Experiment 3: Electromagnetic Damping

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Objective

To study the effects of electromagnetic damping on a rotating metal disc.

Apparatus

Two bar magnets, Flywheel disc assembly mounted on the wall, Slotted masses, Meter scale, Tapes, Phone camera, Slate and chalk, Tracker software, Gaussmeter.

Part A

In this part, we determined the moment of inertia of the flywheel

Introduction

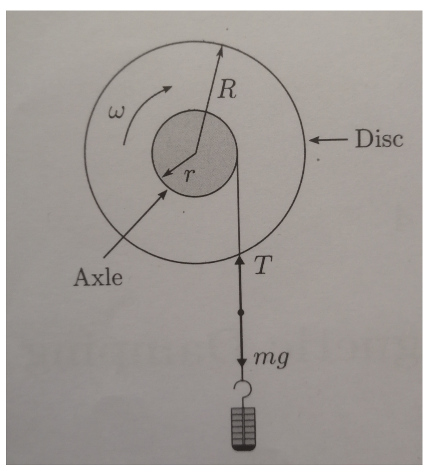

Figure 1: Free body diagram of the flywheel assembly: the disc accelerates due to the falling mass

Let us consider an aluminium disc mounted on a horizontal axle as shown in Figure 1. A cord is wound around the horizontal axle. A slotted mass m is attached to the cord, which starts to fall due to the effect of gravity, rotating the disc in the process.
Now, the torque acting on the disc,

$$\tau = I\alpha \tag{1}$$

Where, I is the moment of inertia of the rotating system and α is the angular acceleration of the disc.

From the free body diagram of the flywheel (Fig. 1), we can write,

$$ma = mg - t \implies T = m(g-a) \tag{2}$$

where, a is the acceleration of the slotted mass.
While rotating, the disc will experience a frictional torque τ_f
Now, the torque produced by the slotted mass on the disc is Tr', where r is the radius of the axle around which the string is wound.
Therefore, the total external torque on the object,

$$\tau = Tr - \tau_f \tag{3}$$

Using equations (1) and (2), we can write,

$$\begin{aligned} I &= mr(g-a) - \tau_f \\ \implies I\frac{a}{r} &= mrg - mra - \tau_f \\ \implies Ia &= mr^2g - mr^2a - \tau_f r \\ \implies a(I + mr^2) &= mr^2g - \tau_f r \end{aligned}$$

Therefore,

$$a = g\left(\frac{mr^2}{I + mr^2}\right) - \frac{\tau_f r}{I + mr^2} \tag{4}$$

We know that the moment of inertia of a disc of radius R and mass M is $\frac{1}{2}MR^2$
For a massive ($M >> m$) disc , we can write,

$$I + mr^2 \approx I$$

Therefore,

$$a \approx m\left(\frac{gr^2}{I}\right) - \frac{\tau_f r}{I} \tag{5}$$

Preliminary calculation: radius of the axle used

Least count of the Vernier caliper = $0.02mm$

Table 1: Measuring the diameter of the axle used

MSR (mm)	VSD	Total (mm)	Average
18	3	18.06	
18	4	18.08	
18	5	18.10	
18	3	18.06	**18.06**
18	4	18.08	
18	5	18.10	
18	0	18.00	
18	1	18.02	
18	2	18.04	

From the data above we can observe that the length measurement ranges from 18.00 to 18.10

$$\therefore \delta d = \pm \frac{18.10 - 18.00}{2} mm = \pm 0.05 mm$$

Now, radius, $r = \frac{d}{2} = \frac{18.06}{2} \pm \frac{0.05}{2} mm = 9.03 \pm 0.025 mm$

Procedure

We attached a slotted mass to an axle of the flywheel using a cord of negligible mass and wound it on the axle. We dropped the mass very carefully so that there is minimal oscillation while the mass is falling. We used a camera to to take a video of the falling mass and analysed it to find the acceleration. We than repeated this procedure for different masses.

Observations

Table 2: The variation of acceleration with mass

Mass $\pm$ 0.1 (g)	Acceleration (m/s^2)
102.8	0.0091
151.7	0.0149
200.0	0.0210
250.0	0.0239
299.5	0.0307
350.5	0.0348
400.5	0.0427
450.3	0.0482
499.4	0.0533
547.5	0.0587

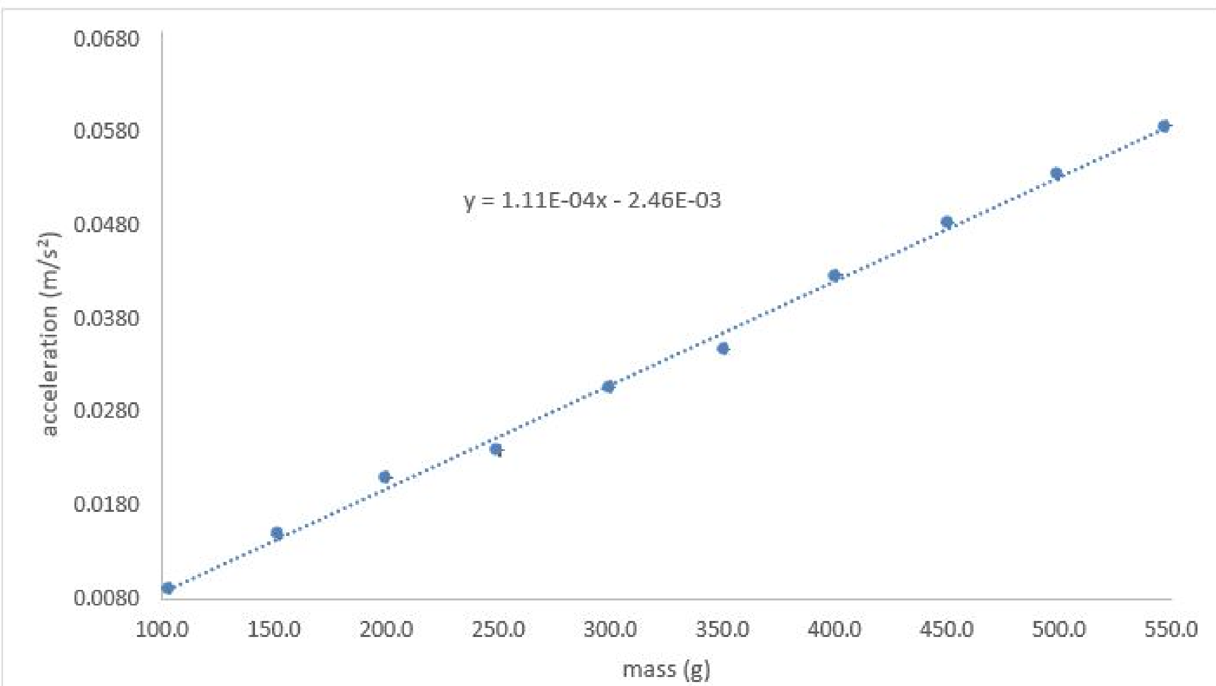

Figure 2: The graph shows that acceleration is proportional to mass

Using the LINEST function in excel we got, the uncertainty in the slope of the trendline =2.19E-06 and the uncertainty in the intercept =7.78E-04

Analysis

From eq. (5),

$$a \approx m\left(\frac{gr^2}{I}\right) - \frac{\tau_f r}{I}$$

We have the slope of the trendline equation, p = 1.11E-04
Therefore,

$$\frac{gr^2}{I} = 1.11 \times 10^{-4}$$
$$\implies I = \frac{gr^2}{1.11 \times 10^{-4}} = \frac{9.81 \times 9.03^2 \times 10^{-6}}{1.11 \times 10^{-4}} gm^2 = 7.02gm^2$$

We have, $\delta p = \pm\, 2.19 \times 10^{-6}$, $\delta r = \pm\, 0.025mm \implies \delta(r^2) = 2r\delta r =$ and $g = 9.81m/s^2$

$$\therefore\ \delta I = \ gI\sqrt{\left(\frac{\delta p}{p}\right)^2 + \left(\frac{\delta r^2}{r^2}\right)^2} = \ gI\sqrt{\left(\frac{\delta p}{p}\right)^2 + \left(\frac{2r\delta r}{r^2}\right)^2}$$
$$\implies \delta I = \ 9.81 \times 7.02\sqrt{\left(\frac{2.19 \times 10^{-6}}{1.11 \times 10^{-4}}\right)^2 + \left(\frac{2 \times 9.03 \times 0.025}{9.03^2}\right)^2} gm^2 = 1.4gm^2$$

Therefore, $I = 7 \pm 1\ gm^2$

Again, we have the intercept of the trendline equation, q = 2.46E-03 Therefore,

$$\frac{\tau_f r}{I} = 2.46 \times 10^{-3}$$
$$\implies \tau_f = \frac{2.46 \times 10^{-3} \times I}{r} = \frac{2.46 \times 10^{-3} \times 7.02 \times 10{-}3}{9.03 \times 10^{-3}} Nm = 1.9 \times 10^{-3}\ Nm$$

We have $\delta q = \pm 7.78 \times 10^{-4}$, $\delta I = \pm 0.02gm^2$, $\delta r = 0.025mm$

$$\therefore\ \delta\tau_f = \ \tau_f\sqrt{\left(\frac{\delta q}{q}\right)^2 + \left(\frac{\delta r}{r}\right)^2 + \left(\frac{\delta I}{I}\right)^2}$$
$$\implies \delta\tau_f = \ 1.9 \times 10^{-3}\sqrt{\left(\frac{7.78 \times 10^{-4}}{2.46 \times 10^{-3}}\right)^2 + \left(\frac{0.025}{9.03}\right)^2 + \left(\frac{1}{7}\right)^2} Nm = 0.3 \times 10^{-3} Nm$$

Therefore, $\tau_f = (1.9 \pm 0.3) \times 10^{-3}\ gm^2$

Part B

In this part, we introduced electromagnetic damping to the flywheel system.

Introduction

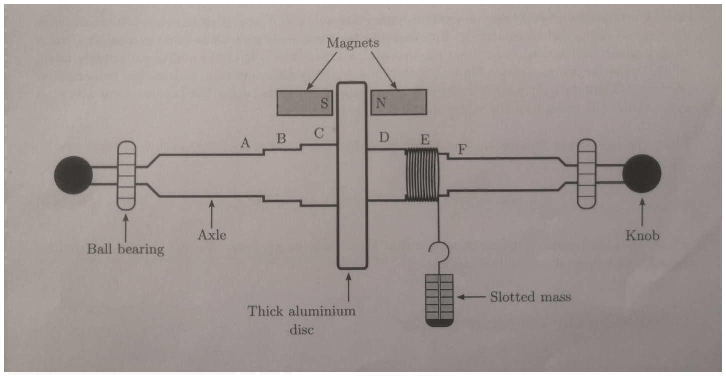

Figure 3: Schematic diagram of the disc and magnets assembly

A pair of magnets is placed in the system as shown in Fig. 2. The aluminium disc is a conductor. So it has free electrons in it. When the slotted mass starts to fall, the disc starts to rotate, which creates induced eddy currents in the disc. This eddy current produces a magnetic field with opposite polarity of the original magnets. This introduces an electromagnetic damping in the system.
In this case,

$$I\alpha = m(g-a)r - \tau_f - C\omega B^2 \tag{6}$$

Where, ω is the angular velocity of the disc, B is the magnetic field acting on the disc, and C is a constant. When the disc reaches its terminal angular velocity ω_t and the mass its terminal velocity v_t, $\alpha = 0$ and $a = 0$
In this case,

$$\omega_t = \frac{1}{CB^2}(mgr - \tau_f)$$

Therefore,

$$v_t = w_t r = \frac{gr^2}{CB^2}\left(m - \frac{\tau_f}{gr}\right) \tag{7}$$

Procedure

We clamped a pair of magnets in the holders. We adjusted the mounts so that the axes of the magnets are aligned and are perpendicular to the disc. We also made sure that the magnets are equally spaced from the disc. We then repeated the procedure in Part A. But in this case, electromagnetic damping causes the falling mass to attain a terminal velocity after some time.

Observations

Table 3: The variation of terminal velocity with mass

Mass $\pm$ 0.1 (g)	Terminal velocity v_t (m/s)
102.8	0.0200
200.1	0.0408
250.6	0.0520
301.6	0.0625
352.1	0.0733
402.7	0.0835
450.4	0.0904
498.9	0.1001
549.1	0.1147
598.5	0.1201

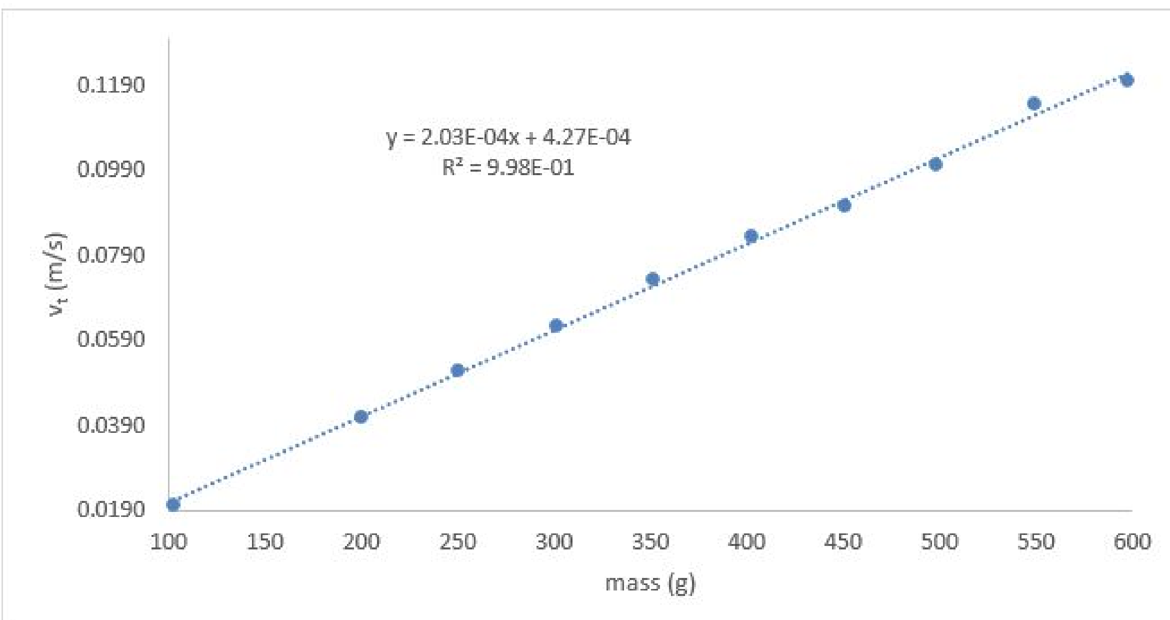

Figure 4: The graph shows that terminal velocity is proportional to mass.

Analysis

The above graph satisfies equation (7)

Part C

In this part we studied how terminal velocity changes with magnetic field.

Procedure

We first varied the distance between the magnets and measured the magnetic field exactly between the magnets using a gauss-meter.

Then we kept the magnets some known distance apart and calculated the terminal velocity for a fixed slotted mass. Keeping the slotted mass constant, we changed the separation between the magnets and calculated the terminal velocity.

Observations

Table 4: The variation of magnetic field with the distance between the magnets

Distance between the magnets (mm)	Magnetic field (Gauss)
6.52 $\pm$ 0.02	221 $\pm$ 1
7.40 $\pm$ 0.02	203 $\pm$ 1
8.38 $\pm$ 0.02	188 $\pm$ 1
9.46 $\pm$ 0.02	165 $\pm$ 1
10.40 $\pm$ 0.02	153 $\pm$ 1
11.40 $\pm$ 0.02	138 $\pm$ 1
12.42 $\pm$ 0.02	126 $\pm$ 1
13.50 $\pm$ 0.02	117 $\pm$ 1
14.30 $\pm$ 0.02	108 $\pm$ 1
15.44 $\pm$ 0.02	100 $\pm$ 1
16.44 $\pm$ 0.02	86 $\pm$ 1
17.28 $\pm$ 0.02	80 $\pm$ 1
18.36 $\pm$ 0.02	74 $\pm$ 1
19.46 $\pm$ 0.02	70 $\pm$ 1
20.46 $\pm$ 0.02	64 $\pm$ 1
21.42 $\pm$ 0.02	59 $\pm$ 1
22.70 $\pm$ 0.02	55 $\pm$ 1
23.80 $\pm$ 0.02	50 $\pm$ 1
24.82 $\pm$ 0.02	47 $\pm$ 1
25.96 $\pm$ 0.02	41 $\pm$ 1

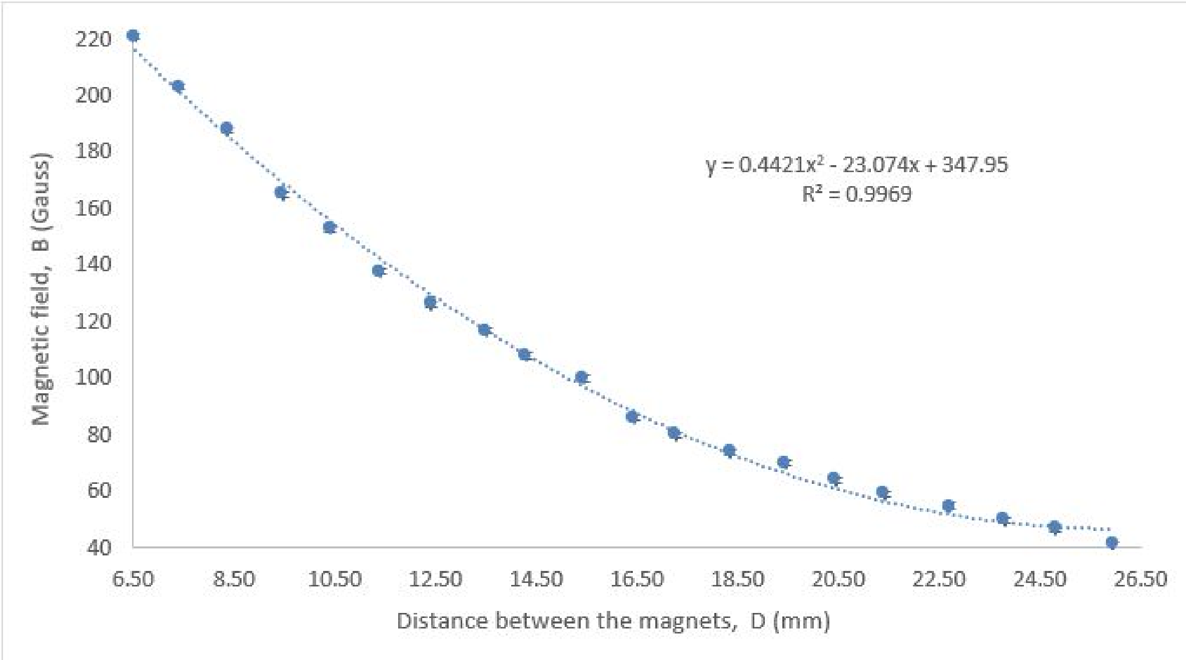

Figure 5: The graph shows that when the distance between the magnets increases, the magnetic field experiences a decrease that is not linear. The quadratic trendline equation gives us a very good approximation of the relation between the two variables within the range of our data.

Using the trendline equation of the above graph we can find the magnetic field at the center of the disc for different distances between the magnets ranging from 6.52 mm to 25.96 mm.

Table 5: The variation of terminal velocity with magnetic field

Distance between the magnets (mm)	Magnetic field B (Gauss)	Terminal velocity V_t (m/s)	**ln (B)**	**ln (V_t)**
16.10	91	0.047	**4.511**	**-3.058**
16.90	84	0.062	**4.434**	**-2.785**
18.00	76	0.076	**4.329**	**-2.571**
19.30	67	0.093	**4.209**	**-2.371**
20.40	61	0.108	**4.115**	**-2.226**
21.42	57	0.133	**4.035**	**-2.021**
22.76	52	0.148	**3.947**	**-1.911**
23.60	50	0.160	**3.905**	**-1.833**
24.64	48	0.177	**3.867**	**-1.734**
25.80	47	0.184	**3.848**	**-1.694**

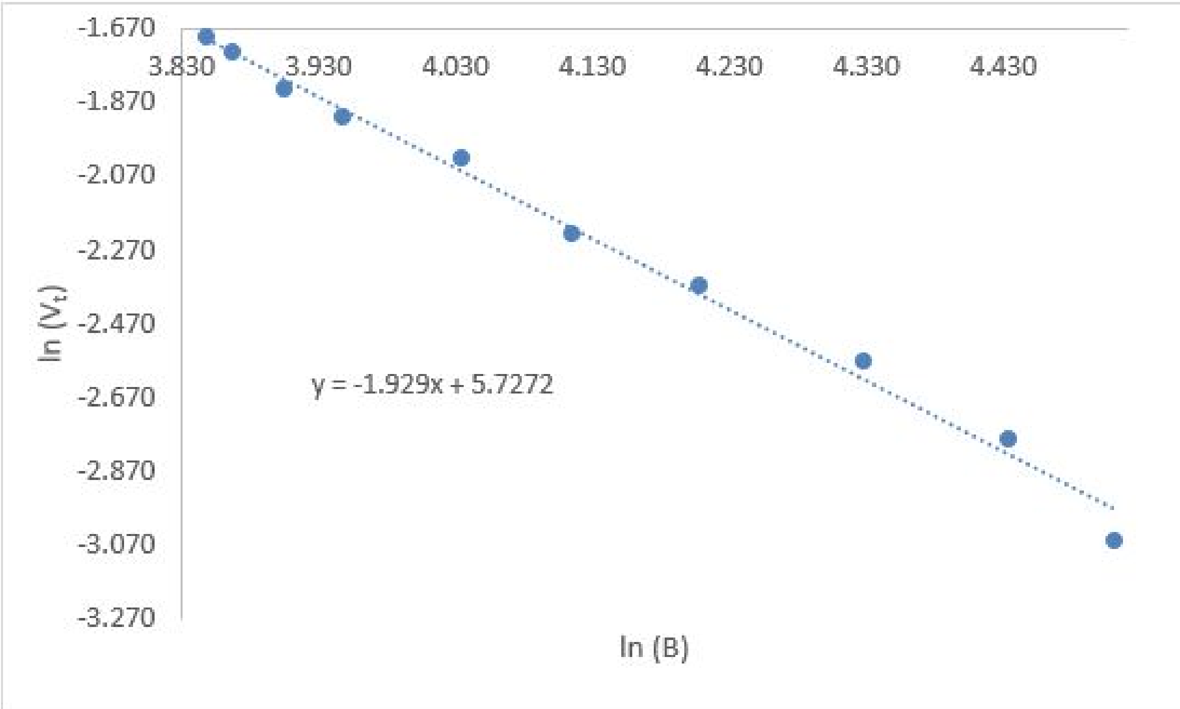

Figure 6: The log-log graph shows that terminal velocity decreases with magnetic field.

Using the LINEST function in excel we got the uncertainty in the slope of the trendline equation =0.07

Analysis

From the above graph we have the slope of the trendline equation, $\alpha = -1.92 \pm 0.07$

Therefore, $v_t \propto B^{-1.92 \pm 0.07}$

From eq. (7), we have,$v_t \propto B^{-2}$, which is consistent with our experimental finding.

Therefore, our experiment proves the theoretical prediction.