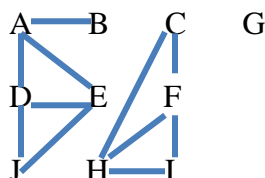


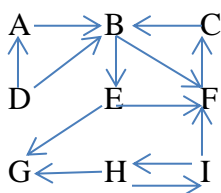
# CS372 Spring 2019 Assignment #4.

**Due:** at the beginning of the lecture on Thursday, March 14th.

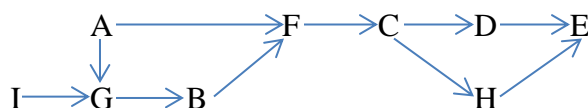
1. Perform a depth-first search on the following graph; whenever there's a choice of vertices, pick the one that is alphabetically first. Classify each edge as a tree edge or back edge, and give the `pre` and `post` number of each vertex.



2. Perform depth-first search on the following graph; whenever there's a choice of vertices, pick the one that is alphabetically first. Classify each edge as a tree edge, forward edge, back edge, or cross edge, and give the `pre` and `post` number of each vertex.



3. Run the DFS-based topological ordering algorithm on the following graph. Whenever you have a choice of vertices to explore, always pick the one that is alphabetically first.
  - (a) Indicate the `pre` and `post` numbers of the nodes.
  - (b) What are the sources and sinks of the graph?
  - (c) What topological ordering is found by the algorithm?
  - (d) How many topological orderings does this graph have? List all of them.



4. Directed graph  $G$  is represented by the adjacency matrix below. Draw the directed graph  $G$  and run the strongly connected components algorithm on it (use the algorithm from p.94 of the textbook). When doing DFS on  $G^R$ : whenever there is a choice of vertices to explore, always pick the one that is alphabetically first. Answer the following questions.
  - (a) In what order are the strongly connected components (SCCs) found?
  - (b) Which are source SCCs and which are sink SCCs?

- (c) Draw the “metagraph” (each meta-node is an SCC of G).  
 (d) What is the minimum number of edges you must add to this graph to make it strongly connected?

	A	B	C	D	E	F	G	H	I	J
A	0	0	1	0	0	0	0	0	0	0
B	1	0	0	0	0	0	0	1	0	0
C	0	0	0	0	0	0	0	0	0	1
D	0	1	0	0	1	0	1	0	0	0
E	1	0	0	1	0	0	0	0	0	0
F	0	0	0	0	0	0	0	0	1	0
G	0	0	0	0	0	1	0	1	0	0
H	0	0	1	0	0	0	0	0	0	0
I	0	0	1	0	0	0	1	0	0	0
J	1	0	0	0	0	0	0	1	0	0

(Recall that 1 in row  $i$  and column  $j$  means that there is an edge from vertex  $i$  to vertex  $j$ . For instance, there is an edge from A to C.)

5. Exercise 3.15. Note: “Formulate this problem graph-theoretically” means that you need to explain what the vertices in the graph are and what the edges are (when exactly two vertices are connected by an edge).

6. Exercise 3.24. Note: You need to explain the algorithm and analyze its running time.

**What to submit:**

- Submit answers to all the questions on paper.