REPRESENTING KNOWLEDGE ABOUT KNOWLEDGE AND MUTUAL KNOWLEDGE

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ABSTRACT

In order to represent speech acts, in a multi-agent context, we choose a knowledge representation based on the modal logic of knowledge KT4 which is defined by Sato. Such a formalism allows us to reason about knowledge and represent knowledge about knowledge, the notions of truth value and of definite reference.

I INTRODUCTION

Speech act representation and the language planning require that the system can reason about intensional concepts like knowledge and belief. A problem resolver must understand the concept of knowledge and know for example what knowledge it needs to achieve specific goals. Our assumption is that a theory of language is part of a theory of action (Austin [4]).

Reasoning about knowledge encounters the problem of intensionality. One aspect of this problem is the indirect reference introduced by Frege [7] during the last century. Mc Carthy [15] presents this problem by giving the following example :
Let the two phrases : Pat knows Mike's telephone number   (1)
               and   Pat dialled Mike's telephone number (2)
The meaning of the proposition "Mike's telephone number" in (1) is the concept of the telephone number, whereas its meaning in (2) is the number itself.
Then if we have : "Mary's telephone number = Mike's telephone number",
we can deduce that :
               "Pat dialled Mary's telephone number"
but we cannot deduce that :
               "Pat knows Mary's telephone number",
because Pat may not have known the equality mentioned above.

Thus there are verbs like "to know", "to believe" and "to want" that create an "opaque" context. For Frege a sentence is a name, reference of a sentence is its truth value, the sense of a sentence is the proposition. In an oblique context, the reference becomes the proposition. For example the referent of the sentence p in the indirect context "A knows that p" is a proposition and no longer a truth value.

Mc Carthy [15] and Konolige [11] have adopted Frege's approach. They consider the concepts like objects of a first-order language. Thus one term will denote Mike's telephone number and another will denote the concept of Mike's telephone number. The problem of replacing equalities by equalities is then avoided because the concept of Mike's telephone number and the number itself are different entities.

Mc Carthy's distinction concept/object corresponds to Frege's sense/reference or to modern logicians' intension/extension.

Maida and Shapiro [13] adopt the same approach but use propositional semantic networks that are labelled graphs, and that only represent intensions and not extensions, that is to say individual concepts and propositions and not referents and truth values. We bear in mind that a semantic network is a graph whose nodes represent individuals and whose oriented arcs represent binary relations.

Cohen [6], being interested in speech act planning, proposes the formalism of partitioned semantic networks as data base to represent an agent's beliefs. A partitioned semantic network is a labelled graph whose nodes and arcs are distributed into spaces. Every node or space is identified by its own label. Hendrix [9] introduced it to represent the situations requiring the delimitation of information sub-sets. In this way Cohen succeeds in avoiding the problems raised by the data base approach. These problems are clearly identified by Moore [17,18]. For example to represent 'A does not believe P', Cohen asserts ∿ Believe (A,P) in a global data base, entirely separated from any agent's knowledge base. But as Appelt [2] notes, this solution raised problems when one needs to combine facts from a particular data base

with global facts to prove a single assertion.
For example, from the assertion :

$\sim$ know (John,Q) & know (John,P $\supset$ Q)

where P $\supset$ Q is in John's data base and $\sim$ know
(John,Q) is in the global data base, it should
be possible to conclude $\sim$ know (John,P) but
a good strategy must be found !

In a nutshell, in this first approach
which we will call a syntactical one, an a-
gent's beliefs are identified with formulas
in a first-order language, and propositional
attitudes are modelled as relations between
an agent and a formula in the object langua-
ge, but Montague showed that modalities can-
not consistently be treated as predicates ap-
plying to nouns of propositions.

The other approach no longer considers
the intension as an object but as a function
from possible worlds to entities. For ins-
tance the intension of a predicate P is the
function which to each possible world W (or
more generally a point of reference, see
Scott [23]) associates the extension of P
in W.

This approach is the one that Moore
[17,18] adopted. He gave a first-order axio-
matization of Kripke's possible worlds seman-
tics [12] for Hintikka's modal logic of know-
ledge [10].

The fundamental assumption that makes
this translation possible, is that an attri-
bution of any propositional attitude like
"to know", "to believe", "to remember", "to
strive" entails a division of the set of pos-
sible worlds into two classes : the possible
worlds that go with the propositional attitu-
de that is considered, and those that are in-
compatible with it. Thus "A knows that P" is
equivalent to "P is true in every world com-
patible with what A knows".

We think that possible worlds language
is complicated and unintuitive, since, rather
than reasoning directly about facts that some-
one knows, we reason about the possible worlds
compatible with what he knows. This transla-
tion also presents some problems for the plan-
ning. For instance to establish that A knows
that P, we must make P true in every world
which is compatible with A's knowledge. This
set of worlds is a potentially infinite set.

The most important advantage of
Moore's approach [17,18] is that it gives
a smart axiomatization of the interaction
between knowledge and action.

II PRESENTATION OF OUR APPROACH

Our approach is comprised in the general
framework of the second approach, but in-
stead of encoding Hintikka's modal logic
of knowledge in a first-order language,
we consider the logic of knowledge propo-
sed by Mc Carthy, the decidability of
which was proved by Sato [21] and we pro-
pose a prover of this logic, based on na-
tural deduction.

We bear in mind that the idea of u-
sing the modal logic of knowledge in A.I.
was proposed for the first time by Mc Car-
thy and Hayes [14].

A. Languages

A language L is a triple (Pr,Sp,T)
where :
.Pr is the set of propositional va-
riables,
.Sp is the set of persons,
.T is the set of positive integers.
The language of classical proposi-
tional calculus is L = (Pr,ϕ,ϕ). So$\in$ Sp
will also be denoted by O and will be
called "FOOL".

B. Well Formed Formulas

The set of well formed formulas is
defined to be the least set Wff such as :
(W$_1$) Pr $\subseteq$ Wff
(W$_2$) a,b$\in$ Wff implies a$\supset$b $\in$ Wff
(W$_3$) S$\in$ Sp, t $\in$ T,a $\in$ Wff implies(St)a$\in$ Wff
The symbol $\supset$ denotes "implication".

(St)a means "S knows a at time t"
<St>a (= $\sim$ (St) $\sim$ a) means "a is pos-
sible for S at
time t".
{St}a (= (St)a $\lor$ (St) $\sim$ a) means
"S knows whether
a at time t".

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C. Hilbert-type System KT4

The axiom schemata for KT4 are :

A1. Axioms of ordinary propositional logic
A2. $(St)a \supset a$
A3. $(Ot)a \supset (Ot)(St)a$
A4. $(St)(a \supset b) \supset ((Su)a \supset (Su)b)$, where $t \leqslant u$
A5. $(St)a \supset (St)(St)a$
A6. If a is an axiom, then $(St)a$ is an axiom.

Now, we give the meaning of axioms :

(A2) says that what is known is true, that is to say that it is impossible to have false knowledge. If P is false, we cannot say : "John knows that P" but we can say "John believes that P". This axiom is the main difference between knowledge and belief.

This distinction is important for planning because when an agent achieves his goals, the beliefs on which he bases his actions must generally be true.

(A3) says that what FOOL knows at time t, FOOL knows at time t that anyone knows it at time t. FOOL's knowledge represents universal knowledge, that is to say all agents knowledge.

(A4) says that what is known will remain true and that every agent can apply modus ponens, that is, he knows all the logical consequences of his knowledge.

(A5) says that if someone knows something then he knows that he knows it. This axiom is often required to reason about plans composed of several steps. It will be referred to as the positive introspective axiom.

(A6) is the rule of inference.

D. Representation of the notion of truth value.

We give a great importance to the representation of the notion of truth value of a proposition, for example the utterance :

John knows whether he is taller than Bill (1)

can be considered as an assertion that mentions the truth value of the proposition P = John is taller than Bill, without taking a position as to whether the latter is true or false.

In our formalism (1) is represented by :

$\{John\}$ P

This disjunctive solution is also adopted by Allen and Perrault [1]. Maida and Shapiro [13] represent this notion by a node because the truth value is a concept (an object of thought).

The representation of the notion of truth value is useful to plan questions : A speaker can ask a hearer whether a certain proposition is true, if the latter knows whether this proposition is true.

E. Representing definite descriptions in conversational systems :

Let us consider a dialogue between two participants : A speaker S and a hearer H. The language is then reduced to :

$Sp = \{O,H,S\}$ and $T = \{1\}$

Let P stand for the proposition : "The description D in the context C is uniquely satisfied by E".
Clark and Marshall [5] give examples that show that for S to refer to H to some entity E using some description D in a context C, it is sufficient that P is a mutual knowledge; this condition is tantamount to $(O)P$ is provable. Perrault and Cohen [20] show that this condition is too strong. They claim that an infinite number of conjuncts are necessary for successful reference :

$(S) P \& (S)(H) P \& (S)(H)(S) P \& ...$

with only a finite number of false conjuncts. Finally, Nadathur and Joshi [19] give the following expression as sufficient condition for using D to refer to E :

$\vdash (S) B \supset (S)(H) P \& \nvdash ((S) B \supset (S)\sim(O)P)$

where B is the conjunction of the set of sentences that form the core knowledge of S and $\vdash$ is the inference symbole.

III SCHÜTTE - TYPE SYSTEM KT4'

Gentzen's goal was to build a formalism reflecting most of the logical reasonings that are really used in mathemati-

cal proofs. He is the inventor of natural deduction (for classical and intuitionistic logics). Sato [21] defines Gentzen - type sysmen GT4 which is equivalent to KT4. We consider here, schütte-type system KT4' [22] which is a generalization of S4 and equivalent to GT4 (and thus to KT4), in order to avoid the thinning rule of the system GT4 (which introduces a cumbersome combinatory). Firstly, we are going to give some difinitions to introduce KT4'.

A. Inductive definition of positive and negative parts of a formula F

Logical symbols are $\sim$ and V.
a. F is a positive part of F.
b. If $\sim$ A is a positive part of F, then A is a negative part of F.
c. If $\sim$ A is a negative part of F, then A is a positive part of F.
d. If A V B is a positive part of F, then A and B are positive parts of F.

Positive parts or negative parts which do not contain any other positive parts or negative parts are called minimal parts.

B. Semantic property

The truth of a positive part implies the truth of the formula which contains this positive part.

The falsehood of a negative part implies the truth of the formula which contains this negative part.

C. Notation

$F[A+]$ is a formula which contains A as a positive part

$F[A-]$ is a formula which contains A as a negative part.

$F[A+,B-]$ is a formula which contains A as a positive part and B as a negative part where A and B are disjoined (i. e. one is not a subformula of the other).

D. Inductive definition of F $[.]$

From a formula F $[A]$, we build another formula or the empty formula F $[.]$ by deleting A :

a. If F $[A]$ = A, then F $[.]$ is the empty formula.

b. If $F[A]$ = $G[\sim A]$ then $F[.]$ = $G[.]$
c. If $F[A]$ = $G[A V B]$ or $F[A]$ = $G[B V A]$ then $F[.]$ = $G[B]$.

E. Axiom

An axiom is any formula of the form $F[P+,P-]$ where P is a propositional variable.

F. Inference rules

(R1) $F[(A V B)]$ V $\sim$ A, $F[(A V B)]$
v $\sim$ B $\vdash$ $F[(A V B)]$

(R2) $F[(St)A]$ V $\sim$ A $\vdash$ $F[(St)A]$

(R3) $\sim(Su)A_1$ V ... V $\sim(Su)Am$ V
$\sim(Ou)B_1$ V ... V $\sim(Ou)Bn$ V C
$\vdash$ $F[(St) C_+]$
where $(Su)A_1$,..., $(Su)Am$, $(Ou)B_1$,
..., (Ou) Bn must appear as negative parts in the conclusion, and
$u \leq t$

(R4) $F_1[C+]$, $F_2[C-]$ $\vdash$ $F_1[.]$ V $F_2[.]$
(cut)

G. Cut-elimination theorem (Hauptsatz)

Any KT4' proof-figure can be transformed into a KT4' proof-figure with the same conclusion and without any cut as a rule of inference (hence, the rule (R4) is superfluous. The proof of this theorem is an extension of Schütte's one for S4'. This theorem allows derivations "without detour".

IV DECISION PROCEDURE

A logical axiom is a formula of the form $F[P+,P-]$. A proof is an single-rooted tree of formulas all of whose leaves are logical axioms. It is grown upwards from the root, the rules (R1), (R2) and (R3) must be applied in a reverse sense. These reversal rules will be used as "production rules". The meaning of each production expressed in terms of the programming language PROLOG is an implication.

It can be shown [24] that the following strategy is a complete proof procedure :

. The formula to prove is at the star-

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ting node;
. Queue the minimal parts in the given for-
mula;
. Grow the tree by using the rule (R1) in
priority , followed by the rule (R2), then
by the rule (R3).

The choice of the rule to apply can be
done intelligently. In general, the choice of
(R1) then (R2) increases the likelihood to
find a proof because these (reversal) rules
give more complex formulas. In the case where
(R3) does not lead to a loss of formulas, it
is more efficient to choose it at first. The
following example is given to illustrate this
strategy :

Example

Take (A4) as an example and let Fo deno-
tes its equivalent version in our language
(Fo is at the start node) :

$Fo = \sim(St)(\sim a \lor b) \lor \sim(Su)a \lor (Su)b$ where
$t \leqslant u$
P_i^+ denotes positive parts and P_i^- denotes
negative parts
$P_o^+ = \{\sim(St)(\sim a \lor b), \sim(Su)a,(Su)b\}$;
$P_o^- = \{(St)(\sim a \lor b),(Su)a\}$;

By (R3) we have (no losses of formulas) :
$F_1 = \sim(St)(\sim a \lor b) \lor \sim(Su)a \lor b$

$P_1^+ = \{\sim(St)(\sim a \lor b), \sim(Su)a,b\}$
$P_1^- = \{(St)(\sim a \lor b),(Su)a\}$
By (R2) we have :
$F_2 = F_1 \lor \sim(\sim a \lor b)$
$P_2^+ = P_1^+ \cup \{\sim(\sim a \lor b)\}$
$P_2^- = P_1^- \cup \{\sim a \lor b\}$

By (R1) we have :
$F_3 = F_2 \lor \sim\sim a$
$P_3^+ = P_2^+ \cup \{\sim\sim a,a\}$
$P_3^- = P_2^- \cup \{\sim a\}$
and :
$F_4 = F_2 \lor \sim b$
$P_4^+ = P_2^+ \cup \{\sim b\}$
$P_4^- = P_2^- \cup \{b\}$

F_4 is a logical axiom because $P_4^+ \cap P_4^- = \{b\}$

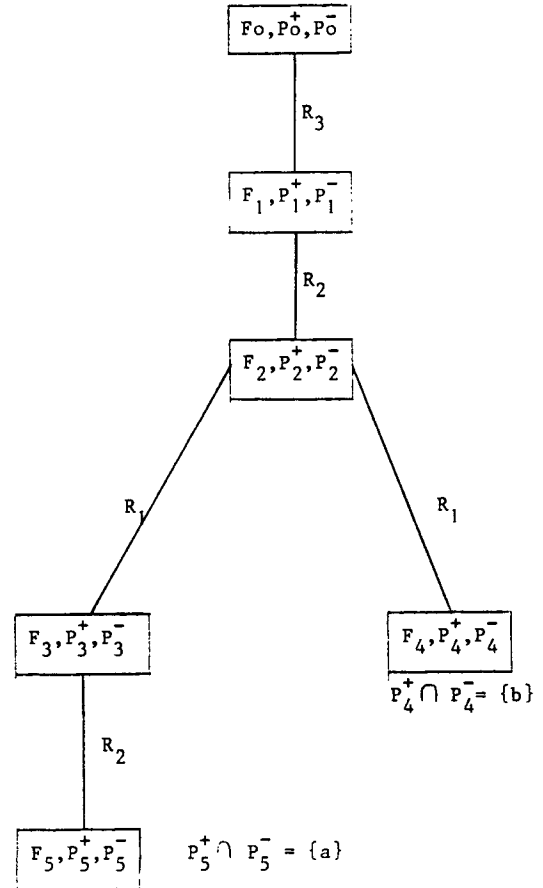Finally, we have to apply (R2) to the last but
one node :

$F_5 = F_3 \lor \sim a$
$P_5^+ = P_3^+ \cup \{\sim a\}$
$P_5^- = P_3^- \cup \{a\}$
F_5 is a logical axiom because $P_5^+ \cap P_5^- = \{a\}$

The generated derivation tree is then :

Derivation tree

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