Experiment 2: Air track

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Date of performing the experiment: 17-09-19 to 27-09-19
Date of submission: 01-10-19

Objectives

Studying motion in one dimension and Newton's laws.

Introduction

In this experiment we studied the motion of bodies in one dimension. We used an air track to minimize the frictional force on the test object. The test object (rider) floats on a layer of air, which eliminates most of the frictional force between the rider and the air track. So that the rider can move freely with almost no horizontal force.

Apparatus

Air track, Blower, Riders, Magnetic buffers, Brass buffers, Velcro buffers, Two vernier photogates, Retort stands, Vernier caliper, Weighing scale.

Preliminary calculations

The length of the prongs

We used an interrupter with two prongs of small width in Part A and Part B. We named the two prongs A and B. We calculated the width of the prongs using a Vernier caliper of least count 0.02 mm.

Table 1: Table for the length measurement of prongs A and B

Prong	MSR (mm)	VSD	VSR = (VSD×LC)	Total (mm)	Average (mm)
A	10	4	0.08	10.08	X_a = 10.11
	10	5	0.10	10.10	
	10	6	0.12	10.12	
	10	5	0.10	10.10	
	10	6	0.12	10.12	
	10	7	0.14	10.14	
	10	5	0.10	10.10	
	10	6	0.12	10.12	
	10	7	0.14	10.14	
B	10	2	0.04	10.04	X_b =10.07
	10	3	0.06	10.06	
	10	4	0.08	10.08	
	10	3	0.06	10.06	
	10	4	0.08	10.08	
	10	5	0.10	10.10	
	10	3	0.06	10.06	
	10	4	0.08	10.08	
	10	5	0.10	10.10	

From the data above we can observe that in the case of prong A the length measurement ranges from 10.08 to 10.14 and for prong B it ranges from 10.04 to 10.10.

$$\therefore\ \delta x_a = \pm\frac{10.14 - 10.08}{2} mm = \pm\ 0.03mm$$
$$and,\ \delta x_b = \pm\frac{10.10 - 10.04}{2} mm = \pm\ 0.03mm$$

The width of prong A, $x_a = 10.11\ \pm\ 0.03\ mm$
The width of prong B, $x_b = 10.07\ \pm\ 0.03\ mm$

Error in time measurement using the photo-gates

We first leveled the air track using a spirit level and a rider. We attached a pulley at one end of the air track. We passed a long unstretchable string across the pulley and connected the rider at one end and a slotted mass at the other end. We attached an interrupter with prongs on top of the rider which will be detected by the photo-gates. We set up two photo-gates at two different points in the trajectory of the rider. With the rider at one end and the mass hanging on the other end we turned on the air blower. The air track surface became friction-less and the rider started moving with an acceleration. We measured the time taken by the prongs (attached to the rider) to pass two different points using the photo-gates and Logger Pro 3.15 software. Keeping all the initial conditions constant we repeated this 10 times and compared the time readings collected using the photo-gates.

Figure 1: Rider of mass M on the horizontal air track, connected by a string to mass m that falls vertically under the inuence of gravity

Table 2: Time taken by the rider to pass the photogates for the same initial conditions

Trials	Time taken by prong A to pass the 1st photo-gate (s)	Time taken by prong B to pass the 1st photo-gate (s)	Time taken by prong A to pass the 2nd photo-gate (s)	Time taken by prong B to pass the 2nd photo-gate (s)
1	0.020223	0.019407	0.012732	0.012475
2	0.020452	0.019619	0.012765	0.012504
3	0.020197	0.019432	0.012678	0.012428
4	0.019658	0.018922	0.012353	0.012117
5	0.019524	0.018814	0.012296	0.012036
6	0.019524	0.018814	0.012296	0.012036
7	0.019658	0.018922	0.012353	0.012117
8	0.020197	0.019432	0.012678	0.012428
9	0.020452	0.019619	0.012765	0.012504
10	0.020223	0.019407	0.012732	0.012475

From the data gathered above we can see that the third decimal point of the time measurement is fluctuating by one digit.
Therefore, the error in time measurement,

$$\delta t = \pm \frac{0.001}{2}\ s\ =\ \pm 0.0005\ s$$

Part A: Newton's first law of motion

Theory

According to Newton's first law of motion, an object moving with an initial velocity will move at constant velocity if no external force is applied on it.

For an object moving under the force of an ideal spring,

$$\text{Initial energy, } E_i = \tfrac{1}{2}kx^2$$

$$\text{Final energy, } E_f = \tfrac{1}{2}mv^2$$

Using the principle of conservation of energy we can write,

$$\frac{1}{2}kx^2 = \frac{1}{2}mv^2 \implies kx^2 = mv^2 \implies v = \sqrt{\frac{k}{m}x}$$

Therefore, The velocity of the object is directly proportional to the displacement of the spring.
If, $x_2 = 2x_1$;

$$kx_2^2 = mv_2^2 \implies k(2x_1)^2 = mv_2^2 \implies 8(\frac{1}{2}kx_1^2) = mv_2^2 \implies 8(\frac{1}{2}mv_1^2) = mv_2^2 \implies 4v_1^2 = v_2^2$$

$$\therefore v_2 = 2v_1 \tag{1}$$

Therefore, when the initial displacement is increased by a factor of 2, the velocity of the object also gets increased by a factor of 2.

Procedure

We attached two metal springs at both ends of the air track and switched on the air blower.We placed the rider at one end of the air track. While gently pushing the rider against the spring we let it move with an initial velocity. The velocity of the prongs (A and B) attached to the rider at two different points was calculated using the data gathered by the photo-gates. We repeated this for different initial velocities.

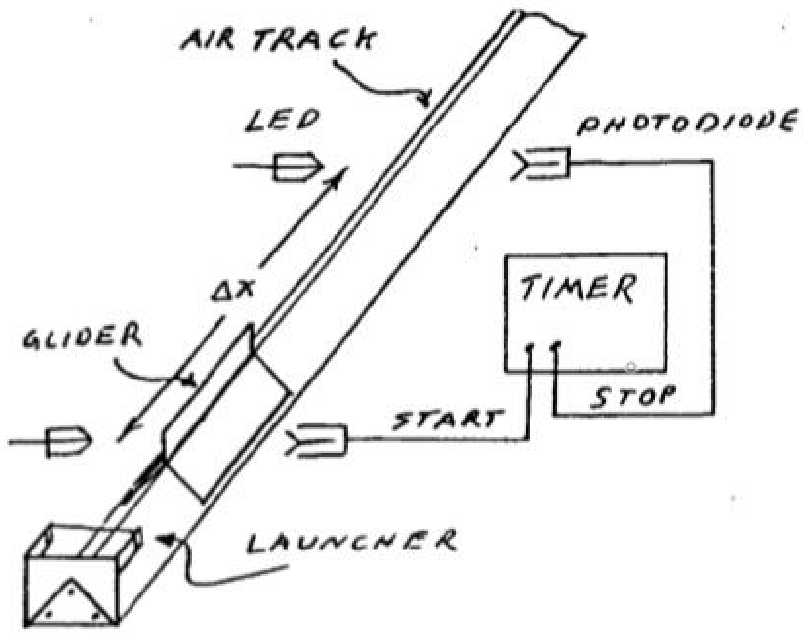

Figure 2: Experimental setup for Part A

Error analysis

$$v = \frac{x}{t}$$

$$\therefore\ \delta v = v\sqrt{\left(\frac{\delta x}{x}\right)^2 + \left(\frac{\delta t}{t}\right)^2}$$

Now, $\delta x_a = \delta x_b = 0.03\ mm$ and $\delta t = 0.0005\ s$
Therefore,

$$\delta v = v\sqrt{\left(\frac{0.03}{x}\right)^2 + \left(\frac{0.0005}{t}\right)^2} \tag{2}$$

Observations

Table 3: Velocity of prong A attached to the rider at two different points

Serial No.	Velocity at the 1st photo-gate V_1 (mm/s)	$\pm\,\delta v_1$ (s)	Velocity at the 2nd photo-gate V_2 (mm/s)	$\pm\,\delta v_2$ (s)	$\lvert v_1 - v_2\rvert$ (mm/s)	$\delta\lvert v_1 - v_2\rvert = \pm\sqrt{(\delta v_1)^2 + (\delta v_2)^2}$ (s)
1	109.54	0.68	122.87	0.83	**13.33**	**1.07**
2	177.77	1.65	166.77	1.46	**11.00**	**2.20**
3	202.77	2.12	205.35	2.17	**2.58**	**3.04**
4	240.37	2.95	234.13	2.80	**6.24**	**4.06**
5	253.49	3.27	251.01	3.20	**2.48**	**4.57**
6	327.93	5.41	325.04	5.31	**2.89**	**7.58**
7	399.24	7.97	392.33	7.70	**6.91**	**11.08**
8	400.87	8.04	398.22	7.93	**2.65**	**11.29**
9	549.19	15.01	530.32	14.00	**18.87**	**20.52**

Table 4: Velocity of prong B attached to the rider at two different points

Serial No.	Velocity at the 1st photo-gate V_1 (mm/s)	$\pm\,\delta v_1$ (s)	Velocity at the 2nd photo-gate V_2 (mm/s)	$\pm\,\delta v_2$ (s)	$\lvert v_1 - v_2\rvert$ (mm/s)	$\delta\lvert v_1 - v_2\rvert = \pm\sqrt{(\delta v_1)^2 + (\delta v_2)^2}$ (s)
1	115.91	0.72	131.99	0.90	**16.08**	**1.15**
2	186.18	1.71	176.64	1.55	**9.54**	**2.31**
3	214.29	2.24	217.89	2.31	**3.60**	**3.22**
4	251.87	3.06	247.49	2.95	**4.38**	**4.25**
5	266.48	3.41	265.59	3.39	**0.89**	**4.81**
6	344.84	5.65	344.76	5.65	**0.08**	**7.99**
7	421.31	8.39	415.53	8.16	**5.78**	**11.70**
8	422.51	8.44	419.90	8.33	**2.60**	**11.86**
9	577.91	15.70	561.18	14.81	**16.73**	**21.58**

Analysis

For both prongs (A and B) the differences in velocity at the two photo-gates are higher than the corresponding error margins for lower velocities (approximately less than 250 mm/s). But for higher velocities, we see that the velocities do not differ more than the corresponding error margins.

As the air track surface is not completely friction-less, the effect of friction becomes significant for lower velocities. As we increase the initial velocity of the test object, the effect of friction becomes insignificant and we see Newton's first law of motion in action.

Part B: Newton's second law of motion

Theory

According to Newton's second law of motion

$$\boldsymbol{a} = \frac{1}{m}\,\boldsymbol{F} \tag{3}$$

If we consider an object of mass m with a constant force F_0 applied to it,

$$\begin{aligned}
\boldsymbol{a} = \frac{\mathrm{d}v}{\mathrm{d}t} &= \frac{\boldsymbol{F_0}}{m} \\
\implies \int dv &= \frac{\boldsymbol{F_0}}{m}\int dt \\
\implies v(t) &= \frac{\boldsymbol{F_0}}{m}t + c
\end{aligned}$$

When $t = 0, v = v(0), \implies c = v(0)$

Therefore,

$$v(t) = v(0) + \frac{\boldsymbol{F_0}}{m}t \tag{4}$$

$$\begin{aligned}
\implies \frac{\mathrm{d}x}{\mathrm{d}t} &= v(0) + \frac{\boldsymbol{F_0}}{m}t \\
\implies \int dx &= v(0)\int dt + \frac{\boldsymbol{F_0}}{m}\int tdt \\
\implies x(t) &= v(0)t + \frac{1}{2}\left(\frac{F_0}{m}\right)t^2 + k
\end{aligned}$$

When $t = 0, x = x(0), \implies k = v(0)$

Therefore,

$$x(t) = x(0) + v(0)t + \frac{1}{2}\left(\frac{F_0}{m}\right)t^2 \tag{5}$$

For a hanging mass m connected to the rider (of mass M) with an inextensible string (Figure 1),
The horizontal tension force on the rider, $T = Ma$
As both the masses are connected by an inextensible string, both the masses will experience the same acceleration.

The force on the mass m,

$$\begin{aligned} F &= mg - T \\ \implies ma &= mg - T \\ \implies ma &= mg - Ma \\ \implies a\,(M+m) &= mg \end{aligned}$$

Therefore,

$$\frac{1}{a} = \frac{M}{g}\left(\frac{1}{m}\right) + \frac{1}{g} \tag{6}$$

Procedure

We made the same experimental setup as we did while calculating the margin in time measurement (Figure 1).We then measured the the change in velocity (for prongs A and B) between two points along the trajectory of the rider for different masses.

Error analysis

$$a = \frac{v_2 - v_1}{t_2 - t_1}$$

$$\therefore\ \delta a = a\sqrt{\left(\frac{\delta(v_2 - v_1)}{(v_2 - v_1)}\right)^2 + \left(\frac{\delta(t_2 - t_1)}{(t_2 - t_1)}\right)^2}$$

Where,

$$\delta(v_2 - v_1) = \sqrt{(\delta v_2)^2 + (\delta v_1)^2}$$

δv_2 and δv_1 can be calculated using equation 2.
Now,

$$\delta(t_2 - t_1) = \sqrt{(\delta t_2)^2 + (\delta t_1)^2}$$

But, $\delta t_1 = \delta t_2 = 0.0005\ s$

$$\implies \delta(t_2 - t_1) = \sqrt{(0.0005)^2 + (0.0005)^2} = 0.0007$$

Least count of the weight scale is 0.1g.
Therefore, $\delta m\ = \pm\, 0.1g$
Now,

$$\delta\left(\frac{1}{a}\right) = \left(\frac{1}{a}\right)\frac{\delta a}{a} \tag{7}$$

And,

$$\delta\left(\frac{1}{m}\right) = \left(\frac{1}{m}\right)\frac{\delta m}{m} \tag{8}$$

Observations

Mass of the rider, $M = 407.5 \pm 0.1\ g$

Table 5: Variation of acceleration of prong A attached to the rider for different masses

Mass m (g)	$\pm$ δm (g)	Acceleration a (mm/s^2)	$\pm$ δa (mm/s^2)	$\boldsymbol{\frac{1}{m}}$ **(g^{-1})**	$\pm\delta\left(\frac{1}{m}\right)$ (g^{-1})	$\boldsymbol{\frac{1}{a}}$ **(mm/s^2)$^{-1}$**	$\pm\delta\left(\frac{1}{m}\right)$ (mm/s^2)$^{-1}$
10.2	0.1	231.05	19.05	**0.098039**	0.000961	**0.004328**	0.000357
15.3	0.1	351.44	35.36	**0.065359**	0.000427	**0.002845**	0.000286
20.2	0.1	462.64	50.96	**0.049505**	0.000245	**0.002162**	0.000238
25.3	0.1	573.06	74.78	**0.039526**	0.000156	**0.001745**	0.000228
30.3	0.1	678.83	95.92	**0.033003**	0.000109	**0.001473**	0.000208
35.5	0.1	781.09	122.37	**0.028169**	0.000079	**0.00128**	0.000201
40.4	0.1	892.32	145.33	**0.024752**	0.000061	**0.001121**	0.000183
45.5	0.1	997.10	168.73	**0.021978**	0.000048	**0.001003**	0.000170
49.9	0.1	1082.63	189.16	**0.020040**	0.000040	**0.000924**	0.000161
55	0.1	1165.91	214.78	**0.018182**	0.000033	**0.000858**	0.000158
60.1	0.1	1278.40	249.58	**0.016639**	0.000028	**0.000782**	0.000153

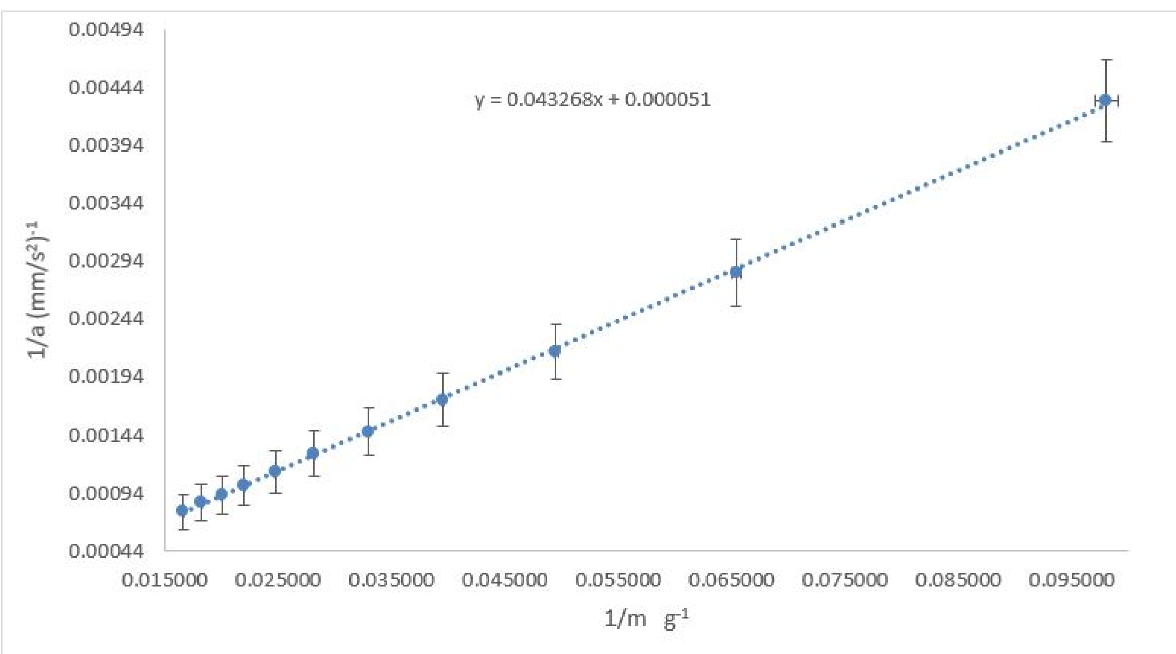

Figure 3: The graph shows a linear relationship between $\frac{1}{m}$ and $\frac{1}{a}$, which was expected from Newton's 2nd law.

Using the LINEST function in excel we got, the uncertainty in the slope of the trendline = 0.000278

Analysis

From eq. (6),

$$\frac{1}{a} \;=\; \frac{M}{g}\left(\frac{1}{m}\right) \;+\; \frac{1}{g}$$

We have the slope of the trendline equation, a = 0.043268

$$\frac{M}{g} = 0.043268 \implies g = \frac{407.5}{0.043268} = 9418\ mm/s^2 = 9.41 m/s^2$$

Now we have, $\delta a = \pm\ 0.000278$, $\delta M = \pm\ 0.1$ and $g = \frac{M}{a}$

$$\therefore\ \delta g = g\ \sqrt{\left(\frac{\delta M}{M}\right)^2 + \left(\frac{\delta a}{a}\right)^2} = \ 9.41\sqrt{\left(\frac{0.1}{407.5}\right)^2 + \left(\frac{0.000278}{0.043268}\right)^2}\ m/s^2 = 0.06\ m/s^2$$

Therefore, from the slope of the trendline equation, we get the value of acceleration due to gravity, $g = 9.41\ \pm\ 0.06\ m/s^2$

Part C: Newton's third law of motion

Theory

According to Newton's third law of motion, if two objects moving at constant velocities v_1 and v_2 collide with each other, the force of the first object on the second object is equal and opposite to the force of the second object on the first object.
Thus,

$$F_{12} = -F_{21} \tag{9}$$

$$\implies m_1 \frac{(v_1' - v_1)}{\Delta t} = m_2 \frac{(v_2' - v_2)}{\Delta t}$$

Where, v_1' and v_2' are the velocity of the objects after the collision.

$$\implies m_1\,(v_1' - v_1) = m_2\,(v_2' - v_2)$$

$$\implies m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2' \tag{10}$$

These type of collisions are called perfectly elastic collision, where there is no loss of kinetic energy.
The kinetic energy in this case is,

$$\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}m_1{v_1'}^2 + \frac{1}{2}m_2{v_2'}^2 \tag{11}$$

However, we don't encounter perfectly elastic collisions in our everyday life. These type of collisions usually occur at atomic scales. If we take example of two billiard balls colliding with each other, a small amount of energy gets lost during the collision, which generates heat and sound.
Now, if we consider a case where the two objects stick together and move at the velocity V after the collision. Then there will be some kinetic lost during the collision. These type of collisions are called perfectly inelastic collision. In this case:

$$m_1v_1 + m_2v_2 = (m_1 + m_2)\,V \tag{12}$$

The kinetic energy,

$$\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 \neq \frac{1}{2}(m_1 + m_2)V^2 \tag{13}$$

Procedure

We put the steel springs back at the ends of the air track. We put two riders with interrupter cards on top of them. We adjusted the heights of the photo-gates in such a way that they detect the interrupter cards. We attached magnetic buffers and counterweights to the riders to simulate elastic collision. We placed the photo-gates in such a way that the the initial and the final velocities of both the riders can be determined. We measured the velocities of the riders before and after the collision for different initial conditions.
Then we replaced the magnetic buffers with velcro buffers to simulate inelastic collisions. We measured the velocities of the riders before and after the collision for different initial conditions.

Error analysis

Momentum

Momentum, $p = mv$
Therefore,

$$\delta p = mv\sqrt{\left(\frac{\delta m}{m}\right)^2 + \left(\frac{\delta v}{v}\right)^2} \tag{14}$$

Kinetic energy

Kinetic energy, $E_k = \frac{1}{2}mv^2$
Now,

$$\begin{aligned}
\delta(v^2) &= 2\frac{\delta v}{v}v^2 = 2v\delta v \\
\implies \delta E_k &= \delta\left(\frac{1}{2}mv^2\right) \\
&= \frac{1}{2}\delta\left(mv^2\right) \\
&= \frac{1}{2}mv^2\sqrt{\left(\frac{\delta m}{m}\right)^2 + \left(\frac{\delta(v^2)}{v^2}\right)^2} \\
&= \frac{1}{2}mv^2\sqrt{\left(\frac{\delta m}{m}\right)^2 + \left(\frac{2v\delta v}{v^2}\right)^2}
\end{aligned}$$

Therefore,

$$\delta E_k = \frac{1}{2}mv^2\sqrt{\left(\frac{\delta m}{m}\right)^2 + \left(2\frac{\delta v}{v}\right)^2} \tag{15}$$

In the case of the kinetic energy after an inelastic collision, $E_k = \frac{1}{2}(m_1 + m_2)V^2$
Here,

$$\delta E_k = \frac{1}{2}mV^2\sqrt{\left(\frac{\delta(m_1 + m_2)}{(m_1 + m_2)}\right)^2 + \left(2\frac{\delta V}{V}\right)^2} = \frac{1}{2}mV^2\sqrt{\left(\frac{\sqrt{(\delta m_1)^2 + (\delta m_2)^2}}{(m_1 + m_2)}\right)^2 + \left(2\frac{\delta V}{V}\right)^2} \tag{16}$$

In each case δv can be calculated using the method used for Part A.

Observations

Elastic collision

Mass of rider 1, $m_1 = 281.4 \pm 0.1\ g$
Mass of rider 2, $m_2 = 448.6 \pm 0.1\ g$
Length of the interrupter card on top of rider 1, $x_1 = 22.1 \pm 0.1\ cm$
Length of the interrupter card on top of rider 2, $x_2 = 21.1 \pm 0.1\ cm$

Inelastic collision

Mass of rider 1, $m_1 = 226.3 \pm 0.1\ g$
Mass of rider 2, $m_2 = 394.5 \pm 0.1\ g$

Length of the interrupter card on top of rider 1, $x_1 = 22.1 \pm 0.1\ cm$
Length of the interrupter card on top of rider 2, $x_2 = 21.1 \pm 0.1\ cm$
The combined effective length of the riders after the collision, $x = 47.5 \pm 0.1\ cm$

Table 6: The initial and final kinetic energies for elastic collision

Total K.E. before collision E_k^i (g cm²/s²)	$\pm\delta E_k^i$ (g cm²/s²)	Total K.E. after collision E_k^f (g cm²/s²)	$\pm\delta E_k^f$ (g cm²/s²)	$\left\|E_k^i - E_k^f\right\|$ (g cm²/s²)	$\pm\delta\left\|E_k^i - E_k^f\right\| = \pm\sqrt{(\delta E_k^i)^2 + (\delta E_k^f)^2}$ (g cm²/s²)
36861	250	31343	158	5518	217
44843	308	40843	201	4000	276
53081	349	48670	318	4411	319
95485	631	85469	591	10016	565
125046	823	111811	893	13235	762
148939	992	131275	1154	17665	921
232625	1540	185856	1459	46769	1255
236367	1616	189086	1869	47281	1351
312687	2071	178155	1765	134532	1197
345569	2343	324510	3033	21059	2431

Table 7: The initial and final kinetic energies for inelastic collision

Total K.E. before collision E_k^i (g cm²/s²)	Total K.E. after collision E_k^f (g cm²/s²)	Loss in K.E. $E_k^i - E_k^f$ (g cm²/s²)
38232	6997	31235
77584	9837	67747
100975	15163	85813
115841	16395	99446
135698	21194	114505
155529	17309	138221
176150	26022	150128
197122	35006	162116
223609	37348	186261
254200	52666	201535

Analysis

For elastic collision, we observe that the kinetic energies before and after the collision differ more than the corresponding error margins. So kinetic energy is not conserved here. Therefore, the collisions were not elastic and this part of the experiment needs further investigation.
For inelastic collision, we noticed a significant amount of loss in kinetic energy.