first, set an order on the vertex by
clockwise direction
for a point p inside the polygon, the angle
θ from $\vec{PA_i}$ to $\vec{PA_{i+1}}$, $\theta < \pi$, by cross product
the vector $\vec{PA_i} \times \vec{PA_{i+1}}$ goes negative way.

$\vec{PA_i} = (x_i, y_i, 0), \vec{PA_{i+1}} = (x_{i+1}, y_{i+1}, 0)$

$\vec{PA_i} \times \vec{PA_{i+1}} = \begin{vmatrix} i & j & k \\ x_i & y_i & 0 \\ x_{i+1} & y_{i+1} & 0 \end{vmatrix} = (0, 0, \begin{vmatrix} x_i & y_i \\ x_{i+1} & y_{i+1} \end{vmatrix})$

that is $\begin{vmatrix} x_i & y_i \\ x_{i+1} & y_{i+1} \end{vmatrix} < 0$, for all i

similarly, if vertex is arranged in counter-clockwise way, $\left|\begin{matrix} x_i & y_i \\ x_{i+1} & y_{i+1} \end{matrix}\right| > 0$

Hence, if all $\left|\begin{matrix} x_i & y_i \\ x_{i+1} & y_{i+1} \end{matrix}\right|$ has the same sign

p is inside the polygon

elif $\left|\begin{matrix} x_i & y_i \\ x_{i+1} & y_{i+1} \end{matrix}\right| = 0$ for any i, that is

$\theta = \pi$, p is on the polygon

else, p is outside the polygon

$$\left|\begin{matrix} x_i & y_i \\ x_{i+1} & y_{i+1} \end{matrix}\right| = x_i y_{i+1} - x_{i+1} y_i$$