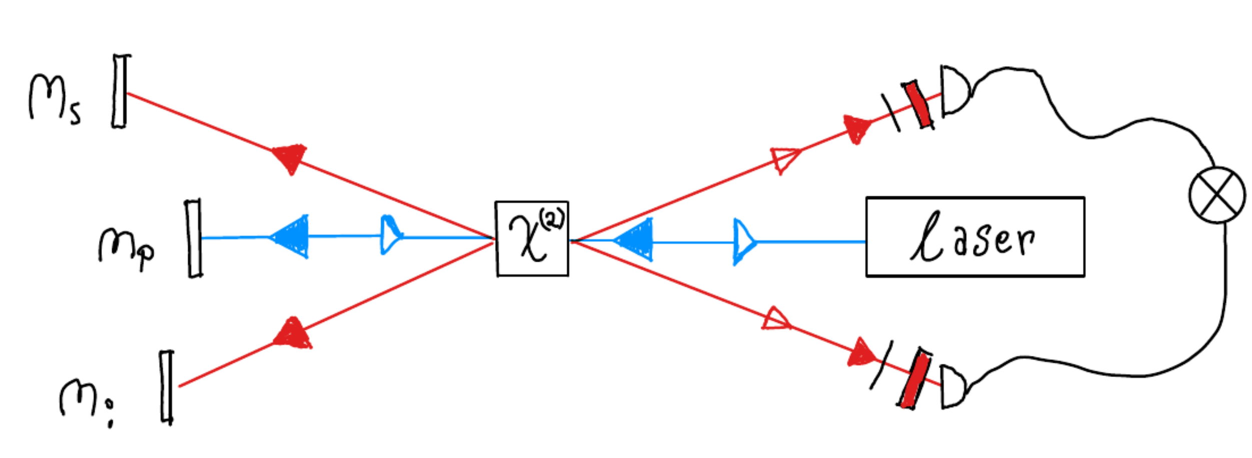
FIELD AT THE DETECTORS

Field operators

$$R_{si}(t,t+\tau)=\alpha_s\alpha_i\,\langle\psi|\hat{E}_s^{(-)}(t)\hat{E}_i^{(-)}(t+\tau)\hat{E}_i^{(+)}(t+\tau)\hat{E}_s^{(+)}(t)|\psi\rangle$$

- α_i,α_s Quantum efficiency off detectors
- Amplitude filters f

$$|\psi\rangle=|\psi_{tp1}\rangle+|\psi_{tp2}\rangle$$



$$\hat{E}_s^{(+)}(t)=\hat{E}_{s1}^{(+)}(t-\tau_{s1})+\hat{E}_{s2}^{(+)}(t-\tau_{s2})$$

$$=c_{s1}e^{i\phi_{s1}}\int_0^\infty d\omega_s f_s(\omega_s-\omega_{s0})e^{-i\omega_s(t-\tau_{s1})}\hat{a}_{s1}(\omega_s)$$

$$+c_{s2}e^{i\phi_{s2}}\int_0^\infty d\omega_s f_s(\omega_s-\omega_{s0})e^{-i\omega_s(t-\tau_{s2})}\hat{a}_{s2}(\omega_s).$$

COINCIDENCES DETECTION

What is the probability to detect a coincidence in a given window of time?

$$R_{si}(t,t+\tau) = \alpha_s\alpha_i \langle\psi|\hat{E}_s^{(-)}(t)\hat{E}_i^{(-)}(t+\tau)\hat{E}_i^{(+)}(t+\tau)\hat{E}_s^{(+)}(t)|\psi\rangle \rightarrow \text{16 terms}$$

$$R_{si}(t,t+\tau) = \alpha_s\alpha_i \langle\psi_{tp1}|\,\hat{E}_{s1}^{(-)}\hat{E}_{i1}^{(-)}\hat{E}_{i1}^{(+)}\hat{E}_{s1}^{(+)}\,|\psi_{tp1}\rangle + \alpha_s\alpha_i \langle\psi_{tp2}|\,\hat{E}_{s2}^{(-)}\hat{E}_{i2}^{(-)}\hat{E}_{i2}^{(+)}\hat{E}_{s1}^{(+)}\,|\psi_{tp2}\rangle$$

$$+\alpha_s\alpha_i \langle\psi_{tp1}|\,\hat{E}_{s1}^{(-)}\hat{E}_{i1}^{(-)}\hat{E}_{i2}^{(+)}\hat{E}_{s2}^{(+)}\,|\psi_{tp2}\rangle + \text{H.c.}$$

$$\equiv R_{si}^{11}(t,t+\tau) + R_{si}^{22}(t,t+\tau) + R_{si}^{12}(t,t+\tau) + \text{H.c.}$$

$$\hat{E}_{s\mathbf{a}}^{(\pm)} = \hat{E}_{s\mathbf{a}}^{(\pm)}(t-\tau_{s2})$$

$$\hat{E}_{i\mathbf{a}}^{(\pm)} = \hat{E}_{i\mathbf{a}}^{(\pm)}(t+\tau-\tau_{i2})$$

FOURTH-POINT CORRELATION FUNCTION

Finding correlation

$$\hat{E}_{i2}^{(+)}(t+\tau-\tau_{i2})\hat{E}_{s2}^{(+)}(t-\tau_{s2})\left|\psi_{tp2}\right\rangle = Ac_2 e^{i\phi_2} e^{-i\omega_0(t+\frac{\tau}{2}-\tau_2)} e^{i\omega_{d0}(\tau+\tau_2')} \times v_2\left(t+\frac{\tau}{2}-\tau_2\right) g_2^*(\tau+\tau_2')\left|\text{vac}\right\rangle_{s2}\left|\text{vac}\right\rangle_{i2}$$

$$\tau_2 = \tau_{p2} + \frac{\tau_{s2}+\tau_{i2}}{2}, \quad \text{and} \quad \tau_2' = \tau_{s2}-\tau_{i2}$$

$$g_2^*(t) \equiv \int_{-\infty}^{\infty} d\omega_d' \Phi_2(\omega_d'+\omega_{s0}, -\omega_d'+\omega_{i0}) f_s(\omega_d') f_i(-\omega_d') e^{-i\omega_d' t} \qquad v_2(t) \equiv \int_{-\infty}^{\infty} d\omega_p' V_2(\omega_p'+\omega_0) e^{-i\omega_p' t}$$

$$\omega_p = \omega_p' + \omega_0 \quad \text{and} \quad \omega_d = \omega_d' + \omega_{d0}, \quad \text{where} \quad \omega_{d0} = \frac{\omega_{s0}-\omega_{i0}}{2}$$

$$R_{si}^{12}(t,t+\tau) = Kc_1^*c_2\left\langle e^{-i(\omega_0\Delta\tau+\omega_{d0}\Delta\tau'+\Delta\phi)} v_1^*\left(t+\frac{\tau}{2}-\tau_1\right) v_2\left(t+\frac{\tau}{2}-\tau_2\right) g_1(\tau+\tau_1') g_2^*(\tau+\tau_2')\right\rangle_{t,\tau}$$

FOURTH-POINT CORRELATION FUNCTION

Finding correlation

Detection of coincidences

$$R_{si}^{12}(t,t+\tau) = Kc_1^* c_2 \left\langle e^{-i(\omega_0 \Delta\tau + \omega_{d0}\Delta\tau' + \Delta\phi)} v_1^* \left(t + \frac{\tau}{2} - \tau_1\right) v_2 \left(t + \frac{\tau}{2} - \tau_2\right) g_1(\tau + \tau_1') g_2^*(\tau + \tau_2') \right\rangle_{t,\tau}$$

$$R_{si}^{12} = Kc_1^* c_2 e^{-i(\omega_0 \Delta\tau + \omega_{d0}\Delta\tau' + \Delta\phi)} \sqrt{|v_1|^2 |v_2|^2} \sqrt{|g_1|^2 |g_2|^2} \gamma(\Delta\tau)\gamma(\Delta\tau')$$

$$R_{si} = R_1 + R_2 + 2\sqrt{R_1 R_2}\, \gamma(\Delta\tau)\gamma'(\Delta\tau') \cos(\omega_0 \Delta\tau + \omega_{d0}\Delta\tau' + \Delta\phi)$$

$$R_1 = K|c_1 g_1 v_1|^2 \quad \text{and} \quad R_2 = K|c_2 g_2 v_2|^2$$

$$R_{si} = R_1 + R_2 + 2\sqrt{R_1 R_2}\, \gamma(\Delta L)\gamma'(\Delta L') \cos(k_0 \Delta L + k_{d0}\Delta L' + \Delta\phi)$$

FOURTH-POINT CORRELATION FUNCTION

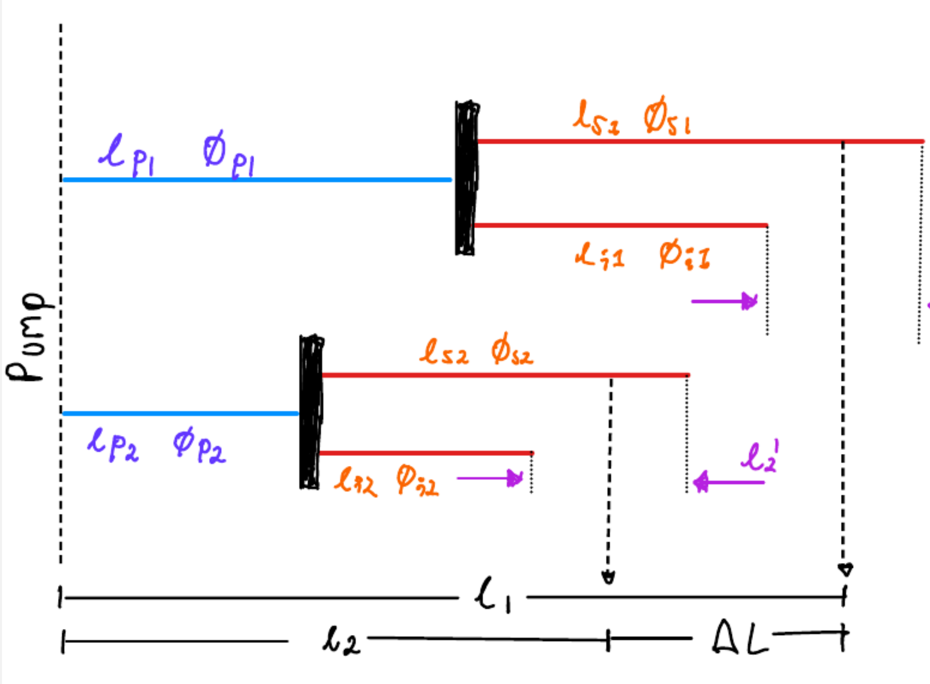
Defining geometrical parameters

$$R_{si} = R_1 + R_2 + 2\sqrt{R_1 R_2}\,\gamma(\Delta L)\gamma'(\Delta L')\cos(k_0 \Delta L + k_{d0}\Delta L' + \Delta\phi)$$

$$\Delta L \equiv l_1 - l_2 = \left(\frac{l_{s1} + l_{i1}}{2} + l_{p1}\right) - \left(\frac{l_{s2} + l_{i2}}{2} + l_{p2}\right)$$

$$\Delta L' \equiv l_1' - l_2' = (l_{s1} - l_{i1}) - (l_{s2} - l_{i2})$$

$$\Delta\phi \equiv \phi_1 - \phi_2 = (\phi_{s1} + \phi_{i1} + \phi_{p1}) - (\phi_{s2} + \phi_{i2} + \phi_{p2})$$



GAUSSIAN-SHAPE CASE

Shape of interference pattern

ASSUMING A GAUSSIAN SPECTRAL DISTRIBUTION

$$R_{si} = R_1 + R_2 + 2\sqrt{R_1 R_2}\,\gamma(\Delta L)\gamma'(\Delta L')\cos(k_0 \Delta L + k_{d0}\Delta L' + \Delta\phi)$$

$$k_0 = \frac{\omega_0}{c} \quad \text{and} \quad k_{d0} = \frac{\omega_{d0}}{c} \rightarrow \omega_s = \omega_i$$

$$R_{si} = C[1 + \gamma'(\Delta L')\gamma(\Delta L)\cos(k_0 \Delta L + \Delta\phi)].$$

- when the pump is a stationary (continuous-wave) field having a Gaussian spectrum of rms frequency width $\Delta\omega p$, the time-averaged degree of correlation of the pump field

$$\gamma(\Delta L) = \exp\left[-\frac{1}{2}\left(\frac{\Delta L}{l^p_{\mathrm{coh}}}\right)^2\right]. \qquad l^p_{\mathrm{coh}} = \frac{c}{\Delta\omega_p}$$

- In situations in which the signal-idler field has a Gaussian spectrum of width $\Delta\omega$,

$$\gamma'(\Delta L') = \exp\left[-\frac{1}{2}\left(\frac{\Delta L'}{l_{\mathrm{coh}}}\right)^2\right], \qquad l_{\mathrm{coh}} = \frac{c}{\Delta\omega}$$

$$\boxed{\begin{array}{c}\Delta L \ll l^p_{\mathrm{coh}} \\ \Delta L' \ll l_{\mathrm{coh}}\end{array}}$$

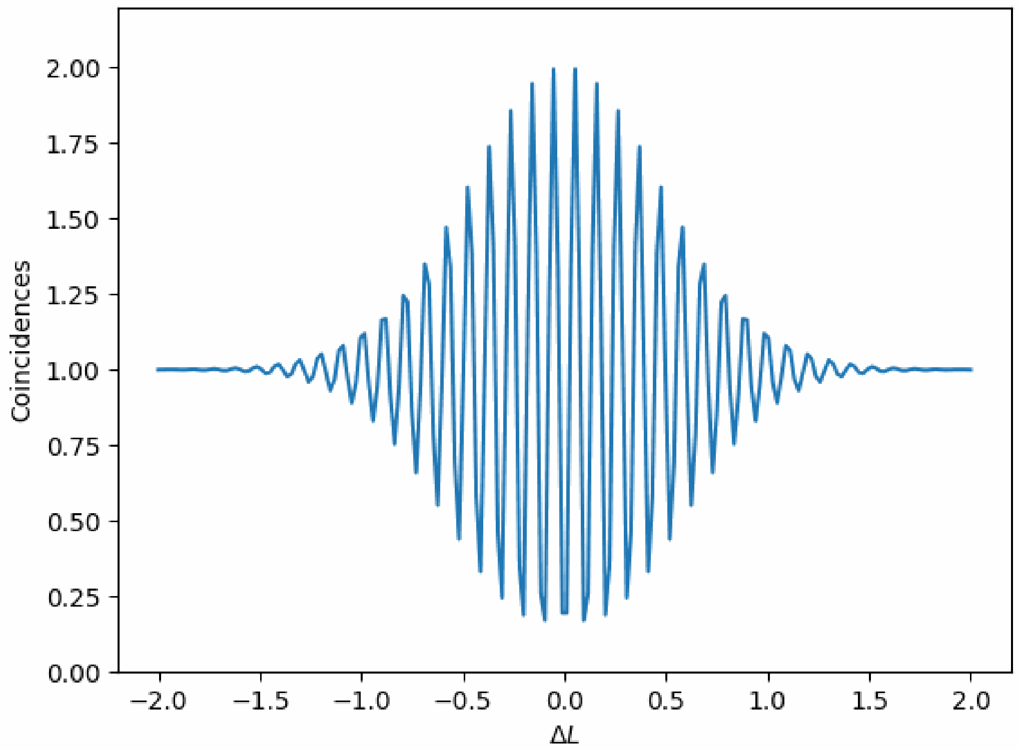
LIMITING CASES

$$\Delta L' = 0 \qquad \Delta\phi = 0$$

$$R_{si} = C\left[1 + \gamma(\Delta L)\cos(k_0 \Delta L)\right].$$

$$\gamma(\Delta L) = \exp\left[-\frac{1}{2}\left(\frac{\Delta L}{l^p_{\mathrm{coh}}}\right)^2\right].$$

$$\boxed{\begin{array}{c} \Delta L \ll l^p_{\mathrm{coh}} \\ \Delta L' \ll l_{\mathrm{coh}} \end{array}}$$



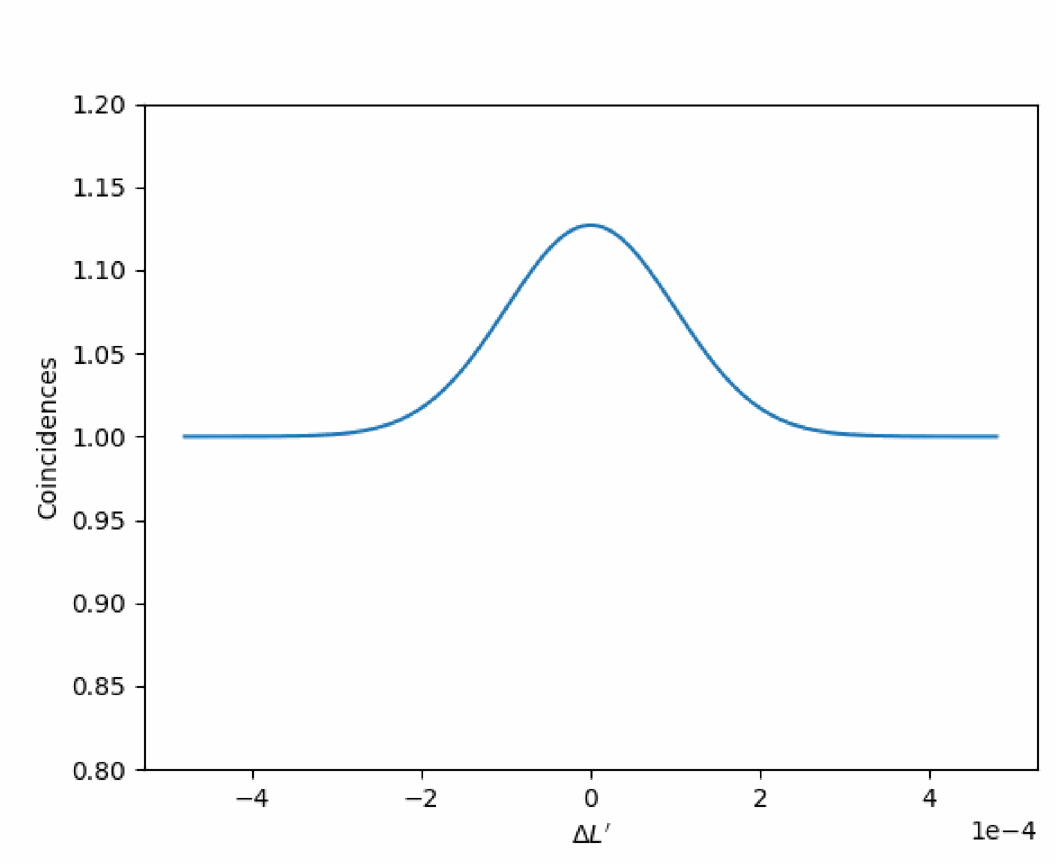
LIMITING CASES

ΔL $\Delta\phi$ Fixed

$$R_{si} = C\left[1 + K\gamma'(\Delta L')\right]$$

$$K = \gamma(\Delta L)\cos(k_0\Delta L + \Delta\phi)$$

- The coincidence count rate can show a dip when the two alternatives interfere destructively (K < 0)
- Hump when the two alternatives interfere constructively (K > 0)



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LIMITING CASES

ΔL $\Delta\phi$ Fixed

$$R_{si} = C\left[1 + K\gamma'(\Delta L')\right]$$

$$K = \gamma(\Delta L)\cos(k_0\Delta L + \Delta\phi)$$

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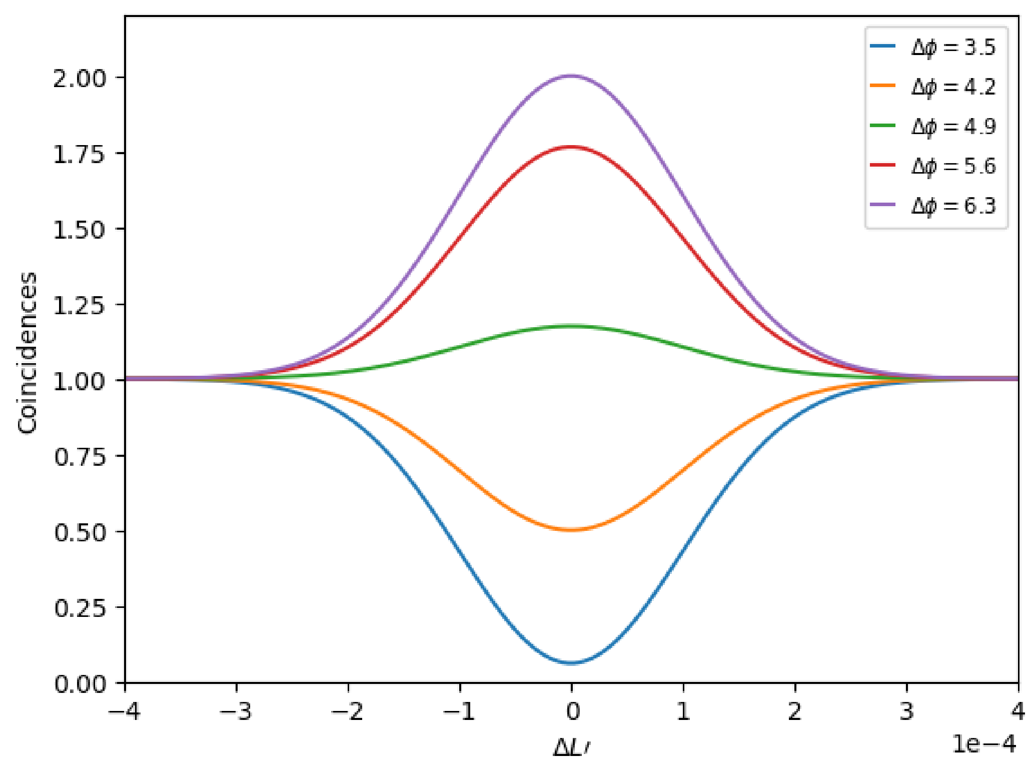

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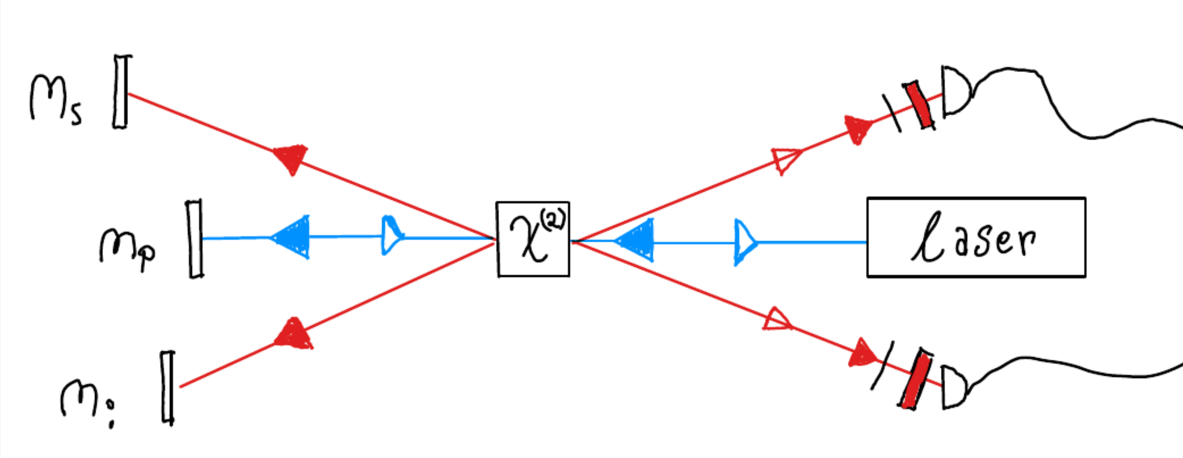
CONCLUSIONS

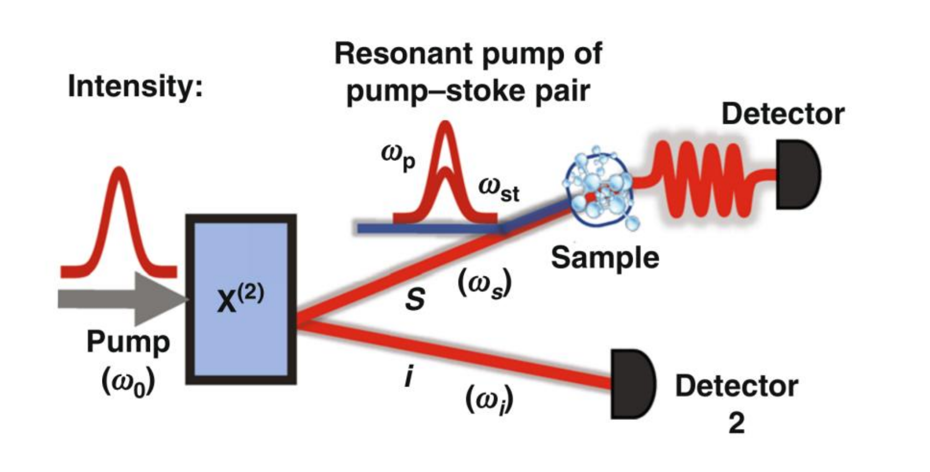
- Interference is a group effect due to interference of individual photon (two-photon).
- photon interference arises from the indistinguishability of quantum states and the superposition of probability amplitudes.
- With gaussian-shape any two-photon interference effect is a superposition of the two limiting cases presented.
- Spontaneous Parametric Down-Conversion (SPDC) is a crucial process for generating biphoton states with tunable spectral and spatial properties.
- This theory works for many different interferometer structure.
- Calculation of quantum correlations is not that simple.



PERSPECTIVES

spectroscopy

Quantum CARS



$$V(t)=\sum_{j=1}^{N}\sum_{b}\alpha_{bg}^{(j)}(|b\rangle\langle g|)_j E_s(t)E_s^{\dagger}(t)e^{i(\omega_b-\omega_g)t}+h.c.$$

Spectral and time resolution at same time

Zhang, Z., Peng, T., Nie, X. *et al.* Entangled photons enabled time-frequency-resolved coherent Raman spectroscopy and applications to electronic coherences at femtosecond scale. *Light Sci Appl* **11**, 274 (2022).

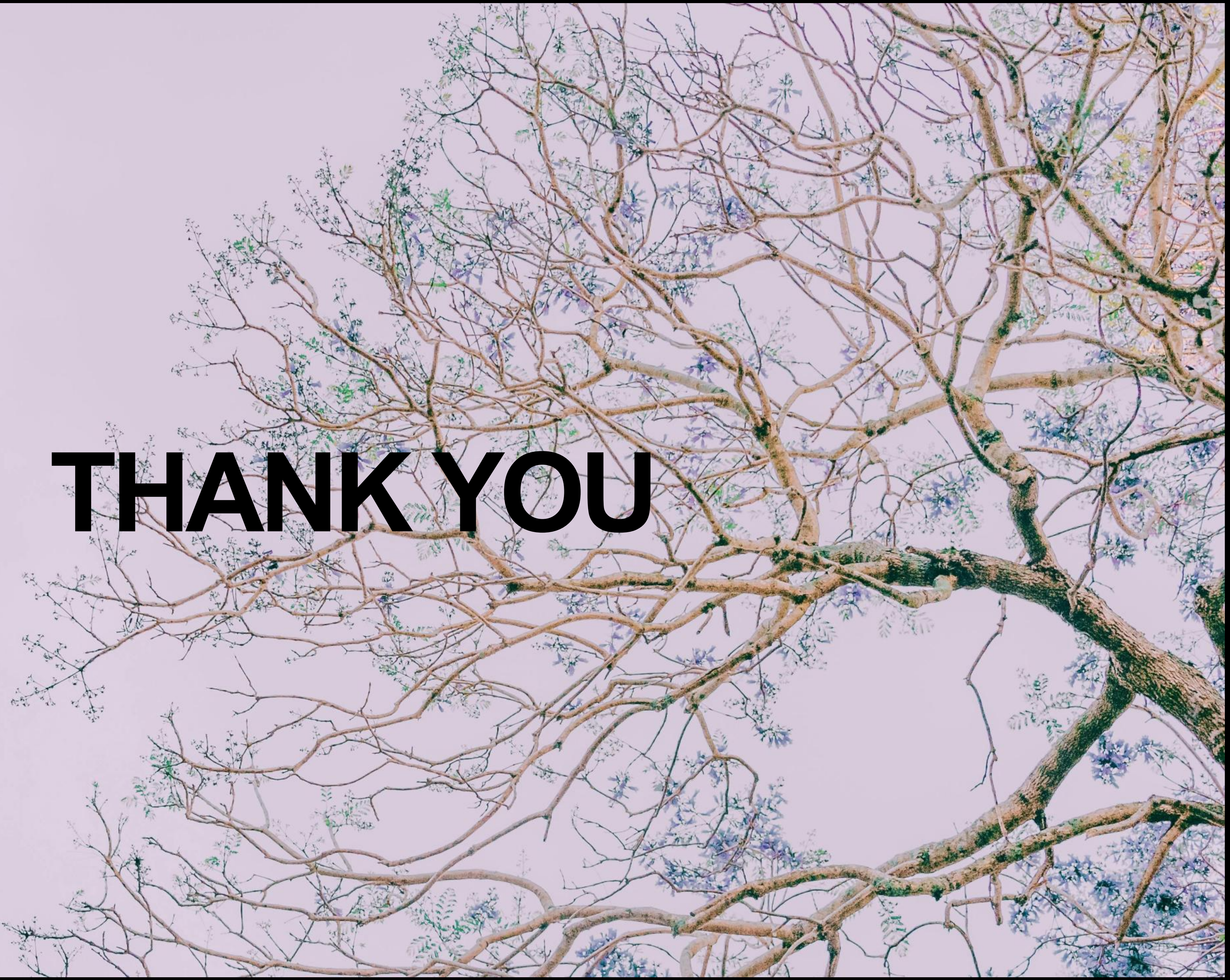
Absorption spectroscopy with quantum light with arbitrary spectral shape

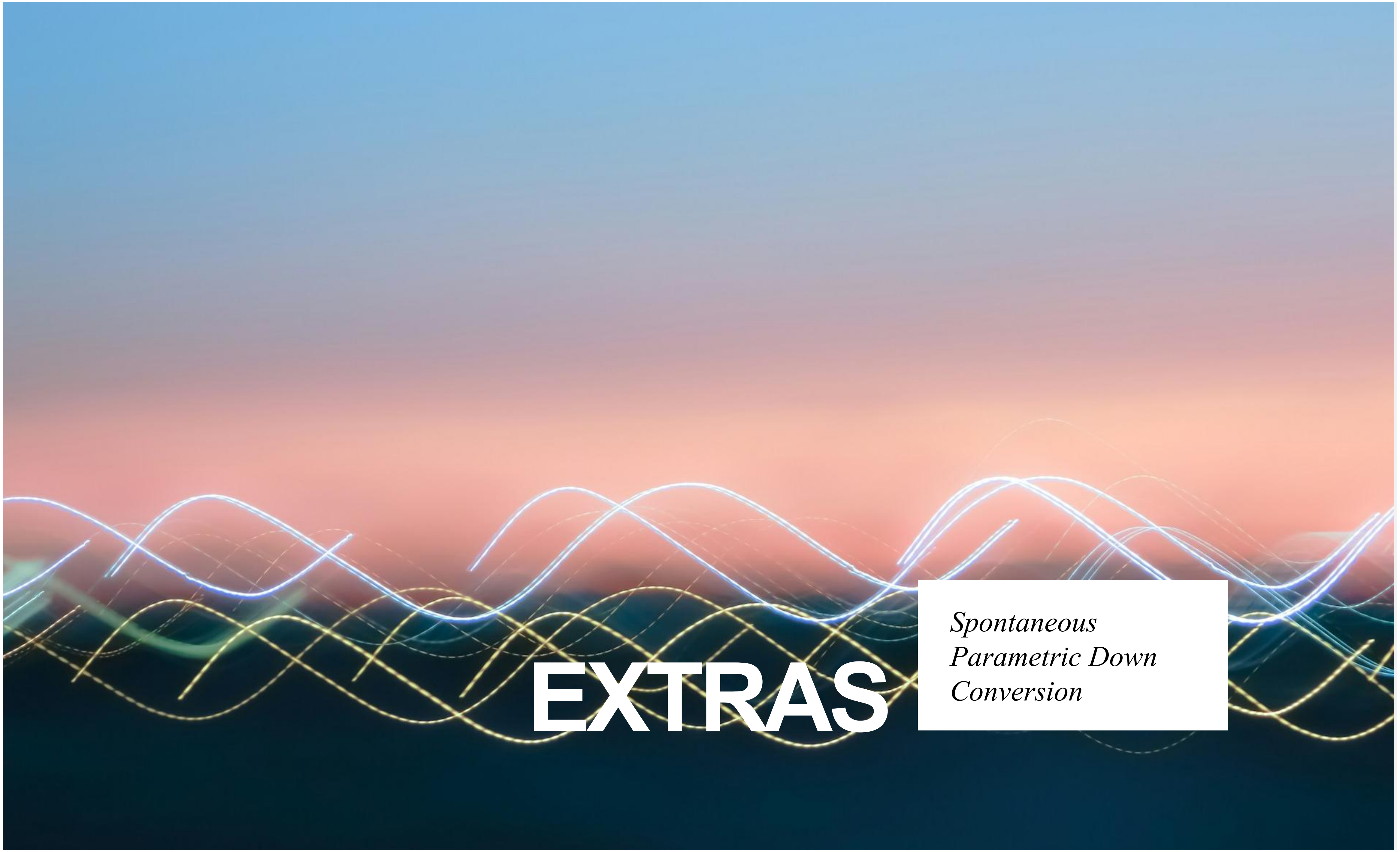
Study two scheme with and without interference

- Advanced optical filtering technology at the telecom frequency can be leveraged to analyse absorption at different frequencies,
- The research includes theoretical analysis in the Heisenberg picture, stablishing high-resolution spectroscopy using quantum interference.

Padilla Camargo, A. A. (2023). Absorption Spectroscopy with Quantum Light: Experimental and Theoretical Simulations Based on Arbitrary Spectral Shapes Generated with a Programmable Filter (Master's thesis). Universitat Politècnica de Catalunya.

THANK YOU





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EXTRAS

Spontaneous Parametric Down Conversion

LIMITING CASES

Where the biphoton comes from

Company Name

- Explicar la fisica
- Mensaje
- Enque trabajo y motivacion
- Hice recapitulacion, que he estudiado
- Perspectivas con interes

SPDC

Process that produces entangled two-photon field.

Electromagnetic field in a non-linear medium

$$P(\mathbf{r},t) = \epsilon_0\chi^{(1)}E(\mathbf{r},t) + \epsilon_0\chi^{(2)}E^2(\mathbf{r},t) + \cdots$$

$$D(\mathbf{r},t) = \epsilon_0 E(\mathbf{r},t) + P(\mathbf{r},t)$$

$$D(\mathbf{r},t)\cdot E(\mathbf{r},t) = \left[\epsilon_0 E(\mathbf{r},t) + \epsilon_0\chi^{(1)}E(\mathbf{r},t)\right]\cdot E(\mathbf{r},t) + \epsilon_0\chi^{(2)}E^2(\mathbf{r},t)\cdot E(\mathbf{r},t)\cdots$$

- Nulla a erat eget nunc hendrerit ultrices eu nec nulla. Donec viverra leo aliquet, auctor quam id, convallis orci.
  - Sed in molestie est. Cras ornare turpis at ligula posuere, sit amet accumsan neque lobortis.
  - Maecenas mattis risus ligula, sed ullamcorper nunc efficitur sed.

Ambiente
Motivacion
Interference

SPDC

Process that produces entangled two-photon field.

Electromagnetic field in a non-linear medium

$$D(\mathbf{r},t)\cdot E(\mathbf{r},t) = \left[\epsilon_0 E(\mathbf{r},t) + \epsilon_0 \chi^{(1)} E(\mathbf{r},t)\right]\cdot E(\mathbf{r},t) + \epsilon_0 \chi^{(2)} E^2(\mathbf{r},t)\cdot E(\mathbf{r},t)\cdots$$

$$H(t) = \int_V P^{(2)}(\mathbf{r},t) E(\mathbf{r},t)\, d^3r \; = \epsilon_0 \int_V \chi^{(2)} E_p(\mathbf{r},t) E_s(\mathbf{r},t) E_i(\mathbf{r},t)\, d^3r$$

$$\hat{H}(t) = \epsilon_0 \int_V \chi^{(2)} \hat{E}_p(\mathbf{r},t)\hat{E}_s(\mathbf{r},t)\hat{E}_i(\mathbf{r},t)\, d^3r$$

$$\hat{E}_p(\mathbf{r},t) = \hat{E}_p^{(+)}(\mathbf{r},t) + \hat{E}_p^{(-)}(\mathbf{r},t),$$

$$\hat{E}^{(+)}(\mathbf{r},t) = \int A_{\mathbf{k}} \hat{a}_{\mathbf{k}}(t) e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)} d^3k.$$

$$\hat{H}(t) = \epsilon_0 \int_V d^3r\, \chi^{(2)} \hat{E}_p^{(+)}(\mathbf{r},t)\hat{E}_s^{(-)}(\mathbf{r},t)\hat{E}_i^{(-)}(\mathbf{r},t) + \mathrm{H.c.}$$

SPDC

Process that produces entangled two-photon field.

Field In detectors

$$
\begin{aligned}
R_{si}(t,t+\tau) = \alpha_s\alpha_i \langle\psi_{tp1}|\, \hat{E}_{s1}^{(-)}(t-\tau_{s1})\hat{E}_{i1}^{(-)}(t+\tau-\tau_{i1})\hat{E}_{i1}^{(+)}(t+\tau-\tau_{i1})\hat{E}_{s1}^{(+)}(t-\tau_{s1})\,|\psi_{tp1}\rangle \\
+\alpha_s\alpha_i \langle\psi_{tp2}|\, \hat{E}_{s2}^{(-)}(t-\tau_{s2})\hat{E}_{i2}^{(-)}(t+\tau-\tau_{i2})\hat{E}_{i2}^{(+)}(t+\tau-\tau_{i2})\hat{E}_{s1}^{(+)}(t-\tau_{s1})\,|\psi_{tp2}\rangle \\
+\alpha_s\alpha_i \langle\psi_{tp1}|\, \hat{E}_{s1}^{(-)}(t-\tau_{s1})\hat{E}_{i1}^{(-)}(t+\tau-\tau_{i1})\hat{E}_{i2}^{(+)}(t+\tau-\tau_{i2})\hat{E}_{s2}^{(+)}(t-\tau_{s2})\,|\psi_{tp2}\rangle + \text{H.c.}
\end{aligned}
$$

COINCIDENCES DETECTION

Process that produces entangled two-photon field.

$$R_{si}(t,t+\tau)=\alpha_s\alpha_i\,\langle\psi|\hat{E}_s^{(-)}(t)\hat{E}_i^{(-)}(t+\tau)\hat{E}_i^{(+)}(t+\tau)\hat{E}_s^{(+)}(t)|\psi\rangle \rightarrow \textit{16 terms}$$

$$R_{si}(t,t+\tau)=\alpha_s\alpha_i\,\langle\psi|\,[\hat{E}_{s1}^{(-)}(t-\tau_{s1})+\hat{E}_{s2}^{(-)}(t-\tau_{s2})][\hat{E}_{i1}^{(-)}(t+\tau-\tau_{i1})+\hat{E}_{i2}^{(-)}(t+\tau-\tau_{i2})]$$

$$\times[\hat{E}_{i1}^{(+)}(t+\tau-\tau_{i1})+\hat{E}_{i2}^{(+)}(t+\tau-\tau_{i2})][\hat{E}_{s1}^{(+)}(t-\tau_{s1})+\hat{E}_{s2}^{(+)}(t-\tau_{s2})]\,|\psi\rangle$$

$$\equiv R_{si}^{11}(t,t+\tau)+R_{si}^{22}(t,t+\tau)+R_{si}^{12}(t,t+\tau)+\mathrm{H.c.}$$

SPDC

Process that produces entangled two-photon field.

Detection of coincidences

$$g_2^*(t) \equiv \int_{-\infty}^{\infty} d\omega_d' \Phi_2(\omega_d' + \omega_{s0}, -\omega_d' + \omega_{i0}) f_s(\omega_d') f_i(-\omega_d') e^{-i\omega_d' t}$$

$$v_2(t) \equiv \int_{-\infty}^{\infty} d\omega_p' V_2(\omega_p' + \omega_0) e^{-i\omega_p' t}$$

$$\hat{E}_{i2}^{(+)}(t+\tau-\tau_{i2}) \hat{E}_{s2}^{(+)}(t-\tau_{s2}) \left|\psi_{tp2}\right\rangle = A c_2 e^{i\phi_2} e^{-i\omega_0 (t+\frac{\tau}{2}-\tau_2)} e^{i\omega_{d0}(\tau+\tau_2')} \times v_2\left(t+\frac{\tau}{2}-\tau_2\right) g_2^*(\tau+\tau_2') \left|\text{vac}\right\rangle_{s2} \left|\text{vac}\right\rangle_{i2}$$

$$R_{si}^{12}(t, t+\tau) = K c_1^* c_2 \left\langle e^{-i(\omega_0 \Delta\tau + \omega_{d0}\Delta\tau' + \Delta\phi)} v_1^*\left(t+\frac{\tau}{2}-\tau_1\right) v_2\left(t+\frac{\tau}{2}-\tau_2\right) g_1(\tau+\tau_1') g_2^*(\tau+\tau_2') \right\rangle$$