CHAPTER QUIZZES

QUIZ A

For the matrix A = $\begin{pmatrix} 4 & 1 & -2 \\ -3 & 2 & 0 \\ -3 & 0 & 2 \end{pmatrix}$, answer the following questions:

a) Determine if $\begin{pmatrix} -3 \\ 3 \\ 3 \end{pmatrix}$ is an eigenvector for A. If it is, find the corresponding eigenvalue.

b) Find all the eigenvalues for the matrix A.

QUIZ B

Given the matrix A = $\begin{pmatrix} 1 & -3 & 0 \\ -4 & 5 & 0 \\ 0 & -4 & -2 \end{pmatrix}$. Answer the following questions:

a) Find all the eigenvalues for the matrix A.

b) Is the matrix A diagonalizable? Explain your answer.

c) Find all the eigenvectors corresponding to the smallest eigenvalue.

QUIZ C

For the matrix A = $\begin{pmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 1 & 1 & 1 \end{pmatrix}$, answer the following questions:

a) Find all the eigenvalues for the matrix A.

b) Find all the eigenvectors corresponding to the eigenvalues in (a).

c) From your answers above, is the matrix diagonalizable? If yes, state the matrices P and D such that $P^{-1}AP = D$.

CHAPTER TESTS

TEST A

Consider the matrix $A = \begin{pmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{pmatrix}$

a) Determine whether $\begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix}$ is an eigenvector of A.

b) Find all the eigenvalues of A.

c) Find the basis for the eigenspace corresponding to the smallest eigenvalue.

TEST B

Consider the matrix $A = \begin{pmatrix} 7 & 0 & 2 \\ 0 & 5 & 0 \\ -4 & 0 & -2 \end{pmatrix}$

a) Find all the eigenvalues of A

b) Is the matrix diagonalizable? Give reasons for your answer.

c) Find the eigenvectors corresponding to the largest eigenvalue.

TEST C

Given the matrix $A = \begin{pmatrix} 4 & 3 & 0 \\ 2 & 3 & 0 \\ 0 & 0 & 6 \end{pmatrix}$

a) Find all the eigenvalues for the matrix above.

b) Find all the eigenvectors corresponding to the eigenvalues found in (a).

c) Is the matrix diagonalizable? Explain your answer.

If it is diagonalizable, state the matrices P and D such that $P^{-1}AP = D$.

TEST A

a) Since $\begin{pmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{pmatrix}\begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix} = \begin{pmatrix} -2 \\ 4 \\ 4 \end{pmatrix} = 2\begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix}$, $\therefore \begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix}$ is an eigenvector of A.

b) $|A-\lambda I| = \begin{vmatrix} 4-\lambda & 0 & 1 \\ -2 & 1-\lambda & 0 \\ -2 & 0 & 1-\lambda \end{vmatrix}$

$= (1-\lambda)\begin{vmatrix} 4-\lambda & 1 \\ -2 & 1-\lambda \end{vmatrix}$ (cofactor expansion along the 2^{nd} column)

$= (1-\lambda)\,[(4-\lambda)(1-\lambda) - 2(-1)]$

$= (1-\lambda)\,(4 - 5\lambda + \lambda^2 + 2)$

$= (1-\lambda)\,(\lambda^2 + 5\lambda + 6)$

$= (1-\lambda)\,(\lambda-2)(\lambda-3)$

$|A-\lambda I| = 0 \Rightarrow (1-\lambda)(\lambda-2)(\lambda-3) = 0 \;\Rightarrow\; \lambda = 1, 2, 3$

c) For $\lambda = 1$

$(A-I)\mathbf{v} = \mathbf{0}$

Form the augmented matrix and row reduce to row echelon form

$$\left(\begin{array}{ccc|c} -3 & 0 & 1 & 0 \\ -2 & 0 & 0 & 0 \\ -2 & 0 & 0 & 0 \end{array}\right) \xrightarrow{R_3+(-1)R_2 \to R_3} \left(\begin{array}{ccc|c} -3 & 0 & 1 & 0 \\ -2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \xrightarrow{\left(-\frac{1}{2}\right)R_2 \to R_2}$$

$$\left(\begin{array}{ccc|c} -3 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \xrightarrow{R_1+(-3)R_2 \to R_1} \left(\begin{array}{ccc|c} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \xrightarrow{R_1 \leftrightarrow R_2} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

Let $x_2 = r$

$x_1 = 0$

$x_3 = 0$

$$\therefore \text{Eigenvector } \mathbf{v} = \left\{\begin{pmatrix} 0 \\ r \\ 0 \end{pmatrix},\ r \in R,\ r \neq 0\right\} = \left\{r\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix},\ r \in R,\ r \neq 0\right\}.$$

A basis for the eigenspace $E_{\lambda=1} = \left\{\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}\right\}$.

TEST B

a) $\lambda = -1, 5, 6$

b) Yes, the matrix is diagonalizable because it is of size 3x3 and has 3 distinct eigenvalues

c) For $\lambda = 6$, eigenvector $\mathbf{v} = \left\{\begin{pmatrix} -2r \\ 0 \\ r \end{pmatrix},\ r \in R,\ r \neq 0\right\} = \left\{r\begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix},\ r \in R,\ r \neq 0\right\}$

TEST C

a) $\lambda = 1, 6, 6$

b) For $\lambda = 1$, eigenvector $\mathbf{v} = \left\{\begin{pmatrix} -r \\ r \\ 0 \end{pmatrix},\ r \in R,\ r \neq 0\right\} = \left\{r\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix},\ r \in R,\ r \neq 0\right\}$

For $\lambda = 6$, eigenvector $\mathbf{v} = \left\{\begin{pmatrix} 3r \\ 2r \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ t \end{pmatrix},\ r, t \in R,\ r \neq 0, t \neq 0\right\}$

$$= \left\{r\begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix} + t\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix},\ r, t \in R,\ r \neq 0, t \neq 0\right\}$$

c) Yes, the matrix is diagonalizable because it is of size 3x3 and has 3 linearly independent eigenvectors.

$$P = \begin{pmatrix} -1 & 3 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{pmatrix}$$