

and Baeza-Yates [44]. Quicksort was originally invented by Hoare [55] and heapsort by Williams [132]. The taxonomy of sorting algorithms presented in this chapter is based on the one by Azmoodeh [5].

On the functional programming side, Paulson [93] describes most of the sorting algorithms in the strict functional language ML and discusses their performance based on profiling information. He also describes the bottom-up mergesort which was originally devised by O'Keefe [92]. A derivation of quicksort from tree sort through tree deforestation is described by Bird [14].

## Chapter 7

# Graph algorithms

7.1	Definitions and terminology	135
7.2	The graph ADT	136
7.3	Depth-first and breadth-first search	140
7.4	Topological sort	144
7.5	Minimum spanning tree	146
7.6	Depth-first search trees and forests†	149
7.7	Conclusion	152
	Exercises	153
7.8	Bibliographical notes	154

This chapter introduces the concept of a graph and describes some of the algorithms that operate on graphs and their applicability to a variety of problems.

### 7.1 Definitions and terminology

First, we need to introduce some of the terminology associated with graphs. Formally, a graph denoted  $G = \langle V, E \rangle$  consists of a finite set of *nodes* (or *vertices*)  $V$  and a set of *edges*  $E$ , where an edge  $ij$  represents a connection between two nodes,  $i$  and  $j$ . In other words, a graph can be defined as a set and a relation over that set, where every element in the set corresponds to a node in the graph, and where there is a relation between two elements if there is an edge between the corresponding nodes. If the edge has a direction, it is called an *arc* (that is, the relation is not symmetric). In the rest of the chapter, the number of nodes and the number of edges in the graph are respectively denoted as  $|V|$  and  $|E|$ .

Informally, graphs are represented visually as the examples in Figure 7.1 show. The graph on the left is *undirected* (that is, the edges have no direction) and the graph in the