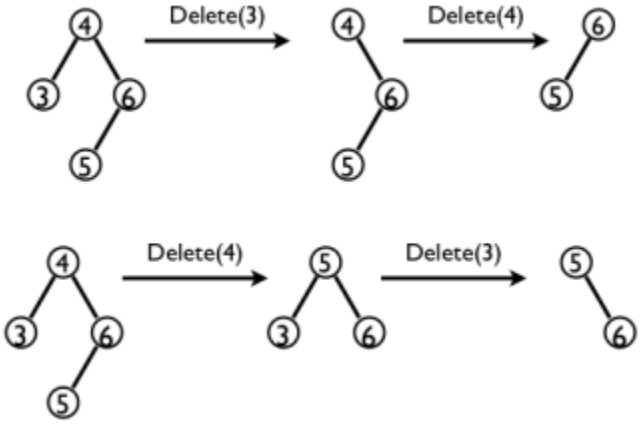
CS372 Spring 2018 Assignment #6 solutions.

1.

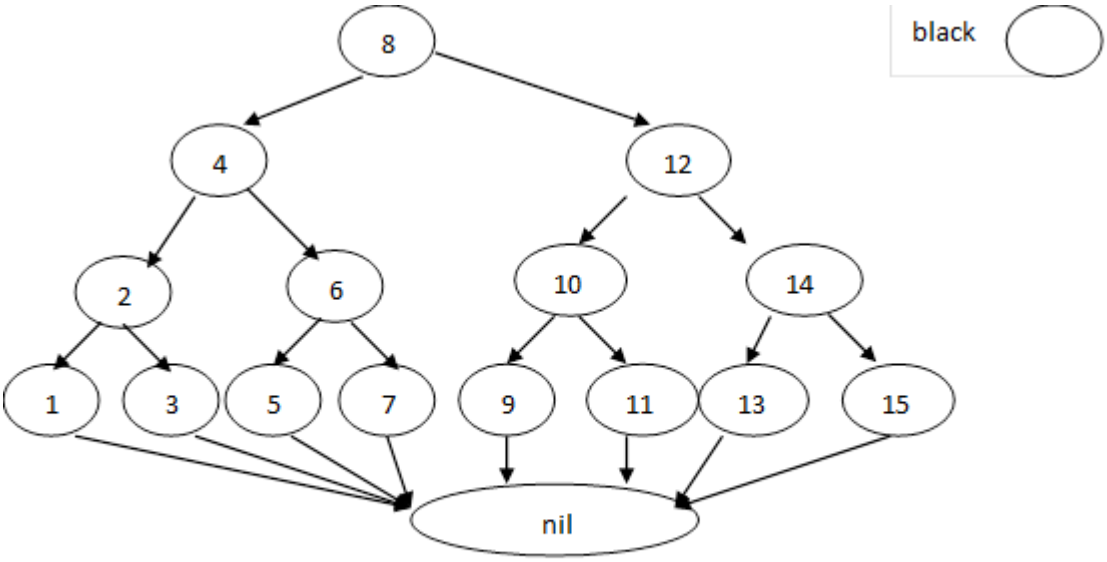
Solution: Deletion in BSTs is not commutative as illustrated by the following counterexample:



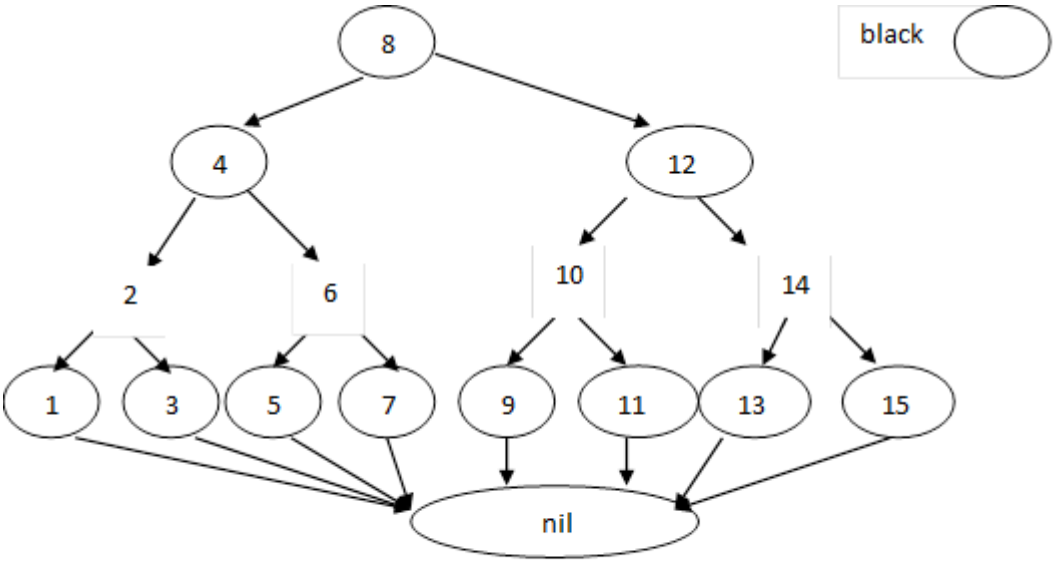
2.

Solution:

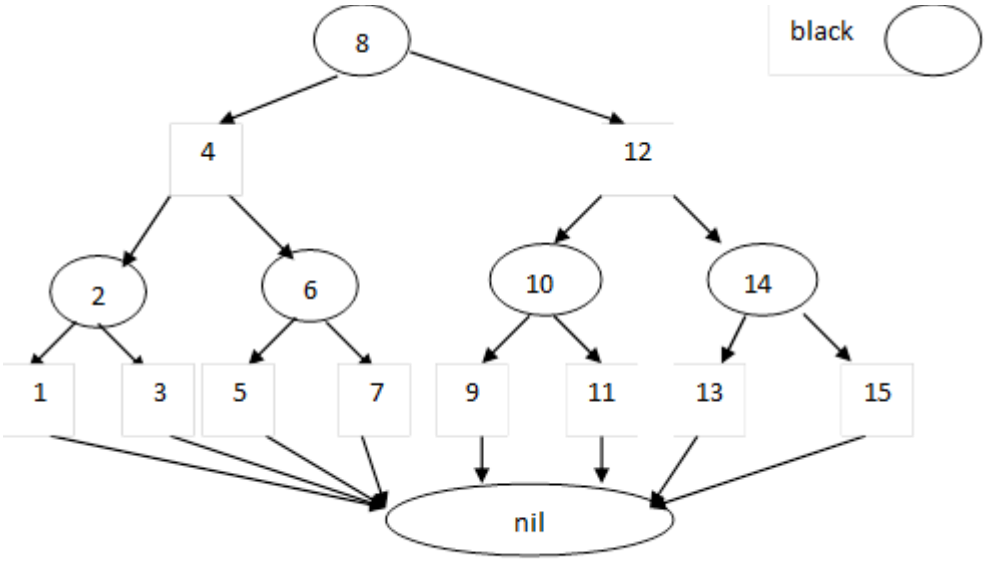
Complete binary tree with bh=4:



Complete binary tree with bh=3 (one of several possible trees):



Complete binary tree with bh=2:

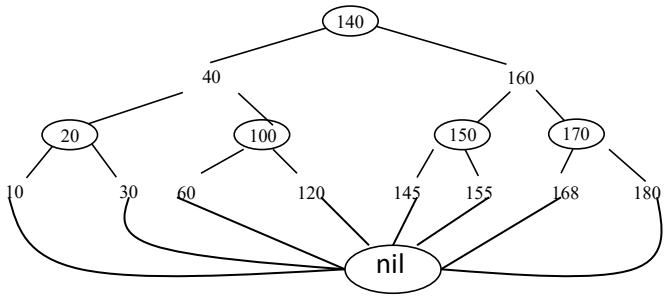


3.

Solution:
Consider a red-black tree with black-height k.
The smallest number of internal nodes will be when all the nodes in the tree are black. The number of nodes in such a tree is
1 (root) + 2 (nodes on level 1) + 2^2 (nodes on level 2) + … + 2^{k-1} (nodes on the last level) = 2^k - 1.
The largest possible number of internal nodes will be when every other level of nodes in the tree is red. The number of nodes in such a tree is $1 + 2 + \ldots + 2^{2k-1} = 2^{2k} - 1$.

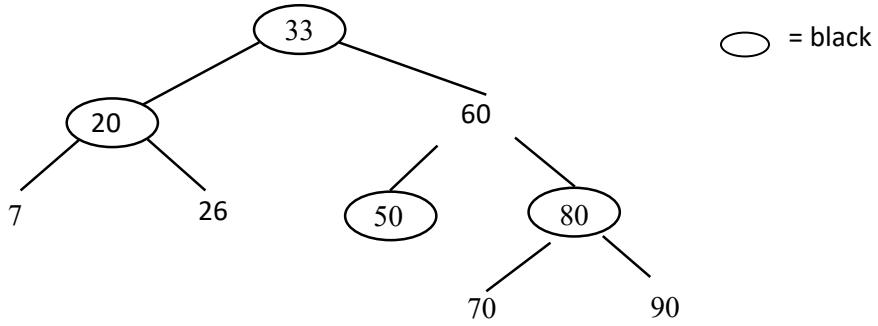
4. The smallest possible ratio of red internal nodes to black internal nodes is when we have a complete tree where all the nodes are black. Then, the number of red nodes is 0 and the ratio is 0.

The largest possible ratio is when we have a complete tree where colors of internal nodes alternate from level to level, that is, internal nodes of height 1 are colored red, internal nodes of height 2 are colored black, internal nodes of height 3 are colored red, etc. For instance, like the following tree:

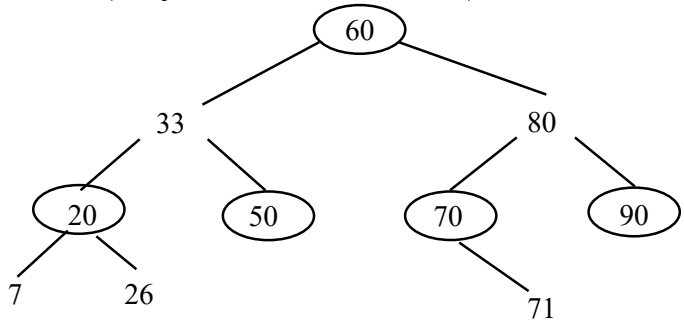


Then, for every black internal node x there are two red internal nodes on the next level (x's children). Hence, the ratio of red internal nodes to black internal nodes is 2:1.
(In the example above there are 10 red internal nodes and 5 black internal nodes.)

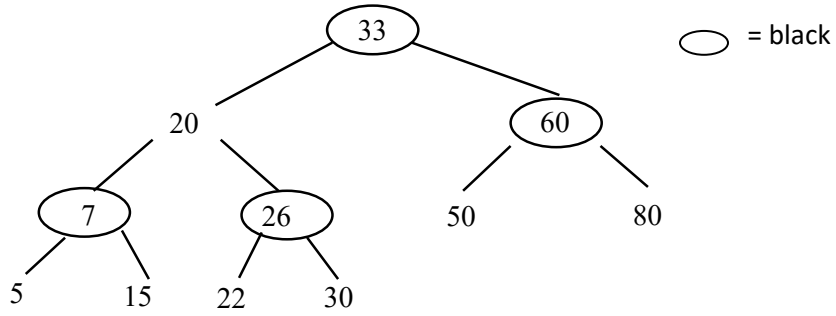
5.1.
Insert 71 into the following red-black tree. Show all your steps. (Note that the leaves (nil) are not shown)



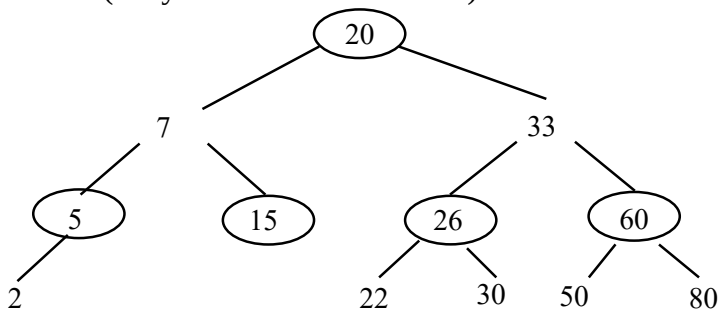
Solution: (Only the final tree is shown)



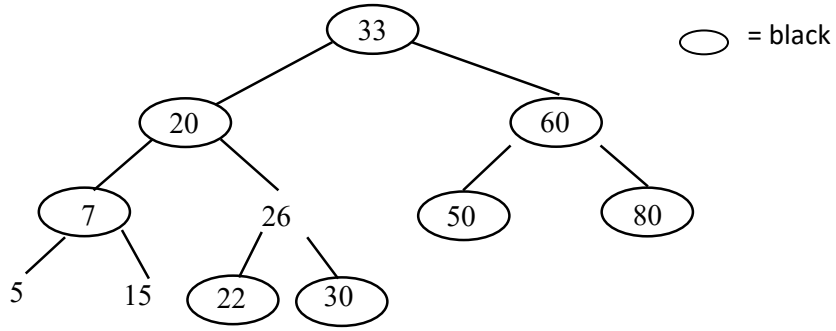
5.2. Insert 2 into the following red-black tree. Show all your steps. (Note that the leaves (nil) are not shown)



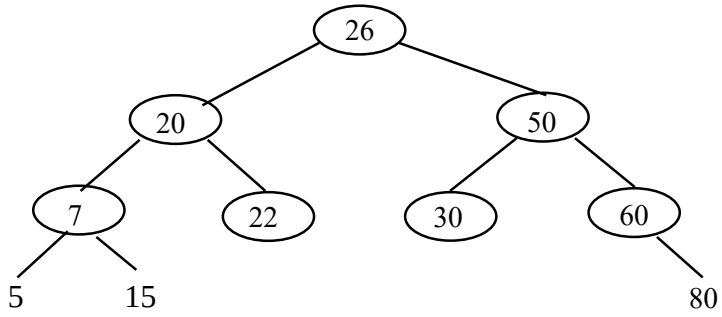
Solution: (Only the final tree is shown)



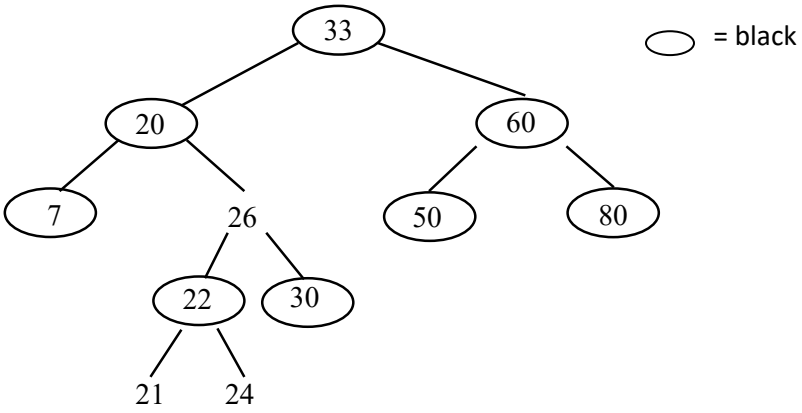
5.3. Delete 33 from the given red-black tree (use successor). Show all your steps.



Solution: (Only the final tree is shown)



5.4. Delete 7 from the given red-black tree. Show all your steps.



Solution: (Only the final tree is shown)