CS372 Assignment #6.

Due: on Thursday, April 4, at the beginning of the lecture.

You may do this assignment in groups of 2 or individually.

1. Recall that deletion in binary search trees (BST) is done in the following manner. Let z be a node that we want to delete. There are three cases:

Case 1: If z has no children, then we simply remove it.
Case 2: If z has only one child, then we elevate that child to take z's position in the tree by modifying z's parent to replace z with z's child.
Case 3. If z has two children, then we find z's successor s (s will have one or no children), copy s's key and data into z, and perform case 1 or 2 to delete s.

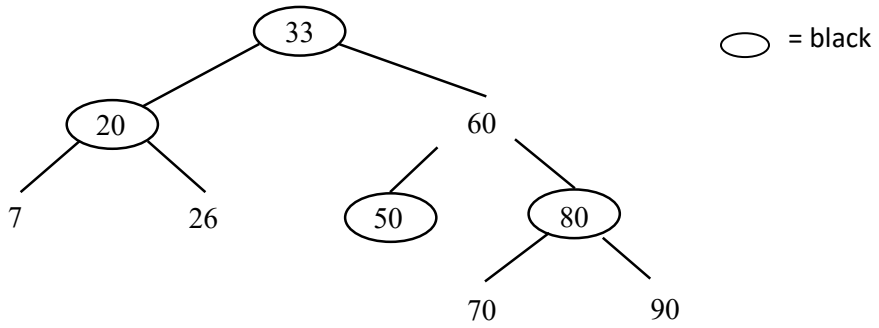
Is the operation of deletion "commutative" in the sense that deleting x and then y from a BST leaves the same tree as deleting y and then x? Argue why it this is the case or give a counterexample.

2. A binary tree is complete (or full) if all its leaves have the same depth and all internal nodes have two children. Draw the complete binary search tree of height 3 on the keys {1,2,3,4,5,6,7,8,9,10,11,12,13,14,15}. Add the NIL leaves and color the nodes in three different ways such that the black-heights of the resulting red-black trees are 2, 3, and 4. (The black-height of a red-black tree is the black-height of its root)

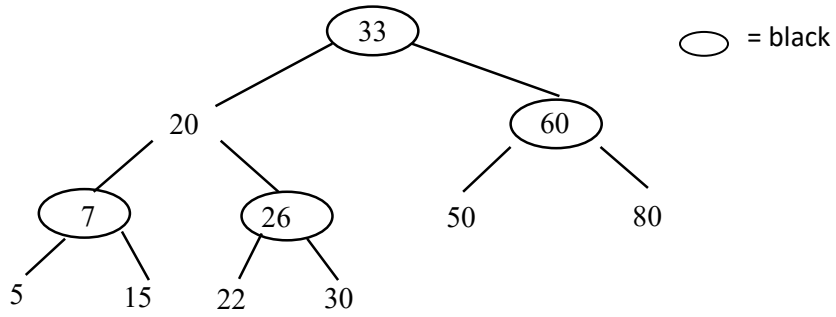
3. Recall that the black-height of a tree is the black-height of its root. What is the largest possible number of internal nodes in a red-black tree with black-height k? What is the smallest possible number?

4. Describe a red-black tree on n keys that realizes the largest possible ratio of red internal nodes to black internal nodes. What is this ratio? What tree has the smallest possible ratio, and what is the ratio?

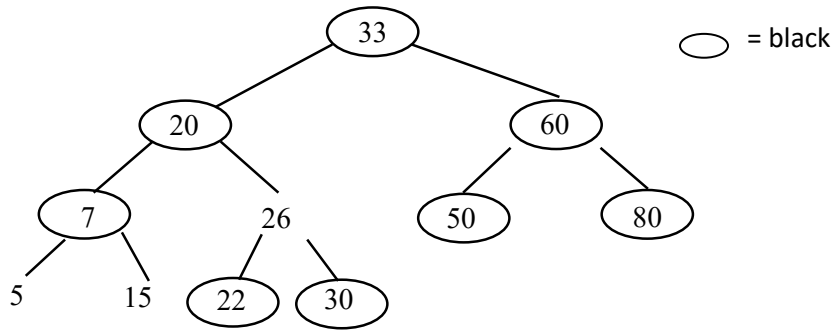
5.1. Insert 71 into the following red-black tree. Show all your steps. (Note that the leaves (nil) are not shown)



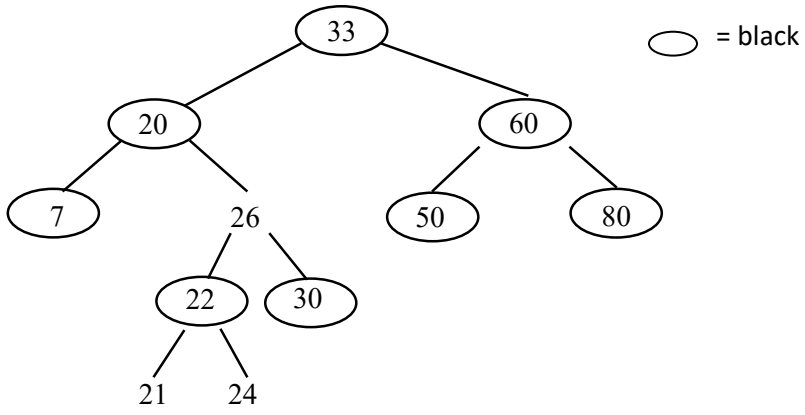
5.2. Insert 2 into the following red-black tree. Show all your steps. (Note that the leaves (nil) are not shown)



5.3. Delete 33 from the given red-black tree (use successor). Show all your steps.



5.4. Delete 7 from the given red-black tree. Show all your steps.



What to submit:

- Submit answers to the questions on paper.